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Abstract

The Consumption-Capital Asset Pricing Model is a statement about the mean of asset returns and does not provide any information on the returns' quantiles. Using quantile maximization decision theory, this paper considers a quantile-based Euler equation that states that the asset price is a function of the quantiles of the payoff, consumption growth, stochastic discount factor and risk aversion. Assuming that the consumption growth rate is log-elliptically distributed, we show that returns' quantiles are non-monotone functions of the consumption growth volatility. Using data from the United States and United Kingdom, empirical evidence validates our theoretical results and shows that this volatility is a driving factor of the returns' distribution.

Keywords: Asset Pricing, CCAPM, Consumption Volatility, Quantile-based Euler Equation, Quantile Utility Function, stochastic volatility.
JEL: C21, C58, G12

1. Introduction

The relationship between consumption and asset prices has long been a topic of interest in the literature (Lucas Jr, 1978; Breeden, 1979). The consumption based asset pricing model (CCAPM) states that the conditional expected return on a risky asset should be proportional to its conditional consumption beta. The empirical validity was initially weak (Blume and Friend, 1973; Stambaugh, 1982; Shanken, 1985; Shapiro and Mankiw, 1985, among others), but a recent wave of empirical work reports encouraging results using different measures of consumption (Savov, 2011; Chen and Lu, 2017) and supporting the relationship on the cross section (Jagannathan and Wang, 2007; Boguth and Kuehn, 2013; Tédongap, 2014; Delikouras, 2017; Dittmar and Lundblad, 2017; Delikouras and Kostakis, 2019), spurring new interest in this fundamental relationship.

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The CCAPM is considered to be one of the most important models in the asset pricing literature, because first, it incorporates the intertemporal nature of portfolio decisions (as in Merton (1973) and Breeden (1979)) and second, it implicitly integrates other forms of wealth beyond stock market wealth that are relevant for measuring systematic risk.[1] However, the CCAPM model is only a statement about the conditional mean of asset prices and it does not provide any information on other levels of the conditional distribution of asset prices such as its quantiles. The conditional mean delivers only limited information on the conditional distribution of asset prices, and therefore, it does not allow concluding about the overall risk that an investor might face.

In this paper, we extend the classic consumption capital asset pricing model (CCAPM) by considering a quantile version of the CCAPM as in Giovannetti (2013). We use the quantile-based Euler equation obtained from solving the standard intertemporal problem of a consumer–investor agent under a quantile maximization decision theory as in Manski (1988) and Rostek (2010). Under the assumption that the consumption growth rate process is log-elliptically distributed, we show that the quantiles of the stock returns conditional distribution can be expressed as a function of the consumption growth rate volatility. To test our model, we use data on U.S. consumption to estimate the consumption growth rate volatility and several U.S. stock market indexes. Consumption growth rate volatility is assumed to follow a stochastic volatility process similar in spirit to that of Tauchen (2011), which includes an asymmetric response of volatility to positive and negative shocks of the same magnitude, the so-called *leverage effects* (see Christie, 1982; Campbell and Hentschel, 1992; Bollerslev et al., 2006).[2] The quantile regression analysis suggests that the volatility of consumption growth rate affects the tails of the conditional distribution of stock returns. The effect is both economically and statistically significant in the tails, but not at the center of the conditional distribution of asset returns. Consistent with the model predictions, the results show that the volatility of consumption growth rate affects negatively the lower quantiles of stock returns, and positively the upper quantiles. In the robustness analysis, we extend the analysis to other U.S. stock market indexes and also to U.K. data and various volatility measures, and the results remain supportive of the previous findings.

Our paper relates with several strands of research, first, to the vast literature on the standard consumption-based CAPM (CCAPM) introduced by Lucas Jr (1978); Breeden (1979). Broadly, that model considers the risk of a security by the covariance of its return with per capita consumption, also known as the consumption beta. It postulates that the expected equity premium is proportional to the consumption beta, which measures the systematic tendency of individual securities returns

[1]Furthermore, Kocherlakota (1996) asserts that the CCAPM is actually more important than the CAPM due to its integral role in modern macroeconomics and international economics.

[2]Although asymmetry and leverage are not exactly the same, we use them interchangeably hereinafter. Leverage is a special case of asymmetry; see, for example, McAleer (2014).

to follow aggregate consumption growth.[3] Second, it relates with the evidence that shows that the compensation for risk depends on the state of financial markets, in line with the time-varying risk premium literature. For example, Rossi and Timmermann (2010)[4] find a non-monotonic relationship between volatility and expected stock returns. In particular, for low and medium values of the conditional volatility (bull market with positive returns), they find a positive relationship between risk and expected returns, while for high levels of volatility (bear market with negative returns) this relationship might be negative. Duffee (2005) argues that the conditional covariance between aggregate stock returns and aggregate consumption growth varies substantially over time. When stock market wealth is high relative to consumption, both the conditional covariance and correlation are high, consistent with the composition effect, that is, agent's consumption growth is more closely tied to stock returns when stock wealth is a larger share of total wealth. Therefore, the consumption growth rate and its volatility can be used to test asset pricing models in which the price of consumption volatility is a negatively priced source of risk for asset stock market returns.

Third, our work relates to the literature that models quantile preferences such as Manski (1988); Chambers (2007, 2009); Rostek (2010); Giovannetti (2013). In particular, Giovannetti (2013) applies the quantile maximization decision theory to the standard intertemporal problem of a consumer–investor agent. In this framework, the representative agent makes decisions about the consumption–investment looking at the worst-case scenarios, which depend on his degree of pessimism. Solving the consumption-investment problem leads to the quantile-based Euler equations that the agent must satisfy in equilibrium.

Finally, our work is also close to the literature that uses quantile estimation to study the intersection of macroeconomic variables and asset prices. Taamouti (2015) investigates the link between monetary policy measures and quantiles of stock market returns, and finds that money supply affects the lower and upper extreme quantiles of the stock return distribution but not its center. The results suggest that the monetary policy measure is effective only during recessions and expansions. Gonzalo and Taamouti (2017) examine the short-run impact of anticipated and unanticipated unemployment rates on quantiles of stock prices. Their quantile regression analysis shows that the causal effects of anticipated unemployment rate on stock returns are usually heterogeneous across quantiles.

Our paper contributes to the literature on the theoretical link between asset prices and economic fluctuations. It provides evidence on the view that aggregate economic uncertainty has sizeable effects on asset valuations, and that financial markets dislike uncertainty in macroeconomic variables (Bansal et al., 2005). Our empirical evidence validates our theoretical results and shows that the consumption growth rate volatility is a driving factor of the quantiles of stock market returns.

The paper is organized as follows. In section 2, we derive the quantile version of the consumption

[3] A recent review on the state of the art on the CCAPM model can be found in Breeden et al. (2015).
[4] See also Rossi and Timmermann (2015).

capital asset pricing model. In section 3, we present the stochastic volatility models for the consumption growth rate and the quantile regressions. In section 4, we provide an empirical application using data on the U.S. and U.K. consumption and on several stock market indexes. In this same section, we perform various robustness checks by considering different deflators of the nominal variables and different stock market price indexes. Finally, we conclude in section 5.

2. Quantile-based asset pricing Euler equation

Understanding the relationship between economic fundamentals and asset prices has been an important question in financial economics. Cochrane (2008) surveys work on the intersection between macroeconomics and finance and points out the asset pricing Euler equation as the central idea of modern finance. Prices are generated by expected discounted payoffs, and the present asset price p_t is expressed as the conditional expectation of the future payoff of the asset x_{t+1} and the stochastic discount factor m_{t+1}. The latter is equal to the growth in the marginal value of wealth. Thus, the fundamental asset pricing relationship is given by:

$$E_t\left(m_{t+1}r_{t+1}\right) = 1,$$

where in the consumption-based capital asset pricing model under the power utility form m_{t+1} is equal to $\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, C_{t+1} is the aggregate consumption, γ is the coefficient of relative risk-aversion and δ is a subjective time-discount factor.

If we rewrite the previous equation in terms of excess return, we can see that the "risk premium" is higher for assets that have a large negative covariance with the discount factor. Furthermore, the risk premium is driven by the covariance of returns with the marginal value of wealth. Given the asset returns' uncertainty, investors would rather do well when they need a little bit of extra wealth, and they would rather do worse when they do not as eagerly value extra wealth. Thus, investors prefer assets whose payoffs have a positive covariance with the marginal utility of wealth, and they avoid assets having a negative covariance.

Recall that the above Euler equation, which leads to the CCAPM, is a statement about the conditional mean of stock returns, but does not say anything about what happens at other levels of the conditional distribution of asset prices such as its quantiles. Therefore, in the next section we consider a quantile-based Euler equation that leads to a quantile version of the CCAPM model, in which the conditional quantiles of stock market returns can be expressed as functions of future payoffs and on the consumption-based stochastic discount factor.

2.1. Quantile-based Euler equation

From our remark at the end of the previous section, we see that there is a need to build a quantile-based Euler equation that leads to a quantile version of the CCAPM model, in which the quantiles of the conditional distribution of stock returns can be expressed as functions of the future payoff of the stock and of the consumption-based stochastic discount factor. We follow Giovannetti (2013) who considers a framework in which the representative agent makes decisions about the consumption-investment considering the worst-case scenarios, which depend on his degree of pessimism. Using the quantile utility maximizer agent of Manski (1988) and Rostek (2010), Giovannetti (2013) expresses the quantile Euler equation at the equilibrium as follows:

$$P_t = \delta \left(Q_t^{\alpha} \left(\frac{C_{t+1}}{C_t} \right) \right)^{-\gamma} Q_t^{\alpha} \left(X_{t+1} \right), \tag{1}$$

where $Q_t^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)$ is the conditional αth quantile of consumption growth and $X_{t+1} = P_{t+1} + D_{t+1}$, where P_{t+1} is the price of the asset at $t+1$ and D_{t+1} is the dividend at $t+1$. In the absence of dividend $X_{t+1} = P_{t+1}$. Given the scale equivalence property of quantiles and that P_t is known at time t, Equation 1 can be written as

$$1 = \delta \left(Q_t^{\alpha} \left(\frac{C_{t+1}}{C_t} \right) \right)^{-\gamma} Q_t^{\alpha} \left(R_{t+1} \right),$$

where R_{t+1} is the gross return at time $t+1$ and it is equal to $\frac{X_{t+1}}{P_t}$.

