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STRATEGIC PROFIT SHARING BETWEEN FIRMS: A PRIMER *

Roberts Waddle [1]

Abstract

This paper builds a theory of profit sharing between two firms in a duopoly market through which firms seek to increase their profits and, in turn, to limit the competition. We use a general model to show the *direct* (negative) and *indirect* (positive) effects of this strategy. We then focus on some oligopolistic models to analyze more deeply and more precisely these two opposite effects in search of the dominant one.

We thus show that *giving away profits is a rewarding strategy* for firms in some (but not all) models of oligopolistic competition.

Keywords: Profit sharing, Oligopoly, Collusion, Competition, Psychology and economics.

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Strategic Profit Sharing Between Firms: A Primer[1]

Roberts Waddle[2]

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Preliminary- Comments welcome!
(*please, do not circulate*)

[1] I'm very grateful to my supervisor José Luis Ferreira for his numerous helpful suggestions. I'm also indebted to Paul Belleflamme, Jean Hindricks, Dominique Demouguin, Paolo Piacentini, Sergio Bruno for useful discussion and comments on an earlier draft and also for their support during my stay in London, in Berlin and in Rome. I also wish to thank especially Diego Moreno for giving me literature related to this topic. Nevertheless, all remained errors are my own.

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Abstract

This paper builds a theory of profit sharing between two firms in a duopoly market through which firms seek to increase their profits and, in turn, to limit the competition. We use a general model to show the *direct* (negative) and *indirect* (positive) effects of this strategy. We then focus on some oligopolistic models to analyze more deeply and more precisely these two opposite effects in search of the dominant one.

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JEL Classification: A12, C72, D21, L13.

1 Introduction

The problem of firms in oligopoly situation has always attracted attention from economists. As attests the huge literature on the models of Cournot, Bertrand and the like. In recent years, this type of model has been applied to a wide range of economic issues such as industrial organisation, macroeconomics, public economics and international trade.

However, most of the economists emphasizes the firms' *classical strategies*. Some for example using the Cournot model or a variant describe firms as competing in quantities. Others using the Bertrand model or a similar present firms as competing in prices. Several consider a combination of these two models. Many using a Stackelberg model introduce the notion of timing in the way firms are competing in the markets. Finally, some recent economists (Kemplerer and Meyer (86), Fudenberg and Tirole (83), Vives (84, 90), Kihlstrom and Vives (89), Jun and Vives (2004)) introduce some grains of sand into those well-known models by pointing the role of uncertainty, the role of information and the role of experience in the behavior of firms[1].

The present paper, by contrast focuses on a particular strategy of firms in a duopoly situation, which until now has surprisingly received very little attention in economics[2]. It corresponds to the fact that each firm decides *unilaterally* to cede *voluntarily* a part of its profit to its rival. Thus, firms first (in the first stage) decide simultaneously the optimal part of their profits to give away to their rivals and then (in the second stage) determine the equilibrium price. The rationale of the "unilateral-decision" assumption is to support the legality of this strategy. Consequently, our firms should not be treated as a cartel or as colluding firms or as joint ventures[3].

We develop a theory of profit sharing between two firms in a duopoly market through which firms seek to increase their profits and, in turn, to limit the competition. We use a general model to show the direct (negative) and indirect (positive) effects of this strategy. We then specialize on some oligopolistic models (Cournot, Bertrand, Hotelling) with two-stage game and linear demand to analyze more deeply these two effects in quest of the dominant one. We thus show that giving away profits is a rewarding strategy for firms in some of (but not all) the above oligopolistic competition models. For instance, *firms are better off to share profits when markets are differentiated*

[1] The terms "grains of sand" is borrowed from Benabou-Tirole (2001).

[2] This idea has received much greater emphasis in psychology. For instance, many psychologists have always encouraged to share because of the positive effect on the human well-being.

[3] In a companion paper (Waddle 2005d), we will totally relax this assumption allowing firms to invest (rather than to share) a portion of their profits in a joint venture.

whether by location or by demand function. Conversely, *firms have no incentive to cede profits when goods are homogeneous* like in the classical Cournot model. Indeed, this is the main difference with those oligopolistic models.

The article proceeds as follows. Section 2 relates our work to the existing literature. Section 3 builds the basic model of profit sharing between two firms in a duopoly market. Section 4 focuses on the classical Cournot model and points out that it is worthless for firms to give away profits. Section 5 moves to the Cournot model with heterogenoeous goods and thus shows that firms win by giving away profits. Section 6 turns to the Hotelling model where goods are differentiated by location and comes to the same conclusion of the previous section. Finally, section 7 explores the Bertrand model with heterogeneous goods and, as in the two previous sections, brings to light that sharing profits is a rewarding strategy for firms. Section 8 concludes with suggestions for future research. Details of proofs are gathered in appendix.

2 Review of the literature

Our paper is connected to three lines of research in the oligopoly theory: partial ownership, competition and collusion.

Partial Ownership. Some economists have recently integrated into their model the notion of partial ownership in the strategies of firms in an oligopoly situation.

Malhueg (92) shows that if firms interact repeatedly, increasing cross ownership may reduce the likelihood of collusion. A high level of cross ownership may even entail a lower likelihood of collusion than would no cross ownership. We differ from this article in two respects: First, Malhueg's paper considers that each firm is entitled to a common fraction of the profit earned by the other firm. Here, we consider that each firm voluntarily gives up a part of its profit to the other firm. In others terms, each firm behaves as if it unilaterally ceded a share of its profit to its rival. Second, our objective is just different. Malhueg investigates wether increasing cross ownership among rivals increase the likelihood of collusion while we examine whether a firm is better off by sharing its profit to its rival in two different situations. We don't pose the problem of collusion which rather is exogenous to our model.

Farell and Shapiro (90) studies a one-way cross-ownership where a big firm wants to acquire assets from an other firm. Through a single-period Cournot oligopoly model, they show that, as the degree of cross ownership among rivals increases, the equilibrium in the market become less competitive in the sense that aggregate output falls towards the monopoly level. However, there are two differences with our model. First, we employ a two-stage game to analyze the profitability of sharing profit between two firms in a duopoly situation. In other terms, we investigate in which conditions firms are better off to share profits. We are not concerned by the effects of increased cross ownership on the competition. At least, the impact on the competition is internalized in our model. Another key difference is that it is always profitable for our firms to share profits when competing in prices.

Caminal and Vives (96) considers a duopoly market where in which consumers have heterogeneous information about one good quality. Their article sustains that the market tends to become more competitive if consumers compete for markets shares beyond the level explained by short-run profit maximization. Here, in contrast, competition has not always increased. Besides, we do not have any information problem. On the contrary, our firms are well informed.

Finally, FritzRoy and Kraft (86) exhibits strong effects of profit-sharing and worker ownership shares on residual owners' return on capital. Although we both analyze the impact of profit-sharing on the profitability, we differ from our targets. Their model is applied to organisation while ours is firms oriented. Furthermore, we don't always have strong effects.

Collusion. The other literature to which our paper belongs to is the one on collusion between oligopoly models.

Brock and Scheinkman (85) investigates the role of industry capacity in enforcing collusion in the context of repeated games. Their article shows that a variation in the number of firms induces a non-monotone effect on the best enforceable price. The distinctive feature of our work is that our firms are not capacity constrainted. We do not analyze the impact of a deviating firm on the cartel. On the contrary, in our model firms do not form a cartel and do not even collude. Of course, the results of our model show eventual consequences on collusion.

Salant et al. (83) points out that some exogeneous mergers may decrease the endogeneous joint profits of colluding firms. Our model shares with Salant et al. (83) a concern with the impact on the other firms' profits though our impact was targeted at each firm' profit. However, we do not consider firms in a merging situation.

Competition. Our work is also related to competition in oligopoly models. More specifically, a line of research has emphasized different strategies for firms competing in oligopoly markets.

For instance, Singh and Vives (84) proves that firms must choose either the quantity contract in case of goods substitutes or the price contract in case of goods complements. Contrary to Singh and Vives, we are not interested to know in which case firms are better off to use prices as dominant strategies and in which one they should use quantities. Rather, we seek to elucidate the impact of *sharing profits* strategy on firms' profits when competing in prices.

Likewise, Bulow et al. (85) studies a situation similar except that it considers two markets and emphasizes how a firm's action in one market can change competitors' strategies in the second market. It investigates under what conditions the action might provide costs or benefits in that second market. This paper and ours share the basic objective of deriving the impact of a firm's action on the other firms. However, our firms' strategies are different. They are not involved in underinvesting in capital to reduce the ferocity of future competition. They simply share a part of their profits to their rivals. Moreover, we do consider only one market where the timing is superfluous.

Reynolds and Snapp (86) shows that in markets entry is difficult, partial ownership arrangements could result in less output and higher prices than otherwise, even if the ownership shares are relatively small. There are two differences with our model. The first one is free-entry in our model. The second one is our firms do not form a cartel and nor involved in any partial ownership arrangements in the sense of Reynolds and Snapp. Furthermore, profits have not always increased.

Finally, Klemperer and Meyer (86) points out the role of uncertainty in the choice of strategic variables (prices, quantities). This uncertainty problem absent in our model gives firms strict preferences between setting price and quantity. There are also other significant differences between our models. First, we consider a two-stage game where firms are competing in a duopoly market. Second, firms use the same strategic variables (price, price) or (quantity,quantity).

3 The basic model

Consider two firms 1 and 2 in a duopoly market with s_1 and s_2 their two respective variables of strategy. As we will see later, these variables can be prices or quantities. Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) wants to share with firm 2 (resp. firm 1). Without loss of generality, we suppose that $\alpha_i \in [0, 1]$. Finally, assume that the respective profit functions of the two firms $\Pi_1(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2))$ and

$\Pi_2(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2))$ (hereafter $\Pi_1(s_1, s_2)$ and $\Pi_1(s_1, s_2)$) are strictly positive. For simplicity, we will also assume that $\Pi_1(s_1, s_2)$ and $\Pi_2(s_1, s_2)$ are strictly concave in α_1 and α_2 and that the functions $s_1(.)$ and $s_2(.)$ are differentiable.

We consider a two-stage game whose sequences are later defined. In the first stage of the game, firms are searching which α_i maximizes their profit. In the second stage of the game, firms are seeking out s_i that maximizes their profits. We solve the problem by backwards induction.

