Working Paper 11-22
Statistics and Econometrics Series 17
July 2011

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-49

The International Stock Pollutant Control: A Stochastic Formulation with Transfers

Omar J. Casas[1] and Rosario Romera[2]

Abstract

This paper provides a formulation of a stochastic dynamic game that arise in the real scenario of international environmental agreements on the transnational pollution control. More specifically, this agreements try to reduce the environmental damage caused by the stock pollutant that accumulates in the atmosphere, such as *CO*2. To improve the non-cooperative equilibrium among countries, we propose the criteria of the minimization of the expected discounted total cost with monetary transfers between the countries involved as an incentive to cooperation. Moreover, it considers the formulation of Stochastic Dynamic Games as Markov Decision Processes, using tools of Stochastic Optimal Control and Stochastic Dynamic Programming. The performance of the proposed schemes is illustrated by its application to such environmental problem.

Keywords: Environmental Pollutant Control, Markov Decision Processes, Stochastic Dynamic Programming, Stochastic Dynamic Games, Optimal Abatement Policies.

(1) Omar J. Casas, Departamento de Estadística e Investigación Operativa II, Universidad Complutense de Madrid, Campus de Somosaguas, 28223 Madrid, Spain, e-mail: ocasas@emp.ucm.es

(2) Rosario Romera, Departamento de Estadística, Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), e-mail: rosario.romera@uc3m.es

The International Stock Pollutant Control: A Stochastic Formulation with Transfers

Omar J. Casas * and Rosario Romera †

July 13, 2011

Abstract

This paper provides a formulation of a stochastic dynamic game that arise in the real scenario of international environmental agreements on the transnational pollution control. More specifically, this agreements try to reduce the environmental damage caused by the stock pollutant that accumulates in the atmosphere, such as CO_2. To improve the non-cooperative equilibrium among countries, we propose the criteria of the minimization of the expected discounted total cost with monetary transfers between the countries involved as an incentive to cooperation. Moreover, it considers the formulation of Stochastic Dynamic Games as Markov Decision Processes, using tools of Stochastic Optimal Control and Stochastic Dynamic Programming. The performance of the proposed schemes is illustrated by its application to such environmental problem.

Keywords: Environmental Pollutant Control, Markov Decision Processes, Stochastic Dynamic Programming, Stochastic Dynamic Games, Optimal Abatement Policies

*Address: Departamento de Estadística e Investigación Operativa II, Universidad Complutense de Madrid, Campus de Somosaguas, 28223 Madrid, Spain. E-mail: ocasas@emp.ucm.es

†Address: Departamento de Estadística, Universidad Carlos III Madrid, Calle Madrid 126, 28903 Getafe, Spain. E-mail: rosario.romera@uc3m.es

1 Introduction

Recently, the theory on international environmental agreements (IEA) and the prospect of climate change have motivated many game theoretic studies, often focused on cooperation and core solutions.

The necessity of cooperation among the countries involved, if a social optimum is to be achieved, has already been addressed in the literature in terms of Game Theory concepts; see e.g. Barrett (2003), Finus (2001) and references therein for a review on these topics. With a few exceptions this literature works with simple static models of pollution despite the fact that many of the important environmental problems, such as climate change, the depletion of the ozone layer or the acid rain problem, are caused by a stock pollutant. However, the stock of pollution may change in the course of the game, as a result of both a positive rate of natural decay and emissions of the countries. Thus, the presence of a stock pollutant leads to a dynamic game that is not strictly repeated.

In the framework of a deterministic cooperative game with a dynamic, multi-regional integrated assessment model, Eyckmans and Tulkens (2003) calculated the optimal path of abatement and aggregated discounted welfare for each region by application of the transfer scheme advocated in previous work by Chander and Tulkens (1997). They defined six regions in the Climate Negotiation World Simulation Model (abbreviated as CWSM), a model which was derived from the seminal multi-region economy-climate Regional Integrated model of Climate and the Economy (RICE) of Nordhaus and Yang (1996), they use of the idea of surplus sharing allowed them to determine first the transfer scheme and then to compute all possible partial agreement Nash equilibria. It was found that allocation in the full cooperation lies in the core of the emission abatement game under this specific transfer scheme.

The transfer schemes are based on a single year for assigning the permits or shares in the surplus. Such static schemes are also observed in reality, e.g. the reduction targets in the Kyoto Protocol (1997) are designed in terms of reduction with respect to 1990 levels. These static schemes, however, do not take into account that the future growth paths of emissions are expected to diverge substantially between regions. Therefore historically large emitters obtain relatively large shares of the permits/surplus, while fast-growing developing countries, such as China or India, obtain relatively small shares. This leads to increasing burdens on these developing countries to reduce their emissions; a notion brought forward by many developing countries in their argumentation on why they do not agree on any reduction targets in the Kyoto Protocol (1997).

The role of transfers in the analysis of self-enforcing International Environmental Agreements (IEA) was developed in Carraro et al (2006). They proposed transfers using internal and external financial resources in order to make welfare optimal agreements. To illustrate the relevance of their transfer scheme, they used a stylized integrated assessment simulation model of climate change to show how appropriate transfers may induce almost all countries into signing a self-enforcing climate treat.

The studies by Germain et al (2003) addressed the issue of how many countries would be interested in signing an IEA with stock pollutant, adopting a cooperative game-theory approach. They extended the result established by Chander and Tulkens (1995) and (1997) for flow pollutants to the larger context of closed-loop (feedback) dynamic games with a stock pollutant. In this context, cooperation is negotiated at each period but financial transfers provide incentives to the countries that ensure the implementation of the grand coalition at each period. Their model thus yields a sequence of full cooperative international agreements so that cooperation is also achieved in a dynamic setting with a stock pollutant.

Another paper related to this issue using a cooperative game-theory approach is Petrosjan and Zaccour (2003). However, in this paper the authors assume that all countries decide to cooperate at the initial time-consistent decomposition of each player's total cost, as given by Shapley value, so that the countries stick at each moment to the full cooperative solution agreed at initial time, supposing that the global allocation problem has been solved.

Stochastic Programming was considered by Dechert and O'Donnell (2006) in a particular application that explored some fundamental issues of the optimal level of pollution in a lake with competing uses. They showed how the model can be interpreted as an open loop dynamic game, where the control variables are the levels of phosphorus discharged into the watershed of the lake, the state of the system is the accumulated level of phosphorus in the lake and the random shock (a multiplicative noise factor on the control variables of the players) is the rainfall that washes the phosphorus in the lake.

The use of stochastic control models to develop climate-economy models was advocated by Haurie and Viguier (2003) to represent the possible competition between Russia and China on the international market of carbon emissions permits their model includes a representation as an event tree of the uncertainty concerning the date of entry of developing countries on this market. Also Bahn et al (2008) showed how a piecewise deterministic stochastic control model, over an infinite time horizon, could be used as a paradigm for the design of an efficient climate policy. Keller et al (2004) had already explored the combined effects of uncertainty and learning about a climate threshold (an uncertain ocean thermohaline circulation collapse) in an economic optimal growth model.

The stability of an International Environmental Agreement among n countries that emit pollutant has been studied using differential games, defined in continuous time, by Jorgensen et al (2003) and (2004), Rubio and Casino (2005), among others.

As far as we know, the only stochastic formulation for the finite horizon dynamic analysis of international agreements on transnational pollution control introduced as an extension of the issues presented in Germain et al (2003) is a recent paper by Casas and Romera (2011). In this paper, the stochastic formulation for the Stock Pollutant Control Model involves the use of Stochastic Dynamic Programming with discrete and finite planing horizon, for searching both cooperatives and non cooperatives equilibrium with transfers. Stochastic optimization problems should be solved by Stochastic Dynamic Programming Techniques (see Bertsekas (2000) and Birge and Louveaux (1997)).

The model proposed in this paper is directly linked to the Kyoto or post-Kyoto agreement mechanisms. We use financial transfers as additional elements in the game for the design of international agreements that achieve global optimality in stock pollutant control problems.

