Optimal coordination of directional overcurrent protections considering the occurrence probability of different configurations and the effect of grouping cases

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Abstract

This article shows the optimal coordination of directional overcurrent protections considering the occurrence probability of different system configurations and the effect of grouping cases. The optimization is formulated for specific subgroups of topologies to find the optimal Bundle of Protection Settings (BPS) for each subgroup. The number of BPS is limited by the available setting groups in the relays. The objective function is the average of main protection tripping times, taking the occurrence probability of each topology as weighting factor. A slowness index σ of each subgroup solution, regarding the optimal solution of a *topology of reference*, is computed. If σ is greater than a given threshold σ_0, some topologies must be taken off of the subgroup until obtaining σ less than σ_0. Thus, the computed settings are optimal for that subset of configurations, without an inadmissible increment of delays for main protections. The most probable topology should be in the first subgroup, and the process is repeated for the next subgroups until covering the total number of topologies. Some topologies could remain out of the solved optimization problems if the number of available BPS is low, and the trade-off between selectivity and speed should be solved for them. Numerical results for an example are shown, as an efficient way to explain the analyzed problems and solutions.

Keywords:
Directional overcurrent protection
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Effect of pre-fault system topology
Effect of grouping cases

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1. Introduction

The application of optimization to coordinate directional overcurrent protections (DOCP) was originally described in 1986 and 1988 [1,2]. Many papers on this topic have been written, often related to optimization tools (e.g., [3-11]). The inclusion of different system topologies in the problem formulation was considered from the beginning [1,2], but the analysis of this problem has been usually omitted. The adaptive solutions are attractive for this problem [12-31]; in this article, an adaptive solution means that protection settings can be changed (manually or automatically) according to the system topology, whereas an adaptive protection implies that the change is automatic. In general, the adaptive solutions in the coordination of protective relays have been proposed to improve desirable features of the protection system (e.g., speed, selectivity) by adapting the settings of the relays to changes in the power system. Nowadays, the possible use of adaptive solutions has been facilitated by the increasing availability of communication links between substations and control centers, but other additional topics must be considered in order to implement an adaptive solution in each specific real case, such as the security in the detection of the valid conditions to change the active setting groups of the relays, the certainty of changing the active setting groups of all the relays simultaneously, and the differences between the behaviour of relays from different manufacturers during the transition from an active setting group to another. Despite the need of solving the specific implementation details of adaptive solutions, the benefits of adapting the protection settings according to the system topology can be theoretically very important, from the perspective of protection speed and/or selectivity, and this fact typically justifies the analysis of this topic.

Many papers about the inclusion of different system topologies in optimal coordination of DOCP deal with adaptive protection in microgrids or in systems with distributed generators (e.g., [12-20]). These papers usually consider few changes in the system topology [12-17], mainly due to the normal grid operation modes or due to the different ratings of generators. These cases are quite different from the analysis of contingencies in traditional interconnected systems, featured by many different topologies (which complicates or makes infeasible the finding of solutions), whose probability of occurrence is very low since contingencies are not normal conditions. Some protection engineers consider that the coordination should be performed only for normal conditions, which are valid during almost all the time (i.e., from this perspective, the loss of selectivity during contingencies is undesired but admissible); whereas other protection engineers try to consider in some extent the coordination during contingencies because the lack of selectivity during a contingency increments the risk of general blackouts, affecting wider areas. Despite the fact of existing multiple possible criteria in the art and science of coordinating protective relays, the possibility of avoiding loss of selectivity during different possible system configurations can be attractive, especially if the coordination of the protections during the normal condition is not excessively degraded by this fact.

