Sección Especial: Óptica No Lineal / Special Section: Non-linear Optics

Shortcuts to adiabaticity

Atajos a la adiabaticidad

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ABSTRACT:

Different ways to accelerate adiabatic processes in cold atom physics and atomic state preparation are reviewed. The invariant-based inverse engineering approach is applied to trap expansions and contractions, and to atomic transport. Berry's Hamiltonian is applied to produce fast versions of adiabatic passage methods.

Keywords: Adiabatic methods, invariants, expansions and transport of atoms, third principle.

RESUMEN:

Se resumen distintas maneras de acelerar procesos adiabáticos en la física de átomos fríos y en la preparación de estados atómicos. Se aplica un método inverso basado en invariantes a expansiones y contracciones de trampas y al transporte atómico. También se describen versiones rápidas de los métodos de paso adiabático basadas en un Hamiltoniano propuesto por Berry.

Palabras clave: Adiabatic methods, invariants, expansions and transport of atoms, third principle.

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1. Introduction

An "adiabatic process" in quantum mechanics is a slow change of Hamiltonian parameters that keeps the populations of the instantaneous eigenstates constant. These processes are frequently used to drive or prepare states in a robust and controllable way, and have also been proposed to solve complicated computational problems, but they are, by definition, slow. A natural objective is to find "shortcuts to adiabaticity", i.e., to cut down the time to arrive at the same final state, possibly up to phase factors, either by designing optimal adiabatic pathways, or by admitting transient excitations in the "instantaneous basis" that diagonalizes the Hamiltonian. Several works have recently proposed different ways to achieve this goal for general or specific cases. One of the early applications considered was particle transport without heating, see [1] and references therein. Another important case is frictionless harmonic trap compressions or expansions for state preparation. They were first addressed with "bang-bang" (piecewise constant frequency) methods [2]. Other route is to design by inverse engineering techniques a time dependent frequency for which the expanding modes associated with Lewis-Riesenfeld invariants [3] take the state from the initial to the final potential configuration without transitions [4,5]. This has been implemented experimentally to decompress ^{87}Rb cold atoms in a harmonic magnetic trap [6]. The extension to Bose-Einstein condensates may be carried out with a variational ansatz [7] and has been realized experimentally as well [8]. Invariant-based inverse engineering has been also proposed to cool mechanical resonators [9], and indeed can be also applied to design efficient transport [1]. Much of this review is devoted to invariant-based inverse engineering: section 2 sets the general formalism, sections 3 and 4 deal with expansions, and section 5 with transport.

A different approach to shortcuts to adiabaticity is due to Berry [10]. He has proposed a Hamiltonian $\mathcal{H}(t)$ for which the adiabatic approximation of the state evolution under a time-dependent reference Hamiltonian $H_0(t)$ becomes the exact dynamics with $\mathcal{H}(t)$. This has been applied at least at a formal level to spins in magnetic fields [10], harmonic oscillators [5], or to speed up adiabatic state-preparation methods such as Rapid Adiabatic Passage (RAP), Stimulated Rapid Adiabatic Passage (STIRAP) and its variants [11]. Section 6 summarizes this application.

2. Inverse invariant methods

Lewis and Riesenfeld derived a simple relation between the solutions of the Schrödinger equation of a system with time-dependent Hamiltonian and the eigenstates of the corresponding invariants [3]. They paid special attention to the time-dependent harmonic oscillator and its invariants quadratic in position and momentum, which are related to earlier work by Ermakov on the classical oscillator. Lewis and Leach found, in the framework of classical mechanics, the general form of the Hamiltonian compatible with invariants

quadratic in momentum [12], including non harmonic potentials. This is the result of interest to us here, together with the corresponding quantum results by Dhara and Lawande [13]. In this Section we shall state the main concepts and equations.

