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Strategic Incentives for Keeping One Set of Books under the Arm's Length Principle*

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Abstract

The OECD's recommendation that transfer prices between multinational enterprises and their subsidiaries be consistent with the Arm's Length Principle (ALP) for tax purposes does not restrict internal pricing policies. However, we show that under imperfect competition firms may choose to keep one set of books (i.e., to set transfer prices consistent with the ALP), as a way of softening competition in the external market. As a result, firms' profits are greater, and the surplus is smaller, than under vertical integration. In contrast, when firms keep two sets of books (i.e., their transfer prices differ from those used for tax purposes), competition intensifies in both markets relative to vertical integration.

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1 Introduction

Transfer pricing policies of multinational enterprises have important implications since exports and imports from related parties are a dominant share of the trade flows – see Bernard, Jensen and Schott (2009). Transfer prices serve the purpose of both allocating costs to subsidiaries and determining the tax liabilities of parents and subsidiaries. Policy makers are aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions, and tend to follow the OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the *Arm's Length Principle* (ALP); i.e., that transfer prices be established, for tax purposes, on a market value basis, thus comparable to transactions between independent (unrelated) parties -see [31]. Multinational enterprises must therefore choose whether to use a single transfer price for both internal transactions and tax purposes (i.e., keep *one set of books*), or use internal transfer prices different from those used for tax purposes (i.e., keep *two sets of books*).

In this paper, assuming that transfer prices are consistent with the ALP for tax purposes, we study firms' choices of accounting policies under vertical separation, imperfect competition and quantity setting, and identify the properties of the ensuing market equilibria. While under vertical integration the choice between keeping one or two sets of books is irrelevant, under delegation this choice affects managerial incentives, and hence firms' consolidated profits, even when tax rates are equal across jurisdictions. Also, quantity setting provides a reduced form model for the analysis of more complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition, see, e.g., Kreps and Scheinkman (1983) and Moreno and Ubeda (2006), or competition via supply functions, see Delgado and Moreno (2004).

We abstract from many distinctive features of the activities of multinational enterprises – see, e.g., Markusen (2002) – and take as given the market structure, including firms' decisions to become multinational enterprises (rather than, e.g., licensing their products), the location of their headquarters, etc. While incorporating these features into the model, and explaining firms' choices, would undoubtedly be of interest, it would make the analysis overly complex.

In our framework, there are two multinational enterprises producing a homoge-

neous good that is sold in two markets, which we refer to as the Latin (home) market and the Greek (external) market. Parents engage in Cournot competition in the Latin market, while subsidiaries, in turn, engage in Cournot competition in the Greek market. Parents simultaneously choose their accounting policies and, upon observing accounting policy choices, simultaneously make output and, when relevant, internal transfer pricing decisions. Competition in the Latin market provides a reference price on comparable market transactions, and hence determines firms' tax liabilities. Subsidiaries, upon observing parents' accounting policies, outputs, and internal transfer pricing decisions, simultaneously make output decisions.

Since changing a firm's accounting policy typically involves high administrative and consulting costs, in our setting the choice of accounting policy serves as a commitment device – see, e.g., Göx (2000) and Arya and Mittendorf (2008). The literature has studied other commitment instruments such as distorting managerial compensation, e.g., Fershtman and Judd (1987), Sklivas (1987), sinking capacity investments, e.g., Dixit (1980), Spence (1977), building inventories, e.g., Ware (1985), limiting information acquisition, e.g., Einy, Moreno and Shitovitz (2002), Gal-Or (1988), or using cost allocation rules, e.g., Gal-Or (1993), Hughes and Kao (1998). Moreover, since accounting policies tend to be public, we assume that they are observed by competing firms prior to making output and transfer pricing decisions. (Accounting policies are disclosed in management discussions and annual reports, and are reported to securities and exchange commissions, tax authorities, etc.) Also, following the literature, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits – see Gal-Or (1993).

Thus, firms face a three stage non-cooperative game of complete information. A subgame perfect equilibrium (SPE henceforth) of this game identifies the firms' accounting policies as well as their outputs in the Latin and Greek market. Using backward induction, we identify the SPE of this game. Given parents accounting policies, outputs, and transfer pricing decisions, which determine the subsidiaries' costs, subsidiaries compete a la Cournot. It is therefore straightforward to identify the ensuing equilibrium in the Greek market. We proceed to study the equilibria that arise in subgames identified by parents accounting policy decisions.

In a subgame in which both parents keep one set of books, i.e., the good is trans-

ferred to subsidiaries at the price it is sold in the Latin market, a parent's output decision must internalize its impact on the transfer price of its subsidiary, as well as its subsidiary's rival. Thus, parents' efforts to alleviate double marginalization intensify competition in the Latin market. As a result, in equilibrium the output served to the Latin (Greek) is larger (smaller), and hence the surplus realized is larger (smaller), than under vertical integration. However, we show that firms' consolidated profits are greater, although the total surplus is smaller, than under vertical integration. Thus, keeping one set of books serves parents as an instrument to soften competition in the Greek market, and ultimately lead to larger consolidated profits. Hence the adoption of the ALP, when it leads firms to keep one set of books, provides a rationale for vertical separation. Consequently, adopting the ALP as a guideline for regulating transfer prices, when it lead to both firms keep one set of books, has negative welfare implications for the external country, i.e., reduces the surplus. (Interestingly, adopting the ALP does not achieve the objective of protecting the external country's tax base either – see Lemus and Moreno (2019).)

In a subgame in which both parents keep two sets of books, i.e., transfer prices differ from those used for tax purposes, internal transfer prices give parents an instrument to try and gain an advantage in the external market. As a result, competition intensifies in the Greek market. Interestingly, the ALP creates a subtle link between the two markets, which intensifies competition in the Latin market as well: since the transfer price for tax purposes is the price in the Latin market, each parent can improve the competitive advantage of its subsidiary by increasing the tax liability of its subsidiary's rival, which can be achieved by increasing output in the Latin market, thus reducing the price. As a result, the output in both markets are greater, and hence the surplus is larger, than under vertical integration. Therefore, adopting the ALP improves market efficiency when firms keep two sets of books.

In a subgame in which parents accounting policies are asymmetric, i.e., one parent keeps one set of books and the other keeps two sets of books, the subsidiary of the firm keeping two sets of books becomes dominant in the external market, while the parent using one set of books becomes dominant in the home market. In these equilibria, the firm choosing to keep one set of books is a weak position given the intensity of competition in both the home and the external market. We show that in both

markets the output and the surplus are larger, while the sum of the firms' profits is smaller, than under vertical integration. Thus, when at least one firm chooses to keep two sets of books, the adoption of the ALP leads to a surplus increase in both markets.

With these results in hand, we identify firms' accounting policies in the SPE of the dynamic game. We restrict attention to pure strategy equilibria. Assuming that following parents' accounting policy choices play forms a SPE, we show that parents' interaction at the stage of choosing their accounting policies is described by a simple symmetric two-action static game G . A Nash equilibrium of G corresponds to a SPE of the dynamic game.

Depending of the parameters of the model, which in our framework are the tax rate common to both markets and the size of the Latin market relative to the Greek market, G is either a prisoners' dilemma game, a game of chicken, a coordination game, or a cooperation game. When G is a prisoners' dilemma game, the unique SPE involves both firms keeping two sets of books. When G is a game of chicken, there are two pure strategy SPE, the both of which involve firms using asymmetric accounting policies. When G is a coordination game, there are two SPE: one in which both firms keep one set of books, and another one in which both firms keep two sets of books. Finally, When G is a cooperation game, the unique SPE involves both firms keeping one set of books.

When G is a cooperation game the welfare consequences of adopting the ALP are negative. When G is a coordination game the equilibrium in which both firms keep one set of books Pareto dominates that in which both firms keep two sets of books, i.e., the firms' profits are larger in the former than in the latter. Thus, even when G is a coordination game, any equilibrium concept that accounts for firms' communication opportunities, e.g., Ferreira (1996)'s communication equilibrium, will select this equilibrium as the more likely. Hence, also in the case the welfare consequences of adopting the ALP are negative. When G is either a prisoners' dilemma game or a game of chicken, the welfare consequences of adopting the ALP are positive. It is therefore useful to explore the parameter constellations that give rise to the different types of games.

We show that all four types of games may arise. For intermediate values of the

relative size of the Latin market and the tax rate, a prisoners' dilemma arises, whereas for larger values of these parameters a game of chicken arises. A coordination or a cooperation game arises when the relative size of the Latin market is small, or when both the tax rate is large, and the Latin and Greeks markets are of similar size. For these parameter constellations a firm has no incentives to switch to keeping two sets of books when the rival keeps one set of books: A deviating firm stands to gain at most the profit of the Stackelberg leader in this market (which is equal to half of the monopoly profits), but incurs the cost of losing entirely its position in the Latin market; such deviation is not profitable since the Greek market's equilibrium outcome is close to monopoly, which profits are shared equally by firms. See Figure 1 in Section 4 for a precise description of the parameter constellations leading to each type of game.

RELATED LITERATURE

While adopting the ALP is inconsequential under perfect competition, see Hirshleifer (1956), or when firms are vertically integrated, under imperfect competition and vertical separation its neutrality is lost: the adoption of the ALP significantly affects firms' behavior and market outcomes.

The literature studying the consequences of the ALP under these conditions has considered alternative market structures. In a monopoly setting, Baldenius, Melumad and Reichelstein (2004) show that keeping two sets of books is optimal as it allows the firm to deal with conflicting managerial objectives. A number of papers study strategic transfer pricing and tax distortions in a differentiated good symmetric duopoly of price competition, assuming that firms keep one set of books: Narayanan and Smith (2000) consider the firms' organizational structure. Göx (2000) shows that strategic transfer pricing softens competition. This literature tends to conclude that under quantity competition, because quantities are strategic substitutes, strategic transfer pricing intensifies competition. In contrast, our results show that the conclusion that under the ALP quantity competition intensifies competition is not warranted.

Closer to our work, Dürr and Göx (2011) establishes conditions under which keeping one set of books is a dominant strategy in a differentiated goods duopoly of price competition. In contrast, we study a homogeneous good symmetric duopoly of quantity competition. (As argued above, quantity competition serves as a reduced

form of more complex models of competition.) Further, we focus on the analysis of the game G that multinational enterprises face when choosing their accounting policies, and show that depending on the parameters G is one of four well-known two-player two-action games: a cooperation, coordination, chicken or prisoners' dilemma game. We provide an equilibrium analysis of G and identify firms' equilibrium accounting policies, i.e., the conditions under which firms keep one set of books, or two sets of books, or asymmetric accounting policies. We find that firms keep one set of books not just when G is a cooperation game, i.e., when keeping one set of books is a dominant strategy as in Dürre and Göx (2011), but also when G is a coordination game. Moreover, the space of parameters in which G is a coordination game is considerably larger than that in which it is a cooperation game. Thus, in a significant part of the parameter space firms keep one set of books even though this is not a dominant strategy. Also, while in Dürre and Göx (2011) the set of possible transfer prices is exogenously given, in our framework transfer prices for tax purposes are endogenously determined. In addition, assuming a linear demand allows us to identify the parameter regions in which the different equilibria arise. (And because these results involve evaluating the sign of certain polynomials, continuity warrants that similar results arise for demand functions in the neighborhood of the set of linear demand functions.) Dürre and Göx (2011) instead consider twice differentiable demand functions and impose conditions on their derivatives. Their main result (Proposition 4), however, involves assumptions regarding equilibrium objects, and hence is not informative of the conditions on primitives these assumptions involve.