On the other hand, some examples related to microgrids and systems with distributed generators consider an important number of operation modes [18] or consider contingencies [19], but without considering the possible infeasibility of the optimization problem. Several papers have shown that including transmission and generation contingencies considerably increases the problem dimension, due to the high number of possible topologies [19-31]. A subset of those papers [19-22] only considers examples which have feasible solutions with all the possible topologies included in the coordination problem (but this is not a general case, as it has been theoretically shown [32]). Some papers show the infeasibility related to include all the topologies in the problem formulation [23-29,31]. A proposed way to avoid the infeasibility is the substitution of the incompatible constraints by penalties functions which can be added to the original objective function [23], but the violation of original constraints is not avoided. A proper adaptive solution which considers the relatively low number of available setting groups in the relays is proposed in [24], and the problem formulation for each set of setting groups only contains topologies whose constraints are not incompatible from each other. Some possible complements for this idea are: a) formulating a bi-objective problem, by combination of the maximization of protection speed and minimization of the number of required setting groups [25]; b) combining the individual setting groups to try to increment the number of available options [26]; c) using alternative clustering techniques to organize the topologies covered by each set of setting groups [27-30]. On the other hand, an optimal-probabilistic method is proposed in [30], and it is shown that the loss of selectivity could be allowed if the occurrence probability is not high. The idea of allowing some loss of selectivity, to consider other protection objectives and the occurrence probability of events, had been shown for distance relays [33], but not for DOCP. Solutions shown in [24] and [30] are valuable, and they are conceptually related to the method herein proposed because in this article: a) the final optimization problems for each bundle of setting groups only contain topologies whose constraints are not incompatible from each other, as in [24]; b) the most probable system configurations have priority for their inclusion in optimization problems and for their weights in the objective function (i.e., the fact of having topologies with low probability of occurrence is considered, as in [30]). That is, both valuable conceptual features have been merged in a practical and understandable way, and that is one of the main contributions of this article. From this perspective, Table I is useful to summarize the ways to solve this problem and to highlight one novelty feature of the method proposed in this article.

Table I. Summary of ways to solve the possible infeasibility of having solutions for the optimal coordination of DOCP with multiple topologies simultaneously included in the problem formulation.

Without avoiding the violation of the original constraints, and without considering the occurrence probability of each topology	(e.g., [23])
Without avoiding the violation of the original constraints, but considering the occurrence probability of each topology	(e.g., [31])
Avoiding the violation of the original constraints, using a small number of sets of settings groups, without considering the occurrence probability of each topology	(e.g., [24-30])
Avoiding the violation of the original constraints, using a small number of sets of settings groups, and considering the occurrence probability of each topology	[only this article]

This article shows that considering contingencies in optimal coordination of DOCP can imply infeasibility or excessive delays, and a novel solution method is proposed. The proposed method considers that the optimal solutions must be specified for subgroups of topologies and considers the probability of occurrence of each topology. A slowness index is defined, as the additional delay due to the grouping of different topologies, regarding the average operation time for a topology of reference, in order to keep under control those additional delays. That is, this slowness index is herein applied as a way to assess how much loss of speed is being accepted in order to obtain selectivity by considering other possible system topologies, and this is another contribution of this article. Conceptually, the extreme solutions for these problems are: a) optimal solution only for the normal configuration, without adaptive solutions, accepting the lack of selectivity in other configurations; b) individual optimal solutions for each topology, accepting the high number of adaptive solutions and assuming the feasibility of having enough setting groups in the relays. Thus, the proposed method is a proper tool to help to decide an intermediate solution, considering the usual low number of available setting groups and the topologies' occurrence probability.

The next Sections of this paper have been organized in the following way. Section 2 shows a summarized description of the problem of optimal coordination of DOCP. Section 3 describes the proposed method. Section 4 describes the system taken as an example for the solutions and explanations shown in Sections 5 and 6. Section 5 describes the possible consequences of using only one set of adjustments for the DOCP of the system relays, as a way to facilitate the understanding of the problem and the possible solutions. Section 6 shows different analyzed options to solve the problem, using two or more sets of adjustments for the DOCP of the system relays, as well as ways to compare these solutions from the perspective of the coordination of protections of electric power systems.

