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Notes, Comments, and Letters to the Editor

Strategic interaction between futures and spot markets

José Luis Ferreira*

Departamento de Economía, Universidad Carlos III de Madrid, Getafe, Madrid 28903, Spain

Abstract

We study an oligopolistic industry where firms are able to sell in a futures market at infinitely many moments prior to the spot market. A kind of Folk-theorem is established any outcome between perfect competition and Cournot can be sustained in equilibrium. We then find that the Cournot outcome can be sustained by a renegotiation-proof equilibrium. However, this is not true for the competitive outcome. Furthermore, only the monopolistic outcome is renegotiation-proof if firms can buy and sell in the futures market. These results suggest, contrary to existing literature, that the introduction of futures markets may have an *anti-competitive* effect.

JEL classification: C72; G13; L13

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1. Introduction

Future markets for contracting output may arise as a consequence of firms' attempt to insure themselves against price fluctuations or to take advantage of arbitrage opportunities. The strategic consequences of the introduction of futures markets, however, are unclear. Williams [9] studies this issue in a model where firms' positions in the futures market affect their cost of production in the spot market, thus altering the circumstances of competition. The strategic interaction between the two markets in a Cournot duopoly is studied by Allaz and Villa [1] (A&V

*Fax: +34-91-624-9875.
E-mail address: jlferr@eco.uc3m.es.

henceforth). They find that the introduction of a futures market induces firms to compete more aggressively. Specifically, A&V show that as the number of periods prior to the spot market at which firms can contract in the futures market increases, the equilibrium outcome approaches the competitive outcome.

In this paper we study a model identical to A&V's, except that the number of periods prior to the spot market where firms can contract in the futures market is infinite. Contrary to A&V, we find that any price between the competitive and Cournot prices can be sustained by a subgame perfect equilibrium. A result that resembles a Folk theorem is thus obtained. (Note, however, that our model is not a repeated game.) We then study the robustness of equilibria to renegotiation, and show that the Cournot outcome can be sustained by a *renegotiation-proof equilibrium* (RPE), whereas the competitive outcome cannot. Furthermore, the Cournot outcome is the unique outcome that can be sustained as a symmetric RPE. Finally, we show that if firms are allowed to buy (as well as to sell) in the futures market, then the unique outcome that can be sustained by an RPE is the monopoly outcome. Contrary to A&V, our findings suggest that the introduction of a futures market is likely to have an anti-competitive effect.

The paper is organized as follows: In Section 2 we present the benchmark model. In Section 3 the infinite case is analyzed and the main results are presented. Section 4 concludes.

2. The basic model

In this section, we present the model in [1]. Consider a Cournot duopoly for a homogeneous good where firms can produce at zero cost, and face a market demand given by $p = A - q$ if $q \in [0, A]$, and $p = 0$ if $q > A$ $(p, q \geqslant 0)$. Prior to competing in the spot market, firms may sell part of their production in a futures market. Denote by $s_i \geqslant 0$ and $f_i \geqslant 0$ the quantities that Firm i sells in the spot and futures market, respectively, and write $q_i = s_i + f_i$. We assume that positions in the futures market are observable. To find the subgame perfect equilibria of this game we proceed backwards. In the second stage, given (f_1, f_2) Firm i solves

$$\max_{s_i} \quad ps_i$$
$$\text{s.t.} \quad p = A - f_1 - f_2 - s_1 - s_2.$$

The solution gives us the reaction function $s_i = \frac{A-f_1-f_2-s_j}{2}$. Solving for s_1, s_2, q_1, q_2 and p we get $s_1 = s_2 = p = \frac{A-f_1-f_2}{3}$ and $q_i = \frac{A+2f_i-f_j}{3}$.