Second stage of game

Let us write each firms's maximisation problem:

$$\begin{aligned} Max_{s_1} \quad P_1(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) &= (1-\alpha_1)\Pi_1(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) + \\ &\quad \alpha_2\Pi_2(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) \end{aligned}$$

$$\begin{aligned} Max_{s_2} \quad P_2(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) &= (1-\alpha_2)\Pi_2(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) + \\ &\quad \alpha_1\Pi_1(s_1(\alpha_1, \alpha_2), s_2(\alpha_1, \alpha_2)) \end{aligned}$$

The first-order conditions with respect to s_1 and s_2 can be written:

$$(1-\alpha_1)\frac{\partial \Pi_1(s_1, s_2)}{\partial s_1} + \alpha_2 \frac{\partial \Pi_2(s_1, s_2)}{\partial s_1} = 0 \tag{1}$$

$$(1-\alpha_2)\frac{\partial \Pi_2(s_1, s_2)}{\partial s_2} + \alpha_1 \frac{\partial \Pi_1(s_1, s_2)}{\partial s_2} = 0 \tag{2}$$

By rearranging the equations (1) and (2), we have:

$$\alpha_1 \frac{\partial \Pi_1(s_1, s_2)}{\partial s_1} - \alpha_2 \frac{\partial \Pi_2(s_1, s_2)}{\partial s_1} = \frac{\partial \Pi_1(s_1, s_2)}{\partial s_1} \tag{3}$$

$$\alpha_2 \frac{\partial \Pi_2(s_1, s_2)}{\partial s_2} - \alpha_1 \frac{\partial \Pi_1(s_1, s_2)}{\partial s_2} = \frac{\partial \Pi_2(s_1, s_2)}{\partial s_2} \tag{4}$$

These last two equations define implicitly $s_1^*(\alpha_1, \alpha_2)$ and $s_2^*(\alpha_1, \alpha_2)$. From where, we can write the reduced-form profit function $P_1(s_1^*(\alpha_1, \alpha_2), s_2^*(\alpha_1, \alpha_2))$ that we use to solve the first stage of the game.

First stage of the game

Let us write each firm's maximisation problem:

$$Max_{\alpha_1} \quad P_1(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2)) = (1-\alpha_1)\Pi_1(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2)) + \alpha_2\Pi_2(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2))$$

$$Max_{\alpha_2} \quad P_2(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2)) = (1-\alpha_2)\Pi_2(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2)) + \alpha_1\Pi_1(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2))$$

Now, we can write the first-order conditions for the firm 1 in the first stage of the game. For that, we derive $P_1(s_1^*(\alpha_1,\alpha_2), s_2^*(\alpha_1,\alpha_2))$ (henceforth P_1) with respect to α_1 by applying the chain rule and we get:

$$\frac{\partial P_1}{\partial \alpha_1} = -\Pi_1 + (1-\alpha_1)\left(\frac{\partial \Pi_1}{\partial s_1}\frac{\partial s_1^*}{\partial \alpha_1} + \frac{\partial \Pi_1}{\partial s_2}\frac{\partial s_2^*}{\partial \alpha_1}\right) + \alpha_2\left(\frac{\partial \Pi_2}{\partial s_1}\frac{\partial s_1^*}{\partial \alpha_1} + \frac{\partial \Pi_2}{\partial s_2}\frac{\partial s_2^*}{\partial \alpha_1}\right)$$

$$\frac{\partial P_2}{\partial \alpha_2} = -\Pi_2 + (1-\alpha_2)\left(\frac{\partial \Pi_2}{\partial s_1}\frac{\partial s_1^*}{\partial \alpha_2} + \frac{\partial \Pi_2}{\partial s_2}\frac{\partial s_2^*}{\partial \alpha_1}\right) + \alpha_1\left(\frac{\partial \Pi_1}{\partial s_1}\frac{\partial s_1^*}{\partial \alpha_2} + \frac{\partial \Pi_1}{\partial s_2}\frac{\partial s_2^*}{\partial \alpha_2}\right)$$

and by rearranging, we have:

$$\frac{\partial P_1}{\partial \alpha_1} = -\Pi_1 + \frac{\partial s_1^*}{\partial \alpha_1}\left((1-\alpha_1)\frac{\partial \Pi_1}{\partial s_1} + \alpha_2\frac{\partial \Pi_2}{\partial s_1}\right) + \frac{\partial s_2^*}{\partial \alpha_1}\left((1-\alpha_1)\frac{\partial \Pi_1}{\partial s_2} + \alpha_2\frac{\partial \Pi_2}{\partial s_2}\right)$$

$$\frac{\partial P_2}{\partial \alpha_2} = -\Pi_2 + \frac{\partial s_1^*}{\partial \alpha_2}\left((1-\alpha_2)\frac{\partial \Pi_2}{\partial s_1} + \alpha_2\frac{\partial \Pi_1}{\partial s_1}\right) + \frac{\partial s_2^*}{\partial \alpha_2}\left((1-\alpha_2)\frac{\partial \Pi_2}{\partial s_2} + \alpha_1\frac{\partial \Pi_1}{\partial s_2}\right)$$

By checking that the terms in the first bracket of the two equations are equal to zero from (1) and (2) and by developing the terms in the second bracket, we have:

$$\frac{\partial P_1}{\partial \alpha_1} = -\Pi_1 + \frac{\partial s_2^*}{\partial \alpha_1}\left(\frac{\partial \Pi_1}{\partial s_2} - \alpha_1\frac{\partial \Pi_1}{\partial s_2} + \alpha_2\frac{\partial \Pi_2}{\partial s_2}\right)$$

$$\frac{\partial P_2}{\partial \alpha_2} = -\Pi_2 + \frac{\partial s_2^*}{\partial \alpha_2}\left(\frac{\partial \Pi_2}{\partial s_2} - \alpha_2\frac{\partial \Pi_2}{\partial s_2} + \alpha_1\frac{\partial \Pi_1}{\partial s_2}\right)$$

Finally, by using the equations (3) and (4), we can write:

$$\frac{\partial P_1}{\partial \alpha_1} = -\Pi_1 + \frac{\partial s_2^*}{\partial \alpha_1}\left(\frac{\partial(\Pi_1+\Pi_2)}{\partial s_2}\right)$$

$$\frac{\partial P_2}{\partial \alpha_2} = -\Pi_2 + \frac{\partial s_2^*}{\partial \alpha_2}\left(\frac{\partial(\Pi_1+\Pi_2)}{\partial s_2}\right)$$

Using the properties of first-order conditions, we have:

$$-\Pi_1 + \frac{\partial s_2^*}{\partial \alpha_1}\left(\frac{\partial(\Pi_1+\Pi_2)}{\partial s_2}\right) = 0 \quad (5)$$

$$-\Pi_2 + \frac{\partial s_2^*}{\partial \alpha_2}\left(\frac{\partial(\Pi_1 + \Pi_2)}{\partial s_2}\right) = 0 \quad (6)$$

The equations (5) and (6) give α_1^* and α_2^*

Proposition 1 *In the Subgame Perfect Equilibrium (SPE),* α_1^* *and* α_2^* *are given by* (5) *and* (6) *respectively where the terms* s_1^*, s_2^* *are the implicit solutions to* (3) *and* (4).

Proof. The proof is straightforward. Equations (3) and (4) define implicitly s_1^* and s_2^* whereas α_1^* and α_2^* can be easily found by solving the equations (5) and (6). ■

Note that:

i) if the equilibrium is given by interior solutions, then α_1^* and α_2^* are greater than 0.

This suggests that *firms are willing to give away profits* (fact that the literature has surprisingly, until now, ignored). For instance, in the next sections, we will show examples of interior solutions where firms are even better off by sharing profits.

i) $\frac{\partial P_i}{\partial \alpha_i} = -\Pi_i + \frac{\partial s_j^*}{\partial \alpha_i}\left(\frac{\partial(\Pi_i+\Pi_j)}{\partial s_j}\right)$ where $\alpha_i \in [0,1]$ & $i,\, j = 1, 2$ $(i \neq j)$

This insinuates that sharing profits between firms has two opposite effects. First, a *direct* or *negative* effect given by the firs term $-\Pi_i$ and then, a *strategic* or *positive* effect given by the second term $\frac{\partial s_j^*}{\partial \alpha_i}\left(\frac{\partial(\Pi_i+\Pi_j)}{\partial s_j}\right)$. Hence, the *total effect is crucial* when firms decide whether or not to give away profits to their rivals.

As we will see in the next sections, this effect is not always positive. For instance, in a homogenoeus market, we found it negative when firms compete in quantities[4], but positive when firms compete in prices.

[4]This article analyzes only quantity competition in an homogeneous market whereas a companion article *Profit Sharing Between Firms: II. The Bertrand Model* (Waddle 2005b) focuses essentially on price competition.

4 The classical Cournot model

In the last section, we have seen intuitively that it is possible for firms to give away profits and that depended on whether or the total effect of this strategy was positive. In this section, we will show that it is not profitable for firms to share profits when competing in quantities.

For that, we consider two firms 1 and 2 in a homogeneous market. We suppose that the marginal cost of production c is, without loss of generality, 0. Let the demand curve be $D(p) = 1 - p$ or $p = P(q_1 + q_2) = 1 - q_1 - q_2$ where q_1 (resp. q_2) are firm 1's (resp. firm 2's) output. Assume that the profit of the two firms $\Pi_1(q_1, q_2)$ and $\Pi_2(q_1, q_2)$ are concave in q_i $(i = 1, 2)$.

Now, let us introduce a grain of novelty in the basic Cournot model. Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) wants to share with firm 2 (resp. firm 1). We suppose that $\alpha_i \in [0, 1]$. Consequently, we can write the *new profit function* $P_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1 - \alpha_i)\Pi_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j) + \alpha_j \Pi_j(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firms choose α_i. In the second stage of the game, firms select p_i.

In the *first stage of the game*, for α_1 and α_2 firms simultaneously solve:

$$Max_{a_1} \quad P_1 = (1 - \alpha_1)\Pi_1 + \alpha_2 \Pi_2$$

$$Max_{\alpha_2} \quad P_2 = (1 - \alpha_2)\Pi_2 + \alpha_1 \Pi_1$$

In the *second stage of the game*, for q_1 and q_2 firms simultaneously solve:

$$Max_{q_1} \quad P_1 = (1 - \alpha_1)\Pi_1 + \alpha_2 \Pi_2$$

$$Max_{q_2} \quad P_2 = (1 - \alpha_2)\Pi_2 + \alpha_1 \Pi_1$$

4.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximizes their profits.