2. Fundamentals of optimal coordination of DOCP

The basic problem formulation was developed in [1,2]. For a given fault k, the tripping time t_{jk} of DOCP at backup relay j must be slower than the tripping time t_{ik} of DOCP at main relay i, considering a safety margin S_M to guarantee selectivity. A selectivity constraint can be formulated for each pair i-j:

$$t_{jk} - t_{ik} \geq S_M \qquad (1)$$

The tripping time of each DOCP depends on the current seen by each DOCP for the specific fault k, and on settings of pick-up current, curve type and time multiplier TM of each DOCP. Herein, it is assumed that the pick-up currents and curve types are selected before of solving the optimization problem. Thus, the tripping times are proportional to the correspondent TM values and the set of selectivity constraints is linear (complemented by the range of feasible settings: $TM_{MINi} \leq TM_i \leq TM_{MAXi}$). The basic problem formulation has been described in many papers (e.g., [1-31]), and more detailed descriptions according to the nomenclature of this article can be found in [34-36] (the details of the basic problem formulation can be easily found in the cited references and were herein omitted for the sake of brevity).

The objective function is the maximization of protection speed. Different functions can be formulated for this objective. Herein, the objective function is related to the minimization of the average tripping times of main protections for faults very near to them (details are shown in the next Section).

The coordination of DOCP has many interesting details [32] but the simultaneous inclusion of them would complicate unnecessarily this article. The following conditions were assumed for the sake of simplicity: a) Only phase DOCP are analyzed; b) Only 3-ϕ solid faults are considered; c) Instantaneous functions are not considered; d) Coordination with radial feeders is not considered [34]; e) Faults are simulated in transmission lines, very near to main relays (i.e., effect of fault location [35] was not considered); f) Transient configurations, due to sequential trips at both line ends [36,37], are not considered; g) Stability constraints [38] are not considered; h) Transient evolution of short-circuit currents [38] is not considered.

3. Description of the proposed method

2.1. Formulation of the optimization problem

A power system configuration is henceforth called *topology*, or simply *configuration*. The number of possible topologies is N_T, and it is herein assumed that all of them cannot be simultaneously considered in a single optimization problem (due to infeasibility or excessive delays). Thus, the topologies must be grouped, and a problem of optimal coordination of DOCP is formulated for each Group of Topologies (GT), considering all the selectivity constraints of that GT.

Considering the N_T topologies, the occurrence probability of each configuration is γ_c. Each GT has a specific number of configurations N_{GT}. The occurrence probability of one specific GT is Γ_{GT}. λ_c is the occurrence probability of the configuration c of that GT, in per unit of Γ_{GT}. That is, the sum of λ_c must be equal to 1 for each analyzed GT:

$$\Gamma_{GT} = \sum_{c=1}^{c=N_{GT}} \gamma_c \qquad (2)$$

$$\lambda_c = \gamma_c / \Gamma_{GC} \qquad (3)$$

$$\sum_{c=1}^{c=N_{GT}} \lambda_c = 1 \qquad (4)$$

The total number of analyzed relays is N_R. The tripping time of main DOCP i, for faults very near to it and for configuration c, is $t_{i,m,c}$. $N_{R,c}$ is the number of main relays which can trip for faults very near to them, for the system configuration c. The average of $t_{i,m,c}$ for that configuration is $t_{avg,m,c}$:

$$t_{avg,m,c} = (1/N_{R,c}) \sum_{i=1}^{i=N_{R,c}} t_{i,m,c} \qquad (5)$$

The average of $t_{avg,m,c}$ for the N_{GT} topologies of a GT, taking λ_c as weighting factor, is the objective function:

$$\boldsymbol{Min}\ t_{avg,m} = \boldsymbol{Min} \sum_{c=1}^{c=N_{GT}} \lambda_c\, t_{avg,m,c} \qquad (6)$$

Thus, the objective function is the average of tripping times of main protections, for faults very near to them, taking the occurrence probability of each topology as weighting factor. The computed optimal settings for the N_R relays are herein called the Bundle of Protection Settings (BPS) for that GT. The maximum number of BPS is N_{BPS}, and it is limited by the number of available setting groups in the applied relays.

3.2. Algorithm to define the groups of topologies

A configuration must be selected as *topology of reference* for each GT. An optimization problem is solved considering only the topology of reference and the corresponding optimal value of $t_{avg,m,c}$ is taken as reference (t_{REF}). The optimal value of the objective function considering the different topologies of the GT is t_σ. The Slowness Index of a Group of Multiple Arrangements (SIGMA, or simply σ) is defined as t_σ/t_{REF}. That is, σ indicates how slow the solution for the GT is, in per-unit of the optimal solution for the topology of reference.

