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Model uncertainty and the forecast accuracy of ARMA models: A survey

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Abstract

The objective of this paper is to analyze the effects of uncertainty on density forecasts of linear univariate ARMA models. We consider three specific sources of uncertainty: parameter estimation, error distribution and lag order. For moderate sample sizes, as those usually encountered in practice, the most important source of uncertainty is the error distribution. We consider alternative procedures proposed to deal with each of these sources of uncertainty and compare their finite properties by Monte Carlo experiments. In particular, we analyze asymptotic, Bayesian and bootstrap procedures, including some very recent procedures which have not been previously compared in the literature.

Keywords: Bayesian forecast, Bootstrap, Model misspecification, Parameter uncertainty.

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1. Introduction

The time series forecasting literature has traditionally focused on point forecasts. However, many aspects of the decision making process require making forecasts of an uncertain future and, consequently, forecasts ought to be probabilistic in nature, taking the form of probability distributions over future events; see, for example, Tay and Wallis (2000), Timmermann (2000), Greenspan (2004), Elliott and Timmermann (2008), Gneiting (2008) and Manzan and Zerom (2013) who discuss several issues related with density forecasts in economics and finance, and Chatfield (1993) and Christoffersen (1998), who stress the importance of interval forecasts for decision makers. Analytic construction of density forecasts has historically required restrictive and sometimes dubious assumptions, such as no parameter and/or model uncertainty and Gaussian innovations. However, in practice, any forecast model is an approximation to the data generating process (DGP); see the discussions by Wallis (1989), Onatski and Stock (2002) and Jordá et al. (2014). Furthermore, even if the model is correctly specified and time invariant, its parameters need to be estimated. Finally, density forecasts often rely on assumptions about the error distribution that might not be good approximations to the data distribution.

Model uncertainty may have important implications when forecasts are used in decision making processes; see Granger and Machina (2006). For example, in the context of economic problems, Draper (1995) shows that ignoring model uncertainty can seriously underestimate the uncertainty in forecasting oil prices, leading to forecast intervals that are too narrow. Onatski and Stock (2002) and Onatski and Williams (2003) show that monetary policy may perform poorly when faced with a different error distribution or with slight variations of the model. Onatski and Williams (2003) conclude that uncertainty about the parameters and the lag structure have the largest effects, whereas uncertainty about the serial correlation of the errors has minor effects. Brock et al. (2007) also explore ways to integrate model uncertainty into monetary policy evaluation. Finally, some spectacular failures in risk management have also emphasized the consequences of neglecting model uncertainty in the context of financial models; see, for example, Avramov (2006), Cont (2006), Schrimpf (2010) and Boucher et al. (2014).

In this paper, we analyze the effects of uncertainty on the forecast accuracy of univariate ARMA models. We show that the most important distortions appear in the context of short run forecasting with non-Normal errors. We also compare the finite sample performance of the main alternative asymptotic, Bayesian and bootstrap procedures proposed to construct forecast densities that incorporate these uncertainties. Asymptotic methods are usually designed to incorporate the parameter uncertainty assuming a given error distribution and a given model specification; see, for example, Yamamoto (1976) and Fuller and Hasza (1981) for early references. More recently, Hansen (2006) proposes an asymptotic procedure to construct forecast intervals that does not rely on a particular assumption about the error distribution. In the context of Bayesian methods, several authors propose incorporating the parameter and lag order uncertainties using procedures based, for instance, on Bayesian Model Averaging or Reversible Jump Markov Chain Monte Carlo, which usually assume that the true model is within the model set considered; see Draper (1995) for an example of the use of Bayesian Model Averaging in economic problems. In order to be computationally feasible, Bayesian methods often assume a known error distribution, usually Gaussianity; see, for example, Monahan (1983), Le et al. (1996) and Ehlers and Brooks (2008). Alternatively, nonparametric Bayesian mixture procedures relax the distributional assumption; see, for instance, the proposal by Tang and Ghosal (2007). Nevertheless, Bayesian methods are often computationally intensive and time demanding. A competitive alternative to compute forecast densities that incorporate simultaneously the parameter, error distribution and lag order uncertainties is based on bootstrap procedures; see for example, Kilian (1998a,b), Alonso et al. (2004, 2006), Pascual et al. (2001, 2004) and Manzan and Zerom (2008). The latter authors propose a non-parametric bootstrap technique that does not assume any particular specification of the conditional moments.

As main results, we found that asymptotic methods are able to provide reliable density forecasts only in large sample sizes and with known error distribution. On the other hand, Bayesian procedures are able to provide very accurate density forecasts in small sample sizes, but require the correct error distribution and a large computing effort when the sample size is large. It is also difficult to make them to take into account simultaneously all the uncertainties. As a simple alternative, the Bootstrap is able to provide reliable forecasts, regardless of the sample size and the error distribution.

The rest of the paper is organized as follows. Section 2 introduces notation by describing the traditional construction of forecast densities and intervals in the context of univariate linear ARMA models. It also analyzes the effects of the uncertainties involved in the estimation of ARMA models on the forecast densities. Section 3 is devoted to the asymptotic, Bayesian and bootstrap procedures designed to incorporate these uncertainties in the forecasts of ARMA models, and finally, Section 4 concludes the paper.

2. Forecast uncertainty in the context of univariate linear ARMA models

In this section, we introduce notation by describing the traditional procedure to construct forecast densities in the context of univariate linear ARMA models. The sources of uncertainty and their effects on forecast densities are also described.

2.1. Known error distribution, model specification and parameters

Consider the following ARMA(p, q) model

$$(1 - \phi_1 L - ... - \phi_p L^p) y_t = \mu + (1 - \theta_1 L - ... - \theta_q L^q) \varepsilon_t, \tag{1}$$

where y_t is the observation of the series of interest at time t, L is the lag operator, such that $L^i y_t = y_{t-i}$, for i=1,2,..., and ε_t is a strict white noise process with distribution F_ε and variance σ_ε^2. The polynomials $\phi(L) = (1 - \phi_1 L - ... - \phi_p L^p)$ and $\theta(L) = (1 - \theta_1 L - ... - \theta_q L^q)$ have all their roots outside the unit circle and no common roots between them. The autoregressive and moving average orders are p and q, respectively. Note that if ε_t is Gaussian, the polynomial $\theta(L)$ is not identifiable using second order moments, unless the invertibility assumption is imposed. However, for non-Gaussian models, model (1) becomes identifiable on the basis of higher-order moments; see, for example, Breidt and Hsu (2005) and Hsu and Breidt (2009). The invertibility assumption in the non-Gaussian case is entirely artificial and removing it leads to a broad class of useful models. However, in this paper, we assume invertibility.

If the loss function is quadratic[1] and the objective is to predict y_{T+h} given the information available at time T for $h > 0$, then the point forecast with minimum mean square forecast error (MSFE) is given by the conditional mean, denoted by $y_{T+h|T} = E(y_{T+h}|y_1, .., y_T)$; see Granger (1969). For the ARMA model in (1) with Gaussian errors and/or the MA parameters satisfying the invertibility condition, the conditional mean is a linear function of $\{y_1, ..., y_T\}$; see Rosenbaltt (2000).[2] Then, given the information observed up to time T and assuming that the errors are observable within the sample period and have a Gaussian distribution, the h-step-ahead forecast density of y_{T+h}, for $h = 1, 2, ...$, is given by

$$y_{T+h}|y_1, ..., y_T \sim N(y_{T+h|T}, MSFE(e_{T+h|T})), \tag{2}$$

where $y_{T+h|T}$ can be obtained recursively from

$$(1-\phi_1 L - ... - \phi_p L^p) y_{T+h|T} = \mu + (1-\theta_1 L - \ldots - \theta_q L^q)\varepsilon_{T+h|T}, \quad (3)$$

where $\varepsilon_{T+j|T} = 0$ for $j > 0$ and $\varepsilon_{T+j|T} = \varepsilon_{T+j}$ and $y_{T+j|T} = y_{T+j}$ for $j \leq 0$. $MSFE(e_{T+h|T})$ is the mean square forecast error of the h-step-ahead forecast error, $e_{T+h|T} = y_{T+h} - y_{T+h|T}$, which is given by

$$MSFE(e_{T+h|T}) = \sigma_\varepsilon^2 \sum_{i=0}^{h-1} \psi_i^2. \quad (4)$$

The corresponding $(1-\alpha)\%$ forecast intervals are given by

$$y_{T+h|T} \pm z_{\alpha/2}(MSFE(e_{T+h|T}))^{1/2}, \quad (5)$$

where $z_{\alpha/2}$ is the $1-\alpha/2$ quantile of the standard Normal distribution; see Granger et al. (1989) for a clear description of the construction of forecast intervals.