The paper is organized as follows: in Section 2 we present the international stock pollutant model with its components, the cost functional components and their elements; the description of the modes of countries behaviour; the stock pollutant control models (cooperative and non cooperative) and the underlaying Markov Decision Process (MDP). In Section 3, we define the monetary transfers to ensure that each country is not worse off when it participates and we report on a non cooperative model with the monetary transfer corresponding to each country in each period of time, with the necessary definitions and results. In Section 4, we present an algorithm which solves the problem of minimizing the expected discounted total cost with transfers, the optimal action sets, and optimal policies for a finite horizon model for each period of time and for every country. In Section 5, we present a numerical example based on real scenarios borrowed from the work by Eyckmans and Tulkens (2003) and Casas and Romera (2011). In Section 6, we present conclusions and extensions.

2 The Stock Pollutant Control Model

In this section, we formulate a discrete-time stochastic dynamic game with finite planning horizon. We refer to the reader to the modelization in Germain et al (2003) for a deterministic counterpart of our setting. We consider a Markovian Game described by a tuple $G = \{J, \mathcal{T}, S, E, p,\}$, with the following elements; n players where $J = \{1, 2, ..., n\}$ denotes the set of countries (regions). A finite planing horizon with discrete-time periods t, such that $t \in \mathcal{T} = \{1, 2, ..., T\} \subset \mathbb{Z}^+$. The state space of the game, S, with elements s, is a Borel subset of some Polish (i.e., complete, separable, metric) countable and non empty space. The control variables or actions (emissions) are $e_{it} \in E$, where E is the countable and non empty overall control space or action space, and $E = \bigcup_{s \in S} E(s)$, where $E(s)$ is the finite set of *admissible actions*. The law of motion (or transition probabilities) p for the game defined for each $(s, e) \in S \times E$ is the conditional probability $p(\cdot|s, e)$ over the Borel sets of S.

The state of the system is the accumulated level of pollution in the atmosphere given as stock of pollutant at each period t, $s_t \in S$, which evolves according to the state equation

$$s_t = (1-\delta)s_{t-1} + \sum_{i=1}^{n} e_{it} + \xi_t \quad , \qquad 1 \leq t \leq T, \tag{1}$$

where s_0 is the initial given stock of pollutant or preindustrial level and δ is the pollutant's natural rate of atmospheric absorption of CO_2 between two periods of time, such that $0 < \delta < 1$. The random disturbance ξ_t is a noise process: a sequence of independent and identically distributed (i.i.d.) random variables, and independent of the initial state s_0, with

$$\mathbb{E}\left[\xi_t\right] = 0, \qquad \sigma^2 = \mathbb{E}\left[\xi_t^2\right] < \infty, \qquad \forall t = 1, 2, ..., T-1. \tag{2}$$

Following Jorgensen and Zaccour (2001) among many others, we assume that the emissions are proportional to productions. The emissions are aggregated and considered as CO_2 equivalents.

We consider that future costs are discounted by the constant and positive *discount factor* β with $0 < \beta \leq 1$. In addition, function $c_i(e_{it})$ measures in monetary terms the total cost incurred by country $i \in J$ at period $t \in \mathcal{T}$ from limiting its own industrial emissions to e_{it}; it is a differentiable, decreasing ($c_i' < 0$) and strictly convex function ($c_i'' > 0$). We assume that the only way to control the stock of pollution is through the control of emissions, therefor reducing pollution is done through the reduction of emissions, and not through the cleaning of the environment. The marginal cost c_i of reducing emissions is higher for lower levels of emissions. Function $d_i(s_t)$ measures in monetary terms the damages caused by the stock of pollutant s_t during the time period t for the i-th country; it is a differentiable, increasing ($d_i' > 0$) and convex function ($d_i'' \geq 0$).

The damages in each country's environment depend on the emissions of pollutant of all different countries at each time-period t that contribute to a stock s_t. In cooperative form the countries jointly choose at each period their emissions levels in order to minimize the expected total discount costs, then the resulting trajectories of emissions and stock constitute the international optimum. In non-cooperative form, each country considers only the damages of the stock of pollutant over itself. In the sense of a Nash equilibrium, the countries minimize, at each period, only its own expected discounted costs, with knowledge of the emissions vector e_{jt}, with $j \neq i$, of the other countries. Both the cooperative and non-cooperative frameworks constitute the standard settings based on average costs optimality. Although our approach is different to the former one, we develop some previous results for further comparison to our proposed model.

2.1 The Cooperative Model

One assumes that the countries behave in an internationally optimal way, and the damages to the environment of country i will depend on the emissions of all countries. We solve the following problem

$$
\begin{aligned}
(P1) \qquad & \min_{\{e_{it}\}} \quad \mathbb{E}\left[\sum_{t=1}^{T}\sum_{i=1}^{n}\beta^t\left(c_i(e_{it})+d_i(s_t)\right)\right] \\
& \text{s.t.} \quad s_t = (1-\delta)^t s_{t-1} + \sum_{i=1}^{n} e_{it} + \xi_t \\
& \qquad\quad e_{it} \geq 0 \quad \forall t \in \mathcal{T}; \quad \forall i \in J \\
& \qquad\quad s_0 > 0
\end{aligned}
$$

The convexity of the functions $c_i(e_{it})$ and $d_i(s_t)$ suffices to guarantee that the solution exists and is unique (see Casas and Romera (2011)). Thus, Problem $(P1)$ has an equilibrium $\{e_{it}^W\}$. Note that the resulting family of trajectories of emissions (policies) e_{it}^W for all players

$i \in J$ with the resulting stock s_t^W, constitute the international optimum or the cooperative equilibrium for all periods $t \in \mathcal{T}$. By following basic principals of Dynamic Programming (see Puterman (2005)) we obtain the value function $W(t, s_t)$, $\forall t \in \mathcal{T}$. Let denote by $W_i(t, s_t^W)$, for all $t \in \mathcal{T}$, the contribution of the country i to the expected discounted total cost evaluated at the international optimum s_t^W. Let W_i be the marginal expected discounted cooperative total cost for all country $i \in J$, then

$$W_i \equiv \sum_{t=1}^{T} W_i(t, s_{t-1}^W). \tag{3}$$

Note that this marginal value is the overall cost incurred by country i under the cooperative paradigm.

2.2 The Non-Cooperative Model

In this mode of behaviour, one may assume that countries behave non cooperatively in the sense of Nash equilibrium, where each of them minimizes at each period only its own discounted costs, taking into account the emissions of the other countries. In such an equilibrium, no individual country has an incentive to deviate as long as the other countries stick to their equilibrium strategies. Formally, there are n $(P2)$ problems to solve one for each country $i \in J$

$$\begin{aligned} (P2) \qquad \min_{\{e_{it}\}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} \beta^t \left[c_i(e_{it}) + d_i(s_t)\right]\right] \\ \text{s.t.} \quad & s_t = (1-\delta)s_{t-1} + \sum_{i=1}^{n} e_{it} + \xi_t \\ & e_{it} \geq 0 \quad \forall t \in \mathcal{T}; \quad \forall i \in J \\ & s_0 > 0 \end{aligned}$$

According to Bellman's principle and using Stochastic Dynamic Programming, see Puterman (2005) and Hernández-Lerma (1999), we obtain the optimal expected value function, $N_i(t, s_t)$, for all $t \in \mathcal{T}$ and for each country $i \in J$.

Note that the resulting vector of trajectories of emissions e_{it}^N, for all country $i \in J$, together with the resulting stock s_t^N, constitute a non-cooperative Nash equilibrium for all periods $t \in \mathcal{T}$. The convexity of the function $c_i(e_{it})$ and $d_i(s_t)$ suffices to guarantee that the Nash equilibrium exists and is unique, see Casas and Romera (2011).