The above equation can be written in terms of continuously compounded returns, $r_{t+1} = \ln\left(R_{t+1}\right)$, using the invariance property of the quantile and the fact that the logarithm function is monotonically increasing, such as

$$Q_t^{\alpha} \left(r_{t+1} \right) = -\ln\left(\delta\right) + \gamma \ln \left(Q_t^{\alpha} \left(\frac{C_{t+1}}{C_t} \right) \right).$$

Using again the scale equivalence property of quantiles, we obtain

$$Q_t^{\alpha} \left(r_{t+1} \right) = -\ln\left(\delta\right) + \gamma Q_t^{\alpha} \left(\ln \left(\frac{C_{t+1}}{C_t} \right) \right). \tag{2}$$

Equation 2 shows that the αth quantile of stock returns is a linear function of the αth quantile of the consumption growth rate and the slope is given by the coefficient of relative risk-aversion, that is,

$$\frac{dQ_t^{\alpha}\left(r_{t+1}\right)}{dQ_t^{\alpha}\left(g_{t+1}\right)} = \gamma,$$

where $g_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right)$.

We next assume that the consumption growth rate g_{t+1} follows the process in Bansal et al. (2005), that is:

$$g_{t+1} = \mu + \sigma_{c,t}\eta_{t+1}, \tag{3}$$

where $\eta_{t+1} \sim iid\ D\left(0,1\right)$. If the distribution function D belongs to the family of elliptical distributions[5] then we can show that

$$Q_t^{\alpha}(g_{t+1}) = \mu + \sigma_{c,t} D^{-1}(\alpha).$$

For example, if D is a normal distribution, then $D^{-1}(\alpha) = \Phi^{-1}(\alpha)$, where Φ is the cumulative distribution of the normal distribution.

Proposition 1. *If consumption growth rate process g_{t+1} is log-elliptically distributed, following (3), and the pricing is given by (1), then $\forall \alpha \in (0,1)$,*

$$Q_t^{\alpha}(r_{t+1}) = \mu_r(\alpha) + \beta_r(\alpha)\ \sigma_{c,t},$$

with

$$\mu_r(\alpha) = -\ln(\delta) + \gamma\mu,$$

$$\beta_r(\alpha) = \gamma D^{-1}(\alpha), \forall \alpha \in (0,1),$$

where $D^{-1}(\alpha)$ is the αth quantile of the elliptical distribution D.

The above result indicates that the coefficient of the impact of consumption growth volatility varies across the quantiles and has the same direction as the function $D^{-1}(\alpha)$, whereas the constant term $\mu_r(\alpha)$ does not change with quantiles. Thus, the model predicts that for the lower quantiles of the conditional distribution of stock returns, the sign of $\beta(.)$ is negative and for the higher quantiles the sign is positive, suggesting a non-monotonic relationship between stock market returns and the volatility of the consumption growth rate (at the quantiles of the conditional distribution of stock returns). Rossi and Timmermann (2010, 2015) also find a non-monotonic relationship between expected stock returns and conditional market volatility. In particular, Rossi and Timmermann (2010) report that for low and moderate values of the conditional market volatility the relationship between risk and expected returns is positive, but it becomes negative for high levels of the conditional volatility.

Hereinafter, the consumption log-volatility will be modeled as a latent variable that follows an AR(1) process. Equation 3 together with the specification of the log-volatility compose the stochastic volatility model for the consumption growth rate that is presented in section 3.

3. Empirical Methodology

For Cochrane (2008) the main challenge in formulating the complex relationship between macroeconomics (consumption) and asset prices "*is to find the right measure of "bad times" rises in the*

[5]The family of elliptical distributions is a class of symmetric distributions, in which the normal distribution is included, which provides more flexibility in modeling the tails of the consumption growth rate distribution since it accommodates, if necessary, the excess of kurtosis of its distribution. The elliptical distribution assumption has been used in modeling stock market returns (Hodgson et al., 2002; Hamada and Valdez, 2008) and consumption growth rates (Tauchen, 2011; Dionne et al., 2012).

marginal value of wealth, so that we can understand high average returns or low prices as compensation for assets' tendency to pay off poorly in "bad times". In our context, this measure is given by the volatility of consumption growth rate that is assumed to follow a Stochastic Volatility (SV) model (Tauchen, 2011). SV models provide more flexibility in capturing the high kurtosis of the financial time series and the high persistence of the volatility simultaneously (Carnero et al., 2004). These characteristics are also common to consumption growth rates; see Table 2 and Figure 1.

[Table 2 around here]

[Figure 1 around here]

In the context of stock markets, stock returns and volatility are negatively correlated. In this paper, we also consider that the growth rate of consumption can have an asymmetric response to volatility by following an asymmetric SV model. The most popular asymmetric SV models are those proposed by Harvey and Shephard (1996) and So et al. (2002). The first specifies the asymmetric response of the volatility through a correlation between the level and log-volatility disturbances, while the second consists of a Threshold SV model, in which the parameters of the log-volatility equation change depending on whether the past consumption growth rate is positive or negative.

3.1. Consumption growth rate volatility

Using the notation in subsection 2.1, let g_t be the consumption growth rate at time t, $\sigma_{c,t}$ its volatility and η_{t+1} follows an elliptical distribution. The asymmetric SV model considered in this paper is given by

$$g_{t+1} = \mu + \sigma_{c,t}\eta_{t+1}, \qquad t = 1, \dots, T-1$$

$$\ln \sigma^2_{c,t+1} - \omega = \phi(\ln \sigma^2_{c,t} - \omega) + m(\eta_t; \psi) + z_{t+1},$$

where $m(\cdot)$ represents the leverage (asymmetry) in the volatility process that might be discontinuous and depends on η_t, ψ is the vector of parameters to be estimated, ω is related to the marginal variance of the consumption growth rate, and ϕ controls the persistence of volatility. The disturbance z_{t+1} is assumed to be Gaussian with mean zero and variance σ^2_z, and it is independent of η_{t+1}. When looking at the distribution of the consumption growth rate used in the empirical application, we see that the series are leptokurtic; see Table 2. Therefore, we consider the possibility that η_{t+1} follows a Student-t distribution with ν degrees of freedom. In addition, we consider the following functions of m:

Asymmetric Autoregressive SV model (AARSV): $m(\eta_t; \psi) = \gamma_1 \eta_t$

Threshold SV model (TSV): $m(\eta_t) = \delta_1 I(\eta_t < 0)$,

where $I(\cdot)$ is an indicator function that takes value 1 when the argument is true and zero otherwise. Notice that the AARSV specification corresponds to the asymmetric autoregressive SV model of

Harvey and Shephard (1996), and the TSV is a threshold SV model in which only a constant changes depending on the sign of the past consumption growth rate. The latter specification is based on a restricted version of the threshold SV proposed by So et al. (2002) (see Mao et al., 2017, for more details on this restricted TSV model) .

3.2. Quantile regressions

To empirically study the impact of the consumption growth rate volatility, σ_c, on the quantiles of stock returns r, we consider the following quantile regression model:

$$r_t = \theta(\alpha)' w_{t-1} + \varepsilon_{r,t}^{(\alpha)}, \text{ for } \alpha \in (0,1), \tag{4}$$

where $w_{t-1} = \left(1, \sigma_{c,t-1}^2\right)'$, $\theta(\alpha) = (\mu_r(\alpha), \beta_r(\alpha))'$ is an unknown vector of parameters associated with the αth quantile, $\varepsilon_{r,t}^{(\alpha)}$ is an unknown error term also associated with the αth quantile, and it satisfies the unique condition:

$$Q_\alpha\left(\varepsilon_t^{(\alpha)} \mid \sigma_{c,t-1}\right) = 0, \text{ for } \alpha \in (0,1), \tag{5}$$

that is, the αth quantile of the conditional distribution of the error term is equal to zero. In the present paper and for the purposes of estimation and inference, the i.i.d. (independently and identically distributed) assumption of the error terms $\varepsilon_{r,t}^{(\alpha)}$ is not needed. Under assumption (5), the αth quantile of r_t given $\sigma_{c,t-1}$ can be written as:

$$Q_\alpha(r_t \mid \sigma_{c,t-1}) = \theta(\alpha)' w_{t-1}, \text{ for } \alpha \in (0,1).$$

Using Koenker and Bassett Jr (1978), the quantile regression estimator of the vector of parameters $\theta(\alpha)$ in Equation 4 is the solution to the following minimization problem:

$$\hat{\theta}(\alpha) = \underset{\theta(\alpha)}{\arg\min}\left(\sum_{t:r_t>\theta(\alpha)'w_{t-1}} \alpha \mid r_t - \theta(\alpha)' w_{t-1} \mid + \sum_{t:r_t<\theta(\alpha)'w_{t-1}} (1-\alpha) \mid r_t - \theta(\alpha)' w_{t-1} \mid\right). \tag{6}$$

