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Abstract

In this paper we propose factor-models assembled out of three new factors and evaluate them on European Equities. The new factors are built from statistical measurements on stock prices, in particular, coefficient of variation, skewness and kurtosis. Data come from Reuters, correspond to nearly 2000 EU companies and span from Jan-2008 to Feb-2018. Regarding methodology, we propose a non-parametric resampling procedure that accounts for time dependency in order to test the validity of the model and the significance of the parameters involved. We compare our bootstrap-based inferential results with classical proposals (based on F-statistics). Methods under assessment are Time-series regression, Cross-Sectional regression and the Fama-MacBeth procedure. The main findings indicate that the two factors that better improve the CAPM-model with regard to the adjusted R^2 in the time-series regressions are the skewness and the coefficient of variation. For this reason, a model including those two factors together with the market is thoroughly studied.

Keywords: Asset pricing, Bootstrap, Cross-Sectional regression, Factor models, Time series

1 Introduction

Understanding why and how certain assets go up in price while others go down is a major concern for both Industry and Academia. There is an overwhelming number of research articles regarding asset pricing; however, for the moment, no model has been able to explain the behaviour of the Stock Market in a fully satisfactory manner. Some of the classical proposed models rely on financial measures, such as Price to Book ratio, Market Capitalisation or Profitability to predict future asset returns (Fama and French, 1992). Others, incorporate also macro or industry-related measures, such as Interest Rates levels (Viale et al., 2009), or Oil Price (Ramos et al., 2017). Lately, such measures are combined with others involving psychological factors, such as Momentum, in order to build more sophisticated models

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(Carhart, 1997).

Markowitz (1952) determined that rational investors use diversification to optimize the profitability of their portfolios. This implies that the return required by an investor does not depend on the risk of a certain stock because part of that risk is diversifiable. The only important factor is, thus, non-diversifiable risk. Markowitz assumes that investors are risk-averse so, when they choose a portfolio they tend to minimize the variance of the return of the portfolio for a given expected return and maximize the expected return for a given variance.

The Capital Asset Pricing Model, CAPM, was developed by Sharpe (1964), Lintner (1965), and Mossin (1966) and represents the expected profitability of the i-th stock, $E(R_i)$, in the following manner:

$$E(R_i) = r_f + \beta_i(E(r_m) - r_f),$$

where r_f is the risk-free rate, $E(r_m) - r_f$ is the Market Risk Premium, and β_i the sensitivity of expected excess asset's return associated with the i-th asset.

CAPM is used to predict stock profitabilities, not portfolio profitabilities. According to Kristjanpoller and Liberona (2010), the best strategy under the assumptions of CAPM is a passive one: buy the Market portfolio in the long run. This conclusion is even stronger if we take into account transaction costs.

Even though CAPM is well formulated on a theoretical level, the assumptions behind the model limit its practical application. For instance, CAPM's β is static and only applies for 1 year. Indeed, Lintner (1965) and Miller and Scholes (1972) tested the model with NYSE stocks obtaining certain inconsistencies which led to the definition of multifactor models.

Multifactor models should explain better the behaviour of assets' returns. The identification of these factors is a key issue in the process of building the model. The seminal example of multifactor models is the one by Fama and French (1992). The asset's returns are explained by a linear model that depends on the size of the company (market capitalization), the Price to Book ratio and, as in CAPM, the Market risk premium:

$$E(R_i) = r_f + \beta_i(E(r_m) - r_f) + \beta_{i1}SMB + \beta_{i2}HML,$$

where SMB is the difference between the return of a portfolio of small companies and one of big companies (Small minus Big), HML is is the difference between the return of a portfolio of companies with high Price to Book ratios and one of companies with low Price to Book ratios (High Minus Low), while r_f and $E(r_m)$ are as before. Adding the two factors to the regressions results in large increases in the coefficient of determination. According to Fama and French (1993), the Market factor alone produces only two (of 25) R^2 values greater than 0.9; in the three-factor regressions R^2 values greater than 0.9 are usual (21 of 25).

Inferential procedures about multifactor models strongly rely on assumptions regarding the data: for instance, factors being uncorrelated over time, normally distributed errors which are i.i.d. over time and independent of the factors, etc. When these assumptions are not satisfied, classical estimators may be biased. To circumvent such a drawback, non-parametric techniques can be used.

Efron (1972) developed the Bootstrap methodology as a resamplig technique to approximate the distribution of test statistics. In the Asset Pricing literature, Kosowski et al. (2006) propose a bootstrap procedure by which they resample from the returns of mutual funds in order to test that all portfolios' returns are explained by the factors. Fama and French (2010) modify the process to jointly resample both fund and explanatory returns. This additional step is important as it takes into account possible correlation. However, none of these procedures consider dependency over time. Grané and Veiga (2008) explore the block bootstrap for computing the unconditional distribution of returns. These authors found huge differences in the MCRRs estimates when using conditional approaches (such as GARCH type models and stochastic volatility models) specially for long positions and larger investment horizons.

The objectives of this paper are: (1) to propose a multifactor model based on naive statistical factors, (2) develop non-parametric resampling techniques that account for time dependency in order to test the validity of the model and the significance of the parameters involved, and (3) to compare bootstrap-based inferential techniques with the classical proposals. These procedures are tested on a real dataset of assets' returns of over $2,000$ European companies extracted from Reuters and spanning from Jan-2008 to Feb-2018.