Let define N_i for each country $i \in J$ and for all $t \in \mathcal{T}$, as the expected discounted non cooperative total cost by

$$N_i \equiv \sum_{t=1}^{T} N_i(t, s_{t-1}^N). \tag{4}$$

Algorithms for solving problems $(P1)$ and $(P2)$ can be found in Casas and Romera (2011) with their corresponding numerical results.

2.3 The underlying MDP Model

The models considered up here in this work are discrete-time, finite-horizon and stationary MDP, described by the tuple

$$\Gamma = (S, E, R, P, \beta), \tag{5}$$

where the *state space* S and the overall *action space* $E = \bigcup_{s \in S} E(s)$ are both countable and nonempty, $E(s)$ is the set of *admissible actions* (emissions), when the system is in each state (pollutant level) s. For each $s \in S$ the set $E(s)$ is finite. The *cost set* R is a bounded countable subset of $\mathbb{R}$. For each $t \geq 1$, let s_t, e_t and r_{it}, with $r_{it} = c_i(e_{it}) + d_i(s_t)$, denote the state (pollutant level) of the system, the action (emissions) taken by the decision maker

i (pays), and the cost incurred at period of time t, respectively.

The stationary, single-stage, conditional *transition probabilities* are defined by

$$p^e_{i,j,r} := Prob\left(s_{t+1} = j, r_t = r / s_t = i, e_t = e\right), \tag{6}$$

$$\forall i, j \in S, \quad e \in E(i), \quad r \in R, \quad t \geq 1, \sum_{j \in S, r \in R} p^e_{i,j,r} = 1, \quad i \in S, \quad e \in E(i).$$

The parameter β with $0 < \beta \leq 1$, is the constant and positive *discount factor*.

3 The Transfers Definition

At the international optimum ($P1$), and contrary to what happens at the Nash equilibrium ($P2$) in Casas and Romera (2011), each country takes account of the impact of its pollution on the environment of all other countries. Therefore, from a collective point of view, the optimum is better than the Nash equilibrium. Nothing ensures that this is also true at the individual level. Indeed, countries being different, it is possible that some country i at some period of time t is better off at the non cooperative equilibrium than the optimum, so that cooperation is not profitable for this country, at least at time t. The same can occur for subsets of countries (i.e. coalitions) in the sense that, by limiting cooperation to such coalitions, the members of the latter could be better off than at the international optimum.

In a deterministic dynamic programming framework, Germain et al (2003) propose a mechanism of financial transfers between countries that can make them interested in achieving the international optimum at all periods t (*individual rationality*). This mechanism has the additional property that no subgroup of countries has ever an incentive to form a coalition and enact an optimum for itself only (*coalition rationality*). In the present paper, our aim is to apply this mechanism to the climate change stochastic model introduced by Casas and Romera (2011).

At each time period t one could take the non cooperative Nash equilibrium from t onwards as such point of reference, and determine the transfers accordingly. However, one should not neglect the fact that countries know that later on, thanks to the cooperative transfers to which they will have access, they will be better off than at the non cooperative Nash equilibrium. Hence, a better point of reference at period of time t is non cooperation at time t, followed by cooperation afterwards. We are thus aiming at a cooperative international optimum.

3.1 The Transfers at final period T

We start by determining which transfers yield for all countries when they cooperate in the last period $t = T$, of the finite horizon $\mathcal{T}$, for any level of the stock of pollution s_{T-1} inherited from the past.

In the non cooperative equilibrium the countries are supposed to solve problem $(P2)$, the country i's expected total cost at period final T is then

$$N_i(T, s_{T-1}^N) = c_i(e_{iT}^N) + d_i(s_T^N),$$

where $e_{iT}^N = (e_{1T}^N, e_{2T}^N, \ldots, e_{nT}^N)$ denotes the vector of emissions equilibrium level of each countries and s_T^N denotes the resulting stock of pollutant given by

$$s_T^N = [1-\delta]s_{T-1}^N + \sum_{i=1}^{n} e_{iT}^N,$$

where s_{T-1}^N is the inherited stock of pollutant at the begin of period T.

If countries cooperate, they jointly solve $(P1)$. The country i's expected total cost at final period T is

$$W_i(T, s_T^W) = c_i(e_{iT}^W) + d_i(s_T^W),$$

where $e_{iT}^W = (e_{1T}^W, e_{2T}^W, \ldots, e_{nT}^W)$ is the vector of optimal emission level (policy) and s_T^W is the optimal stock of pollutant at final period T, given by

$$s_T^W = [1-\delta]s_{T-1} + \sum_{i=1}^{n} e_{iT}^W,$$

where s is the inherited stock of pollutant at the begin of period T.

Let defines $W(T, s_T^W)$ and $N(T, s_T^N)$ as the expected total cost cooperative and non cooperative, respectively, at final period of time T. By definition of the optimum, one verifies that

$$W(T,s) \equiv \sum_{i=1}^{n} W_i(T,s) \quad \leq \quad \sum_{i=1}^{n} N_i(T,s) \equiv N(T,s). \tag{7}$$

The difference $W(T,s) - N(T,s)$, between the two sides of this inequality (7) measures the *ecological surplus* resulting from international cooperation.

However, the inequality (7) is not sufficient to ensure cooperation. Indeed, if $\exists i \in J$ such that $W_i(T,s) > N_i(T,s)$, then country i will not cooperate without financial compensation for higher cost it incurs. Since stochastic dynamic programming reduces the choice of emissions to one period at the time, one can use the transfers formula in a static framework.

Following the transfers formula proposed by Chander and Tulkens (1997) in a static framework, we can use this transfers formula in our stochastic dynamic framework at final period T

$$\theta_i(T,s) \quad = \quad -[W_i(T,s) - N_i(T,s)] + \mu_{iT}[W(T,s) - N(T,s)], \tag{8}$$

with

$$\sum_{i=1}^{n} \theta_i(T,s) = 0.$$

The transfer (8) is < 0 if received and > 0 if paid to country i at period T, and it satisfies that $\mu_{iT} \in]0;1[$, $\forall i \in J$, and

$$\sum_{i=1}^{n} \mu_{iT} = 1.$$

The fact that μ_{iT} cannot be equal to 0 ensure that country i will benefit from cooperation if $W_i(T,s) < N_i(T,s)$. The fact that μ_{iT} cannot be equal to 1 exclude that country i monopolizes all the gains of cooperation.

Then country i's total cost including transfers at final period T becomes

$$\tilde{W}_i(T,s) = W_i(T,s) + \theta_i(T,s). \tag{9}$$

The cooperation with transfers is *individually rational* at final period of time T, in the sense that each country have interest to participate whatever the inherited stock of pollutant s, since by construction

$$\tilde{W}_i(T,s) - N_i(T,s) = \mu_{i,T} \left[W(T,s) - N(T,s)\right] \leq 0, \quad \forall i \in J.$$

3.2 The Transfers at period $T-1$

The countries know that, whatever they do at period $T-1$, financial transfers exist defined by (8), that make the international optimum (cooperative) at period T preferable for each of them with respect to the non cooperative equilibrium. Lets assume that these transfers induce cooperation, following Chander and Tulkens (1997), one could indeed obtain the cooperative optimum with transfers as an equilibrium, called *ratio equilibrium*, and that countries therefore expect, at period of time $T-1$, that they will cooperate in period T. This is the rational expectations assumption. The problem we wish to consider now is whether under assumption transfers can be designed that make the countries interested to cooperate at period $T-1$ as well.

In absence of cooperation at $T-1$, each country $i \in J$ minimizes its own expected discounted total cost over two periods $T-1$ and T, expecting cooperation and transfers at period T. Thus, given the emissions of the other countries, the country i solves problem $(P2)$ for $t = T-1$ with expected transfers at period T.