If σ is greater than a given threshold σ_0, some topologies must be taken off of the GT, until the obtaining of σ less than σ_0. Thus, the selected settings are optimal for the resultant GT, without an inadmissible increment of t_σ regarding t_{REF}.

As many topologies as possible should be included in a GT (the number of available setting groups in relays is limited). The priority to be in the GT should be given to the topologies with higher probability of occurrence. The limit in the number of topologies in each GT is defined by the fact that σ must be lower than σ_0. After finding the optimal solution for the first GT, the optimal solution for the second GT is searched, and the process is repeated until N_{BPS} is covered ($n_{BPS}=N_{BPS}$).

Fig. 1 summarizes the algorithm. The condition $\sigma>\sigma_0$ is another constraint but has been kept out of the optimization because monitoring σ is useful to define possible actions (e.g., a change of σ_0). After finding a feasible option (reducing the number of topologies in the GT), the analysis of more grouping options could be still necessary due to the combinatorial nature of this problem.

A way to finish the algorithm is by reaching the maximum number of BPS ($n_{BPS}=N_{BPS}$). If all the topologies are included in the optimal solutions, there is no need of a special solution for any remaining topology; otherwise, settings for remaining topologies must be still selected. These remaining topologies: a) have low γ_c; b) cannot be included in the optimization related to any of the defined GT because σ would be excessive; c) cannot be selectively protected using any of the defined BPS. The solution for these remaining topologies implies a trade-off between speed and selectivity.

The algorithm can also be ended because all the topologies were included in the BPS without reaching the maximum number of BPS. In this case, the algorithm can be run again, with a lower σ_0, to take advantage of the available number of BPS in order to increase the speed of the final solutions.

The trade-off between speed and selectivity is not unusual in coordination of protections since the problem is conceptually multi-objective (speed and selectivity are two features to be maximized). Thus, the possible solutions conceptually imply Pareto frontiers (i.e., increase in selectivity implies decrease in speed). That is, speed and severity of loss of selectivity should be assessed for each option, in order to make the decision.