Proposition 2 *If* $\alpha_i \in [0,1]$, *then any quantities* $(q_1^*,\ q_2^*)$ *such that* $q_1^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_2)}{3-\alpha_1-\alpha_2}$ *and* $q_2^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_1)}{3-\alpha_1-\alpha_2}$ *constitute a NE in the second stage of the game.*

Proof. To see that, it suffices to solve the game backwards.

Second stage of the game

$Max_{q_1} P_1(q_1, q_2) = (1-\alpha_1)pq_1 + \alpha_2 pq_2$

or $Max_{q_1}\ P_1(q_1, q_2) = (1-\ \alpha_1)(1-q_1-q_2)q_1 +\ \alpha_2(1-q_1-q_2)q_2$

The first-order conditions with respect to q_1 give:

$(1-\alpha_1)(1-q_1-q_2) - (1-\alpha_1) - \alpha_2 q_2 = 0$

$q_1(q_2) = \frac{(1-\alpha_1)-(1-\alpha_1+\alpha_2)q_2}{2(1-\alpha_1)}$

and by analogy:

$q_2(q_1) = \frac{1}{2} - \frac{1-\alpha_2+\alpha_1}{2(1-\alpha_2)} q_1$

By solving the system of the two previous equations, we have[5]:

$$q_1^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_2)}{3-\alpha_1-\alpha_2} \tag{7}$$

$$q_2^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_1)}{3-\alpha_1-\alpha_2} \tag{8}$$

Conclusion: *If* $\alpha_i \in [0,1]$, then $(q_1^*,\ q_2^*)$ satisfying equations (7) & (8) constitute a NE in the second stage of the game. ■

The second-stage being entirely solved and NE being found, we can thus move to the first-stage of the game in order to find SPNE

[5] Note that:
i) If $\alpha_1 = \alpha_2 = 0$ we have: $q_1^* = q_2^* = \frac{1}{3}$ and $P_1 = P_2 = \frac{1}{9}$ (*Cournot's case*)
ii) If $\alpha_1 = \alpha_2 = \frac{1}{2}$ we have: $q_1^* = q_2^* = \frac{1}{2}$ and $P_1 = P_2 = \frac{1}{2}$ *(Monopoly's case)*

4.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profits to share with their rivals.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE in prices summarized below:

$q_1^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_2)}{3-\alpha_1-\alpha_2}$ and $q_2^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_1)}{3-\alpha_1-\alpha_2}$ *if* $\alpha_i \in [0, 1]$

Since $p^* = 1 - q_1^* - q_2^*$ we have:

$$p^*(\alpha_1, \alpha_2) = \frac{1}{3 - \alpha_1 - \alpha_2} \quad (9)$$

Using equations $(7) - (9)$, we can rewrite the profit function :

$P_1(\alpha_1, \alpha_2) = (1 - \alpha_1)p^*q_1^* + \alpha_2 p^* q_2^*$

$P_1(\alpha_1, \alpha_2) = \frac{1-\alpha_1}{(3-\alpha_1-\alpha_2)^2}$

Now, in the current section, we draw our attention to the first-stage of the game searching for SPNE in α_i.

Proposition 3 *$(\alpha_1,\ \alpha_2) : \alpha_i^* = 0$; $(q_1,\ q_2) : q_1^* = \frac{1}{3}$ and $q_2^* = \frac{1}{3}$ if $\alpha_i \in [0, 1]$ are SPNE of the game. Besides, profits in the SPNE decreases with α_i making it worthless for firms to give away profits.*

Proof. The proof is straightforward. It suffices to solve the rest of the game.

First-stage of the game

$Max_{\alpha_1}\ P_1(\alpha_1, \alpha_2) = \frac{1-\alpha_1}{(3-\alpha_1-\alpha_2)^2}$

The first-order conditions with respect to α_1 give:

$\frac{\partial P_1}{\partial \alpha_1} = \frac{\partial}{\partial a_1}\left[\frac{1-\alpha_1}{(3-\alpha_1-\alpha_2)^2}\right]$

$\frac{\partial P_1}{\partial \alpha_1} = \frac{-1-\alpha_1+\alpha_2}{(3-a_1-\alpha_2)^2}$

$\frac{\partial P_1}{\partial \alpha_1} = 0 \Rightarrow -1 - \alpha_1 + \alpha_2 = 0 \Rightarrow \alpha_1 = -1 + \alpha_2$

By solving the above equation, we find $\alpha_1^* = 0$

Using the same reasoning as before for firm 2, we easily show that $\alpha_2^* = 0$

Substituying α_1^* and α_2^* in equations (7) and (8), we have: $q_1^* = q_2^* = \frac{1}{3}$

Likewise, one can check that: $\frac{\partial P_1}{\partial \alpha_1} < 0$

Conclusion: $(\alpha_1, \alpha_2) : \alpha_i^* = 0$; $(q_1, q_2) : q_1^* = \frac{1}{3}$ and $q_2^* = \frac{1}{3}$ if $\alpha_i \in [0, 1]$ are SPNE of the game. Besides, profits in the SPNE decreases with α_i making it worthless for firms to give away profits. ■

In the next section, we will turn to the Cournot model but with differentiated markets. Interestingly, we will show that sharing profits is a rewarding strategy for firms.

5 The Cournot model with heterogeneous goods

In this section, we consider a model similar as the one described in the previous section except that we introduce the heterogeneity in the market.

We thus consider two firms 1 and 2 in a heterogeneous market. The marginal cost of production c is, without loss of generality, 0. We still assume that the profit of the two firms $\Pi_1(q_1, q_2)$ and $\Pi_2(q_1, q_2)$ are concave in q_i $(i = 1, 2)$.

Without loss of generality, we suppose that $\alpha_i \in [0, 1]$, $i = 1, 2$. Let $p_1 = 1 - q_1 - \gamma q_2$ and $p_2 = 1 - \gamma q_1 - q_2$ the respective inverse demand functions of firm 1 and firm 2 where q_1(resp. q_2) are firm 1's (resp. firm 2's) output.

As before, we introduce a grain of novelty in this Cournot model (with heterogeneous goods). Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) wants to share with firm 2 (resp. firm 1). We suppose that $\alpha_i \in [0, 1]$. Consequently, we can write the *new profit function* $P_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1 - \alpha_i)\Pi_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j) + \alpha_j \Pi_j(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firms choose α_i. In the second stage of the game, firms select p_i.

In the *first stage of the game*, for α_1 and α_2 firms simultaneously solve:

$$Max_{a_1} \quad P_1 = (1-\alpha_1)\Pi_1 + \alpha_2\Pi_2$$
$$Max_{\alpha_2} \quad P_2 = (1-\alpha_2)\Pi_2 + \alpha_1\Pi_1$$

In the *second stage of the game*, for q_1 and q_2 firms simultaneously solve:

$$Max_{q_1} \quad P_1 = (1-\alpha_1)\Pi_1 + \alpha_2\Pi_2$$
$$Max_{q_2} \quad P_2 = (1-\alpha_2)\Pi_2 + \alpha_1\Pi_1$$

5.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximizes their profits.

Proposition 4 *$\forall\ \gamma$, if $\alpha_i \in [0,1]$ & $\alpha_1+\alpha_2 \neq 1$, then any quantities $(q_1^*,\ q_2^*)$ / $q_1^* = \frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ & $q_2^* = \frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ constitute a NE in the second stage of the game.*

Proof. To see that, it suffices to solve the game backwards.

Second stage of the game

$Max_{q_1} P_1(q_1, q_2) = (1-\alpha_1)pq_1 + \alpha_2 pq_2$

or $Max_{q_1}\ P_1(q_1, q_2) = (1-\ \alpha_1)(1-q_1-\gamma q_2)q_1+\ \alpha_2(1-\gamma q_1 - q_2)q_2$

The first-order conditions with respct to q_1 give:

$\frac{\partial P_1}{\partial q_1} = (1-\ \alpha_1)(1-2q_1-\gamma q_2)-\ \alpha_2\gamma q_2 = 0$

$q_1 = \frac{(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2)q_2}{2(1-\alpha_1)}$

and by analogy:

$q_2 = \frac{(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1)q_1}{2(1-\alpha_2)}$

By solving the system of the last two previous equations, we have[6]:

$$q_1^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \tag{10}$$

and by analogy,

$$q_2^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \tag{11}$$

Conclusion: *If* $\alpha_i \in [0,1]$ & $\alpha_1+\alpha_2 \neq 1$, then $(q_1^*,\, q_2^*)$ satisfying equations (10) & (11) constitute a NE in the second stage of the game. ■

The second-stage being entirely solved and NE being found, we can thus move to the first-stage of the game in order to find SPNE

5.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profits to share with their rivals.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE in prices summarized below:

$q_1^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ and

$q_2^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ *if* $\alpha_i \in [0,1]$ & $\alpha_1+\alpha_2 \neq 1$

Since $p_1^* = 1 - q_1^* - \gamma q_2^*$ and $p_2^* = 1 - \gamma q_1^* - q_2^*$ we have:

$$p_1^*(\alpha_1,\alpha_2) = \frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_2(1-\alpha_2+\alpha_1)+\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \tag{12}$$

$$p_2^*(\alpha_1,\alpha_2) = \frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_1(1-\alpha_1+\alpha_2)+\gamma(1-\alpha_1)(1-\alpha_2+\alpha_1)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)} \tag{13}$$

Using equations $(10)-(13)$, we can rewrite the profit function:

$P_1(\alpha_1,\alpha_2) = (1-\alpha_1)p_1^* q_1^* + \alpha_2 p_2^* q_2^*$

Now, in the current section, we draw our attention to the first-stage of the game searching for SPNE in α_i.

[6]We can easily check that if $\gamma = 1$, we obtain the same result already found in the previous section with homogeneous product, that is, $q_1(a_1,a_2) = \frac{(1-a_2)}{(3-a_1-a_2)}$

Proposition 5 *For $\gamma = -1$, $(\alpha_1, \alpha_2) : \alpha_i^* = 0.45$; $(q_1, q_2) : q_1^* = q_2^* \approx 5.5$ if $\alpha_i \in [0,1]$ are SPNE of the game. Besides, profits in the SPNE is by far higher than in the case where $\alpha_1 = \alpha_2 = 0$, making it worthwile for firms to share profits.*

Proof. The proof is straightforward. It suffices to solve the rest of the game.

