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Valence, Complementarities, and Political Polarization*

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Abstract

I study a model of electoral competition where two parties that care about both the spoils of office and policy compete by announcing policy platforms. Parties are characterized by their valence on the one hand and by their policy platforms on the other. Unlike in the extant literature, I assume that valence and policy are *complements* (instead of substitutes) from the voter's perspective. I generally characterize electoral equilibrium and show that in such a framework increasing one or both parties' valence level(s) leads to policy moderation. To the contrary, if both parties have minimal valence policy platforms are maximally polarized. The model hence uncovers valence as an important determinant of political polarization.

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1 Introduction

One of the core characteristics of democracies is that voters are free to elect the party they favor, which in turn incentivizes parties to choose policies that are appealing to voters. This logic brings about the famous median voter theorem (Hotelling, 1929), which implies that competing parties choose identical platforms at the center of the electoral spectrum. Of course, in real elections parties usually do not choose identical platforms but differentiate along ideological lines. The literature has uncovered different reason for this, for example policy motivations of parties, uncertainty about voters' preferences, differences in valence, or electoral rules.[1] In this paper I describe another determinant of policy polarization: absolute valence or competence levels.

Starting with the seminal paper by Stokes (1963), political economists and political scientists have begun to study non-positional—or valence—issues. According to Stokes such valence issues are issues that '*'merely involve the linking of the parties with some condition that is positively or negatively valued by the electorate.*" With very few exceptions, the literature has operationalized valence by assigning to every party or party a valence level that additively changes voters' evaluation of this party.[2] Because valence enters voters' utility functions in an additively separable way, policy platforms are determined by valence *advantages*, whereas absolute valence levels play no role. In this paper I show that this conclusion is an artifact of the assumption that valence and policy are additively separable and thus perfect substitutes. Allowing for some degree of complementarity between policy and valence changes this conclusion and absolute valence levels begin to matter. In particular, as this paper shows, increasing parties' valence leads to policy moderation: parties adopt more centrist policy platforms.

Other authors have discussed the possibility of a utility function where valence and policy are not separable before. In the appendix of his paper, Groseclose (2001) discusses the case that valence is interpreted as competency, and hence influences the probability that the party actually fulfills its policy promises. Under this interpretation, valence and policy are complements from the voter's perspective. He provides conditions such that the party with a small valence advantage adopts a more moderate position than the low valence party. Hollard and Rossignol (2008) study a model with multiplicative valence and show that extreme voters vote based on valence, while moderate voters vote based on policy. Gouret et al. (2011) estimate how well different specifications of utility functions perform in explaining voter behavior in France. They show that a model they call the intensity model, where policy and valence are complements, explains voter behavior best, which suggests that the implications of complementarities need to be studied in more detail.

So why should valence and policy be considered complements rather than substitutes, at least in some cases? Groseclose (2001) provided us already with one rationale. When valence is interpreted

[1] See for example Calvert (1985), Groseclose (2001), or Matakos et al. (2016).

[2] See Aragones and Palfrey (2002), Meirowitz (2008), Herrera et al. (2008), Ashworth and Bueno de Mesquita (2009), Serra (2010), Denter and Sisak (2015), or Aragones and Xefteris (2017).

as a parties competence, then the probability that a party, once in office, actually manages to successfully implement its policy platform depends on valence. In particular, greater valence increases the change that the proposed policy can be implemented. Similarly, valence can be interpreted as the ability to run a public administration efficiently. For example, the degree to which taxation is distortionary might depend on the governing party's valence, as greater valence implies smaller welfare losses due to taxation. In a Meltzer and Richard (1981) style model of redistribution we should then expect the tax rate (policy) and party's valence to be complements.[3]

In this paper I develop a model of electoral competition with probabilistic voting in which parties, that are both policy and office motivated, choose policy platforms in an effort to win an election. The baseline model is deliberately starkly simplified: valence and policy are *perfect complements* for the voter, parties have equal valence, and parties' preferred policy positions are equidistant from and on different sides of the voter's preferred policy. This model is very tractable and is sufficient to show that a greater common level of valence leads to weakly more moderate policy choices. As I show in the sequel, this conclusion remains valid as long as policy and valence are complementary and is also robust to allowing for various kinds of asymmetries between parties. Hence, the model uncovers a simple but novel determinant of policy polarization.

Related Literature. The paper contributes to different strands of literature. First, it contributes to the extensive literature studying the implications of valence issues for policy making, for example Aragones and Palfrey (2002), Beniers and Dur (2007), Serra (2010), Xefteris (2014), or Aragones and Xefteris (2017). Like in the current paper, the focus of this literature is to characterize equilibria of an electoral competition game with a focus on how valence impacts policy choice. Modelling voter preferences for policy and valence as perfect substitutes, these papers show that only *differences* in valence matter, while absolute valence levels are irrelevant for policy choices. The current paper innovates by studying the implications of complementarities between valence and policy platforms. The model reveals that greater valence has a moderating effect on parties' policy platforms, even if parties have identical valence. This is similar to a finding by Ansolabehere and Snyder (2000), who study a model of office motivated parties who differ in valence and who choose a multi-dimensional policy vector. Voters' preferences are common knowledge. This implies that in equilibrium the party with the valence advantage locates close to the electoral center, while the disadvantaged party's policy is not clearly determined and it may choose any policy in the policy space, because it loses for sure. Hence, the advantaged party tends to be more moderate. In contrast, in the current paper I study parties with both office and policy motivation and show that in the presence of complementarities *both* parties tend to become more moderate as parties' valence increases. Bernhardt et al. (2011) study a dynamic model where parties are distinguished by both their

[3]More precisely, we should expected them to be q-complements as defined by Hicks (1970): greater valence implies greater marginal benefits from taxation.

valence levels and policies. Voters observe the valence only of incumbents, but not of challengers. They show that so far unknown challengers may choose more moderate policies when their valence increases. However, if the probability of being reelected is not too small, in the long run a majority of incumbents with greater valence tends to be more extreme. This is different in the current paper. Greater valence leads *uniformly* to policy moderation when valence and policy are complements.

Some authors have studied more general preference relations that go beyond the case of perfect substitutes before. As mentioned earlier, Groseclose (2001) discusses how small valence advantages affect policy choices when policy and valence are complements. He studies a model where parties cannot observe the voter's bliss point and derives conditions such that the favored party chooses a more moderate policy, while the disadvantaged party chooses a more extreme policy. In the current paper, parties know the voter's policy preferences but voting is probabilistic. This changes conclusions, because greater valence of one or both parties leads to generally greater policy moderation.

Krasa and Polborn (2012) also study equilibria of a platform determination game when some of the parties' characteristics are immutable while others can be flexibly adjusted. They allow for general voter preferences and assume that parties are office motivated and cannot perfectly observe the distribution of voters' preferences. While they do not provide conditions for the existence of equilibrium, they show that, if voter preferences satisfy what they call 'uniform candidate ranking' (UCR),[4] then any strict Nash equilibrium of the platform determination game is convergent, and if a strict Nash equilibrium exists, then it is the unique Nash equilibrium of the game. Therefore, in their framework there cannot be policy divergence in any strict Nash equilibrium, if voter preferences satisfy UCR. This differs in the current paper, even though voter preferences throughout the paper satisfy UCR. The reason is that parties also care about policies, which importantly impacts equilibrium behavior. This, together with a more restrictive voter utility function, allows me to study how changing valence affects equilibrium policy polarization. In an extension, I show that when parties care only about the spoils of office, then there is indeed policy convergence in any strict Nash equilibrium.

Hollard and Rossignol (2008) study a model of multiplicative valence and show that the set of voters, who prefer the high valence candidate, is non-convex: voters with extreme policy preferences prefer the high valence candidate, while moderates prefer the candidate who is ideologically closer to them. Gouret et al. (2011) study how well different utility function specifications, among others multiplicative valence, perform in explaining electoral data from France. They show that a model with complementarities between valence and policy—they call it the intensity valence model—outperforms a model of perfect substitutes. This intensity valence model violates the UCR property by Krasa and Polborn (2012). Gouret and Rossignol (2019) theoretically study the implications

[4]According to Krasa and Polborn (2012) a voter has UCR preferences, if, when both candidates choose the same policy p, the voter always prefers the same candidate independent of p. A formal definition of UCR can be found in Footnote 11.

of the intensity valence model when there is no electoral uncertainty and parties are purely office motivated. They show that when the electorate is sufficiently homogeneous (heterogeneous), the party with higher (lower) valence wins the election with certainty by choosing the median's preferred policy platform. For intermediate levels of voter heterogeneity, no pure strategy equilibrium exists and both parties win with positive probability. Thus, unlike in most of the literature, the whole distribution of voter preferences matters and not just the location of the median voter. Krasa and Polborn (2010) also study a model with complementarities between valence and policy. In their paper, a policy platform is a division of a fixed government budget over two different issues and is used to produce an issue specific public good. The amount of the public good a party produces in each issue dimension is the product of the party's issue specific ability and the respective budget allocation, and hence there are complementarities between policy and valence/ability. Voter preferences in Krasa and Polborn (2010) also violate UCR, which implies there must be policy divergence. The model studied in the current paper differs from the ones in the above mentioned articles in that preferences satisfy UCR, and thus, absent parties' policy motivation, there tends to be policy convergence in equilibrium. However, the focus is on situation in which parties have policy preferences themselves, and therefore electoral equilibria usually display policy divergence. The main focus of the paper is to show how the degree of policy divergence, or policy polarization, depends on parties' valence levels.

Ahn and Oliveros (2012) study voting equilibria when voters vote on multiple policy issues at the same time and issues can be both substitutes and complements. In their paper, policies are fixed. In contrast, the current paper is concerned with endogenous policy platforms when policies and party valence are complements.

The paper also relates to a literature studying the origins of policy platform polarization. For example, Matakos et al. (2015, 2016) or Bol et al. (2019) analyze both theoretically and empirically how electoral rule disproportionality impact parties' equilibrium policy platforms. They show that greater rule disproportionality goes hand in hand with greater platform polarization. The current paper also studies platform polarization, but suggests that the reason is that voters perceive valence and policy to some degree as complements. There are also a papers that study policy platforms when valence is partly endogenous. Herrera et al. (2008) study a model of electoral competition in which there is uncertainty about parties' exogenous valence levels, and where parties care about both the spoils of office and policy. They show that an increase in the accuracy of campaign spending leads to policy moderation, polarization decreases. To the contrary, an increase in electoral uncertainty also increases policy polarization. Ashworth and Bueno de Mesquita (2009) and Balart et al. (2018) study parties' investments in valence when parties are purely policy motivated and increasing platform polarization shifts voters attention from valence to policy issues. Importantly, in these papers the mechanism is not from valence to endogenous platform polarization but from platform polarization to endogenous valence. Increasing platform polarization leads to less investment in valence, which means parties economize on effort in the endogenous valence game and hence save costs. In the current paper, valence is exogenous, but nevertheless impacts a party's platform choice. In particular, greater exogenous valence leads to platform moderation even when both parties have identical valence.

2 The Model

In this section I describe the basic model. There is one voter and two political parties, $i = 1, 2$. Each party is described by a fixed immutable characteristic v_i (valence) and an endogenous policy platform $p_i \in [0, 1]$. In the baseline model and in some of the extensions, I will assume that $v_1 = v_2 = v$ and hence that valence is common. The voter prefers higher valence over lower valence and has Downsian policy preferences. Her bliss point in the policy dimension is $b^* = \frac{1}{2}$. The utility she receives if party i is elected is

$$u(p_i, v) = \min\left\{\frac{1}{2} - \left|\frac{1}{2} - p_i\right|, v\right\}.$$

Thus, valence and policy are perfect complements to the voter. This is the main difference in comparison to the extant literature, which usually studies perfect substitutes.