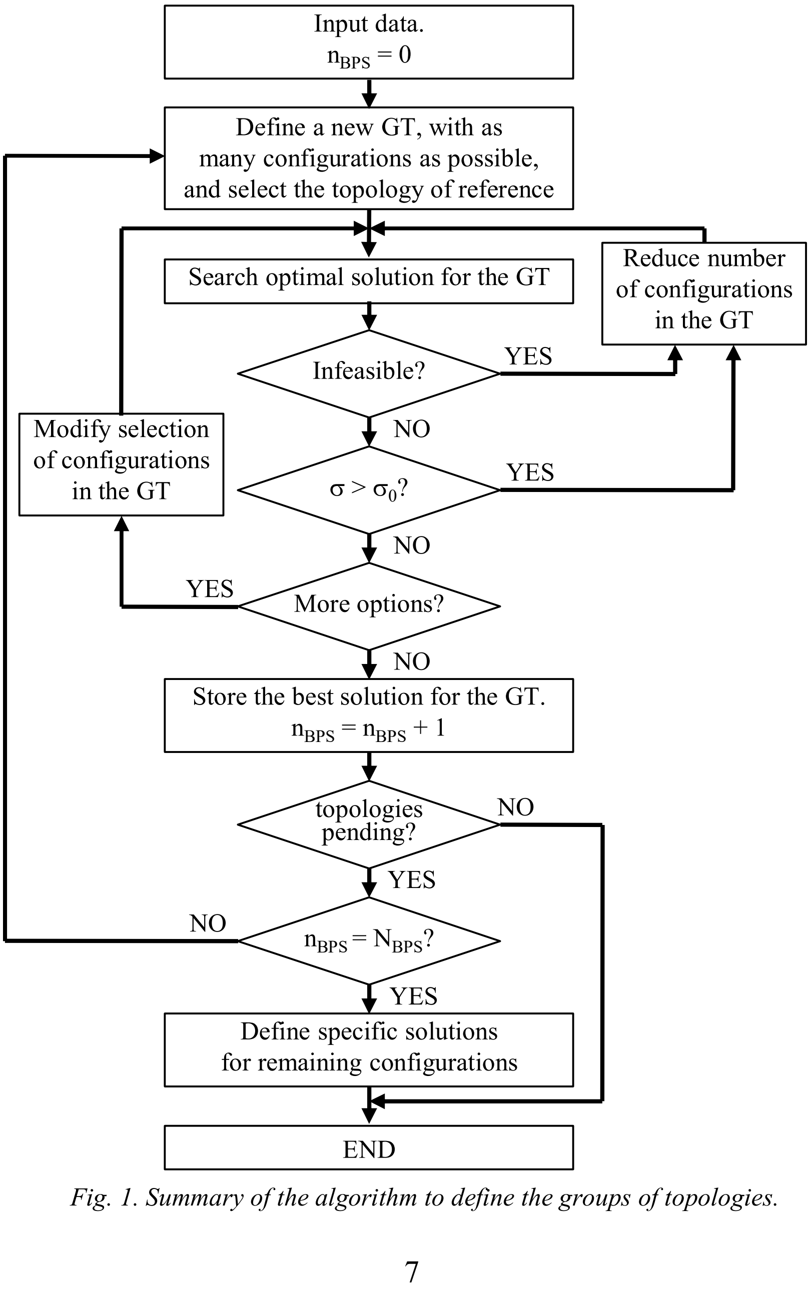

Fig. 1. Summary of the algorithm to define the groups of topologies.

3.3. Selection of the topology of reference

The topology of reference can be selected in different ways. For example, the base case can be selected as unique reference in a traditional interconnected system. Another option is the selection of a topology of reference for each GT. For example, the most probable configuration of all the analyzed ones can be the reference for the first GT, and the optimal solution is the first BPS ($n_{BPS}=1$). The most probable configuration of the remaining ones can be selected as the topology of reference for the second GT, and the process is repeated until $n_{BPS}=N_{BPS}$. This option could be preferred for microgrids or for systems with distributed generators since the speed of DOCP could be quite different depending on the connection or not to a strong system (because short-circuit levels can be quite different).

3.4. Suitability of this method to different problems of optimal coordination of DOCP

This method is suitable for optimal coordination of DOCP considering contingencies in interconnected power systems or considering different topologies in microgrids or systems with distributed generators. In traditional interconnected systems, the weight of contingencies in the objective function is often much lower than the normal system configuration weight. However, the normal configuration is only considered for the first BPS; that is, the weights of different topologies in the objective function can be similar to each other for $n_{BPS}\neq1$. In microgrids or in systems with distributed generators, the weight factors for the topologies in the objective function can be similar to each other even for $n_{BPS}=1$. The condition $\sigma<\sigma_0$ avoids an excessive increment of t_σ in both cases.

In systems with high-penetration of inverter-interfaced generation, the possible application of DOCP could face diverse inconveniences [39], such as low values of short-circuit currents and polarization issues. In such cases, the suitability of using DOCP should be assessed before formulating the problem of optimal coordination of DOCP.

4. System taken as an example

Fig. 2 shows the power system taken as an example. Optimal results with a set of pickup currents and NI curves are shown in [40] and were useful for an initial verification. However, those pickup currents should not be applied if contingencies are considered because some DOCP would trip during normal load conditions.

In general, the selection of pickup settings must consider all the possible topologies because a change of topology should not imply the risk of incorrect trip (i.e., the load current must be always lower than the pickup setting). Adaptive solutions can imply non-selective TM settings during the transition from a BPS to another BPS. The risk of incorrect trips due to this fact is low, because the probability of occurrence of a fault just during that transition is low. However, adaptive solutions must avoid the risk of incorrect trips due to incorrect sensitivity of pickup currents because this risk could be relatively high.

