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METHODOLOGY FOR BUS STRUCTURE TORSION STIFFNESS AND NATURAL VIBRATION FREQUENCY PREDICTION BASED ON A DIMENSIONAL ANALYSIS APPROACH

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ABSTRACT−Engineering bus design requires testing of bus structures prototypes in order to guarantee a certain level of strength and an appropriate static and dynamic behavior of the bus superstructure when exposed to road loads. However, experimental testing of real bus structures is very expensive as it requires expensive resources and space. If testing is done on a scale bus model the previous required expenses are considerably reduced. Therefore, a novel methodology based on dimensional analysis applied to bus structure prediction to evaluate the bus structure static and dynamic performance is proposed. The static performance is evaluated attending to torsion stiffness and the dynamic in terms of the natural vibration frequencies and rollover threshold. A scale bus has been manufactured and dimensionless parameters have been defined in order to project the results obtained in the scale bus model to a larger model. Validation of the proposed methodology has been carried out under experimental and finite element analysis.

KEY WORDS: bus, dimensional analysis, static torsion test, modal analysis

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1. INTRODUCTION

Bus structures have to comply with certain regulations and directives in order to obtain type approval. Regulation R66 (*UNECE No. 66*, 2006) ("Uniform Technical Prescriptions concerning the approval of large passenger vehicles with regard to the strength of their superstructure") requires testing of bus structures prototypes in order to guarantee a certain level of strength of the bus superstructure. Directive 2001/85/CE (*Directiva 2001/85*,2001) (*"Special provisions for vehicles used for the carriage of passengers comprising more than eight seats in addition to the driver's seat"*) is of direct application in Spain and includes Regulations R66 and R36, relative to general construction of large passenger vehicles. In addition, bus manufactures may want to comply with internal standards and therefore, may carry out their own tests, especially at the end of bus design stage and before manufacturing it, in order to evaluate the static and dynamic performance of the bus chassis (*Gauchía et al.*, 2009; *Olatunbosun et al.*, 2011; *Gauchia et al.*, 2010). It is of special importance rollover limit evaluation (*Gauchia et al.*, 2011; *Belingardi et al.*, 2007). However, experimental testing of bus prototypes, or parts of their structure (*Diaz et al.*, 2007), and manufacturing of even one of them just for testing issues requires of big installations and space being, therefore, very expensive. Experimental testing of scale bus structure models seems to be the most suitable solution. Prior to any test, the dimensions of the scale model must be set. Its dimensions are computed by means of the dimensional analysis theory (*Buckingham*, 1914). Once the scale model has been tested a set of equations must allow projection of the result's test on the scale model to the prototype. This set of equations is derived from the dimensional analysis.

Examples of successful application of scale vehicle systems such as the anti-locking braking system (ABS) have proved to be a great aid for the prototyping of vehicle systems (*Longoria*, 2004). Scale modeling has also been applied to other vehicle components such as the tire (*Polley* et al., 2004). Polley *et al.* proved application of dimensional analysis of tires for vehicle dynamic studies. Recent researches (*Liburdi*, 2010) designed a scale vehicle dynamic test bed for dimensional analysis. In (*Lapapong et al.*, 2009) a deep analysis of fidelity of scaled vehicles for chassis dynamic evaluation has been done. Dimensional analysis has also proven to be a successful tool in vehicle dynamic control (B*rennan, S. and Alleyne, A.*, 2001). Recently, dimensional analysis has been applied to surface treatment, such as single shot peening (*Wu et al.*, 2012). However, dimensional analysis has never been applied to predict the static and dynamic bus structure behavior. The aim is to predict by means of dimensional analysis the static torsion stiffness and the natural vibration frequencies and modes of a bus structure by means of static and dynamic simulation and testing of a scale bus structure. Regarding the static torsion test, a vertical force was applied in one of the scale bus structure corners while the other three were constrained. Regarding the dynamic test, the scale bus was hanged from a crane and an impact modal test was performed. By means of an accelerometer the vertical acceleration was measured, being able to identify the scale bus natural vibration frequencies and modes. The static torsion stiffness is important for bus structure integrity and rollover. Identification of natural mode frequencies and modes are also important to avoid resonance and for improving bus NVH (Noise, vibration and harshness). The proposed methodology has not been applied before to predict bus structure static and dynamic behavior. Therefore, in this paper, a new methodology to evaluate the bus static and dynamic performance of its structure is proposed, based on a dimensional analysis approach.

2. METHODOLOGY TO CONDUCT SCALE MODEL OF THE BUS STRUCTURE

Nowadays, bus structures are made of two main parts: a chassis and a superstructure. Figure 1 depicts a real bus superstructure of one of the main body builders of Spain (Castrosua). Because low floor city busses with portal driving axle do not have chasis, the proposed methodology is also valid. Some of the beams of the superstructure are placed as support of other components such as seats, etc., and others, are placed in order to provide a certain amount of stiffness and to guarantee bus structural integrity. For this reason, only the beams of the real bus structure of Figure 1 that provide stiffness to the whole structure have been selected, in order to create a simplified bus structure which has the same number of modules. This simplified bus structure (from now onwards the word simplified is omitted) is named prototype bus and it is at scale 1:1. The aim is to be able to predict the static and dynamic performance of this prototype bus structure, by projecting the experimental results measured on a scale bus structure. The bus prototype (scale 1:1) measures 8483 mm long, 2490 mm height and 2335 mm wide. The beams have a hollow rectangular cross section and the joining technique is welding. The prototype longitudinal beams of the roof have a section of 40x40x2 mm and the cross beams measure 40x40x3 mm. The bus prototype vertical pillars measure 70x40x3 mm and beams placed at the bottom 80x80x4 mm.