The main findings are that the two factors that better complement the market (only factor at the CAPM) are the skewness and the coefficient of variation. With regard to the time-series regressions, the conclusions drawn from the newly proposed bootstrap inferential techniques are somehow coherent with the ones obtained from classical inference procedures. Indeed, since the used data does not fulfill all the classical distributional requirements, the bootstrap conclusions tend to be less strict with departures from then benchmark described at the null hypotheses. In particular, our bootstrap inferences suggest that all adjusted time-series regression models have intercepts that are jointly equal to zero, which is not the case for the classical method. With respect to the specific models estimated for each portfolio, those with high loading on the coefficient of variation (respectively skewness) have a higher coefficient of variation (resp. skewness) coefficient, while market βs are all close to 0.5. The cross-section model built from the estimated coefficients presents some deficiency, since the coefficient of variation does not contribute significantly to it, while the market and skewness do. The underlying reason might be some multicollinearity problem. However, the performance of the model seems reasonably good in spite of the period analyzed, that comprises the European debt crisis.

The remainder of this paper is organized as follows. In Section 2, we discuss our data and the construction of portfolio returns. Section 3 explains the methodology, first the classical (Time-Series, Cross-Sectional and Fama-MacBeth) and second the resampling technique developed for the analysis. Section 4 reports the results of the analysis and compares different methodologies. We close the paper with some conclusions in Section 5.

2 Data and Portfolio Returns

2.1 Data

We start with 2,012 European companies that have been selected from all EU countries. As is usual in the Factor literature, we exclude Financials as they usually have high leverage ratios, affecting several financial ratios. We extract Reuters prices at the end of each month from Jan-2008 to Feb-2018 for these companies. We apply several filters to the Data: first, we delete all the datapoints where Price is not available; second, we apply a transaction filter, excluding all companies that do not show transactions for a whole semester; third, we exclude companies with no Market Cap or P/B info for more than 2 consecutive years. Finally, we exclude companies with non-positive Equity at the end of any year. After all these filters have been passed, we end up with 1,913 companies. Next, we calculate monthly returns for all the companies and estimate the Risk Free Rate (r_f) with the 2 year German Bond yield and Market (r_m) through the SP600. Monthly returns were calculated with the natural logs of the market price divided by the market price in the previous month.

Table 1 contains the descriptive statistics by year. In particular, the table provides statistics on the number of firms, prices, and the monthly raw returns.

Year	companies	months	mean	SD	min	median	max
2009	1631	12	69.55	1619.75	0.01	5.59	92316.88
2010	1643	12	47.39	765.65	0.01	6.48	42907.84
2011	1672	12	36.20	516.56	0.00	6.50	26316.80
2012	1706	12	23.77	120.80	0.00	5.48	4203.77
2013	1729	12	24.98	109.47	0.00	5.98	3400.01
2014	1737	12	28.37	123.28	0.00	7.20	3789.00
2015	1789	12	30.98	139.74	0.00	7.25	3999.00
2016	1833	12	32.39	164.73	0.01	7.20	6000.00
2017	1864	12	38.99	195.06	0.00	8.74	6149.00
2018	1878	2	41.34	204.65	0.00	9.08	6000.00

Table 1: Descriptive statistics of the data by year

We can observe that average price decreases until 2012 (from 69.55 in 2009 to 23.77 in 2012) and then increases until 2018 (41.34). This behaviour is related to the Financial Crisis. In terms of standard deviation, we can observe a similar pattern.

2.2 Statistical Factors

We calculated different measures for each stock in the sample, namely:

1. Coefficient of variation (CV) of prices for each year and company:

$$CV = \frac{s_n}{\bar{x}},$$

where $s_n^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $\bar{x}$ is the sample mean. CV represents the relative spread for positive random variables and it can take values in $(0, +\infty)$.

2. Skewness (Skew) of prices for each year and company:

$$Skew = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{s_n^3}$$

Positive values indicate that the distribution is positively skewed, that is, the right tail is longer than the left one, while the contrary occurs for negative values. When both tails are similar, the skewness is roughly 0.

3. Excess Kurtosis (Kurt) of prices for each year and company:

$$Kurt = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{s_n^4} - 3$$

measures how fat are the tails of a distribution in comparison of those of a normal random variable. Positive (negative) values indicate heavier (thinner) tails than those of a normal random variable.

In general, factors are positively correlated, except for Kurt, which is negatively correlated with Market and CV. As can be seen in Table 2, Market and CV on one side and CV and Kurt on the other are highly correlated. This could lead to multicolineality in models built with these factors and, thus, to larger Confidence Intervals.

	Market	CV	Kurt	Skew
Market	1.0000	0.5970	-0.4210	0.1770
CV	0.5970	1.0000	-0.5450	0.2930
Kurt	-0.4210	-0.5450	1.0000	0.1650
Skew	0.1770	0.2930	0.1650	1.0000

Table 2: Correlation among the factors

2.3 Portfolio Returns

Next, we calculate percentiles for each of these measures and assign to portfolios accordingly following Fama and French (1992). There are two portfolios for each measure, except for Skew, where we calculate three.

- Stocks with low CV will be included in portfolios 1-y-z, while stocks with high CV will be included in portfolios 2-y-z.
- Stocks with low Kurt will be included in portfolios x-1-z, while stocks with high Kurt will be included in portfolios x-2-z.
- Stocks with low Skew (less than the 30-th percentile, P30) will be included in portfolios x-y-1, while stocks with high Skew (greater than P70) will be included in portfolios x-y-3. The rest will be included in portfolios x-y-2.

The resulting portfolios are summarized in Table 3. Portfolios are updated yearly based on the previous year measurements (2008 data is only used to built the starting portfolios). We proceed to calculate average monthly returns for the stocks included in each portfolio. Additionally, we calculate each factor as the excess return of the higher portfolio in each category minus the return of the lower portfolio. All returns are calculated for equally-weighted portfolios at $t + 1$. In figure 1, we have plotted the cumulative returns of all the portfolios.