However, if the errors have a known but non-Gaussian distribution, then explicit expressions of the conditional forecast density can only be obtained for $h = 1$. When $h > 1$, there are not analytical expressions of the density. In this case, the forecast densities can be approximated by simulating $e_{T+h|T}$ using the true parameters and the forecast intervals are given by

$$\left[y_{T+h|T} + p_{\alpha/2}(MSFE(e_{T+h|T}))^{1/2}, y_{T+h|T} + p_{1-\alpha/2}(MSFE(e_{T+h|T}))^{1/2}\right], \quad (6)$$

where p_i is the ith percentile of the known error distribution when $h = 1$ and of the simulated distribution of $e_{T+h|T}$ when $h > 1$.

The construction of forecast densities described above requires the unrealistic assumption of a known forecast model, i.e, without parameter and/or lag order uncertainty and with a known error distribution. However, in practice, the forecast model is an approximation to the true DGP. Next, we revise the effects of neglecting the parameter, the error distribution and the lag order uncertainties on the construction of standard forecast densities and intervals.

2.2. Parameter uncertainty

Consider that the error distribution and the lag orders are known. When the parameters are unknown, the h-step-ahead forecast of y_{T+h} is obtained from equation (3) with the true parameters substituted by consistent estimates. In particular, in this paper, we consider the Quasi-Maximum Likelihood (QML) estimator obtained by maximizing the Gaussian Likelihood. Hannan (1973) establishes the consistency and asymptotic Normality of the QML estimator when the model is stationary and invertible with finite second order moment and does not have a constant; see also Yao and Brockwell (1988) for a direct proof.[3] Recently, Bao (2015a) considers the ARMA model with a constant and derives a compact analytical representation of the asymptotic covariance matrix of the QML estimator. Note that, if there is not a MA part, the QML estimator reduces to Least Squares (LS). It is well known that in finite samples the LS estimator is biased; see, among others, Shaman and Stine (1988), Patterson (2000) and Ledolter (2009). For example, in an AR(1) model, the bias tends to shrink the LS estimator toward zero, with larger bias when the autoregressive coefficient is larger in absolute value. Ledolter (2009) shows that the effect of bias on point forecasts is small. However, when $h > 1$, it affects the coverage of forecast intervals since forecast intervals become too narrow. The coverage is improved when bias-adjusted estimates of the autoregressive parameter are used. Kim and Durmaz (2012) show that substantial gains of correcting for bias can be obtained when the true AR model is very persistent and/or the forecast horizon is fairly short. The results about biases of the QML estimator when the model contains a MA component are much more scarce. Some examples regarding simple MA(1) models are Tanaka (1984) and Cordeiro and Klein (1994) that derive the approximated bias of the QML estimator under the data assumption of Normality. Other examples are Bao and Ullah (2007) that consider the case when the data is not Normal, but restrict it to a zero mean MA(1) model, and Demos and Kyriakopoulou (2013) that derive the bias of the QML estimator for a MA(1) model with a known or unknown intercept. More recently, Bao et al. (2014) derive the approximated bias of the QML estimator of the parameters in an invertible MA(1) model with a possible non-zero mean and non-Normal distributed data. They show that the feasible multi-step-ahead forecasts are unbiased under any non-Normal distribution while the one-step-ahead forecast is unbiased under symmetric distributions. Finally, results for general ARMA(p,q) models are given by Bao (2015b).

In practice, the forecast density of y_{T+h} is obtained as in (2) with the unknown parameters involved in $y_{T+h|T}$ and $MSFE(e_{T+h|T})$ substituted by the corresponding QML estimates corrected by bias. Denote by $\hat{y}_{T+h|T}$ and $\widehat{MSFE}(e_{T+h|T})$ the point forecast and estimated MSFE, respectively. The latter is not the MSFE of $\hat{y}_{T+h|T}$, which is given by

$$MSFE(\hat{y}_{T+h|T}) = MSFE(e_{T+h|T}) + E_T[(y_{T+h|T} - \hat{y}_{T+h|T})^2], \quad (7)$$

where $MSFE(e_{T+h|T})$ is defined as in equation (4) and the last term, which is of order $O(T^{-1})$, depends on the mean square error (MSE) of the parameter estimator; see Fuller (1996). To illustrate the underestimation effect of the MSFE of $\hat{y}_{T+h|T}$ on the forecast densities when it is obtained by $\widehat{MSFE}(e_{T+h|T})$, we carry out Monte Carlo experiments based on $R = 1000$ replicates generated by two AR models. The first DGP is given by an AR(1) model with parameters $\mu = 0$, $\phi = 0.8$ and $\sigma_\varepsilon^2 = 1$.

The second DGP is a persistent AR(2) model with parameters $\mu = 0$, $\phi_1 = 0.6$, $\phi_2 = 0.3$ and $\sigma_\varepsilon^2 = 1$. The disturbances are either Gaussian, Student-5 or $\chi^2_{(5)}$. For each replicate, the parameters are estimated by LS and corrected from bias. The bias correction is carried out using the procedure proposed by Orcutt and Winokur (1969) with the expression of Shaman and Stine (1988) and Stine and Shaman (1989) for the first order bias of the LS estimator of an AR model of known and finite order; see Patterson (2000) and Kim (2004) for implementations of this procedure.[4] It is important to note that the bias correction can push estimates into the non-stationarity region, mainly when the model is highly persistent. Consequently, the stationarity correction proposed by Kilian (1998b) is implemented.[5] Then, the estimated conditional forecast densities are computed, for h =1, 6 and 12, as in (2) when the errors are Gaussian or by simulation when they are not Gaussian and h >1, using $\hat{y}_{T+h|T}$ and $\widehat{MSFE}(e_{T+h|T})$. These densities, denoted as EST, are constructed assuming that both the lag-order and the error distribution are known; see Table 1 for a summary of all procedures considered in this paper to construct density and interval forecasts, their acronyms, properties and some references. We also obtain the corresponding 80% and 95% forecast intervals. Finally, for each replicate, we generate 1000 values of y_{T+h}, and construct their empirical forecast density and count how many of these values lie inside the EST intervals. Table 2 reports the Monte Carlo averages and standard deviations of the Mallows Distances (MD) between the empirical and EST h-step-ahead forecast densities when the DGP is the AR(2) model; see Czado and Munk (1998) and Levina and Bickel (2001) for some properties of the MD distance and Lopes et al. (2013) and Fresoli et al. (2015) for applications of the MD in the context of Gaussian and non-Gaussian VARFIMA(0,d,0) and VAR models, respectively.[6] It is shown that, as expected, regardless of the error distribution, the MDs of EST decrease with the sample size and increase with forecast horizon. Moreover, the averages and standard deviations of the distances have similar magnitudes for the different error distributions considered.

We also analyze the finite sample coverages of the forecast intervals obtained by EST. Table 3 reports the Monte Carlo average coverages of the EST forecast intervals when the nominal coverage is 80%. Note that regardless of the distribution, if the sample size is T=50, the empirical coverages of EST are around 77% slightly smaller than the nominal level. The undercoverage is slighly larger for non-Gaussian distributions. However, if the sample size is T=100 or larger, the coverage rates are very close to the nominal level. Consequently, the parameter uncertainty is not an important issue when constructing forecast intervals as far as the sample size is moderate or large.[7]

2.3. Uncertainty about the error distribution

Traditional forecasting procedures in the context of linear time series models assume Gaussian forecast errors. However, often, the variables under analysis do not have a Gaussian distribution; see, for example, Li and McLeod (1988), Kilian (1998b) and Harvey and Newbold (2003) for departures from Gaussianity in the context of economic time series. Note that when the errors are non-Gaussian, it is not always clear which distribution should be assumed.

If the forecast densities and the corresponding intervals are constructed as in (2) and (5), the quantile of the Normal distribution could not be appropriate any longer. Denote by BJ the forecast densities constructed as in (2) with the parameters substituted by their corresponding QML estimates corrected from bias. Note that when the errors are Gaussian, the EST and BJ procedures coincide. Table 2, which reports the Monte Carlo averages and standard deviations of the MD distances, shows that when the errors are Student-5 and $\chi^2_{(5)}$, the distances are larger for the BJ than for the EST densities, especially for asymmetric errors. Moreover, the difference between the distances of the EST and BJ densities increases with the sample size and decreases with h. Note that when h=12 the MDs of the BJ densities are very similar to those of the EST densities. For example, when the errors are $\chi^2_{(5)}$ and T=300, the increase in the average MD is $\frac{0.217-0.075}{0.075} = 203\%$ when $h = 1$ while the increase is $\frac{0.277-0.221}{0.221} = 25.34\%$ when $h = 12$. Therefore, it seems that assuming Normal forecast errors when they are non-normal has an important effect on the construction of forecast densities mainly when the sample size is large and the forecast horizon is small.

The Monte Carlo averages and standard deviations of the coverage rates of the BJ forecast intervals are reported in Table 3, when the nominal coverage is 80%. In both cases, one-step-ahead BJ intervals have average coverages that tend to overestimate the nominal level. The overcoverage is larger as T increases. Furthermore, when the errors follow a $\chi^2_{(5)}$ distribution, we observe that the coverage in the left tail is much smaller than the coverage in the right tail. In accordance with the results in Table 2, these problems decrease when h increases, that is, the coverages tend to the nominal level, suggesting that the effect of assuming wrongly Normality is less important in the long term.