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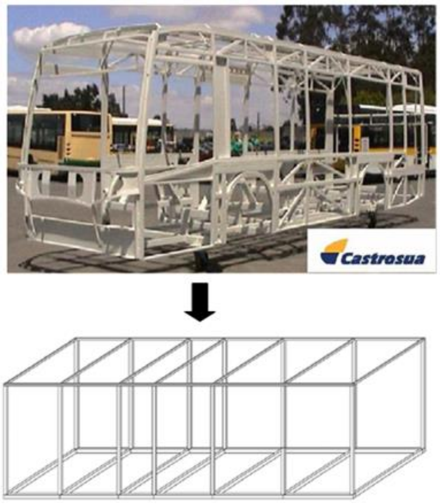


Figure 1. Real bus structure (top) and prototype simplified structure (bottom).

To prove that the simplified bus structure complies with the R66 in terms of superstructure strength, a rollover platform tilt test has been simulated in Ls Dyna, as depicted in Figure 2. The bus properties have been defined and gravity has been applied. The ground and platform have been defined with rigid material and the ground displacements and rotations have been restrained. In addition, the survival space has been defined attending to R66 and it has been meshed with element Mesh 200, which does not modify the structure dynamics and it is only used to visualize if any of the beams of the bus enters the survival space.

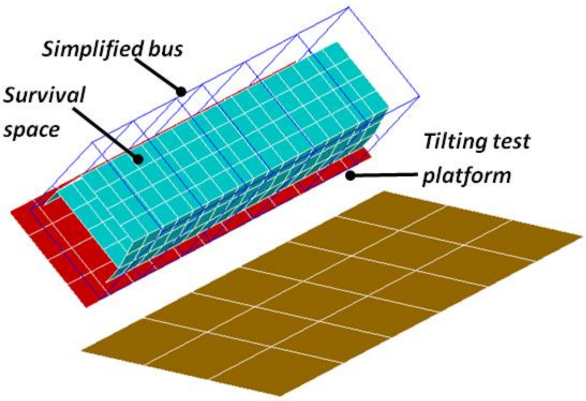
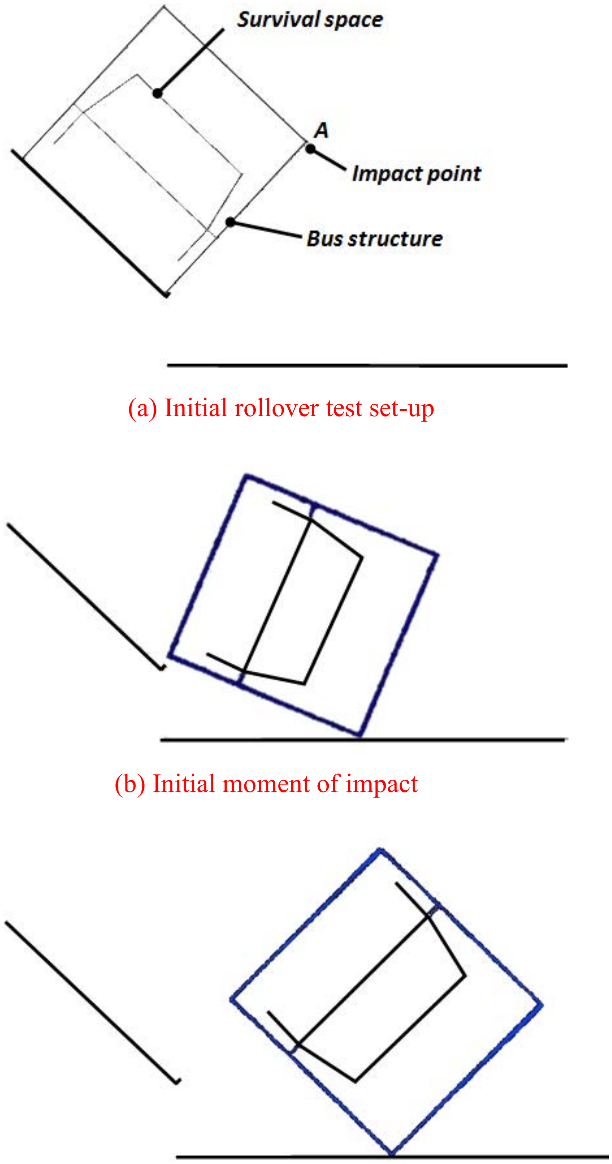


Figure 2. Rollover test of the simplified bus structure

In addition, contact between surfaces has been defined. Intrusion inside the survival space can be visually checked after running the simulation. The moment of impact is captured in Figure 3. Results of Ls Dyna simulation show that that the beams do not enter the survival space, therefore, the bus complies with R66. It must be noted that initially the bus structure (represented by a thin line) is afterwards represented with a thicker line during impact. The reason is that the bus structure safety rings are placed along the bus at different distances, therefore, during impact it is subjected to a small torsion. Thus, front and rear structure safety rings are not longer overlapped, as in the initial rollover test set-up, but slightly displaced one with respect to the other.



(a) Initial rollover test set-up

(b) Initial moment of impact

(c) Impact against the ground

Figure 3. Rollover animation of the simplified bus structure under R66 test

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Solution of the vertical displacement of one of the roof corners (Point A in Figure 2) is depicted in Figure 4.

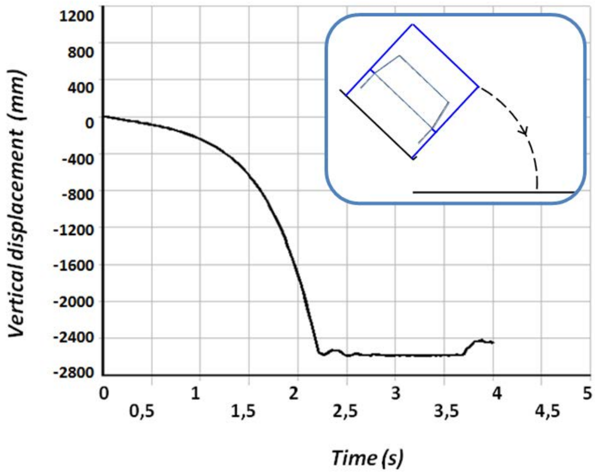

Figure 4. Vertical displacement of roof corner

Therefore, the selected simplified bus structure is appropriate to conduct the novel proposed dimensional methodology described in the paper.