A one-dimensional Hamiltonian with an invariant which is quadratic in momentum must have the form $H = p^2/2m + V(q,t)$,[1] with the potential [12,13]

$$V(q,t) = -F(t)q + \frac{m}{2}\omega^2(t)q^2 + \frac{1}{\rho(t)^2}U\left[\frac{q-\alpha(t)}{\rho(t)}\right], \quad (1)$$

ρ, α, ω and F are arbitrary functions of time that satisfy two auxiliary equations

$$\ddot{\rho} + \omega^2(t)\rho = \frac{\omega_0^2}{\rho^3}, \quad (2)$$

$$\ddot{\alpha} + \omega^2(t)\alpha = \frac{F(t)}{m}, \quad (3)$$

with ω_0 constant. Their physical interpretation depends on the application as explained below. The quadratic dynamical invariants, up to a constant factor, are given by

$$I = \frac{1}{2m}[\rho(p - m\dot{\alpha}) - m\dot{\rho}(q-\alpha)]^2 + \frac{1}{2}m\omega_0^2\left(\frac{q-\alpha}{\rho}\right)^2 + U\left(\frac{q-\alpha}{\rho}\right), \quad (4)$$

and verify

$$\frac{dI}{dt} = \frac{\vartheta I(t)}{\vartheta t} + \frac{1}{i\hbar}[I(t), H(t)] = 0, \quad (5)$$

so that $\frac{d}{dt} = \langle\psi(t)|I(t)|\psi(t)\rangle = 0$, for any wave function $\psi(t)$ that evolves with the Hamiltonian H. The invariants are useful in different ways. For example we may expand $\psi(t)$ in terms of constant coefficients c_n and eigenvectors ψ_n of I,

$$\psi(t) = \sum_n c_n e^{i\alpha_n}\psi_n(q,t), \quad (6)$$

$$I(t)\psi_n(q,t) = \lambda_n\psi_n(q,t), \quad (7)$$

where the λ_n are time-independent eigenvalues of the invariant. The spectrum of I may be discrete and/or continuous, so ψ_n may have continuum normalization or be normalized to one as in most applications discussed here. The phases α_n satisfy [3,13]

$$\hbar\frac{d\,\alpha_n}{dt} = \langle\psi_n\left|i\hbar\frac{\vartheta}{\vartheta t} - H\right|\psi_n\rangle, \quad (8)$$

$$\alpha_n = -\frac{1}{\hbar}\int_0^t dt'\left(\frac{\lambda_n}{\rho^2} + \frac{m(\dot{\alpha}\rho - \alpha\dot{\rho})^2}{2\rho^2}\right). \quad (9)$$

The ψ_n are in practice obtained as [13]

$$\psi_n(q,t) = e^{\frac{im}{\hbar}\left[\dot{\rho}q^2/2\rho + (\dot{\alpha}\rho - \frac{\alpha\dot{\rho})q}{\rho}\right]}\frac{\phi_n\left(\frac{q-\alpha}{\rho}\right)}{\rho^{1/2}}, \quad (10)$$

with $(q-\alpha)/\rho := \sigma$, from the solutions $\phi_n(\sigma)$ (normalized in σ-space) of the auxiliary stationary Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{\vartheta^2}{\vartheta\sigma^2} + \frac{1}{2}m\omega_0^2\sigma^2 + U(\sigma)\right]\phi_n = \lambda_n\phi_n \quad (11)$$

The basic strategy of invariant-based inverse engineering methods is to design the auxiliary functions ρ and α first to achieve desired objectives, and deduce the Hamiltonian afterwards. In most applications so far the key point is to control the boundary conditions of the auxiliary functions and their time derivatives at initial and final times. In particular, they may be set so that the eigenvectors of H and I coincide at initial and final times, and the process produces no final excitation. It is however not adiabatic in the usual sense, as excitations in the instantaneous basis at intermediate times are allowed.

3. Fast expansions

Performing fast expansions of trapped atoms without losses or vibrational excitation is important, for example to reduce velocity dispersion and collisional shifts in spectroscopy and atomic clocks, to reach extremely low temperatures unaccessible by standard cooling techniques or, in experiments with optical lattices, to broaden the atomic cloud before turning on the lattice. Trap contractions are also common to prepare the state. For a harmonic oscillator trap we may consider these expansion or contraction processes by setting $\alpha = U = F = 0$. Then Eq. (3) does not play any role and the important auxiliary equation is the Ermakov equation (2). Moreover ρ is

[1] q and p will denote operators or numbers, and the context should clarify their meaning

proportional to the standard deviation of the "expanding (or contracting) modes" $e^{i\alpha_n}\psi_n$.