	Portfolio	CV	Kurt	Skew
(1)	1-1-1	Low	Low	Low
(2)	1-1-2	Low	Low	Medium
(3)	1-1-3	Low	Low	High
(4)	1-2-1	Low	High	Low
(5)	1-2-2	Low	High	Medium
(6)	1-2-3	Low	High	High
(7)	2-1-1	High	Low	Low
(8)	2-1-2	High	Low	Medium
(9)	2-1-3	High	Low	High
(10)	2-2-1	High	High	Low
(11)	2-2-2	High	High	Medium
(12)	2-2-3	High	High	High

Table 3: Description of portfolios

The three portfolios with the best performance are 1-1-2, 1-2-1 and 2-1-1. The first two share the feature of having low CV. There is one portfolio that ends with a negative cumulative return (2-2-3); it includes stocks with high CV, high Kurt and high Skew. Portfolios 2-2-1 and 2-2-2 (high CV and high Kurt) are also among the worse performers.

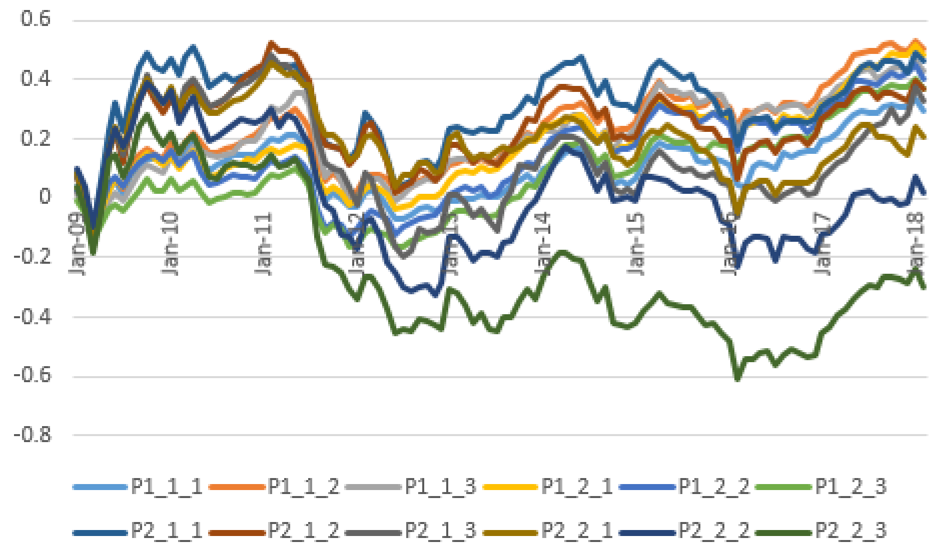

Figure 1: Portfolios' cumulative returns

3 Methodology

3.1 Classical Methodologies

Linear Factor Models have gained great popularity lately and this has lead to a vast literature on estimating and testing such models. Typically, these models take the form of the following equation:

$$E(R_i) = \alpha_i + \beta_i' \lambda.$$

The expected return for the i-th portfolio has a linear relationship with a set of factors λ. The simplest of such models is the one-factor model developed by Jensen (1968) and based on CAPM, where asset risk premiums are linear functions of Market risk premium and the systematic risk of the asset (β_i), that is:

$$E(R_i) = \alpha_i + \beta_i (E(r_m) - r_f),$$

where α is referred as the Jensen's Alpha of the asset and is commonly used as a performance measure. In general, we will consider that a model works when the expected value of α is zero. The model is extendable, mainly, by including additional factors as independent variables. Feng et al. (2019) have made a great effort to systematically evaluate the contribution to asset pricing of any new factor, above and beyond what a high-dimensional set of existing factors explains.

Several procedures to estimate the parameters α, β, and λ have been developed in the literature. After reviewing two analytic procedures which assume a certain behaviour of the underlying data (Time-Series Regression and Cross-Sectional Regression), we propose a

Block Bootstrap method that works even if the previous assumptions are violated.

The setup of the experiment includes N assets (portfolios of equities), K factors ($K = 1$ for CAPM) and T time periods. The factors are excess returns. We use portfolios instead of stocks because: (1) the errors of α and β are higher for individual stocks as their volatility is higher and (2) portfolios have more stable characteristics while stocks vary over time. All the statistics below are derived from the classical assumptions that returns are independent over time, correlated across assets, and (for finite sample results) normally distributed.

3.1.1 Time-Series Regression (TS)

A time series regression is run for each portfolio $i = 1, \ldots, N$ as

$$R_t^i = \alpha_i + \beta_i' f_t + \epsilon_t^i, \quad t = 1, 2, \ldots, T.$$

The estimates of α_i and β_i and their standard errors are those of the time-series regression (OLS), while λ is estimated as $\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T} f_t = \overline{f}$. Assuming factors are uncorrelated over time, its standard error is estimated as $\hat{\sigma}(\hat{\lambda}) = \hat{\sigma}(f_t)/\sqrt{T}$.

To assess the ability of a model to explain excess returns, we use the GRS test proposed by Gibbons et al. (1989) whose null hypothesis is $H_0 : \alpha_1 = \ldots = \alpha_N = 0$. Under the assumption that ϵ are normally distributed, the test statistic is:

$$\frac{T - N - K}{N}[1 + \overline{f}'\Sigma_f^{-1}\overline{f}]^{-1}\hat{\alpha}'\Sigma^{-1}\hat{\alpha} \sim F_{N,T-N-K}, \tag{1}$$

where Σ_f is the covariance matrix of the factors and Σ the residual covariance matrix.