Voting is probabilistic and the voter casts a vote for party 1 if $u(p_1, v) - u(p_2, v) - \epsilon > 0$, and votes for party 2 otherwise. $\epsilon \in \mathbb{R}$ is a continuous random variable with the $\mathcal{C}^2$ cumulative distribution function (c.d.f.) G and density g. I assume $g(\epsilon)$ is symmetric around zero, strictly positive on at least $[-1, 1]$, and quasi-concave.[5] Moreover, I assume that (i) $|g'(\epsilon)| < g(\epsilon)$ and (ii) $g(0) \in [\frac{1}{4}, \frac{1}{2}]$. (i) assures a well behaved optimization problem for the parties, while (ii) is useful to avoid corner solutions, which simplifies exposition.[6] Distributions fulfilling the assumption include for example the uniform on $[-m, m]$, $m \in [1, 2]$, or a truncated normal distribution with support $[-2, 2]$ and variance $\sigma^2 = \sqrt{2}$.

The political parties care about both the spoils of office, which have value 1, and about the policy platform that wins. The framework is thus similar to for example Whitman (1983), Groseclose (2001), or Herrera et al. (2008). Parties' utility functions are as follows:

$$\pi_i = \begin{cases} 1 - |b_i - p_i| & \text{if } i \text{ wins,} \\ -|b_i - p_j| & \text{if } j \text{ wins.} \end{cases}$$

b_i is party i's bliss point in the policy dimension. I assume that $b_1 = 1 = 1 - b_2$. Letting $\varphi(p_1, p_2, v)$

[5]Absent the randomness caused by ϵ, an electoral equilibrium in pure strategies might not exist or multiple equilibria might exist, similar to Ansolabehere and Snyder (2000). ϵ assures that the electoral competition game has a unique Nash equilibrium.

[6]For all $g(0) \geq \frac{1}{2}$ in the symmetric and unconstrained equilibrium described below in Proposition 1, both parties will choose a policy $p_1 = p_2 = \frac{1}{2}$. Similarly, for all $g(0) \leq \frac{1}{4}$ both will choose an extreme policy equal to the respective individual bliss point. Assuming $g(0) \in [\frac{1}{4}, \frac{1}{2}]$ still allows for these equilibria but simplifies the notation.

denote the probability that party 1 wins the election, we can now describe the parties' expected utility functions as follows:

$$\begin{aligned} E\pi_1(p_1,p_2,v) &= \varphi(p_1,p_2,v)\,(1-|1-p_1|)-(1-\varphi(p_1,p_2,v))\,|1-p_2| \\ E\pi_2(p_1,p_2,v) &= (1-\varphi(p_1,p_2,v))\,(1-|0-p_2|)-\varphi(p_1,p_2,v)\,|0-p_1| \end{aligned} \tag{1}$$

Parties choose platforms to maximize their expected utility. The solution concept is Nash equilibrium.

The model is quite stylized and many of the assumptions imposed appear quite strict. I chose this simple framework because it allows me to show the important intuitions in a straightforward and intuitive way. Nevertheless, before moving to the equilibrium analysis, at this point it is worthwhile to pause shortly and to discuss the model's main assumptions.

In the first part of the paper, both parties have equal valence. This is of course not very realistic, but I chose this assumption to highlight an important consequence of complementarities between valence and policy: absolute valence level matter for policy choices. In the extant literature, where valence and policy are usually perfect substitutes, this is different, because the additive separability of utility from v and utility from p_i implies that both parties valence levels cancel each other out. In Sections 4.1 and 4.2 I show that the main result, that greater valence leads to policy moderation, remains robust also when parties have different valence levels.

Another strict assumption that I have imposed is that preferences are *perfect* complements. This assumption is not supposed to be taken literally, but it helps to develop the important intuitions in a concise way. Nevertheless, in Section 4.2 I show that this assumption is innocuous and that the main result carries over to the case of *imperfect* complements.

Another simplifying assumption is that parties are homogeneous in their ideological distance from the voter. While this assumption may seem strict, as the voter might be ideologically closer to one of the two parties, in Section 4.3 I show that this assumption does not matter for conclusions and that the paper's main result remains valid also when parties differ in their ideological distance from the voter.

Finally, I have assumed that the value of office is fixed and equal to 1. This implies that the spoils of office have greater weight in parties' utility function than policy differences. This assumption is useful to avoid having to deal with corner equilibria. In Section 4.4 I show that relaxing this assumption does not impact the model's main results.

3 Equilibrium

Before starting with the equilibrium analysis, I establish three useful lemmas:

Lemma 1. *Any policy* $p_1 \in [0, \max\{\frac{1}{2}, 1-v\})$ *is strictly dominated for party 1. Any policy* $p_2 \in (\min\{\frac{1}{2}, v\}, 1]$ *is strictly dominated for party 2.*

Lemma 2. *Each party i's expected utility is strictly concave in the own policy* p_i *whenever* $v \geq \frac{1}{2} - \left|\frac{1}{2} - p_i\right|$.

Lemma 3. *A pure strategy Nash equilibrium exists.*

Lemma 1 helps us to narrow down the relevant policy space for the each party. It tells us in particular that as long as a party is judged based on its valence, that is when $\frac{1}{2} - \left|\frac{1}{2} - p_i\right| > v$, then this party has an incentive to deviate from p_i and to choose a policy closer to b_i. Lemma 2 shows that when $\frac{1}{2} - \left|\frac{1}{2} - p_i\right| \leq v$, i.e., when party i is evaluated by his policy choice, i's utility is strictly concave in p_i. This is important to prove Lemma 3.

Let us now begin with the equilibrium analysis. If both parties have high valence, $v \geq \frac{1}{2}$, there will be intense policy competition between both, as both parties will be judged based on their policies. The probability that party 1 wins the election is then $G(|\frac{1}{2}-p_2|-|\frac{1}{2}-p_1|) = G(1-p_1-p_2)$. To the contrary, if valence is low, $v \leq 0$, the lack of valence makes both parties seem unattractive, independent of their policy choices, and that destroys competition. In that case, independent of the chosen policy platforms, each party wins with probability $G(0) = \frac{1}{2}$. Finally, if $\frac{1}{2} > v > 0$, competition is intermediate and the winning probabilities depend on both v and the respective policy choices.

With low valence, the unique electoral equilibrium follows directly from Lemma 1: both parties choose their respective bliss points. Hence, low valence induces great policy divergence, $p_1^* = 1 = 1 - p_2^*$. To the contrary, with high valence parties may move closer to the center. The reason is that in this case the voter reacts entirely to parties' policy platforms. When $g(0)$ is large and thus large electoral shocks are unlikely, both parties move to the electoral center. Finally, with intermediate valence, $v \in (0, \frac{1}{2})$, there is competition between parties and whether the voter judges them by their policy or valence is determined endogenously. Choosing an extreme policy will appal the voter, but moving all the way to the center is to no avail as the low valence prevents the voter to esteem the parties higher. Proposition 1 formalizes these intuitions:

Proposition 1. *When parties have equal valence v, the electoral competition game has a unique Nash equilibrium in which parties locate symmetrically around the voter's bliss point,* $p_1^* = 1 - p_2^*$, *where*

$$p_1^* = \max\left\{1 - v, \frac{1}{4g(0)}\right\}.$$

Since no party has an electoral advantage through higher valence, the result that parties locates symmetrically around the voters's bliss point is intuitive. Moreover, when valence v is large, it has no impact on policy platforms, as the voter focusses on the respective policy platforms. However, as valence decreases this changes at some point, and parties choose more *radical* platforms. The

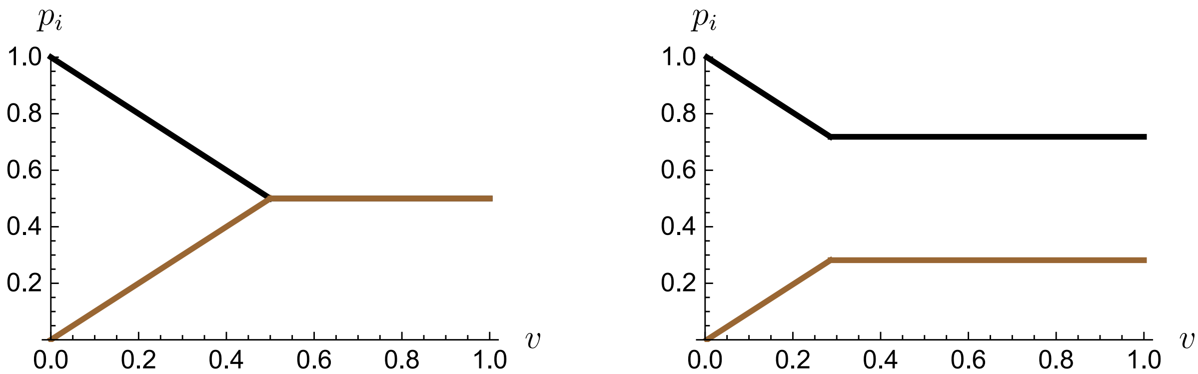

Figure 1: p_1^* and p_2^* as a function of v when $g(0) = \frac{1}{2}$ (left panel) and when $g(0) = 0.35$ (right panel). Platform polarization clearly decreases as valence increases.

reason is that the benefit of choosing a more moderate platform is dampened by the low valence, while the costs of a moderate platform remain unchanged. Figure 1 shows how policies change with v for different values of $g(0)$.

A direct implication of Proposition 1 is that policy platforms tend to become more polarized as the *common* valence level v decreases. Define equilibrium platform polarization as follows:

$$\Delta(v) := |p_1^* - p_2^*| .$$

Then the following proposition is a direct corollary of Proposition 1:

Proposition 2. *Platform polarization $\Delta(v)$ weakly decreases in the valence level v. $\Delta(v)$ is maximized and equal to 1 when $v \leq 0$.*

When valence is low, increasing valence means platforms become strictly more moderate, but when valence is large, it has no impact on the electoral equilibrium anymore. Hence, greater valence weakly decreases platform polarization. The proposition provides us with a new determinant for the degree of platform polarization. When valence and policy are complements, a greater common valence level leads to platform moderation. In the extant literature equal valence levels imply that policies are independent of valence, because they are perfect substitutes from the voter's perspective. Of course the model is very stylized, because even if complementarities are probably relevant, it is unlikely that policy and valence are *perfect* complements. I chose this simple model because it allows to very transparently derive the important intuitions and mechanisms. In Section 4.2 I study the more realistic but also more complicated case of imperfect complements.

Also the assumption of common changes in valence is of course a strict one. One possible justification for such a model could be a comparison between two countries. If parties' valence levels in country i are drawn from a distribution F_i with support $\mathcal{S} \subseteq \mathbb{R}$, and if F_1 first-order stochastically dominates F_2, then expected valence $\bar{v}_1$ in country 1 is greater than in country 2, $\bar{v}_2$.

Going from F_2 to F_1 is one way to think about a common change in valence. In the limit,[7] when the variance of valence in both countries i converges to zero, valence levels in country i equal their respective expected values $\bar{v}_i$. Our analysis then suggests that equilibrium policy polarization will be lower in country 1 than in country 2. This remains true even without taking the limit, if valence levels become common knowledge after they are drawn and if policy polarization also decreases in *individual* valence levels v_1 and v_2. In Section 4.1 I show that this is indeed the case, and hence Proposition 2 generalizes.