To achieve the proposed aim of the research the following steps have been followed:

1. Fix a value of the scale for the scale bus in order to manufacture the scale bus structure.
2. By means of dimensional analysis a scale bus model has been manufactured with the same material as the original one.
3. Finite element models (FEM) of the scale bus model and the prototype bus have been developed in ANSYS. The FEM of the scale bus model will allow validation when comparing with experimental measurements.
4. Static analysis:
    a. An experimental torsion stiffness analysis has been performed in the scale bus structure.
    b. FEM of the scale bus model has been validated with static torsion tests.
    c. The results of the FEM of the scale bus have been projected to the bus prototype by means of dimensional analysis.
    d. To validate the proposed dimensional analysis the results of the FEM of the bus prototype have been compared with those obtained using dimensional analysis.
5. Dynamic analysis:
    a. An experimental modal analysis has been performed in the scale bus model.
    b. FEM of scale bus has been validated with experimental modal tests of bus scale model.
    c. The results of the FEM of the bus scale have been projected to the bus prototype by means of dimensional analysis.
    d. To validate the proposed dimensional analysis the results of the FEM of the bus prototype have been compared with those obtained using dimensional analysis.
6. The rollover limit of the scale bus has been computed and the result has been projected to the bus prototype.

In Figure 5 and Figure 6 a flow diagram for the static and dynamic analysis, used to conduct the proposed dimensional analysis, are depicted.

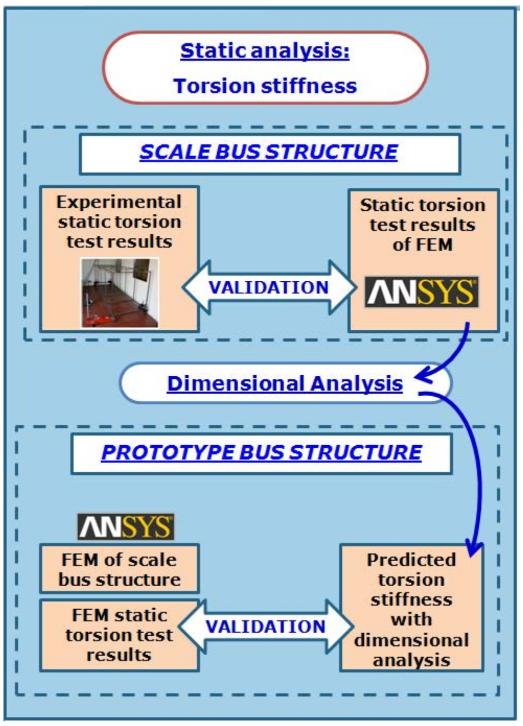

Figure 5. Flow diagram for the static analysis.

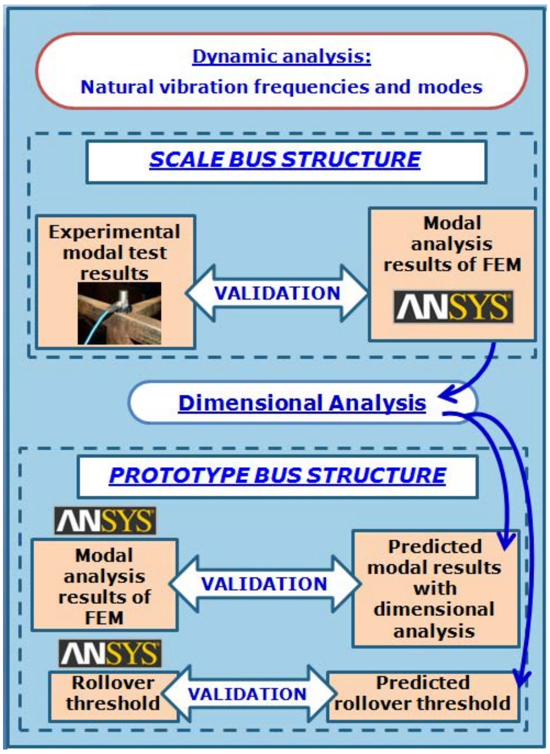

Figure 6. Flow diagram for the dynamic analysis.

3. DIMENSIONAL ANALYSIS TO DEFINE THE SCALE BUS MODEL

Prior to any experimental testing is the design and specification of the dimensions of each of the beams and of the external dimensions of the scale bus model structure. Limitations due to the available space and minimum allowable thickness of the beams, due to welding, must be considered. In order to carry out the experimental tests the available space at the lab is of 5000 mm. Secondly, due to the fact that the scale bus model beams will be joined by welding the minimum allowable thickness is restricted to 1 mm. The bus prototype has a length of 8483 mm and the minimum thickness of the bus prototype beams is 2 mm. Therefore, a scale of 1:2 for the bus scale model is selected as it complies with both conditions. The bus scale model was manufactured at a scale 1:2 using the original type of material of the bus structure. However, the thickness and lengths of each of the beams had still to be computed before its manufacturing. The dimensions of the beams of the scale bus model requires of the application of the Buckingham Theorem (*Buckingham*, 1914) .