First-stage of the game

$$Max_{\alpha_1}\ P_1(\alpha_1, \alpha_2) = (1 - \alpha_1)p_1^* q_1^* + \alpha_2 p_2^* q_2^*$$

The first-order conditions with respect to α_1 give:
$\frac{\partial P_1}{\partial \alpha_1} = 0$

At the equilibrium we have $\alpha_1 = \alpha_2 = \alpha$:

$\Rightarrow \frac{\partial P_1}{\partial \alpha_1}\rfloor_{\alpha_1 = \alpha_2 = \alpha} = 0$

By solving the above equation, we find: $\alpha^* = 45\%$ and $\gamma = -1$

By replacing $\alpha_1 = \alpha_2 = \alpha$ in the profit function, we have:

$$\begin{aligned} P_1^*(\alpha, \alpha) &= (1-\alpha)^2 \tfrac{2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)}{4(1-\alpha)^2 - \gamma^2} \tfrac{2-2\alpha-\gamma}{4(1-\alpha)^2 - 1} + \\ & \alpha \tfrac{2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)}{4(1-\alpha)^2 - \gamma^2} (1-\alpha) \tfrac{2-2\alpha-\gamma}{4(1-\alpha)^2 - 1} \end{aligned}$$

Finally, substituing $\alpha^* = 0.45$ and $\gamma = -1$ in the above equation and in equation (10) & (11) we find $P_1^* = 5.5$ and $q_1^* = q_2^* = 5.5$

However, if firms decide not to share their profits, that is, $\alpha^* = 0$ and $\gamma = -1$, we find $P_1 = 1$

Conclusion: For $\gamma = -1$, $(\alpha_1, \alpha_2) : \alpha_i^* = 0.45$; $(q_1, q_2) : q_1^* = q_2^* \approx 5.5$ if $\alpha_i \in [0,1]$ are SPNE of the game. Profits in the SPNE are thus by far higher than in the case where $\alpha_1 = \alpha_2 = 0$, making it worthwile for firms to share profits. ■

In the next section, we will turn to a Hotelling model where goods are differentiated by location. As before, we will show that sharing profits is a winning strategy for firms.

6 The Hotelling model

In this section, we consider a model similar as the one described in the previous section except that goods are differentiated by location.

For that, consider a model in which a linear city of length 1 lies on the abscissa of a line and consumers are uniformly distributed with density 1 along this interval. There are two firms which sell the same physical good. For simplicity, these two firms are located at the extremes[7] of the city; firm 1 at $x = 0$ and firm 2 at $x = 1$.

Consumers incur a transportation cost t ($t > 0$) per unit of length (this cost may include the value of time spent in travel). Thus, a consumer living at x incurs a cost of tx to go to firm 1 and a cost of $t(1 - x)$ to go to firm 2. Consumers have unit demands; that is, each consumes one or zero unit of the good. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to s

We take the firms' location as given and look for the Nash equilibrium in prices. Assuming that firms choose their prices p_1 and p_2 simultaneously, we derive the demand function for linear transportation costs. A consumer who is indifferent between the two firms is located at $x = D_1(p_1, p_2)$, where x is given by equating generalized costs; i.e.,

$$p_1 + tx = p_2 + t(1 - x)$$

The firms' respective demand are:

$$D_1(p_1, p_2) = x = \tfrac{1}{2} + \tfrac{p_2 - p_1}{2t}$$

$$D_2(p_1, p_2) = 1 - x = \tfrac{1}{2} - \tfrac{p_2 - p_1}{2t}$$

Firms' profit functions can thus be written as:

$$\Pi_1 = p_1 D_1 (p_1, p_2)$$

$$\Pi_2 = p_2 D_2 (p_1, p_2)$$

[7] One can easily check that the results do not change if firms are located symmetrically anywhere in the linear city as long as the transportation cost is linear.

We will consider a *variant* of this model in which firm 1 (resp. firm 2) decides *unilaterally* and *voluntarily* to share a part of α_1 (resp. α_2) of its profit with firm 2 (resp.firm1). We suppose that $\alpha_i \in [0,1]$, $i = 1, 2$, $(i \neq j)$. Consequently, we can write the *new profit function* $P_i(p_i(\alpha_i, \alpha_j), p_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1-\alpha_i) p_i D_i(p_i(\alpha_i, \alpha_j), p_j(\alpha_i, \alpha_j) + \alpha_j p_j D_j(p_i(\alpha_i, \alpha_j), p_j(\alpha_i, \alpha_j))$$

As in the previous section, we look at a two-stage game. In the first stage of the game, each firm *simultaneously* chooses α_i optimum to share with its rival. In the second stage, each one *simultaneously* chooses the price p_i to charge given the price p_j (the part α_j of the profit) charged (offered) by its rival and also the part α_i of the profit it itself gives away. As usual, we solve backwards to find the subgame-perfect equilibrium.

6.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximizes their profits.

Proposition 6 *$\forall$ t, if $\alpha_i \in [0,1]$ & $\alpha_1 + \alpha_2 \neq 1$, then any prices $(p_1^*,\ p_2^*)$ / $p_1^*(\alpha_1, \alpha_2) = t\,(1-\alpha_2)\,\frac{3-3\alpha_1+\alpha_2}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$ & $p_2^*(.,.) = t\,(1-\alpha_1)\,\frac{3-3\alpha_2+\alpha_1}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$ constitute a NE in the second stage of the game.*

Proof. To see that, it suffices to solve the game backwards.

Second stage of the game
$Max_{p_1} \quad P_1(p_1, p_2) = (1-\alpha_1) p_1 D_1(p_1, p_2) + \alpha_2 p_2 D_2(p_1, p_2)$

or $Max_{p_1} \quad P_1(p_1, p_2) = (1-\alpha_1) p_1 (\frac{1}{2} + \frac{p_2 - p_1}{2t}) + \alpha_2 p_2 (\frac{1}{2} - \frac{p_2 - p_1}{2t})$

The first-order conditions with respct to p_1 give:

$\frac{\partial P_1}{\partial p_1} = (1-\alpha_1)\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right) + \frac{\alpha_2 p_2}{2t} = 0$

$p_1(p_2) = \frac{t}{2} + \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)} p_2$

and by analogy:

$p_2(p_1) = \frac{t}{2} + \frac{1-\alpha_2+\alpha_1}{2(1-\alpha_2)} p_1$

By solving the system of the last two previous equations, we have:

$$p_1^*(\alpha_1, \alpha_2) = t\,(1-\alpha_2)\,\tfrac{3-3\alpha_1+\alpha_2}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)} \tag{14}$$

and by analogy,

$$p_2^*(\alpha_1, \alpha_2) = t\,(1-\alpha_1)\,\tfrac{3-3\alpha_2+\alpha_1}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)} \tag{15}$$

Conclusion: *If* $\alpha_i \in [0, 1]$ & $\alpha_1+\alpha_2 \neq 1$, then $(p_1^*,\, p_2^*)$ satisfying equations (14) & (15) constitute a NE in the second stage of the game. ■

6.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profits to share with their rivals.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE in prices summarized below:

$p_1^*(\alpha_1, \alpha_2) = t\,(1-\alpha_2)\,\frac{3-3\alpha_1+\alpha_2}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$ and

$p_2^*(\alpha_1, \alpha_2) = t\,(1-\alpha_1)\,\frac{3-3\alpha_2+\alpha_1}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$ *if* $\alpha_i \in [0, 1]$ & $\alpha_1+\alpha_2 \neq 1$

Since $D_1(p_1, p_2) = x = \frac{1}{2} + \frac{p_2-p_1}{2t}$ and $D_2(p_1, p_2) = 1 - x = \frac{1}{2} - \frac{p_2-p_1}{2t}$, we have:

$$D_1(p_1^*, p_2^*) = \left(\tfrac{1}{2}\tfrac{3-2\alpha_2}{3-\alpha_1-\alpha_2}\right) \tag{16}$$

$$D_2(p_1^*, p_2^*) = \left(\tfrac{1}{2}\tfrac{3-2\alpha_1}{3-\alpha_1-\alpha_2}\right) \tag{17}$$

Using equations $(14) - (17)$, we can rewrite the profit function:

$$P_1(\alpha_1, \alpha_2) = \tfrac{t(1-\alpha_1)}{2(3-\alpha_1-\alpha_2)^2(1-\alpha_2-\alpha_1)}\left(2\alpha_2^3 - 8\alpha_2^2 - 3\alpha_2 + 12\alpha_1\alpha_2 - 2\alpha_1^2\alpha_2 + 9 - 9\alpha_1\right)$$

Now, in the current section, we draw our attention to the first-stage of the game searching for SPNE in α_i.

Proposition 7 *For* $t > 0$, $(\alpha_1,\, \alpha_2) : \alpha_i^* = 0.15$*;* $(p_1,\, p_2) : p_1^* \approx 1.2\ t$ *and* $p_2^* \approx 1.2\ t$ *if* $\alpha_i \in [0, 1]$ *are SPNE of the game. Besides, profits in the SPNE is* 20% t *higher than in the case where* $\alpha_1 = \alpha_2 = 0$*, making it rewarding for firms to share profits.*

Proof. The proof is straightforward. It suffices to solve the rest of the game.