4 Discussion

In this section I extend the model along several dimensions and show that the main conclusion drawn from the simple model, i.e., that greater valence leads to policy moderation, is valid more generally.

4.1 Valence Differences

We now turn to analyze a situation in which parties differ in their valence levels. Without loss of generality, let $v_1 > v_2$. With unequal valence levels, equilibrium depends, as before, on the absolute valence levels. If v_2 is large, both parties will strictly be evaluated by their policies. We may say that in this case parties are not constrained by their valence levels. Then the equilibrium described in Proposition 1 remains valid and parties choose platforms equidistant around $\frac{1}{2}$, even though they differ in their valences:

Proposition 3. *Let $v_1 > v_2 \geq 0$. If $v_2 \geq 1 - \frac{1}{4g(0)}$, the electoral competition game has a unique Nash equilibrium, in which parties locate symmetrically around the voter's bliss point, $p_1^* = 1 - p_2^*$, where $p_1^* = \frac{1}{4g(0)}$.*

The proposition shows that valence differences need not imply asymmetric policy platforms. If both parties have valence high enough such that the voter strictly focuses on policy, then having greater valence does not translate into an electoral advantage. Consequently, the electoral equilibrium is symmetric.

When the condition in Proposition 3 is violated, party 2 is constrained in its policy choice by its low valence. It will then choose $p_2 = v_2$, while party 1 reacts optimally. The optimal reaction will generally lead to a policy p_1 that is more moderate than p_2, i.e., it is closer to the voter's bliss point than v_2. Hence, having a valence advantage has a moderating effect. If v_1 is large, this optimal reaction will be somewhere in $[\frac{1}{2}, 1 - v_2)$. Otherwise, when party 1 has low valence, it might also be constrained in equilibrium and the optimal policy then is in $[1 - v_1, 1 - v_2)$:

[7] For example, think of the limits of two shifted Dirac delta functions that converge to $\bar{v}_1$ and $\bar{v}_2$ respectively.

Proposition 4. *Let $v_1 > v_2 \geq 0$. If $v_2 < 1 - \frac{1}{4g(0)}$, the electoral competition game has a unique Nash equilibrium, in which party 2 chooses policy platform $p_2^* = v_2$. Moreover, if $\frac{G(v_1 - v_2)}{g(v_1 - v_2)} > 2 - v_1 - v_2$, party 1 chooses a policy $p_1^*(v_2) \in (1 - v_1, 1 - v_2)$ that decreases in v_2. If $\frac{G(v_1 - v_2)}{g(v_1 - v_2)} \leq 2 - v_1 - v_2$, party 1 chooses policy $p_1^* = 1 - v_1$.*

While equilibrium is strictly symmetric if no party is constrained by its valence, constrained electoral equilibria are generally asymmetric. When only the low valence party is constrained in its policy choice, it chooses a more extreme policy platform than party 1. This resembles earlier findings in the literature, see for example Ansolabehere and Snyder (2000) or Groseclose (2001). However, the intuition is a different one. In Ansolabehere and Snyder (2000) voters' preferences are known with certainty, and therefore the disadvantaged party is unconstrained in its policy choice because it will lose the election anyway. As a consequence, any policy, also an extreme one, could be an equilibrium outcome. In Groseclose (2001) this is the case because parties do not know the exact position of the voter's bliss point, and the disadvantaged party strategically chooses an extreme position in an effort to "gamble for resurrection." In the current paper the intuition follows from a standard microeconomic reasoning: With complementarities between valence and policy, the marginal benefit of moving closer to the center is increasing in the own valence.[8]

In Section 3 we saw that with equal valence, $v_1 = v_2 = v$, raising v uniformly weakly decreased policy polarization. Proposition 3 shows that when valence is unequal but high, this remains true, because policies are unaffected by v. Further, Proposition 4 shows that this remains true also when valence is low and therefore one or both parties are constrained in their policy choices. Define equilibrium platform polarization analogous to before as $\Delta(v_1, v_2) := |p_1^* - p_2^*|$. Then we can conclude the following:

Proposition 5. *Equilibrium platform polarization $\Delta(v_1, v_2)$ weakly decreases in both v_1 and v_2.*

The proposition shows that the result established with equal valence carries over to the case where parties have valence differences. Increasing valence, i.e., higher v_1, higher v_2, or both, leads to policy moderation. The left panel of Figure 2 shows this relation between valence and equilibrium policies.

4.2 Imperfect Complements

So far the results were derived under the simplifying assumption that valence and policy are *perfect* complements. It hence stands to reason to ask whether the moderating effect of greater valence is an artifact of this somewhat extreme utility function, or whether this is a robust result. To this end, assume that the voter has utility function $u(v_i, d_i)$, which strictly increases in both v_i

[8]Note that in models where uncertainty is not regarding the voter's bliss point, but regarding the size and direction of the valence advantage, the predictions in all standard models, for example Whitman (1983), go in the opposite direction: the advantaged party takes the more radical policy position.

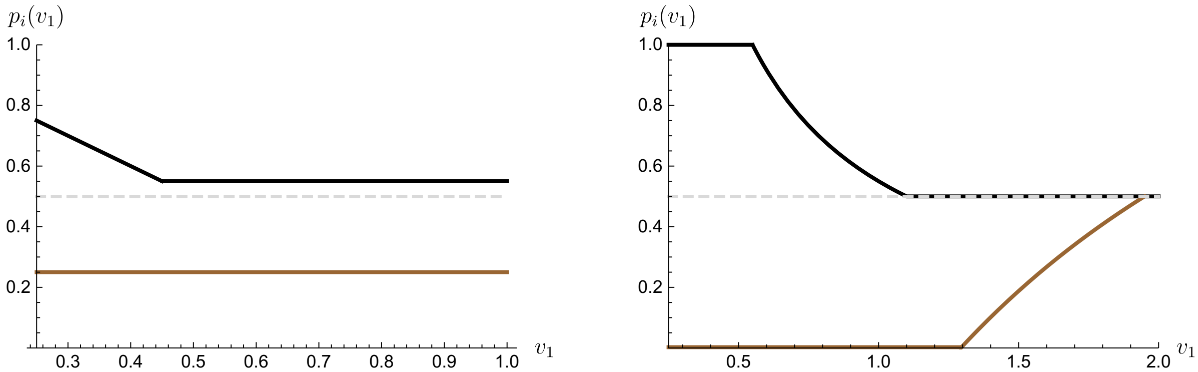

Figure 2: p_1^* and p_2^* as functions of $v_1 \in [\frac{1}{4}, 2]$ when $v_2 = \frac{1}{4}$ and $\epsilon \sim U[-\frac{11}{10}, \frac{11}{10}]$ for the case of perfect complements (left panel) and a Cobb-Douglas utility function with $u(v_i, p_i) = v_i\left(\frac{1}{2} - \left|\frac{1}{2} - p_i\right|\right)$ (right panel; see Section 4.5). Under both specifications, the lower valence party 2 chooses more extreme positions and policy polarization weakly decreases in v_1.

and $d_i \equiv -|\frac{1}{2} - p_i|$ and is weakly concave in d_i. If the cross-derivative $\frac{\partial^2 u}{\partial v_i \partial d_i}$ is positive, which I assume throughout this section, valence and policy are (imperfect) complements for the voter. If the cross-derivative is zero, $\frac{\partial^2 u}{\partial v_i \partial d_i} = 0$, which is the standard formulation in the literature, valence and policy are perfect substitutes and additively separable.

Our above analysis suggests that whenever there is (some form) of complementarity, greater valence v should have a moderating effect on policy platforms. The next proposition shows that this is indeed the case:

Proposition 6. *If an interior Nash equilibrium exists, it is symmetric, $p_1^* = 1 - p_2^*$, and unique. Moreover, if $\frac{\partial^2 u(v,d_i)}{\partial v \partial d_i} > 0$ (i) both parties move closer to the center as the common level of valence v increases and consequently (ii) platform polarization $\Delta(v)$ decreases in v. If $\frac{\partial^2 u(v,d_i)}{\partial v \partial d_i} = 0$, both platforms and polarization are invariant to changes in v.*

The proposition shows that the results derived with perfect complements carry over to the more general case of imperfect complements, at least when a symmetric interior equilibrium exists.[9] The absolute value of the common valence level v importantly impacts parties' strategic policy choices and greater v leads to greater policy moderation. This is in stark contrast to the existing literature, where v generally has no impact on policy outcomes. Proposition 6 reveals that the reason for this is that policy and valence are *perfect* substitutes from the voter's perspective.

Of course, the assumption of a common level of valence is strict, but the results derived in Section 4.1 indicate that greater valence should also have a moderating factor when $v_1 \neq v_2$, at least when valence and policy are perfect complements. The next proposition shows that this result generalizes at least locally around the symmetric equilibrium:

[9] It is straightforward to show that for example when G is a uniform distribution on $[-1, 1]$ and if the voter's utility function is of the CES type, $u(v, p_i) = \left(\alpha v^\gamma + (1-\alpha)\left(\frac{1}{2} - \left|\frac{1}{2} - p_i\right|\right)^\gamma\right)^{\frac{1}{\gamma}}$, then parties' expected utility functions are strictly concave in the own strategy and a symmetric pure strategy equilibrium exists.

Proposition 7. *In a neighborhood around the symmetric equilibrium with $v_1 = v_2$ and $p_2^* = 1 - p_1^*$, equilibrium platform polarization $\Delta(v_1, v_2)$ decreases in v_i, $i = 1, 2$:*

$$\left.\frac{\partial \Delta(v_1, v_2)}{\partial v_i}\right|_{v_1 = v_2} < 0, \ i = 1, 2.$$

If $\frac{\partial^2 u(v_i, d_i)}{\partial v_i \partial d_i} = 0$, platform polarization is invariant to changes in v_i, $i = 1, 2$.

Note that the proposition does not imply that *both* platforms, p_1 and p_2, move closer to the electoral center. If v_i increases, p_j becomes more moderate, but p_i might actually become more radical. Indeed, if $\frac{\partial^2 u(v_i, d_i)}{\partial v_i \partial d_i} \approx 0$ but positive, it always does. This is an important difference to our above analysis with perfect complements: Only if the degree of complementarity $\frac{\partial^2 u(v_i, d_i)}{\partial v_i \partial d_i}$ is sufficiently large will *both* parties choose more moderate platforms when one party's valence increases. When complementarity is low, a party may indeed adopt a more extreme platform when its valence increases. However, even when that is the case, polarization decreases because p_j reacts stronger to the changing valence than does p_i.