Let us discuss an expansion process from $\omega(0)=\omega_0$ to $\omega(t_f)=\omega_f$. The treatment for contractions is very similar. By choosing $\rho(0)=1$, $\dot{\rho}(0)=0$, $H(0)$ and $I(0)$ commute and have common eigenfunctions, at $t=0$. $\ddot{\rho}(0)=0$ holds as well, consistent with the Ermakov equation. At the final time t_f we impose $\rho(t_f)=\gamma=(\omega_0/\omega_f)^{1/2}$, $\dot{\rho}(t_f)=0$, $\ddot{\rho}(t_f)=0$. The consequence is that the expanding mode is an instantaneous eigenvector of H at $t=0$ and t_f. This is so regardless of the form of $\rho(t)$ in between. In practice one chooses a functional form to interpolate between these two times, with enough flexibility to satisfy the imposed boundary conditions. A simple option is the polynomial ansatz

$$\rho(t)=\sum_{j=0}^{5} a_j\, t^j, \tag{12}$$

where the coefficients are determined by solving the equations set by the boundary conditions,

$$\rho(t)=6(\gamma-1)s^5-15(\gamma-1)s^4 +10(\gamma-1)s^3+1. \tag{13}$$

Here $s=t/t_f$.

The next step is to solve for $\omega(t)$ in the Ermakov equation (2). This procedure poses in principle no fundamental lower limit to t_f, which could be arbitrarily small. There are of course practical limitations, and two warnings: the first one is that for short enough t_f, $\omega(t)$ may become purely imaginary at some t [4], which corresponds to a parabolic repeller configuration; the second one is that the energy required may be too high, as analyzed in detail in the following section.

4. Transient energy excitation in shortcuts to adiabaticity for the time dependent harmonic oscillator

In this section we shall examine the energy "cost" of shortcut processes; more precisely, their transient excitation energies [14]. Our central study case is the expansion (or compression) of a harmonic oscillator, which is a basic model for many operations in any cold atoms laboratory. One may expect the transient system energy and the time of the process to be "conjugate", i.e., an increase of the former when decreasing the later, but the details of this relation, and the role played by other parameters defining the process (such as initial and final frequencies) have to be clarified for fundamental reasons and for the applications. Clearly, the energy excitation will set limits to the possible speed-up. Actual traps are only approximately harmonic so large transient energies will imply perturbing effects of anharmonicities and thus undesired excitations of the final state, or even atom loss.

The transient excitation energy is also important to quantify the principle of unattainability of zero temperature, first enunciated by Nernst. It is usually formulated as the impossibility to reduce the temperature of any system to the absolute zero in a finite number of operations, and identified with the third law of thermodynamics although this is sometimes disputed. Kosloff and coworkers [2] have restated the unattainability principle as the vanishing of the cooling rate in quantum refrigerators when the temperature of the cold bath approaches zero, and quantify it by the scaling law relating cooling rate and cold bath temperature. We shall examine the consequences of the transient energy excitation on the unattainability principle at two levels, namely, for a single, isolated expansion, and considering the expansion as one of the branches of a quantum refrigerator cycle.

A lower bound for the time-averaged energy of the n-th expanding mode is found by calculus of variations [14],

$$\mathcal{B}_n=\frac{(2n+1)\hbar}{2\omega_0 t_f^2}\Big\{\left(B^2-\omega_0^2 t_f^2\right)-2\omega_0 t_f \times\Big[\operatorname{arctanh}\left(\frac{B^2+B-\omega_0^2 t_f^2}{\omega_0 t_f}\right)- -\operatorname{arctanh}\left(\frac{B}{\omega_0 t_f}\right)\Big]\Big\}, \tag{14}$$

such that $\overline{E_n}\geq\mathcal{B}_n$. Here $B=-1+\left(\gamma^2+\omega_0^2 t_f^2\right)^{1/2}$. When the final frequency ω_f is small enough to satisfy $t_f\ll 1/\sqrt{\omega_0\omega_f}$, and $\gamma\gg 1$, the lower bound has the asymptotic form