There are n problems to solve at period $T-1$, each country $i \in J$ solves the following problem

$$\min_{e_{i,T-1}} \quad \mathbb{E}\left[c_i(e_{i,T-1}) + d_i(s_{T-1}) + \beta \tilde{W}_i(T, s_{T-1})\right] \tag{10}$$

$$\text{s.t.} \quad s_{T-1} = (1-\delta)s_{T-2} + \sum_{i=1}^{n} e_{i,T-1} + \xi_{T-1}$$

$$e_{i,T-1} \geq 0, \quad \forall i \in J.$$

Since the expected value functions $\tilde{W}_i(T, s_{T-1})$ contain transfers that sum up to zero, convexity of the cost functions c_i and damages function d_i ensure that the objectives in (10) are convex, unlike what happens for the period T.

This yields an equilibrium characterized at period $T-1$ by emissions level $e^V_{i,T-1}$ as functions of initial stock s at period $T-1$. The expected value functions $V_i(T-1, s)$ denotes country i's expected discounted non cooperative equilibrium costs including future transfers at final period T.

The *expected value functions* V_i, according to Bellman's principle of optimality

$$V_i(T-1,s) = c_i(e^V_{i,T-1}) + d_i(s^V_{T-1}) + \beta \tilde{W}_i(T, s^V_{T-1}), \quad \forall i \in J, \tag{11}$$

where

$$s^V_{T-1} = (1-\delta)s_{T-2} + \sum_{i=1}^{n} e^V_{i,T-1},$$

denotes country i's expected discounted equilibrium costs. We will call this equilibrium the non cooperative equilibrium with transfers at period $T-1$.

In the case where all countries cooperate, they solve problem $(P1)$ for $t = T-1$. Optimal levels of emissions and of the resulting stock of pollutant are denoted by $e^W_{i,T-1}$ and s^W_{T-1}, respectively, both are function of the initial stock s at period $T-1$. This yields

$$W_i(T-1,s) = c_i(e^W_{i,T-1}) + d_i(s^W_{T-1}) + \beta \tilde{W}_i(T, s^W_{T-1}), \quad \forall i \in J, \tag{12}$$

which is country i's part in the optimal expected total discounted costs, taking into account the transfers and the resulting cooperation expected at final period T.

As in period T, see (7), one verifies that

$$W(T-1,s) \equiv \sum_{i=1}^{n} W_i(T-1,s) \leq \sum_{i=1}^{n} V_i(T-1,s) \equiv V(T-1,s). \tag{13}$$

The difference $W(T-1,S) - V(T-1,s)$, between the two sides of this inequality (13) measures the *ecological surplus* induced by extending from international cooperation to period $T-1$, with respect to alternative scenario where cooperation is limited to period T.

However, (13) is again not sufficient to induce cooperation at time $T-1$, if exist $i \in J$ such that $W_i(T-1,S) > V_i(T-1,s)$, then country i will not want to extend cooperation to period $T-1$ without financial compensation.

To induce country i to participate at period $T-1$, let

$$\theta_i(T-1,s) = -[W_i(T-1,s) - V_i(T-1,s)] + \mu_{i,T-1}[W(T-1,s) - V(T-1,s)], \tag{14}$$

be the transfer paid or received by country i at period $T-1$ where $\mu_{i,T-1} \in]0,1[$, for all $i \in J$ and

$$\sum_{i=1}^{n} \mu_{i,T-1} = 1 \quad \text{and} \quad \sum_{i=1}^{n} \theta_i(T-1,s) = 0.$$

Then country i's expected total cost including transfers becomes

$$\tilde{W}_i(T-1,s) = W_i(T-1,s) + \theta_i(T-1,s).$$

It is clear that this transfers defined by (14) make cooperation individually rational at period $T-1$, whatever the inherited stock of pollutant s.

3.3 The Transfers at period t

We can repeat the preceding analysis for all earlier periods. The final result will be that the countries cooperate in each period t. This determines the emissions levels in each period for each country, and also the trajectory of the stock of pollutant, given its initial value s_0. In turn these trajectory determines the expected value functions V_i, W_i and $\tilde{W}_i$, and therefore also the value of the transfers θ_i.

There are n problems to solve, one for each country i, named $(P3)$

$$\begin{aligned} (P3) \qquad \min_{e_{it}} \quad & \mathbb{E}\left[(c_i(e_{it}) + d_i(s_t)) + \beta \tilde{W}_i(t+1, s_t)\right] \\ \text{s.t.} \quad & s_t = (1-\delta)s_{t-1} + \sum_{i=1}^{n} e_{it} + \xi_t \\ & e_{it} \geq 0 \quad \forall t \in \mathcal{T}, \quad \forall i \in J \\ & s_0 > 0 \end{aligned}$$

This yields an equilibrium characterized at period t by emissions level e_{it}^V as functions of initial stock s_0 at each period t.

The value functions $V_i(t,s)$ denotes the country i's expected discounted non cooperative equilibrium costs including transfers at $t+1$ period

$$V_i(t,s_t) = c_i(e_{it}^V) + d_i(s_t) + \beta \tilde{W}_i(t+1,s_t), \qquad \forall i \in J,$$

where the transfer paid or received by country i at period t

$$\theta_i(t,s) = -[W_i(t,s) - V_i(t,s)] + \mu_{i,t} \left[\sum_{i=1}^{n} W_i(t,s) - \sum_{i=1}^{n} V_i(t,s) \right]. \tag{15}$$

be the transfer paid or received by country i at period t where $\mu_{i,t} \in]0,1[$, for all $i \in J$,

$$\sum_{i=1}^{n} \mu_{i,t} = 1 \quad \text{and} \quad \sum_{i=1}^{n} \theta_i(t,s) = 0.$$

Then country i's expected total cost including transfers becomes

$$\tilde{W}_i(t,s) = W_i(t,s) + \theta_i(t,s).$$

It is clear that these transfers (15) make cooperation individually rational at period t, whatever the inherited stock of pollutant s.

3.4 The Problem with transfers Alternative

Following Casas and Romera (2011), and bearing in mind the explicit recursive expression (1) obtained for the stock pollutant s_t, we have to solve for each country $i \in J$ the alternative problem with future transfers, as following

$$\min_{\{e_{it}\}} \quad \mathbb{E}\left[\left[c_i(e_{it}) + \tilde{d}_i(s_0, e_{it}, \xi_t) \right] + \beta \tilde{W}_i(t+1, s_t) \right]$$

$$\begin{aligned} \text{s.t.} \qquad M_i e_{it} &= b_i + \xi \\ e_{it} &\geq 0 \quad \forall t \in \mathcal{T}, \quad \forall i \in J \\ s_0 &> 0 \end{aligned}$$

where

$$\begin{aligned} e_i' &= (e_{i1}; e_{i2}; \cdots; e_{i,T-1}) \\ b_i' &= (b_{i1}; b_{i2}; \cdots; b_{i,T-1}) \\ b_{it} &= -(1-\delta)^t s_0 - \sum_{\tau=1}^{t} \sum_{j \neq i}^{n} (1-\delta)^{t-\tau} e_{j\tau} \\ \xi' &= \big(\xi_1; (1-\delta)\xi_1 + \xi_2; \ldots; \ldots; (1-\delta)^{T-1}\xi_1 + (1-\delta)^{T-2}\xi_2 + \cdots + \xi_T\big) \end{aligned}$$

The matrix M_i is a square matrix, lower triangular, of order T, with ones in the principal diagonal. The vector b and the random disturbance ξ have order T. The structure of the matrix M_i is as follows

$$M_i = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ (1-\delta) & 1 & 0 & 0 & 0 & \cdots & 0 \\ (1-\delta)^2 & (1-\delta) & 1 & 0 & 0 & \cdots & 0 \\ (1-\delta)^3 & (1-\delta)^2 & (1-\delta) & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ (1-\delta)^{t-1} & (1-\delta)^{t-2} & (1-\delta)^{t-3} & \cdots & \cdots & (1-\delta) & 1 \end{pmatrix}$$

then we can may obtain the inverse matrix of the matrix B_i, which is quasi diagonal

$$M_i^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -(1-\delta) & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & -(1-\delta) & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -(1-\delta) & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \cdots & -(1-\delta) & 1 \end{pmatrix}$$

As in the cooperative model solution $(P1)$ and in the non cooperative model solution $(P2)$, by using the development presented in this section one can find the parameters e_{it}^V of optimal emissions for each country $i \in J$, and one obtains the stock levels of contamination s_t^V at each period of time $t \in \mathcal{T}$.