To be able to derive results that hold more generally, that is, not only in a neighborhood of the symmetric equilibrium, we impose another assumption: $\epsilon \sim U[-m, m]$. This assumption is already sufficient to guarantee existence of equilibrium, but for uniqueness we need an additional assumption:

Assumption 1.

$$1 > \frac{\frac{\partial u(v_j, d_j)}{\partial d_j} - \frac{\partial u(v_i, d_i)}{\partial d_i}}{(2 - d_i - d_j)\frac{\partial^2 u(v_i, d_i)}{\partial d_i^2} - 2\frac{\partial u(v_i, d_i)}{\partial d_i}} > -1 \ \forall (v_i, v_j) \in \mathbb{R}^2 \wedge \forall (d_i, d_j) \in \left[0, \frac{1}{2}\right]^2.$$

If this holds, the slope of the best response functions is strictly between 1 and -1, which implies uniqueness of Nash equilibrium. Under these assumptions we can prove the following result:

Proposition 8. *Let $\epsilon \sim U[-m, m]$. If Assumption 1 holds, then the game has a unique Nash equilibrium in pure strategies. Moreover, in any interior Nash equilibrium platform polarization $\Delta(v_1, v_2)$ strictly decreases in v_i, $i = 1, 2$, if $\frac{\partial^2 u(v_i, d_i)}{\partial v_i \partial d_i} > 0$ is sufficiently large.*

The proposition shows that when complementarities between policy and valence are sufficiently strong, then greater valence of one of the parties generally decreases platform polarization. A sufficient condition for this to hold is $\frac{\partial^2 u(v_i, d_i)}{\partial d_i^2} \approx 0$. The right panel of Figure 2 shows this graphically.

4.3 Asymmetric Bliss Points

So far both parties' bliss points were located symmetrically around the voter's bliss point, $b_1 = 1 = 1 - b_2 \Leftrightarrow b_1 + b_2 = 1$. This of course promotes symmetric equilibria like the ones we have discovered so far. If we assume asymmetrically located party bliss points, this may beget asymmetric equilibria, and as parties' bliss points move closer to the electoral center, polarization will decrease. Nevertheless, increasing valence still leads to policy moderation if valence and policy are complements:

Proposition 9. *Assume $b_1 \in (\frac{1}{2}, 1)$ and $b_2 = 0$ as before. Then $\Delta(v)$ weakly decreases in v.*

Therefore, the conclusion that greater valence leads to policy moderation is robust to allowing for asymmetric bliss points.

Note that this result holds true more generally, i.e., also when preferences can be represented by the general utility function from Section 4.2. To see this, assume an interior equilibrium, i.e., a situation such that $b_1 > p_1^* > \frac{1}{2} > p_2^* > 0$. Now note that neither parties' stakes in the competition are affected by b^*. This is clear for party 2, who only cares about party 1's policy platform and not about the underlying preferences. But also the stakes of party 1 are unaffected by changing b_1. To see this, note that the stakes of party 1 can be measured by the difference between the utility conditional on being elected, $1 - |p_1 - b_1|$, and the utility conditional on losing the election, $-|p_2 - b_1|$. This difference equals $1 + p_1 - p_2$, which is independent of b_1. Hence, we can conclude that neither parties' stakes are affected by changing b_1, and thus that our comparative static results from Section 4.2 remain true in any interior equilibrium, even when parties' bliss points change. However, also note that whether an interior equilibrium exists does depend on the bliss points.

4.4 Value of Spoils of Office

Up to this point, the spoils of office had value 1 and where hence weakly more important to the parties than policy platforms. This excluded the extreme cases of purely office and purely policy motivated parties. Assume now instead that the value of the spoils of office is $W \in [0, 1] \cup \{\infty\}$.[10] When $W = 0$, parties are purely policy motivated as in Section 2 of Calvert (1985) or in Duggan and Fey (2005). Increasing W increases the payoff of winning the election, independent of the policies chosen. This changes how both parties' trade off a marginal increase in the probability of winning and the payoff conditional on winning, with greater emphasis on the latter. Hence, parties tend to move to the electoral center as W increases and therefore become more moderate.

When $W \to \infty$, politicians become purely office motivated. Theorem 1 of Krasa and Polborn (2012) shows that in this case all strict Nash equilibria must be convergent if $v_1 = v_2 = v$ and if voters' preferences satisfy their uniform party ranking (UCR) condition.[11] They also show that if a

[10] When $W \in (1, \infty)$ the assumptions made are not sufficient to guarantee that expected utility is quasi-concave in the own policy platform.However, if $\epsilon \sim U[-1, 1]$ results are easy to come by. In this case $p_2^* = 1 - p_1^*$ and $p_1^* = \min\left\{\max\left\{\frac{1}{2}, 1 - v, \frac{2-W}{2}\right\}, 1\right\}$, implying policy polarization still decreases in valence.

[11] Using the notation of the current paper, preferences satisfy UCR when, if parties choose equal policies $p_1 = p_2 = p$, $u(v_1, p) \geq u(v_2, p)$ or $u(v_1, p) \leq u(v_2, p)$ for all $p \in [0, 1]$. This is the case with perfect complements, because when $v_1 \geq v_2$, it also holds that $\min\left\{v_1, \frac{1}{2} - \left|\frac{1}{2} - p\right|\right\} \geq \min\left\{v_2, \frac{1}{2} - \left|\frac{1}{2} - p\right|\right\}$ for all $p \in [0, 1]$.

strict Nash equilibrium exists, this is the unique Nash equilibrium of the game. Finally, their Theorem 5 also shows that the unique equilibrium is convergent in a version of the probabilistic voting model with Euclidian preferences studied by Persson and Tabellini (2002). Perfect complements do satisfy UCR. Nevertheless, the next proposition shows that not all equilibria are convergent, i.e., equilibria with $\Delta(v) > 0$ may exist. The reason is that no strict Nash equilibrium exists when valence is small, $v < \frac{1}{2}$. In this case, choosing the voter's ideal policy is not a strict Nash equilibrium, because $\min\{v, \frac{1}{2}\} = v$, implying both politicians can change their policy without changing the voter's utility. Indeed, any $(p_1, p_2) \in [\frac{1}{2}, 1 - v] \times [v, \frac{1}{2}]$ is an equilibrium when $v < \frac{1}{2}$. With a continuum of equilibria, comparative statics of $\Delta(v)$ are not really meaningful. Nevertheless, we can derive an upper bound $\bar{\Delta}(v)$ on polarization and see how this changes with valence.

Proposition 10. *If $W \in [0, 1]$, then platform polarization $\Delta(v)$ weakly decreases in v. If $W \to \infty$, then $\Delta(v) \in [0, \max\{0, 1 - 2v\}]$, and thus the upper bound on polarization is $\bar{\Delta}(v) = \max\{0, 1-2v\}$, which also weakly decreases in v.*

Note that the potential multiplicity of Nash equilibria is an artifact of the assumption of *perfect* complements, which implies that when valence is small, the voter's utility is flat in the politicians' policy choices over some range of policies in a neighbourhood of $\frac{1}{2}$. With imperfect complements as defined in Section 4.2 and pure policy motivation ($W \to \infty$), a unique strict Nash equilibrium exists, and it follows from Krasa and Polborn (2012) that in this case both parties always choose $p_1^* = p_2^* = \frac{1}{2}$, implying $\Delta(v) = 0$. To the contrary, when politicians become purely policy motivated ($W = 0$), then a convergent equilibrium never exists and $\Delta(v) > 0$. For intermediate values of W, such that an interior equilibrium exists, and it remains true that equilibrium platform polarization decreases in valence v. The next proposition formalizes this result.

Proposition 11. *Assume preferences are as defined in Section 4.2. When $W \to \infty$, then $\Delta(v) = 0$ $\forall v \geq 0$. Moreover, if W is finite and if an interior equilibrium exists, then this equilibrium is unique and symmetric, $p_2^* = 1 - p_1^*$, and $\Delta(v)$ decreases in v. Finally, if $W = 0$, then $\Delta(v) \in (0, 1]$ $\forall v \geq 0$.*

4.5 An Example: Cobb-Douglas Utility and Uniform Noise

In this section I study the above questions by way of an example. The purpose of the example is to show in a concise way how all the effects studied above work out together, and that the paper's main result remains valid in such a scenario. In the example, I allow for potentially asymmetric bliss points, an arbitrary value of the spoils of office, potentially differing valence levels, and the utility specification is one with imperfect complements. To be able to solve the game in closed form, I also assume $\epsilon \sim U[-m, m]$. To keep the exposition simple, I focus on interior Nash equilibria.

Example 1: *Let $u(p_i, v_i) = v_i \cdot (\frac{1}{2} - |\frac{1}{2} - p_i|)$ and $\epsilon \sim U[-m, m]$. Moreover, let the spoils of office be $W \geq 0$, let valence levels be $v_1 > 0$ and $v_2 > 0$, and let party 1's bliss point be $b_1 \in (\frac{1}{2}, 1]$. In an* *interior equilibrium, policy platforms are*

$$p_1^* = \frac{(v_1+v_2)(v_1-v_2W)-m(v_1-3v_2)}{(v_1+v_2)^2} \in \left(\tfrac{1}{2}, b_1\right),$$
$$p_2^* = \frac{(v_1+v_2)v_1(1+W)+m(v_2-3v_1)}{(v_1+v_2)^2} \in \left(0, \tfrac{1}{2}\right),$$

and equilibrium policy polarization is

$$\Delta(v_1, v_2) = \frac{2m}{v_1+v_2} - W > 0.$$

Thus, policy polarization decreases in both v_1 and v_2.

In the example platform polarization generally decreases in v_1 and v_2 in any interior equilibrium. Moreover, it decreases in the value of the spoils of office W and increases in m, which is a measure of the importance of randomness in the campaign. Party 1's bliss point b_1 has no influence on polarization in interior equilibrium, but influences whether equilibrium is interior.

5 Conclusion

This paper presents a simple model of electoral competition when valence and policy are complements from the voter's perspective. The model shows that these complementarities induce parties to choose policy positions that are closer to the electoral center when one party's or both parties' valence increases. In particular, the model shows that valence or party quality is an important predictor of electoral polarization even if no party has a valence advantage.

A Mathematical Appendix

A.1 Proof of Lemmas 1 - 3

In this section I prove the lemmas from the main text. I will prove them for the case when valence levels are potentially different, $v_1 \geq v_2$, as this will be useful for the proofs of later propositions.

Proof of Lemma 1.

Proof. I prove the result for party 1. The proof for party 2 is analogous.

Assume party 1 choose some policy $p_1' \in [0, \frac{1}{2})$. Then the voter is indifferent between p_1' and $p_1'' = 1 - p_1'$, and hence both yield identical and strictly positive winning probabilities. At the same time, party 1 strictly prefers p_1'' over p_1'. Therefore, p_1'' strictly dominates p_1'. This implies that party 1 never chooses any policy $p_1 \in [0, \frac{1}{2})$. Next assume party 1 chooses a policy $p_1' \in (\frac{1}{2}, 1-v_1)$. This is only possible if $v_1 < \frac{1}{2}$. Then $\min\left\{v_1, \frac{1}{2} - \left|\frac{1}{2} - p_1\right|\right\} = v_1$. Hence, increasing p_1 on $(\frac{1}{2}, 1-v_1)$ does not alter how the voter evaluates party 1, but it increases party 1's utility in case of winning the election, which happens with strictly positive probability, as the support of ϵ is at least $[-1,1]$ due to the assumption that $g(0) \leq \frac{1}{2}$. Hence, the party has always an incentive to choose $p_1'' = 1 - v_1 > p_1'$ instead. Therefore, if $v_1 < \frac{1}{2}$, any $p_1 \in (\frac{1}{2}, 1 - v_1)$ is strictly dominated. These two parts together prove the lemma. □

Proof of Lemma 2.

Proof. I prove the result for party 1, the proof for party 2 is analogous.