First-stage of the game

$$Max_{\alpha_1}\ P_1(\alpha_1,\alpha_2) \text{ or } (1-\alpha_1)\, p_1^* D_1(p_1^*,p_2^*) + \alpha_2 p_2^* D_2(p_1^*,p_2^*)$$

The first-order conditions with respect to α_1 give:
$\frac{\partial P_1}{\partial \alpha_1} = 0$

At the equilibrium we have $\alpha_1 = \alpha_2 = \alpha$:

$\Rightarrow \frac{\partial P_1}{\partial \alpha_1}\rfloor_{\alpha_1=\alpha_2=\alpha} = -\frac{1}{2}\left(4\alpha^3 - 10\alpha^2 + 8\alpha - 1\right)\frac{t}{(2\alpha-1)^2(2\alpha-3)} = 0$

By solving the above equation, we find: $\alpha^* = 0.15$

By replacing $\alpha_1 = \alpha_2 = \alpha^*$ in the profit function, we have:

$$P_1^*(\alpha^*,\alpha^*) = \frac{1}{2}(1-\alpha)\frac{t}{1-2\alpha^*}$$

Finally, replacing $\alpha^* = 0.15$ in the above equation and equations (14) & (15), we find $P_1^* = 0.60\ t$ and $p_1^* = p_2^* = 1.2\ t$

However, if firms decide not to share their profits, that is, $\alpha^* = 0$, we find $P_1^* = 0.50\ t$

Conclusion: For $t > 0$, $(\alpha_1,\, \alpha_2) : \alpha_i^* = 0.15$; $(p_1,\, p_2) : p_1^* \approx 1.2\ t$ and $p_2^* \approx 1.2\ t$ if $\alpha_i \in [0,1]$ are SPNE of the game. Besides, profits in the SPNE is 20% t higher than in the case where $\alpha_1 = \alpha_2 = 0$, making it rewarding for firms to share profits. ■

7 The Bertrand model with heterogenous goods

In this section, we study a problem similar to the one described in section 5, introducing the heterogeneity in the market. The only difference is that the strategic variable is now price. The latter matches with the case of direct demand which we are, here, interested to.

We consider two firms 1 and 2 in a heterogeneous market. We suppose that the marginal cost of production c is, without loss of generality, 0. Let $p_1 = 1 - q_1 - \gamma q_2$ and $p_2 = 1 - \gamma q_1 - q_2$ the respective inverse demand functions of firm 1 and firm 2 where q_1(resp. q_2) are firm 1's (resp. firm 2's) output. So, their respective demand functions are $q_1 = \frac{1}{1-\gamma^2}(1 - \gamma - p_1 + \gamma p_2)$ and $q_2 = \frac{1}{1-\gamma^2}(1 - \gamma - p_2 + \gamma p_1)$. Finally, assume that the profit of the two firms $\Pi_1(q_1, q_2) = p_1 q_1$ and $\Pi_2(q_1, q_2) = p_2 q_2$ are concave in q_i $(i = 1, 2)$.

As before, we introduce a grain of novelty in this sort of Bertrand model (with heterogeneous goods). Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) wants to share with firm 2 (resp. firm 1). We suppose that $\alpha_i \in [0, 1]$, $i = 1, 2$ $(i \neq j)$. Consequently, we can write the *new profit function* $P_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1 - \alpha_i)\Pi_i(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j) + \alpha_j \Pi_j(q_i(\alpha_i, \alpha_j), q_j(\alpha_i, \alpha_j))$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firms choose α_i. In the second stage of the game, firms select p_i.

In the *first stage of the game*, for α_1 and α_2 firms simultaneously solve:

$$Max_{a_1} \quad P_1 = (1 - \alpha_1)\Pi_1 + \alpha_2 \Pi_2$$

$$Max_{\alpha_2} \quad P_2 = (1 - \alpha_2)\Pi_2 + \alpha_1 \Pi_1$$

In the *second stage of game*, for p_1 and p_2 firms simultaneously solve:

$$Max_{p_1} \quad P_1 = (1 - \alpha_1)\Pi_1 + \alpha_2 \Pi_2$$

$$Max_{p_2} \quad P_2 = (1 - \alpha_2)\Pi_2 + \alpha_1 \Pi_1$$

7.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximizes their profits.

Proposition 8 *$\forall\ \gamma$, if $\alpha_i \in [0, 1]$ & $\alpha_1 + \alpha_2 \neq 1$, then any prices $(p_1^*,\ p_2^*)$ / $p_1^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_2)(1-\gamma)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ &*
$p_2^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_1)(1-\gamma)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ constitute a NE in the second stage of the game.

Proof. To see that, it suffices to solve the game backwards.

Second stage of the game

$$Max_{p_1}\ P_1(p_1,p_2) = (1-\alpha_1)pq_1 + \alpha_2 pq_2 \quad (18)$$

or $Max_{p_1}\ P_1(p_1,p_2) = \frac{1}{1-\gamma^2}\left[(1-\alpha_1)p_1(1-\gamma-p_1+\gamma p_2) + \alpha_2 p_2(1-\gamma-p_2+\gamma p_1)\right]$

The first-order conditions with respct to q_1 give:

$\frac{\partial P_1}{\partial p_1} = (1-\alpha_1)(1-\gamma-2p_1+\gamma p_2) + \alpha_2\gamma p_2 = 0$

$p_1(p_2) = \frac{1-\gamma}{2} + \gamma\frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)}p_2$

and by analogy:

$p_2(p_1) = \frac{1-\gamma}{2} + \gamma\frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)}p_1$

By solving the system of the last two previous equations, we have:

$$p_1^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_2)(1-\gamma)\left[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)\right]}{\left[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)\right]} \quad (19)$$

and by analogy,

$$p_2^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_1)(1-\gamma)\left[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)\right]}{\left[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)\right]} \quad (20)$$

Conclusion: *If* $\alpha_i \in [0,1]$ & $\alpha_1+\alpha_2 \neq 1$, then any prices (p_1^*, p_2^*) satisfying equations (19) & (20) constitute a NE in the second stage of the game. ■

The second-stage being entirely solved and NE being found, we can thus move to the first-stage of the game in order to find SPNE

7.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profits to share with their rivals.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE in prices summarized below:

$p_1^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_2)(1-\gamma)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ and

$p_2^*(\alpha_1,\alpha_2) = \frac{(1-\alpha_1)(1-\gamma)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ *if* $\alpha_i \in [0,1]$ & $\alpha_1+\alpha_2 \neq 1$

Since $q_1^* = \frac{1}{1-\gamma^2}(1-\gamma-p_1^*+\gamma p_2^*)$ and $q_2^* = \frac{1}{1-\gamma^2}(1-\gamma-p_2^*+\gamma p_1^*)$ we have:

$$q_1^*(\alpha_1,\alpha_2) = \frac{1}{1+\gamma}\frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]} \qquad (21)$$

$$q_2^*(\alpha_1,\alpha_2) = \frac{1}{1+\gamma}\frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_1(1-\alpha_1)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]} \qquad (22)$$

Using equations (19) – (22), we can rewrite the profit function[8]:

$P_1(\alpha_1,\alpha_2) = (1-\alpha_1)p_1^*q_1^* + \alpha_2 p_2^* q_2^*$

Now, in the current section, we draw our attention to the first-stage of the game searching for SPNE in α_i.

Proposition 9 *For $\gamma \approx -1$, $(\alpha_1, \alpha_2) : \alpha_i^* = 0.43$; $(p_1, p_2) : p_1^* = p_2^* \approx \frac{1}{2}$ if $\alpha_i \in [0,1]$ are SPNE of the game. Besides, profits in the SPNE is 12% higher than in the case where $\alpha_1 = \alpha_2 = 0$, making it rewarding for firms to share profits.*

Proof. The proof is straightforward. It suffices to solve the rest of the game.

First-stage of the game

$$Max_{\alpha_1}\ P_1(\alpha_1,\alpha_2) = (1-\alpha_1)p_1^*q_1^* + \alpha_2 p_2^* q_2^*$$

The first-order conditions with respect to α_1 give: $\frac{\partial P_1}{\partial \alpha_1} = 0$

At the equilibrium we have $\alpha_1 = \alpha_2 = \alpha$: $\Rightarrow \frac{\partial P_1}{\partial \alpha_1}\rfloor_{\alpha_1=\alpha_2=\alpha} = 0$

By solving the above equation, we find: $\alpha^* = 0.43$ and $\gamma \approx -1$

By replacing $\alpha_1 = \alpha_2 = \alpha$ in the profit function, we have:

$$\begin{aligned} P_1^*(\alpha,\alpha) \;\; = \;\; & (1-\alpha)^2(1-\gamma)\frac{[2-2a+\gamma]}{\left[4(1-\alpha)^2-\gamma^2\right]^2(1+\gamma)} \\ & \left((2+\gamma)(1-\alpha)^2 - \gamma(1+\gamma)\alpha(1-\alpha) - \gamma^2\alpha^2\right) \\ & \alpha(1-\alpha)(1-\gamma)\frac{[2-2a+\gamma]}{\left[4(1-\alpha)^2-\gamma^2\right]^2(1+\gamma)} \\ & \left((2+\gamma)(1-\alpha)^2 - \gamma(1+\gamma)\alpha(1-\alpha) - \gamma^2\alpha^2\right) \end{aligned}$$

[8] See Appendix for the complete expression of the profit function and other computation details.

Finally, substituing $\alpha^* = 0.43$ and $\gamma = -1$ in the above equation and in equation (19) & (20) we find $P_1^* = 26.4$ and $p_1^* = p_2^* = \frac{1}{2}$

However, if firms decide not to share their profits, that is, $\alpha^* = 0$ and $\gamma = -1$, we find $P_1 = 23.6$

Conclusion: For $\gamma \approx -1$, $(\alpha_1, \alpha_2) : \alpha_i^* = 0.43$; $(p_1, p_2) : p_1^* = p_2^* \approx \frac{1}{2}$ if $\alpha_i \in [0, 1]$ are SPNE of the game. Besides, profits in the SPNE is 12% higher than in the case where $\alpha_1 = \alpha_2 = 0$, making it rewarding for firms to share profits. ■

8 Conclusion

This paper has shown how two firms in a duopoly market may be able to increase their profits and, in turn, to limit the competition through the profit-sharing strategy, and identified the direct and indirect effects of such a strategy. Using different oligopolistic models, we have brought to light when give away profits could be a rewarding strategy for firms.

There are many dimensions along which this simple model can be improved and extended. For instance, since firms are not enforced by any contract, one can wonder what will happen if a firm stop unilaterally sharing its profit with its rival (while it itself keeps on receiving a part of its rival's profit). Waddle (2005d) *"Profit Sharing Between Firms: IV. A Win-Win Strategy"* investigates such an opportunistic behavior.

Likewise, a number of concerns occurs from the fact that firms' successful strategies have generally influenced other firms outside the market. For instance, subsequent profits might attract new firms in the market, ready to divide the cake. What should the incumbents do face to those eventual entrants? Should they block them or accommodate them? In the latter case, should they share their profits with those new arrivals? Waddle (2005b) *"Profit Sharing Between Firms: II. The Bertrand Model"* implicitly analyses these issues.