A previous literature has established that, in the absence of the ALP, vertical separation intensifies or alleviates competition depending on whether firms compete in quantities or prices – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998). Interestingly, our results showing that under certain conditions the ALP softens competition in the external market recovers a rationale for vertical separation under quantity competition. (Obviously, the result that vertical separation intensifies competition with quantity setting does not hold for other market structures; e.g., Moresi and Schwartz (2017), shows that a vertically integrated input monopolist supplying to a differentiated downstream rival may prefer the rival to expand even under Cournot competition. Also, the organization literature has studied

alternative schemes for managers' remuneration that overcome the shortcoming of delegation under quantity competition, see, e.g., Jansen *et al.* (2007, 2009).

The rest of the paper is organized as follows. We introduce the basic setup in Section 2. In Section 3 we study the equilibria of the subgames given the firms' choice of accounting policies. In Section 4 we study the choice of accounting policies. We conclude in Section 5. The proofs of our results are relegated to the Appendix.

2 Model and Preliminaries

A good is sold in two markets, the *Latin* market and the *Greek* market, in which the inverse demand functions are $d(q) = \max\{0, 1 - bq\}$ and $\delta(\chi) = \max\{0, 1 - \beta\chi\}$, respectively. In these formulae, q and χ are the units of good demanded in the Latin and Greek markets at prices $p = d(q)$ and $\pi = \delta(\chi)$, respectively, and b and β are positive real numbers. There are two firms producing the good with the same constant marginal cost, which is assumed to be zero without loss of generality.

Assuming linear demands and identical maximum willingness to pay in both markets reduces notation, simplifies the analysis, and facilitates interpreting the results. (The equality of the maximum willingness to pay implicitly assumes that the range of per capita income are similar in both markets.) A measure of the size of the Latin (Greek) market is provided by the demand at price zero, $1/b$ ($1/\beta$). The smaller is b (β), the larger is the Latin (Greek) market. The parameter $s := \beta/b$ is thus a proxy for the size of Latin market relative to that of the Greek market. Also, we denote by $t, \tau \in (0, 1)$ the tax rates on profits in the Latin and Greek markets, respectively.

Under vertical integration and Cournot competition, in equilibrium the output of each firm, the price, and the surplus in the Latin and Greek markets are readily calculated as

$$(q_{VI}^*, p_{VI}^*, S_{VI}^*) = \left(\frac{1}{3b}, \frac{1}{3}, \frac{4}{9b} \right),$$

and

$$(\chi_{VI}^*, \pi_{VI}^*, \Sigma_{VI}^*) = \left(\frac{1}{3\beta}, \frac{1}{3}, \frac{4}{9\beta} \right),$$

respectively. Note that the tax rates do not affect market outcomes. Each firm's *consolidated profit* (i.e., its total revenue minus its total tax bill) is

$$C_{VI}^* = \frac{1-t}{9b} + \frac{1-\tau}{9\beta}.$$

We consider a setting of vertical separation in which firms engage in Cournot competition in the Latin market, and have *subsidiaries* which in turn engage in Cournot competition in the Greek market. Each *parent* firm seeks to maximize its consolidated profit, while each subsidiary seeks to maximize its *own profit* (i.e., its revenue, minus the total transfer paid to the parent, minus its tax bill). Also, we assume throughout that transfer prices must be consistent with the ALP for tax purposes, i.e., if a parent and its subsidiary serve q and χ units of output to the Latin and Greek markets at prices p and π , respectively, then the parent's tax bill is $tp(q + \chi)$, and the subsidiary's tax bill is $\tau(\pi - p)\chi$. Hence consolidated profit is

$$pq + \pi\chi - tp(q + \chi) - \tau(\pi - p)\chi = (1 - t)pq + (1 - \tau)\pi\chi + (\tau - t)p\chi.$$

Further, if the parent uses p as the transfer price, a situation we refer to as *keeping one set of books*, then its subsidiary's profit is

$$\Pi_I(p, \pi, \chi) = (1 - \tau)(\pi - p)\chi, \quad (1)$$

whereas if the parent uses an internal transfer price r that differs from p , a situation we refer to as *keeping two sets of books*, then the subsidiary's profit is

$$\Pi_{II}(p, r, \pi, \chi) = (\pi - r)\chi - \tau(\pi - p)\chi. \quad (2)$$

In order to avoid results merely driven by differences in tax rates, we assume throughout that $t = \tau$, and henceforth we denote by τ the common tax rate. Under this assumption consolidated profit is

$$C(p, q, \pi, \chi) = (1 - \tau)(pq + \pi\chi), \quad (3)$$

and therefore, maximizing consolidated profit amounts to maximizing revenue, i.e., the tax rate does not affect the parent's incentives. However, note that the tax rate affects the subsidiary's incentives only when the parent keeps two sets of books.

Assuming that $t = \tau$ facilitates our analysis and the interpretation of our results, which as we shall see are driven by only two parameters, the relative market size of the Latin market s , and the common tax rate τ . Our qualitative results are, nonetheless, robust to small differences in tax rates. Moreover, it is not difficult to see how tax differences affect the results, and issue on which we comment below.

In this setting of vertical separation, we will assume that parents first decide simultaneously their accounting policies, i.e., whether they keep one set of books or two sets of books. Then, upon observing the accounting policies chosen, parents simultaneously decide how much output to supply to the Latin market as well as their transfer prices. The subsidiaries decide how much output to serve to the Greek market upon observing the parents' decisions.

As argued in the introduction, assuming that firms commit to their accounting policies is reasonable if, for example, the costs associated with changing the accounting policy are sufficiently high. Göx (2000) notes that a new accounting policy usually requires substantial investments in developing or acquiring software and in training employees and/or hiring consultants. By choosing to keep one set of books, a parent commits to use the Latin market price as the transfer price per intrafirm transaction. Likewise, a parent that chooses to keep two sets of books allows itself the flexibility of using an internal transfer price different from the price in the Latin market.

Our objective is to identify the conditions under which alternative accounting policies can be sustained by subgame perfect equilibria (SPE henceforth), and to study the properties of the outcomes they generate. We restrict attention to SPE in pure strategies.

3 Equilibria with Alternative Accounting Policies

The parents' accounting policies, their decisions about the output they serve to the Latin market and, when relevant, their transfer prices, identify the subgame in which subsidiaries decide how much output to serve to the Greek market. In a SPE the subsidiaries' output decisions must form an equilibrium of the corresponding subgame. Moreover, in the subgame following firms' accounting policy decisions, parents' outputs and transfer pricing decisions (when they apply) must form an equilibrium when parents anticipate the ensuing equilibrium outcome in the Greek market following their decisions. We identify the market equilibria that arise under alternative accounting policies.

ONE SET OF BOOKS

Consider a SPE in which both parents keep *one set of books*, i.e., they commit to

using the price in the Latin market p as the transfer price per intrafirm transaction. Thus, the constant marginal cost of both subsidiaries is p . Then the subsidiary's output at the Cournot equilibrium of the Greek market, $\chi_I(p)$, and the equilibrium price, $\pi_I(p)$, can be readily calculated as

$$\chi_I(p) = \frac{1-p}{3\beta}, \quad (4)$$

and

$$\pi_I(p) = \delta(2\chi_I(p)) = \frac{1+2p}{3}. \quad (5)$$

A parent's output in the equilibrium of the subgame, q_I^* , maximizes consolidated profits (i.e., revenue), when the output of the competitor is q_I^* , and the subsidiaries outputs following the parents' decisions are $(\chi_I(p), \chi_I(p))$, where $p = d(q_I^* + q_I^*)$. That is, q_I^* solves the problem

$$\max_{q \in \mathbb{R}_+} C(d(q + q_I^*), q, \pi_I(d(q + q_I^*)), \chi_I(d(q + q_I^*))).$$

The first order condition for a solution to this problem identifies q_I^* . Thus, the equilibrium price in the Latin market is $p_I^* = d(2q_I^*)$. Substituting the value of p_I^* into the formulae for $\chi_I(p)$ and $\pi_I(p)$ given above, we obtain the subsidiaries' output, χ_I^* . Finally, the equilibrium price in the Greek market is $\pi_I^* = \delta(2\chi_I^*)$. The consolidated profits are readily calculated using equation (3).

We describe our results in Proposition 1. This description involves the function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ given for $s \in \mathbb{R}_+$ by

$$f(s) := \frac{2+9s}{8+27s}.$$

Note that f is strictly increasing and satisfies $f(0) = 1/4$ and $\lim_{s \rightarrow \infty} f(s) = 1/3$.

Proposition 1. *In a SPE in which both parents keep one set of books the output of each parent is*

$$q_I^* = \frac{1-f(s)}{2b} > \frac{1}{3b} = q_{VI}^*,$$

and the price in the Latin market is $p_I^ = f(s) < 1/3 = p_{VI}^*$, while the output of each subsidiary is*

$$\chi_I^* = \frac{1-f(s)}{3\beta} < \frac{1}{3\beta} = \chi_{VI}^*,$$

and the price in the Greek market is

$$\pi_I^* = \frac{1 + 2f(s)}{3} > \frac{1}{3} = \pi_{VI}^*.$$

Moreover, the firms' consolidated profits C_I^* are greater, and the total surplus $S_I^* + \Sigma_I^*$ is smaller, than under vertical integration, i.e., $C_I^* > C_{VI}^*$ and $S_I^* + \Sigma_I^* < S_{VI}^* + \Sigma_{VI}^*$.

When parents keep one set of books, they supply more output to the (home) Latin market, while their subsidiaries supply less output to the (external) Greek market, than under vertical integration. The intuition of this results is simple: parents' incentives to alleviate double marginalization leads them to increase their output in the Latin market above q_{VI} . Keeping one set of books, i.e. setting internal prices consistently with the ALP, allows parents to soften competition in the Greek market. The incentive to keep one set of books is sharper the larger the size of the Greek market relative to that of the Latin market, i.e., the smaller is s . Note that as s tends to zero, i.e., β approaches zero or b becomes arbitrarily large, the equilibrium of the Greek market approaches the monopoly outcome. (A related result of Choe and Matsushima (2013) shows that the ALP facilitates tacit collusion in dynamic imperfectly competitive markets.)