For any profile of platforms $(p_1, p_2) \in [\max\{\frac{1}{2}, 1 - v_1\}, 1] \times [0, \min\{v_2, \frac{1}{2}\}]$, party 1's expected utility is

$$\begin{aligned} E\pi_1(p_1, p_2, v) &= G(1 - p_1 - \kappa)(1 - |1 - p_1|) - (1 - G(1 - p_1 - \kappa))|1 - p_2| \\ &= G(1 - p_1 - \kappa)p_1 - (1 - G(1 - p_1 - \kappa))(1 - p_2) \\ &= G(1 - p_1 - \kappa)(1 + p_1 - p_2) - (1 - p_2) \end{aligned}$$

where $\kappa = \min\{v_2, \frac{1}{2} - \left|\frac{1}{2} - p_2\right|\}$. Taking the necessary derivatives with respect to p_1 reveals that

$$\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1} = -g(1 - p_1 - \kappa)(1 + p_1 - p_2) + G(1 - p_1 - \kappa) \tag{A.1}$$

and

$$\frac{\partial^2 E\pi_1(p_1, p_2, v)}{\partial (p_1)^2} = g'(1 - p_1 - \kappa)(1 + p_1 - p_2) - 2g(1 - p_1 - \kappa). \tag{A.2}$$

$g(1 - p_1 - \kappa)$ is strictly positive, whereas $g'(1 - p_1 - \kappa)$ is positive when $1 - p_1 - \kappa < 0$, negative when $1 - p_1 - \kappa > 0$, and zero when $1 - p_1 - \kappa = 0$. Note that $1 + p_1 - p_2 \leq 2$, because the greatest distance in policies possible is 1. The second derivative is most likely to be positive when $1 + p_1 - p_2$ is large. Thus, if it is negative for $1 + p_1 - p_2 = 2$, then it is generally negative:

$$2g'(1 - p_1 - \kappa) - 2g(1 - p_1 - \kappa) < 0 \Leftrightarrow g'(1 - p_1 - \kappa) < g(1 - p_1 - \kappa).$$

When $g' \leq 0 \Leftrightarrow 1 - p_1 - \kappa \geq 0$, this is obviously true. When $g' > 0 \Leftrightarrow 1 - p_1 - \kappa < 0$, this is true by Assumption 1. Hence, over this policy range party 1's expected utility is strictly concave in p_1, which proves the lemma. □

Proof of Lemma 3.

Proof. I prove the result for party 1, the proof for party 2 is analogous.

To prove the lemma we only need to show that the party's expected utility is continuous and quasi-concave in its own policy platform p_1. Continuity follows from the fact that $\min\{v_i, \frac{1}{2} -$

$\left|\frac{1}{2} - p_i\right|\}$ is continuous in p_i, $i = 1, 2$. When valence is large, expected utility is strictly quasi-concave as the voter evaluates the party strictly by its policy platform, and Lemma 2 showed us than in this case expected utility is strictly concave. When valence is low, party 1 will be evaluated by its valence when policy is close to $\frac{1}{2}$ and (potentially) by its policy when p_1 is close enough to 1. For low valence the party's utility increases linearly until $p_1 = 1$ or until $p_1 = 1 - v_1$. For larger p_1 utility is strictly concave by Lemma 2. Since expected utility is continuous at $p_1 = 1 - v_1$, it must be generally quasi-concave. Thus, a pure strategy Nash equilibrium exists (see for example Theorem 1.2 in Fudenberg and Tirole, 1991), which proves the lemma. □

A.2 Proof of Proposition 1

Proof. First, we show that any equilibrium of the game must be symmetric. Note that both fight for the same "prize," that is the utility difference between winning and losing is $1 + p_1 - p_2$ for both. Assuming party 2 is judged by its policy, the first derivative of its utility function is

$$\frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2} = -g\left(1 - p_1 - p_2\right)\left(1 + p_1 - p_2\right) + 1 - G\left(1 - p_1 - p_2\right). \tag{A.3}$$

Hence, if both parties' optimal policies are decided by their respective first-order condition (FOC), it is easy to show that it must be the case that

$$G(1 - p_1 - p_2) = 1 - G(1 - p_1 - p_2) \Leftrightarrow G(1 - p_1 - p_2) = \frac{1}{2} \Leftrightarrow 1 - p_1 - p_2 = 0 \Leftrightarrow p_1 = 1 - p_2,$$

implying parties locate symmetrically around $\frac{1}{2}$.

Next we show that a situation, in which one party chooses policy based on the FOC while the other is in a corner solution is impossible. To do so note that

$$\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1} + \frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2} = 2G(1 - p_1 - p_2) - 1. \tag{A.4}$$

Thus,

$$\text{Sign}\left[2G(1 - p_1 - p_2) - 1\right] = \text{Sign}\left[1 - p_1 - p_2\right].$$

Now assume party 1 chooses a policy $p_1' \in \{\frac{1}{2}, 1 - v\}$, while party 2 chooses $p_2' < 1 - p_1'$. For this to be an equilibrium it must be that party 1's marginal expected utility is negative, whereas the marginal expected utility of party 2 is zero. Thus, the sum is negative. We just saw that if this sum is negative, it must be the case that $1 - p_1 - p_2 < 0 \Leftrightarrow p_1 > 1 - p_2$. But this contradicts that $p_2' < 1 - p_1'$, and thus p_1' and p_2' cannot constitute a Nash equilibrium. In a similar way we can show that $p_1' = 1$ and $p_2' > 0$ cannot be an equilibrium. In that case the sum of the first derivatives is positive, implying $1 - p_1 - p_2 > 0 \Leftrightarrow 1 > p_1 + p_2$, which contradicts $p_1' + p_0' > 1$. Thus, asymmetric equilibria are not possible, and any equilibrium must have both parties locate symmetrically around

$\frac{1}{2}$, $p_1^* = 1 - p_2^*$.

Uniqueness follows from the monotonicity of the FOC when $p_1^* = 1 - p_2^*$. Again, I show this for party 1 only. The FOC when $p_2 = 1 - p_1$ is

$$\left.\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1}\right|_{p_2 = 1 - p_1} = -2p_1 g(0) + \frac{1}{2}.$$

This strictly and linearly decreases in p_1, and hence there exists a unique policy p_1 such that $\frac{\partial E\pi_1(p_1,p_2,v)}{\partial p_1}|_{p_2=1-p_1} = 0$. The solution to this equation is $p_1^* = \frac{1}{4g(0)} \in [\frac{1}{2}, 1]$. However, note that when $\frac{1}{4g(0)} < 1 - v$ this policy is strictly dominated. In that case $p_1^* = 1 - v$. □

A.3 Proof of Proposition 2

Proof. This follows immediately from the discussion. □

A.4 Proof of Proposition 3

Proof. This follows immediately from the discussion. □

A.5 Proof of Proposition 4

Proof. When $v_2 < 1 - \frac{1}{4g(0)}$ it is clear that the symmetric equilibrium determined by the system of FOCs cannot exist, because the implied policy is strictly dominated for party 2. A situation in which some party's policy is determined by its valence I call a constrained equilibrium.

I start by showing how party 1 optimally reacts to a policy $p_2 = v_2$. The high valence party reacts optimally by choosing the more moderate platform. To see this first consider again the FOC in the symmetric unconstrained Nash equilibrium. There,

$$\left.\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1}\right|_{p_1 = 1 - p_2^*} = 2p_2^* g(0) - 2g(0) + \frac{1}{2} = 0. \quad \text{(A.5)}$$

Now contrast this with the first derivative of party 1's expected utility function when $p_2 = v_2$:

$$\left.\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1}\right|_{p_2 = v_2} = -(1 + p_1 - v_2)g(1 - p_1 - v_2) + G(1 - p_1 - v_2). \quad \text{(A.6)}$$

Evaluating this at $p_1 = 1 - p_2 = 1 - v_2$ yields

$$\left.\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1}\right|_{p_1 = 1 - v_2 \wedge p_2 = v_2} = 2v_2 g(0) - 2g(0) + \frac{1}{2}. \quad \text{(A.7)}$$

We know that (A.5) holds with equality, as it is the equilibrium condition in the unconstrained case. (A.7) differs only because we replace p_2^* by v_2. Moreover, (A.7) increases in v_2. In the constrained equilibrium, we know that $p_2 = v_2 < p_2^*$. But then (A.7) must be negative, implying party 1 would like to choose a policy $p_1(v_2) < 1 - v_2$. Thus, if party 2 is constrained and chooses $p_2 = v_2$, party 1 reacts and chooses a policy $p_1(v_2) < 1 - v_2$, if it is itself unconstrained, that is if $p_1(v_2) > 1 - v_1$. To check whether this is the case we resort once again to the derivative of party 1's expected utility. Evaluated at $p_1 = 1 - v_1$ and $p_2 = v_2$, we find

$$\left.\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1}\right|_{p_1 = 1 - v_1 \wedge p_2 = v_2} = G(v_1 - v_2) + (-2 + v_1 + v_2)g(v_1 - v_2). \tag{A.8}$$

If this is positive, the concavity of the expected utility function implies that the optimal reaction of party 1 to party 2's platform of $p_2 = v_2$ is some $p_1(v_2) > 1 - v_1$, determined by the FOC above. Otherwise, if (A.8) is negative, the optimal reaction is $p_1(v_2) = 1 - v_1$.

Note that the optimal reaction always must be a policy that is weakly larger than $\frac{1}{2}$, which follows from Lemma 1. To show that this is indeed the case consider the following:

$$\left.\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1}\right|_{p_1 = \frac{1}{2}} = -\left(\frac{3}{2} - p_2\right) g\left(\frac{1}{2} - p_2\right) + G\left(\frac{1}{2} - p_2\right).$$

This decreases in p_2, as

$$\left.\frac{\partial^2 E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1 \partial p_2}\right|_{p_1 = \frac{1}{2}} = \left(\frac{3}{2} - p_2\right) g'\left(\frac{1}{2} - p_2\right) \leq 0.$$

Next note that when $p_2 = \frac{1}{2}$, $\left.\frac{\partial E\pi_1(p_1,p_2,v_1,v_2)}{\partial p_1}\right|_{p_1 = p_2 = \frac{1}{2}} \geq 0$. This implies that as we decreases p_2 from $\frac{1}{2}$ the derivative cannot become negative. Thus, the FOC uniquely identifies the optimal unconstrained policy response by party 1.

Next I show that given party 1's optimal reaction to $p_2 = v_2$, party 2 has no incentive to change its policy position. In order to do this consider first the FOC of party 1 when $p_2 = v_2$, i.e., set (A.6) equal to zero, and solve for $g(1 - p_1 - v_2)$:

$$g(1 - p_1^* - v_2) = \frac{G(1 - p_1^* - v_2)}{(p_1^* - v_2)} \tag{A.9}$$

We know that party 2 will never choose a policy greater than v_2. Moreover, a policy $p_2 = v_2$ is at the boundary of the interval of relevant strategies $[0, v_2]$ and thus we can check the first derivative of party 2's expected utility to check for deviation incentives. Evaluated at $p_2 = v_2$, this derivative needs to be weakly positive, as otherwise party 2 would like to deviate to a policy $p_2' < v_2$. Thus, we need that

$$\left.\frac{\partial E\pi_2(p_1, p_2, v_1, v_2)}{\partial p_2}\right|_{p_2 = v_2 \wedge p_1 = p_1(v_2))} = -1 + (1 + p_1(v_2) - v_2)g(1 - p_1(v_2) - v_2) + G(1 - p_1(v_2) - v_2) \geq 0.$$

Using (A.9), this simplifies to

$$G(1-p_1(v_2)-v_2) \geq \frac{1}{2}.$$

Note that we have established above that $p_1(v_2) < 1-v_2 \Leftrightarrow 1-p_1(v_2)-v_2 > 0$. This implies that $G(1-p_1(v_2)-v_2) > \frac{1}{2}$, and thus party 2 has no deviation incentive. Accordingly, $(p_1(v_2), v_2)$ is a Nash equilibrium of the semi-constrained game. If also party 1 is constrained in its policy choice, that is when the optimal reaction to $p_2 = v_2$ is $p_1 = 1 - v_1$, we need to check whether

$$\left.\frac{\partial E\pi_2(p_1,p_2,v_1,v_2)}{\partial p_2}\right|_{p_2=v_2 \wedge p_1=1-v_1} = -1 + G(v_1-v_2) + (2-v_1-v_2)g(v_1-v_2) \leq 0$$

is possible. However, this cannot happen if (A.8) is negative. The reason is, as we have seen in (A.4), that the sum of the first derivatives when $p_1 = 1-v_1$ and $p_2 = v_2$ is $2G(v_1-v_2)-1 > 0$, which follows from $v_1 > v_2$. But then it is impossible that for both parties the first derivative is negative. Thus, party 2 has no incentive to deviate from $p_2 = v_2$.