3.1.2 Cross-Sectional Regression (CS) and Fama-MacBeth (FM) Procedure

This is a two-step procedure:

- First, a Time-Series regression is run to obtain the sensitivity of the expected excess return of each of the portfolios. For each $i = 1, \ldots, N$ estimate β_i in the model

  $$R_t^i = a_i + \beta_i' f_t + \epsilon_t^i, \quad t = 1, 2, \ldots, T.$$

  Note that the intercept here is denoted by a instead of α.

- Run then a Cross-Sectional regression to get λ

  $$E(R^i) = \hat{\beta}_i'\lambda + \alpha_i, \quad \text{for } i = 1, \ldots, N.$$

where the estimates for $\hat{\beta}_i$ were obtained in the TS regression at the first step. In conclusion $\hat{\lambda}$ represents the slope coefficients in the CS regression, while $\hat{\alpha}$ are the residuals in the CS regression.

Fama and MacBeth (1973) developed a simplified process. After running the TS regression, they run a CS regression at each time period to get λ.

$$R_t^i = \hat{\beta}_i' \lambda_t + \alpha_{it}, \quad \text{for } i = 1, \ldots, N.$$

With FM it is easy to do models in which β change over time. Additionally, it can be used when there is a big cross-section where elements are correlated among them (but not across time).

Our estimates for λ and α are the averages across time. Note that if the β are the same over time, λ and α estimates are identical to those in the CS regression.

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t \quad \hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^{T} \hat{\alpha}_{it}$$

with the following variance for λ:

$$\hat{\sigma}_\lambda^2 = \frac{1}{T^2} \sum_{t=1}^{T} (\hat{\lambda}_t - \hat{\lambda})^2.$$

If the estimated β are important determinants of average returns, then the risk premiums should be statistically significant. The Cross-Section standard error formulas are a little bit better than the ones presented before because they include the error of estimating β (Shanken correction, Shanken (1992)), although the difference may be very small in practice.

3.2 Resampling Techniques: Bootstrap

A naive bootstrap procedure for factor models was proposed by Kosowski et al. (2006) and later modified by Fama and French (2010) to sample fund residuals and factors returns jointly in order to preserve cross correlation among the different portfolios (iid-bootstrapping pairs). This second procedure was used by Sørensen (2009) in order to determine if a certain fund generates alpha (skill). However, in this paper we are interested in determining whether the model works (all α are jointly zero). We propose a block-bootstrapping pairs scheme to preserve the time correlation of the data of B bootstrap samples of block size b.

First Step Estimate benchmark regression models, one for each portfolio. For each portfolio, we save α, β, their corresponding t-statistics, residuals, the estimates of risk factors and the GRS statistic according to the Time-series regression:

$$R_{i,t}^e = \alpha_i + \sum_{j=1}^{K} \hat{\beta}_{i,j} f_{j,t} + e_{i,t}$$

where K is the number of factors and index i is used to denote the i-th portfolio.

Second Step Produce a set of simulation runs (in our case, $B = 10,000$) which is the same for every portfolio to preserve the cross-correlation of the returns. Draw a vector $\tilde{T}_s$ of T/b independent components from a discrete uniform distribution on $\{1, 2, \dots, T - b + 1\}$. Each component represents the initial time point for a block of size b.

Third Step Use the simulated time indices to build a new series of α-free portfolio returns which has the property that α is zero. If the factor model adequately describes returns, then, the expected value of the intercept is zero.

$$\tilde{R}_i^e = f(\tilde{T}_s)\hat{\beta}_i + e_i\tilde{T}_s$$

Fourth Step Run the Time-Series factor model regression on the artificially constructed returns. Obtain $\hat{\alpha}$, $\hat{\beta}$ and the corresponding confidence intervals. Finally, generate $B = 10,000$ samples of a GRS Statistic, calculate different percentiles from the bootstrapped distribution and compare them to the original GRS statistic.

Regarding Cross-sectional regression, we have used βs and average returns from each bootstrapped sample to determine the significance of the risk factors' estimates (λs). Given that βs are estimated, confidence intervals for λs present a lot of bias. We have decided to use a Reverse bootstrap percentile interval to determine the significance of the factors. The reverse percentile bootstrap interval is not transformation respecting, neither is it very asymptotically efficient, but, at least, is not affected by bootstrap bias. To be consistent, we have also applied this type of interval to all the bootstrapped distributions.

In the empirical application, we have set the number of items per block $b = 2$ months. We noticed that when we set the number of months of the block to $b = 1$ (iid-bootstrapping pairs as in Fama and French, 2010; Sørensen, 2009), the confidence intervals shrink. The process is developed through parallel computing, using several cores to improve calculation time.

4 Empirical results

4.1 Time-Series regressions

In this section, we present the results for the Time-Series regressions of five models, considering several combinations of the statistical factors described before. Other combinations have been considered, but were outperformed by the ones shown below.

4.1.1 Model 1: CAPM

In Table 4 we present the results of the time series estimation using only the Market Factor. The dependent variables are the returns of the the twelve portfolios.