Since the future and spot prices are equal (no-arbitrage) the problem for Firm i in the first stage is

$$\max_{f_i} \quad pq_i$$
$$\text{s.t.} \quad q_i = \frac{A+2f_i-f_j}{3},$$
$$p = \frac{A-f_1-f_2}{3}.$$

First-order conditions give reaction functions $f_i = \frac{A-f_j}{4}$ for $i=1,2$. Therefore, equilibrium quantities and price are $f_1^* = f_2^* = \frac{A}{5}$, $p^* = s_1^* = s_2^* = \frac{A}{5}$, $q_1^* = q_2^* = \frac{2}{5}A$. Firms' profits are $\Pi_1^* = \Pi_2^* = \frac{2}{25}A^2$.

Note that in this framework the introduction of a futures market has a pro-competitive effect (the Cournot equilibrium, in the absence of the futures market, is $q_i^C = p^C = \frac{A}{3}$, with $\Pi_i^C = \frac{A^2}{9}$). To understand the source of this effect, suppose that $f_2 = 0$, then Firm 1 sets $f_1 = \frac{A}{4}$ resulting in $p = s_i = \frac{A}{4}$, $q_1 = \frac{1}{2}A$, and $q_2 = \frac{1}{4}A$. In this case firms behave as if Firm 1 is the Stackelberg leader and Firm 2 the follower. This is not an equilibrium as Firm 2 also has an incentive to take positions in the futures market. The result is that firms enter a prisoners' dilemma resulting in a higher quantity being produced by the industry in equilibrium.

If firms are allowed to hold positions at $T < \infty$ periods in the futures market, the equilibrium is calculated in a similar fashion, by simply adding additional stages and solving backwards. The equilibrium outcome (see A&V for further details) is given by $p = s_i = f_i^t = \frac{A}{3+2T}$, $q_i = A\frac{1+T}{3+2T}$, where f_i^t is the futures position of Firm i at time t. Thus, the limit of the equilibrium outcome as T goes to infinity is the competitive outcome ($q = q_1 + q_2 = A$ and $p = 0$).

3. The infinite case

In this section we study the equilibria of an industry identical to the one described in Section 2, introducing an infinite number of periods in the futures market. Specifically, the futures market is open at times in $\mathscr{T} = \{t_1, t_2, \dots\}$, with $t_k = \frac{k}{k+1}$ and $k \in \{1, 2, \dots\}$. The spot market takes place at time $t = 1$. Recall that in Section 2 we found that the limit of the equilibria as the number of periods in the futures market goes to infinity gives the competitive outcome. Although this limit is one of the equilibria in the infinite case, in Proposition 2 we show that there are many others.

For $k \in \{1, 2, \dots, \infty\}$, denote by $f_i^k \geqslant 0$ the quantity that Firm i sells in the futures market at time t_k. Let $F_i^k = \sum_{n \leqslant k} f_i^n$ be the accumulated futures positions by Firm i at time t_k and define $F^k = F_1^k + F_2^k$, $F_i = F_i^\infty$, and $F = F_1 + F_2$ (i.e., F_i is the accumulated futures positions by Firm i at the time of the spot market, and F is the total by both firms—note that F_i and F are well defined). If F is sold in the futures market, firms compete *a la* Cournot in the spot market for the residual demand $q = A - F - p$. Because there is no discount, no-arbitrage requires that prices in all markets be the same.

The actions of firms at different stages are defined recursively. At time t_1 Firm i chooses $f_i^1 \geqslant 0$. At any other time $t_k \in \mathscr{T}$ Firm i chooses $f_i^k \geqslant 0$ as a function of vectors $(f_1^h)_{h<k}$ and $(f_2^h)_{h<k}$, with $h \in \{1, 2, \dots\}$; i.e., $f_i^k = f_i^k\ ((f_1^h, f_2^h)_{h<k})$. At time $t = 1$ (spot market) Firm i chooses $s_i = s_i((f_1^h, f_2^h)_h)$. A strategy for Firm i is a pair $((f_i^h)_h, s_i)$, where s_i is a function and $(f_i^h)_h$ a sequence of functions as defined above. Payoffs are profits defined by $\Pi_i = pq_i$, where $q_i = F_i + s_i$, and $q = q_1 + q_2$. Proposition 1 characterizes the set of payoffs that can be sustained by *subgame*