To prove the comparative statics result I use the implicit function theorem:

$$\frac{\partial p_1(v_2)}{\partial v_2} = -\frac{\frac{\partial^2 E\pi_1(p_1,p_2,v_1,v_2)}{\partial p_1 \partial v_2}}{\frac{\partial^2 E\pi_1(p_1,p_2,v_1,v_2)}{\partial (p_1)^2}} = -\frac{(p_1(v_2)-v_2)g'(1-p_1(v_2)-v_2)}{(p_1(v_2)-v_2)g'(1-p_1(v_2)-v_2) - 2g(1-p_1(v_2)-v_2)}$$

Note that we know that $p_1(v_2) < 1-v_2$, implying $g'(1-p_1(v_2)-v_2) \leq 0$. Moreover, we know that $p_1(v_2) > \frac{1}{2} > v_2$. Thus, $\frac{\partial p_1(v_2)}{\partial v_2} \leq 0$. Therefore, in any semi-constrained equilibrium, party 1 chooses more and more extreme positions as party 2's valence decreases. □

A.6 Proof of Proposition 5

Proof. Taking into account that policies are continuous in both v_1 and v_2, the proof follows from the discussion and Propositions 3 and 4. □

A.7 Proof of Proposition 6

Proof. The proof follows from the proof of Proposition 11 when $W = 1$. □

A.8 Proof of Proposition 7

Proof. The probability that party 1 wins the election is $\varphi(p_1, p_2, v_1, v_2) = G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))$. Then:

$$\begin{aligned}
E\pi_1(p_1, p_2, v_1, v_2) &= G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))p_1 \\
&- (1 - G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))))\,(1 - p_2) \\
&= G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\,(1 + p_1 - p_2) - (1 - p_2) \\
E\pi_2(p_1, p_2, v_1, v_2) &= -G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))p_1 \\
&+ (1 - G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))))\,(1 - p_2) \\
&= -G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\,(1 + p_1 - p_2) + (1 - p_2)
\end{aligned}$$

The respective FOCs for an interior optimum are

$$\begin{aligned}
\frac{\partial E\pi_1(p_1, p_2, v_1, v_2)}{\partial p_1} &= g(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\tfrac{\partial u(v_1, d_1(p_1))}{\partial d_1}\tfrac{dd_1}{dp_1}\,(1 + p_1 - p_2) \\
&\quad + G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))) = 0 \\
&= -g(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\tfrac{\partial u(v_1, d_1(p_1))}{\partial d_1}\,(1 + p_1 - p_2) \\
&\quad + G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))) = 0 \\
\frac{\partial E\pi_2(p_1, p_2, v_1, v_2)}{\partial p_2} &= -g(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\tfrac{\partial u(v_2, d_2(p_2))}{\partial d_2}\tfrac{dd_2}{dp_2}\,(1 + p_1 - p_2) \\
&\quad + G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))) - 1 = 0 \\
&= -g(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2)))\tfrac{\partial u(v_2, d_2(p_2))}{\partial d_2}\,(1 + p_1 - p_2) \\
&\quad + G(u(v_1, d_1(p_1)) - u(v_2, d_2(p_2))) - 1 = 0
\end{aligned} \tag{A.10}$$

where we used that $\frac{dd_1}{dp_1} = -1$ and $\frac{dd_2}{dp_2} = 1$ in an interior equilibrium. To derive the comparative static result, we totally differentiate the system of FOCs. Define

$$M = \begin{pmatrix} \frac{\partial E\pi_1}{\partial p_1^2} & \frac{\partial E\pi_1}{\partial p_1 \partial p_2} \\ \frac{\partial E\pi_2}{\partial p_1 \partial p_2} & \frac{\partial E\pi_2}{\partial p_2^2} \end{pmatrix}, \quad M_{1i} = \begin{pmatrix} -\frac{\partial E\pi_1}{\partial p_1 \partial v_i} & \frac{\partial E\pi_1}{\partial p_1 \partial p_2} \\ -\frac{\partial E\pi_2}{\partial p_2 \partial v_i} & \frac{\partial E\pi_2}{\partial p_2^2} \end{pmatrix}, \quad M_{2i} = \begin{pmatrix} \frac{\partial E\pi_1}{\partial p_1^2} & -\frac{\partial E\pi_1}{\partial p_1 \partial v_i} \\ \frac{\partial E\pi_2}{\partial p_1 \partial p_2} & -\frac{\partial E\pi_2}{\partial p_2 \partial v_i} \end{pmatrix}.$$

Comparative statics are then

$$\frac{\partial p_1^*}{\partial v_i} = \frac{|M_{1i}|}{|M|} \quad \text{and} \quad \frac{\partial p_2^*}{\partial v_i} = \frac{|M_{2i}|}{|M|}. \tag{A.11}$$

Evaluating everything at the symmetric equilibrium, that is, when $v_1 = v_2$ and $p_2^* = 1 - p_1^*$, and simplifying, we get

$$\left.\frac{\partial p_i^*}{\partial v_i}\right|_{v_2 = v_1 \wedge p_2 = 1 - p_1} = \frac{\frac{\partial u(v_1, d_1(p_1))}{\partial v_1} - 2p_1 \frac{\partial^2 u(v_1, d_1(p_1))}{\partial v_1 \partial d_1}}{2\left(\frac{\partial u(v_1, d_1(p_1))}{\partial d_1} - p_1 \frac{\partial^2 u(v_1, d_1(p_1))}{\partial d_1^2}\right)}$$

and

$$\left.\frac{\partial p_j^*}{\partial v_i}\right|_{v_2=v_1\wedge p_2=1-p_1} = \frac{\frac{\partial u(v_1,d_1(p_1))}{\partial v_1}}{2\left(\frac{\partial u(v_1,d_1(p_1))}{\partial d_1} - p_1\frac{\partial^2 u(v_1,d_1(p_1))}{\partial d_1^2}\right)},$$

where we used that $\frac{dd_1}{dp_1} = -1$ and $\frac{dd_2}{dp_2} = 1$ in an interior equilibrium and that $g'(0) = 0$, $\frac{\partial u(v_1,d_1(p_1))}{\partial p_1} = -\left.\frac{\partial u(v_1,d_2(p_2))}{\partial p_2}\right|_{p_2=1-p_1}$, and $\frac{\partial^2 u(v_1,d_1(p_1))}{\partial (p_1)^2} = \left.\frac{\partial^2 u(v_1,d_2(p_2))}{\partial (p_2)^2}\right|_{p_2=1-p_1}$. Note that $\left.\frac{\partial p_j^*}{\partial v_i}\right|_{v_2=v_1\wedge p_2=1-p_1} > 0$ because $\frac{\partial u(v_1,d_1(p_1))}{\partial v_1} > 0$. This implies that party j, whose position deteriorates a bit, chooses a more moderate policy platform. How party i reacts to the increase in v_i is unclear, as $\left.\frac{\partial p_i^*}{\partial v_i}\right|_{v_2=v_1\wedge p_2=1-p_1}$ might be positive, negative, or zero, depending on the relative size of $\frac{\partial u(v_1,d_1(p_1))}{\partial v_1}$ and $\frac{\partial^2 u(v_1,d_1(p_1))}{\partial p_1 \partial v_1}$. However, we can show that platform polarization strictly decreases. For this, the following derivative needs to be negative:

$$\begin{aligned}\left.\frac{\partial \Delta(v_1,v_2)}{\partial v_1}\right|_{v_2=v_1\wedge p_2=1-p_1} &= \left.\frac{\partial p_i^*}{\partial v_i}\right|_{v_2=v_1\wedge p_2=1-p_1} - \left.\frac{\partial p_i^*}{\partial v_j}\right|_{v_2=v_1\wedge p_2=1-p_1} \\ &= \frac{-2p_1\frac{\partial^2 u(v_1,d_1(p_1))}{\partial v_1 \partial d_1}}{2\left(\frac{\partial u(v_1,d_1(p_1))}{\partial d_1} - p_1\frac{\partial^2 u(v_1,d_1(p_1))}{\partial d_1^2}\right)}\end{aligned}$$

Note that $\frac{\partial^2 u(v_1,d_1)}{\partial d_1^2} \leq 0$. Moreover, $\frac{\partial u(v_1,d_1)}{\partial d_1} > 0$. Therefore, the denominator is strictly negative. Because $p_1 > 0$, polarization decreases when $\frac{\partial^2 u(v_1,d_1)}{\partial d_1 \partial v_1} > 0$, which is the case. Moreover, if instead $\frac{\partial^2 u(v_1,p_1)}{\partial p_1 \partial v_1} = 0$, increasing valence has no influence on polarization, as in this case $\left.\frac{\partial \Delta(v_1,v_2)}{\partial v_1}\right|_{v_2=v_1\wedge p_2=1-p_1} = 0$. This proves the proposition. □

A.9 Proof of Proposition 8

Proof. Using that $g(u(v_1,d_1(p_1)) - u(v_2,d_2(p_2))) = \frac{1}{2m}$ and

$$G(u(v_1,d_1(p_1)) - u(v_2,d_2(p_2))) = \frac{m + u(v_1,d_1(p_1)) - u(v_2,d_2(p_2))}{2m},$$

the respective FOCs for an interior optimum become

$$\begin{aligned}\frac{\partial E\pi_1(p_1,p_2,v_1,v_2)}{\partial p_1} &= \frac{u(v_1,d_1(p_1)) - u(v_2,d_2(p_2)) + m - (1+p_1-p_2)\frac{\partial u(v_1,d_1(p_1))}{\partial d_1}}{2m} = 0 \\ \frac{\partial E\pi_2(p_1,p_2,v_1,v_2)}{\partial p_2} &= \frac{u(v_1,d_1(p_1)) - u(v_2,d_2(p_2)) - m + (1+p_1-p_2)\frac{\partial u(v_2,d_2(p_2))}{\partial d_2}}{2m} = 0\end{aligned}$$

The respective second-order conditions are

$$\begin{aligned}\frac{\partial^2 E\pi_1(p_1,p_2,v_1,v_2)}{\partial p_1^2} &= \frac{(1+p_1-p_2)\frac{\partial^2 u(v_1,d_1(p_1))}{\partial d_1^2} - 2\frac{\partial u(v_1,d_1(p_1))}{\partial d_1}}{2m} < 0 \\ \frac{\partial^2 E\pi_2(p_1,p_2,v_1,v_2)}{\partial p_2^2} &= \frac{(1+p_1-p_2)\frac{\partial^2 u(v_2,d_2(p_2))}{\partial d_2^2} - 2\frac{\partial u(v_2,d_2(p_2))}{\partial d_2}}{2m} < 0\end{aligned}$$

where the signs follow from $(1+p_1-p_2)>0$ for all $p_1 \in [\frac{1}{2},1]$ and $p_2 \in [0,\frac{1}{2}]$ as well as $\frac{\partial^2 u(v_i,d_i)}{\partial d_i^2} \leq 0$. Because strategy spaces are closed and convex sets and because payoffs are continuous and concave, a pure strategy equilibrium exists (see for example Fudenberg and Tirole, 1991).