	Estimates		t-statistic		bootstrap CI (2.5%,97.5%)		
Portfolio	alpha	market	alpha	market	alpha	market	Adj. R^2
1-1-1	-0.0005	0.558	-0.282	14.287	-0.005,0.003	0.467,0.668	0.651
1-1-2	0.0010	0.627	0.661	19.470	-0.001,0.005	0.555,0.699	0.776
1-1-3	0.0003	0.623	0.163	13.442	-0.004,0.006	0.492,0.744	0.622
1-2-1	0.0011	0.567	0.749	17.456	-0.001,0.006	0.482,0.647	0.736
1-2-2	0.0000	0.637	-0.015	18.314	-0.004,0.003	0.534,0.738	0.754
1-2-3	0.0003	0.530	0.192	14.014	-0.003,0.004	0.437,0.627	0.642
2-1-1	-0.0011	0.919	-0.358	13.760	-0.008,0.004	0.765,1.075	0.633
2-1-2	-0.0022	0.962	-0.758	15.497	-0.010,0.002	0.819,1.109	0.687
2-1-3	-0.0022	0.896	-0.603	11.332	-0.012,0.003	0.729,1.080	0.539
2-2-1	-0.0030	0.848	-1.014	13.132	-0.012,0.000	0.692,1.016	0.611
2-2-2	-0.0046	0.824	-1.201	9.916	-0.017,-0.002	0.649,1.009	0.472
2-2-3	-0.0080	0.918	-2.355	12.478	-0.023,-0.009	0.731,1.112	0.587

Notes. GRS: 1.9420, P-value: 0.038, P-value Boot.: 0.054

Table 4: Results for Time-Series estimation for Model 1 (CAPM)

We find that the coefficient estimates for Market are always positive and statistically significant. Moreover, the portfolios with high loading on the CV factor have a higher market coefficient, while the adjusted coefficients of determination indicate that Market factor by itself explains between 47.2% and 77.6% of the variation in the returns of the portfolios. Additionally, under the classical methodology, we find that α are not statistically significant (except for portfolio 2-2-3). Finally, the GRS test rejects the null hypothesis at 5% (but not at 2.5%) that all pricing errors are equal to zero which might indicate that other factors are missing.

In order to confirm if the OLS hypotheses are fulfilled (normality, homoscedasticity and independence), we performed several analysis (Soumaré et al., 2013). A Shapiro-Wilk test was run on each set of residuals, suggesting that, in general, the residuals of the TS regression are normally distributed, except for portfolios 1-1-1 (p-value 0.007), 1-2-1 (p-value 0.037), and 1-2-2 (p-value 0.024). Additionally, charts of residuals vs. fitted values show that the homoscedasticity assumption is violated for portfolios 7 to 12 (high CV). The Durbin-Watson test also suggests that errors may be autocorrelated for portfolios 2 to 6 (low CV). The autocorrelation charts indicate that residuals are especially correlated for lag equal to 1.

These three facts suggest that bootstrap might be useful to approximate the distribution of the GRS statistic. We obtained a bootstrap p-value of 5.4%, so we cannot reject that the α

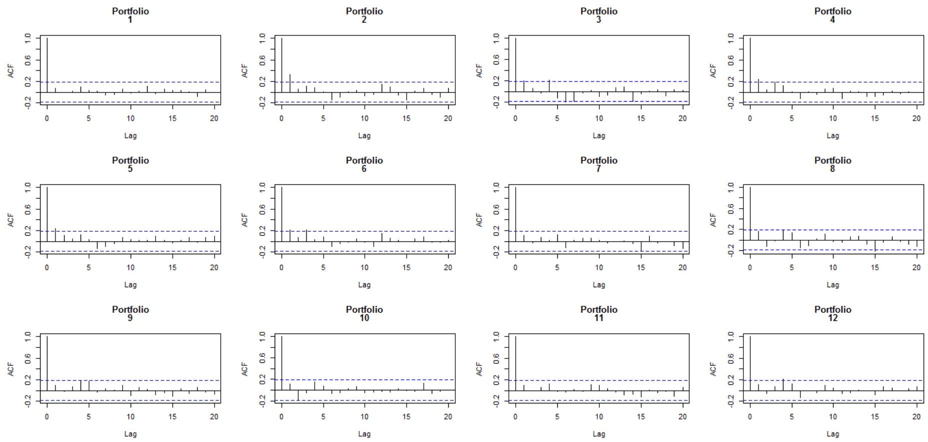

Figure 2: Autocorrelation charts for errors in TS regression for CAPM

are jointly zero. We then observe the coefficients (α and β) generated for the 12 portfolios and we notice that: (1) all the bootstrap α CIs include zero except for the last two portfolios (αs are not only jointly zero, but individually zero also) and (2) all the β CIs do not include zero (in fact, β is positive in all of them).

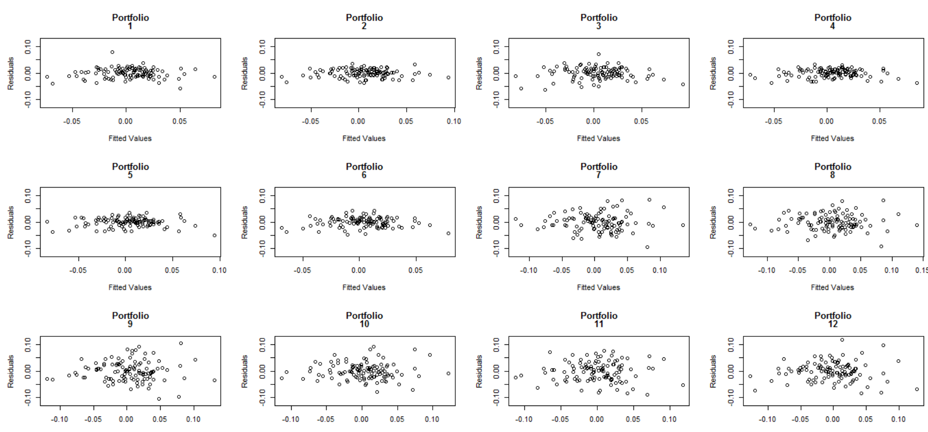

Figure 3: Fitted values vs. Residuals in TS regression for CAPM

Next, we incorporate one by one the statistical factors (coefficient of variation, skewness and kurtosis). We find that the market coefficient estimates continue to be positive and statistically significant. In general, α are not statistically significant in any of the models.