3.5 The infinite horizon case

In the infinite horizon case $(T = \infty)$, the backward reasoning considered above no longer applies. However, we can consider the stationary solution by taking advantage of the fact that the cost functions c_i and d_i as well as the sharing parameters μ_i do not depend directly on time. The functional forms of the solutions thus only vary in time through the varying stock of pollutant s.

In both the cooperative $(P1)$ and non cooperative $(P2)$ problems, identical steps are repeated over all time periods. The solution functions of these problems are now constant, and only the value of the stock s must be calculated at each poriod of time.

In the case where $T = \infty$, the expected discounted total cost of the cooperative problem $(P1)$ does not depend explicitly of time, and its value function W can be written as

$$W(s) = \min_{e_i} \mathbb{E}\left[\sum_{i=1}^{n} c_i(e_i) + d_i(\bar{s}) + \beta W(\bar{s})\right]$$

$$\begin{aligned} \text{s.t.} \qquad \bar{s} &= (1-\delta)s + \sum_{i=1}^{n} e_i + \xi \\ e_i &\geq 0 \qquad \forall i \in J \end{aligned}$$

The First Order Conditions of Bellman equations lead to the optimal emission levels e^* and stock s^*, both depending on s, which solve the system

$$c_i'(e_i^*) + \sum_{j=1}^{n} d_j'(s^*) + \beta W'(\bar{s}) = 0 \qquad \forall i \in J \tag{16}$$

$$s^* = [1-\delta]s + \sum_{i=1}^{n} e_i^* + \xi \tag{17}$$

$$W(s) = \sum_{i=1}^{n} c_i(e_i^*) + d_i(s^*) + \beta W(s^*) \tag{18}$$

where c_i' and d_j' are the derivatives of the functions c_i and d_j respectively. Optimal values of emissions e_i^* and stock pollution s^* are obtained by solving these equations.

The main problem is the identification of a value function W such that First Order Conditions (16), (17) and (18) are met. This system can be considered as a functional equations system, where functions W, s^* and e_i^* are unknown for all $i \in J$. This system can be solved whenever functions c_i and d_j are know for all $i \in J$.

The same reasoning can be applied to the country cost functions V_i and W_i. Total cost of country i with no present cooperation can be obtained in the case of infinite horizon, in analogy with (8), (9) and $(P3)$, assuming the country will cooperate in the future

$$V_i(s) = \min_{e_i} \quad \mathbb{E}\left[(c_i(e_i) + d_i(\bar{s})) + \beta \left[V_i(\bar{s}) + \mu_i[W(\bar{s}) - V(\bar{s})]\right]\right]$$

$$\begin{aligned} \text{s.t.} \quad & s^V = (1-\delta)s + \sum_{i=1}^{n} e_i^V + \xi \quad s\text{given} \\ & e_i \geq 0 \qquad \forall i \in J \\ & e_j \quad \text{with} j \neq i \quad \text{given} \end{aligned}$$

In the non-cooperative equilibrium with transfers problem, emission levels e_i^V and stock s^V can be obtained in terms of stock s by solving the First Order equations

$$c_i'(e_i^V) + d_j'(s^V) + \beta\left[V_i'(s^V) + \mu_i[W'(s^V) - V'(s^V)]\right] = 0 \qquad \forall i \in J \tag{19}$$

$$s^V = [1-\delta]s + \sum_{i=1}^{n} e_i^V + \xi \tag{20}$$

$$V_i(s) = c_i(e_i^V) + d_i(s^V) + \beta\left[V_i(s^V) + \mu_i[W(s^V) - V(s^V)]\right] \qquad \forall i \in J \tag{21}$$

where c_i' and d_j' are the derivatives of the functions c_i and d_j respectively.

As in the previous case, the problem now is again the identification of the value functions V_i such that First Order Conditions (19), (20) and (21) are met. Let V be $V = \sum_{i=1}^{n} V_i$ and recall that W has been calculated in problem $(P1)$. First Order Conditions can then be considered as a functional equations system where functions V_i, V, s^V and e_i^V are unknown for all $i \in J$. This system can be solved whenever functions c_i and d_j are known for all $i \in J$.

If

$$W_i(s) = c_i(e_i^V) + d_i(s^*) + \beta \tilde{W}_i(s^*) \leq V_i(s^V) \tag{22}$$

holds for each $i \in J$ then every country will be interested in cooperate. This means that the cost corresponding to each country i under cooperation improves the cost without cooperation. This can be considered as a signal of individual rationality.

On the other hand, if (22) does not hold, transfer functions are defined:

$$\theta_i(s) = -[W_i(s) - V_i(s)] + \mu_i \, [W(s) - V(s)]$$

where $\mu_i \in]0;1[$, for all $i \in J$ and $\sum_{i=1}^{n} \mu_i = 0$. My construction of these functions $\sum_{i=1}^{n} \theta_i(s) = 0$. Furthermore, if country i receives $\theta_i(s)$ under cooperation, then

$$\tilde{W}_i(s) = W_i(s) + \theta_i(s) = V_i(s) + \mu_i[W(s) - V(s)] \leq V_i(s) \tag{23}$$

This can be taken as a proof that individual rationality is still under cooperation for every value of the inherited stock of pollutant s.

4 The Algorithm

The aim of this section is to describe a numerical algorithm that calculates the expected value and the transfers functions when the cost functions c_i and d_i are convex. The algorithm is written for finite horizon problems, which is appropriate for most practical applications. The emissions and stock trajectories associated with the international cooperative optimum $(P1)$ can be calculated by non linear stochastic programming techniques. The transfers are more difficult to calculate, because they make use of values calculated at non cooperative equilibrium $(P2)$, so that one must proceed by backward induction. The basic idea is to construct explicit approximations for the surfaces or value functions $\tilde{W}_i(t, s_{t-1})$, for all $i \in J$, as polynomial functions of s_{t-1}, by using classical regression.

The first step of the algorithm is to solve the cooperative problem $(P1)$ associated with the international optimum. This is done by using non linear stochastic programming techniques and leads to the optimal trajectories of the abatement rates e_{it}^W for all $i \in J$ and $t \in \mathcal{T}$ and of the CO_2 stock s_t^W with $t \in \mathcal{T}$.

The second step computes the financial transfers. This solves the stochastic dynamic programming problems associated with the non cooperative equilibrium. As the algorithm proceeds backwards, this is first done at the final time T using the system of first order conditions associated to the non-cooperative problem $(P2)$.

Hence one obtains the non cooperative abatement rates e_{jT}^N, the transfers $\theta_i(s, T)$ and the expected discounted total costs (transfers included) $\tilde{W}_i(T, s)$ for all $t \in \mathcal{T}$ as functions of the CO_2 stock s_T inherited at the beginning of time T.

The resolution for period T of problems $(P2)$ is repeated for a set $\mathcal{S}_T$ of given values of the inherited CO_2 stock. This set is chosen in order to be representative of the interval of possible values of s_T. Once the values $\tilde{W}_i(T, s)$ for all $i \in J$ have been calculated on the set $\mathcal{S}_T$, the value functions $\tilde{W}_{iT}$ are written as polynomials of s_T and regressed on the set $\mathcal{S}_T$. So that the functions $\tilde{W}_{iT}$ are approximated by explicit analytical functions of s_T.

The step 2 of the algorithm is repeated for each period of time $T - 1$, $T - 2$, ... until period 1. At each period, one calculates the non cooperative equilibrium by solving the system of first order conditions associated to problems with monetary transfers $(P3)$. To do so at time t, the algorithm makes use of the polynomials regressed for period of time $t + 1$.