Sometimes the criteria to coordinate protections have been inherited from the past. For example, the pickup currents of electromechanical DOCP were traditionally selected greater than load currents, regardless of pre-fault load flow direction. The result of applying this procedure to the analyzed example is labeled as *1^{st} set* in Table I. In the original example (without contingencies [40]), the pre-fault load flow direction was considered, and the minimum pickup current was set in 120A. The result of an equivalent procedure, but considering contingencies, is labeled as *2^{nd} set* in Table I. Another option, slightly different, considers that the minimum pickup current is 180A instead of 120A, and it is labeled as *3^{rd} set* in Table I.

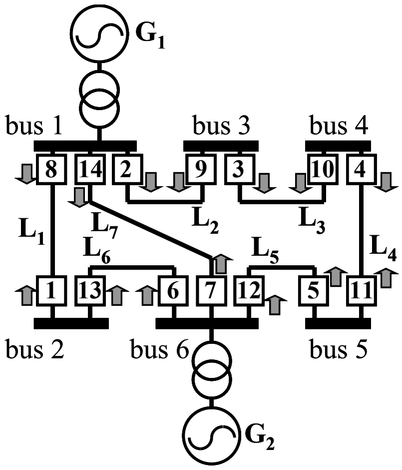

Fig. 2. System taken as an example (main data are shown in [40]).

Table I. Sets of pickup currents of DOCP to analyze the effect of contingencies in transmission and generation.

Relays	Pick-up of DOCP (A)		
	1^{st} set	*2^{nd} set*	*3^{rd} set*
R_1, R_{13}	240	120	180
R_5, R_9	1140	120	180
R_6, R_8	240	240	240
R_7, R_{14}	280	280	280
R_{11}	720	720	720
R_3	800	800	800
R_4	720	420	420
R_{10}	800	360	360
R_2, R_{12}	1140	1140	1140

Each DOCP could be set with an independent curve type, but it is assumed herein the same curve type in all the relays, for the sake of simplicity. Standardized IEC curves (Normal Inverse NI, Very Inverse VI, and Extremely Inverse EI) were considered. S_M is 0.3s, TM_{MINi} is 0.05, and TM_{MAXi} is 1.2.

All the contingencies in transmission lines and generators were considered, one by one (N-1 criterion). The probability of occurrence of a generator outage is considered to be much lower than the one of a transmission line outage. For the sake of simplicity, the occurrence probability γ_c is herein in per unit of the sum of γ_c (i.e., the sum of γ_c is equal to 1). The assumed values of γ_c are 0.929 for the normal condition, 0.01 for each line outage, and 0.0005 for each generator outage. The base case was firstly selected as topology of reference.

In case of a generator outage in the original example, the capacity of the remaining generator is lower than the system load. Thus, the load data were herein modified only to analyze generation contingencies (i.e., load data were not modified for other topologies): the original load data were simply divided by two for the cases of generation contingencies.

5. Solutions with only one BPS

The case with only one BPS is herein shown to facilitate the understanding of the problem and the possible solutions. This case (N_{BPS}=1) implies that the relays' settings will not be changed due to changes in the power system configuration.

5.1. Infeasibility and/or excessive delays related to selection of pick-up currents, considering only transmission contingencies

Considering the normal condition and all the transmission contingencies in one GT, the optimal results were obtained with the three sets of pickup currents (relaxing the constraints related to TM_{MAXi}). For this GT, Table II shows the results of t_{REF} and t_σ. The *1st set* of pickup currents must be discarded because implies excessive delays (t_{REF} is greater than 1 second and t_σ is greater than 1.2 seconds for all the curve types). The results with the *2nd set* and the *3rd set* of pickup currents are similar to each other, and results with NI and VI curves imply very slow protections in both cases, since t_σ is in the order of 1.2 and 0.5 seconds, respectively. Thus, the next results will be only obtained with EI curves.

The detailed results of TM are not included here for the sake of brevity. Such results would show that, with the *2nd set* of pickup currents and EI curves, two TM results are greater than 1.7. These values could be feasible, because some relays have extended range for TM_{MAX}, but the *3rd set* of pickup currents (whose results do not have that problem) has been herein preferred for the next results.

The relaxing of constraints related to TM_{MAXi} is convenient because some feasible practical solutions could be hidden in mathematically infeasible cases. In this example, the infeasibility is not inherent to incompatible constraints but mainly related to TM_{MAXi}. These results indicate that the selection of pickup currents can have an important effect on having infeasibility or excessive delays.