Of course, since the transfer price is determined by parents' outputs in the Latin market, competition in this market is more aggressive than under vertical integration. Nevertheless, consolidated profits are greater than under vertical integration. Thus, this accounting policy may provide a rationale for vertical separation.

When parents keep one set of books, however, both the surplus in the Greek market and the total surplus are smaller than under vertical integration. This conclusion raises questions about the social benefits of adopting the ALP as a guideline for regulating transfer prices when this policy induces firms to keep one set of books. (In addition, Lemus and Moreno (2019) show that in this case the ALP fails to protect the external country tax base.)

Finally, it is easy to assess the impact on equilibrium of differences in the tax rates: if the tax rate is larger in the Greek market than in the Latin market, then parents can reduce the tax liability in the Greek market by reducing their output in the Latin market (thus increasing the price); i.e., parents incentives to avoid double marginalization are diminished, making the equilibrium output smaller in both

markets; that is, under one set of books increasing (decreasing) the tax rate in the external market above that of the home market alleviates (intensifies) competition in both markets.

TWO SETS OF BOOKS

Let us now consider a SPE in which both parents keep *two sets of books*. If the price in the Latin market is p , and parent i uses the transfer price r_i , while its competitor uses the transfer price r_{-i} , then subsidiaries compete a la Cournot maximizing their profit as given by the function Π_{II} . In an interior equilibrium of the Greek market following these decisions, subsidiary i supplies the output

$$\chi_{II}(p, r_i, r_{-i}) = \frac{1}{3\beta} + \frac{\tau p - 2r_i + r_{-i}}{3\beta(1-\tau)}, \quad (6)$$

and the price in the Greek market is

$$\pi_{II}(p, r_i, r_{-i}) = \frac{1}{3} - \frac{2\tau p - r_i - r_{-i}}{3(1-\tau)}. \quad (7)$$

Thus, a parent's output q_{II}^* and transfer price r_{II}^* solve the problem

$$\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + q_{II}^*), q, \pi_{II}(d(q + q_{II}^*), r, r_{II}^*), \chi_{II}(d(q + q_{II}^*), r, r_{II}^*)).$$

Solving the system of first order conditions for an interior solution to this problem we get (q_{II}^*, r_{II}^*) , and $p_{II}^* = d(2q_{II}^*)$. It turns out that if $s \leq g(\tau)$, where $g : [0, 1] \rightarrow \mathbb{R}$ is giving for $s \in \mathbb{R}_+$ by

$$g(\tau) = \frac{2\tau}{5(1-\tau)},$$

then in equilibrium $p_{II}^* = 0$, and the parents' consolidated profit are independent of their output in the Latin market. In this case, r_{II}^* solves problem

$$\max_{r \in \mathbb{R}_+} C(0, q, \pi_{II}(0, r, r_{II}^*), \chi_{II}(0, r, r_{II}^*)).$$

In either case, substituting p_{II}^* and r_{II}^* in the formulae above we get χ_{II}^* and π_{II}^* . Consolidated profits are readily calculated using equation (3). We describe our results in Proposition 2.

Proposition 2. *In a SPE in which both parents keep two sets of books the output of each subsidiary is*

$$\chi_{II}^* = \frac{2}{5\beta} > \chi_{VI}^*,$$

and the price in the Greek market is $\pi_{II}^* = 1/5 < \pi_{VI}^*$. Further:

(P2.1) If $s > g(\tau)$, then the output of each parent is

$$q_{II}^* = \frac{2s + g(\tau)}{6bs} > q_{VI}^*,$$

and the price in the Latin market is

$$p_{II}^* = \frac{s - g(\tau)}{3s} < p_{VI}^*.$$

(P2.2) If $s \leq g(\tau)$, then the price in the Latin market is $p_{II}^* = 0$, and parents serve the demand at this price.

Moreover, in either case consolidated profits, C_{II}^* , are smaller than under vertical integration, i.e., $C_{II}^* < C_{VI}^*$, while the surpluses in both the Latin and Greek markets, S_{II}^* and Σ_{II}^* , respectively, are larger than under vertical integration, i.e., $S_{II}^* > S_{VI}^*$ and $\Sigma_{II}^* > \Sigma_{VI}^*$.

When firms keep two sets of books, the ALP creates a subtle link between the Latin and Greek markets, which intensifies competition in both markets: because for tax purposes the transfer price is the price in the Latin market, each parent has an incentive to improve the competitive advantage of its subsidiary by increasing the output it supplies to the Latin market, thus reducing the price and increasing the tax liability of its subsidiary's rival. This incentive is not present when both firms keep one set of books, or when they are vertically integrated. In addition, each parent has incentives to reduce the internal transfer price in order to give a cost advantage to its subsidiary, which makes competition more aggressive in the Greek market as well.

These incentives give rise to an equilibrium with better welfare properties than that arising when parents keep one set of books: in this scenario the output in both markets are greater (rather than smaller), and hence the surplus is larger than under vertical integration. Therefore, adopting the ALP as a guideline for regulating transfer prices improves market efficiency when firms keep two sets of books. On the other hand, consolidated profits are smaller than under vertical integration. Hence, when the adoption of the ALP give rise to an equilibrium in which firms keep two sets of books, vertical separation is hardly an optimal organizational design.

As for the impact on equilibrium of differences in tax rates, for tax rates below $1/2$ it is easy to see that under two sets of books an increment of the tax rate in

the Greek market above that of the Latin market creates an incentive for a parent to decrease its output in the Latin, in order to reduce its tax liability in the Greek market. Unlike under one set of books, however, this incentive is partially counter by a parent incentive to increase the tax liability of its subsidiary's rival.

ASYMMETRIC ACCOUNTING POLICIES

Let us consider now the case in which one parent (which we identify with the subindex I) keeps *one set of books*, while the other parent (II) keeps *two sets of books*. Consider a subgame in which the subsidiaries compete in quantities after observing the price in the Latin market, \hat{p} , and the transfer price chosen by Parent II , \hat{r}_{II} . Subsidiary I (the subsidiary of Parent I) supplies the output $\hat{\chi}_I$ that solves the problem

$$\max_{\chi \in \mathbb{R}_+} \Pi_I(\hat{p}, \delta(\chi + \hat{\chi}_{II}), \chi).$$

The solution to this problem gives us the *reaction function* of Subsidiary I to $\hat{\chi}_{II}$, given \hat{p} and \hat{r}_{II} . Likewise, Subsidiary II (the subsidiary of Parent II) supplies the output $\hat{\chi}_{II}$ that solves the problem

$$\max_{\chi \in \mathbb{R}_+} \Pi_{II}(\hat{p}, \hat{r}_{II}, \delta(\hat{\chi}_I + \chi), \chi).$$

The solution to this problem gives us the reaction function of Subsidiary II to $\hat{\chi}_I$, given \hat{p} and \hat{r}_{II} . The solution to the system of equations formed by these reaction functions is

$$\begin{aligned} \hat{\chi}_I(\hat{p}, \hat{r}_{II}) &= \max \left\{ \frac{1 - \hat{p}}{3\beta} - \frac{(\hat{p} - \hat{r}_{II})}{3\beta(1 - \tau)}, 0 \right\}, \\ \hat{\chi}_{II}(\hat{p}, \hat{r}_{II}) &= \max \left\{ \frac{1 - \hat{p}}{3\beta} + \frac{2(\hat{p} - \hat{r}_{II})}{3\beta(1 - \tau)}, 0 \right\}. \end{aligned}$$

Let us consider parents' decisions when that they anticipate that the subsidiaries' output is given by the functions $(\hat{\chi}_I, \hat{\chi}_{II})$. Parent I supplies the output \hat{q}_I that solves the problem

$$\max_{q \in \mathbb{R}_+} C(d(q + \hat{q}_{II}), q, \hat{\pi}(d(q + \hat{q}_{II}), \hat{r}_{II}), \hat{\chi}_I(d(q + \hat{q}_{II}), \hat{r}_{II})).$$

The first order condition for a solution to this problem identifies Parent I 's reaction function to the output and the transfer price set by Parent II . Likewise, Parent II 's output and transfer price, $(\hat{q}_{II}, \hat{r}_{II})$, solves the problem

$$\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + \hat{q}_I), q, \hat{\pi}(d(q + \hat{q}_I), r), \hat{\chi}_{II}(d(q + \hat{q}_I), r)).$$

The system of first order conditions for a solution to this problem gives \hat{q}_{II} and \hat{r}_{II} as a function of \hat{q}_I .

In the proof of Proposition 3 (see the Appendix) we establish that there is no interior SPE sustaining firms' asymmetric accounting policies: the solution of the system of equations formed by the parents' reaction functions leads to a negative value for the output of Subsidiary I whenever $\tau < 1/2$. Since $\tau < 1/2$ in just about every country in the world, we proceed to identify a corner pure strategy SPE assuming that $\tau \in [0, 1/2]$. (According to Jahnsen and Pomerleau (2017), only two countries in the world, UAE and Comoros, have corporate tax rates above 50%. Auerbach *et al.* (2009) show that corporate tax rate are falling across G7 economies, and provide some evidence suggesting convergence of corporate tax rates to values between 30% and 40%.)

Also, we show in the proof of Proposition 3 that in a SPE where firms use asymmetric accounting policies the output of Parent II is zero, i.e., $\hat{q}_{II}^* = 0$, while the output of Parent I as well as that of the subsidiaries, $(\hat{q}_I^*, \hat{\chi}_I^*, \hat{\chi}_{II}^*)$, are positive. The intuition behind this result is that when parents use asymmetric accounting policies, Parent I 's incentive to increase its output in order to alleviate double marginalization is counterbalanced by output reductions of Parent II that seeks to increase the cost of its subsidiary's rival. When the tax rate is below $1/2$, Parent II decreases its output all the way to zero. Thus, Parent I becomes the dominant producer in the Latin (home) market, whereas Parent II becomes the dominant producer in the Greek (external) market. Perhaps not surprisingly, in a SPE in which firms use asymmetric accounting policies, the consolidated profit of Parent II is greater than that of Parent I .

Identifying the market outcomes in a SPE involves simple but tedious calculations – see the Appendix. Proposition 3 summarizes our results. The following notation will be useful in describing the equilibria arising in this setting, as well as in the analysis of the next section. Write

$$h(\tau) := \frac{1 + \tau}{12(1 - \tau)}, \quad l(\tau) := \frac{2 - \tau}{3(1 - \tau)}.$$

For $\tau \in [0, 1/2]$, the functions h and l are both decreasing, and satisfy $h(\tau) < l(\tau)$.