Note that the quantile regression estimator defined in Equation 6 corresponds to minimizing a weighted sum of the absolute errors $\varepsilon_{r,t}^{(\alpha)}$, where the weights α and $(1-\alpha)$ are symmetric and equal to $\frac{1}{2}$ for the median regression case and asymmetric otherwise. This estimator can be obtained as the solution to a linear programming problem. Several algorithms for obtaining a solution to this problem have been proposed in the literature (see Barrodale and Roberts, 1974; Koenker and d'Orey, 1987; Portnoy et al., 1997; Koenker and Hallock, 2001, among others). Under some regularity conditions, the estimator defined in Equation 6 is asymptotically normally distributed with different forms of the asymptotic covariance matrix depending on the model assumptions (Koenker, 2005), that is:

$$\sqrt{T}\left(\hat{\theta}(\alpha) - \theta(\alpha)\right) \overset{d}{\sim} \mathcal{N}(0, \Sigma_\alpha), \text{ for } T \to \infty, \tag{7}$$

where "$\overset{d}{\sim}$" denotes the convergence in distribution, Σ_α is the covariance matrix of the estimator $\hat{\theta}(\alpha)$, and T is the sample size. Thus, tests for statistical significance of the parameter estimates can be constructed using critical values from the normal distribution with asymptotic justification. Computation of an estimator of covariance matrix Σ_α is important in quantile regression analysis. Generally speaking, we distinguish between three classes of estimators for Σ_α: **(1)** methods for estimating the Σ_α in i.i.d. settings; **(2)** methods for estimating Σ_α for independent but not-identical distribution; **(3)** bootstrap resampling methods for both i.i.d. and independent and non identically distributed settings (Koenker, 2005). The estimator most commonly used in practice, and the most efficient in small samples, is based in the design matrix bootstrap (see Buchinsky, 1995). The design matrix bootstrap estimator of Σ_α that was suggested initially by Efron (1979) and Efron (1982) is given by:

$$\hat{\Sigma}^*_\alpha = \frac{T}{B}\sum_{j=1}^{B}\left(\hat{\theta}^*_j(\alpha) - \hat{\theta}(\alpha)\right)\left(\hat{\theta}^*_j(\alpha) - \hat{\theta}(\alpha)\right)', \tag{8}$$

where $\hat{\theta}^*_j(\alpha)$ is the quantile regression estimator of $\theta(\alpha)$ based on the jth bootstrap sample, for $j = 1, ..., B$. The bootstrap samples $\left\{\left(r^*_t, \sigma^*_{c,t}\right)'\right\}_{t=1}^{T}$ are drawn from the empirical joint distribution of r and σ_c. The design matrix bootstrap is the most natural form of bootstrap resampling, and is valid in settings where the error term $\varepsilon^{(\alpha)}_{r,t}$ and regressor $\sigma_{c,t-1}$ are not independent. Using Monte Carlo simulations, Buchinsky (1995) examined six different estimation procedures of the asymptotic covariance matrix Σ_α: design matrix bootstrap; error bootstrapping; order statistic; sigma bootstrap; homoskedastic kernel and heteroskedastic kernel. In his study, the Monte Carlo samples are drawn from real data sets and the estimators are evaluated under various realistic scenarios. His results favor the design bootstrap estimation of Σ_α for the general case. Thus, in our application we use standard errors obtained from the design matrix bootstrap estimator.

4. Empirical results

4.1. Data description

We use U.S. consumption data for the period 1955:I to 2018:I[6], which we obtain from the U.S. national accounts. Following prior work (e.g. Hansen and Singleton, 1983; Yogo, 2006; Feunou et al., 2014), aggregate consumption is measured as the seasonally adjusted real per capita consumption of non-durables plus services. The quarterly real per capita consumption data are taken from the NIPA tables available from the Bureau of Economic Analysis.[7]

Regarding stock market data, we use the return of a stock market index that represents the whole total stock market index.[8] All these data are obtained from Datastream. The starting periods of

[6] The use of data only after 1955 has been common practice because of the effect of the world wars (Boguth and Kuehn, 2013). The period immediately after the war is associated with unusually high durable consumption growth due to the rapid restocking of durable goods.

[7] A detailed description of the data can be found in Tables A.1, A.2 and A.3 in the Appendix.

[8] In the robustness section, we show the results for the stock market indexes S&P 500 and NASDAQ.

these indexes are presented in Table A.3. Furthermore, all variables are measured at constant prices, nominal values are adjusted using the associated Personal Consumption Expenditures (PCE) from the NIPA tables. All variables are also multiplied by 100 and expressed in logarithmic form.

Panel A of Table 2 presents the summary statistics of real stock market return (r) and consumption growth rate (g). The unconditional means of consumption growth rate and stock market indices in the U.S. are positive. The standard deviation of stock returns is higher than the standard deviation of the consumption growth rate (around 13 times). Similar values are reported by Duffee (2005). Furthermore, the series exhibit negative skewness and excess kurtosis.

To strengthen our empirical analysis, we use data from the U.K. The U.K. consumption data are from the U.K. National Statistics Office. We use Final Consumption Expenditure, Households, Household and Non-Profit Institutions Serving Households's Expenditure, which is a long series that starts in the first quarter of 1955. Regarding the stock market returns, we use the returns on the Datastream stock market index. To deflate returns we use the Consumer Price Index All Items (UK_CPI). All series are converted to the same base date. Series are described in Table A.3.

We follow the same procedure that we use for the U.S. data.[9] A summary of the descriptive statistics for the U.K. data is presented in Panel B of Table 2. We observe that the unconditional means of consumption growth rate and stock market indices are positive. U.K. stock markets provide lower returns than U.S. stock markets. The volatility is greater in the U.K. than in the U.S. stock market. Furthermore, the unconditional distributions of both consumption growth rate and stock market indices exhibit excess kurtosis and negative skewness. The sample kurtosis for all variables is greater than three, the kurtosis of a Gaussian random variable. The U.K. consumption growth rate volatility is often higher than the volatility of the U.S. consumption growth rate. Finally, Figure 1 shows the autocorrelation of the squares of consumption growth rates. We see that the autocorrelation functions decay very slowly towards zero, which suggest that consumption growth rates' volatilities are quite persistent.

Next, we use the above data and different volatility models to estimate consumption growth rate volatility.

4.2. Volatility estimation results

Several estimation methods have been proposed in the literature to estimate asymmetric SV models. For example, the importance sampling techniques are used by Shephard and Pitt (1997), Durbin and Koopman (1997) and Richard and Zhang (2007); while other studies are based upon

[9]A strand of work has sought to use different measures of aggregate consumption. Yogo (2006) derives a model with nonseparable consumption of nondurable goods. Jagannathan and Wang (2007) find that measuring consumption growth as the growth in year-on-year fourth quarter consumption explains a substantial portion of cross-sectional variation in returns. Savov (2011) uses garbage as a measure of consumption.

Markov chain Monte Carlo (MCMC) estimators, such as the single-move Metropolis-Hastings algorithm (Jacquier et al., 2004) and the multi-move algorithms by Kim et al. (1998) and Omori et al. (2007). In this paper, we use a Bayesian approach, just another Gibbs sampler (JAGS) to estimate the asymmetric SV alternatives. The advantage of this estimator is its programming simplicity, but similar to OpenBugs the algorithm is single-move, and therefore not simulation-efficient. However, Meyer and Yu (2000) and Yu (2005) find that the simulation-efficiency is clearly not a problem for asymmetry SV models.

Regarding the prior distributions, we use the prior specifications of Meyer and Yu (2000) and Yu (2005). Furthermore, we assume that $\delta_1 \sim N(0.05, 10)$ and $\nu \sim \Gamma(2, 0.1)$, where Γ refers to the Gamma distribution. All priors are assumed to be independent and quite non-informative. For all asymmetric SV models, we use three chains with a burn-in period of 15,000 and a follow-up of 30,000 iterations.

Table 3 reports the estimation results of fitting the two asymmetric SV models to the consumption growth rate. We see that the volatility persistence parameter ϕ is always higher than 0.9, confirming that volatility is quite persistent. The asymmetry parameters (γ_1 and δ_1) are also higher in absolute value for the U.K. than U.S. consumption growth rates, but they are always statistically insignificant. However, excluding the asymmetric effects from the models leads to an increase of the Deviance Information Criterion (DIC). Furthermore, the models are quite similar in terms of goodness-of-fit.

[Table 3 around here]

Finally, Figure 2 shows the estimated volatilities for the U.S. (left panel) and U.K. (right panel) consumption growth rates using the models described in subsection 3.1. The volatility of U.S. consumption growth rate shows a decreasing trend, with occasional spikes such as the one around the 2008 financial crisis, while the pattern of the U.K. volatility is characterized by an increase until the 1980's, and then a decrease with occasional spikes that often coincide with recession periods. Furthermore, the estimated volatility is much larger for the U.K., especially until the 1990's.

[Figure 2 around here]

4.3. Regression estimation results

4.3.1. Mean Estimation

We first re-examine the impact of the consumption growth rate volatility on the conditional *mean* of stock market returns using the following two mean regression models:

$$r_t = \mu_r + \beta_r \sigma_{c,t-1} + \varepsilon_{r,t}, \tag{9}$$

$$r_t = \mu_r + \beta_r \sigma_{c,t-1} + \phi_r r_{t-1} + \varepsilon_{r,t}, \tag{10}$$

where r_t is the stock market return at time t, $\sigma_{c,t-1}$ is the volatility of consumption growth rate at time $t-1$, r_{t-1} is the stock market return at time $t-1$ and $\varepsilon_{r,t}$ is an error term. The coefficients μ_r and α_r are estimated using OLS and the tests for statistical significance of the coefficient estimates are performed using the conventional t-statistic calculated based on standard errors robust to heteroskecasticity and autocorrelation.