perfect equilibria (SPE). The following sets in $\Re^2_+$ will play an important role:

$$\Phi = \{(f_1,f_2) \mid f_1 + f_2 \leqslant A\},$$

$$\Omega = \{(q_1,q_2) \mid q_1 + q_2 \leqslant A, q_2 \geqslant A - 2q_1, q_1 \geqslant A - 2q_2\},$$

$$\Pi = \{(\Pi_1,\Pi_2) \mid \Pi_1 \leqslant A\sqrt{\Pi_2} - 2\Pi_2, \Pi_2 \leqslant A\sqrt{\Pi_1} - 2\Pi_1\}.$$

Simple algebra shows that $(f_1,f_2) \in \Phi$ if and only if $(q_1,q_2) = (f_1 + s_1, f_2 + s_2) \in \Omega$ with $s_i = \frac{A-f_1-f_2}{3}$, and that $(\Pi_1,\Pi_2) \in \Pi$ if and only if there exists a $(f_1,f_2) \in \Phi(f_1,f_2)$ such that $pq_i = \Pi_i$ with $p = A - q$, $q_i = f_i + s_i$ and $s_i = \frac{A-f_1-f_2}{3}$.

Proposition 1. *A pair of profits*, (Π_1,Π_2), *may be sustained by an* SPE *if and only if it belongs to* Π.

Proof. Let $(\Pi_1^*,\Pi_2^*) \in \Pi$, let $(q_1^*,q_2^*) \in \Omega$ be the unique solution to $\Pi_1^* = (A - q_1 - q_2)q_1$, $\Pi_2^* = (A - q_1 - q_2)q_2$, and let $(f_1^*,f_2^*) \in \Phi$ be the unique solution to

$$q_1^* = s_1 + f_1,$$

$$q_2^* = s_2 + f_2,$$

$$s_1 = s_2 = \frac{A - f_1 - f_2}{3}.$$

Define $\bar{f}_i^k = f_i^*$ if $k = 1$, and $\bar{f}_i^k = 0$ if $k > 1$. Define also $F^0 = 0$. In (i)–(iv) below we construct a strategy profile that gives payoffs (Π_1^*,Π_2^*) and show that it is an SPE: (i) defines actions for every stage of the game in the different subgames, (ii)–(iv) classify subgames in three groups (states), C, P_1 and P_2, according to history.

(i) At time t_k, if the game is at state C, firms play $(f_1^k,f_2^k) = (\bar{f}_1^k,\bar{f}_2^k)$, whereas if the game is at state P_j firms play $f_i^k = \frac{1}{4}(A - F^{k-1})$ and $f_j^k = 0$. At time $t = 1$ (spot market), firms play $s_1 = s_2 = \frac{A-F}{3}$.

(ii) At t_1 the game is said to be at state C.

(iii) If the game is in state C at time t_k, then, at time t_{k+1} (a) it remains at state C if $(f_1^k,f_2^k) = (\bar{f}_1^k,\bar{f}_2^k)$ or $f_i^k \neq \bar{f}_i^k$ for $i = 1,2$, or (b) it goes to state P_j if $f_i^k = \bar{f}_i^k$ and $f_j^k \neq \bar{f}_j^k$.

(iv) If the game is in state P_j at time t_k, then, at time t_{k+1} (a) it goes to state C if $f_i^k = \frac{1}{4}(A - F^{k-1})$ and $f_j^k = 0$ or if $f_i^k \neq \frac{1}{4}(A - F^{k-1})$ and $f_j^k \neq 0$, (b) it remains in state P_j if $f_i^k = \frac{1}{4}(A - F^{k-1})$ and $f_j^k \neq 0$ or (c) it goes to state P_i if $f_i^k \neq \frac{1}{4}(A - F^{k-1})$ and $f_j^k = 0$.