In Proposition 3 we assume that $1/8 \leq s \leq l(\tau)$, since otherwise a pure strategy equilibrium does not exist. Note that $l(1/2) = 1$, and therefore the assumption

$s \leq l(\tau)$ limits the scope of Proposition 3 to multinational enterprises for which the external market is larger than the home market. Nevertheless, when the external market is large relative to the home market, managerial incentives have significant impact on the firms' tax liabilities and revenues. Hence this is the region of the parameter space most interesting to consider. Moreover, many multinational enterprises sell more than half of their products away from their home region, including nearly 40% of the Fortune 500 firms, operating in industries such as oil (BP), electronics (Sony, Intel), food (Coca-Cola), cars (Toyota, Honda), banking (ING, Santander), etc. See, e.g., Rugman and Verbeke (2004). Interpreting the external market as corresponding to that of the rest of world would allow us to consider such examples within the scope of Proposition 3.

Proposition 3. *Assume that $\tau \in [0, 1/2]$ and $s \in [1/8, l(\tau)]$. In a SPE in which Parent I keeps one set of books and Parent II keeps two sets of books, the output of Parent II is $\hat{q}_{II}^* = 0$. Moreover:*

(P3.1) *If $s > h(\tau)$, then the output of Parent I is*

$$\hat{q}_I^* = \frac{2}{3b} \left(1 + \frac{2(2-\tau)}{l(\tau)} \frac{l(\tau) - s}{5 - 7\tau + 24(1-\tau)s} \right),$$

and the price in the Latin market satisfies $0 < \hat{p}^ < p_{VI}^*$, while the subsidiaries' outputs are*

$$\begin{aligned} \hat{\chi}_I^* &= \frac{1}{4\beta} \left(1 - \frac{3(1+\tau)}{h(\tau)} \frac{s - h(\tau)}{5 - 7\tau + 24(1-\tau)s} \right), \\ \hat{\chi}_{II}^* &= \frac{1}{2\beta} \left(1 + \frac{1+\tau}{h(\tau)} \frac{s - h(\tau)}{5 - 7\tau + 24(1-\tau)s} \right). \end{aligned}$$

and the price in the Greek market is $\hat{\pi}^ < \pi_{VI}^*$*

(P3.2) *If $s \leq h(\tau)$, then the price in the Latin market is $\hat{p}^* = 0$, while the subsidiaries' outputs are*

$$(\hat{\chi}_I^*, \hat{\chi}_{II}^*) = \left(\frac{1}{4\beta}, \frac{1}{2\beta} \right),$$

and the price in the Greek market is $\hat{\pi}^ = 1/4 < \pi_{VI}^*$.*

Moreover, in either case the surpluses in the Latin and Greek markets, \hat{S}^ and $\hat{\Sigma}^*$, respectively, are larger than under vertical integration, i.e., $\hat{S}^* > S_{VI}^*$ and $\hat{\Sigma}^* > \Sigma_{VI}^*$, whereas the consolidated profit of the industry, $\hat{C}_I^* + \hat{C}_{II}^*$, is smaller than under vertical integration, i.e., $\hat{C}_I^* + \hat{C}_{II}^* < 2C_{VI}^*$.*

When firms maintain asymmetric accounting policies, Parent *II* (the firm keeping two sets of books) supplies no output to the Latin market in order to warrant its subsidiary a dominant position in the Greek market. Parent *I* (the firm keeping one set of books), besides its apparent dominant position in the Latin market, supplies a large output in order to counterbalance its subsidiary disadvantage. The smaller the relative size of the Latin market s , the larger is the output of Parent *I*: by increasing its output in the Latin market, thus reducing the price, Parent *I* alleviates double marginalization, and increases the tax bill of its subsidiary's rival. Moreover, the benefits to Parent *I* of increasing its output are larger the larger is the tax rate τ , and hence \hat{q}_I^* increases with τ .

The equilibrium output (price) in both markets are greater (smaller), and hence the surplus is larger, than under vertical integration. Thus, adopting the ALP to regulate transfer prices improves market efficiency when at least one firm keep two sets of books.

4 The Choice of Accounting Policies

We study parents' choice of accounting policies. In a pure strategy SPE, following accounting policy decisions the parents' payoffs in the space of parameters

$$A := \{(\tau, s) \in \mathbb{R}^2 \mid \tau \in [0, 1/2], s \in [1/8, l(\tau)]\},$$

are identified by propositions 1-3. (Assuming that $s \in [1/8, l(\tau)]$ is required to remain within the scope of Proposition 3.) For $(\tau, s) \in A$, parents' payoffs when they choose to keep either one set of books (action *I*) or two sets of books (action *II*) are described in Table I below. In this table, C_I^* and C_{II}^* , are the parents' consolidated profits in the equilibrium arising when both parents keep one set of books, and when both parents keep two sets of books, respectively. These values are readily calculated using propositions 1 and 2, respectively. Likewise, \hat{C}_I^* (\hat{C}_{II}^*) is the consolidated profit of the parent that keeps one (two) set(s) of books in the equilibrium arising when parents maintain asymmetric accounting policies. These values are calculated using the results of Proposition 3.

G	I	II
I	C_I^*, C_I^*	$\hat{C}_I^*, \hat{C}_{II}^*$
II	$\hat{C}_{II}^*, \hat{C}_I^*$	C_{II}^*, C_{II}^*

Table 1: Parents Choice of Accounting Policies

Thus, at the stage of choosing their accounting policies, anticipating their ensuing output choices and transfer pricing decisions, as well as the output choices of their subsidiaries following these decisions, parents face the two-player two-action symmetric game G described in Table 1. A pure strategy Nash equilibrium (NE henceforth) of G identifies a SPE equilibrium of the dynamic game played by parents and subsidiaries. The nature of the game G is determined by the sign of the differences $\hat{C}_{II}^* - C_I^*$ and $C_{II}^* - \hat{C}_I^*$, as shown in the following table.

G	$\hat{C}_{II}^* - C_I^*$	$C_{II}^* - \hat{C}_I^*$
Prisoners' Dilemma	+	+
Chicken	+	-
Coordination	-	+
Cooperation	-	-

Table 2: The Nature of Game G

When G is a *prisoners' dilemma game*, its unique NE involves both firms using the dominant strategy II . When G is a *game of Chicken*, both (II, I) and (I, II) are NE. When G is a *coordination game*, both (I, I) and (II, II) are NE. (Recall that $C_I^* > C_{II}^*$, and hence (I, I) Pareto dominates (II, II) .) Finally, when G is a *cooperation game*, its unique NE involves both firms using the dominant strategy I .

We study the sign of the differences $(\hat{C}_{II}^* - C_I^*, C_{II}^* - \hat{C}_I^*)$ across the space A , and show that all four types of games may arise. To study this issue, it is useful to

partition A in four regions:

$$\begin{aligned} A_1 &:= \{(\tau, s) \in A \mid s > \max\{g(\tau), h(\tau)\}\}, \\ A_2 &:= \{(\tau, s) \in A \mid h(\tau) < s \leq g(\tau)\}, \\ A_3 &:= \{(\tau, s) \in A \mid g(\tau) < s \leq h(\tau)\}, \\ A_4 &:= \{(\tau, s) \in A \mid s \leq \min\{g(\tau), h(\tau)\}\}. \end{aligned}$$

Figure 1 below shows the graphs of the functions g , h , and l , and identifies these regions. In region A_1 , consolidated profits are identified in propositions 1, (P2.1) and (P3.1). In region A_2 , consolidated profits are identified in propositions 1, (P2.2) and (P3.1). In region A_3 , consolidated profits are identified in propositions 1, (P2.1) and (P3.2). In region A_4 , consolidated profits are identified in propositions 1, (P2.2) and (P3.2). Signing the differences $(\hat{C}_{II}^* - C_I^*, C_{II}^* - \hat{C}_I^*)$ in these regions is simple but tedious – see the Appendix. Our results are summarized in Proposition 4.

Proposition 4. *Assume that $\tau \in [0, 1/2]$ and $s \in [1/8, l(\tau)]$.*

(P4.1) If $s > \max\{g(\tau), h(\tau)\}$, then all four types of games may arise: G is a coordination game when s is small; for intermediate values of s and τ , G is a prisoners' dilemma game; for larger values of s and τ , G becomes a chicken game; and for very large values of s and τ , G is a cooperation game.

(P4.2) If $s \leq h(\tau)$, then G is a coordination game.

(P4.3) If $h(\tau) < s \leq g(\tau)$, then G is either a coordination or a cooperation game.

Hence, when s is small both symmetric accounting policies are sustained by SPE. For intermediate values of s and τ , firms keep two sets of books. For larger values of s and τ , firms use asymmetric accounting policies. And for very large values of s and τ , firms keep one set of books.

Figure 1 offers a precise description of our results. In this figure, the signs “+/-” in the first and second position of the parenthesis (\cdot, \cdot) placed at different regions of the parameter space correspond to the signs of $\hat{C}_{II}^* - C_I^*$ and $C_{II}^* - \hat{C}_I^*$, respectively, in such regions.

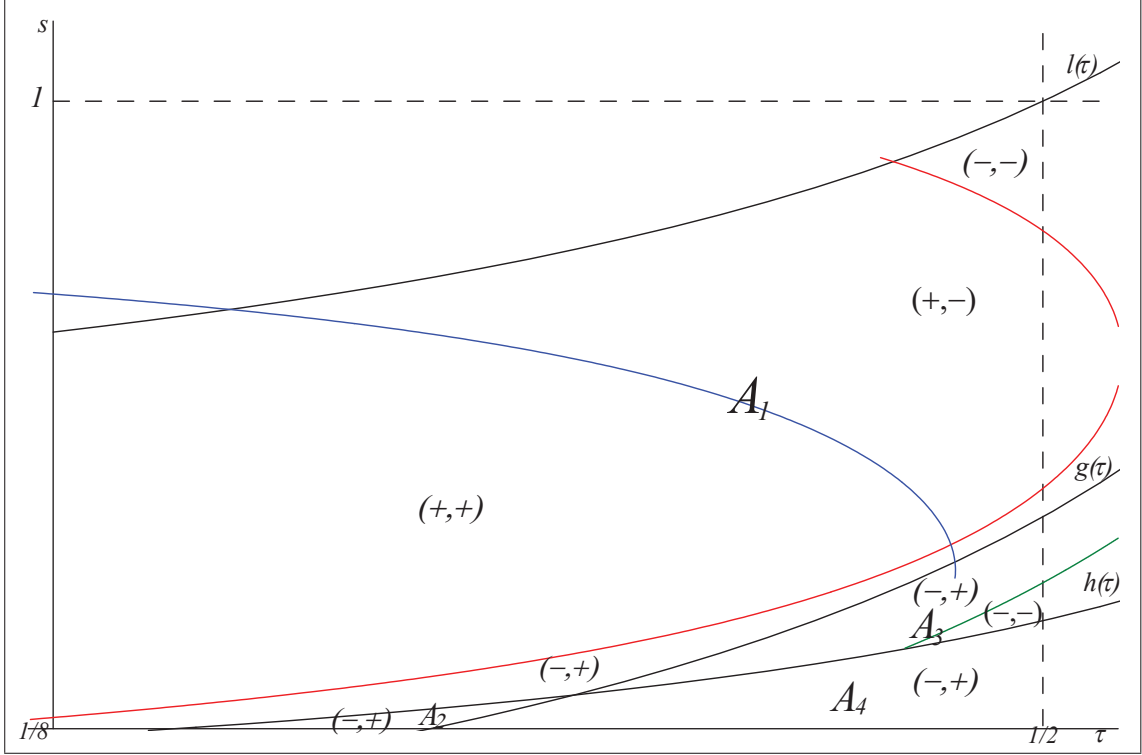


Figure 1: Accounting Policies in a SPE

Proposition (P4.1) describes our results for $(\tau, s) \in A_1$. In A_1 the sign of $\hat{C}_{II}^* - C_I^*$ is positive (negative) inside (outside) the red parabola, while the sign of $C_{II}^* - \hat{C}_I^*$ is positive (negative) inside (outside) the blue parabola. Across A_1 the differences $(C_{II}^* - \hat{C}_I^*, C_{II}^* - \hat{C}_I^*)$ take all possible signs. Proposition (P4.1) notices that as τ and s increase along the line $s = 2\tau$, the sign of $\hat{C}_{II}^* - C_I^*$ switches from negative to positive, and then back to negative, while the sign of $C_{II}^* - \hat{C}_I^*$ switches from positive to negative. Thus, in this region all four types of games arise.