Using the implicit function theorem, we can calculate the slopes of both parties' reaction functions. In particular,

$$\begin{aligned}\frac{\partial p_1(p_2)}{\partial p_2} &= \frac{\frac{\partial u(v_2,d_2(p_2))}{\partial d_2} - \frac{\partial u(v_1,d_1(p_1))}{\partial d_1}}{(1+p_1-p_2)\frac{\partial^2 u(v_2,d_2(p_2))}{\partial d_2^2} - 2\frac{\partial u(v_1,d_1(p_1))}{\partial d_1}}, \\ \frac{\partial p_2(p_1)}{\partial p_1} &= \frac{\frac{\partial u(v_1,d_1(p_1))}{\partial d_1} - \frac{\partial u(v_1,d_1(p_1))}{\partial d_1}}{(1+p_1-p_2)\frac{\partial^2 u(v_1,d_1(p_1))}{\partial d_1^2} - 2\frac{\partial u(v_2,d_2(p_2))}{\partial d_2}}.\end{aligned}$$

If $1 > \frac{\partial p_i(p_j)}{\partial p_i} > -1$, Nash equilibrium is unique. After noting that in an interior equilibrium $d_1 = \frac{1}{2} - \left|\frac{1}{2} - p_1\right| = 1 - p_1$ and $d_2 = \frac{1}{2} - \left|\frac{1}{2} - p_2\right| = p_2$, it is clear that also $1+p_1-p_2 = 2-d_1-d_2$. Using this together with $1 > \frac{\partial p_i(p_j)}{\partial p_i} > -1$ we get the condition stated in the proposition.

As before, to derive the comparative static result we totally differentiate the system of FOCs. Using the same notation as before, comparative statics follow from $\frac{\partial p_1^*}{\partial v_i} = \frac{|M_{1i}|}{|M|}$ and $\frac{\partial p_2^*}{\partial v_i} = \frac{|M_{2i}|}{|M|}$, and thus

$$\frac{\partial \Delta(v_1,v_2)}{\partial v_i} = \frac{|M_{1i}| - |M_{2i}|}{|M|}.$$

Lemma 4. *$|M| > 0$.*

Proof. Let $u(v_i,d_i) = u_i$. Then

$$\begin{aligned}|M| &= \frac{1}{4m^2}\Big[(1+p_1-p_2)^2\frac{\partial^2 u_1}{\partial d_1^2}\frac{\partial^2 u_2}{\partial d_2^2} - 2(1+p_1-p_2)\frac{\partial^2 u_1}{\partial d_1^2}\frac{\partial u_2}{\partial d_2} \\ &\quad +2\frac{\partial u_1}{\partial d_1}\left(\frac{\partial u_2}{\partial d_2} - (1+p_1-p_2)\frac{\partial^2 u_2}{\partial d_2^2}\right) + \left(\frac{\partial u_1}{\partial d_1}\right)^2 + \left(\frac{\partial u_2}{\partial d_2}\right)^2\Big]\end{aligned}$$

Because $1+p_1-p_2>0$, $\frac{\partial u_i}{\partial d_i} > 0$, and $\frac{\partial^2 u_i}{\partial d_i^2} \leq 0$, $|M| > 0$. □

Thus, for polarization to be decreasing in v_i, it has to hold that $|M_{1i}| - |M_{2i}| < 0$. I prove the proposition for v_1 only. The proof for v_2 is analogous. After some simplifications, $|M_{11}| - |M_{21}|$ looks as follows:

$$|M_{11}| - |M_{21}| = \frac{1+p_1-p_2}{4m^2}\left[\frac{\partial u_1}{\partial v_1}\left(\frac{\partial^2 u_1}{\partial d_1^2} - \frac{\partial^2 u_2}{\partial d_2^2}\right) - \frac{\partial^2 u_1}{\partial d_1 \partial v_1}\left(\frac{\partial u_1}{\partial d_1} + \frac{\partial u_2}{\partial d_2} - (1+p_1-p_2)\frac{\partial^2 u_2}{\partial d_2^2}\right)\right].$$

This is negative if

$$\frac{\partial u_1}{\partial v_1}\left(\frac{\partial^2 u_1}{\partial d_1^2} - \frac{\partial^2 u_2}{\partial d_2^2}\right) < \frac{\partial^2 u_1}{\partial d_1 \partial v_1}\left(\frac{\partial u_1}{\partial d_1} + \frac{\partial u_2}{\partial d_2} - (1+p_1-p_2)\frac{\partial^2 u_2}{\partial d_2^2}\right).$$

The sign of the LHS is unclear, because it is not clear whether $\left(\frac{\partial^2 u_1}{\partial d_1^2} - \frac{\partial^2 u_2}{\partial d_2^2}\right)$ is positive, negative, or zero. With complementarities, $\frac{\partial^2 u_1}{\partial d_1 \partial v_1} > 0$, and hence the sign of the RHS depends on the sign of $\left(\frac{\partial u_1}{\partial d_1} + \frac{\partial u_2}{\partial d_2} - (1 + p_1 - p_2)\frac{\partial^2 u_2}{\partial d_2^2}\right)$. It is easy to see that this is positive, and thus the RHS is positive. When $\frac{\partial^2 u_1}{\partial d_1 \partial v_1}$ is sufficiently large, that is, when complementarities are strong enough, then the inequality holds, which implies that $\Delta(v_1, v_2)$ decreases in v_1. Note that when $\frac{\partial^2 u_i}{\partial d_i^2} = 0$, the inequality holds for any $\frac{\partial^2 u_1}{\partial d_1 \partial v_1} > 0$. □

A.10 Proof of Proposition 9

Proof. I prove the proposition without explicitly characterizing equilibrium. Party 2's expected payoff function has not changed. Party 1's expected payoff function now is

$$\begin{aligned} E\pi_1(p_1, p_2, v) &= \varphi(p_1, p_2, v)(1 + p_1 - b_1) - (1 - \varphi(p_1, p_2, v))(b_1 - p_2) \\ &= \varphi(p_1, p_2, v)(1 + p_1 - p_2) - (b_1 - p_2). \end{aligned}$$

b_1 enters as a constant, and hence has no impact on equilibrium policy choices in interior equilibrium. However, the set of dominated policies clearly depends on it, as no policy $p_1 \notin [\frac{1}{2}, b_1]$ will ever be chosen by party 1. Any policy $p_1 > b_1$ is strictly dominated by policy $p_1 = b_1$, whereas any policy $p_1 < \frac{1}{2}$ is strictly dominated by $p_1 = \frac{1}{2}$. Moreover, any policy $p_1 < 1 - v$ is dominated if $1 - v < b_1$, so that the relevant set of policies for party 1 is $[\max\{1 - v, \frac{1}{2}\}, b_1]$ if $b_1 > 1 - v$ and b_1 else. Valence affects party 2's policy only when $p_2^* = v$ and it affects party 1's policy only when $p_1^* = 1 - v$. It is possible that only party 2 chooses this policy while party 1 chooses $p_1^* = b_1$, because—relative to the electoral center—party 1's ideology b_1 is less extreme than party 2's ideology $b_2 = 0$. Nevertheless, whenever valence affects policy, increasing valence must decrease policy polarization $\Delta(v)$. This proves the proposition. □

A.11 Proof of Proposition 10

Proof. We prove the proposition in two steps:

1. $W \in [0, 1]$**:** The respective FOCs follow from (A.10) using $v_1 = v_2 = v$ and equal

$$\begin{aligned} \frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1} &= G(1 - p_1 - p_2) - (W + p_1 - p_2)g(1 - p_1 - p_2) = 0, \\ \frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2} &= G(1 - p_1 - p_2) - 1 + (W + p_1 - p_2)g(1 - p_1 - p_2) = 0. \end{aligned} \quad (A.12)$$

This system of equations has a unique solution, where $p_2^* = 1 - p_1^*$ and

$$p_1^* = \frac{1}{4}\left(2 - 2W + \frac{1}{g(0)}\right). \quad (A.13)$$

Recall that any $p_1 \in [0, \frac{1}{2})$ is strictly dominated, and the same is true for any $p_1 > 1$. Thus, if $\frac{1}{4}\left(2 - 2W + \frac{1}{g(0)}\right) \in [\max\{\frac{1}{2}, 1 - v\}, 1]$, equation (A.13) describes the unique equilibrium. Recall that both parties' expected utility functions are quasi-concave and that $\frac{\partial E\pi_2(p_1,p_2,v)}{\partial p_2}\Big|_{p_1=1-p_2} = -\frac{1}{2} + (2p_1 + W - 1)g(0) = -\frac{\partial E\pi_1(p_1,p_2,v)}{\partial p_1}\Big|_{p_1=1-p_2}$. It then follows that if $\frac{1}{4}\left(2 - 2W + \frac{1}{g(0)}\right) > 1$, equilibrium policies are $p_1^* = 1$ and $p_2^* = 0$. Moreover, if $v \geq \frac{1}{2}$ and $\frac{1}{4}\left(2 - 2W + \frac{1}{g(0)}\right) \leq \frac{1}{2}$, the equilibrium is both parties choosing the voter's bliss point, $p_1^* = p_2^* = \frac{1}{2}$. Finally, if $v \geq \frac{1}{2}$ and $\frac{1}{4}\left(2 - 2W + \frac{1}{g(0)}\right) \leq 1 - v$, equilibrium is constrained by parties' low valence and the platforms are $p_1^* = 1 - v$ and $p_2^* = v$. Consequently, the greater is W, the closer platforms tend to be to the electoral center, which implies they are more likely to be determined by valence. Comparative statics with respect to v do not change, though, and $\Delta(v)$ keeps decreasing in v.

2. $W \to \infty$**:** Recall that when $W \to \infty$, parties maximize the probability to win, because they are not willing to trade off winning probability and policy utility anymore. When valence is large, $v \geq \frac{1}{2}$, both parties will be judged based on policy. Consequently, they choose the policy that maximizes the probability to win the election, which is $p_1 = p_2 = \frac{1}{2}$. When valence is low, $v < \frac{1}{2}$, $\min\{v, \frac{1}{2} - \left|\frac{1}{2} - p\right|\}$ equals v for $p \in [v, \frac{1}{2}] \cup [\frac{1}{2}, 1 - v]$. When $p \in [0, v] \cup [1 - v, 1]$, utility is below v. Thus, the probability to win is maximized when $p \in [v, \frac{1}{2}] \cup [\frac{1}{2}, 1 - v]$. This implies that *any* $(p_1, p_2) \in [\frac{1}{2}, 1 - v] \times [v, \frac{1}{2}]$ constitutes an equilibrium. The greatest possible policy polarization in these equilibria is $\bar{\Delta}(v) = 1 - v - v = 1 - 2v$, which decreases in v. □

A.12 Proof of Proposition 11

Proof. I prove the proposition in four parts.