2.4. Uncertainty about the orders p and q of the ARMA process

Besides the uncertainty about the error distribution, when fitting an ARMA stationary model to a data set, the true orders of the underlying stochastic process are often unknown and should be determined. In practice, the model used for forecasting is chosen by using a selection criterion and forecasts are obtained conditional on the selected model which is considered as being the true one. The most popular selection criteria are the Akaike (1973) information criterion (AIC), its bias-corrected version (AICC) proposed by Hurvich and Tsai (1989, 1991), which penalizes larger models to counteract the overfitting nature of AIC, and the Bayesian information criterion (BIC) of Schwarz (1978); see Bhansali (1993) for a review of other selection procedures. Several authors have studied the effects of order misspecification on conditional forecasts. For instance, Tanaka and Maekawa (1984), assuming Gaussian errors, assess analytically the asymptotic MSFE when the forecasts are obtained from an AR(1) and the true model is an ARMA(1,1). For h=1, they derive expressions for the bias and the MSFE when the wrong model is assumed and conclude that, in this situation, the MSFE is underestimated. Davies and Newbold (1980) also show that although a MA(1) model can be approximated arbitrarily closely by an high order AR model, the finite sample effect of estimating additional parameters is that the forecast error variance increases.

Nevertheless, Chatfield (1996, 2000) warns about the forecast biases generated by formulating and fitting a model to the same data. He argues that those forecasts will be over-optimistic when the data-dependent model-selection process is ignored, leading to forecast intervals that are generally too narrow and fail to take into account the model uncertainty. In other words, it is expected that a model fitted to the same data used to formulate it will provide the best fit among the alternative models; see also Clements and Hendry (1998, 2001) for a detailed taxonomy of uncertainty applied to forecast errors in economic stationary and non-stationary time series.

In order to analyse the impact of the lag order uncertainty of an ARMA model on the density and interval forecasts, we carry out Monte Carlo experiments by generating replicates from the same AR(2) model described above. In each simulation, we assume an AR(p) model and select p using the AICC criterion with $p_{max} = T/10$ as recommended by Bhansali (1983). The parameters of the selected model are estimated by LS and corrected from bias, and the forecast densities and the corresponding forecast intervals are constructed assuming Gaussian errors. This procedure is denoted as $\mathrm{BJ}_{\mathrm{aicc}}$.[8]

Table 2 provides the Monte Carlo MD averages and standard deviations of the BJ_{aicc} densities. We observe that, regardless of the error distribution, the distances between the true and the BJ_{aicc} densities are larger than those obtained with the BJ procedure and they decrease with the sample size, since the AICC criterion is asymptotically efficient. Furthermore, the MD differences between the BJ and BJ_{aicc} densities also decrease with the forecast horizon.

Analysing the Monte Carlo average coverages reported in Table 3, we observe that the coverages of the BJ_{aicc} intervals are similar to those of the EST and BJ intervals when the errors are Gaussian and non-Gaussian, respectively.

3. Procedures to incorporate the forecast uncertainties of ARMA models

In the previous section, we have seen that the effects of parameter and lag-order uncertainties on the forecast densities are neglegible in moderate sample sizes. However, assuming wrongly Normality may generate important distortions mainly when forecasting in the short run. In this section, we revise the procedures proposed in the literature to incorporate the types of uncertainties described in the previous section and analyze their finite sample performance. We classify them in three categories: asymptotic, Bayesian and bootstrap procedures.

3.1. Asymptotic methods

To correct the biases of the MSFE caused by parameter uncertainty, many authors propose using asymptotic approximations of the MSE of the QML estimator to compute the MSFE of $\hat{y}_{T+h|T}$ in (7). The derivation of the asymptotic MSFE (AMSFE) is usually based on assuming that the sample data used to estimate the parameters are statistically independent of the data used to construct the forecasts. Although Phillips (1979) points out that this assumption is quite unrealistic in practical situations, Maekawa (1987) shows that the AMSFE of AR(p) processes is the same regardless of whether the data used for parameter estimation is dependent on that used for forecasting. The expression of the AMSFE of AR(p) models has been derived by Fuller and Hasza (1981) who extend the results of Phillips (1979) for the AMSFE of AR(1) processes while Ansley and Kohn (1986) extend it to state-space models. As the general ARMA model can be formulated as a state-space model, the latter results also cover ARMA models as a special case. It is worth noting that the above results on the AMSFE have been derived in the context of Gaussian errors. Bao (2007) study the MSFE of the AR(1) model with non-Normal distributed errors and shows that it coincides with the unconditional AMSFE of Box and Jenkins (1970) and Yamamoto (1976). Bao and Zhang (2014) point out that results for AMSFE in the context of non-Normal data are not available for MA models.

In this paper, we consider the conditional asymptotic approximation proposed by Fuller and Hasza (1981) and Fuller (1996). If the forecast errors are Gaussian, the conditional forecast density of y_{T+h} can be constructed as in (2) with $MSFE(e_{T+h|T})$ substituted by $AMSFE(\hat{y}_{T+h|T})$. Analogously, in the case of non-Gaussian errors, the distribution of $y_{T+1}|y_1, ..., y_T$ could be approximated by the distribution assumed for the error if h=1. For h >1, the forecast distribution of $y_{T+h}|y_1, ..., y_T$ could be simulated using the true or estimated parameters adjusted by bias as described above. The estimated AMSFE is denoted by $\widehat{AMSFE}$. Since the term associated to the parameter uncertainty in the AMSFE is of order T^{-1}, the impact of the parameter uncertainty to the MSFE of $\hat{y}_{T+h|T}$ is negligible when the sample size is relatively large. On the other hand, for a given sample size, the parameter uncertainty contribution increases with the forecast horizon.

The Monte Carlo results for the MD distances when the MSFE is replaced by the $\widehat{AMSFE}$ are approximately identical to those reported in Table 2. Table 5 reports the

MC average coverages and standard deviations of the EST, BJ and BJ_{aicc} intervals, computed with the MSFE substituted by the $\widehat{AMSFE}$ and denoted by AEST, ABJ and ABJ_{aicc}, respectively. The results show that using the asymptotic correction, the coverages are only slighly larger than those reported in Table 3 without the correction. In general, when the coverage is bellow the nominal, we obtain coverages closer to the nominal. However, when the error distribution is non-Normal and the forecast density is assumed to be Normal, the overcoverage is even larger than that obtained with the asymptotic correction. Therefore, it seems that the asymptotic correction of the MSFE is not useful to obtain forecast intervals with better coverages. Furthermore, the computation of the AMSFE can become difficult in high order autoregressive or general ARMA models.

When constructing forecast intervals using the AMSFE, we need to assume a particular distribution for the errors. Alternatively, Hansen (2006) proposes the Simple Reference Adjustment (SRA) procedure to construct conditional asymptotic forecast intervals.[9] Unlike the asymptotic methods described above, the SRA procedure only requires i.i.d errors, without relying on any particular assumption about the error distribution. The SRA intervals are based on direct forecast autoregressions whose forecast interval endpoints depend on the sample size and the empirical distribution of the residuals. In order to analyze the finite sample performance of the SRA procedure when constructing forecast intervals, consider again the same AR(2) model used in the previous Monte Carlo simulations. Table 5, which reports the Monte Carlo averages and standard deviations of the coverages of the SRA forecast intervals, implemented without estimating the lag order, shows that, regardless of the error distribution, the empirical coverages are close to the nominal when h =1, but they decrease substantially for h = 6 and 12. We also implement the SRA procedure after estimating the lag order and denote it by SRA_{aicc}. Comparing the coverages of the ABJ_{aicc} and SRA_{aicc} densities, we observe that the latter only provides accurate coverages for h =1. The poor performance of the SRA forecast intervals in the long run may be due to the fact that SRA is based on direct forecasts rather on iterated forecasts, as the previous procedures are. Ing (2003) shows that when $\hat{p} > p$ is fixed, the relative performance of direct forecasts, in terms of mean square prediction error, deteriorates as the forecast horizon increases. Similar conclusions are found by Marcellino et al. (2006) who compares iterated and direct forecasts in macroeconomic time series. Therefore, it seems that the SRA intervals may only be applicable to sample sizes rather large and/or short horizons. For all procedures including the SRA, we have calculated the 95% and 99% interval coverages and lengths and the conclusions are similar.[10] However, we observe that for these two significance levels the SRA procedure often provides intervals with lengths that are unrealistically large.