Once the value functions $\tilde{W}_{it}$ are known as functions of s_t for all times of the planning period $t \in \mathcal{T}$, the algorithm performs its third step, i.e. the computation of the actual values of these value functions and of the transfers for all regions all along the optimal trajectory $e_1^W, e_2^W, ..., e_T^W$ calculated at first step.

4.1 The Statement of the Algorithm

The algorithm consists of several steps

Step 1

The algorithm solves the stochastic optimization cooperative problem $(P1)$ associated with the international optimum. The first order conditions are

$$c_i'(e_{it}^W) + \sum_{\tau=t}^{T} \beta^{\tau-t}[1-\delta]^{\tau-t} \sum_{j=1}^{n} d_j'(s_\tau^W) = 0 \qquad \forall i \in J; \forall t \in \mathcal{T} \tag{24}$$

$$s_t^W = [1-\delta]s + \sum_{i=1}^{n} e_{it}^W \qquad \forall t \in \mathcal{T} \tag{25}$$

where c_i' and d_j' are the derivatives of functions c_i and d_j respectively. Solving these $T(n+1)$ equations yields to the optimal values of emissions e_{it}^W and the stock of pollutant s_t^W for all periods of time $t \in \mathcal{T}$.

Step 2a

To calculate the transfers, one must first solve the stochastic dynamic programming problems associated with the non cooperative equilibrium. The algorithm proceeds backwards, starting from the last period. At final time T, given the inherited stock of pollutant s, the non cooperative equilibrium, which coincides with the Nash equilibrium at last period, has to satisfy the conditions

$$c_i'(e_{iT}^N) + d_i'\left([1-\delta]s + \sum_{j=1}^{n} e_{jT}^N\right) = 0 \qquad \forall i \in J \tag{26}$$

Once this system of equations has been solved for the variables e_{jT}^N, and knowing by Step 1 the optimal emissions at time T, it is possible to calculate the transfers $\theta_i(s,T)$ using

$$\tilde{\theta}_i(s) = -[c_i(e_i^W) - c_i(e_i^V)] + \tilde{\mu}_i(s^W)\sum_{j=1}^{n}[c_j(e_j^W) - c_j(e_j^V)] \tag{27}$$

with

$$\tilde{\mu}_i(s_T^W) = \frac{d_i'(s_T^W)}{\sum_{j=1}^{n} d_j'(s_T^W)} \tag{28}$$

as well as the value functions $\tilde{W}_i(T,S)$ defined by

$$\tilde{W}_i(s) = W_i(s) + \tilde{\theta}_i(s), \quad \forall i \in J. \tag{29}$$

The **Step 2a** is done for a set Ω_T of given values of the inherited stock of pollutant s. This set is chosen to be representative of the interval of possible values of s. This interval is bounded below by the value of s that would be obtained with zero emissions during periods $1, 2, \ldots, T-1$ given s_0, and bounded above by the value of s that would be obtained with maximum emissions during the same period of time.

Step 2b

We now assume that the total costs with transfers at time T have the following polynomial form

$$\tilde{W}_i(T,s) = k_{iT,m}s^m + k_{iT,m-1}s^{m-1} + ... + k_{iT,0} \qquad \forall i \in J$$

where m is the order of the polynomial chosen so that the fit is good enough. To identify the parameters $k_{iT,m}, k_{iT,m-1}, ..., k_{iT,0}$, we use an ordinary least square regression method implemented in the software Matlab, so that the functions $\tilde{W}_i(T,s)$ are now approximated by explicit analytical functions of s.

The Steps 2a and 2b are then repeated for period $T-1$. Given (10) and the inherited stock of pollution s, the first order conditions associated to the non cooperative equilibrium are

$$c_i'(e_{it}^V) + d_i'\left([1-\delta]s + \sum_{j=1}^{n} e_{j,T-1}^V\right) + \beta\tilde{W}_i'\left(\left[[1-\delta]s + \sum_{j=1}^{n} e_{j,T-1}^V\right], T\right) = 0 \tag{30}$$

for all $i \in J$.

Once this system of equations has been solved for the $e_{j,T-1}^V$, it is possible to calculate the transfers $\theta_i(s, T-1)$ using (27)-(28) and the value functions $\tilde{W}_i(s, T-1)$ using (29). This is done for a set Ω_{T-1} of given values of the inherited stock of pollutant s, so that by regression of the calculated values $\tilde{W}_i(s, T-1)$ on Ω_{T-1}, they can be approximated by explicit analytical functions of s.

The algorithm continues backwards by repeating Steps 2a and 2b until the first time period $t = 1$ is reached.

Step 3

Once the value functions $\tilde{W}_i$ are known as functions of S for all periods of time of the planning period $\mathcal{T} = \{1, 2, ..., T\}$, the algorithm calculates the actual values of both these value functions and the transfers for all countries all along the optimal trajectory calculated at Step 1.

5 A numerical Example

In this section, we show some numerical results obtained from solving the problem ($P3$) in a real scenario. The simulations are made for a time horizon of 100 years, but we give the results only up to 2030, in order to avoid boundary problems. All computations were made by use of the software Matlab 7.3.0 (R2006b). We have implemented the equivalent formulation of problem ($P3$) given in Section 3. Thus, we have developed specific code for our example.

The cost and damage functions used in this case are nonlinear and the arguments of these functions are selected according to climate and economic principles. The temperature change equation is taken from the climate economy model RICE (Regional Integrated model of Climate and the Economy), as well as most of the parameter values and all basic data on GDP, population, capital stock, carbon emissions and concentration and global mean temperature. A complete overview of the equations and parameter values of the Climate Negotiation (CLIMNEG) World Simulation Model (abbreviated as CWSM) can be found in Eyckmans and Tulkens (2003) and Casas and Romera (2011). The partition of the world is the same as in the RICE model. There are 6 countries or regions: USA, Japan, European Union (EU), China, Former Soviet Union (FSU) and Rest of the World (ROW). The time is divided in years and the initial period (period $t = 0$) refers to year 1990.

Let consider optimal cooperative W and non cooperative N value functions respectively of Casas and Romera (2011), for each country and for each period of time. We conclude that countries like USA or ROW have not incentive to cooperate, and the other hand Japan has from the beginning interest in cooperation. Thus, without any modification of the initial game the most rational behavior of countries will probably be the non cooperation one, and the Nash equilibrium will result in the optimal non-cooperative stock pollutant $\{s_t^N\}$. It means that the stock pollutant will be international optimum $\{s_t^W\}$.

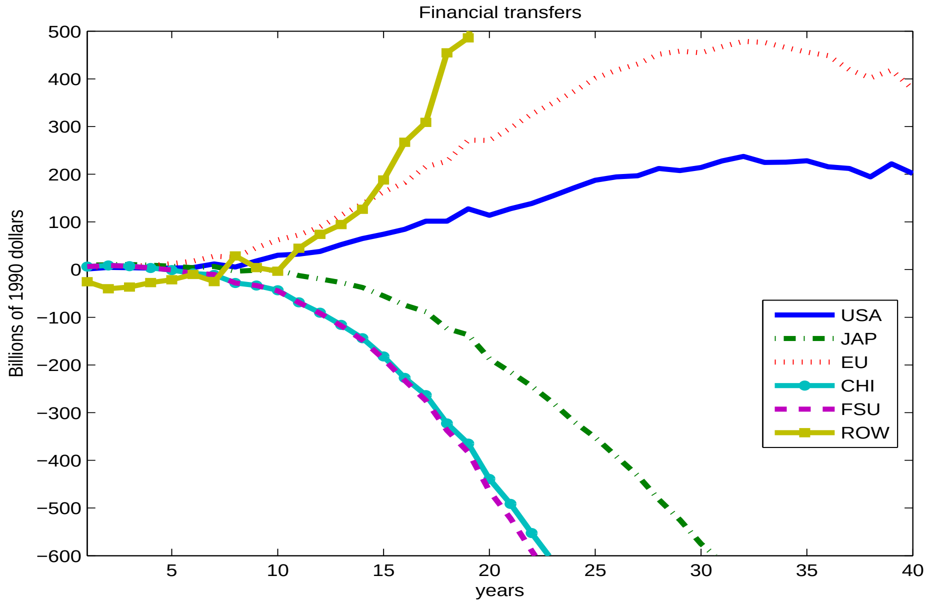

Figure 1: Financial transfers θ_{it} of country i for each period of time t in billions of 1990 USA dollars.