Notice that the strategy profile described in (i)–(iv) is well defined, and that, by construction, it gives payoffs (Π_1^*,Π_2^*). Now we show that this strategy is an SPE, i.e., a Nash equilibrium in every subgame.

Consider, first, one-stage deviations at time t_k.

Case 1: *Subgames in state C*. Assuming no deviations, $(f_1^h, f_2^h) = (0,0)$ for all $h \geqslant k$, in the remaining of the game, Firm j gets $\frac{(A-F^{k-1})^2}{9}$. If Firm j chooses $f_j^k \neq 0$ it can get at most the solution to the following problem:

$$\begin{aligned}
\max_{f_j^k} \quad & \Pi_j^k = p(f_j^k + s_j) \\
\text{s.t.} \quad & p = A - (F^k + s_j + f_i^{k+1} + s_i), \\
& f_i^{k+1} = \frac{1}{4}(A - F^k), \\
& s_i = s_j = \frac{A - F^k - f_i^{k+1}}{3}, \\
& F^k = F^{k-1} + f_j^k, \\
& p \geqslant 0, \quad s_i \geqslant 0, \quad f_i^{k+1} \geqslant 0, \quad f_j^k \geqslant 0,
\end{aligned}$$

where Π_j^k are Firm j's profits derived from selling in the futures market from time k on, and from selling in the spot market. The solution gives $f_j^k = \frac{A-F^{k-1}}{3}$ and $f_i^{k+1} = \frac{A-F^{k-1}}{6}$, with the other variables following the expressions $s_1 = s_2 = p = \frac{A-F^{k-1}}{6}$, $q_j = \frac{A-F^{k-1}}{2}$, $q_i = \frac{A-F^{k-1}}{3}$, $\Pi_i^k = \frac{(A-F^{k-1})^2}{18}$ and $\Pi_j^k = \frac{(A-F^{k-1})^2}{12} < \frac{(A-F^{k-1})^2}{9}$. Hence we conclude that no one-stage deviations are profitable at state C.

Case 2: *Deviation by Firm j in state P_j*. Repeat Case 1 except that now $F^k = F^{k-1} + f_j^k + f_i^k$, with $f_i^k = \frac{1}{4}(A - F^{k-1})$. The solution gives $f_j^k = \frac{A-F^{k-1}-f_i^k}{3}$ and $f_i^{k+1} = \frac{A-F^{k-1}-f_i^k}{6}$, with $s_1 = s_2 = p = \frac{A-F^{k-1}-f_i^k}{6}$, $q_j = \frac{A-F^{k-1}-f_i^k}{2}$, $q_i = \frac{A-F^{k-1}-f_i^k}{3}$, $\Pi_j = \frac{(A-F^{k-1}-f_i^k)^2}{12}$, and $\Pi_i = \frac{(A-F^{k-1}-f_i^k)^2}{18}$. Again Firm j enjoys smaller profits than if it complies with the strategy, $\Pi_j = \frac{(A-F^{k-1}-f_i^k)^2}{9}$.

Case 3: *Deviation by Firm i in state P_j*. To see that Firm i does not have a profitable deviation when it is punishing Firm j notice that $f_i^k = \frac{1}{4}(A - F^{k-1})$ solves the following problem:

$$\begin{aligned}
\max_{f_i^k} \quad & \Pi_i^k = p(f_i^k + s_i) \\
\text{s.t.} \quad & p = A - (F^{k-1} + f_i^k + s_i + s_j), \\
& s_1 = s_2 = \frac{A - F^{k-1} - f_i^k}{3}, \\
& p \geqslant 0, \quad s_i \geqslant 0, \quad f_i^k \geqslant 0.
\end{aligned}$$