3.2. Bayesian forecasts

One of the earliest references using Bayesian procedures to forecast in the context of time series models is Monahan (1983), who constructs forecast densities that take into account parameter and lag order uncertainties. Monahan (1983) uses numerical integration techniques, restricting the analysis to models with no more than two parameters, that is, $p + q \leq 2$. Thompson and Miller (1986) overcome some of the computational difficulties and simulate future paths of time series for ARMA(p, q) models and h-steps-ahead forecasts, simulating from the predictive distribution rather than trying to obtain its analytical form. The Bayesian forecasting procedure of Thompson and Miller (1986) allows to assume other error distributions and they show, explicitly, how to construct forecast densities and intervals for ARMA models; see also Geweke and Whiteman (2006) for the principles of Bayesian forecasting. Later, Chib and Greenberg (1994) and Marriott et al. (1996) propose MCMC samples for ARMA models which enforce stationarity and invertibility, but they rely on Gaussian errors.

The Bayesian procedure of Thompson and Miller (1986) is illustrated by implementing it to construct forecast intervals for the AR(2) model considered previously. When the Bayesian procedure is implemented assuming Gaussian forecast errors it is denoted as BAYESN while, if the errors are assumed to be Student-ν, it is denoted as BAYEST.[11] When Gaussianity is assumed, it is well known that any diffuse prior for ϕ and σ_ε^2 leads to Normal and Inverse Gamma posterior distributions, respectively. These posterior distributions are obtained using Gibbs sampler. Regarding the Student-ν case, we are not able to identify the posteriors of all parameters and therefore the Metropolis-Hasting algorithm is implemented. Following Sahu et al. (2003), we assume an exponential prior distribution with parameter 0.1 truncated in the region $\nu > 2$ for the degrees of freedom (ν) of the Student-ν.[12] We run 11000 iterations for the MCMC algorithms of BAYESN and BAYEST and save the last 1000 iterations to construct the forecast densities and intervals. Table 4 reports the Monte Carlo averages and standard deviations of the MD distances between the Bayesian and the true forecast densities. These distances should be compared with those of EST densities reported in Table 2 as in both cases the lag order and error distribution are assumed to be known. We observe that when the errors are Normal, the Bayesian distances are slightly larger for h =1. However, when the forecast horizon increase to h =6 and 12, the distances decrease. Note that their standard errors are also smaller. Similar results are obtained when the errors are Student-5. We also compute the Bayesian densities assuming Normality when the errors are truly Student-5 or $\chi^2_{(5)}$. In this case, the distances should be compared with those reported as BJ in Table 2. Regardless of whether the true distribution is Student-5 or $\chi^2_{(5)}$, when the densities are constructed assuming Normality, the averages and standard deviations are almost identical to those obtained by the BJ procedure when h=1. However, the averages and standard deviations are smaller for h=6 and 12.

Consider now the results for the coverages of the corresponding forecast intervals in Table 6. We observe that, if the true error distribution is known, the Bayesian procedure is able to provide coverages closer to the nominal level than those of the asymptotic methods. On the other hand, if we misspecify the error distribution when using the Bayesian procedure, we can have distorted coverages for 80% intervals in the short term, as happens to the BJ and ABJ intervals. Furthermore, note that the overcoverage can be even larger than those of the BJ intervals. Finally, when the true errors are $\chi^2_{(5)}$, the Bayesian intervals based on Gaussian errors are asymmetric when h=1.

The Bayesian procedures described above assume that the error distribution is known. However, some Bayesian approaches are able to incorporate the error distribution uncertainty in their forecasts. They are based on nonparametric Bayesian mixture of models, but their main drawback is that they are intensive computationally and the construction of forecast intervals and densities are not straightforward; see Tang and Ghosal (2007) for applications in the context of autoregressive models.

Finally, some Bayesian procedures are designed to take into account the uncertainty about the lag order of ARMA models. For example, applications of Bayesian model averaging to AR processes are reported by Schervish and Tsay (1988) and Le et al. (1996). Other fixed-dimensional MCMC algorithms are proposed in Barnett et al. (1996) and Huerta and West (1999). In Barnett et al. (1996) the AR coefficients are reparameterized in terms of the partial correlation coefficients so that the AR model can be treated as a nested model and model-order selection is performed by associating a binary indicator variable with each coefficient, and using these to perform subset selection. Barnett et al. (1997) extend the procedure of Barnett et al. (1996) for ARMA models. On the other hand, Huerta and West (1999) define a prior structure directly on the roots of the AR characteristic polynomial and the model uncertainty is, then, naturally accounted for by allowing the roots to have zero moduli. Nevertheless, in the above procedures the maximum orders of the models are fixed and the estimations are made on the saturated model, which may lead to a parameter space of a very large dimension and, consequently, the estimation becomes difficult. To avoid this problem, some authors propose to apply the Reversible Jump Markov Chain Monte Carlo (RJMCMC) algorithm proposed by Green (1995) that is a generalisation of the Metropolis Hasting algorithm that allows jumps between states of different dimensions. It can jointly estimate the orders p and q and the parameters ϕ, θ and σ_ε^2 of an ARMA model and the lag order uncertainty is accounted explicitly in terms of the posterior distributions of p and q; see Troughton and Godsill (1998), Vermaak et al. (2004) and Ehlers and Brooks (2008) for applications to AR(p) models. An alternative to the RJMCMC algorithm is proposed by Stephens (2000), whose procedure is based on the simulation of a continuous time birth and death Markovian process between-model moves. Philippe (2006) adapts such algorithm to ARMA models and denotes it as the birth and death MCMC (BDMCMC) algorithm. Her choice is based on Brooks et al. (2003), whose numerical results favour the BDMCMC algorithm against the RJMCMC in terms of convergence assessment in the particular case of AR models. However, a comparison about forecast performance was not assessed.

A simpler alternative to the previous Bayesian procedures is based on the Bayesian LASSO regression. The LASSO operator, proposed by Tibshirani (1996) is a shrinkage method that was originally used for variable selection in the linear regression. Its main advantage is that it can be directly implemented in the full model and no model search is needed. Schmidt and Makalic (2013) adapt the Bayesian LASSO to AR models and their simulations demonstrate that their procedure performs well in terms of forecast errors when compared with a standard autoregresion order selection method and they suggest its extension to ARMA models. Nevertheless, it worth noting that the above Bayesian methods so far developed for AR or ARMA processes rely on the Gaussian assumption of the errors and they are very intensive computationally.

The procedure of Schmidt and Makalic (2013), called BAYESL, is illustrated with Monte Carlo experiments, using $p_{max} = T/10$; see Tables 4 and 6 for the implementation of the BAYESL procedure. As well as for the BAYESN and BAYEST procedures, we run 11000 iterations and discard the first 10000. In Table 4 we observe that, when the errors are Normal the distances are reduced with respect to BJ_{aicc} if T=50 and h=6 and 12. However, for T=100 and 300, the distances are larger. Similar results are obtained for the other two distributions considered. Looking at the Monte Carlo results of the interval coverages in Table 6 we observe that BAYESL generates coverages close to the nominal level for h=1, but as the forecast horizon increases, BAYESL underestimates the nominal level, regardless of the error distribution. Moreover, since BAYESL assumes Gaussianity, it presents distorted coverages as T increases for h=1 when the errors are non-Gaussian.

Finally, we can conclude that, unlike the asymptotic methods, the Bayesian methods are able to provide accurate forecast densities in moderate sample sizes and mainly in the short term when the true distribution is known. The drawback is that they may demand a large computing effort when the sample size is large. In our study, for example, the BAYEST and BAYESL procedures take approximately 18 and 39 hours, respectively, to compute the MD values and coverage rates of one Monte Carlo simulation of sample size $T = 300$; see Table 8 for a detailed time comparison between Bayesian and alternative procedures.

3.3. Bootstrap forecasts

A simple alternative to construct forecast densities that take into account the parameter, error distribution and lag-order uncertainties is based on bootstrap procedures. They are attractive because they use computationally simple algorithms. The original bootstrap procedure to obtain forecast densities is proposed by Thombs and Schucany (1990) in the context of AR(p) models to incorporate the parameter uncertainty. Extensions of their work include Masarotto (1990), Kabaila (1993), McCullough (1994), Breidt et al. (1995), Grigoletto (1998) and Kim (1999). Pascual et al. (2001, 2004) propose an alternative procedure that does not require bootstrap re-sampling through the backward representation of the process and, consequently, it can be applied to models with moving-average components. The procedure by Pascual et al. (2001, 2004) is implemented by Clements and Taylor (2001) and Kim (2001) who apply the bootstrap-after-bootstrap of Kilian (1998a) in order to take into account the small sample bias of the parameter estimators to construct AR forecasts.

In this paper, we consider the bootstrap procedure proposed by Pascual et al. (2001, 2004) with the analytical parameter bias correction method described in Section 2.2 for an AR(p) model, whose advantage over the bootstrap bias correction of Kilian (1998a) is its computational efficiency; see Kim (2004) for the same bias correction procedure. In the literature, we can find other alternatives to the bootstrap parameter bias correction. For instance, Clements and Kim (2007) show that when the process is near unit root or non-stationary, the parameter estimation proposed by Roy and Fuller (2001) performs better and is computationally cheaper. Another alternative is the grid bootstrap method of Gospodinov (2002), but it only applies to AR(1) models; see Kim and Durmaz (2012).