4.1.2 Model 2: Market and Coefficient of variation

The coefficient estimates for Market and CV are always positive and statistically significant. Moreover, the portfolios with high loading on the CV factor have higher CV coefficients (interestingly, Market coefficients have stabilized around 0.5 with respect to the previous model). Adjusted R^2 ranges between 63.9% and 92.9% and increases with respect to the previous model (especially in portfolios with high CV loading). Finally, the GRS test rejects the null hypothesis at 5% that all pricing errors are equal to zero.

After applying our bootstrap procedure, we find that 13.26% of the bootstrap values are greater than the obtained GRS statistic, so we cannot reject the hypothesis that all the α are jointly zero. Actually, the Bootstrap methodology offers a higher p-value than the one of the traditional methodology. Indeed, when we try to fit an F distribution to the sampled GRS statistic, we find that the MLEs of degrees of freedom are 12.8 and 33.6, where the degrees of freedom in the denominator are far less than those in equation (1). We then observe the coefficients (α and β) generated for the 12 portfolios and we notice that: (1) 6 of the α CIs include zero and (2) none of the β's CIs (both Market and CV) include zero, being β positive in all of them.

	Estimates			t-statistic			bootstrap CI (2.5%,97.5%)			
Portfolio	alpha	market	CV	alpha	market	CV	alpha	market	CV	Adj. R^2
1-1-1	0.0008	0.441	0.332	0.447	9.776	4.322	-0.002,0.005	0.344,0.562	0.209,0.430	0.700
1-1-2	0.0021	0.523	0.297	1.537	14.263	4.765	0.001,0.007	0.441,0.607	0.178,0.407	0.814
1-1-3	0.0012	0.541	0.235	0.587	9.562	2.449	-0.002,0.007	0.393,0.689	0.054,0.405	0.639
1-2-1	0.0022	0.471	0.275	1.539	12.520	4.300	0.001,0.007	0.386,0.561	0.140,0.390	0.773
1-2-2	0.0008	0.559	0.222	0.527	13.401	3.139	-0.002,0.005	0.453,0.669	0.087,0.326	0.773
1-2-3	0.0013	0.443	0.247	0.753	9.791	3.218	-0.001,0.006	0.346,0.552	0.061,0.395	0.670
2-1-1	0.0036	0.484	1.238	2.139	10.709	16.133	0.004,0.011	0.387,0.574	1.097,1.374	0.892
2-1-2	0.0024	0.539	1.202	1.746	14.576	19.131	0.002,0.008	0.462,0.608	1.065,1.317	0.929
2-1-3	0.0031	0.404	1.400	1.419	6.850	13.948	0.002,0.011	0.286,0.551	1.197,1.570	0.835
2-2-1	0.0016	0.427	1.198	0.949	9.783	16.144	0.000,0.007	0.345,0.518	1.062,1.341	0.886
2-2-2	0.0006	0.345	1.364	0.232	4.963	11.550	-0.004,0.006	0.196,0.514	1.115,1.564	0.763
2-2-3	-0.0033	0.490	1.217	-1.463	8.056	11.780	-0.012,-0.002	0.334,0.666	1.007,1.409	0.818

Notes. GRS: 1.9437, P-value: 0.038, P-value Boot.: 0.1326

Table 5: Results for Time-Series estimation of Model 2 (Market and CV)

4.1.3 Model 3: Market and Kurtosis

The coefficient estimates for Market are always positive and statistically significant. Moreover, the portfolios with high loading on the CV factor have higher coefficients. However, coefficient estimates for kurtosis change sign between portfolios (negative for low kurtosis ones and positive in the rest) and are only significant regarding classical methodology at standard levels for 5 of them (portfolios with low CV and high Kurt and portfolios with high CV and low Kurt). Adjusted R^2 registers lower values than in Model 2, going from 46.7% to 77.6%, with marginal gains, in general, with respect to Model 1. The GRS test does not reject the null hypothesis at 5% significance level.

When we apply the bootstrap methodology, we cannot reject the null hypothesis that all αs are jointly zero, since we obtain a p-value of 8.6% (higher than the one of the traditional methodology). Again, the second degree of the fitted F distribution is much lower than the one suggested by formula 1 (now the MLEs degrees of freedom are 12.2 and 35.2). Analogously as before, (1) seven of the α CI include zero, (2) all the market β CI do not include zero (in fact, β is positive in all of them) and (3) eight out of twelve of the kurtosis β CI include zero, corresponding, in general, to those with low CV loading and those with high CV and high Kurt loading. This last fact emphasizes the small contribution of this statistical factor in explaining the returns of the twelve portfolios.