Firstly, the 9 variables of the system are defined: displacement (U), force (F), width of the beam (b), the beam depth of the beam (h), height of the structure (H), width of the structure (W), length of the structure (L), thickness of the beam (t) and elastic modulus (E). The units of these variables are two: meters (m) and newtons (N). The number of dimensionless variables is obtained by subtraction of the total number of variables (9) and the number of units (2). Therefore, 7 dimensionless variables can be defined. Because there are 9 variables and the set of dimensionless variables must produce a square matrix, a set of 7 dimensionless variables will be defined as a function of 2 selected variables which are: thickness of the beam (t) and elastic modulus (E). In Table 1 the dimensionless variables are defined as πi and the aim is to find the value of each dimensionless variable as a function of t and E.

Table 1. Identification of dimensionless variables for static analysis

	U	F	b	h	H	W	L	t	E
m	1	0	1	1	1	1	1	1	-2
N	0	1	0	0	0	0	0	0	1
π1	1	0	0	0	0	0	0	?	?
π2	0	1	0	0	0	0	0	?	?
π3	0	0	1	0	0	0	0	?	?
π4	0	0	0	1	0	0	0	?	?
π5	0	0	0	0	1	0	0	?	?
π6	0	0	0	0	0	1	0	?	?
π7	0	0	0	0	0	0	1	?	?

The dimensionless variable π_1 is obtained by:

$$\pi_1 = U \cdot t^a \cdot E^b \tag{1}$$

where constants *a* and *b* are computed by taking into account that π_1 must be dimensionless. Therefore:

$$N^0 \cdot m^0 = (m) \cdot (m)^a \cdot (N \cdot m^{-2})^b \tag{2}$$

A set of two equations is obtained:

$$\begin{aligned} 0 &= b \\ 0 &= 1 + a - 2 \cdot b \end{aligned} \tag{3}$$

Solving the equations *a*=-1 and *b*=0. Therefore, the first dimensionless variable, π_1, is:

$$\pi_1 = \frac{U}{t} \tag{4}$$

Dimensionless variable π_2 is obtained by:

$$\pi_2 = F \cdot t^a \cdot E^b \tag{5}$$

where constants a and b are computed by taking into account that π_2 must be dimensionless. Therefore:

$$N^0 \cdot m^0 = (N) \cdot (m)^a \cdot (N \cdot m^{-2})^b \tag{6}$$

The set of two equations is obtained:

$$\begin{aligned} 0 &= 1 + b \\ 0 &= a - 2 \cdot b \end{aligned} \tag{7}$$

Therefore, a=-2 and b=-1. Dimensionless variable π_2 is:

$$\pi_1 = \frac{F}{t^2 \cdot E} \tag{8}$$

Carrying out the same operations for the rest of the parameters, the corresponding dimensionless variables coefficients can be computed and are shown in Table 2.

The bus prototype, whose performance is to be predicted by dimensional analysis, has a length of 8483 mm. A length of 4241.5 mm (scale 1:2) for the scale bus is defined. The rest of the dimensions of the scale bus are obtained by means of the dimensionless variables shown in Table 2.

Table 2. Computation of dimensionless variables for static analysis

	U	F	b	h	H	W	L	t	E
m	1	0	1	1	1	1	1	1	-2
N	0	1	0	0	0	0	0	0	1
π1	1	0	0	0	0	0	0	-1	0
π2	0	1	0	0	0	0	0	-2	-1
π3	0	0	1	0	0	0	0	-1	0
π4	0	0	0	1	0	0	0	-1	0
π5	0	0	0	0	1	0	0	-1	0
π6	0	0	0	0	0	1	0	-1	0
π7	0	0	0	0	0	0	1	-1	0

The dimensionless variables have the same value for the prototype bus (define by subscript *PB*) and for the scale bus model (define by subscript *SB*):

$$\pi_7 = \frac{L_{SB}}{t_{SB}} = \frac{L_{PB}}{t_{PB}} \tag{9}$$

Therefore, the thicknesses of the beams of the scale bus model have to be selected attending to the following relationship:

$$t_{SB} = \frac{L_{SB} \cdot t_{PB}}{L_{PB}} = \frac{\cancel{L_{SB}} \cdot t_{PB}}{2 \cdot \cancel{L_{SB}}} = \frac{t_{PB}}{2} \tag{10}$$

The dimensionless variables π_3 to π_6 follow the same relationship. Therefore, all the scale bus beams where manufactured at a scale 1:2. The prototype bus had a length, width and height of 8483 mm, 2335 mm and 2490 mm, respectively. In Figure 7 the final scale bus dimensions are depicted. The scale bus dimensions have been computed applying the dimensionless variables, which in this particular case is equivalent to applying a scale 1:2 to the bus prototype dimensions.

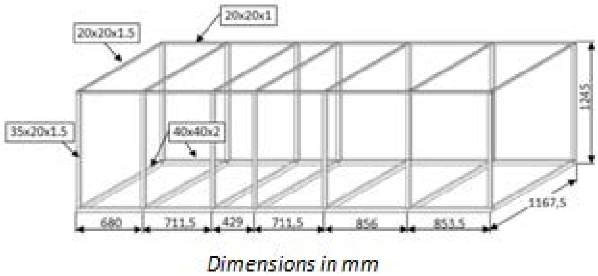

Figure 7. Scale bus model dimensions (in mm).

4. RESULTS AND DISCUSSION

4.1. Torsion test of the model at static conditions

The aim of the static analysis is to validate the proposed dimensional analysis which will predict the torsion stiffness of the prototype bus by projecting FEM torsion static results carried out on the scale bus. Once the scale bus was manufactured it was subjected to torsion static tests in order to measure torsion stiffness. A load of 100 N was applied in one of the bottom corners of the bus structure while in the other three points vertical displacement was restrained. This test was carried out 10 times in order to have a correct statistical number of measurements. The applied force was measured by means of a load cell, as depicted in Figure 8.