$$\mathcal{B}_n\approx\frac{(2n+1)\hbar}{2\omega_f t_f^2}, \tag{15}$$

A consequence of this is

$$t_f \geq \sqrt{\frac{(2n+1)\hbar}{2\omega_f \overline{E_n}}}. \qquad (16)$$

If in Eq. (16) $\overline{E_n}$ is limited by some maximal value, because of anharmonicities or a finite trap depth, the scaling is fundamentally the same as for bang-bang methods [2], and leads to a cooling rate $R \propto T_c^{3/2}$ in an inverse quantum Otto cycle, although an opportunity is offered to improve the proportionality factor by increasing the allowed $\overline{E_n}$. This dependence had been previously conjectured to be a universal dependence characterizing the unattainability principle for any cooling cycle [15]. The present results provide strong support for the validity of this conjecture within the set of processes defined exclusively by time-dependent frequencies and call for further testing and study.

Independently of the participation of the harmonic trap expansion as a branch in a refrigerator cycle, we may apply the previous analysis on a single expansion by assuming that the initial and final states are canonical density operators characterized by temperatures T_0 and T_f , related by $T_f = (\omega_f/\omega_0)T_0$ for a population-preserving process. For a parabolic potential expansion, the unattainability of a zero temperature can be thus reformulated as follows: The transient excitation energy becomes infinite for any population-preserving and finite-time process if the final temperature is zero (which requires $\omega_f = 0$). This excitation energy has to be provided by an external device, so there remains a fundamental obstruction to reach $T_f = 0$ in a finite time, in the form of the need for a source of infinite power.

The standard deviation can also be studied [14]. The dominant dependences of the time averages found numerically scale on ω_f and t_f in the same way as the average energy. These dependences differ from the ones in the Anandan-Aharonov relation [16]

$$\overline{\Delta H} t_f \geq \frac{h}{4}, \qquad (17)$$

where

$$\overline{\Delta H} = \frac{\int_0^{t_f} \Delta H(t) dt}{t_f}. \qquad (18)$$

The AA bound, although correct, is not tight.

We have considered first simple processes in which the only external manipulation consists in shaping $\omega(t)$. As shown in [14] one could design even faster processes by adding terms to the harmonic oscillator Hamiltonian [10], but their physical implementation remains a challenge.

5. Fast transport

A key element for controlling the states and dynamics of cold neutral atoms and ions is their efficient transport by moving the confining trap. In spite of the broad span of conditions, heating mechanisms, transport distances (from microns to tens of centimeters), transport times, and accelerations that can be found, there are common elements that allow for a rather generic theoretical treatment as the one presented in [1] and summarized here. Transport should ideally be fast, lossless, and lead to a final state as close as possible ("faithful") to the initial one, up to global phase factors, in the transporting trap frame.

As done for expansions, we may use the dynamical invariants associated with the Hamiltonian of an atom in a one dimensional moving trap to inverse engineer the trap motion and perform fast atomic transport without final vibrational heating. The atom is driven non-adiabatically through a shortcut to the result of adiabatic, slow trap motion. For harmonic potentials this only requires designing appropriate trap trajectories, whereas perfect transport in anharmonic traps may be achieved by applying an extra field to compensate the forces in the rest frame of the trap. The results can be extended to atom stopping or launching. The limitations due to geometrical constraints, energies and accelerations involved are analyzed in [1], as well as the relation to previous approaches (based on classical trajectories or "fast-forward" and "bang-bang" methods) which can be integrated in the invariant-based framework.

Whereas trap expansions and contractions imply a time dependent function [4], a large family of transport problems may be described by taking

$$\rho(t) = 1, \omega^2(t) = \omega_0^2, \qquad (19)$$

so the auxiliary Eq. (2) plays no role and only Eq. (3) is relevant. We shall assume that the conditions (19) hold from now on, and consider two main reference cases.