Table (1) and Figure (1) show the profile of financial transfers θ_{it} received and given for each country $i \in J$ and each period of time $t \in \mathcal{T}$. A negative transfer is a transfer received, and a positive transfer is a transfer given by the country i at period t. Note that ROW, EU and USA pay at each period a transfer to China, Japan and FSU in order to induce these regions to cooperate.

Table (2) shows the optimal cooperative value function $\tilde{W}_{it}$ with transfers θ_{it} of country i for each period of time t. Let compare these results with the optimal cooperative value function W_{it} and N_{it} in Casas and Romera (2011). As it is expected the total value function is the same because the total sum of the transfers is equal to zero. Let compare the marginal total value function of country under both cooperative models summarized in these tables. We observe that USA, EU and ROW increase these values when transfers are considered in comparison to the basic cooperative game, while Japan, China and FSU are net receivers of transfers.

Table (3) resumes all the results related to the marginal optimal value function per country under the cooperative, non cooperative and cooperative with transfers games. Note that the distribution of the total value function in the model with transfers, the last column in Table (3), provides net given countries like USA, EU and ROW. This is necessary in order to achieve the international optimum $\{s_t^W\}$ which is the same as in the cooperative model $(P1)$ of Casas and Romera (2011). This is in fact a weakness of this proposal, the cooperative model with transfers $(P3)$. A question arises, *Are there enough incentives in order to ensure that these countries will be able to accept that solution?*

Table 1: Transfers received or given θ_{it} per country i for each period of time t in billions of 1990 USA dollars.

t	USA	Japan	EU	China	FSU	ROW	Total
1	1,344	9,448	1,950	6,337	6,438	-25,518	0
2	4,502	9,972	8,562	8,561	8,582	-40,179	0
3	3,897	10,566	7,657	7,252	7,199	-36,570	0
4	3,136	9,226	8,295	3,207	3,422	-27,286	0
5	3,280	7,255	12,482	-0,970	-0,847	-21,200	0
6	4,370	3,886	17,805	-8,220	-7,681	-10,160	0
7	11,772	6,548	28,113	-10,999	-10,233	-25,201	0
8	5,460	-3,537	25,595	-28,118	-28,005	28,605	0
9	18,018	-1,396	45,842	-33,272	-33,347	4,155	0
10	29,970	-0,824	62,288	-43,405	-44,534	-3,495	0
11	32,436	-12,435	72,837	-68,624	-69,029	44,814	0
12	38,045	-19,538	89,594	-90,332	-91,572	73,802	0
13	52,374	-27,270	114,094	-115,961	-117,777	94,539	0
14	64,968	-37,387	137,561	-143,903	-147,685	126,446	0
15	74,123	-55,283	162,851	-182,176	-187,505	187,990	0
16	84,494	-74,426	182,412	-227,146	-232,343	267,010	0
17	101,462	-88,426	215,900	-263,159	-274,450	308,674	0
18	101,608	-123,503	226,931	-322,520	-337,154	454,638	0
19	127,461	-136,978	271,382	-365,000	-382,995	486,131	0
20	113,754	-186,970	270,760	-439,026	-464,389	705,871	0
21	127,734	-214,839	296,716	-491,414	-521,973	803,776	0
22	138,693	-244,477	326,143	-552,522	-589,603	921,766	0
23	154,696	-279,573	349,452	-611,008	-655,119	1041,551	0
24	171,396	-319,129	373,623	-674,143	-723,614	1171,865	0
25	187,499	-351,904	401,119	-736,773	-793,565	1293,623	0
26	194,499	-391,849	418,039	-804,2897	-866,545	1450,145	0
27	196,856	-431,922	431,069	-862,506	-942,539	1609,042	0
28	211,961	-481,956	451,534	-931,934	-1020,664	1771,060	0
29	207,796	-524,782	458,192	-997,308	-1099,344	1955,446	0
30	214,195	-575,060	454,540	-1060,708	-1174,514	2141,547	0
31	228,066	-612,957	467,591	-1115,824	-1242,638	2275,761	0
32	237,386	-662,788	478,792	-1170,096	-1311,934	2428,640	0
33	224,590	-716,412	476,237	-1235,415	-1391,554	2642,554	0
34	225,200	-759,759	465,807	-1290,270	-1465,844	2824,866	0
35	228,321	-809,238	456,043	-1342,846	-1536,995	3004,715	0
36	215,579	-866,480	448,571	-1395,338	-1599,790	3197,458	0
37	212,161	-927,964	419,422	-1454,819	-1676,990	3428,191	0
38	194,319	-973,168	401,847	-1492,729	-1730,234	3599,965	0
39	221,677	-984,480	418,007	-1496,011	-1757,176	3597,984	0
40	201,541	-1052,285	378,299	-1558,071	-1833,559	3864,077	0
Total	4870,641	-12892,096	10333,951	-23591,495	-26338,098	47617,097	0

Table 2: Optimal Cooperative Value Function $\tilde{W}_{it}$ with transfers θ_{it} per country i for each period of time t in billions of 1990 USA dollars.

t	USA	Japan	EU	China	FSU	ROW	Total
1	11,713	12,493	13,424	7,128	7,260	14,587	66.607
2	28,207	19,968	34,779	10,677	11,149	40,399	145.178
3	50,170	33,001	63,877	11,363	12,306	78,134	248.851
4	78,112	48,018	101,561	10,428	11,903	128,446	378.469
5	113,754	63,781	149,067	10,096	11,943	193,851	542.492
6	154,415	81,375	203,179	7,314	9,726	271,426	727.435
7	205,217	107,883	269,183	9,623	12,303	367,301	971.510
8	248,125	121,759	327,812	-1,806	0,048	462,235	1158.172
9	307,766	149,995	404,808	-0,990	0,507	580,153	1442.238
10	367,301	174,926	481,586	-5,076	-4,924	706,112	1719.925
11	417,462	187,171	549,274	-23,807	-24,051	830,075	1936.124
12	470,951	202,598	620,323	-39,558	-41,182	966,771	2179.902
13	527,332	215,311	694,166	-58,167	-62,694	1109,769	2425.717
14	577,722	223,345	760,871	-80,148	-88,469	1252,451	2645.772
15	619,311	222,145	821,521	-112,676	-124,533	1396,334	2822.102
16	658,402	215,144	874,042	-152,595	-166,349	1542,044	2970.687
17	703,078	210,715	934,467	-184,326	-204,952	1695,016	3153.998
18	724,793	183,688	965,247	-238,731	-266,190	1834,475	3203.282
19	764,166	174,115	1019,765	-277,295	-310,298	1993,415	3363.869
20	760,703	129,847	1028,551	-348,459	-389,777	2120,401	3301.268
21	784,681	100,961	1058,533	-397,593	-447,326	2266,549	3365.805
22	798,736	69,107	1085,503	-456,175	-514,123	2418,482	3401.530
23	808,947	31,018	1100,829	-512,848	-581,356	2560,294	3406.884
24	817,373	-13,358	1113,577	-575,314	-651,136	2709,680	3400.821
25	821,474	-54,283	1122,285	-637,198	-722,511	2850,642	3380.409
26	818,029	-103,617	1118,464	-702,792	-797,140	2990,516	3323.459
27	801,938	-154,773	1108,514	-763,070	-875,387	3124,254	3241.477
28	793,182	-218,269	1090,459	-835,192	-955,750	3251,602	3126.033
29	767,898	-273,973	1076,702	-901,501	-1037,413	3376,618	3008.332
30	754,921	-333,611	1051,298	-965,577	-1114,717	3497,996	2890.311
31	742,584	-385,926	1031,607	-1022,805	-1186,220	3627,795	2807.036
32	720,020	-445,725	1007,040	-1081,844	-1258,747	3751,389	2692.133
33	686,851	-514,676	968,473	-1149,871	-1339,950	3859,373	2510.199
34	664,655	-571,019	939,731	-1206,772	-1417,620	3977,051	2386.026
35	632,763	-631,237	896,446	-1265,289	-1490,979	4097,682	2239.386
36	604,897	-695,347	864,043	-1318,900	-1557,640	4208,091	2105.144
37	575,247	-756,824	812,375	-1379,047	-1635,655	4308,736	1924.833
38	550,947	-810,619	788,701	-1418,248	-1690,059	4431,372	1852.094
39	551,345	-834,034	774,573	-1427,287	-1719,593	4563,275	1908.279
40	517,082	-904,602	723,369	-1488,542	-1796,068	4668,961	1720.200
Total	22002,271	-4723,526	30050,023	-20962,870	-24395,662	88123,753	90093.990