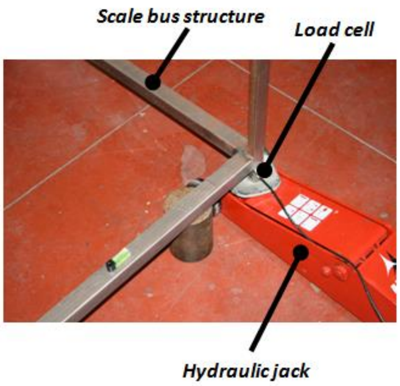

Figure 8. Load cell and hydraulic jack for static torsion test.

Twenty five vertical displacement measurements, for an applied load of 100 N, along the length of the bus structure were also registered. Vertical displacement decreased to zero in a linear trend along the bus length.

Vertical displacements were measured by means of a Vernier caliper. Errors due to measurements were computed as:

$$\sigma = \pm\sqrt{\sum_{i=1}^{N}\frac{(\bar{x}-x_i)^2}{N(N-1)}} \tag{11}$$

where $\bar{x}$ is the mean value of the measurements, xi is the individual value and N is the number of measurements. Systematic error due to precision measurement equipment is 0,01 mm. Torsion stiffness is computed as (*Gauchia et al.,* 2010):

$$K_T = \frac{M}{\Delta\theta} = \frac{F \cdot W}{arctg\left(\frac{U}{W}\right)} \tag{12}$$

where F is the measured force, W the width of the scale bus structure and U the vertical displacement measured in the corner where load is applied. From the 10 measurements, an arithmetic mean value of torsion stiffness was computed. The experimental measured torsion stiffness of the scale bus model (KT,Exp,SB) is:

$$K_{T,Exp,SB} = 300,3 \pm 6,0 \ \frac{\mathrm{N{\cdot}m}}{\mathrm{deg}} \tag{13}$$

4.2. Finite element model of the scale bus structure for torsion stiffness computation

A finite element model (FEM) of the scale bus and of the bus prototype have been developed by means of ANSYS. In this section, the scale bus FEM will be developed and validated with the experimental scale bus results. The overall aim is to project the FEM results of the scale bus to the prototype bus, by means of dimensional analysis. Therefore, a validation of the FEM technique of the scale bus with the experimental results must be previously done.

The finite element model has been carried out by means of beam elements and the geometry of each cross section has also been defined. Afterwards, the model material properties of steel have been specified. Mechanical properties were defined with an elasticity modulus of 210 GPa and a poisson ratio of 0,3. The FEM has been meshed with BEAM 44 elements and has 52 elements and 28 nodes. In addition, in order to take into account structure self weight density has been defined as 7850 kg/m3. In Figure 9 the finite element model of the scale bus is depicted.

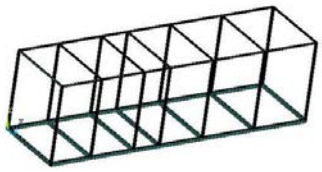

Figure 9. Finite element model of the scale bus model.

A static torsion test was also performed in ANSYS and compared with the experimental value. The computed torsion stiffness of the scale bus structure by means of the FEM (KT,FEM,SB) is.

$$K_{T,FEM,SB} = 325,8 \ \frac{\mathrm{N{\cdot}m}}{\mathrm{deg}} \tag{14}$$

The error between the finite element model torsion stiffness calculation and the experimental measurements is lower than 10%, therefore, the finite element model of the scale bus is validated. Table 3 compares the torsion stiffness obtained with the developed FEM and the measured one in the static torsion test.

Table 3. Validation of the static FEM of the scale bus with static torsion test

Bus structure	**Experimental torsion test** (N·m/deg)	**Finite element analysis** (N·m/deg)	**Difference** (%)
Scale bus	300,3±6,0	325,8	6 - 10

4.3. Dimensional analysis prediction and validation of torsion stiffness

The aim of this section is to predict the torsion stiffness of the prototype bus (scale 1:1) by projecting the FEM torsion stiffness results (already validated) of the scale bus (scale 1:2) to the bus prototype by means of dimensional analysis. The projected FEM torsion stiffness of the prototype bus will be compared with the FEM of the prototype bus.

Applying the dimensionless variables defined in section 3, the torsion stiffness of the prototype bus ($K_{T,Pred,PB}$) is predicted by projecting the FEM results obtained previously in the scale bus ($K_{T,FEM,SB}$):

$$K_{T,Pred,PB} = \frac{F_{PB} \cdot W_{PB}}{arctg\left(\frac{U_{PB}}{W_{PB}}\right)} = \frac{4 \cdot F_{SB} \cdot 2 \cdot W_{SB}}{arctg\left(\frac{2 \cdot U_{SB}}{2 \cdot W_{SB}}\right)} = 8 \cdot K_{T,FEM,SB} \quad (15)$$

Therefore, the torsion stiffness of the prototype bus is computed as eight times the torsion stiffness measured in the scale bus:

$$K_{T,Pred,PB} = 8 \cdot K_{T,Exp,SB} = 8 \cdot 325,8 = 2606,4 \ \frac{\mathrm{N \cdot m}}{\mathrm{deg}} \quad (16)$$

In order to validate this dimensional analysis the torsion stiffness of the prototype bus was calculated by means of ANSYS FEM and a value of 2606,7 N·m/deg was obtained. In Table 4 the results obtained for static torsion stiffness estimation by means of dimensional analysis as well as those computed by ANSYS are summarized.

Table 4. Validation of dimensional analysis of static torsion test

Bus structure	**Prediction with dimensional analysis** (N·m/deg)	**Finite element analysis** (N·m/deg)	**Difference** (%)
Prototype bus	2606,4	2606,7	0,012

4.4. Experimental modal testing of scale bus structure

The aim of the dynamic analysis is to predict the natural vibration frequencies of the prototype bus (scale 1:1) by projecting the results found on a scale bus (scale 1:2) (*Gauchia et al.*, 2011).