5.1 Main cases

(i) Rigid harmonic oscillator driven by the "transport function" $q_0(t)$ ("harmonic transport" for short). Suppose that a harmonic trap is moved from $q_0(0)$ at time $t = 0$ to $d = q_0(t_f)$ at a time t_f. In Eq. (1) this corresponds to

$$F = m\omega_0^2 q_0(t), \omega(t) = \omega_0, U = 0. \tag{20}$$

Adding to V the irrelevant time dependent global term $m\omega_0^2 q_0^2/2$, the trap potential can be written as a moving harmonic oscillator $m\omega_0^2[q - q_0(t)]^2/2$,

$$H = \frac{p^2}{2m} + m\omega_0^2[q - q_0(t)]^2/2, \tag{21}$$

and α may be identified with a classical trajectory q_c. Eq. (3) becomes

$$\ddot{q}_c + \omega_0^2(q_c - q_0) = 0. \tag{22}$$

In this case $\lambda_n = E_n = (n + 1/2)\hbar\omega_0$, and the transport mode $e^{i\alpha_n}\psi_n$ takes the form

$$e^{i\alpha_n}\psi_n = \exp\left\{-\frac{i}{\hbar}\left[E_n t + \int_0^t \frac{m\dot{q}_c{}^2}{2} dt'\right]\right\} \exp\left\{\frac{im\dot{q}_c q}{\hbar}\right\}\phi_n(q - q_c), \tag{23}$$

Efficient transport may be engineered by designing first an appropriate classical trajectory $q_c(t)$, from which the trap motion trajectory $q_0(t)$ is deduced via Eq. (22).

A variant of this case is vertical transport with a gravity force, so that $F = m\omega_0^2 q_0 - mg$ and Eq. (22) becomes

$$\ddot{q}_c + \omega_0^2(q_c - q_0) = -g. \tag{24}$$

It is also possible to design stopping or launching processes without losses or heating apart from pure rest-to-rest transport [1].

A major practical concern in all these applications is to keep the harmonic approximation valid. This may require an analysis of the actual potential in each case and of the excitations taking place along the non-adiabatic transport process. Without performing such detailed analysis, the feasibility of the approach for a given transport objective set by the pair d, t_f can be estimated simply by comparing lower excitation bounds. These are obtained using calculus of variations as discussed before for expansions. Writing the average potential energy for a given transport mode as $\langle V(t)\rangle = (n + 1/2) + E_P$ we find [1]

$$\overline{E_P} \geq \frac{6md^2}{t_f^4\omega_0^2}. \tag{24}$$

This bound describes the relevant dependences, as shown by numerical comparisons with actual time-averaged energies for polynomial trajectories, and sets a rather strong t_f^{-4} scaling, compare this with the milder dependence on t_f^{-2} of the time-averaged transient energy in invariant-based inverse-engineered expansion processes [14].

(ii) Arbitrary-trap driven transport with compensating force ("compensating force approach" for short). Now, in Eq. (1)

$$\omega = \omega_0 = 0, \tag{26}$$

$$F = m\ddot{q}_0. \tag{27}$$

In this case the trap potential $U[q - q_0(t)]$ is arbitrary (in particular it could be harmonic), and it is rigidly displaced along $q_0(t)$, so that α in Eq. (3) may be now identified with the transport function q_0. In addition to U, there is a compensating time dependent linear potential term $-mq\ddot{q}_0$ in H,

$$H = p^2/2m - mq\ddot{q}_0 + U(q - q_0). \tag{28}$$

The corresponding force compensates exactly the inertial force due to the trap motion in the rest frame of the trap, in such a way that the wave function in that frame is not modified up to a time dependent global phase factor. This Hamiltonian has been proposed by Mashuda and Nakamura following a very different "fast-forward" scaling technique [17].

Inverse engineering in this case is based on choosing the boundary conditions for q_0 [1].

6. Accelerating adiabatic passage in two and three level atoms

There are two major routes for manipulating the state of a quantum system with interacting fields: resonant pulses, or adiabatic methods such as "Rapid" Adiabatic Passage (RAP), Stimulated Raman Adiabatic Passage (STIRAP), and their