Table II. Optimal results of t_{REF} and t_σ (in seconds) with the pickup currents of Table I, considering the base case and all the transmission contingencies in the GT.

Case	NI		VI		EI	
	Base t_{REF}	$T_{B,T}$ t_σ	Base t_{REF}	$T_{B,T}$ t_σ	Base t_{REF}	$T_{B,T}$ t_σ
1st set	1.044	1.273	1.058	1.237	1.976	2.065
2nd set	0.473	1.308	0.192	0.505	0.105	0.257
3rd set	0.529	1.211	0.215	0.495	0.116	0.256

$T_{B,T}$: the normal condition and all the transmission contingencies in the GT.

5.2. Inclusion of generation contingencies

Table III shows the optimal results of t_{REF} and t_σ with the *3rd set* of pickup currents and EI curves, considering the base case, all the transmission contingencies and three options related to generation contingencies: none ($T_{B,T}$), only G1 ($T_{B,T,G1}$) , only G2 ($T_{B,T,G2}$) and both ($T_{B,T,BG}$). The values of σ are also shown in Table III. The selection of the threshold σ_0 depends on engineer criterion (i.e., this selection can be seen as part of the art and science of coordination of protections). σ_0 equal to 2 is used herein as an example. That is, the results of Table III do not satisfy the criterion about σ_0.

Table III. Optimal results of t_{REF} and t_σ (in seconds) with the 3rd set of pickup currents and EI curves, considering transmission and generation contingencies.

Base, t_{REF}	$T_{B,T}$, t_σ	$T_{B,T,G1}$, t_σ	$T_{B,T,G2}$, t_σ	$T_{B,T,BG}$, t_σ
0.116	0.256	0.260	0.261	0.327
σ=1	σ=2.20	σ=2.31	σ=2.31	σ=2.90

$T_{B,T,[G]}$: the base case, all the transmission contingencies, and the generation contingency [G] are in the GT.
[G] can be none, only G1, only G2, or both generation contingencies (BG).

5.3. Possible infeasibility intrinsically related to generation contingencies

Fig. 3 shows a simple system which was useful to explain that contingencies in generation can imply infeasibility in an intrinsic way [32]. For a fault in F1 without G2, R_4 should be slower than R_2, whereas for a fault in F2 without G1, R_2 should be slower than R_4. As the currents seen by the relays can be very similar in both cases, it is simply impossible to obtain simultaneously selectivity for both configurations [32].

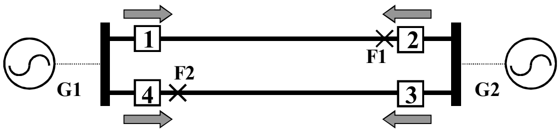

Fig. 3. Simple system that shows that generation contingencies can cause intrinsic infeasibility of obtaining selectivity for all the configurations [32].

Considering generation contingencies in the system shown in Fig. 3 seems similar to do that for the system shown in Fig. 2, since both systems have only two buses with generation. In Fig. 2, for a fault in L_1 very near to R_8, without G_1, R_7 should be slower than R_8, whereas for a fault in L_7 very near to R_7, without G_2, R_8 should be slower than R_7. However, the system shown in Fig. 2 is more meshed than the one of Fig. 3, and the current in the relay nearest to the fault is greater than the current in the backup relay. This fact helped to the obtaining of feasible solutions in the example shown in Fig. 2 when both generation contingencies were simultaneously included.

5.4. Examples of possible solutions

In order to obtain σ smaller than 2, only 6 transmission contingencies can be included in the GT, and there are two options: a) all the transmission contingencies except L7 outage, $T_{B,Tw/oL7}$, which is the option with lower σ; b) all the transmission contingencies except L2 outage. Table IV shows the detailed results for case $T_{B,Tw/oL7}$ and for all the combinations of this case with the inclusion of generation contingencies. Only one generation contingency can be included in the GT to obtain $\sigma < 2$. The settings of case $T_{B,Tw/oL7,G1}$ are selected for the first BPS because imply a lower σ than $T_{B,Tw/oL7,G2}$. Thus, the available number of BPS was reached (N_{BPS}=1) and some topologies were not included in the optimal results; thus, the solution implies a trade-off between speed and selectivity (as described in Section 3.2).