The scale bus structure was subjected to an experimental modal test in order to identify the natural vibration frequencies and modes. Free-free boundary conditions were applied. Free-free boundary conditions in real life are impossible to achieve, being the closest possibility hanging the scale bus structure from a bridge crane as shown in Figure 10.

Figure 10. Scale bus structure hanged from a bridge crane.

The modal experimental test was done by means of an impact hammer and an accelerometer that measures in the vertical direction, as depicted in Figure 11.

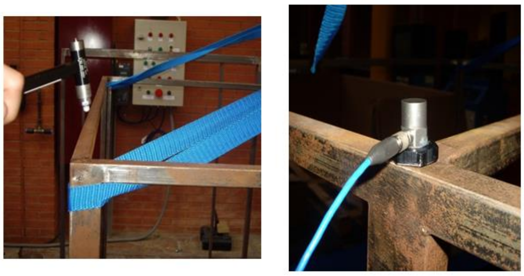

Figure 11. Left: Impact hammer. Right: Accelerometer for modal experimental test.

The impact hammer and the accelerometer were connected to an acquisition system and, by means of Oros, the signal and data was registered and saved in a computer. To discretize correctly the structure the accelerometer was placed in all joints of the bus structure. The analog signal of the accelerometers is registered by an FFT (Fast Fourier Transformation) spectrum analyzer to compute the FRF (Frequency Response Function) that digitalizes, conditions and filters the signal. By means of Test Lab FRF are analyzed to obtain the natural vibration frequencies and modes. The technical features of the impact hammer used in the modal test are described in Table 5.

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Table 5. Hammer technical features

Feature	Value
Sensitivity (±15%)	11.2 mV/N
Non linearity	≤1%
Frequency range (-10 dB)	2.5 KHz
Mass of the hammer	0.1 kg

The accelerometers employed in the modal test were ICP (Integrated circuit piezoelectric) with technical features shown in Table 6.

Table 6. Accelerometer technical features

Feature	Value
Sensitivity (±10%)	10.2 mV/(m/s^2)
Lateral sensitivity	≤ 5 %
Frequency range (±5%)	0.5 to 5000 Hz
Non linearity	≤1%
Mass of the hammer	7.4 kg

The analyzer used in the modal test is the OROS (model OR25) with 4 channels with technical features shown in Table 7.

Table 7. Analizer technical features

Feature	Value
Amplitude: Dynamic range	96 dB
Amplitude: Distorsion	< -85 dB at 1 kHz complete scale (<0.006%)
FFT Analysis: Band	FFT in real time: From 0-0.5 Hz to 0-20 kHz
FFT Analysis: Resolution	From 101 to 3201 lines
FFT Analysis: Precision	± 0.04%

The software employed for modal analysis is LMS Test Lab, which allows, between other features, 3D visualization of the modes. The geometry of the structure has to be previously defined in terms of nodes, lines and surfaces."

The natural vibration frequencies for the scale bus model are shown in Table 8.

Table 8. Experimental natural vibration frequencies for the scale bus.

Mode	Frequency (Hz)
Torsion	13,54
Bending	17,87
Torsion	19,65
Bending	31,48
Torsion	47,75
Bending	63,13

As an example, bending mode (17,87 Hz) is shown in Figure 12.

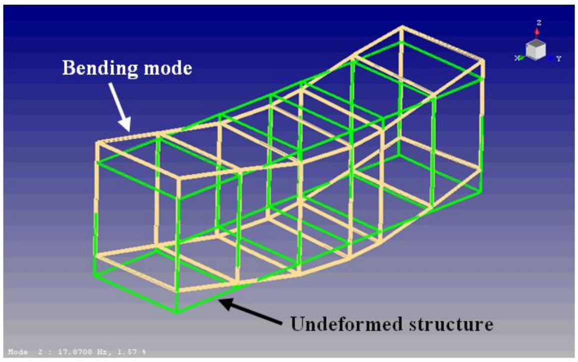

Figure 12. Bending mode shape (17,8 Hz) obtained by means of the experimental modal analysis.

4.5. Finite element model of the scale bus structure for modal analysis

The aim of this section is to develop a FEM which will be validated with the experimental modal analysis previously described. The results of the FEM will be projected from the scale bus to the prototype bus by means of dimensional analysis in order to predict its dynamic behavior. The employed FEM has already been described in section 4c. No boundary conditions have been applied to the bus and, therefore, free-free conditions have been applied in order to carry out a modal analysis. Table 9 resumes the natural vibration frequencies obtained for the scale bus model.

Table 9. FEM natural vibration frequencies for the scale bus.

Mode	Frequency (Hz)
Torsion	11,05
Bending	16,9
Torsion	22,72
Bending	32,09
Torsion	50,96
Bending	68,52

As an example, Figure 13 depicts the bending mode (16,9 Hz) obtained by means of the FEM.

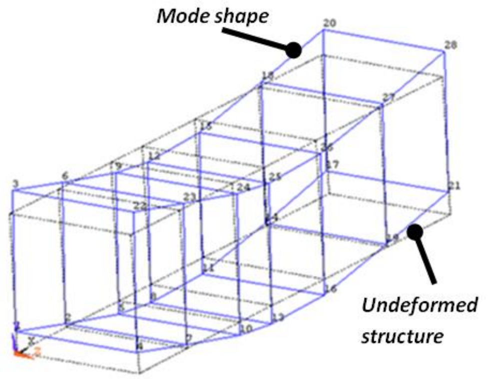

Figure 13. Bending mode of scale bus model (16,9 Hz) obtained by means of FEM.