Analysing the Monte Carlo results of Table 4, we observe that BOOT shows lower MDs than BJ and ABJ as T increases when there is error distribution uncertainty and mainly the true errors are asymmetric. Regarding the coverage rates (Table 7), for T=50, the BOOT intervals already have coverages very close to the nominal levels for all forecast horizons, outperforming AEST and the Bayesian procedures that use the correct error distribution and lag-order. Note that the coverages BOOT do not decrease with the forecast horizon. This is a result of the implemented bias correction. The gain of bias correction can be substantial in small samples, when the AR root of the model is close to one and when the forecast horizon is larger; see Kim (2003, 2004).

Finally, the uncertainty associated with the lag order can be incorporated by using the endogenous lag-order bootstrap algorithm of Kilian (1998a), the sieve exogenous order bootstrap of Alonso et al. (2004) and the moving blocks bootstrap of Alonso et al. (2006). Clements and Kim (2007) show that incorporating the lag order selection has marginal small improvements when the true process is highly persistent. The results of Clements and Kim (2007) also warn against the use of bootstrap techniques for highly persistent processes with non-Gaussian distributions.

We apply the sieve exogenous order bootstrap of Alonso et al. (2004) with the bias-correction procedure described in Section 2.2, denoted here as BOOTEX. The proposal of Alonso et al. (2004) is easier to implement than the moving blocks bootstrap of Alonso et al. (2006) and both procedures provide similar coverage results; see Alonso et al. (2006). The advantage of the latter is that it is less dependent on the initial selected order $\hat{p}$ than the sieve exogenous order bootstrap. Yet, it introduces the sampling variability of the model that is less dependent on the initial $\hat{p}$ order than the endogenous order bootstrap of Kilian (1998b) and it is more efficient computationally since it skips the step of re-estimating in each bootstrap resample the lag order by the same method used to estimate the initial lag order. Alonso et al. (2004) find in their Monte Carlo study that their proposal outperforms the endogenous lag order bootstrap and provides consistent forecast intervals for ARMA processes.

Looking at the results of Tables 4 and 7 we observe that the BOOTEX procedure yields MDs and coverages very close to those obtained with BOOT, which assumes the correct lag-order, showing a clear advantage over the asymptotic methods that incorporate only the parameter variability in the forecasts, such as AEST, or also the error distribution in the forecast intervals as SRA, and over more complex methods that incorporate both parameter and lag-order uncertainties, as the Bayesian procedures. Moreover, it is worth noting that bootstrap procedures usually require less computational effort in comparison with Bayesian procedures. In our study, for example, the bootstrap procedure takes approximately 1 hour for computing the MD values and coverage rates of one Monte Carlo simulation when T=300, whereas the Bayesian procedures, as BAYEST and BAYESL, take more than 12 hours to compute the same measures.

Alternatively, we can use forecasting methods, which are not model based, such as the nonparametric bootstrap of Manzan and Zerom (2008). Their method just requires that the time series under analysis follows a Markovian process. Consequently, it encompasses a wide range of relevant structures implied by various commonly used linear and non-linear models. Manzan and Zerom (2008) adapt the local bootstrap approach of Paparoditis and Politis (2001, 2002) to the context of out-of-sample forecast density estimation. For one-step-ahead forecasts, their proposed non-parametric procedure reduces to the well known conditional density estimator; see, for example, De Gooijer and Zerom (2003). The nonparametric bootstratp of Manzan and Zerom (2008) is denoted here as BOOTNP. It uses the nonparametric method of Diks and Manzan (2002) to select p, but it still depends on the choice of p_{max}, which we have considered as $p_{max} = T/10$. The simulation results report that BOOTNP provides the highest distances, and it is able to provide close coverages only for T=300 and h=1, since it requires larger sample sizes in order to obtain good performance; see Manzan and Zerom (2008).

Given the good results reported by BOOTEX in comparison with the asymptotic and Bayesian procedures, we highlight the importance of considering resample methods for taking into account model uncertainty when constructing density and forecast intervals.

4. Conclusions

In this paper, we compare alternative procedures to construct density and interval forecasts that deal with model uncertainty on the forecast accuracy of univariate ARMA models. We show that the most important source of uncertainty when constructing density forecasts for small forecast horizons is the error distribution. However, as the forecast horizon increases, the normal approximation of the density is more appropriate. Consequently, the asymptotic correction of the MSFE is not useful. Furthermore, it is only available for relatively simple ARMA models. The SRA procedure to construct asymptotic forecast intervals is sensible for small forecast horizons but does not work for large ones. Moreover, it requires large samples and small nominal coverages. Alternatively, Bayesian procedures are time consuming and computationally complicated when incorporating simultaneously parameter and lag order uncertainties without assuming a particular error distribution. Finally, bootstrap procedures seem to be a feasible alternative if the sample size is large even when the error distribution is unknown.

Along this work, we have enconttered several gaps in the literature that could be the focus for further research. First, the results on forecasting with MA models are scarce with most of the literature focusing on AR models. For example, it could be interesting to analyse the effects of this bias correction on point and interval forecasts. Furthermore, the bias correction usually implemented to the AR parameters is based on a known and finite order. However, in practice, the order is also unknown and, consequently, the bias correction could not be appropriate. These corrections could also be important when implemented in the context of bootstrap forecasts.

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Notes

[1]When the loss function is non-quadratic, constructing forecasts using the conditional mean is inappropriate, since the mean of the predictive distribution is not optimal as a point predictor; see Granger (1969), Christoffersen and Diebold (1997), Granger and Pesaran (2000a,b), Patton and Timmermann (2007a,b) and Gneiting (2011) for prediction problems involving asymmetric loss functions.

[2]See Breidt and Hsu (2005) and Lanne et al. (2012) for forecasting in the context of non-invertible non-Gaussian MA models.

[3]Hsu and Breidt (2009) propose an exact ML estimator that does not require invertibility; see also Lii and Rosenblatt (1992, 1996), Huang and Pawitan (2000) and Gospodinov and Ng (2015) for alternative estimators.

[4]Shaman and Stine (1988) derive a simple analytical expression of the first order bias of the LS estimator of the parameters of an AR(p) model. Their expression can be extended to other estimators of similar design. Kiviet and Phillips (1994) also provide an alternative expression for the bias of the LS estimator of an autoregressive model with normal errors. Alternatively, Tanizaki et al. (2005) provide a bootstrap bias correction for the LS estimator in AR(p) models. Finally, Kim and Durmaz (2012) describe alternative bias-correction proposals.

[5]Alternatively, Kim et al. (2010) propose a stationarity correction based on the stable spectral factorization of Poskitt and Salau (1993).

[6]The MD is computed as follows. Let $x_{(1)} \leq ... \leq x_{(N)}$ and $y_{(1)} \leq ... \leq y_{(N)}$ be ordered

realizations of the random variables X and Y, with absolutely continuous distributions F and G, respectively. The MD between F and G is given by $MD(F,G) = \left(\frac{1}{N}\sum_{i=1}^{N}|x_{(i)} - y_{(i)}|^{\alpha}\right)^{1/\alpha}$. In this paper we use $\alpha = 1$.

[7]The results for the AR(1) DGP and 95% nominal coverages are similar and not reported to save space. They are available from the authors upon request.

[8]Noting that the bias correction procedure of Shaman and Stine (1988) and Stine and Shaman (1989), in the case of lag order misspecification, just hold when the order is overspecified.

[9]Note that the procedure proposed by Hansen (2006) does not allow the construction of forecast densities. It is also worth noting that there is no bias correction method available for direct forecast regressions.

[10]Results available from the authors upon request.

[11]We did not consider $\chi^2_{(5)}$ errors since as far as we know there is not any proposal in the literature to deal with this distribution in the context of Bayesian forecasting.

[12]Alternatively, as proposed by Jacquier et al. (2004), we also consider a truncated discrete uniform prior distribution for ν, so that $\nu \sim U[3,40]$. Although using the latter prior we obtain similar MD distances, the coverage rates of the model with truncated exponential prior are closer to the nominal level than those of the latter model. Consequently, the subsequent results are based on the truncated exponential prior.

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Table 1: Procedures considered in the paper.