	Estimates			t-statistic			bootstrap CI (2.5%,97.5%)			
Portfolio	alpha	market	kurt	alpha	market	kurt	alpha	market	kurt	Adj. R^2
1-1-1	-0.0007	0.541	-0.168	-0.396	12.563	-0.923	-0.005,0.002	0.436,0.674	-0.563,0.211	0.650
1-1-2	0.0008	0.614	-0.131	0.545	17.275	-0.871	-0.002,0.005	0.537,0.696	-0.486,0.191	0.776
1-1-3	0.0003	0.623	-0.004	0.158	12.130	-0.018	-0.004,0.006	0.482,0.767	-0.505,0.487	0.619
1-2-1	0.0014	0.588	0.213	0.925	16.507	1.421	0.000,0.006	0.506,0.681	-0.152,0.564	0.738
1-2-2	0.0005	0.682	0.452	0.348	18.387	2.892	-0.002,0.005	0.595,0.780	0.033,0.857	0.770
1-2-3	0.0008	0.563	0.337	0.437	13.692	1.942	-0.002,0.005	0.478,0.667	-0.106,0.757	0.651
2-1-1	-0.0026	0.796	-1.232	-0.921	11.644	-4.279	-0.011,0.001	0.655,0.952	-2.078,-0.541	0.684
2-1-2	-0.0037	0.841	-1.210	-1.392	13.373	-4.567	-0.013,-0.002	0.700,1.001	-1.977,-0.571	0.736
2-1-3	-0.0037	0.779	-1.179	-1.048	9.344	-3.357	-0.015,0.000	0.620,0.983	-2.103,-0.372	0.579
2-2-1	-0.0038	0.783	-0.656	-1.304	11.192	-2.224	-0.014,-0.002	0.654,0.933	-1.506,0.028	0.625
2-2-2	-0.0045	0.832	0.075	-1.162	9.037	0.194	-0.017,-0.001	0.647,1.045	-0.951,0.936	0.467
2-2-3	-0.0079	0.923	0.057	-2.305	11.337	0.166	-0.023,-0.009	0.751,1.127	-0.998,0.956	0.583

Notes. GRS: 1.767, P-value: 0.065, P-value Boot.: 0.086

Table 6: Results for Time-Series estimation of Model 3 (Market and Kurt)

4.1.4 Model 4: Market and Skewness

Using the classical methodology, the coefficient estimates for Market are always positive and statistically significant. The portfolios with high loading on the CV factor have the highest Market coefficients. Coefficient estimates for skewness are always positive and are, in general, significant at standard levels (10 out of 12 portfolios). Adjusted R^2 levels (between 52.1% and 82.0%) are better than those in Model 3, but worse than those in Model 2. Finally, the GRS test does not reject the null hypothesis at 5% .

On the other hand, 7.8% of the bootstrapped values of the sample are higher than the GRS statistic. We then observe the coefficients (α and β) generated for the 12 portfolios and we notice that: (1) seven out of twelve of the α CI include zero, (2) all the market β CI do not include zero (in fact, β is positive in all of them) and (3) two of the skewness' β CI include zero.

4.1.5 Model 5: Market, Coefficient of Variation and Skewness

According to the classical methodology, we find that the coefficient estimates for Market and CV are always positive and statistically significant (except for portfolio 1-1-3, where the

Portfolio	Estimates alpha	Estimates market	Estimates skew	t-statistic alpha	t-statistic market	t-statistic skew	bootstrap CI (2.5%,97.5%) alpha	bootstrap CI (2.5%,97.5%) market	bootstrap CI (2.5%,97.5%) skew	Adj. R^2
1-1-1	0.0002	0.543	0.292	0.116	13.924	2.229	-0.003,0.004	0.461,0.645	0.070,0.602	0.663
1-1-2	0.0022	0.600	0.516	1.661	20.448	5.228	0.002,0.007	0.546,0.657	0.362,0.690	0.820
1-1-3	0.0021	0.586	0.704	1.052	13.703	4.893	0.000,0.008	0.490,0.676	0.439,1.010	0.689
1-2-1	0.0019	0.551	0.309	1.276	17.242	2.877	0.001,0.007	0.467,0.626	0.114,0.533	0.753
1-2-2	0.0011	0.612	0.478	0.760	18.687	4.335	-0.001,0.005	0.513,0.700	0.269,0.693	0.789
1-2-3	0.0023	0.489	0.784	1.561	15.590	7.430	0.002,0.007	0.418,0.559	0.578,0.996	0.762
2-1-1	-0.0004	0.904	0.289	-0.126	13.357	1.268	-0.007,0.005	0.746,1.065	-0.178,0.778	0.635
2-1-2	-0.0006	0.928	0.637	-0.216	15.306	3.121	-0.006,0.005	0.786,1.079	0.231,1.066	0.710
2-1-3	0.0011	0.827	1.329	0.323	11.617	5.549	-0.004,0.008	0.665,1.003	0.847,1.878	0.639
2-2-1	-0.0026	0.839	0.177	-0.852	12.762	0.801	-0.011,0.001	0.677,1.013	-0.344,0.723	0.610
2-2-2	-0.0023	0.775	0.943	-0.617	9.635	3.484	-0.012,0.003	0.594,0.964	0.322,1.651	0.521
2-2-3	-0.0040	0.832	1.638	-1.471	14.217	8.322	-0.013,-0.002	0.684,0.978	1.244,2.090	0.747

Notes. GRS: 1.8035, P-value: 0.051, P-value Boot.: 0.078

Table 7: Results for Time-Series estimation of Model 4 (Market and Skew)

t-statistic for β_{CV} is 1.53). As in Model 2, the portfolios with high loading on the CV factor have a higher CV coefficients and Market coefficients have stabilized around 0.5. Coefficient estimates for Skewness are positive except for portfolios 2-1-1 and 2-2-1 and are, in general, significant at standard levels (9 out of 12 portfolios). Adjusted R^2 ranges between 69.2% and 93.0% and reaches the highest values among the considered models. The GRS test does not reject the null hypothesis at 5% significance levels.