variants. In general simple fixed-area resonant pulses may be fast if they are intense enough, but also unstable with respect to errors or fluctuations of the parameters, whereas adiabatic passage is slow but robust. For many applications the ideal method should be fast and robust, two requirements that are particularly demanding if quantum computing is to become feasible. Pulse sequences may be more stable than single pulses, but their use is limited by technical reasons. In NMR, composite pulses are being superseded by adiabatic passage methods, which have also been very successful in chemical reaction dynamics, laser cooling, atom optics, metrology, interferometry, or cavity quantum electro-dynamics. When robustness is the primary concern, they are quite sufficient, and have as well become basic operations for quantum information processing. If speed is also important, however, the limitations may be severe. Given the difficulties of composite pulses, it is natural to look for robustness and high operation rates by shortening the duration of adiabatic methods. A shortcut to adiabatic passage [11] ("SHAPE" hereafter), which is not only fast but remarkably stable, may be found using the "transitionless quantum driving" algorithm proposed by Berry [10]. The specific applications we shall discuss is the speeded-up version of (2-level) Rapid Adiabatic Passage (RAP). 3-level systems and STIRAP are worked out in [11], and variants such as fractional RAP or fractional STIRAP, and multilevel schemes may be treated similarly.

The transitionless quantum driving algorithm [10] provides Hamiltonians $\mathcal{H}(t)$ for which the adiabatic approximation for the time-dependent wavefunction evolving with a reference Hamiltonian $H_0(t)$ becomes exact. The simplest Hamiltonian, $H_1(t)$, steers the dynamics along the instantaneous eigenstates $|\lambda_n(t)\rangle$ of $H_0(t)$ without transitions among them and without phase factors, formally in an arbitrary time,

$$H_1(t) = i\hbar \sum_n |\vartheta_t \lambda_n\rangle \langle \lambda_n|. \qquad (29)$$

Concerning the populations, the addition of H_0 is possible, but not necessary, since it only affects the phases. H_1 may thus supplement H_0, when $\mathcal{H} = H_1$, or substitute it, if we take $\mathcal{H} = H_0 + H_1$. The physical feasibility of H_1 has to be examined in each system. For example, when H0 describes a particle in a time-dependent harmonic potential, H_1 becomes a non-local interaction [5]. For a particle with spin in a time dependent magnetic field, H_1 becomes a time-dependent magnetic field [10]. For the atomic two- and three-level systems, H_1 will involve auxiliary laser or microwave interactions. The optional addition of H_0 will imply different physical implementations [11].

The technique known as Rapid Adiabatic Passage inverts the population of two-levels, $|1\rangle$ and $|2\rangle$, by sweeping the radiation slowly through resonance. This broad spread method originated in Nuclear Magnetic Resonance but is used in virtually all fields where 2-level systems may be controlled by external interactions. The term "rapid" may sound contradictory here. It refers to the case in which the frequency sweep is shorter than the life-time of spontaneous emission and other relaxation times.

We shall formulate H_0, H_1, or $\mathcal{H}$ in a peculiar interaction picture so they will carry an I superscript, to distinguish them from Schrödinger picture Hamiltonians (with S subscript). This also applies to wave functions. To set H_0^I let us assume a semiclassical description of the (electric dipole) interaction with the electric field **E**. Using the rotating wave approximation (RWA), the Hamiltonian for a laser interaction with linear polarization in x-direction is $H^S = (\hbar/2)\big[|2\rangle\langle 1|\Omega_R e^{-i\omega_L t} + |1\rangle\langle 2|\Omega_R e^{i\omega_L t} + \omega_0(|2\rangle\langle 2| - |1\rangle\langle 1|)\big]$, with R real. In a laser-adapted interaction picture based on a time dependent $h_0 = \hbar(\omega_L(t)/2)(|2\rangle\langle 2| - |1\rangle\langle 1|)$, the dynamics of the wave function $\psi^I(t) = e^{ih_0 t/\hbar}\psi^S(t)$ is governed by the Hamiltonian $H_0^I = e^{ih_0 t/\hbar}\big(H^S - h_0 - t\dot{h_0}\big)e^{-ih_0 t/\hbar}$, where the dot denotes time derivative.