In this section the FEM of the bus scale model is validated with the modal experimental analysis. In Table 10 the modes and natural vibration frequencies obtained by means of the FEM and by means of experimental modal analysis are compared. It can be seen that the finite element model (FEM) clearly describes the dynamic performance of the bus structure, and therefore, the FEM is validated. The biggest relative error between the FEM results and the experimental modal analysis is 18,3 % for the first mode. The reason for this deviation might be due to the fact that the FEM is a beam model where joints are represented as perfectly rigid. Last but not least, boundary conditions in the FEM are completely free-free whereas in the experimental modal analysis it is very difficult to obtain such boundary conditions, although the bus was hanged from a bridge crane.

Table 10. Validation of the dynamic FEM results of the scale bus with experimental modal analysis.

	Frequency (Hz)		
Mode	**Finite element results (FEM)**	**Experimental modal results**	**Error (%)**
Torsion	11,05	13,54	18,3
Bending	16,9	17,87	5,4
Torsion	22,72	19,65	15,6
Bending	32,09	31,48	1,9
Torsion	50,96	47,75	6,7
Bending	68,52	63,13	8,5

4.6. Dimensional analysis prediction and validation of natural vibration frequencies

In this section the bus prototype (scale 1:1) dynamic performance is predicted by projecting the FEM results obtained in the scale bus by means of the proposed dimensional analysis. The aim is to validate the proposed methodology not only for static behavior prediction but also for dynamic one.

The 9 variables of the system are: frequency (*f*), width of the beam (*b*), the beam depth (*h*), height of the structure (*H*), width of the structure (*W*), length of the structure (*L*), density of material (ρ), thickness of the beam (t) and elastic modulus (*E*). The units of these variables are three: millimeters (*mm*), kilograms (*kg*) and seconds (*s*). The number of dimensionless variables is obtained by subtraction of the total number of variables (9) and the number of units (3). Therefore, 6 dimensionless variables (9-3=6) can be defined. Because there are 9 variables and the set of dimensionless variables must define a square matrix, a set of 6 dimensionless variables will be defined as a function of 3 selected variables which are: thickness of the beam (*t*), material density (*ρ*) and elastic modulus (*E*). Table 11 resumes the dimensional analysis.

As an example, the dimensionless variable $\pi 1$ is obtained by:

$$\pi_1 = f \cdot t^a \cdot \rho^b \cdot E^c \tag{17}$$

where constants a, b and c are computed by taking into account that $\pi 1$ must be dimensionless. Therefore:

$$kg^0 \cdot m^0 \cdot s^0 = (s^{-1}) \cdot (m)^a \cdot (kg \cdot m^{-3})^b \cdot (kg \cdot m \cdot s^{-2})^c \tag{18}$$

Table 11. Computation of dimensionless variables for dynamic analysis.

	f	b	h	H	W	L	ρ	t	E
s	-1	0	0	0	0	0	0	0	-2
mm	0	1	1	1	1	1	-3	1	-1
kg	0	0	0	0	0	0	1	0	1
π1	**1**	0	0	0	0	0	**1/2**	**1**	**-1/2**
π2	0	**1**	0	0	0	0	**0**	**-1**	**0**
π3	0	0	**1**	0	0	0	**0**	**-1**	**0**
π4	0	0	0	**1**	0	0	**0**	**-1**	**0**
π5	0	0	0	0	**1**	0	**0**	**-1**	**0**
π6	0	0	0	0	0	**1**	**0**	**-1**	**0**

A set of three equations is obtained:

$$\begin{gathered} 0 = b + c \\ 0 = a - 3 \cdot b + c \\ 0 = -1 - 2 \cdot c \end{gathered} \tag{19}$$

Solving the equations a=1, b=1/2 and c=-1/2. Therefore, the first dimensionless variable, $\pi 1$, is:

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$$\pi_1 = f \cdot t \cdot \sqrt{\frac{\rho}{E}} \tag{20}$$

The width of the beams (b), the depth of the beam (h), the height of the bus structure (H) and the width of the bus structure (W) depend on parameter "t". On the other hand, the frequency of the prototype bus fPB can be estimated by means of the measured frequency on the scale bus model:

$$f_{PB} \cdot t_{PB} \cdot \sqrt{\frac{\rho}{E}} = f_{SB} \cdot t_{SB} \cdot \sqrt{\frac{\rho}{E}} \tag{21}$$

Due to the fact that both bus structures use the same material and taking into account equation (20), equation (21) yields:

$$f_{PB} = \frac{f_{SB}}{2} \tag{22}$$

This means that the natural vibration frequencies of the prototype bus are half of the natural vibration frequencies of the scale bus model for the same vibrating mode.

Taking as a starting point the validated results of the FEM of the bus scale model the bus dynamic performance of the bus prototype will be predicted by means of the dimensional analysis. From the previous sections and applying equation (22) it was found that the bus prototype natural frequencies can be computed as half of the natural vibration frequencies of the scale bus structure. Therefore, using equation (22) the prediction of the bus vibration natural frequencies are resumed in Table 12.

Table 12. Prediction of the natural vibration frequencies for the prototype bus by means of dimensional analysis.

Modes	Frequency (hz)	
	FEM results for scale bus structure Scale 1:2	**Dimensional analysis prediction for prototype bus Scale 1:1**
Torsion	11,05	5,53
Bending	16,9	8,45
Torsion	22,72	11,36
Bending	32,09	16,05
Torsion	50,96	25,48
Bending	68,52	34,26

Next, prediction of natural vibration frequencies of the prototype bus, by means of dimensional analysis, is validated with the FEM modal results of the prototype bus. In Table 13 a comparison between the natural vibration frequencies of the bus prototype obtained by the FEM and the results predicted by dimensional analysis is shown, revealing that the dimensional analysis is able to predict the natural vibration frequencies with a maximum relative error of 16,7 % in the last mode.

Table 13. Validation of the proposed dimensional analysis.