When using the bootstrap methodology, we find that 9.70% of the values of the sample are higher than the GRS statistic, so we cannot reject the hypothesis that all the α are jointly zero. We then observe the coefficients (α and β) generated for the 12 portfolios we notice that: (1) four of the α CIs include zero (mainly those with high CV and high Kurt), (2) none of the market β CIs include zero (in fact, β is positive in all of them), (3) two of the Coefficient of Variation β CIs include zero (portfolios 1-1-3 and 1-2-3) and (4) only three of the skewness β CIs include zero.

4.2 Cross-Sectional regression

The only model that we consider in the present section is Model 5. Its GRS statistic (see Table 9) suggest that we cannot reject that all αs are jointly zero and further it registers the highest explained variability. Additionally, CV and Skew are statistically significant in most of the portfolios.

We use the βs estimated in Table 9 to examine if the factors are priced on the cross-section of returns. We must take into account that despite the fact that we are working with portfolios, risk premia will be estimated from the estimated coefficients, which can lead to an important bias, especially if the model is not well specified.
The highest risk premium is the Market one, which is 0.0102, while the CV premium is estimated as -0.0015 and Skew premium is -0.0026. As a consequence, returns depend positively on the Market, and negatively on the CV and skewness factors. The reported

		t-statistic		
	Estimate	FM	CS	bootstrap basic 95% CI
Market	0.0102	2.0582	1.4853	0.0039, 0.0245
CV	-0.0015	-0.5816	-0.4020	-0.0070, 0.0039
Skew	-0.0026	-1.807	-1.2878	-0.0006, -0.0060

Table 8: Results for Cross-Sectional estimation of Model 5

t-statistics (which in the case of the CS ones include the Shanken correction as in Shanken (1992)) suggest that none of the factors are significant at conventional significance levels. However, bootstrap intervals indicate that Market and Skew λs are significantly different from zero (at 5% level) contributing significantly to the model. The non significance of CV may be related to some multicollinearity problem. Note that the correlation between the market and the CV is around 0.6 (see 2). However, we cannot reach the same conclusion for the CV factor, we still consider the current model (Model 5) superior to the other ones due to the superior explanatory power of its associated time-series regression models.

5 Conclusions

We have defined three factors based on several statistical moments of stock prices. That is, the factors do not rely on external data and thus can be used at any market. Based on them and in order to evaluate their relevancy, we have built 12 portfolios, and conducted a time-series and cross-section regression analysis.

In order to overcome the limitations of the classical tests commonly used in the factor-model literature, due to the strong assumptions about the data, we have proposed a block bootstrap inferential procedure. This procedure accounts for both potential cross sectional and time series correlation among returns and factors, while it allows to run all usual tests in factor models (GRS test on the intercepts of TS regression, test on the TS regression coefficients, and test on the CS regression coefficients).

The introduced technique has been used with data from several European stock markets which does not fulfill the requirements of the classical inferential techniques. After testing several models, we propose one with three factors: market, skewness, and CV. This model has some drawback, which we conjecture that can be due to multicollinearity, but an important explanatory power. In general, the model performance seems reasonably good in such a turbulent economic period in the Euro zone.

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Appendix

	Estimates				t-statistic				bootstrap CI (2.5%,97.5%)				
Portfolio	alpha	market	CV	skew	alpha	market	CV	skew	alpha	market	CV	skew	Adj. R^2
1-1-1	0.001	0.441	0.306	0.175	0.639	9.815	3.889	1.381	-0.001,0.005	0.348,0.563	0.171,0.417	-0.063,0.485	0.702
1-1-2	0.003	0.522	0.234	0.427	2.282	15.469	3.962	4.476	0.003,0.009	0.455,0.593	0.115,0.351	0.257,0.608	0.842
1-1-3	0.002	0.540	0.140	0.651	1.252	10.347	1.528	4.420	0.001,0.009	0.425,0.666	-0.043,0.304	0.349,0.991	0.692
1-2-1	0.003	0.470	0.243	0.216	1.838	12.706	3.749	2.070	0.002,0.008	0.387,0.554	0.102,0.370	0.022,0.428	0.779
1-2-2	0.002	0.558	0.161	0.416	1.080	14.183	2.341	3.743	0.000,0.006	0.460,0.654	0.020,0.277	0.187,0.643	0.797
1-2-3	0.003	0.442	0.140	0.730	1.853	11.680	2.113	6.832	0.002,0.008	0.365,0.526	-0.015,0.255	0.522,0.959	0.769
2-1-1	0.003	0.484	1.267	-0.196	1.913	10.782	16.143	-1.551	0.003,0.010	0.387,0.575	1.125,1.403	-0.442,0.055	0.894
2-1-2	0.003	0.539	1.175	0.187	2.002	14.727	18.352	1.813	0.003,0.008	0.465,0.607	1.022,1.303	-0.022,0.402	0.930
2-1-3	0.005	0.404	1.276	0.840	2.412	7.795	14.082	5.749	0.005,0.013	0.308,0.514	1.094,1.445	0.528,1.176	0.873
2-2-1	0.001	0.427	1.242	-0.298	0.613	10.021	16.643	-2.476	-0.001,0.005	0.339,0.523	1.083,1.394	-0.479,-0.065	0.891
2-2-2	0.001	0.344	1.299	0.446	0.561	5.058	10.897	2.320	-0.002,0.008	0.196,0.498	1.067,1.487	0.141,0.844	0.772
2-2-3	-0.001	0.489	1.034	1.242	-0.598	11.200	13.550	10.084	-0.005,0.001	0.391,0.595	0.900,1.159	1.009,1.510	0.906

Notes. GRS: 1.7759, P-value: 0.063, P-value Boot.: 0.097

Table 9: Results for Time-Series estimation of Model 5 (Market, CV and Skew)