This means that, in these subgames, Firm i is already playing its best reply given no further future positions, which is the case if firms follow the described strategy. Unless Firm i plays $f_i^k \geqslant A - F^{k-1}$, in which case $p = 0$ and $\Pi_i^k = 0$, as described in the strategy, any other action by Firm i, $f_i^k = \hat{f}_i^k \neq \frac{1}{4}(A - F^{k-1})$, will have the consequence of Firm j playing $f_j^{k+1} > 0$. Then we can write

$$\Pi_i^k(f_i^k = \tfrac{1}{4}(A - F^{k-1}), f_j^{k+1} = 0)$$
$$< \Pi_i^k(f_i^k = \hat{f}_i^k, f_j^{k+1} = 0)$$
$$< \Pi_i^k(f_i^k = \hat{f}_i^k, f_j^{k+1} > 0).$$

Hence, in any case, Firm i does not gain by deviating.

Case 4: *Subgame at time* $t = 1$ (*spot market*). The only Nash equilibrium is $s_1 = s_2 = \frac{A-F}{3}$, as required by the strategy.

Consider now a deviation in finitely many periods: the last deviation cannot be profitable, as it would contradict one of the cases above.

Finally, if the deviation includes infinitely many periods, it implies $F = A$ and zero profits for the firms. To see this, recall that $F^k = \sum_{n<k}(f_1^n + f_2^n)$ and that, if player j deviates at infinitely many periods, then $f_i^k = \frac{1}{4}(A - F^{k-1})$ for infinitely many moments t_k.

In order to complete the proof, we need to show that profits not in Π cannot be supported by an SPE. However, simple algebra shows that profits not in Π can only be attained with negative futures positions.

The strategy in the proof is simple, although formally requires some elaboration. Firms start by holding enough positions in the futures market to set price and total quantities at the desired level. A firm holds no more positions in the futures market unless the rival does. In this case it sells in this market as well. Note that $q_1 + q_2 = A$ leads to a competitive outcome, $(q_1, q_2) = (A - 2q_2, A - 2q_1)$ leads to a Cournot solution, and that other quantities in Ω lead to intermediate situations. Specifically, the sets Φ, Ω, and Π show the futures positions, total quantities and profits that can be sustained by an SPE (other than $F_1 + F_2 > A$ and $q_1 + q_2 > A$, which are uninteresting ways of obtaining zero profits.)

This result resembles Folk theorems in Game Theory. However, this infinite version of the basic model is not a repeated game, and Folk theorems cannot be invoked in proving Proposition 1. In particular, subgames are different if they come after a different history in terms of the accumulated futures positions. Hence, subgames in later periods are, in general, a reduced version of the original game (because of the reduced residual demand), and one has to make sure that payoffs in the remaining game are sufficient to sustain punishments.

3.1. Renegotiation

To study the robustness of equilibria we concentrate on the property of renegotiation-proofness. An equilibrium is renegotiation-proof if there is no other equilibrium that is better for both players in any subgame. For one-stage games, this idea is captured by selecting, within the set of Nash equilibria, those that give Pareto optimal outcomes. This definition is called Pareto Optimal Nash equilibrium. The definition of *Pareto perfect equilibrium* (PPE) provides the natural extension for finite games. A PPE is an equilibrium that, in the first stage of the game, selects Pareto optimal Nash equilibria within the set of Nash equilibria that are PPE in the continuation of the game. Note that one cannot provide a recursive definition for infinite games. This difficulty has been addressed in different ways. Here we present one of the best known and most demanding definitions: A SPE s is said to be a *strongly renegotiation proof equilibrium* (SRPE) if there is no other SPE s' and a subgame g such that, for all players, $U_i^g(s') > U_i^g(s)$, where $U_i^g(.)$ is the utility to player i of following a given strategy conditioned on the game being at subgame g.