The estimation results of the above models using the data described in subsection 4.1 are reported in Table 4. The results obtained using the AARSV and TSV volatility models are shown in Panels A and B of Table 4, respectively. Regarding the U.S. stock market returns and using the AARSV model, Panel A shows that the impact of consumption volatility on stock market returns has a negative sign, but it is statistically not significant. From Panel B (TSV model), we find that the consumption volatility also has a negative impact on U.S. stock market returns that is weakly statistically significant. For the U.K. stock market returns, we find results similar to those obtained using the U.S. stock market returns, except that now the impact of the consumption volatility on market returns is statistically not significant using both AARSV and TSV models. Hence, the mean regression analysis shows that there *is no relationship* between stock returns and the volatility of consumption growth rate.

[Table 4 around here]

All in all and considering only mean regression analyses, we conclude that there is no relationship between stock market returns and the volatility of consumption growth rate. These results raise the question of whether or not the dependence between the two variables exists at other levels (quantiles) of the conditional distribution of stock market returns which we investigate next.

4.3.2. Quantile Estimation

To examine the impact of volatility of the consumption growth rate on the quantiles of stock market returns, we consider the following quantile regression models:

$$r_t = \mu_r(\alpha) + \beta_r(\alpha)\sigma_{c,t-1} + \varepsilon_{r,t}^{(\alpha)}, \tag{11}$$

$$r_t = \mu_r(\alpha) + \beta_r(\alpha)\sigma_{c,t-1} + \phi_r(\alpha)r_{t-1} + \varepsilon_{r,t}^{(\alpha)}, \quad \text{for } \alpha \in (0,1)\,, \tag{12}$$

with $Q_\alpha\left(\varepsilon_{r,t}^{(\alpha)} \mid \sigma_{c,t-1}\right) = 0$. To estimate the covariance matrix Σ_α in (7) we apply the design matrix bootstrap estimator given by (Equation 8).

The empirical results are presented in Panels A and B of Table 5. For the U.S. stock market returns, we see that the effect of consumption growth rate volatility is both economically and statistically significant at the tails of the conditional distribution of market returns, but not at its center. We also find that the volatility of consumption negatively affects the lower quantiles of stock return, but positively affects the upper quantiles. Thus, the quantile regression analysis suggests that the volatility of consumption growth rate has an impact on the tails of the conditional distribution of stock returns.

[Table 5 around here]

Regarding the U.K. stock market returns, the empirical results show conclusions similar to those we obtain using the U.S. stock market returns, that is, in the lower quantiles, the sign of the impact of the consumption volatility is negative and statistically significant, while at the upper quantiles the sign is statistically significant and positive.

Results from studies that address the relationship between returns and conditional market volatility also find a non-monotonic relationship between volatility and stock returns. Rossi and Timmermann (2010) find for low and medium values of the conditional volatility, a positive relationship between risk and expected returns, while for high levels of volatility this relationship might be negative.[10] Ghysels et al. (2014) also find strong evidence of regime changes in the risk-return relationship. The non-monotonicity in the risk return relationship is also developed in the following works. Whitelaw (2000) proposes a non-linear relationship between returns and risk based on an equilibrium framework that includes a complex, non-linear, and time-varying relationship between expected return and volatility. Similarly, Mayfield (2004) employs a methodology in which states of the market are defined by volatility regimes, and influence the risk-return trade-off to these different states. Differently, Veronesi (2000) states that the covariance between consumption and returns is small or even negative for high levels of risk aversion leading to a decrease of the risk premium. Abel (1988) and Backus and Gregory (1993) develop theoretical models that also support a negative risk-return relationship, arguing that this is consistent with the time-varying dynamics of the economy because both variables are a function of the state of the economy.

Another branch of the literature finds a link between the volatility of macroeconomic fundamentals and of the stock market. An increase in the volatility of the fundamentals translates into an increase of stock market volatility; see Diebold and Yilmaz (2010) for details. Jaccard (2018) argues that the non-monotonic relationship is straightened when the volatility of consumption is used because in recessions in which the consumption rate decreases substantially, risk aversion rises and uncertainty about the future plays an important role in the investment decisions.

Another possible reason for a negative relationship between future stock returns and volatility consumption at low quantiles is the relationship between the elasticity of intertemporal substitution (EIS) and the inverse of the coefficient of risk aversion. If the latter depends on the business cycle (as it seems to be the case), the EIS can be larger than the inverse of the coefficient of risk aversion, which makes the investor demand a negative price risk for changes to the conditional volatility of consumption; see Boguth and Kuehn (2013).

Our results provide new empirical evidence about the joint dynamics of stock returns and volatility

[10]In the working paper version, Rossi and Timmermann (2015) argue that expected stock returns and conditional variance are countercyclical, that is, they tend to be higher during recessions but the effect on the conditional variance dominates the positive increase in expected stock returns, which makes investments in stock markets less attractive during the recessions, and consequently leads to a decrease in the demand for stock assets.

of consumption growth rate. In particular, they show that the compensation for risk depends on the state of financial markets.

4.4. Robustness

For robustness, we provide additional estimation results that are obtained using other U.S. and U.K. stock markets returns and other deflators.

First, we re-estimate the regressions at the mean; see equations (9)-(10). Regarding the U.S. economy, we obtain data for two other stock market indexes such as the S&P 500 and the NASDAQ. Table A.5 shows that results do not change. We also use the Consumer Price Index as a deflator. Table A.6 shows that the results using this deflator are similar to those obtained with the CPE deflator. We proceed similarly for the U.K., we re-estimate using other stock market indexes, such as the MSCI UK and the FTSE. Table A.7 reports that the impact of the consumption growth rate volatility on stock market returns is generally statistically not significant. The robustness results confirm our conclusions of subsubsection 4.3.1.

Second, we re-estimate the main regressions for the quantiles of the conditional distribution of stock market returns; see equations (11)-(12). Tables A.8-A.10 report the main results. We see that for the U.S. stock markets, regardless of the deflator used, the impact of the consumption volatility is economically and statistically significant at the tails of the conditional distribution of all U.S. market returns under consideration, but not at its center. In particular, we find that the volatility of consumption affects negatively the lower quantiles and positively the upper quantiles of stock market returns distribution. Similar conclusions are reached for the U.K. stock markets; see Table A.10. Thus, once again the quantile regression analysis suggests that the volatility of consumption growth rate affects the tails of the conditional distribution of stock market returns differently, which suggests that the compensation for risk depends on the state of financial markets.

5. Conclusion

In this paper, we propose an asset pricing model based on a quantile Euler equation that expresses the asset price as a function of the quantiles of the consumption growth, asset payoff, stochastic discount factor, and relative risk aversion. Assuming that the consumption growth rate process is log-elliptically distributed, we show that the quantiles of asset returns can be expressed as affine functions of the consumption volatility.

Using data on the U.S. and U.K. stock market prices, empirical evidence validates our theoretical results and confirms that the consumption growth rate volatility is a driving factor of the distribution of stock market returns, especially at its tails. Furthermore, the results also show that the sign of the impact of consumption volatility is different across quantiles. A negative sign is obtained for the lower quantiles while the upper quantiles report a positive sign. These results are consistent with the literature on time-varying risk premia.

Overall our paper contributes to the literature on the theoretical link between asset prices and economic fluctuations. The results are that aggregate economic uncertainty has meaningful effects on asset valuations, and that the consumption growth rate volatility is a driving factor of the quantiles of stock market returns.

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Figures and Tables

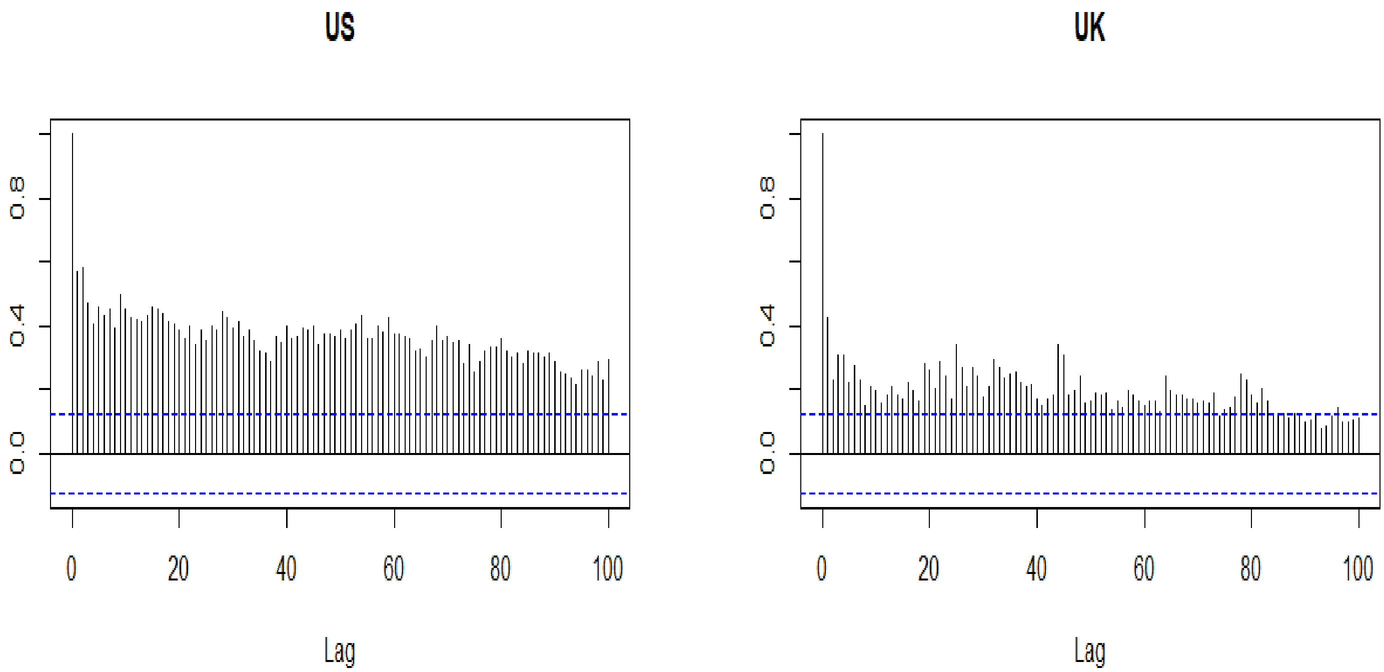

Figure 1: Autocorrelation functions of the squares of the U.S. and U.K. consumption growth rates (g).