Proposition (P4.2) describes our results for $(\tau, s) \in A_2 \cup A_4$. Across this region $C_{II}^* - \hat{C}_I^* < 0$ and $C_{II}^* - \hat{C}_I^* > 0$. Thus, in this region G is a coordination game.

Proposition (P4.3) describes our results for $(\tau, s) \in A_3$. In A_3 , $\hat{C}_{II}^* - C_I^* < 0$, while $C_{II}^* - \hat{C}_I^*$ is positive (negative) above (below) the green curve. Thus, in this region G is either a coordination game or a cooperation game.

Therefore, in region $A_2 \cup A_3 \cup A_4$ only symmetric accounting policies can be sustained by SPE. Moreover, keeping one set of books is sustained by a SPE across the region – uniquely in the subset of A_3 below the green curve. Since $C_I^* > C_{VI}^* > C_{II}^*$ by propositions 1 and 2, i.e., the firms' consolidated profits in the SPE in which both

firms keep one set of books are larger than in the SPE in which both firms keep two sets of books, the former SPE is more likely. (Any notion of equilibrium that accounts for the possibilities of non-binding communication, e.g., see Ferreira (1996)'s communication equilibrium, would identify this SPE as the likely outcome.) Note that the subset of $A_2 \cup A_3 \cup A_4$ in which G is a cooperative game (i.e., keeping one set of books is a dominant strategy) is small. Moreover, this region as well as the subregion of A_1 in which G is a cooperative game (which is also small) involve large tax rates (well above 0.4).

It is interesting to note that even though in $A_2 \cup A_3 \cup A_4$ the Latin market is relatively small, a firm has no incentives to change its accounting policy when both the firm and its competitor keep one set of books: even though a firm may become dominant in the relatively larger Greek market by changing its accounting policy (i.e., by keeping two sets of books rather than one), the maximum profit the firm stands to make is those of a Stackelberg leader, which is half of the monopoly profits. However, in these regions the sum of the firms' profits in the Greek market is approximately the monopoly profits. Hence, the gains in profits in the Greek market to be had from such deviation are small, and do not compensate the losses that the firm incurs in the Latin market. (Recall the equilibrium of a subgame in which firms accounting policies are asymmetric, the firm keeping two sets of books produces zero in the Latin market – see Proposition 3.)

Finally, our observations to propositions 1 and 2 regarding the effect of a tax rate in the Greek market (τ) above that in the Latin market (t) when firms keep either one or two sets of books, respectively, suggest that the region of parameters in which there is a SPE in which firms keep one set of books will be larger in this case than when the tax rates are the same in both markets; i.e., that, *ceteris paribus*, it is more likely that equilibrium involves both firms keeping one set of books when $\tau > t$ than when $\tau = t$.

5 Conclusions

We study the consequences of adopting the ALP in imperfectly competitive markets in which firms are vertical separated. Since transfer prices play the dual role of

allocating costs to subsidiaries and determining the tax liabilities in the jurisdictions where firms operate, the firms' incentives to choose their accounting policies are complex. Since the ALP does not restrict the firms internal pricing decisions, we study the accounting policies that arise in equilibrium, and the properties of the ensuing market outcomes. We show that the accounting policies firms adopt depend on the relative sizes of the home and external markets and on the tax rates, and that all configurations of accounting policies may arise in the parameter space.

The choice of accounting policies serves as a precommitment device. We show that relative to vertically integration, competition in the external (home) market softens (intensifies) when parents keep one set of books, while it intensifies when firms keep two sets of books or when firms adopt asymmetric accounting policies.

Our analysis provides an explanation for the mixed empirical evidence on the use of alternative accounting policies. Further, it contributes to understanding the role of transfer prices, and the incentives present the choice of accounting policies. In contrast to most of the literature on this topic (e.g., Schjelderup and Sorgard (1997), Narayanan and Smith (2000), Göx (2000), Hyde and Choe (2005), Korn and Lengsfeld (2007), Nielsen et al. (2008), Lemus and Moreno (2019)), which takes as given that firms keep one set of books, and focuses on the analysis of the impact of the ALP on tax revenue, we endogenize the choice of the accounting policies, and show conditions under which in equilibrium firms keep one set of book. Under these conditions, the ALP provides a rationale for vertical separation.

6 Appendix

Proof of Proposition 1. A parent solves the problem $\max_{q \in \mathbb{R}_+} B_I(q)$, where

$$\begin{aligned} B_I(q) &= C(d(q + q_I^*), q, \pi_I(d(q + q_I^*)), \chi_I(d(q + q_I^*))) \\ &= (1 - \tau) \left((1 - b(q + q_I^*))q + \frac{b(q + q_I^*)(3 - 2b(q + q_I^*))}{9\beta} \right). \end{aligned}$$

Since q_I^* must satisfy the first order condition for a solution to this problem, we get

$$q_I^* = \frac{3b + 9\beta}{27b\beta + 8b^2} = \frac{1 - f(s)}{2b}.$$

Hence

$$p_I^* = d(2q_I^*) = f(s).$$

Using equations (4) and (5)

$$\chi_I^* = \chi_I(p_I^*) = \frac{1 - f(s)}{3\beta},$$

and

$$\pi_I^* = \pi_I(p_I^*) = \frac{1 + 2f(s)}{3}.$$

Direct calculation yields

$$C_I^* - C_{VI}^* = (1 - \tau) \frac{2}{9\beta} \frac{4 + 22s + 27s^2}{(8 + 27s)^2} > 0,$$

and

$$S_I^* + \Sigma_I^* - (S_{VI}^* + \Sigma_{VI}^*) = -\frac{2}{9\beta} \frac{20 + 155s + 297s^2}{(8 + 27s)^2} < 0. \quad \square$$

Proof of Proposition 2. A parent solves the problem $\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} B_{II}(q, r)$, where

$$\begin{aligned} B_{II}(q, r) &= C(d(q + q_{II}^*), q, \pi_{II}(d(q + q_{II}^*), r, r_{II}^*), \chi_{II}(d(q + q_{II}^*), r, r_{II}^*)) \\ &= (1 - \tau) \left(q - b(q + q_{II}^*)q + \frac{D}{9\beta} \right), \end{aligned}$$

where

$$D = \left(1 + \frac{\tau(1 - b(q + q_{II}^*)) - 2r + r_{II}^*}{1 - \tau} \right) \left(1 - \frac{2\tau(1 - b(q + q_{II}^*)) - r - r_{II}^*}{1 - \tau} \right).$$

Since (q_{II}^*, r_{II}^*) must satisfy the first order condition for a solution to this problem, we get

$$(q_{II}^*, r_{II}^*) = \left(\frac{2s + g(\tau)}{6bs}, \frac{\tau(s - g(\tau))}{3s} - \frac{1 - \tau}{5} \right).$$

Hence

$$p_{II}^* = d(2q_{II}^*) = \frac{s - g(\tau)}{3s}.$$

provided $s > g(\tau)$. Using equations (6) and (7)

$$\chi_{II}^* = \chi_{II}(p_{II}^*, r_{II}^*, r_{II}^*) = \frac{2}{5\beta},$$

and

$$\pi_{II}^* = \pi_{II}(p_{II}^*, r_{II}^*, r_{II}^*) = \frac{1}{5}.$$

If $s \leq g(\tau)$, then in equilibrium $p_{II}^* = 0$, and parents' problem becomes $\max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} \bar{B}_{II}(r)$, where

$$\begin{aligned} \bar{B}_{II}(r) &= C(0, q, \pi_{II}(0, r, r_{II}^*), \chi_{II}(0, r, r_{II}^*)) \\ &= (1 - \tau) \pi_{II}(0, r, r_{II}^*) \chi_{II}(0, r, r_{II}^*) \\ &= \frac{1 - \tau}{9\beta} \left(1 - \frac{2r - r_{II}^*}{1 - \tau}\right) \left(1 + \frac{r + r_{II}^*}{1 - \tau}\right). \end{aligned}$$

Since r_{II}^* must satisfy the first order condition for a solution to this problem, we get

$$r_{II}^* = -\frac{1 - \tau}{5}.$$

Using again equations (6) and (7) we get we get $\chi_{II}^* = 2/5\beta$ and $\pi_{II}^* = 1/5$.

Since the outputs supplied in both the Latin and Greek markets are larger than under vertical separation, and surplus increases with output on $[0, 1]$, the surpluses in both markets are greater than under vertical integration, i.e., $S_{II}^* > S_{VI}^*$ and $\Sigma_I^* > \Sigma_{VI}^*$. Moreover, consolidated profits decrease with output above the monopoly output (which is equal to $1/2$), and hence consolidated profits are smaller than under vertical integration, i.e., $C_{II}^* < C_{VI}^*$. Of course, these inequalities can be directly verified. \square

Proof of Proposition 3. Let us be given a SPE in which one firm keeps one set of books and the other keeps two sets of book, and denote by $(\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*, \hat{q}_I^*, \hat{q}_{II}^*, r_{II}^*, \hat{p}^*)$ the ensuing outputs and prices in the Greek and Latin markets. Then these values satisfy the conditions:

$$\hat{\chi}_I = \max \left\{ \frac{1 - \hat{p} - \beta \hat{\chi}_{II}}{2\beta}, 0 \right\} \quad (8)$$

$$\hat{\chi}_{II} = \max \left\{ \frac{1 - \hat{r}_{II} - (\hat{\pi} - \hat{p})\tau - \beta \hat{\chi}_I}{2\beta}, 0 \right\} \quad (9)$$

$$\hat{\pi} = \delta (\hat{\chi}_I + \hat{\chi}_{II}) \quad (10)$$

$$\hat{p} = d(\hat{q}_I + \hat{q}_{II}) \quad (11)$$

$$\hat{q}_I \in \arg \max_{q \in \mathbb{R}_+} C(d(q + \hat{q}_{II}), q, \hat{\pi}(d(q + \hat{q}_{II}), \hat{r}_{II}), \hat{\chi}_I(d(q + \hat{q}_{II}), \hat{r}_{II})) \quad (12)$$

$$(\hat{q}_{II}, \hat{r}_{II}) \in \arg \max_{(q,r) \in \mathbb{R}_+ \times \mathbb{R}} C(d(q + \hat{q}_I), q, \hat{\pi}(d(q + \hat{q}_I), r), \hat{\chi}_{II}(d(q + \hat{q}_I), r)). \quad (13)$$

We establish a number of preliminary results that will help us proving Proposition 3.