Acronyms	Description	References
EST	Parameters estimated by LS. The error distribution and the lag order are known	
BJ	Parameters estimated by LS. The error distribution is assumed to be Gaussian and the lag order is known	
BJ_{aicc}	Parameters estimated by LS. The error distribution is assumed to be Gaussian and the lag order	
	is estimated by the AICC criterion	
AEST	Parameters estimated by LS with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE.	Fuller and Hasza (1981) and
	The error distribution and the lag order are known	Fuller (1996)
ABJ	Parameters estimated by LS with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE.	Fuller and Hasza (1981) and
	The error distribution is assumed to be Gaussin and the lag order is known	Fuller (1996)
ABJ_{aicc}	Parameters estimated by LS with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE.	
	The error distribution is assumed to be Gaussin and the lag order is estimated by the AICC criterion	
SRA	Conditional forecast intervals that incorporate parameter and error distribution uncertainty	Hansen (2006)
BAYESN	Bayesian procedure to incorporate parameter uncertainty in the forecasts.	Thompson and Miller (1986)
	The errors are assumed to be Gaussian	Thompson and Miller (1986)
BAYEST	Bayesian procedure to incorporate parameter uncertainty in the forecasts.	Thompson and Miller (1986)
	The errors are assumed to be Student-ν	Thompson and Miller (1986)
BAYESL	Bayesian LASSO procedure. It incorporates parameter and lag order uncertainty in the forecasts	Schmidt and Makalic (2013)
BOOT	Bootstrap procedure. It incorporates parameter and error distribution uncertainty in the forecasts	Pascual et al. (2001, 2004)
BOOTEX	Sieve exogenous bootstrap procedure. It incorporates parameter, error distribution and lag order uncertainty	Alonso et al. (2004)
BOOTNP	Non-parametric bootstrap procedure	Manzan and Zerom (2008)

Table 2: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/BJ	0.187	0.462	0.682	0.129	0.302	0.425	0.068	0.152	0.205
	(0.150)	(0.447)	(0.710)	(0.105)	(0.304)	(0.456)	(0.057)	(0.154)	(0.208)
BJ_{aicc}	0.255	0.517	0.698	0.168	0.336	0.452	0.079	0.162	0.215
	(0.201)	(0.443)	(0.678)	(0.163)	(0.319)	(0.461)	(0.074)	(0.156)	(0.209)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.204	0.477	0.682	0.147	0.341	0.472	0.078	0.170	0.224
	(0.182)	(0.440)	(0.668)	(0.124)	(0.347)	(0.536)	(0.067)	(0.162)	(0.216)
BJ	0.234	0.491	0.693	0.185	0.357	0.485	0.136	0.200	0.250
	(0.181)	(0.443)	(0.672)	(0.122)	(0.343)	(0.532)	(0.065)	(0.159)	(0.214)
BJ_{aicc}	0.290	0.541	0.704	0.216	0.391	0.509	0.144	0.212	0.264
	(0.228)	(0.462)	(0.665)	(0.162)	(0.358)	(0.531)	(0.077)	(0.167)	(0.225)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.201	0.501	0.724	0.138	0.330	0.463	0.075	0.167	0.221
	(0.156)	(0.467)	(0.754)	(0.113)	(0.327)	(0.497)	(0.054)	(0.139)	(0.184)
BJ	0.290	0.537	0.750	0.248	0.377	0.495	0.217	0.241	0.277
	(0.133)	(0.457)	(0.750)	(0.087)	(0.311)	(0.486)	(0.040)	(0.120)	(0.168)
BJ_{aicc}	0.335	0.594	0.768	0.273	0.409	0.519	0.224	0.251	0.288
	(0.170)	(0.464)	(0.738)	(0.124)	(0.336)	(0.505)	(0.056)	(0.137)	(0.186)

Table 3: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/BJ	78.06 (0.05)	10.84/11.09	77.39 (0.09)	11.09/11.52	77.55 (0.11)	10.99/11.46
	BJ_{aicc}	76.60 (0.06)	11.69/11.71	77.38 (0.10)	11.18/11.44	76.96 (0.12)	11.37/11.67
100	EST	79.00 (0.04)	10.58/10.43	78.86 (0.06)	10.61/10.53	78.97 (0.08)	10.54/10.49
	BJ_{aicc}	78.18 (0.05)	10.95/10.87	78.72 (0.07)	10.66/10.62	78.58 (0.08)	10.69/10.73
300	EST	79.66 (0.02)	10.18/10.17	79.78 (0.03)	10.2/10.02	79.82 (0.04)	10.16/10.02
	BJ_{aicc}	79.47 (0.02)	10.26/10.26	79.66 (0.03)	10.23/10.11	79.72 (0.05)	10.2/10.08
	Student-5	h=1		h=6		h=12	
50	EST	76.99 (0.07)	11.70/11.31	76.28 (0.10)	12.07/11.64	76.87 (0.12)	11.76/11.37
	BJ	81.33 (0.07)	9.50/9.16	78.02 (0.10)	11.19/10.79	78.06 (0.12)	11.15/10.78
	BJ_{aicc}	79.74 (0.08)	10.21/10.04	77.62 (0.11)	11.16/11.22	77.30 (0.13)	11.33/11.36
100	EST	78.30 (0.05)	10.94/10.76	77.85 (0.07)	11.12/11.02	77.93 (0.09)	11.08/10.99
	BJ	82.52 (0.05)	8.82/8.66	79.49 (0.07)	10.30/10.21	79.09 (0.09)	10.50/10.42
	BJ_{aicc}	81.59 (0.06)	9.29/9.11	79.23 (0.08)	10.56/10.21	78.72 (0.09)	10.80/10.48
300	EST	79.53 (0.03)	10.22/10.26	79.50 (0.04)	10.20/10.3	79.58 (0.05)	10.14/10.28
	BJ	83.65 (0.03)	8.15/8.19	81.11 (0.04)	9.40/9.49	80.68 (0.05)	9.58/9.74
	BJ_{aicc}	83.35 (0.03)	8.29/8.35	80.90 (0.04)	9.49/9.61	80.47 (0.05)	9.67/9.84
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	77.31 (0.09)	11.68/11.01	76.45 (0.10)	11.74/11.80	76.45 (0.13)	11.66/11.88
	BJ	82.52 (0.07)	5.74/11.72	77.51 (0.10)	10.04/12.44	77.15 (0.12)	10.42/12.42
	BJ_{aicc}	80.61 (0.09)	7.13/12.25	77.23 (0.11)	10.20/12.57	76.35 (0.13)	10.83/12.81
100	EST	78.33 (0.07)	11.13/10.53	78.17 (0.07)	11.88/10.94	78.28 (0.09)	10.77/10.93
	BJ	83.80 (0.05)	4.96/11.24	79.24 (0.07)	9.19/11.56	78.93 (0.09)	9.59/11.47
	BJ_{aicc}	82.50 (0.07)	6.00/11.49	78.85 (0.08)	9.47/11.67	78.40 (0.09)	9.95/11.64
300	EST	79.53 (0.04)	10.32/10.13	79.48 (0.04)	10.33/10.18	79.59 (0.05)	10.23/10.17
	BJ	85.29 (0.03)	3.86/10.84	80.57 (0.04)	8.62/10.80	80.25 (0.05)	9.05/10.69
	BJ_{aicc}	84.85 (0.04)	4.23/10.92	80.33 (0.04)	8.81/10.85	80.03 (0.05)	9.21/10.75