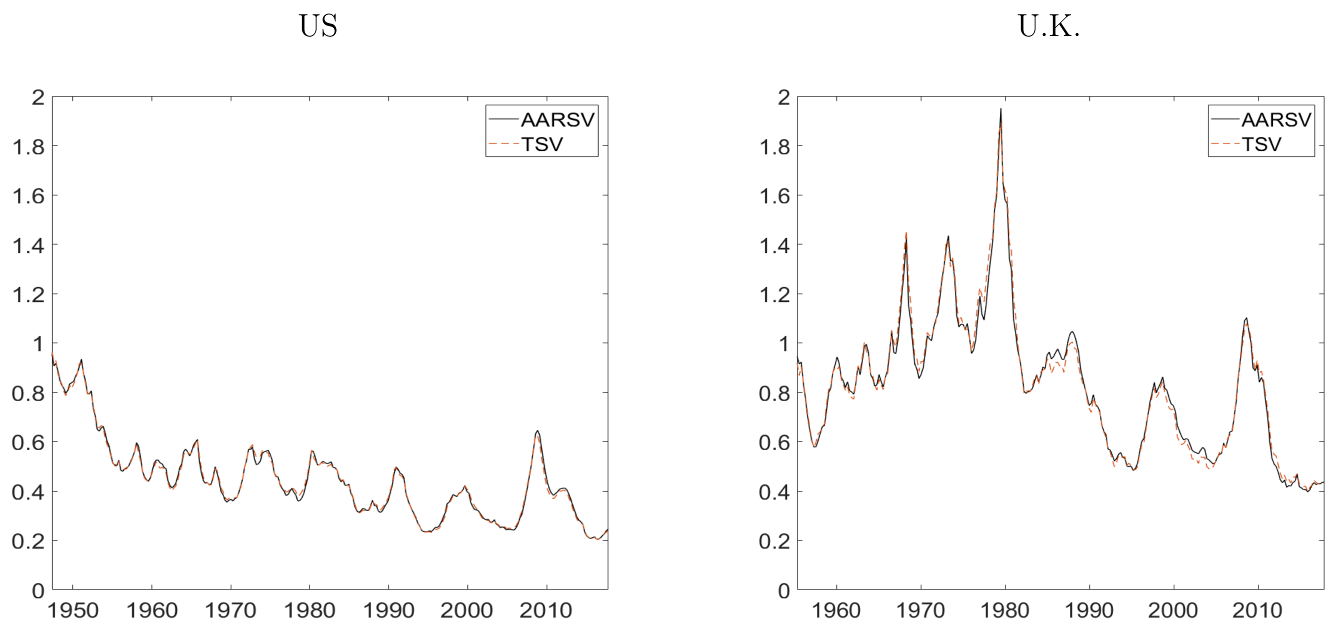

Figure 2: Estimated consumption growth rate volatilities using AARSV and TSV models.

Table 1: Data Labels

Variable	Description	U.S.	U.K.
g	Growth rate of consumption (quarterly variation)	For the U.S. , Consumption data is taken from Table 7.1. Selected Per Capita Product and Income Series in Current and Chained Dollars. We sum Nondurable goods (A796RC0) + Services (A797RC0). Values are deflated with PCE: Personal Consumption Expenditures index (the base year is 2009)	For the U.K., U.K. Consumption data is taken from the U.K. Office National Statistics. It is Final Consumption Expenditure, Households, Household and Non-Profit Institutions Serving Households's Expenditure, British Pound Sterling, Values are deflated with Consumer Price Index All Items (CPI_UK)
r	Stock Market Returns (quarterly variation)	Log returns U.S. stock market index from Datastream. Values are deflated with PCE: Personal Consumption Expenditures index (the base year is 2009)	Log returns U.K. stock market index from Datastream Values are deflated with Consumer Price Index All Items (CPI_UK)

Table 2: Descriptive Statistics of real returns

	Panel A: U.S. Data			
	Mean	Std Dev	Skewness	Kurtosis
Growth rate of consumption (g)	0.531	0.526	-0.461	4.427
stock market returns (r)	0.986	8.528	-0.965	5.088
	Panel B: U.K. Data			
Growth rate of consumption (g)	0.621	1.012	-0.182	7.586
Stock Market returns (r)	0.6291	9.454	-0.339	6.458

Note: Quarterly real growth rate of consumption (g) and real return of the stock market indexes (r). Real rates are multiplied by 100. US data is deflated with PCE and U.K. data is deflated with CPI_UK. Sample period is 1955:I to 2018:I.

Table 3: Parameter estimates (left panel) and DIC values (right panel) of the U.S. and U.K. consumption growth rates

	Panel A: U.S. real growth rate of consumption						
Models	ω	ϕ	δ_1	γ_1	σ_z^2	ν	DIC
TSV	-0.167* (0.100)	0.923*** (0.040)	0.047 (0.097)	–	0.107 (0.068)	23.868* (13.614)	479.6
AARSV	-0.151** (0.079)	0.918*** (0.044)	–	-0.045 (0.052)	0.107 (0.071)	24.751* (14.165)	499.0
	Panel B: U.K. real growth rate of consumption						
Models	ω	ϕ	δ_1	γ_1	σ_z^2	ν	DIC
TSV	0.020 (0.053)	0.929*** (0.042)	-0.133 (0.105)	–	0.099* (0.060)	25.348* (13.894)	694.5
AARSV	-0.042 (0.030)	0.928*** (0.042)	–	0.070 (0.051)	0.097* (0.057)	24.762* (13.696)	699.9

Note: Posterior standard errors of parameter estimates are reported in parentheses. TSV stands for threshold stochastic volatility and AARSV stands for asymmetric autoregressive stochastic volatility model. ν corresponds to the degrees of freedom of the Student-t distribution.*** statistically significant at 1%, ** at 5% and * at 10%. DIC stands for Deviance Information Criteria.

Table 4: Mean Estimation of CCAPM for the U.S. and U.K. Stock Market Returns

	U.S. Stock Market				U.K. Stock Market			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: volatility AARSV								
Const.	5.126**	(2.436)	4.620*	(2.449)	2.589*	(1.551)	2.388	(1.568)
$\sigma_{c,t-1}$	-11.109	(7.103)	-9.779	(7.061)	-2.342	(2.078)	-2.151	(2.059)
r_{t-1}			0.055	(0.085)			0.094	(0.096)
$R^2(\%)$	1.840		1.950		0.550		1.420	
Panel B: volatility TSV								
Const.	5.981**	-(2.403)	5.458**	(2.449)	2.577	(1.525)	2.392	(1.540)
$\sigma_{c,t-1}$	-13.424*	(6.991)	-12.036*	(7.037)	-2.32	(2.026)	-2.148	(2.005)
r_{t-1}			0.048	(0.085)			0.094	(0.096)
$R^2(\%)$	2.600		2.580		0.570		1.450	

Note: Estimation of equations (9)–(10). The dependent variables are log of real returns of the U.S. and U.K. stock market indexes. $\sigma_{c,t-1}$ is the coefficient of the volatility. r_{t-1} is the coefficient of lagged returns. In Panel A, U.S. and U.K. volatilities of consumption growth rates are computed as Asymmetric AutoRegressive stochastic volatility (AARSV). In Panel B, U.S. and U.K. volatilities of consumption growth rates are computed as Threshold Stochastic Volatility (TSV). Impact of volatility (AARSV, TSV) of consumption growth rate on the *mean* of stock market return.

Table 5: Quantile Estimation of CCAPM for the U.S. and U.K. Stock Market

	US Stock market				UK Stock market			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: volatility AARSV								
10**th Quantile**								
Const.	0.035	(4.062)	-3.759	(3.474)	0.035	(4.062)	-3.759	(3.474)
$\sigma_{c,t-1}$	-14.014***	(4.891)	-8.725**	(3.850)	-14.014***	(4.891)	-8.725**	(3.850)
r_{t-1}			0.400**	(0.165)			0.400**	(0.165)
$R^2(\%)$	4.680		9.440		4.680		9.440	
25**th Quantile**								
Const.	4.947**	(2.209)	4.274	(2.708)	4.947**	(2.209)	4.274	(2.708)
$\sigma_{c,t-1}$	-9.977***	(2.661)	-10.412***	(3.155)	-9.977***	(2.661)	-10.412***	(3.155)
r_{t-1}			0.238*	(0.136)			0.238*	(0.136)
$R^2(\%)$	4.430		6.540		4.430		6.540	
50**th Quantile**								
Const.	1.981	(1.486)	2.018	(1.508)	1.981	(1.486)	2.018	(1.508)
$\sigma_{c,t-1}$	-0.439	(2.181)	-0.196	(2.263)	-0.439	(2.181)	-0.196	(2.263)
r_{t-1}			0.040	(0.085)			0.040	(0.085)
$R^2(\%)$	0.080		0.230		0.080		(0.230)	
75**th Quantile**								
Const.	3.833**	(1.475)	3.592**	(1.519)	3.833**	(1.475)	3.592**	(1.519)
$\sigma_{c,t-1}$	2.366	(2.030)	2.647	(2.093)	2.366	(2.030)	2.647	(2.093)
r_{t-1}			0.035	(0.097)			0.035	(0.097)
$R^2(\%)$	0.890		0.990		0.890		(0.990)	
90**th Quantile**								
Const.	4.134*	(2.342)	3.844	(2.624)	4.134*	(2.342)	3.844	(2.624)
$\sigma_{c,t-1}$	7.536**	(2.985)	7.807**	(3.655)	7.536**	(2.985)	7.807**	(3.655)
r_{t-1}			0.023	(0.111)			0.023	(0.111)
$R^2(\%)$	4.180		4.250		4.180		(4.250)	

Note: Estimation of equations (11)–(12). The dependent variables are log real returns of the stock market indexes. Volatility of consumption growth rates are computed as Asymmetric AutoRegressive stochastic volatility. Impact of volatility (PCE, AARSV) of consumption growth rate on *Quantiles* of stock market returns.