Claim 1. If $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$, then

$$\begin{aligned}\hat{\chi}_I^* &= \frac{1 - \hat{p}^*}{3\beta} - \frac{(\hat{p}^* - \hat{r}_{II}^*)}{3\beta(1 - \tau)}, \\ \hat{\chi}_{II}^* &= \frac{1 - \hat{p}^*}{3\beta} + \frac{2(\hat{p}^* - \hat{r}_{II}^*)}{3\beta(1 - \tau)},\end{aligned}$$

and

$$\hat{\pi}^* = \frac{1 + 2\hat{p}^*}{3} - \frac{(\hat{p}^* - \hat{r}_{II}^*)}{3(1 - \tau)} > 0.$$

Proof. If $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$, then equations (8) and (9) become the formulae for $\hat{\chi}_I^*$ and $\hat{\chi}_{II}^*$, respectively, given in the claim. Moreover, $\hat{\chi}_I^* > 0$ and $\hat{p}^* \geq 0$ imply

$$0 < 1 - \hat{p}^* - (\hat{p}^* - \hat{r}_{II}^*)/(1 - \tau) \leq 1 - (\hat{p}^* - \hat{r}_{II}^*)/(1 - \tau),$$

and therefore

$$1 - \beta(\hat{\chi}_I^* + \hat{\chi}_{II}^*) = \frac{1}{3} \left(1 + 2\hat{p}^* - \frac{\hat{p}^* - \hat{r}_{II}^*}{1 - \tau} \right) > 0,$$

which using equation (10) establishes the claim. \square

Claim 2. If $\hat{\chi}_{II}^* > 0$, then $\hat{r}_{II}^* \leq \hat{p}^*$ and $\hat{r}_{II}^* \in (-\frac{1}{2}, \frac{1}{4})$.

Proof. Assume that $\hat{\chi}_{II}^* > 0$.

If $\hat{\chi}_I^* > 0$, then $(\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*)$ are given by the formulae of Claim 1. Using these values, condition (13) for Parent *II*'s profit maximization leads to the equation

$$\hat{r}_{II}^* = -\frac{1 - \tau + (1 - 5\tau)\hat{p}^*}{4}.$$

Hence $\tau \in (0, 1/2)$ and $\hat{p}^* \in [0, 1]$ imply

$$\hat{r}_{II}^* < -\frac{1 - \frac{1}{2} + (1 - \frac{5}{2})\hat{p}^*}{4} < -\frac{\frac{1}{2} + (1 - \frac{5}{2})}{4} = \frac{1}{4},$$

and

$$\hat{r}_{II}^* > -\frac{1 - 0 + (1 - \frac{5}{2}(0))\hat{p}^*}{4} = -\frac{1 + \hat{p}^*}{4} > -\frac{1}{2}.$$

Moreover,

$$\hat{r}_{II}^* - \hat{p}^* = -\frac{1 - \tau + (1 - 5\tau)\hat{p}^*}{4} - \hat{p}^* = -\frac{(1 - \tau)(1 + 5\hat{p}^*)}{4} < 0.$$

If $\hat{\chi}_I^* = 0$, then equations (9) and (10) yield

$$(\hat{\chi}_{II}^*, \hat{\pi}^*) = \left(\frac{1 - \hat{r}_{II}^* - (1 - \hat{p}^*)\tau}{2\beta(1 - \tau)}, 1 - \frac{1 - \hat{r}_{II}^* - (1 - \hat{p}^*)\tau}{2(1 - \tau)} \right).$$

Substituting these values and solving (13), we obtain $\hat{r}_{II}^* = -\tau\hat{p}^* \in (-1/2, 0)$, which establishes the claim. \square

Claim 3. $\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{q}_I^*, \hat{q}_{II}^* > 0$ does not hold; i.e., no interior SPE exists.

Proof. Assume by way of contradiction that $\hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{q}_I^*, \hat{q}_{II}^* > 0$. Using the values given in Claim 1 to solve the parents maximization problems (12) and (13) we get

$$\begin{aligned}\hat{q}_I^* &= \frac{1}{2b} \frac{(1-2\tau)s^2 + 2(5-4\tau)s + 12(1-\tau)}{(1-2\tau)s + 18(1-\tau)}, \\ \hat{q}_{II}^* &= -\frac{1}{2b} \frac{(1-2\tau)s^2 + 2(5-4\tau)s - 12(1-\tau)}{(1-2\tau)s + 18(1-\tau)}, \\ \hat{r}_{II}^* &= -\frac{1}{2} \frac{(1-2\tau)((1-3\tau)s + 12(1-\tau))}{(1-2\tau)s + 18(1-\tau)}.\end{aligned}$$

Substituting these values into equation (11) to obtain \hat{p} , and then into the equation for $\hat{\chi}_I^*$ in Claim 1 we get

$$\hat{\chi}_I^* = -\frac{1}{2\beta} \frac{(1-2\tau)s}{(1-2\tau)s + 18(1-\tau)},$$

which is negative for $\tau \in (0, 1/2)$. This contradiction establishes the claim. \square

Claim 4. If $\hat{q}_{II}^* = 0$, then $\hat{q}_I^* > 0$.

Proof. Assume $\hat{q}_{II}^* = 0$. Then $\hat{p}^* = d(\hat{q}_I^*)$.

Assume that $\hat{r}_{II}^* \leq \hat{p}^* - (1-\tau)(1-\hat{p}^*)$. Then $\hat{\chi}_I^* = 0$, and Parent I 's profit maximization condition (12) implies

$$\hat{q}_I^* = \frac{1}{2b} > 0.$$

Assume that $\hat{p}^* - (1-\tau)(1-\hat{p}^*) < \hat{r}_{II}^* < \hat{p}^* - (1-\tau)(1-\hat{p}^*)/2$. Then $\hat{\chi}_1^*, \hat{\chi}_2^* > 0$ and condition (12) yields

$$\hat{q}_I^* = \frac{1}{2b} \frac{9(1-\tau)^2 + (5(1-2\tau) + 3\tau^2)s + (1+\tau)s\hat{r}_{II}^*}{9(1-\tau)^2 + (1-2\tau)(2-\tau)s} > 0.$$

Since $\hat{r}_{II}^* > -\frac{1}{2}$ by Claim 2,

$$\begin{aligned}\hat{q}_I^* &> \frac{1}{2b} \frac{9(1-\tau)^2 + (5(1-2\tau) + 3\tau^2)s + (1+\tau)s(-\frac{1}{2})}{9(1-\tau)^2 + (1-2\tau)(2-\tau)s} \\ &= \frac{1}{2b} \frac{18(1-\tau)^2s + 3(1-2\tau)(3-\tau)}{18(1-\tau)^2s + 2(1-2\tau)(2-\tau)} > 0.\end{aligned}$$

Finally, assume that $\hat{r}_{II}^* > \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)/2$. Then $\hat{\chi}_{II}^* = 0$ and Parent I 's profit maximization condition (12) yields

$$\hat{q}_I^* = \frac{2s + 1}{b(4s + 1)} > 0. \quad \square$$

Claim 5. $\hat{q}_I^* > 0$.

Proof. Assume by way of contradiction that $\hat{q}_I^* = 0$. Then $\hat{q}_{II}^* > 0$ by Claim 4, and $\hat{p}^* = d(\hat{q}_{II}^*)$ by equation (11).

Assume that $\hat{r}_{II}^* < \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)$. Then $\hat{\chi}_I^* = 0$, and therefore

$$\hat{\chi}_{II}^* = \frac{1 - \hat{p}^*}{2\beta} + \frac{\hat{p}^* - \hat{r}_{II}^*}{2\beta(1 - \tau)},$$

by equation (9). Since $\hat{q}_{II}^* > 0$, then Parent II 's profit maximization condition (13) yields the equations

$$\begin{aligned} \hat{q}_{II}^* &= \frac{2\beta(\tau - 1)^2 - b\tau(\hat{r}_{II}^* - \tau)}{b(b\tau^2 + 4\beta(1 - \tau)^2)}, \\ \hat{r}_{II}^* &= \tau(1 - b\hat{q}_{II}^*). \end{aligned}$$

Solving this system of equations, we get $\hat{q}_{II}^* = 1/(2b)$ and $\hat{r}_{II}^* = \tau/2$. Substituting these values into the equation for $\hat{\chi}_{II}^*$ above, we get $\hat{\chi}_{II}^* = 1/(2\beta)$. For these values, however, Parent I 's profit maximization condition (12) is

$$\hat{q}_I^* = \frac{1}{4b} > 0,$$

contradicting that $\hat{q}_I^* = 0$.

Assume that $\hat{p}^* - (1 - \tau)(1 - \hat{p}^*) < \hat{r}_{II}^* < \hat{p}^* - (1 - \tau)(1 - \hat{p}^*)/2$. Then $\hat{\chi}_I^*, \hat{\chi}_{II}^* > 0$. Using that formulae in Claim 1 and noticing that $\hat{q}_{II}^* > 0$ we can write Parent II 's profit maximization conditions as

$$\begin{aligned} \hat{q}_{II}^* &= \frac{9(1 - \tau)^2 s + 5(1 - 2\tau) + 3\tau^2 - 3(3 - 5\tau) + (1 - 5\tau)\hat{r}_{II}^*}{2b(9(1 - \tau)^2 s - (1 - 2\tau)(1 + \tau))}, \\ \hat{r}_{II}^* &= -\frac{1 - 3\tau}{2} + \frac{b}{4}(1 - 5\tau)\hat{q}_{II}^*, \end{aligned}$$

Solving this system of equations, we get

$$(\hat{q}_{II}^*, \hat{r}_{II}^*) = \left(\frac{2(1 - 2s)}{b(1 - 8s)}, \frac{3s - \tau(7s + 1)}{1 - 8s} \right).$$

Using again the equations for $\hat{\chi}_I^*, \hat{\chi}_{II}^*$ we get

$$(\hat{\chi}_I^*, \hat{\chi}_{II}^*) = \left(\frac{1+s}{\beta(1-8s)}, -\frac{6}{\beta(1-8s)} \right).$$

Hence either $\hat{\chi}_1^* < 0$ or $\hat{\chi}_{II}^* < 0$, which contradicts that $\hat{\chi}_1^*, \hat{\chi}_{II}^* > 0$.