Table 4: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/ABJ	0.187	0.464	0.688	0.130	0.304	0.429	0.068	0.152	0.207
	(0.150)	(0.447)	(0.711)	(0.105)	(0.304)	(0.457)	(0.057)	(0.154)	(0.209)
ABJ_{aicc}	0.255	0.522	0.706	0.168	0.338	0.456	0.079	0.162	0.216
	(0.202)	(0.444)	(0.680)	(0.163)	(0.319)	(0.462)	(0.074)	(0.156)	(0.209)
BAYESN	0.193	0.406	0.561	0.139	0.280	0.363	0.092	0.177	0.225
	(0.148)	(0.348)	(0.616)	(0.094)	(0.214)	(0.303)	(0.057)	(0.132)	(0.174)
BAYESL	0.264	0.430	0.451	0.241	0.386	0.416	0.144	0.243	0.273
	(0.194)	(0.292)	(0.257)	(0.181)	(0.273)	(0.278)	(0.096)	(0.171)	(0.191)
BOOT	0.219	0.476	0.734	0.164	0.334	0.487	0.102	0.187	0.249
	(0.131)	(0.421)	(0.687)	(0.096)	(0.297)	(0.468)	(0.054)	(0.153	(0.215)
BOOTEX	0.251	0.494	0.710	0.190	0.351	0.494	0.110	0.197	0.259
	(0.158)	(0.404)	(0.651)	(0.132)	(0.296)	(0.450)	(0.065)	(0.154)	(0.214)
BOOTNP	0.483	0.812	0.974	0.404	0.623	0.755	0.306	0.411	0.494
	(0.277)	(0.447)	(0.552)	(0.247)	(0.379)	(0.454)	(0.219)	(0.275)	(0.315)
Panel B: Student-5		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.204	0.479	0.687	0.147	0.342	0.477	0.078	0.171	0.225
	(0.183)	(0.441)	(0.670)	(0.124)	(0.348)	(0.538)	(0.067)	(0.163)	(0.218)
ABJ	0.236	0.493	0.699	0.187	0.360	0.491	0.137	0.201	0.252
	(0.182)	(0.443)	(0.673)	(0.122)	(0.343)	(0.534)	(0.066)	(0.161)	(0.217)
ABJ_{aicc}	0.291	0.546	0.713	0.217	0.394	0.515	0.145	0.213	0.266
	(0.229)	(0.463)	(0.673)	(0.162)	(0.358)	(0.533)	(0.077)	(0.169)	(0.227)
BAYEST	0.197	0.376	0.513	0.146	0.287	0.375	0.097	0.178	0.221
	(0.144)	(0.300)	(0.477)	(0.094)	(0.221)	(0.322)	(0.052)	(0.118)	(0.149)
BAYESN	0.234	0.411	0.546	0.181	0.312	0.400	0.138	0.196	0.237
	(0.192)	(0.382)	(0.590)	(0.114)	(0.268)	(0.426)	(0.065)	(0.145)	(0.189)
BAYESL	0.305	0.424	0.443	0.263	0.391	0.413	0.180	0.262	0.287
	(0.244)	(0.332)	(0.287)	(0.184)	(0.283)	(0.278)	(0.095)	(0.187)	(0.217)
BOOT	0.240	0.485	0.721	0.183	0.369	0.532	0.113	0.198	0.258
	(0.160)	(0.415)	(0.652)	(0.111)	(0.333)	(0.538)	(0.054)	(0.150)	(0.213)
BOOTEX	0.270	0.505	0.706	0.211	0.390	0.533	0.122	0.208	0.266
	(0.187)	(0.410)	(0.637)	(0.139)	(0.330)	(0.509)	(0.063)	(0.152)	(0.214)
BOOTNP	0.512	0.805	0.968	0.418	0.645	0.809	0.330	0.436	0.527
	(0.311)	(0.429)	(0.507)	(0.315)	(0.419)	(0.551)	(0.210)	(0.309)	(0.413)
Panel C: $\chi^2_{(5)}$		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.202	0.503	0.730	0.138	0.331	0.466	0.075	0.167	0.223
	(0.156)	(0.468)	(0.755)	(0.113)	(0.328)	(0.498)	(0.054)	(0.139)	(0.185)
ABJ	0.292	0.540	0.757	0.249	0.379	0.500	0.218	0.242	0.279
	(0.134)	(0.458)	(0.752)	(0.087)	(0.311)	(0.488)	(0.040)	(0.120)	(0.169)
ABJ_{aicc}	0.336	0.598	0.777	0.274	0.411	0.524	0.225	0.252	0.290
	(0.170)	(0.468)	(0.756)	(0.124)	(0.337)	(0.507)	(0.056)	(0.137)	(0.187)
BAYESN	0.294	0.479	0.623	0.247	0.336	0.416	0.218	0.238	0.267
	(0.137)	(0.391)	(0.678)	(0.081)	(0.228)	(0.341)	(0.041)	(0.110)	(0.148)
BAYESL	0.341	0.486	0.489	0.303	0.423	0.439	0.246	0.296	0.311
	(0.172)	(0.327)	(0.298)	(0.128)	(0.258)	(0.263)	(0.072)	(0.157)	(0.184)
BOOT	0.229	0.507	0.767	0.167	0.355	0.515	0.103	0.191	0.252
	(0.142)	(0.440)	(0.730)	(0.103)	(0.323)	(0.512)	(0.050)	(0.136)	(0.192)
BOOTEX	0.261	0.529	0.745	0.194	0.376	0.518	0.111	0.200	0.260
	(0.152)	(0.431)	(0.709)	(0.127)	(0.322)	(0.496)	(0.067)	(0.144)	(0.200)
BOOTNP	0.496	0.782	0.951	0.403	0.623	0.780	0.336	0.414	0.497
	(0.284)	(0.435)	(0.545)	(0.253)	(0.393)	(0.498)	(0.221)	(0.257)	(0.309)

Table 5: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/ABJ	78.67 (0.05)	10.54/10.79	78.05 (0.09)	10.76/11.18	78.23 (0.11)	10.65/11.13
	ABJ_{aicc}	77.16 (0.06)	11.42/11.42	78.01 (0.10)	10.88/11.11	77.53 (0.12)	11.08/11.39
	SRA	79.84 (0.06)	8.98/11.18	72.32 (0.13)	12.24/15.44	64.80 (0.15)	15.69/19.51
	SRA_{aicc}	79.26 (0.07)	9.32/11.42	72.57 (0.13)	4.22 (1.25)	65.05 (0.16)	15.60/19.35
100	AEST	79.38 (0.04)	10.38/10.24	79.46 (0.06)	10.30/10.23	79.67 (0.08)	10.19/10.14
	ABJ_{aicc}	78.61 (0.04)	10.74/10.65	79.33 (0.07)	10.36/10.31	79.26 (0.08)	10.35/10.39
	SRA	79.97 (0.04)	9.56/10.47	76.96 (0.08)	11.08/11.96	73.98 (0.11)	12.40/13.62
	SRA_{aicc}	79.40 (0.05)	9.77/10.83	76.42 (0.09)	11.29/12.29	73.17 (0.12)	12.74/14.08
300	AEST	79.80 (0.02)	10.11/10.09	80.05 (0.03)	10.07/9.88	80.13 (0.05)	9.99/9.87
	ABJ_{aicc}	79.65 (0.02)	10.18/10.17	79.94 (0.03)	10.09/9.97	80.03 (0.05)	10.04/9.92
	SRA	79.97 (0.03)	9.86/10.16	79.19 (0.04)	10.37/10.44	78.63 (0.06)	10.64/10.73
	SRA_{aicc}	79.86 (0.03)	9.92/10.22	78.90 (0.04)	10.45/10.65	78.51 (0.06)	10.75/10.74
	Student-5	h=1		h=6		h=12	
50	AEST	77.55 (0.07)	11.42/11.03	76.88 (0.10)	11.76/11.36	77.48 (0.12)	11.44/11.08
	ABJ	81.85 (0.06)	9.24/8.90	78.58 (0.10)	10.91/10.51	78.66 (0.12)	10.83/10.50
	ABJ_{aicc}	80.24 (0.07)	9.98/9.78	78.13 (0.11)	10.91/10.96	77.80 (0.13)	11.08/11.11
	SRA	79.74 (0.07)	9.33/10.94	71.28 (0.13)	13.56/15.17	64.92 (0.16)	16.58/18.50
	SRA_{aicc}	79.25 (0.08)	9.46/11.29	71.45 (0.13)	13.13/15.42	65.27 (0.16)	16.28/18.44
100	AEST	78.62 (0.05)	10.77/10.61	78.40 (0.07)	10.86/10.74	78.61 (0.09)	10.75/10.64
	ABJ	82.81 (0.05)	8.66/8.52	80.02 (0.07)	10.05/9.93	79.76 (0.09)	10.17/10.06
	ABJ_{aicc}	81.94 (0.06)	9.11/8.95	79.82 (0.08)	10.27/9.91	79.38 (0.09)	10.48/10.14
	SRA	79.92 (0.05)	9.67/10.41	76.56 (0.09)	11.28/12.16	73.95 (0.11)	12.68/13.37
	SRA_{aicc}	79.28 (0.06)	10.01/10.71	76.31 (0.09)	11.44/12.25	73.25 (0.12)	12.93/13.82
300	AEST	79.64 (0.03)	10.16/10.20	79.75 (0.04)	10.08/10.16	79.86 (0.05)	10.00/10.14
	ABJ	83.75 (0.03)	8.09/8.14	81.34 (0.04)	9.28/9.37	80.96 (0.05)	9.45/9.59
	ABJ_{aicc}	83.50 (0.03)	8.23/8.27	81.17 (0.04)	9.36/9.47	80.79 (0.05)	9.53/9.68
	SRA	80.09 (0.03)	9.74/10.17	79.19 (0.04)	10.08/10.72	78.64 (0.06)	10.33/11.03
	SRA_{aicc}	79.97 (0.03)	9.79/10.24	79.00 (0.05)	10.29/10.71	78.38 (0.06)	10.57/11.05
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	78.01 (0.09)	11.20/10.77	77.10 (0.10)	11.35/11.53	77.07 (0.13)	11.27/11.65
	ABJ	83.14 (0.07)	5.36/11.49	78.15 (0.10)	9.67/12.17	77.77 (0.13)	10.05/12.17
	ABJ_{aicc}	81.17 (0.08)	6.77/12.05	77.79 (0.12)	9.86/12.34	76.88 (0.14)	10,52/12,60
	SRA	79.78 (0.08)	9.23/10.99	71.21 (0.13)	13.45/15.34	63.78 (0.16)	17.31/18.91
	SRA_{aicc}	78.98 (0.09)	9.76/11.26	71.33 (0.13)	13.58/15.10	64.16 (0.17)	16.93/18.92
100	AEST	78.77 (0.07)	10.82/10.40	78.78 (0.07)	10.52/10.69	78.98 (0.09)	10.37/10.64
	ABJ	84.17 (0.05)	4.72/11.10	79.82 (0.07)	8.85/11.32	79.62 (0.09)	9.20/11.17
	ABJ_{aicc}	82.96 (0.06)	5.70/11.35	79.46 (0.08)	9.11/11.42	79.08 (0.09)	9.57/11.35
	SRA	79.57 (0.05)	9.98/10.45	76.15 (0.08)	11.65/12.2	73.92 (0.11)	12.87/13.21
	SRA_{aicc}	78.98 (0.07)	10.48/10.54	76.04 (0.08)	11.82/12.14	73.25 (0.12)	13.10/13.65
300	AEST	79.71 (0.04)	10.20/10.08	79.78 (0.04)	10.15/10.07	79.91 (0.05)	10.04/10.03
	ABJ	85.43 (0.03)	3.77/10.79	80.86 (0.04)	8.45/10.68	80.58 (0.05)	8.87/10.55
	ABJ_{aicc}	85.05 (0.03)	4.09/10.85	80.65 (0.04)	8.62/10.71	80.39 (0.05)	9.01/10.59
	SRA	79.86 (0.03)	9.99/10.14	78.97 (0.05)	10.54/10.49	78.55 (0.06)	10.71/10.74
	SRA_{aicc}	79.71 (0.04)	10.11/10.18	78.68 (0.05)	10.75/10.57	78.28 (0.06)	10.90/10.82