	US Stock market				UK Stock market			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel B: Volatility TSV								
10**th Quantile**								
Const.	7.375	(6.558)	6.196	(6.165)	-1.476	(4.329)	-3.639	(3.753)
$\sigma_{c,t-1}$	-40.667**	(17.958)	-42.251**	(17.707)	-12.136**	(5.117)	-8.643**	(3.928)
r_{t-1}			0.408	(0.311)			0.416***	(0.155)
$R^2(\%)$	5.710		7.680		4.770		(9.330)	
25**th Quantile**								
Const.	6.976***	(1.673)	6.099**	(2.498)	3.908*	(2.078)	3.998	(2.550)
$\sigma_{c,t-1}$	-27.610***	(4.218)	-24.868***	(6.569)	-9.082***	(2.510)	-10.490***	(2.998)
r_{t-1}			0.209	(0.168)			0.235*	(0.130)
$R^2(\%)$	9.400		9.170		4.570		(6.730)	
50**th Quantile**								
Const.	5.655*	(2.873)	5.076*	(2.662)	1.988	(1.470)	2.022	(1.587)
$\sigma_{c,t-1}$	-11.023	(9.725)	-9.024	(9.198)	-0.457	(2.202)	-0.204	(2.299)
r_{t-1}			0.025	(0.100)			0.040	(0.087)
$R^2(\%)$	0.710		0.540		0.080		(0.230)	
75**th Quantile**								
Const.	3.160*	(1.909)	3.128	(2.064)	3.422**	(1.414)	3.838***	(1.365)
$\sigma_{c,t-1}$	9.284*	(5.610)	9.380	(6.032)	2.926	(1.912)	2.353	(1.856)
r_{t-1}			-0.001	(0.080)			0.030	(0.092)
$R^2(\%)$	1.280		1.430		0.920		(1.000)	
90**th Quantile**								
Const.	5.553*	(2.976)	3.109	(2.854)	3.838	(2.563)	4.200*	(2.529)
$\sigma_{c,t-1}$	11.722	(7.177)	19.708***	(7.199)	7.657**	(3.055)	7.229**	(3.247)
r_{t-1}			-0.093	(0.076)			0.019	(0.109)
$R^2(\%)$	2.980		3.510		3.670		(3.690)	

Note: Estimation of equations (11)–(12). The dependent variables are log real returns of the U.S. and U.K. stock market indexes. Volatility of consumption growth rates are computed as Threshold stochastic

Appendix

A.1.1. Proof of Propositions

Proof of Proposition 1. Since the consumption growth rate is log-elliptically distributed, we have

$$Q_t^{\alpha}\left(g_{t+1}\right) = Q_t^{\alpha}\left(\ln\left(\frac{C_{t+1}}{C_t}\right)\right) = \mu + \sigma_{c,t} D^{-1}\left(\alpha\right). \tag{13}$$

Thus, using Equation (2) and the result in (13), we get

$$\begin{aligned} Q_t^{\alpha}\left(r_{t+1}\right) &= -\ln\left(\delta\right) + \gamma Q_t^{\alpha}\left(\ln\left(\frac{C_{t+1}}{C_t}\right)\right) \\ &= -\ln\left(\delta\right) + \gamma\left(\mu + \sigma_{c,t} D^{-1}\left(\alpha\right)\right) \\ &= -\ln\left(\delta\right) + \gamma\mu + \gamma\sigma_{c,t} D^{-1}\left(\alpha\right) \\ &= \mu_r^{\alpha} + \beta_r \sigma_{c,t}, \end{aligned}$$

where $\mu_r^{\alpha} = -\ln\left(\delta\right) + \gamma\mu$ and $\beta_r = \gamma D^{-1}\left(\alpha\right)$. ■

A.1.2. Data Description

U.S. data.

- U.S. Consumption data is taken from Table 7.1. Selected Per Capita Product and Income Series in Current and Chained Dollars. We sum Nondurable goods (A796RC0) + Services (A797RC0). More details can be seen in Table A.1.

- U.S. market indexes are obtained from Datastream. Stock market series are presented in Table A.3. Series are nominal and were then deflated using one of the deflators and then compute the rate of change in logs.

- We have used as deflators PCE: Personal Consumption Expenditures index (the base year is 2009) and CPI:Consumer Price Index for All Urban Consumers (the base date is 1983). More details can be seen in Table A.1.

U.K. data.

- U.K. Consumption data is taken from the U.K. Office National Statistics. It is Final Consumption Expenditure, Households, Household and Non-Profit Institutions Serving Households's Expenditure, British Pound Sterling, 2015 Chained Prices.

- We withdraw data from several U.K. market indexes from Datastream. Series are presented in Table A.3. All values are nominal. Deflators are applied on nominal series.

- U.K. data is deflated with Consumer Price Index All Items (CPI_UK).

All series are converted to the same base date. We then compute the rate of change in logs.

Table A.1: U.S. Consumption and Deflators Series Data

Series ID:	Title:	Source:	Units	Frequency	Seasonal Adjustment
Consumption Series					
Nominal series					
A796RC0	Personal consumption expenditures per capita Goods: Nondurable goods	US Bureau of Economic Analysis	Nominal	Quarterly	Seasonally adjusted Annual rates
A797RC0	Personal consumption expenditures per capita Services	US Bureau of Economic Analysis	Nominal	Quarterly	Seasonally adjusted Annual rates
Deflator Series					
CPIAUCSL	Consumer Price Index for All Urban Consumers All items	US Bureau of Labor Statistics/FRED	Average (aggregation) Index $1982-1984=100$	Quarterly	Seasonally adjusted
DPCERD3Q086SBEA	Personal consumption expenditures Implicit price deflator		Index $2009=100$	Quarterly	Seasonally adjusted

Table A.2: UK Consumption and Deflator Series Data

Series ID:	**Title:**	**Source:**	**Units**	**Frequency**	**Seasonal Adjustment**	description
Consumption Series						
UKCNPER.D	UK CONSUMER SPENDING CONA	UK ONS	Constant Prices	Quarterly	Seasonally Adjusted	UK EA, Final Consumption Expenditure Household and Non-Profit Institutions serving Households's Expenditure £, 2015 Chained Prices
UKCNPER.B	UK CONSUMER SPENDING CURA	UK ONS	Current Prices	Quarterly	Seasonally Adjusted	UK EA, Final Consumption Expenditure Households, Consumer Spending £
Deflator						
UKQCP009F	UK All Items Consumer Price Index All Items		Index, 2010 = 100			

Table A.3: Stock Market Indexes data

Variable Name	Series ID	Title	Source	Starting Date	Frequency	Units
U.S. Data						
US Stock Market	TOTMKUS	US-DS Market - price index	Datastream	1973:Q1	Quarterly	nominal
S& P 500	S&PCOMP	S&P 500 COMPOSITE - price index	Datastream	1964:Q1	Quarterly	nominal
NASDAQ	NASCOMP	NASDAQ COMPOSITE - price index	Datastream	1971:Q2	Quarterly	nominal
U.K. Data						
UK Stock Market	TOTMKUK	UK-DS Market - price index	Datastream	1965:Q1	Quarterly	nominal
MSCI UK	MSUTDKL	MSCI UK - price index	Datastream	1970:Q1	Quarterly	nominal
FTSE	FTALLSH	FTSE ALL SHARE - price index	Datastream	1962:Q2	Quarterly	nominal

Table A.4: Descriptive Statistics of real stock market returns (r)

	Panel A: US Data 1955:I to 2018:I			
	Mean	Std Dev	Skewness	Kurtosis
S&P 500	0.807	8.064	-0.947	5.030
NASDAQ	1.379	11.948	-0.664	4.684
	Panel B: U.K. Data 1955:I to 2018:I			
UK MSCI	0.313	9.636	-0.308	6.447
FTSE	0.431	10.18	0.403	11.755

Note: Real return of the market portfolio (r). Real rates are multiplied by 100. US data is deflated with PCE and U.K. data is deflated with CPI.

A.2. Internet Appendix: Tables not to be presented with the paper

A.2.1. Robustness

A.2.1.1. Estimation at the mean

Table A.5: Robustness: Mean Estimation of CCAPM for the S&P500 and NASDAQ

	S&P 500				NASDAQ			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: Volatility AARSV								
Const.	4.023**	(1.980)	3.818*	(1.979)	4.075	(2.994)	3.851	(3.117)
$\sigma_{c,t-1}$	-8.251	(5.354)	-7.932	(5.279)	-7.118	(8.197)	-6.667	(8.317)
r_{t-1}			0.068	(0.080)			0.039	(0.098)
$R^2(\%)$	1.200		1.760		0.400		0.560	
Panel B: Volatility TSV								
Const.	2.009	(6.523)	4.593**	(1.955)	5.235*	(2.965)	5.008	(3.113)
$\sigma_{c,t-1}$	-26.787*	(16.127)	-9.905*	(5.178)	-10.189	(8.134)	-9.714	(8.305)
r_{t-1}			0.062	(0.080)			0.035	(0.098)
$R^2(\%)$	1.830		2.330		0.810		0.930	

Note: Estimation of equations (9)–(10). The dependent variables are log returns of S&P 500 and NASDAQ. Nominal values are deflated with CPE. $\sigma_{c,t-1}$ is the coefficient of the volatility. r_{t-1} is the coefficient of lagged returns. In Panel A, U.S. volatility of consumption growth rate is computed as Asymmetric AutoRegressive stochastic volatility (AARSV). In Panel B U.S. volatility of consumption growth rate is computed as Threshold stochastic volatility (TSV).