1. Equilibrium is convergent when $W \to \infty$**:** In the limit, when $W \to \infty$, all the parties care about is the probability to win. Because $\frac{\partial u(v,d_i)}{\partial d_i} > 0$, for any v and p_j the probability to win is maximized when $p_i = \frac{1}{2}$. This implies that the unique equilibrium is $p_1^* = p_2^* = \frac{1}{2}$, which implies the result that $\Delta(v) = 0$ for all v. The result also follows from Theorem 1 of Krasa and Polborn (2012).

2. Uniqueness and symmetry of interior equilibrium: The probability that party 1 wins the election is $\varphi(p_1, p_2, v) = G(u(v, d_1(p_1)) - u(v, d_2(p_2)))$. Expected payoffs are thus as follows:

$$\begin{aligned}
E\pi_1(p_1, p_2, v) &= G(u(v, d_1(p_1)) - u(v, d_2(p_2)))(W - 1 + p_1) \\
&- (1 - G(u(v, d_1(p_1)) - u(v, d_2(p_2))))\,(1 - p_2) \\
&= G(u(v_1, d_1(p_1)) - u(v, d_2(p_2)))\,(W + p_1 - p_2) - (1 - p_2) \\
E\pi_2(p_1, p_2, v) &= -G(u(v, d_1(p_1)) - u(v, d_2(p_2)))p_1 \\
&+ (1 - G(u(v, d_1(p_1)) - u(v, d_2(p_2))))\,(W - p_2) \\
&= -G(u(v, d_1(p_1)) - u(v, d_2(p_2)))\,(W + p_1 - p_2) + (W - p_2)
\end{aligned}$$

The respective FOCs for an interior optimum are

$$\begin{aligned}
\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1} &= -g(u(v, d_1(p_1)) - u(v, d_2(p_2)))\,(W + p_1 - p_2)\,\tfrac{\partial u(v, d_1(p_1))}{\partial d_1} \\
&\quad +G(u(v, d_1(p_1)) - u(v, d_2(p_2))) = 0 \\
\frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2} &= g(u(v, d_1(p_1)) - u(v, d_2(p_2)))\,(W + p_1 - p_2)\,\tfrac{\partial u(v, d_2(p_2))}{\partial d_2} \\
&\quad +G(u(v, d_1(p_1)) - u(v, d_2(p_2))) - 1 = 0
\end{aligned}$$

Adding the two FOCs, we get

$$-g(u(v, p_1) - u(v, p_2))\,(W + p_1 - p_2)\left(\frac{\partial u(v, p_1)}{\partial d_1} - \frac{\partial u(v, p_2)}{\partial d_2}\right) + 2G(u(v, p_1) - u(v, p_2)) - 1 = 0.$$

Assume the equilibrium was asymmetric, that is $p_1 + p_2 \neq 1$. For specificity, assume $p_1 < 1 - p_2$. Then $u(v, p_1) > u(v, p_2)$, implying $2G(u(v, p_1) - u(v, p_2)) - 1 > 0$. Hence, for an interior asymmetric equilibrium to be possible, it must be that

$$-g(u(v, p_1) - u(v, p_2))\,(W + p_1 - p_2)\left(\tfrac{\partial u(v, p_1)}{\partial d_1} - \tfrac{\partial u(v, p_2)}{\partial d_2}\right) < 0 \Leftrightarrow \tfrac{\partial u(v, p_2)}{\partial d_2} < \tfrac{\partial u(v, p_1)}{\partial d_1}.$$

But this contradicts weak concavity of utility in d_i, which implies that $\frac{\partial u(v, p_2)}{\partial d_2} \geq \frac{\partial u(v, p_1)}{\partial d_1}$. Therefore, an asymmetric interior equilibrium cannot exist.

Using symmetry, $p_1 = 1 - p_2$, the FOCs become

$$\begin{aligned}
\left.\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1}\right|_{p_2 = 1 - p_1} &= \tfrac{1}{2} - (W + 2p_1 - 1)g(0)\tfrac{\partial u(v, d_1(p_1))}{\partial d_1} = 0 \\
\left.\frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2}\right|_{p_2 = 1 - p_1} &= -\tfrac{1}{2} - (W + 2p_1 - 1)g(0)\tfrac{\partial u(v, d_2(p_2))}{\partial d_2} = 0 \\
&= -\tfrac{1}{2} + (W + 2p_1 - 1)g(0)\tfrac{\partial u(v, d_1(p_1))}{\partial d_1} = 0
\end{aligned}$$

If $A := \frac{1}{2} - (W + 2p_1 - 1)g(0)\frac{\partial u(v,d_1(p_1))}{\partial d_1}$ is monotone in p_1, the equilibrium is unique:

$$\frac{\partial A}{\partial p_1} = g(0)\left((W + 2p_1 - 1)\frac{\partial^2 u(v, d_1(p_1))}{\partial (d_1)^2} - 2\frac{\partial u(v, d_1(p_1))}{\partial d_1}\right) < 0.$$

Thus, if an interior equilibrium exists, it is symmetric and unique.

3. Comparative statics in interior equilibrium: To derive the comparative statics with respect to v we use the implicit function theorem and, without loss of generality, the first derivative of party 1's expected utility function. Then

$$\frac{\partial p_1^*(v)}{\partial v} = -\frac{\frac{\partial}{\partial v}\left.\frac{\partial E\pi_1(p_1,p_2,v)}{\partial p_1}\right|_{p_2=1-p_1}}{\frac{\partial}{\partial p_1}\left.\frac{\partial E\pi_1(p_1,p_2,v)}{\partial p_1}\right|_{p_2=1-p_1}} = \frac{g(0)(W + 2p_1 - 1)\frac{\partial^2 u(v,d_1(p_1))}{\partial d_1 \partial v}}{g(0)\left((W + 2p_1 - 1)\frac{\partial^2 u(v,d_1(p_1))}{\partial (d_1)^2} - 2\frac{\partial u(v,d_1(p_1))}{\partial d_1}\right)} < 0.$$

Policy divergence in such an interior equilibrium is $\Delta(v) = p_1^*(v) - p_2^*(v) = p_1^*(v) - (1-p_1^*) = 2p_1^*(v) - 1$ and decreases in v when $p_1^*(v)$ decreases. This is indeed the case.

Note that if instead we would assume that valence and policy are perfect substitutes, then $\frac{\partial^2 u(v,d_1(p_1))}{\partial d_1 \partial v} = 0$ and hence policy does not change when valence v changes.

4. Divergent equilibrium when $W = 0$: To prove this part, take the FOCs and evaluate them when $p_1 = p_2 = \frac{1}{2}$:

$$\begin{aligned}
\left.\frac{\partial E\pi_1(p_1, p_2, v)}{\partial p_1}\right|_{W=0 \wedge p_1=p_2=\frac{1}{2}} &= G(u(v, \tfrac{1}{2}) - u(v, \tfrac{1}{2})) = \tfrac{1}{2} > 0 \\
\left.\frac{\partial E\pi_2(p_1, p_2, v)}{\partial p_2}\right|_{W=0 \wedge p_1=p_2=\frac{1}{2}} &= G(u(v, \tfrac{1}{2}) - u(v, \tfrac{1}{2})) - 1 = -\tfrac{1}{2} < 0
\end{aligned}$$

Thus, both parties have an incentive to deviate and therefore $(p_1, p_2) = (\frac{1}{2}, \frac{1}{2})$ cannot be an equilibrium when $W = 0$. □

A.13 Example 1

First note that when $\epsilon \sim U[-m, m]$, the probability that party 1 wins the election becomes

$$\begin{aligned}
G(u(p_1, v_1) - u(p_2, v_2)) &= G\left(v_1 \cdot \left(\tfrac{1}{2} - |\tfrac{1}{2} - p_1|\right) - v_2 \cdot \left(\tfrac{1}{2} - |\tfrac{1}{2} - p_2|\right)\right) \\
&= G\left(v_1 \cdot (1 - p_1) - v_2 \cdot (p_2)\right) \\
&= \tfrac{m + v_1 \cdot (1-p_1) - v_2 \cdot p_2}{2m}
\end{aligned}$$

Conditional on winning, party 1's utility in an interior equilibrium is $W - |b_1 - p_1| = W + p_1 - b_1$, while utility in the case of losing is $p_2 - b_1$. For party 2 we get $W - |0 - p_2| = W - p_2$ and $-p_1$

respectively. Expected utilities then equal the following

$$\begin{aligned}
E\pi_1 &= \tfrac{m+v_1\cdot(1-p_1)-v_2\cdot p_2}{2m}\left(W+p_1-b_1\right)+\left(1-\tfrac{m+v_1\cdot(1-p_1)-v_2\cdot p_2}{2m}\right)(p_2-b_1)\\
&= \tfrac{1}{2}\left(p_1+p_2+W-\tfrac{(p_1-p_2+W)((p_1-1)v_1+p_2v_2)}{m}\right)-b_1\\
E\pi_2 &= \tfrac{m+v_1\cdot(1-p_1)-v_2\cdot p_2}{2m}\left(-\right)p_1+\left(1-\tfrac{m+v_1\cdot(1-p_1)-v_2\cdot p_2}{2m}\right)(W-p_2)\\
&= \tfrac{(p_1-p_2+W)((p_1-1)v_1+p_2v_2)-m(p_1+p_2-W)}{2m}
\end{aligned}$$

The FOCs are

$$\begin{aligned}
\frac{\partial E\pi_1}{\partial p_1} &= \tfrac{v_1(p_2-2p_1-W+1)-p_2v_2+m}{2m}=0\\
\frac{\partial E\pi_2}{\partial p_2} &= \tfrac{v_2(p_1-2p_2+W)+(1-p_1)v_1-m}{2m}=0
\end{aligned}$$

The second-order conditions are $\frac{\partial^2 E\pi_i}{\partial p_i}=-\frac{v_i}{m}<0$, which is true. Solving the system of FOCs then shows that the following platforms constitute the unique Nash equilibrium, if an interior equilibrium exists:

$$p_1^*=\frac{(v_1+v_2)(v_1-v_2W)-m(v_1-3v_2)}{(v_1+v_2)^2}\quad\text{and}\quad p_2^*=\frac{(v_1+v_2)v_1(1+W)+m(v_2-3v_1)}{(v_1+v_2)^2}.$$

Of course, the FOCs only determine the optimal policies in an interior equilibrium. Thus, it has to hold that $p_1^*\in\left(\frac{1}{2},b_1\right)$ and $p_2^*\in\left(0,\frac{1}{2}\right)$.

Policy polarization in an interior equilibrium is

$$\begin{aligned}
\Delta(v_1,v_2) &= p_1^*-p_2^*=\frac{(v_1+v_2)(v_1-v_2W)-m(v_1-3v_2)}{(v_1+v_2)^2}-\frac{(v_1+v_2)v_1(1+W)+m(v_2-3v_1)}{(v_1+v_2)^2}\\
&= \tfrac{(v_1+v_2)(v_1-v_2W)-m(v_1-3v_2)-(v_1+v_2)v_1(1+W)+m(v_2-3v_1)}{(v_1+v_2)^2}=\tfrac{2m}{v_1+v_2}-W.
\end{aligned}$$

Because in an interior equilibrium $\Delta(v_1,v_2)>0$, this clearly decreases in both v_1 and v_2, and hence greater valence leads to policy moderation.

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