Our findings have interesting implications for economic development policy. For instance, a government can give subsidies (airport tax cuts) to airlines companies in order to attract tourists. Interestingly, some empirical evidence shows an increase of tourists in cities with low-cost airlines.

9 Appendix

Proof of Propositions 4 & 5

The demand functions are:

$$D_1(p_1,p_2) = x = \frac{1}{2} + \frac{p_2-p_1}{2t}$$

$$D_2(p_1,p_2) = 1 - x = \frac{1}{2} - \frac{p_2-p_1}{2t}$$

Second stage of the game:

$$Max_{p_1} \quad P_1 = (1-\ \alpha_1)p_1D_1(p_1,p_2)+\ \alpha_2 p_2 D_2(p_1,p_2)\ (1)$$

By replacing $D_1(p_1,p_2)$ and $D_2(p_1,p_2)$ by their values, we have:

$$Max_{p_1} \quad P_1 = (1-\ \alpha_1)p_1(\frac{1}{2} + \frac{p_2-p_1}{2t})+\ \alpha_2 p_2(\frac{1}{2} - \frac{p_2-p_1}{2t})\ (1)$$

The first-order conditions with respect to p_1 give:

$$(1-\alpha_1)\left(\frac{1}{2} + \frac{p_2-p_1}{2t}\right) + \frac{\alpha_2 p_2}{2t} = 0$$

$$\frac{1-\alpha_1}{2} + \frac{(1-\alpha_1)p_2}{2t} + \frac{\alpha_2 p_2}{2t} = \frac{(1-\alpha_1)}{t}p_1$$

By developing, we have:

$p_1 = \frac{t}{2} + \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)}p_2$

To summary, we have:

$p_1(p_2) = \frac{t}{2} + \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)}p_2$ (2)

and by analogy

$p_2(p_1) = \frac{t}{2} + \frac{1-\alpha_2+\alpha_1}{2(1-\alpha_2)}p_1$ (3)

Now, we can find $p_1(\ \alpha_1,\ \alpha_2)$ and $p_1(\ \alpha_1,\ \alpha_2)$. Let us substitute p_2 by its value into (2) and we have:

$p_1 = \frac{t}{2} + \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)} \left(\frac{t}{2} + \frac{1-\alpha_2+\alpha_1}{2(1-\alpha_2)} p_1 \right)$

$p_1 = -\frac{1}{4} \frac{-3t+2t\alpha_2+3t\alpha_1-3t\alpha_1\alpha_2-p_1-2\alpha_1 p_1 \alpha_2+p_1\alpha_1^2+t\alpha_2^2+p_1\alpha_2^2}{(-1+\alpha_1)(-1+\alpha_2)}$

$\Rightarrow 4(-1+\alpha_1)(-1+\alpha_2)p_1 +$

$(-3t+2t\alpha_2+3t\alpha_1-3t\alpha_1\alpha_2-p_1-2\alpha_1 p_1 \alpha_2+p_1\alpha_1^2+t\alpha_2^2+p_1\alpha_2^2) = 0$

Finally, we have:

$p_1 = -t \frac{-3+2\alpha_2+3\alpha_1-3\alpha_1\alpha_2+\alpha_2^2}{3+2\alpha_1\alpha_2+\alpha_1^2-4\alpha_2-4\alpha_1+\alpha_2^2}$

and by factorizing, we have:

$p_1^*(\ \alpha_1,\, \alpha_2) = t(1-\alpha_2) \frac{3-3\alpha_1+\alpha_2}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$

$p_2^*(\ \alpha_1,\, \alpha_2) = t(1-\alpha_1) \frac{3-3\alpha_2+\alpha_1}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)}$

and then :

$D_1(p_1^*, p_2^*) = \left(\frac{1}{2} + \frac{p_2^*-p_1^*}{2t} \right) = \left(\frac{1}{2} \frac{3-2\alpha_2}{3-\alpha_1-\alpha_2} \right)$

$D_2(p_1^*, p_2^*) = \left(\frac{1}{2} - \frac{p_2^*-p_1^*}{2t} \right) = \left(\frac{1}{2} \frac{3-2\alpha_1}{3-\alpha_1-\alpha_2} \right)$

Back to the equation (1), we have:

$P_1 = (1-\alpha_1) p_1^* D_1(p_1^*, p_2^*) + \alpha_2 p_2^* D_2(p_1^*, p_2^*)$

$P_1 = (1-\alpha_1) \left(\frac{1}{2} \frac{3-2\alpha_2}{3-\alpha_1-\alpha_2} \right) \left(t(1-\alpha_2) \frac{3-3\alpha_1+\alpha_2}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)} \right) +$

$\alpha_2 \left(\frac{1}{2} \frac{3-2\alpha_1}{3-\alpha_1-\alpha_2} \right) \left(t(1-\alpha_1) \frac{3-3\alpha_2+\alpha_1}{(1-\alpha_1-\alpha_2)(3-\alpha_1-\alpha_2)} \right)$

By simplifying and rearranging the above equation, we have:

$P_1(\ \alpha_1, \alpha_2) = \frac{t(1-\alpha_1)}{2(3-\alpha_1-\alpha_2)^2(1-\alpha_2-\alpha_1)} \left(2\alpha_2^3 - 8\alpha_2^2 - 3\alpha_2 + 12\alpha_1\alpha_2 - 2\alpha_1^2\alpha_2 + 9 - 9\alpha_1 \right)$ (4)

and by analogy:

$P_2(\ \alpha_1, \alpha_2) = \frac{t(1-\alpha_2)}{2(3-\alpha_1-\alpha_2)^2(1-\alpha_2-\alpha_1)} \left(2\alpha_1^3 - 8\alpha_1^2 - 3\alpha_1 + 12\alpha_1\alpha_2 - 2\alpha_2^2\alpha_1 + 9 - 9\alpha_2 \right)$ (5)

First stage of the game

$Max_{\ \alpha_1}\ P_1(\ \alpha_1, \alpha_2) = \frac{t(1-\alpha_1)}{2(3-\alpha_1-\alpha_2)^2(1-\alpha_2-\alpha_1)}\left(2\alpha_2^3 - 8\alpha_2^2 - 3\alpha_2 + 12\alpha_1\alpha_2 - 2\alpha_1^2\alpha_2 + 9 - 9\alpha_1\right)$

The first-order conditions with respect to α_1 give:

$\frac{dP_1}{d\alpha_1} = \frac{1}{2}t\left(-3\alpha_1 + 3 + \alpha_2\right)\frac{2\alpha_2^4-16\alpha_2^3+4\alpha_1\alpha_2^3-16\alpha_1\alpha_2^2+37\alpha_2^2+2\alpha_1^2\alpha_2^2+12\alpha_1\alpha_2-26\alpha_2+3-3\alpha_1^2}{(-3+\alpha_1+\alpha_2)^3(-1+\alpha_2+\alpha_1)^2}$

With $\alpha_1 = \alpha_2 = \alpha$, we have:

$\frac{dP_1}{d\alpha_1} = \frac{1}{2}t\left(-2\alpha + 3\right)\frac{8\alpha^4-32\alpha^3+46\alpha^2-26\alpha+3}{(2\alpha-3)^3(2\alpha-1)^2}$

$\frac{dP_1}{d\alpha_1} = -\frac{1}{2}\left(4\alpha^3 - 10\alpha^2 + 8\alpha - 1\right)\frac{t}{(2\alpha-1)^2(2\alpha-3)} \ = 0$

By solving the above equation, we find :

$\alpha^* = 15\%$

Now, we can evaluate the corresponding profit. By replacing $\alpha_1 = \alpha_2 = \alpha^*$ in the equation (4) , we find:

$P_1^* = \frac{1}{2}\left(1 - \alpha\right)\frac{t}{1-2\alpha^*}$

By replacing $\alpha^* = 15\%$ in the above equation, we find $P_1 = 60\%$

However, if firms decide not to share their profit, that is, $\alpha = 0$, we find $P_1 = 50\%\ t$.

Proof of Propositions 6 & 7

The inverse demand functions are:

$$p_1 = 1 - q_1 - \gamma q_2 \ (1)$$

$$p_1 = 1 - \gamma q_1 - q_2 \ (2)$$

Second stage of the game

$Max_{p_1}\ P_1 = (1- \alpha_1)p_1q_1 + \alpha_2p_2q_2$ (3)

By replacing q_1 and q_2 by their values, we have:

$Max_{q_1}\ P(q_1, q_2) = (1- \alpha_1)(1 - q_1 - q_2)q_1 + \alpha_2(1 - q_1 - q_2)q_2$

The first-order conditions with respect to q_1 give:

$\frac{\partial P_1}{\partial q_1} = 0$

$\Rightarrow (1-\ \alpha_1)(1 - 2q_1 - \gamma q_2)-\ \alpha_2\gamma q_2 = 0$

$\Rightarrow 2(1-\ \alpha_1)q_1 = (1-\ \alpha_1) - \gamma(1-\ \alpha_1+\ \alpha_2)q_2$ (4)

By simplifying, we have:

$q_1 = \frac{(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2)q_2}{2(1-\alpha_1)}$ (5)

And by analogy, we have:

$q_2 = \frac{(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1)q_1}{2(1-\alpha_2)}$ (6)

Using (5) and (6), we can find $q_1(\ \alpha_1,\ \alpha_2)$ and $q_2(\ \alpha_1,\ \alpha_2)$

$2(1-\ \alpha_1)q_1 = (1-\ \alpha_1) - \gamma(1-\ \alpha_1+\ \alpha_2)\frac{(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1)q_1}{2(1-\alpha_2)}$

$4(1-\ \alpha_1)(1-\ \alpha_2)q_1 = 2(1-\ \alpha_1)(1-\ \alpha_2) - \gamma(1-\ \alpha_1+\ \alpha_2)(1-\ \alpha_2)$
$+\gamma^2(1-\ \alpha_1+\ \alpha_2)(1-\ \alpha_2+\ \alpha_1)q_1$

$q_1^*(\ \alpha_1,\ \alpha_2) = \frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ (7)

and by anology,

$q_2^*(\ \alpha_1,\ \alpha_2) = \frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ (8)

Finally, by replacing $q_1^*(\ \alpha_1,\ \alpha_2)$ and $q_2^*(\ \alpha_1,\ \alpha_2)$ by their value in (1) and (2), we find easily:

$p_1^*(\ \alpha_1,\ \alpha_2) = \frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_2(1-\alpha_2+\alpha_1)+\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ (9)

$p_2^*(\ \alpha_1,\ \alpha_2) = \frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_1(1-\alpha_1+\alpha_2)+\gamma(1-\alpha_1)(1-\alpha_2+\alpha_1)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ (10)

Using the equations (5)-(8), we can write the profit function:

$$P_1(\ \alpha_1,\ \alpha_2) = (1-\ \alpha_1)p^*q_1^*+\ \alpha_2 p^*q_2^* \ (11)$$

$P_1 = (1-\alpha_1)\,\frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_2(1-\alpha_2+\alpha_1)+\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$

$\frac{(1-\alpha_2)(2(1-\alpha_1)-\gamma(1-\alpha_1+\alpha_2))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$

$+\ \alpha_2\frac{2(1-\gamma)(1-\alpha_1)(1-\alpha_2)-\gamma^2\alpha_1(1-\alpha_1+\alpha_2)+\gamma(1-\alpha_1)(1-\alpha_2+\alpha_1)}{4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$

$\frac{(1-\alpha_1)(2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1))}{4(1-\alpha_1)(1-\alpha_2)-(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)}$ (12)

First stage of the game:

$Max_{\alpha_1}\ P_1 = (1-\ \alpha_1)p_1^*q_1^*+\ \alpha_2 p_2^* q_2^*$

The first-order conditions with respect to α_1 give:

$\frac{dP_1}{d\alpha_1} = 0$

$\{(\gamma^2\alpha_2\,(2a_1-\alpha_2)+(1-\alpha_2)\,(-\gamma a_2-2a_1-2a_1\gamma))$
$((1-\alpha_2)\,(2\,(1-\alpha_1)-\gamma\,(1-\alpha_1+\alpha_2)))+(1-\alpha_2)\,(-2+\gamma)$

$((1-\alpha_1)\,(2\,(1-\gamma)\,(1-\alpha_1)\,(1-\alpha_2)-\gamma^2\alpha_2\,(1-\alpha_2+\alpha_1)+\gamma\,(1-\alpha_2)\,(1-\alpha_1+\alpha_2)))$

$+\,(\alpha_2\,(-2\,(1-\gamma)\,(1-\alpha_2)-(\gamma^2+\gamma)\,(1-\alpha_1+\alpha_2)-\gamma^2\alpha_1-\gamma\,(1-\alpha_1)))$

$((1-\alpha_1)\,(2\,(1-\alpha_2)-\gamma\,(1-\alpha_2+\alpha_1)))$

$+\,(-2\,(1-\alpha_2)+\gamma\,(1-\alpha_1+\alpha_2)-\alpha_1\gamma\,(1-\alpha_1))$

$(\alpha_2\,(2\,(1-\gamma)\,(1-\alpha_1)\,(1-\alpha_2)-\gamma^2\alpha_1\,(1-\alpha_1+\alpha_2)+\gamma\,(1-\alpha_1)\,(1-\alpha_2+\alpha_1)))\}$

$(4\,(1-\alpha_1)\,(1-\alpha_2)-\gamma^2\,(1-\alpha_1+\alpha_2)\,(1-\alpha_2+\alpha_1))^2$

$-4\,(4\,(1-\alpha_1)\,(1-\alpha_2)-\gamma^2\,(1-\alpha_1+\alpha_2)\,(1-\alpha_2+\alpha_1))\,(-2\,(1-\alpha_2)+\gamma^2\,(\alpha_1-\alpha_2))$

$\{((1-\alpha_1)\,(2\,(1-\gamma)\,(1-\alpha_1)\,(1-\alpha_2)-\gamma^2\alpha_2\,(1-\alpha_2+\alpha_1)+\gamma\,(1-\alpha_2)\,(1-\alpha_1+\alpha_2)))$

$+\,((1-\alpha_2)\,(2\,(1-\alpha_1)-\gamma\,(1-\alpha_1+\alpha_2)))$

$+\,(\alpha_2\,(-2\,(1-\gamma)\,(1-\alpha_2)-(\gamma^2+\gamma)\,(1-\alpha_1+\alpha_2)-\gamma^2\alpha_1-\gamma\,(1-\alpha_1)))$

$((1-\alpha_1)\,(2\,(1-\alpha_2)-\gamma\,(1-\alpha_2+\alpha_1)))\} = 0$

At the equilibrium we have $\alpha_1 = \alpha_2 = \alpha$:

$\Rightarrow \frac{\partial P_1}{\partial \alpha_1}\rfloor_{\alpha_1=\alpha_2=\alpha} = 0$

$\Rightarrow \left(\gamma^2\alpha^2 + (1-\alpha)(-3a\gamma - 2a)\right)(1-\alpha)(2-2a-\gamma) + (1-\alpha)^2(-2+\gamma)$
$\left(2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)\right) +$
$\alpha^2\left(-2(1-\gamma)(1-\alpha) - \gamma^2 - \gamma - \gamma^2\alpha - \gamma(1-\alpha)\right)$
$(1-\alpha)(2-2a-\gamma)\left(-2+2a+\gamma - a\gamma(1-\alpha)\right)$
$\left(2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)\right)$
$\left(4(1-\alpha)^2 - \gamma^2\right)^2 - 4\left(4(1-\alpha)^2 - \gamma^2\right)(-2+2a)$
$(1-\alpha)\left(2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)\right) +$
$(1-\alpha)(2-2a-\gamma)$
$+\ \alpha\left(-2(1-\gamma)(1-\alpha) - \gamma^2 - \gamma - \gamma^2\alpha - \gamma(1-\alpha)\right)$
$(1-\alpha)(2-2a-\gamma) = 0$

By solving the above equation, we find :

$\{\alpha = 1\}, \{\alpha = \rho\}, \left\{\alpha = 1 + \frac{1}{2}\gamma\right\}, \left\{\alpha = 1 - \frac{1}{2}\gamma\right\}$ where ρ is a root of:

$(8\gamma^4 - 24\gamma^2 + 16\gamma) Z^9 + (-24\gamma^4 + 128\gamma^2 + 32 - 144\gamma + 4\gamma^3 + 4\gamma^5) Z^8$
$+ (38\gamma^4 - 192 - 2\gamma^6 + 512\gamma + 4\gamma^3 - 328\gamma^2 - 8\gamma^5) Z^7$
$+ (-960\gamma + 528\gamma^2 - 56\gamma^3 + 480 + 2\gamma^6 + 3\gamma^5 - 32\gamma^4 - \gamma^7) Z^6$
$+ (1040\gamma - 2\gamma^6 - 640 - 552\gamma^2 + 112\gamma^3) Z^5 +$
$(-100\gamma^3 + 14\gamma^4 - 3\gamma^6 + 352\gamma^2 + 480 + 6\gamma^5 - 632\gamma) Z^4$
$+ (-\gamma^7 + 156\gamma - 200 - 9\gamma^5 + 44\gamma^3 - 108\gamma^2 - 2\gamma^4 + 5\gamma^6) Z^3 +$
$(48 + 36\gamma + 4\gamma^5 - \gamma^6 - 14\gamma^3 + 8\gamma^2 - 2\gamma^4) Z^2 +$
$(7\gamma^3 - 16 - \gamma^4 - 6\gamma^2 - 28\gamma) Z + 8 + 2\gamma^2 + 4\gamma - \gamma^3$ (15)

An approximative solution of the equation (15) is[9]:

$\alpha^* = 50\%$ and $\gamma = -1$

Now, we can evaluate the corresponding profit. By replacing $\alpha_1 = \alpha_2 = \alpha$ in the equation (12), we find:

$P_1^*(\alpha, \alpha) = (1-\alpha)^2 \frac{2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)}{4(1-\alpha)^2 - \gamma^2} \frac{2-2a-\gamma}{4(1-\alpha)^2 - 1} +$
$\alpha \frac{2(1-\gamma)(1-\alpha)^2 - \gamma^2\alpha + \gamma(1-\alpha)}{4(1-\alpha)^2 - \gamma^2} (1-\alpha) \frac{2-2a-\gamma}{4(1-\alpha)^2 - 1}$

By replacing $\alpha^* = 45\%$ and $\gamma = -1$ in the above equation, we find $P_1^* = 5.5$

However, if firms decide not to share their profit, that is, $\alpha^* = 0$ and $\gamma = -1$, we find $P_1 = 1$

[9]We use "Mapple", a program integrated to "Scientific Workplace" to solve this equation.

Proof of Propositions 8 & 9

The inverse demand functions are:

$$p_1 = 1 - q_1 - \gamma q_2 \ (1)$$

$$p_1 = 1 - \gamma q_1 - q_2 \ (2)$$

Let's solve the system of equations (1) and (2) by the Cramer method:

$(1) \Rightarrow q_1 + \gamma q_2 = 1 - p_1$
$(2) \Rightarrow \gamma q_1 + q_2 = 1 - p_2$

$D = 1 - \gamma^2$
$N_{q_1} = 1 - \gamma - p_1 + \gamma p_2$

$N_{q_2} = 1 - \gamma - p_2 + \gamma p_1$
$q_1 = \frac{N_{q_1}}{D} = \frac{1}{1-\gamma^2}(1 - \gamma - p_1 + \gamma p_2) \ (3)$
$q_2 = \frac{N_{q_2}}{D} = \frac{1}{1-\gamma^2}(1 - \gamma - p_2 + \gamma p_1) \ (4)$

The demand functions are:

$$q_1 = \frac{1}{1-\gamma^2}(1 - \gamma - p_1 + \gamma p_2) \ (3)$$

$$q_2 = \frac{1}{1-\gamma^2}(1 - \gamma - p_2 + \gamma p_1) \ (4)$$

Second stage of the game:

$Max_{p_1} \ P_1 = (1 - \alpha_1) p_1 q_1 + \alpha_2 p_2 q_2 \ (5)$

By replacing q_1 and q_2 by their values, we have:

$Max_{p_1} \ P_1 = \frac{1}{1-\gamma^2} \left[(1 - \alpha_1) p_1 (1 - \gamma - p_1 + \gamma p_2) + \alpha_2 p_2 (1 - \gamma - p_2 + \gamma p_1) \right]$

The first-order conditions with respect to p_1 give:

$\frac{\partial P_1}{\partial p_1} = (1 - \alpha_1)(1 - \gamma - 2p_1 + \gamma p_2) + \alpha_2 \gamma p_2 = 0$

$\Leftrightarrow (1 - \alpha_1)(1 - \gamma) - 2(1 - \alpha_1) p_1 + \gamma (1 - \alpha_1 + \alpha_2) p_2 = 0 \ (6)$

We have:

$p_1(p_2) = \frac{1-\gamma}{2} + \gamma \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)} p_2$ (7)

and by analogy,

$p_2(p_1) = \frac{1-\gamma}{2} + \gamma \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)} p_1$ (8)

Solving equations (7) and (8), we can write:

$(7) \Rightarrow 2(1-\alpha_1)p_1 = (1-\alpha_1)(1-\gamma) + \gamma(1-\alpha_1+\alpha_2)\left[\frac{1-\gamma}{2} + \gamma \frac{1-\alpha_1+\alpha_2}{2(1-\alpha_1)} p_1\right]$

$\Rightarrow \left[4(1-\alpha_1)(1-\alpha_2) - \gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)\right] p_1 =$
$(1-\alpha_2)(1-\gamma)\left[2(1-\alpha_1) + \gamma(1-\alpha_1+\alpha_2)\right]$

$p_1^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_2)(1-\gamma)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ (9)

and by anology,

$p_2^*(\alpha_1, \alpha_2) = \frac{(1-\alpha_1)(1-\gamma)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ (10)

Using (3) and (4), we can find $q_1(\alpha_1, \alpha_2)$ and $q_2(\alpha_1, \alpha_2)$

$q_1 = \frac{1}{1-\gamma^2}(1-\gamma - \frac{(1-\alpha_2)(1-\gamma)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]} +$

$\gamma \frac{(1-\alpha_1)(1-\gamma)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]})$

$q_1 = \frac{1}{1+\gamma} \frac{\left[2(1+\gamma)(1-\alpha_1)(1-\alpha_2)+\gamma^2(1-\alpha_2+\alpha_1)(1-\alpha_1-1+\alpha_1-\alpha_2)-\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)\right]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1 = \frac{1}{1+\gamma} \frac{\left[2(1+\gamma)(1-\alpha_1)(1-\alpha_2)+\gamma^2(1-\alpha_2+\alpha_1)(-\alpha_2)-\gamma(1-\alpha_2)(1-\alpha_1+\alpha_2)\right]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1 = \frac{1}{1+\gamma} \frac{(1-\alpha_2)[2(1-\alpha_1)+2\gamma(1-\alpha_1)-\gamma(1-\alpha_1)-\gamma a_2)]-\alpha_2\gamma^2(1-\alpha_2+\alpha_1)}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1 = \frac{1}{1+\gamma} \frac{(1-\alpha_2)[(2+\gamma)(1-\alpha_1)-\gamma a_2)]-\alpha_2\gamma^2(1-\alpha_2+\alpha_1)}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1 = \frac{1}{1+\gamma} \frac{(1-\alpha_2)(2+\gamma)(1-\alpha_1)-\gamma a_2(1-\alpha_2+\gamma(1-\alpha_2+\alpha_1)}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1 = \frac{1}{1+\gamma} \frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma a_2[(1+\gamma)(1-\alpha_2)+\gamma a_1]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$q_1^*(\alpha_1, \alpha_2) = \frac{1}{1+\gamma} \frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ (11)

and by analogy, we have:

$q_2^*(\alpha_1, \alpha_2) = \frac{1}{1+\gamma} \frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_1(1-\alpha_1)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ (12)

Using the equations (9)-(12), we can write the profit function

$P_1 = (1-\alpha_1)p_1^*q_1^* + \alpha_2 p_2^* q_2^*$ (13)

$P_1 = (1-\alpha_1)\frac{(1-\alpha_2)(1-\gamma)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}\frac{1}{1+\gamma}$

$\frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$

$+ \alpha_2 \frac{(1-\alpha_1)(1-\gamma)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}\frac{1}{1+\gamma}$

$\frac{(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_1(1-\alpha_1)-\gamma^2\alpha_1\alpha_2}{[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]}$ (14)

First stage of the game:

$Max_{\alpha_1}\ P_1 = (1-\alpha_1)p_1^*q_1^* + \alpha_2 p_2^* q_2^*$

The first-order conditions with respect to P_1 give:

$\frac{\partial P_1}{\partial \alpha_1} = 0$

$$\begin{aligned}
&\Rightarrow [(1-\alpha_2)(1-\gamma)\{[(1-\alpha_1)(-4-\gamma)-\gamma(1-\alpha_1+\alpha_2)] \\
&[(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2] \\
&-[(2+\gamma)(1-\alpha_2)+\gamma^2\alpha_2](1-\alpha_1)[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)]\} \\
&+\alpha_2(1-\gamma)\{[-2(1-\alpha_2)-\gamma(1-\alpha_2+\alpha_1)+\gamma(1-\alpha_1)] \\
&[(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1+\gamma)\alpha_1(1-\alpha_1)-\gamma^2\alpha_1\alpha_2] \\
&-2[(2+\gamma)(1-\alpha_2)-2\gamma(1+\gamma)\alpha_1+\gamma(1+\gamma)+\gamma^2\alpha_2] \\
&(1-\alpha_1)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)]\}] \\
&[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)]^2(1+\gamma) \\
&-4[4(1-\alpha_1)(1-\alpha_2)-\gamma^2(1-\alpha_1+\alpha_2)(1-\alpha_2+\alpha_1)] \\
&[-2(1-\alpha_2)+\gamma^2(\alpha_1-\alpha_2)](1+\gamma)\{(1-\gamma)(1-\alpha_1)(1-\alpha_2) \\
&[2(1-\alpha_1)+\gamma(1-\alpha_1+\alpha_2)] \\
&[(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1-\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2] \\
&+\alpha_2(1-\gamma)(1-\alpha_1)[2(1-\alpha_2)+\gamma(1-\alpha_2+\alpha_1)] \\
&[(2+\gamma)(1-\alpha_2)(1-\alpha_1)-\gamma(1-\gamma)\alpha_2(1-\alpha_2)-\gamma^2\alpha_1\alpha_2]\} = 0
\end{aligned}$$

$\frac{\partial P_1}{\partial \alpha_1}\rfloor_{\alpha_1=\alpha_2=\alpha} = 0$ gives:

$$\begin{aligned}
&[(1-\alpha)(1-\gamma)\{[(1-\alpha)(-4-\gamma)-\gamma] \\
&[(2+\gamma)(1-\alpha)^2-\gamma(1+\gamma)\alpha(1-\alpha)-\gamma^2\alpha^2] \\
&-[(2+\gamma)(1-\alpha)+\gamma^2\alpha](1-\alpha)[2(1-\alpha)+\gamma]\}
\end{aligned}$$

$+ \alpha (1-\gamma) \{[-2(1-\alpha) - \gamma + \gamma(1-\alpha)]$
$[(2+\gamma)(1-\alpha)^2 - \gamma(1+\gamma)\ \alpha(1-\alpha) - \gamma^2\ \alpha^2]$
$-2[(2+\gamma)(1-\alpha) - 2\gamma(1+\gamma)\ \alpha + \gamma(1+\gamma) + \gamma^2\ \alpha]$
$(1-\alpha)[2(1-\alpha)+\gamma]\}]\ [4(1-\alpha)^2 - \gamma^2]^2\ (1+\gamma) -$
$4[4(1-\alpha)^2 - \gamma^2][-2(1-\alpha)]\ (1+\gamma)\ \{(1-\gamma)\ (1-\alpha)^2$
$[2(1-\alpha)+\gamma][(2+\gamma)(1-\alpha)^2 - \gamma(1-\gamma)$
$\alpha(1-\alpha) - \gamma^2\ \alpha^2] + \alpha(1-\alpha)\ (1-\gamma)\ [2(1-\alpha)+\gamma]$
$[(2+\gamma)(1-\alpha)^2 - \gamma(1-\gamma)\ \alpha(1-\alpha) - \gamma^2\ \alpha^2]\} = 0$

By solving the above equation, we find :

$\left\{\alpha = -\frac{1}{2}\gamma + 1\right\}, \{\alpha = \rho\}, \left\{\alpha = 1 + \frac{1}{2}\gamma\right\}, \left\{\alpha = 1 + \frac{1}{2}\gamma\right\}$ where ρ is a root of:

$(8+8\gamma)\ Z^5 + (-52-48\gamma)\ Z^4 + (-2\gamma^3 + 106\gamma + 128)\ Z^3 +$
$(6\gamma^2 - \gamma^4 + 9\gamma^3 - 110\gamma - 152)\ Z^2 + (-12\gamma^2 + 88 + 54\gamma - 10\gamma^3)\ Z$
$+3\gamma^3 - 20 + 6\gamma^2 - 10\gamma$ (15)

An approximative solution of the equation (15) is[10]:

$\alpha^* = 43\%$ and $\gamma = -0.990\ 57$

Now, we can evaluate the corresponding profit. By replacing $\alpha_1 = \alpha_2 = \alpha$ in the equation (14) , we find:

$P_1(\ \alpha, \alpha) = (1-\alpha)^2\ (1-\gamma)\ \frac{[2-2a+\gamma]}{\left[4(1-\alpha)^2-\gamma^2\right]^2(1+\gamma)}$
$\left((2+\gamma)\ (1-\alpha)^2 - \gamma\ (1+\gamma)\ \alpha\ (1-\alpha) - \gamma^2\alpha^2\right) +$
$\alpha\ (1-\alpha)\ (1-\gamma)\ \frac{[2-2a+\gamma]}{\left[4(1-\alpha)^2-\gamma^2\right]^2(1+\gamma)}$
$\left((2+\gamma)\ (1-\alpha)^2 - \gamma\ (1+\gamma)\ \alpha\ (1-\alpha) - \gamma^2\alpha^2\right)$

By replacing $\alpha^* = 43\%$ and $\gamma = -0.990\ 057$ in the above equation, we find $P_1^* = 26.4$

However, if firms decide not to share their profit, that is, $\alpha^* = 0$ and $\gamma = -0.990\ 057$, we find $P_1 = 23.6$

[10] We use "Mapple", a program integrated to "Scientific Workplace" to solve this equation.

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