Table A.6: Robustness: Mean Estimation of CCAPM for the S&P500, Total Stock Market, and NASDAQ

	S&P500				**Total Stock market**				**NASDAQ**			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: Volatility ARSV												
Const.	3.004	(1.824)	2.776	(1.876)	3.693*	(1.902)	3.361*	(1.977)	2.998	(2.929)	2.826	(3.062)
$\sigma_{c,t-1}$	-4.383	(3.740)	-4.090	(3.754)	-5.589	(4.025)	-4.931	(4.048)	-2.989	(5.729)	-2.753	(5.833)
r_{t-1}			2.530	(1.790)			0.066	(0.086)			0.045	(0.098)
$R^2(\%)$	0.650		1.300		1.080		1.440		0.150		0.360	
Panel B: Volatility TSV												
Const.	2.744	(1.745)	0.078	(0.081)	3.439*	(1.818)	3.127*	(1.883)	2.647	(2.771)	2.483	(2.892)
$\sigma_{c,t-1}$	-4.383	(3.740)	-4.090	(3.754)	-5.589	(4.025)	-4.931	(4.048)	-2.989	(5.729)	-2.753	(5.833)
r_{t-1}			2.530	(1.790)			0.066	(0.086)			0.045	(0.098)
$R^2(\%)$	0.650		1.300		1.080		1.440		0.150		0.360	

Note: Estimation of equations (9)–(10). The dependent variables are log returns of the indexes S&P 500, Total stock market of Datastream and NASDAQ. Nominal values are deflated with CPI. $\sigma_{c,t-1}$ is the coefficient of the volatility. r_{t-1} is the coefficient of lagged returns. In Panel A, U.S. volatility of consumption growth rate is computed as Asymmetric AutoRegressive stochastic volatility (AARSV). In Panel B, U.S. volatility of consumption growth rate is computed as Threshold stochastic volatility (TSV).

Table A.7: Robustness: Mean Estimation of CCAPM for the U.K. MSCI Index, and FTSE

	MSCI UK				**FTSE**			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: Volatility AARSV								
Const.	3.343**	(1.563)	3.151**	(1.572)	2.594	(1.603)	2.667*	(1.537)
$\sigma_{c,t-1}$	-3.701*	(2.167)	-3.442	(2.115)	-2.575	(2.202)	-2.634	(2.071)
r_{t-1}			0.069	(0.102)			-0.022	(0.134)
$R^2(\%)$	1.370		1.830		0.550		0.600	
Panel B: Volatility TSV								
Const.	3.411**	(1.549)	3.208**	(1.556)	2.351	(1.575)	2.422	(1.504)
$\sigma_{c,t-1}$	-3.782*	(2.137)	-3.511*	(2.084)	-2.279	(2.158)	-2.335	(2.024)
r_{t-1}			0.069	(0.102)			-0.022	(0.134)
$R^2(\%)$	1.500		1.940		0.460		0.500	

Note: Estimation of equations (9)–(10). The dependent variables are log returns of the indexes U.K. MSCI, Total stock market of Datastream and FTSE all share. Nominal values are deflated with CPI_UK. $\sigma_{c,t-1}$ is the coefficient of the volatility. r_{t-1} is the coefficient of lagged returns In Panel A Volatility of consumption growth rates is computed as Asymmetric AutoRegressive stochastic volatility (AARSV). In Panel B Volatility of consumption growth rates is computed as Threshold stochastic volatility (TSV).

A.2.1.2. Quantiles estimation for the US

	S&P 500				NASDAQ			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel A: Volatility AARSV								
10th Quantile								
Const.	2.066	(6.412)	-0.135	(5.943)	4.140	(6.160)	0.539	(6.384)
$\sigma_{c,t-1}$	-26.899	(16.411)	-21.969	(15.877)	-45.250***	(14.411)	-37.832***	(13.731)
r_{t-1}			0.426	(0.261)			0.348	(0.217)
$R^2(\%)$	2.570		4.580		6.050		8.360	
25th Quantile								
Const.	5.823***	(1.695)	2.911	(2.617)	4.600	(5.162)	3.319	(5.303)
$\sigma_{c,t-1}$	-24.299***	(5.120)	-16.255**	(6.698)	-24.217	(15.400)	-21.457	(15.851)
r_{t-1}			0.275**	(0.125)			0.088	(0.174)
$R^2(\%)$	5.670		7.730		2.010		2.210	
50th Quantile								
Const.	3.304*	(5.434)	2.794	(1.718)	3.167	(2.820)	5.018*	(2.983)
$\sigma_{c,t-1}$	-3.521	(1.789)	-2.641	(5.512)	-2.535	(7.733)	-5.174	(8.064)
r_{t-1}			0.037	(0.123)			-0.075	(0.105)
$R^2(\%)$	0.230		0.320		0.070		0.330	
75th Quantile								
Const.	4.212**	(1.638)	4.134**	(1.823)	5.515**	(2.471)	5.397*	(2.835)
$\sigma_{c,t-1}$	4.480	(4.627)	4.544	(5.200)	8.113	(7.836)	8.007	(8.724)
r_{t-1}			0.022	(0.071)			0.011	(0.092)
$R^2(\%)$	0.230		0.340		0.650		0.690	
90th Quantile								
Const.	4.929**	(2.414)	6.090***	(2.330)	4.514	(4.379)	7.197*	(4.260)
$\sigma_{c,t-1}$	11.804**	(5.710)	8.915	(5.663)	21.726**	(10.773)	21.306*	(11.283)
r_{t-1}			-0.058	(0.076)			-0.135	(0.120)
$R^2(\%)$	2.180		2.720		2.120		2.870	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes S&P 500 and NASDAQ. Nominal values are deflated with PCE. Volatility of consumption growth rates are computed

	S&P 500				NASDAQ			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
				Panel B: Volatility TSV				
10th Quantile								
Const.	2.066	(6.412)	0.497	(5.620)	6.231	(5.889)	3.486	(6.272)
$\sigma_{c,t-1}$	-26.899	(16.411)	-23.758	(15.063)	-49.792***	(13.914)	-45.222***	(14.093)
r_{t-1}			0.417*	(0.251)			0.201	(0.204)
$R^2(\%)$	2.570		4.870		6.680		8.890	
25th Quantile								
Const.	5.823	(1.695)	3.814	(2.459)	7.058	(5.044)	6.454	(5.045)
$\sigma_{c,t-1}$	-24.299***	(5.120)	-17.958***	(6.179)	-29.895**	(14.962)	-27.980*	(15.023)
r_{t-1}			0.251*	(0.128)			0.029	(0.175)
$R^2(\%)$	5.670		8.340		2.640		2.700	
50th Quantile								
Const.	3.304**	(1.789)	4.094**	(1.873)	4.428	(2.927)	5.636**	(2.781)
$\sigma_{c,t-1}$	-3.521	(5.434)	-6.724	(5.920)	-5.561	(8.068)	-6.515	(7.425)
r_{t-1}			0.036	(0.120)			-0.076	(0.101)
$R^2(\%)$	0.230		0.530		0.210		0.480	
75th Quantile								
Const.	4.212**	(1.638)	4.801***	(1.796)	6.795***	(2.314)	5.730**	(2.759)
$\sigma_{c,t-1}$	4.480	(4.627)	2.897	(5.063)	4.521	(7.363)	6.687	(8.720)
r_{t-1}			0.013	(0.076)			0.022	(0.093)
$R^2(\%)$	0.230		0.110		0.350		0.390	
90th Quantile								
Const.	4.929	(2.414)	5.743**	(2.576)	7.677*	(4.590)	9.640**	(4.464)
$\sigma_{c,t-1}$	11.804**	(5.710)	9.853	(6.395)	15.211	(11.111)	15.502	(11.787)
r_{t-1}			-0.065	(0.077)			-0.110	(0.118)
$R^2(\%)$	2.180		2.300		1.740		2.320	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes S&P 500 and NASDAQ. Nominal values are deflated with PCE. Volatility of consumption growth rates are computed

Table A.9: Robustness: Quantile Estimation of CCAPM for the S&P 500, U.S. Total stock market and NASDAQ