Finally, assume that $\hat{r}_{II}^* > \hat{p}^* - (1-\tau)(1-\hat{p}^*)/2$. Then $\hat{\chi}_{II}^* = 0$, and therefore $\hat{\chi}_I^* = (1-\hat{p}^*)/2\beta$ by equation (8). Since $\hat{q}_{II}^* > 0$, Parent II 's profit maximization condition (13) yields $\hat{q}_{II}^* = 1/2b$. Then Parent I 's profit maximization condition (12) implies

$$\hat{q}_I^* = \frac{1}{2b} \frac{1+2s}{1+4s} > 0,$$

contradicting that $\hat{q}_I^* = 0$. \square

Claim 6. $\hat{\chi}_I^* > 0$.

Proof. Assume by way of contradiction that $\hat{\chi}_I^* = 0$. Then equation (9) yields

$$\hat{\chi}_{II}^* = \frac{1-\hat{p}^*}{2\beta} + \frac{\hat{p}^* - \hat{r}_{II}^*}{2\beta(1-\tau)} > 0,$$

where the inequality holds since $\hat{r}_{II}^* \leq \hat{p}^*$ by Claim 2. Since $\hat{q}_I^* > 0$ by Claim 5, the first-order condition for Parent 1's profit maximization yields

$$\hat{q}_I^* = \frac{1 - b\hat{q}_{II}^*}{2b},$$

and the first-order conditions for Parent II 's profit maximization yields the system

$$\begin{aligned} \hat{q}_{II}^* &= \max \left(0, \frac{(2(1-\tau)^2 s + \tau^2)(1 - b\hat{q}_I^*) - \tau\hat{r}_{II}^*}{b(4s(1-\tau)^2 + \tau^2)} \right), \\ \hat{r}_{II}^* &= \tau(1 - b(\hat{q}_I^* + \hat{q}_{II}^*)). \end{aligned}$$

Solving this system of equations, we get $(\hat{q}_I^*, \hat{q}_{II}^*, \hat{r}_{II}^*) = (1/3b, 1/3b, \tau/3)$. Hence $\hat{p}^* = 1/3$. Substituting these values into equation (8) yields

$$\hat{\chi}_I^* = \frac{1}{12\beta} > 0,$$

contradicting that $\hat{\chi}_I^* = 0$. \square

Claim 7. $\hat{\chi}_{II}^* > 0$.

Proof. Assume by way of contradiction that $\hat{\chi}_{II}^* = 0$. Then

$$\hat{\chi}_I^* = \frac{1-\hat{p}^*}{2\beta} > 0,$$

by equation (8). Since $\hat{q}_I^* > 0$ by Claim 5, Parent I 's profit maximization condition (12) yields

$$\hat{q}_I^* = \frac{(2s+1)(1-b\hat{q}_{II}^*)}{b(4s+1)}.$$

And since $\hat{\chi}_{II}^* = 0$, then the consolidated profit of Parent II is independent of r_{II}^* , and profit maximization condition (13) yields

$$\hat{q}_{II}^* = \max\left(0, \frac{1}{2b}(1-b\hat{q}_I^*)\right).$$

Solving the two equations above for $(\hat{q}_I^*, \hat{q}_{II}^*)$ we get

$$(\hat{q}_I^*, \hat{q}_{II}^*) = \left(\frac{2s+1}{b(6s+1)}, \frac{2s}{b(6s+1)}\right).$$

Using the values for $(\hat{q}_I^*, \hat{q}_{II}^*)$ we readily calculate

$$\begin{aligned}\hat{p}^* &= \frac{2s}{6s+1}, \\ \hat{\chi}_I^* &= \frac{4s+1}{2\beta(1+6s)}.\end{aligned}$$

Using the values of \hat{q}_I^* and $\hat{\chi}_I^*$ to calculate Parent II 's consolidated profit we calculate the first order condition for a solution to this firm's problem as

$$\begin{aligned}\hat{q}_{II}^* &= \frac{36(1-\tau)^2 s^2 + 2(5\tau^2 + 13\tau - 10)s - (1-\tau)(2-\tau) + (6s+1)(1-5\tau)\hat{r}_{II}^*}{2b(9(1-\tau)^2 s^2 - (1-2\tau)(1+\tau)s)(6s+1)}, \\ \hat{r}_{II}^* &= -\frac{2(5-13\tau)s + (1-\tau) + b(6s+1)(1-5\tau)\hat{q}_{II}^*}{4(6s+1)}.\end{aligned}$$

Solving this system, we get

$$\hat{q}_{II}^* = \frac{(10s+1) - 16s^2}{b(6s+1)(1-8s)} \neq \frac{2s}{b(6s+1)},$$

for all $s > 0$, which leads to a contradiction. \square

Claim 8. $\hat{q}_{II}^* = 0$.

Proof. Since $\hat{q}_I^*, \hat{\chi}_{II}^*, \hat{\chi}_I^* > 0$, by claims 5, 6 and 7, Claim 3 implies $\hat{q}_{II}^* = 0$. \square

We complete the proof of Proposition 3. Since $\chi_I^*, \chi_{II}^* > 0$ by claims 6 and 7, we can use the equations in Claim 1 to calculate the consolidated profits of Parent I ,

and derive the first order condition for an interior solution to its profit maximization problem. Solving this equation, we get

$$\hat{q}_I^* = \frac{9(1-\tau)^2 s + 5(1-2\tau) + 3\tau^2 + (1+\tau)r_{II}^*}{2b(9(1-\tau)^2 s + (1-2\tau)(2-\tau))}.$$

Likewise, using the equations in Claim 1 and noting that $\hat{q}_{II}^* = 0$ by Claim 8, we calculate the consolidated profits of Parent *II*, and identify the first order condition for an interior solution to its profit maximization problem. Solving this equation we get

$$\hat{r}_{II}^* = -\frac{1-3\tau}{2} + \frac{b}{4}(1-5\tau)\hat{q}_I^*. \quad (14)$$

Solving the system form by these two equations we get

$$\hat{q}_I^* = \frac{1}{b} \left(1 - \frac{12(s-h(\tau))(2-\tau)}{5-7\tau+24(1-\tau)s} \right), \quad (15)$$

$$\hat{r}_{II}^* = -\frac{(1-\tau)}{4} - \frac{3(s-h(\tau))(2-\tau)(1-5\tau)}{5-7\tau+24(1-\tau)s}. \quad (16)$$

Hence the price in the Latin market is

$$\hat{p}^* = \max\{1 - b\hat{q}_I^*, 0\} = \max\left\{ \frac{12(s-h(\tau))(2-\tau)}{5-7\tau+24(1-\tau)s}, 0 \right\}. \quad (17)$$

Assume that $s > h(\tau)$. Then the value for \hat{p}^* in (17) is positive. Substituting the values of \hat{r}_{II}^* and \hat{p}^* in the formulae of Claim 1 we get

$$\begin{aligned} \hat{\chi}_I^* &= \frac{1}{4\beta} \left(1 - \frac{1}{h(\tau)} \frac{3(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right), \\ \hat{\chi}_{II}^* &= \frac{1}{2\beta} \left(1 + \frac{1}{h(\tau)} \frac{(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right). \end{aligned}$$

Hence the price in the Greek market is

$$\hat{\pi}^* = 1 - \beta(\hat{\chi}_I^* + \hat{\chi}_{II}^*) = \frac{1}{4} \left(1 + \frac{1}{h(\tau)} \frac{(1+\tau)(s-h(\tau))}{5-7\tau+24(1-\tau)s} \right) > 0.$$

In order to verify the values for $(\hat{q}_I^*, \hat{r}_{II}^*, \hat{p}^*, \hat{\chi}_I^*, \hat{\chi}_{II}^*, \hat{\pi}^*)$ given by these formulae form an equilibrium we must show that $\hat{q}_{II}^* = 0$ maximizes Parent *II*'s consolidated profit. Given the value of \hat{q}_I^* above and the formulae of Claim 1 for the subsidiaries' outputs, we calculate the consolidated profit of Parent *II*. Taking derivatives with respect to Parent *II*'s output and transfer price of obtain yields the system of first-order

conditions

$$\begin{aligned}\hat{q}_{II}^* &= \frac{3(1-\tau)(36(1-\tau)^2 s^2 - (27 - (32 + 11\tau)\tau)s + \tau(\frac{29}{3} - \tau)) - 8}{2b(9(1-\tau)^2 s - (1-2\tau)(1+\tau))(24(1-\tau)s + (5-7\tau))} \\ &\quad + \frac{(1-5\tau)\hat{r}_{II}^*}{2b(9(1-\tau)^2 s - (1-2\tau)(1+\tau))}, \\ \hat{r}_{II}^* &= -\frac{3(3-7\tau)(1-\tau)s + 1 - \tau(2-3\tau)}{24(1-\tau)s + (5-7\tau)} + \frac{b}{4}(1-5\tau)\hat{q}_{II}^*.\end{aligned}$$

Solving this system, we get

$$\hat{q}_{II}^* = \frac{1}{b} \frac{4(12(1-\tau)s^2 - 2(5-4\tau)s - (1-2\tau))}{(8s-1)(24(1-\tau)s + (5-7\tau))}.$$

Since $s > 1/8$ by assumption, the denominator of this expression is positive. We show that the numerator is negative, thus establishing that the solution to Parent II 's problem involves $\hat{q}_{II} = 0$. Since $\tau \in [0, 1/2]$ and $s < l(\tau) = (2-\tau)/3(1-\tau)$ by assumption,

$$\begin{aligned}12(1-\tau)s^2 - 2(5-4\tau)s - (1-2\tau) &< (12(1-\tau)s - 2(5-4\tau))s \\ &< \left(12(1-\tau)\frac{2-\tau}{3(1-\tau)} - (10-8\tau)\right)s \\ &= -4\left(\frac{1}{2} - \tau\right)s \\ &< 0.\end{aligned}$$

Now assume that $s \leq h(\tau)$. Then in equilibrium $\hat{p}^* = 0$, and therefore $\hat{q}_I^* \geq 1/b$. Then the first-order condition for Parent II 's profit maximization yields

$$\hat{r}_{II}^* = -\frac{1-\tau}{4},$$

and therefore, using the formulae in Claim 1 we get

$$(\chi_{II}^*, \chi_{II}^*) = \left(\frac{1}{4b}, \frac{1}{2b}\right).$$

The equilibrium price in the Greek markets is

$$\hat{\pi}^* = \frac{1}{4}.$$

In order for $\hat{q}_{II}^* = 0$ to maximize the consolidated profits of Parent II taking as given $\hat{q}_I^* = \frac{1}{b}$, the system defined the first-order conditions that interior solution must

satisfy is

$$\hat{q}_{II} = -\frac{((1-\tau)(2-\tau) + (1-5\tau)\hat{r}_{II}^*)s}{2\beta 9(1-\tau)^2 s - (1-2\tau)(1+\tau)},$$

$$\hat{r}_{II} = -\frac{1-\tau}{4} + \frac{b}{4}(1-5\tau)\hat{q}_{II}^*,$$

which solution involves setting,

$$\hat{q}_{II} = \frac{s}{(1-8s)\beta},$$

which is negative since $s < 1/8$. Hence $\hat{q}_{II}^* = 0$.