Table 6: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BAYESN	79.70 (0.05)	10.10/10.2	79.10 (0.09)	10.29/10.62	78.35 (0.10)	10.68/10.97
	BAYESL	79.94 (0.06)	9.87/10.18	76.99 (0.09)	11.33/11.68	75.29 (0.11)	12.24/12.47
100	BAYESN	79.86 (0.04)	10.17/9.96	79.68 (0.06)	10.22/10.09	79.31 (0.07)	10.35/10.34
	BAYESL	79.72 (0.05)	10.28/10.01	78.09 (0.07)	11.02/10.89	76.72 (0.08)	11.64/11.64
300	BAYESN	79.81 (0.03)	10.10/10.08	79.98 (0.04)	10.05/9.967	79.88 (0.05)	10.10/10.02
	BAYESL	79.87 (0.03)	10.02/10.11	79.31 (0.04)	10.36/10.32	78.81 (0.05)	10.61/10.57
	Student-5	h=1		h=6		h=12	
50	BAYEST	80.12 (0.06)	9.96/9.91	78.38 (0.09)	10.83/10.79	77.82 (0.11)	11.11/11.07
	BAYESN	82.58 (0.06)	8.79/8.63	79.29 (0.09)	10.41/10.3	78.41 (0.11)	10.83/10.76
	BAYESL	82.46 (0.07)	8.56/8.97	77.36 (0.10)	11.26/11.38	75.59 (0.11)	12.21/12.20
100	BAYEST	80.12 (0.04)	10.03/9.85	79.01 (0.06)	10.55/10.44	78.62 (0.08)	10.72/10.65
	BAYESN	83.17 (0.05)	8.50/8.33	80.43 (0.07)	9.85/9.72	79.64 (0.08)	10.23/10.12
	BAYESL	83.09 (0.05)	8.45/8.46	79.23 (0.07)	10.49/10.28	77.66 (0.09)	11.24/11.10
300	BAYEST	80.14 (0.03)	9.89/9.96	79.87 (0.04)	10.06/10.07	79.59 (0.05)	10.16/10.25
	BAYESN	83.89 (0.03)	8.03/8.08	81.37 (0.04)	9.30/9.33	80.72 (0.05)	9.61/9.66
	BAYESL	83.76 (0.03)	8.10/8.14	80.78 (0.05)	9.64/9.58	79.77 (0.05)	10.13/10.10
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	83.77 (0.07)	5.06/11.17	79.28 (0.10)	9.04/11.68	78.12 (0.11)	9.90/11.97
	BAYESL	83.50 (0.08)	5.57/10.93	76.85 (0.10)	10.61/12.54	74.92 (0.11)	11.83/13.24
100	BAYESN	84.60 (0.05)	4.53/10.87	80.18 (0.07)	8.68/11.13	79.36 (0.08)	9.42/11.22
	BAYESL	83.94 (0.06)	5.32/10.73	78.42 (0.08)	9.91/11.66	77.00 (0.09)	10.87/12.14
300	BAYESN	85.37 (0.03)	3.89/10.74	80.83 (0.04)	8.47/10.7	80.29 (0.05)	9.02/10.70
	BAYESL	84.97 (0.04)	4.34/10.68	80.04 (0.05)	9.11/10.85	79.20 (0.06)	9.71/11.09

Table 7: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BOOT	79.08 (0.06)	10.36/10.56	79.64 (0.08)	10.04/10.32	80.69 (0.10)	9.45/9.86
	BOOTEX	78.60 (0.06)	10.68/10.72	80.50 (0.09)	9.67/9.83	80.95 (0.11)	9.39/9.66
	BOOTNP	72.17 (0.17)	13.57/14.26	64.44 (0.15)	17.26/18.30	59.98 (0.15)	19.64/20.38
100	BOOT	79.29 (0.04)	10.37/10.34	80.06 (0.06)	9.97/9.96	80.76 (0.07)	9.62/9.62
	BOOTEX	79.12 (0.05)	10.46/10.42	80.46 (0.06)	9.81/9.72	80.82 (0.08)	9.60/9.58
	BOOTNP	75.77 (0.14)	12.43/11.80	69.50 (0.13)	15.23/15.28	66.63 (0.13)	16.60/16.77
300	BOOT	79.68 (0.03)	10.15/10.17	80.06 (0.04)	10.06/9.877	80.45 (0.05)	9.86/9.69
	BOOTEX	79.66 (0.03)	10.18/10.16	80.12 (0.04)	9.97/9.90	80.50 (0.05)	9.78/9.72
	BOOTNP	79.77 (0.10)	10.04/10.19	74.31 (0.08)	13.02/12.67	72.33 (0.07)	13.97/13.70
	Student-5	h=1		h=6		h=12	
50	BOOT	79.30 (0.06)	10.55/10.15	79.31 (0.09)	10.58/10.11	80.43 (0.11)	9.99/9.57
	BOOTEX	79.18 (0.07)	10.61/10.21	79.75 (0.10)	10.21/10.03	80.34 (0.12)	9.92/9.74
	BOOTNP	71.43 (0.18)	13.40/15.17	63.24 (0.15)	17.60/19.16	59.20 (0.15)	19.84/20.96
100	BOOT	79.62 (0.05)	10.36/10.02	79.66 (0.07)	10.20/10.14	80.24 (0.08)	9.86/9.89
	BOOTEX	79.36 (0.05)	10.47/10.17	79.88 (0.07)	10.16/9.95	80.36 (0.09)	9.92/9.71
	BOOTNP	76.26 (0.16)	12.09/11.65	68.19 (0.13)	16/15.81	65.00 (0.13)	17.36/17.65
300	BOOT	79.90 (0.03)	10.04/10.06	80.00 (0.04)	9.97/10.02	80.40 (0.05)	9.68/9.92
	BOOTEX	79.88 (0.03)	10.03/10.09	80.13 (0.04)	9.84/10.03	80.40 (0.05)	9.718/9.88
	BOOTNP	80.05 (0.12)	9.71/10.24	73.41 (0.09)	13.13/13.46	71.27 (0.09)	14.12/14.61
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	79.38 (0.08)	9.91/10.71	79.28 (0.09)	10.02/10.7	80.11 (0.11)	9.52/10.37
	BOOTEX	79.40 (0.09)	9.62/10.98	80.09 (0.10)	9.37/10.54	80.10 (0.12)	9.37/10.53
	BOOTNP	72.79 (0.17)	12.89/14.32	64.39 (0.14)	17.97/17.64	59.90 (0.14)	20.18/19.92
100	BOOT	79.43 (0.06)	10.21/10.37	79.65 (0.07)	9.86/10.48	80.35 (0.08)	9.49/10.16
	BOOTEX	79.25 (0.07)	10.36/10.39	79.93 (0.07)	9.72/10.35	80.34 (0.09)	9.55/10.11
	BOOTNP	76.43 (0.17)	10.44/13.13	68.80 (0.13)	15.63/15.58	65.53 (0.13)	17.33/17.14
300	BOOT	79.67 (0.04)	10.10/10.22	79.98 (0.04)	9.99/10.03	80.39 (0.05)	9.73/9.88
	BOOTEX	79.55 (0.04)	10.25/10.19	79.97 (0.04)	10.02/10.01	80.27 (0.05)	9.85/9.88
	BOOTNP	81.19 (0.12)	7.57/11.23	74.01 (0.08)	12.40/13.59	72.24 (0.08)	13.19/14.57

Table 8: Simulation time in hours of the most demanding procedures.

Procedure	T=50	T=100	T=300
BAYESN	0.13	0.17	0.41
BAYEST	3.16	6.12	18.59
BAYESL	7.44	13.70	39.35
BOOT	0.6	0.6	1.00
BOOTEX	0.83	0.83	1.23
BOOTNP	0.42	0.83	3.55