	S&P500				**Total Stock market**				**NASDAQ**			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
					Panel A: Volatility AARSV							
10th Quantile												
Const.	-5.380	(5.904)	-6.726	(5.050)	-5.847	(7.388)	-7.421	(7.271)	-7.605	(7.900)	-10.804	(7.706)
$\sigma_{c,t-1}$	-7.582	(9.897)	-4.022	(9.745)	-6.944	(13.920)	-3.142	(15.379)	-12.786	(14.649)	-8.537	(14.282)
r_{t-1}			0.339	(0.216)			0.258	(0.277)			0.380*	(0.214)
$R^2(\%)$	0.950		2.980		0.940		2.610		1.490		4.710	
25th Quantile												
Const.	2.988	(1.818)	0.902	(2.032)	3.492	(2.188)	2.746	(2.229)	2.744	(4.410)	0.251	(4.986)
$\sigma_{c,t-1}$	-13.155***	(4.034)	-9.786**	(3.998)	-14.334***	(4.974)	-12.203***	(4.205)	-15.057	(9.652)	-9.894	(10.674)
r_{t-1}			0.317***	(0.094)			0.237**	(0.118)			0.087	(0.162)
$R^2(\%)$	3.530		7.390		4.860		7.520		1.060		1.300	
50th Quantile												
Const.	3.280*	(1.956)	4.194*	(2.178)	4.704**	(2.248)	4.718*	(2.428)	3.011	(2.962)	3.895	(3.220)
$\sigma_{c,t-1}$	-3.431	(4.705)	-6.210	(5.079)	-6.910	(6.101)	-6.945	(6.238)	-1.728	(6.891)	-2.645	(6.846)
r_{t-1}			0.042	(0.106)			0.005	(0.098)			-0.074	(0.101)
$R^2(\%)$	0.320		0.490		0.800		0.700		0.010		0.250	
75th Quantile												
Const.	4.179**	(1.863)	4.132**	(1.990)	3.801**	(1.823)	3.685*	(1.940)	6.458**	(2.798)	6.546**	(2.965)
$\sigma_{c,t-1}$	3.480	(4.048)	3.343	(4.370)	5.325	(4.204)	5.577	(4.364)	3.969	(6.885)	3.760	(7.034)
r_{t-1}			0.017	(0.073)			0.024	(0.081)			-0.015	(0.094)
$R^2(\%)$	0.140		0.190		0.420		0.560		0.290		0.310	
90th Quantile												
Const.	6.933***	(2.491)	6.950***	(2.541)	6.324**	(3.182)	7.026**	(3.023)	12.164***	(4.303)	13.327***	(4.050)
$\sigma_{c,t-1}$	5.576	(4.562)	5.738	(4.812)	8.139	(5.565)	7.315	(5.450)	3.553	(7.309)	4.626	(7.248)
r_{t-1}			-0.047	(0.082)			-0.061	(0.075)			-0.103	(0.110)
$R^2(\%)$	0.840		1.320		1.060		1.700		0.650		1.310	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes S&P 500, Total stock market of Datastream and NASDAQ. Nominal values are deflated with CPI. Volatility of consumption growth rates is computed as Asymmetric AutoRegressive stochastic volatility.

Table A.9: Robustness: Quantile Estimation of CCAPM for the S&P 500, U.S. Total stock market and NASDAQ (continued)

	S&P500				Total Stock market				NASDAQ			
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
Panel B: Volatility TSV												
10th Quantile												
Const.	-4.852	(6.419)	-6.575	(5.162)	-6.073	(7.531)	-7.324	(6.722)	-7.330	(8.419)	-10.626	(7.820)
$\sigma_{c,t-1}$	-8.451	(10.941)	-4.353	(9.846)	-6.887	(13.961)	-3.336	(14.266)	-13.512	(15.483)	-9.014	(13.927)
r_{t-1}			0.343*	(0.206)			0.260	(0.252)			0.380*	(0.207)
$R^2(\%)$	0.850		2.980		0.840		2.600		1.470		4.670	
25th Quantile												
Const.	3.520*	(1.825)	1.115	(2.075)	3.753*	(2.180)	2.879	(2.359)	2.728	(4.767)	0.481	(4.915)
$\sigma_{c,t-1}$	-14.635***	(4.039)	-10.150**	(4.104)	-15.307***	(4.979)	-12.825***	(4.459)	-14.700	(10.259)	-10.499	(10.626)
r_{t-1}			0.309***	(0.095)			0.251**	(0.120)			0.092	(0.165)
$R^2(\%)$	3.660		7.410		4.890		7.490		1.150		1.350	
50th Quantile												
Const.	4.803**	(2.153)	4.455*	(2.283)	5.037**	(2.396)	5.015**	(2.463)	3.365	(3.002)	5.652*	(3.330)
$\sigma_{c,t-1}$	-7.234	(5.205)	-6.792	(5.245)	-7.784	(6.509)	-7.769	(6.368)	-2.362	(6.866)	-5.984	(7.196)
r_{t-1}			0.044	(0.103)			0.003	(0.101)			-0.075	(0.105)
$R^2(\%)$	0.470		0.620		0.940		0.840		0.040		0.330	
75th Quantile												
Const.	5.398***	(1.928)	4.479**	(2.099)	3.801**	(1.878)	3.550*	(1.960)	6.397**	(2.915)	6.399**	(2.963)
$\sigma_{c,t-1}$	0.508	(4.192)	3.064	(4.710)	5.325	(4.455)	5.904	(4.542)	4.205	(7.359)	4.089	(7.284)
r_{t-1}			0.022	(0.074)			0.024	(0.082)			-0.016	(0.093)
$R^2(\%)$	0.060		0.120		0.280		0.430		0.170		0.200	
90th Quantile												
Const.	7.392***	(2.651)	6.863***	(2.564)	7.299**	(3.408)	6.774**	(3.283)	11.946***	(4.203)	13.573***	(4.163)
$\sigma_{c,t-1}$	4.741	(4.948)	5.923	(5.020)	5.869	(5.975)	8.021	(5.995)	3.992	(7.246)	3.960	(7.728)
r_{t-1}			-0.052	(0.081)			-0.067	(0.078)			-0.104	(0.112)
$R^2(\%)$	0.670		1.150		0.820		1.420		0.580		1.180	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes S&P 500, Total stock market of Datastream and NASDAQ. Nominal values are deflated with CPI. Volatility of consumption growth rates is computed as Threshold stochastic volatility.

	UK MSCI Index				FTSE			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
				Panel A: Volatility AARSV				
10th Quantile								
Const.	0.935	(4.934)	-4.330	(3.789)	-1.376	(4.488)	-0.710	(3.995)
$\sigma_{c,t-1}$	-14.947**	(6.396)	-8.806**	(4.259)	-12.792*	(7.135)	-14.415**	(5.861)
r_{t-1}			0.361**	(0.175)			0.298*	(0.172)
$R^2(\%)$	4.870		10.120		2.530		6.560	
25th Quantile								
Const.	5.141**	(2.368)	3.084	(2.846)	-0.430	(2.227)	-0.935	(2.266)
$\sigma_{c,t-1}$	-10.780***	(2.912)	-10.009***	(3.256)	-4.347*	(2.461)	-3.954	(2.562)
r_{t-1}			0.225	(0.148)			0.069	(0.103)
$R^2(\%)$	5.020		6.380		1.420		1.550	
50th Quantile								
Const.	2.988*	(1.536)	2.969*	(1.512)	2.887**	(1.412)	2.882**	(1.379)
$\sigma_{c,t-1}$	-2.219	(2.309)	-2.077	(2.416)	-2.131	(1.771)	-2.150	(1.775)
r_{t-1}			0.022	(0.101)			0.035	(0.059)
$R^2(\%)$	0.670		0.740		0.380		0.570	
75th Quantile								
Const.	4.639***	(1.579)	4.706***	(1.687)	3.852	(2.454)	3.043	(2.405)
$\sigma_{c,t-1}$	1.337	(2.176)	1.230	(2.273)	2.845	(3.240)	3.753	(3.283)
r_{t-1}			-0.009	(0.089)			-0.052	(0.077)
$R^2(\%)$	0.480		0.510		0.290		0.820	
90th Quantile								
Const.	3.414	(2.877)	1.865	(3.509)	4.519	(2.749)	3.710	(2.872)
$\sigma_{c,t-1}$	9.466**	(4.227)	11.183**	(5.030)	7.727**	(3.782)	8.973**	(4.280)
r_{t-1}			-0.052	(0.115)			-0.146	(0.178)
$R^2(\%)$	3.470		4.090		1.330		1.940	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes U.K. MSCI Index and FTSE. Nominal values are deflated with CPI. Volatility of consumption growth rates

	UK MSCI Index				FTSE			
	(1)		(2)		(3)		(4)	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
				Panel B: Volatility TSV				
10th Quantile								
Const.	-0.400	(5.037)	-4.232	(3.833)	-1.124	(4.431)	-0.070	(3.873)
$\sigma_{c,t-1}$	-13.484**	(6.490)	-8.675**	(4.009)	-13.197*	(7.019)	-15.322***	(5.628)
r_{t-1}			0.384**	(0.169)			0.303*	(0.176)
R^2(%)	5.090		10.250		2.490		6.450	
25th Quantile								
Const.	4.313*	(2.199)	2.764	(2.762)	-0.411	(2.110)	-0.761	(2.146)
$\sigma_{c,t-1}$	-9.971***	(2.619)	-9.762***	(3.087)	-4.323*	(2.348)	-4.170	(2.623)
r_{t-1}			0.218	(0.145)			0.071	(0.109)
R^2(%)	5.260		6.640		1.240		1.390	
50th Quantile								
Const.	3.026**	(1.534)	2.780*	(1.560)	2.845**	(1.255)	2.743**	(1.316)
$\sigma_{c,t-1}$	-2.233	(2.443)	-1.803	(2.449)	-2.056	(1.597)	-2.013	(1.698)
r_{t-1}			0.028	(0.096)			0.035	(0.064)
R^2(%)	0.710		0.780		0.400		0.580	
75th Quantile								
Const.	4.644***	(1.527)	4.742***	(1.553)	3.877	(2.543)	2.796	(2.397)
$\sigma_{c,t-1}$	1.337	(2.120)	1.176	(2.055)	2.889	(3.348)	3.998	(3.214)
r_{t-1}			-0.009	(0.086)			-0.052	(0.078)
R^2(%)	0.470		0.510		0.360		0.880	
90th Quantile								
Const.	4.238	(3.005)	1.350	(3.464)	4.221	(2.650)	4.150	(2.759)
$\sigma_{c,t-1}$	7.318*	(4.274)	12.124**	(4.996)	7.945**	(3.508)	8.153**	(4.135)
r_{t-1}			-0.062	(0.116)			-0.140	(0.177)
R^2(%)	3.210		3.750		1.360		1.920	

Note: Estimation of equations (11)–(12). The dependent variables are log returns of the indexes UK MSCI Index and FTSE. Nominal values are deflated with CPI. Volatility of consumption growth rates