Mode	Frequency		
	Dimensional analysis prediction for bus prototype Scale 1:1	**Fem results for bus prototype Scale 1:1**	**Error (%)**
Torsion	5,53	5,52	0,18
Bending	8,45	8,73	3,2
Torsion	11,36	11,62	2,2
Bending	16,05	16,28	1,41
Torsion	25,48	25,44	0,1
Bending	34,26	41,26	16,9

Therefore, the dimensional analysis has proven to be an adequate technique to predict the dynamic performance of a bus structure prototype from the results obtained of a bus structure scale model.

4.7. Rollover limit prediction by means of dimensional analysis

The aim of this section is to predict the rollover limit threshold by means of dimensional analysis. Rollover limit is defined as the maximum lateral acceleration (ay) a vehicle can withstand without initiating rollover. Rigid suspension rollover limit is computed using the following equation:

$$\frac{a_y}{g} = \frac{W}{2 \cdot h} \tag{23}$$

where g is gravity acceleration, W is the vehicle wheel track and h the height of the center of gravity of the vehicle. The aim is to estimate the rollover limit of the prototype bus by projecting the results of the rollover limit obtained for the scale bus model. From equation (23) note that the rollover limit is a dimensionless parameter. From dimensionless parameter π_4 and remembering that under script *PB* references the

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prototype bus (scale 1:1) and under script *SB*, the scale bus model (scale 1:2):

$$h_{SB} = \frac{t_{SB}}{t_{PB}} \cdot h_{PB} = \frac{\cancel{t_{SB}}}{2 \cdot \cancel{t_{SB}}} \cdot h_{PB} = \frac{h_{PB}}{2} \quad (24)$$

From dimensionless parameter π5:

$$W_{SB} = \frac{t_{SB}}{t_{PB}} \cdot W_{PB} = \frac{\cancel{t_{SB}}}{2 \cdot \cancel{t_{SB}}} \cdot W_{PB} = \frac{W_{PB}}{2} \quad (25)$$

Therefore, the rollover limit of the prototype bus in terms of the rollover limit of the scale bus is:

$$\left.\frac{a_y}{g}\right]_{PB} = \frac{W_{PB}}{2 \cdot h_{PB}} = \frac{\cancel{2} \cdot W_{SB}}{\cancel{2} \cdot (2 \cdot h_{SB})} = \frac{W_{SB}}{2 \cdot h_{SB}} = \left.\frac{a_y}{g}\right]_{SB} \quad (26)$$

Therefore, the rollover limit of the prototype bus must be the same as the rollover limit of the scale bus. In order to compute the rollover limit the height of the centre of gravity must first be found. By means of the FEM of the scale bus model the height of the centre of gravity has been computed. Computations yield results shown in Table 14.

Table 14. Results of the scale bus centre of gravity position.

X (mm)	Y (mm)	Z (mm)
2052,7	398,24	583,75

The height of the centre of gravity is 398,24 mm and the wheel track is 1127,5 mm. Therefore, the rollover limit of the scale bus is:

$$\left.\frac{a_y}{g}\right]_{SB,FEM} = \frac{W_{SB}}{2 \cdot h_{SB}} = \frac{1127.5}{2 \cdot 398,24} = 1,41 \quad (27)$$

Therefore, attending to equation (26) the rollover limit of the prototype bus is:

$$\left.\frac{a_y}{g}\right]_{PB} = \left.\frac{a_y}{g}\right]_{SB} = 1,41 \quad (28)$$

In order to validate the dimensionless analysis, rollover limit for the prototype bus is computed by means of FEM. The height of the centre of gravity of the prototype bus has been found by means of a finite element analysis. Results are resumed in Table 15.

Table 15. Results of the prototype bus centre of gravity position.

X (mm)	Y (mm)	Z (mm)
4105,4	796,48	1167,5

Therefore, the height of the centre of gravity of the prototype bus is 796,48 mm. Rollover limit of the prototype bus is:

$$\left.\frac{a_y}{g}\right]_{PB,FEM} = \frac{W_{PB}}{2 \cdot h_{PB}} = \frac{2255}{2 \cdot 796,48} = 1,41 \quad (29)$$

Thus, dimensionless analysis for bus rollover limit has been validated.

5. CONCLUSION

Bus structures have to comply with certain regulations and directives in order to obtain type approval. These regulations are supported on experimental tests on real bus structures in order to check if the bus meets with certain requirements. In addition, bus manufacturers test their own vehicles in order to prove a certain design. These experimental tests might come up as expensive due to the installation and space required to fulfill the tests. Scale models and dimensional analysis have shown to be a solution. In this research a dimensional analysis has been proposed in order to project the results obtained in a scale bus model to a bus prototype. A scale bus model at scale 1:2 was manufactured with the same material as the original bus. The scale bus has been modeled in Ansys using finite element modeling. The proposed dimensional analysis has been applied in order to predict the static torsion stiffness and natural vibration frequencies of a prototype bus. This prediction is based on projection of the results obtained in the scale bus to the prototype bus. In addition, the finite element model of the bus scale has been validated with static and dynamic experimental tests and has shown to be accurate enough for torsion stiffness and natural vibration frequencies computation. Applying the Buckingham Theorem a set of dimensionless variables was obtained and the static torsion stiffness and natural vibration frequencies of the bus prototype were predicted by means of the results obtained in the scale bus model. This dimensional analysis was validated for both static and dynamic scenarios. From the proposed dimensional analysis it was found that the torsion stiffness of the prototype bus is eight times the torsion stiffness of the scale bus, the natural vibration frequencies of the prototype bus were predicted as half of the natural vibration frequencies found for the scale bus and the rollover limit for the prototype bus was exactly the same as the one for the scale bus. Finally, the proposed dimensional analysis has also been applied for rollover threshold estimation having proved to be a useful tool for bus structure and dynamic analysis.

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