This concept of SRPE is originally defined for repeated games (see [2,3]. We present its natural extension to standard extensive form games. Note that the definition of SRPE is very strong. An SPE may fail to be an SRPE because there exists another SPE which, conditioned on some subgame, gives higher payoffs. Nothing, however, is said about the viability of this other equilibrium. We do not bother about this problem since Propositions 2 and 3 show that SRPE exist and, furthermore, that the non-SRPE are payoff-dominated by SRPE. Again, we need to define some sets. Denote by $fr(\Pi)$ the frontier of the set Π.

$$POF(\Phi) = \left\{ \begin{array}{c} (F_1, F_2) \mid (F_1, F_2) = \lambda(\frac{A}{4}, 0) + (1-\lambda)(0,0), \text{ or} \\ (F_1, F_2) = \lambda(0, \frac{A}{4}) + (1-\lambda)(0,0) \text{ for some } \lambda \in [0,1] \end{array} \right\},$$

$$POF(\Omega) = \left\{ \begin{array}{c} (q_1, q_2) \mid (q_1, q_2) = \lambda(\frac{A}{2}, \frac{A}{4}) + (1-\lambda)(\frac{A}{3}, \frac{A}{3}), \text{ or} \\ (q_1, q_2) = \lambda(\frac{A}{4}, \frac{A}{2}) + (1-\lambda)(\frac{A}{3}, \frac{A}{3}) \text{ for some } \lambda \in [0,1] \end{array} \right\},$$

$$POF(\Pi) = \{(\Pi_1, \Pi_2) \in fr(\Pi) \mid, \Pi_1 \geqslant \tfrac{1}{16}A^2, \Pi_2 \geqslant \tfrac{1}{16}A^2\}.$$

The set $POF(\Pi)$ is the Pareto optimal frontier of Π. That is, if $(\Pi'_1, \Pi'_2) \notin POF(\Pi)$, there exists a $(\Pi_1, \Pi_2) \in POF(\Pi)$ such that $(\Pi_1, \Pi_2) \geqslant (\Pi'_1, \Pi'_2)$, and if $(\Pi_1, \Pi_2) \in POF(\Pi)$ there is no other $(\Pi'_1, \Pi'_2) \in POF(\Pi)$ such that $(\Pi'_1, \Pi'_2) \geqslant (\Pi_1, \Pi_2)$. To see that this is indeed the case observe that $fr(\Pi)$ is the union of the curves $\Pi_1 = A\sqrt{\Pi_2} - 2\Pi_2$ between $\Pi_2 = 0$ and $\frac{1}{9}A^2$, and the curve $\Pi_2 = A\sqrt{\Pi_1} - 2\Pi_1$ between $\Pi_1 = 0$ and $\frac{1}{9}A^2$, and that the function $\Pi_i = A\sqrt{\Pi_j} - 2\Pi_j$ is strictly convex between $\Pi_j = 0$ and $\frac{1}{9}A^2$, with a maximum at $\Pi_j = \frac{1}{16}A^2$. Simple algebra shows that $(q_1, q_2) \in POF(\Omega)$ if and only if $(\Pi_1 = pq_1, \Pi_2 = pq_2) \in POF(\Pi)$, and that $(F_1, F_2) \in POF(\Phi)$ if and only if $(q_1 = F_1 + s_1, q_2 = F_2 + s_2) \in POF(\Omega)$ with $s_1 = s_2 = \frac{A-F}{3}$.

Proposition 2. *A pair of profits*, (Π_1, Π_2), *can be sustained by an SRPE if and only if it belongs to* $POF(\Pi)$.

Proof. First show that $(\Pi_1, \Pi_2) \notin POF(\Pi)$ cannot be sustained by an SRPE: If $(\Pi_1, \Pi_2) \notin POF(\Pi)$ there exists a pair $(\Pi'_1, \Pi'_2) \in POF(\Pi)$ such that $(\Pi'_1, \Pi'_2) > (\Pi_1, \Pi_2)$. By Proposition 1, (Π'_1, Π'_2) can be sustained by an SPE, this implies that no SPE with payoffs (Π_1, Π_2) can be SRPE.