Using $|1\rangle = \binom{1}{0}$, $|2\rangle = \binom{0}{1}$,

$$H_0^I(t) = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega_R(t) \\ \Omega_R(t) & \Delta(t) \end{pmatrix}, \qquad (30)$$

where $\Delta(t) = \omega_0 - \omega_L - t\dot{\omega}_L$ is the effective detuning, controlled by a change in the carrier frequency or an alteration of the Bohr frequency by Zeeman or Stark shifts. Note the inverse relation, $\omega_L = \omega_0 - 1/t\int_0^t \Delta(t')dt'$. The instantaneous eigenvectors are

$$|\lambda_-(t)\rangle = \cos[\theta(t)/2]|2\rangle - \sin[\theta(t)/2]|1\rangle \quad (31)$$

$$|\lambda_+(t)\rangle = \sin[\theta(t)/2]|2\rangle + \cos[\theta(t)/2]|1\rangle \quad (32)$$

with the mixing angle $\theta(t) = \arccos[-\Delta(t)/\Omega(t)]$ and eigenvalues $E_{\mp}(t) = \mp\hbar\Omega/2$, where $\Omega = \sqrt{\Delta^2(t) + \Omega_R^2(t)}$. If the adiabaticity condition $1/2\,|\Omega_a| \ll |\Omega(t)|$, where $\Omega_a \equiv \dot{\theta} = \left|\Omega_R(t)\dot{\Delta}(t) - \Omega\dot{R}t\Delta t/\Omega 2\right.$, is satisfied, the state evolving from $|\psi^I(t_i)\rangle = |\lambda_{\mp}(t_i)\rangle$ follows the adiabatic approximation

$$\left|\psi_{\pm}^I(t)\right\rangle = \exp\left\{-\frac{i}{\hbar}\int_{t_i}^{t} dt'\, E_{\pm}(t')\right\}|\lambda_{\mp}(t)\rangle, \quad (33)$$

whereas transitions will occur otherwise. Different adiabatic passage schemes correspond to $\Omega_R(t)$ and $\Delta(t)$ for which $\psi_{\pm}^I$ goes from one bare state to the other. The simplest one is the Landau-Zener scheme with constant Ω_R and linear-in-time Δ. Regardless of the chosen scheme, $H_1^I(t)$ takes here the form, using Eqs. (29) and (31),

$$H_1^I(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i\Omega_a \\ i\Omega_a & 0 \end{pmatrix}, \quad (34)$$

where (up to a phase factor) Ω_a plays the role of the Rabi frequency for a fast-driving field. In principle $H_1^I(t)$ drives the dynamics along the $H_0^I(t)$-adiabatic path in arbitrarily short times, but there are practical limitations such as the laser power available. Moreover, a comparison with $H_0^I(t)$-dynamics is only fair if $|\Omega_a|$ is smaller or approximately equal to the peak Rabi frequency in the original laser setup. Independently of the scheme chosen and in a range of interaction times that break down the adiabaticity condition, it is remarkable that the dynamics can be driven along the $H_0^I(t)$-adiabatic path while fulfilling the inequalities$|\Omega_a| \leq |\Omega| \leq |\Omega_0|$.

The physical meaning of the fast-driving term is clearer in the Schrödinger picture. For $\mathcal{H}^I = H_0^I(t) + H_1^I(t)$, then

$$\mathcal{H}^S = (\hbar/2)[(\Omega_R + i\Omega_a)|2\rangle\langle 1|\, e^{-i\omega_L t} + (\Omega_R - i\Omega_a)|1\rangle\langle 2|\, e^{-i\omega_L t} + \omega_0(|2\rangle\langle 2| - |1\rangle\langle 1|)], \quad (35)$$

This implies two lasers with the same frequency, orthogonal polarization, and differently shaped time-dependent intensities. Instead, when $\mathcal{H}^I = H_1^I(t)$, $\mathcal{H}^S = (\hbar/2)\big[i\Omega_a|2\rangle\langle 1|e^{-i\omega_L t} - i\Omega_a|1\rangle\langle 2|e^{i\omega_L t} + \omega_L(|2\rangle\langle 2| - |1\rangle\langle 1|)]$, which requires only one interaction and level shift engineering so that the effective detuning vanishes, $\Delta = 0$. In this case the $\pm i$ factors in Eq. (34) can be dropped, which amounts to redefine the states with constant phase factors or to perform an axis rotation, without altering the population transfer.

7. Conclusions

We have reviewed recent techniques to accelerate quantum adiabatic processes. Some of them may be extended to more complicated systems, in particular to condensates [7,8] by using scaling relations, or to other interacting systems such as the Tonks-Girardeau gas. We are currently exploring further aspects, such as stability versus noise and perturbations, and effects of higher dimensions. A promising field is emerging with implications in fundamental and applied physics.

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