Verifying the inequalities relating the surpluses generated in this SPE to those under vertical integration involves direct calculation. Assume that $s > h(\tau)$. Then

$$\hat{q}_I^* + \hat{q}_{II}^* = \hat{q}_I^* = \frac{2}{3b} \left(1 + \frac{2(2-\tau)}{l(\tau)} \frac{l(\tau) - s}{5 - 7\tau + 24(1-\tau)s} \right) > \frac{2}{3b} = 2q_{VI}^*,$$

and

$$\hat{\chi}_I^* + \hat{\chi}_{II}^* = \frac{2}{3\beta} + \frac{1}{3\beta} \frac{(2-\tau) - 3(1-\tau)s}{(5-7\tau) + 24(1-\tau)s} > \frac{2}{3\beta} = 2\chi_{VI}^*.$$

Assume $s \leq h(\tau)$. Then

$$\hat{q}_I^* + \hat{q}_{II}^* = \hat{q}_I^* = \frac{1}{b} > \frac{2}{3b} = 2q_{VI}^*,$$

and

$$\hat{\chi}_I^* + \hat{\chi}_{II}^* = \frac{1}{4\beta} + \frac{1}{2\beta} > \frac{2}{3\beta} = 2\chi_{VI}^*.$$

Hence in either case $\hat{S}^* > S_{VI}^*$ and $\Sigma^* > \Sigma_{VI}^*$.

Likewise, verifying the inequalities relating the industry's consolidated profits to those under vertical integration involves direct calculations. Assume that $s > h(\tau)$.

Then

$$\begin{aligned} \hat{C}_I^* + \hat{C}_{II}^* &= \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} \\ &\quad - \frac{(1-\tau)(2-\tau-3s(1-\tau))}{9bs} \frac{48(1-\tau)s^2 + (73-65\tau)s + 7 - 8\tau}{((5-7\tau) + 24(1-\tau)s)^2} \\ &< \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} \\ &= 2C_{VI}^*. \end{aligned}$$

Assume that $s \leq h(\tau)$. Then

$$\hat{C}_I^* + \hat{C}_{II}^* = \frac{1-\tau}{\beta} \frac{1}{16} + \frac{1-\tau}{\beta} \frac{1}{8} = \left(\frac{27}{32} \right) \frac{2(1-\tau)}{9\beta} < \frac{2(1-\tau)}{9b} + \frac{2(1-\tau)}{9\beta} = 2C_{VI}^*. \quad \square$$

Proof of Proposition 4. Using the results of Proposition 1, we calculate each firm's consolidated profit when both firms keep one set of books, to get

$$C_I^* = \frac{1-\tau}{\beta} \left(\frac{f(s)(1-f(s))s}{2} + \frac{(1+2f(s))(1-f(s))}{9} \right). \quad (18)$$

Likewise, using the results of Proposition 2 we calculate each firm's consolidated profits when both firms keep two sets of books, to get

$$C_{II}^* = \frac{1-\tau}{\beta} \left(\frac{(s-g(\tau))(2s+g(\tau))}{18s} + \frac{2}{25} \right) \quad (19)$$

if $s > g(\tau)$, and

$$C_{II}^* = \frac{1-\tau}{\beta} \left(\frac{2}{25} \right) \quad (20)$$

if $s \leq g(\tau)$. Finally, using the results of Proposition 3 we calculate the each firms' consolidated profit when they use asymmetric accounting policies. Define the functions

$$\begin{aligned} n(\tau, s) &= \frac{2(2-\tau)(s-l(\tau))}{l(\tau)(5-7\tau+24(1-\tau)s)}, \\ r(\tau, s) &= \frac{(1+\tau)(s-h(\tau))}{h(\tau)(5-7\tau+24(1-\tau)s)}. \end{aligned}$$

The consolidated profit of the firm keeping one set of books is

$$\hat{C}_I^* = \frac{1-\tau}{\beta} \left(\frac{2s}{9} (1+2n(\tau, s))(1-n(\tau, s)) + \frac{1}{16} (1+r(\tau, s))(1-3r(\tau, s)) \right) \quad (21)$$

if $s > h(\tau)$, and it is

$$\hat{C}_I^* = \frac{1-\tau}{\beta} \frac{1}{16} \quad (22)$$

if $s \leq h(\tau)$. The consolidated profit of the firm keeping two sets of books is

$$\hat{C}_{II}^* = \left(\frac{1-\tau}{\beta} \right) \frac{1}{8} (1+r(\tau, s))^2 \quad (23)$$

if $s > h(\tau)$, and it is

$$\hat{C}_{II}^* = \frac{1-\tau}{\beta} \frac{1}{8} \quad (24)$$

if $s \leq h(\tau)$. Hence, calculating the consolidated profits in each region involves the formulae described in the table below.

G	\hat{C}_{II}^*	C_I^*	C_{II}^*	\hat{C}_I^*
A_1	(23)	(18)	(19)	(21)
A_2	(23)	(18)	(20)	(21)
A_3	(24)	(18)	(19)	(22)
A_4	(24)	(18)	(20)	(22)

Table 2: Consolidated Profit Formulae

Using equation (24) to calculate \hat{C}_{II}^* we get

$$\hat{C}_{II}^* - C_I^* = -\frac{1 - \tau}{\beta} \frac{3(216s^2 + 117s + 16)s}{8(27s + 8)^2} < 0.$$

Hence $\hat{C}_{II}^* - C_I^*$ is negative in $A_3 \cup A_4$. Likewise, using (20) and to calculate C_{II}^* and (22) to calculate $C_{II}^* - \hat{C}_I^*$ we get

$$C_{II}^* - \hat{C}_I^* = \frac{1 - \tau}{\beta} \left(\frac{2}{25} - \frac{1}{16} \right) > 0.$$

Thus, $C_{II}^* - \hat{C}_I^*$ is positive in A_4 .

The red parabola in Figure 2 shows the locus of points in A for which $\hat{C}_{II}^* - C_I^* = 0$, where \hat{C}_{II}^* and C_I^* are calculated using equations (23) and (18), respectively, which are the relevant formulae in regions A_1 and A_2 . Inside (outside) the red parabola $\hat{C}_{II}^* - C_I^*$ is positive (negative). Thus, $\hat{C}_{II}^* - C_I^*$ is negative in A_2 and takes positive and negative values in A_1 .

The blue parabola in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (19) and (21), respectively, which are the relevant formulae in region A_1 . Inside (outside) the blue parabola $C_{II}^* - \hat{C}_I^*$ is positive (negative). Thus, $C_{II}^* - \hat{C}_I^*$ takes positive and negative values in A_1 .

The brown curve in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (20) and (21), respectively, which are relevant formulae in region A_2 . Above (below) the brown curve $C_{II}^* - \hat{C}_I^*$ is negative (positive). Since A_2 is below the brown curve, $C_{II}^* - \hat{C}_I^*$ is positive in A_2 .

The green curve in Figure 2 shows the locus of points in A for which $C_{II}^* - \hat{C}_I^* = 0$, where C_{II}^* and \hat{C}_I^* are calculated using equations (19) and (24), which are the relevant formulae in region A_3 . Above (below) the green curve $C_{II}^* - \hat{C}_I^*$ is positive (negative). Thus, $C_{II}^* - \hat{C}_I^*$ takes positive and negative values in A_3 .

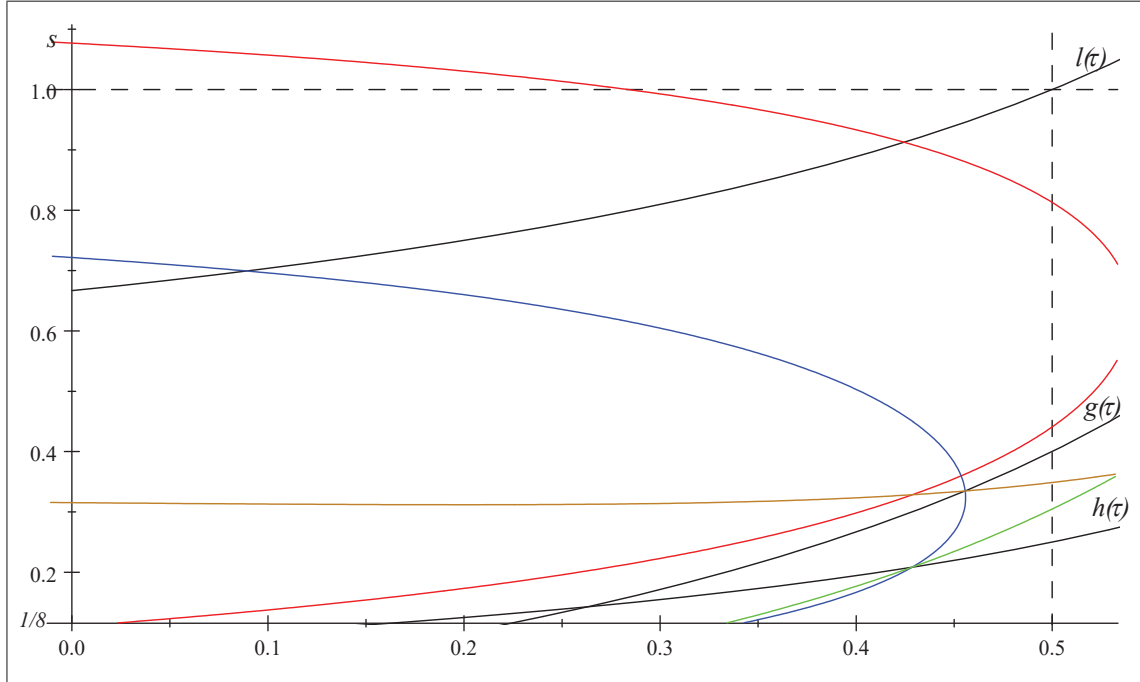


Figure 2: Profit Comparisons

Table 3 summarizes these results.

G	$\hat{C}_{II}^* - C_I^*$	$C_{II}^* - \hat{C}_I^*$
A_1	+/- (Inside/outside the red parabola)	+/- (Inside/outside the blue parabola)
A_2	-	+
A_3	-	+/- (above/below the green curve)
A_4	-	+

Table 3: Signs of Payoffs Differences in G

The results in Proposition 4 are directly implied by the signs given in Table 3. \square

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