Now show that $(\Pi_1, \Pi_2) \in POF(\Pi)$ can be sustained by an SRPE. Let $(\Pi_1^*, \Pi_2^*) \in POF(\Pi)$, and construct the SPE (i)–(iv) in Proposition 1 that has payoffs (Π_1^*, Π_2^*).

Case 1: *Subgames in state C*. At time t_k, the SPE gives firms Cournot profits in the residual demand $p = A - F^k - q$. Any other equilibrium implies positive future positions for some time $t_m > t_k$, with the consequence of smaller total profits. Hence at least one firm loses.

Case 2. *Subgames in state* P_j. As before, total profits decrease if the total quantity sold in the futures market increases. The only possibility then for total profits to increase is that Firm i reduces its position in the futures market (recall that, according to the equilibrium strategy, $f_j^h = 0$ for all $h \geqslant k$). But if Firm i reduces its positions in the futures market, its profits decrease, as shown in Case 3 in the proof of Proposition 1. The case for subgames in state P_i is entirely analogous.

The quantities (q_i, q_j) produced in an SRPE include the Stackelberg outcome with, say, Firm i being the leader, $q_i = \frac{A}{2}$, $q_j = \frac{A}{4}$, the Cournot outcome $q_i = q_j = \frac{A}{3}$, and convex combinations of them. Prices vary between $p = \frac{A}{4}$ and $\frac{A}{3}$, far from the competitive price, $p = 0$. It is also noteworthy that Cournot is the only symmetric outcome in an SRPE.

3.2. *The monopolistic outcome*

In a more general setting, firms would be able to buy as well as to sell in the futures market. To evaluate this new possibility, we first consider the case where firms can buy and sell in the futures market only at t_1, whereas at $t > t_1$ they can only sell. I.e., f_i^1 may be positive or negative, whereas for all $k > 1$, $f_i^k \geqslant 0$. The next proposition shows that any outcome can be supported by an SPE, but that only monopolistic outcomes can be sustained by an SRPE. Moreover, the monopolistic quantity has to be divided with the condition that firms get at least the "Stackelberg follower" profits. First define the sets:

$$\Phi' = \{(f_1, f_2) \mid f_1 + f_2 \leqslant A\},$$

$$\Omega' = \{(q_1, q_2) \mid q_1 + q_2 \leqslant A, q_1 \geqslant 0, q_2 \geqslant 0\},$$

$$\Pi' = \{(\Pi_1, \Pi_2) \mid \Pi_1 + \Pi_2 \leqslant \tfrac{1}{4}A^2, \Pi_1 \geqslant 0, \Pi_2 \geqslant 0\},$$

$$SRPE(\Phi') = \{(f_1, f_2) \mid f_1 + f_2 = -\tfrac{A}{2}, f_i \leqslant -\tfrac{3}{8}A\},$$

$$SRPE(\Omega') = \{(q_1, q_2) \mid q_1 + q_2 = \tfrac{A}{2}, 0 \leqslant q_i \leqslant \tfrac{1}{8}A\},$$

$$SRPE(\Pi') = \{(\Pi_1, \Pi_2) \mid \Pi_1 + \Pi_2 = \tfrac{1}{4}A^2, \Pi_i \geqslant \tfrac{1}{16}A^2\}.$$

As before, Φ' ($SRPE(\Phi')$) contains futures positions that give quantities in Ω' ($SRPE(\Omega')$) in a strategy with $s_1 = s_2 = \frac{A-F}{3}$, and Π' ($SRPE(\Pi')$) are profits supported by elements in Ω' $(SRPE(\Omega'))$. Notice that $SRPE(\Pi')$ is smaller than the Pareto optimal frontier of Π'.