Table 3: Total values per country

Country	W_i	N_i	$\Sigma\theta_i$	$\tilde{W}_i$
USA	17131,631	52801,575	4870,641	22002,271
JAP	8168,569	26075,778	-12892,096	-4723,526
EU	19716,072	60849,327	10333,951	30050,023
CHI	2628,625	9836,434	-23591,495	-20962,870
FSU	1942,436	6403,642	-26338,098	-24395,662
ROW	40506,656	118923,057	47617,097	88123,753
Total	90093,989	274889,813	0,000	90093,989

6 Conclusions and Extensions

In this research we operate under the assumption that countries cooperate because it makes cooperation beneficial for all countries, Flam (2006) arguments can enforce this point.

We develop transfer schemes for individual rationality for the design of international agreements that achieve global optimality in stock pollutant stochastic control problems. Agreements can in principle be negotiated at each period of time. These schemes are also suitable in the context of coalitional rationality. We are going to extend to n coalitions.

The technical complexity of the stochastic algorithm arises because we are not solving Linear Quadratic problems.

Summarizing our results, for each country $i \in J$ and each period $t \in \mathcal{T}$ we obtain the following solutions stocks pollution, emissions and values functions for each model, $\{s_t^*\}$, $\{e_{it}^*\}$, $\{W_i(t, s_{t-1}^*)\}$ corresponding to the solution of the Cooperative Model ($P1$), $\{s_t^N\}$, $\{e_{it}^N\}$, $\{N_i(t, s_{t-1}^N)\}$ corresponding to the solution of the Non-Cooperative Model ($P2$), both in Casas and Romera (2011), and $\{s_t^V\}$, $\{e_{it}^V\}$, $\{V_i(t, s_{t-1}^V)\}$ corresponding to the solution of the Model with Monetary transfers ($P3$).

We find of interest to consider a stochastic model with probability performance criteria and obtain the existence of an optimal policy. In absence of international cooperation, these optimal policies obtained under this new perspective could be an alternative behavior for each country which finally will help reducing the international stock pollutant. Note that the target value (X) could be chosen by each country, according some particular negotiation. Usually the target value (X) should be a quantity ranging between the non-cooperative value function and the cooperative value function. These schemes are also suitable in the context of coalitional rationality.

We find of interest to consider stochastic performance criteria based on bounds of probability, i.e., Markov Decision Problem with percentile performance criteria where the decision-maker wants to find a policy that achieves a specific value (target) at a specified probability level α.

Further research could be done if we consider uncertainty about the random perturbation, say the variance of the i.i.d. sequence. We propose to estimate the parameter recursively and to include the estimation in the stochastic control problem. Stochastic performance criteria based on bounds of probability could be considered.

Acknowledgements. This work was supported by the Spanish Ministry of Education and Science, research projects SEJ2004-03303 and SEJ2007-64500.

References

Bahn O., Haurie A., Malhamé R (2008). A stochastic control model for optimal timing of climate policies. *Automatica* **44**: 1545-1558.

Barrett S. (2003). Enviroment and Statecraft: The Strategy of Environmental Treaty-Making. *Oxford Univertity Press*, Oxford.

Bertsekas D. (2000). Dynamic Programming and Optimal Control, vol I, 2nd edn. *Athena Scientific*, Belmont, Massachusetts.

Birge J.R. and F. Louveaux (1997). Introduction to Stochastic Programming. *Springer-Verlag*, New York.

Carraro C., Eyckmans J., Finus M. (2006). Optimal transfers and participation decisions in international environmental agreements, *Review of International Organisations*. **1(4)**: 379-396.

Casas O. and R. Romera (2011). A Stochastic Model for the Environmental Transnational Pollution Control Problem, *Environmetrics* **22(4)**: 541-552.

Chander P. and Tulkens H. (1995). A core-theoretic solution for design of cooperative agreements on transfrontier pollution. *International Tax and Public Finance* **2**: 279-294.

Chander P. and Tulkens H. (1997). The core of an economy with multilateral environmental externalities. *International Journal of Game Therory* **26**: 379-401.

Dechert W.D. and S.I. O'Donnell (2006). The stochastic lake game: A numerical solution. *Journal of Economic Dynamics and Control* **30**: 1569-1587.

Eyckmans J. and H. Tulkens (2003). Simulating coalitionally stable burden sharing agreements for the climate change problem. *Resource and Energy Economics* **25**: 299-327.

Finus M. (2001). Game Theroy and International Environmental Cooperation. *Edwar Elgar*, Cheltenham UK and Northampton USA.

Flam S.D. (2006). Balanced environmental games. *Computer and Operations Research* **33**: 401-408.

Germain M., Toint P., Tulkens H. and A. de Zeeuw (2003). Transfers to sustain dynamic core-theoretic cooperation in international stock pollutant control. *Journal of Economic Dynamics and Control* **28**: 79-99.

Haurie A. and Viguier L. (2003). A stochastic dynamic game of carbon emission trading. *Environmental Modelling and Assessment* **8**: 239-248.

Hernández-Lerma O. (1999). Further topics on discrete-time Markov control processes. *Springer*, New York.

Hernández-Lerma O. and Romera R. (2001). Limiting discounted-cost control of partially observable stochastic system. *SIAM Journal on Control and Optimization* **40**: 348-369.

Jorgensen S., Martín-Herrán G. and Zaccour G. (2003). Agreeability and time-consitency in linear-state differential games. *Journal of Optimization Theory and Applications* **119**: 49-63.

Jorgensen S. and G. Zaccour (2001). Incentive Equilibrium Strategies and Welfare Allocation in a Dynamic Game of Pollution Control. *Automatica* **37**: 29-36.

Keller K., Bolker B.M. and Bradford D.F. (2004). Uncertain climate thresholds and optimal economic growth. *Journal of Environmental Economics and Management* **48**: 723-741.

Nordhaus W. and Boyer J. (2000). Warming the World: Economic Models of Global Warming. *MIT Press*, Cambridge, MA.

Nordhaus W. and Yang Z. (1996). A regional dynamic general-equilibrium model of alternative climate change strategies. *American Economic Review* **86**: 741-765.

Petrosjan L.A. and G. Zaccour (2003). Time-Consistent Shapley-Value Allocation of Pollution cost Control. *Journal of Economic Dynamics and Control* **27**: 381-398.

Puterman M. (2005). Markov Decision Processes:Discrete Stochastic Dynamic Programming. *John Wiley and Sons*, New Jersey.

Rubio S. and Casino B. (2005). Self-enforcing international enviromental agreements with a stock pollutant. *Spanish Economic Review* **7**: 89-109.