Proposition 3. *If firms can buy in the futures market only at t_1, and can sell in this market at any time, then the pair (Π_1, Π_2) can be sustained*

(i) *by an* SPE *if and only if it belongs to Π', and*
(ii) *by an* SRPE *if and only if it belongs to $SRPE(\Pi')$.*

Proof. Part (i) is proved as in Proposition 1, except that now f_i^1 is not restricted to be non-negative, although this fact does not change the rest of the proof. The "if" part of (ii) is proved as in Proposition 2. We will show that payoffs in Π' but not in $SRPE(\Pi')$ cannot be obtained by an SRPE. By definition of SRPE and by part (i), payoffs such that $\Pi_1 + \Pi_2 < \frac{1}{4}A^2$ cannot be supported by an SRPE. Suppose, then, that firms are playing an SPE that gives payoffs (Π_1, Π_2) such that $\Pi_1 + \Pi_2 = \frac{1}{4}A^2$ and $\Pi_i < \frac{1}{16}A^2$. Simple algebra shows that this strategy requires $-\frac{1}{2}A \leqslant f_i < -\frac{3}{8}A$ and $-\frac{1}{8}A \leqslant f_j \leqslant 0$. Thus, at time t_1 Firm i can deviate from $f_i^1 = f_i$ to $f_i^1 = \epsilon$, where ϵ is a small enough real number. Since total futures positions at time t_1 are still non-positive, at time t_2 Proposition 2 shows that $POF(\bar{\Pi})$ is the set of payoffs in an SRPE at these subgames, where $POF(\bar{\Pi})$ is calculated as $POF(\Pi)$, except that the demand is $p = \bar{A} - q$, with $\bar{A} = A - f_j - \epsilon$. According to this, Firm i can obtain no less that $\frac{1}{16}\bar{A}^2 > \frac{1}{16}A^2$ in such an equilibrium. The deviation is, then, profitable. This is enough to rule out the original strategy as an SRPE. □

Proposition 3 can be generalized for finitely many periods of futures positions not restricted in sign. The case of infinitely many times where firms can buy and sell presents the problem that firms' total futures positions may be undefined for some strategies (for instance, the strategy that requires $f_1^k + f_2^k = c$ if k is even and $f_1^k + f_2^k = -c$ if k is odd).

4. Conclusion

The existing literature attributes a pro-competitive effect to the introduction of a futures market in an oligopolistic industry. We have shown that this need not be the case. In fact, the presence of a futures market may have an anti-competitive effect.

We chose a discrete time model over one with continuous time for two reasons. First, it makes things simpler. Trigger strategies as the one used in Proposition 1 are difficult to define in continuous time. The meaning of a statement like "play s until someone deviates, and then play s' afterwards" is unclear—see [6] for a discussion on this issue. Second, futures and spot markets serve the demand for one period. We assumed no discount within this period. Thus, the choice of the times when to sell in a futures market is irrelevant. Also, if firms are allowed to sell in the futures market at any point in the interval $[0, 1)$, the strategy in Proposition 1 would continue to be an equilibrium—a deviation at a time not in $\mathcal{T}$ would not report greater profits than at some time in $\mathcal{T}$.

Following A&V we assume that future positions are observable. This is important because if committing to sell a given quantity in the futures market puts a firm in a better situation, it is only natural that the firm wants this fact to be known. Otherwise futures position may be overlooked, and may not give the firm any advantage. See [4,5] for further discussions on this issue.

Several extensions are readily suggested, such as the introduction of uncertainty and risk aversion, the study of the role of observability and the information revealed by futures market's prices, and the use of opaque futures markets to conceal collusive behavior. Other lines of research include the extension of our analysis to more than two firms, and to more general demand and cost functions. The main purpose in the present paper is to show a counter-argument to the previous literature, rather than to provide a general result.

Our findings have interesting implications for economic policy towards oligopolies. First, and contrary to A&V, they suggest that the introduction of futures markets is not a substitute for other policies to promote competition. Second, they support the convenience of not allowing producers to have buying positions in future markets.

Interestingly, some empirical evidence for the electricity generation industry in the UK show evidence that competition has not increased after the introduction of futures markets (see reports OXERA [7] and Power UK [8]).

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