The computed BPS for $T_{B,Tw/oL7,G1}$ implies a moderate value of t_σ (since $\sigma < 2$) and the selectivity is not guaranteed for two contingencies (L7 outage or G2 outage). Table III shows that other possible selections are the solutions for: a) $T_{B,T}$, which implies a higher t_σ (σ=2.20) and the selectivity is not guaranteed only for the two less probable contingencies (G1, G2); b) $T_{B,T,G1}$ or $T_{B,T,G2}$, which implies a higher t_σ (σ=2.31) and the selectivity is not guaranteed only for a low probable contingency (G2 or G1). Another possible selection is the solution for $T_{B,Tw/oL7}$ because the disadvantage in comparison with the solution for $T_{B,Tw/oL7,G1}$ (the topology of G1 outage is not included in the GT) could be compensated by the speed improvement (σ is 1.58 instead of 1.92). The theoretical extremes of possible solutions are: a) the computed BPS for the base case, which implies maximum speed but the selectivity is not guaranteed for any contingency; b) the computed BPS for $T_{B,T,BG}$, which implies selectivity for all the analyzed contingencies but the speed is inadequately low.

Table IV. Optimal results with the 3rd set of pickup currents and EI curves, considering transmission and generation contingencies in the GT.

Case	Base	$T_{B,Tw/oL7}$	$T_{B,Tw/oL7,G1}$	$T_{B.Tw/oL7,G2}$	$T_{B,Tw/oL7,BG}$
TM_1	0.117	0.192	0.595	0.222	1.121
TM_2	0.051	0.088	0.088	0.089	0.091
TM_3	0.050	0.081	0.081	0.083	0.088
TM_4	0.050	0.050	0.051	0.059	0.077
TM_5	0.050	0.055	0.077	0.122	0.233
TM_6	0.525	0.569	0.795	0.586	1.090
TM_7	0.178	0.454	0.903	0.457	0.933
TM_8	0.460	0.511	0.522	0.751	1.000
TM_9	0.050	0.055	0.126	0.080	0.245
TM_{10}	0.067	0.073	0.090	0.079	0.118
TM_{11}	0.050	0.105	0.109	0.106	0.116
TM_{12}	0.050	0.093	0.094	0.093	0.097
TM_{13}	0.116	0.197	0.216	0.625	1.069
TM_{14}	0.177	0.468	0.473	0.949	0.981
t_σ	0.116	0.184	0.224	0.228	0.331
σ	1	1.58	1.92	1.96	2.84

$T_{B,Tw/oL7,[G]}$: the normal condition, all the transmission contingencies except L7 outage, and the generation contingency [G] are in the GT; [G] can be none, only G1, only G2, or both generation contingencies (BG).

6. Solutions with multiple BPS

6.1. Basic solutions with two BPS

The computed BPS for $T_{B,Tw/oL7,G1}$ is firstly assumed to be the solution for n_{BPS}=1. The topology correspondent to the L7 outage (T_{L7}) should be in the second GT since it has the highest probability of occurrence among the remaining ones. T_{L7} was selected as topology of reference for the second GT. Table V shows the optimal solutions for T_{L7} and for all the combinations of that case with the inclusion of generation contingencies (only EI curves are considered, because it was verified that the solutions with other curve types are also very slow for these GT). These results indicate that the second GT could be formed with the topologies with L7 outage and G2 outage ($T_{L7,G2}$), and all the topologies would be included in the optimal results for N_{BPS}=2. This solution is named 2A.

Another option is the selection of $T_{B,Tw/oL7,G2}$ for the first BPS and $T_{L7,G1}$ for the second BPS. The main drawback of this option is that $T_{B,Tw/oL7,G2}$ is slower than $T_{B,Tw/oL7,G1}$, and the most probable topology is covered by the first BPS. By coincidence, the solution for $T_{L7,G1}$ is also slower than the one for $T_{L7,G2}$; thus, the selection of $T_{B,Tw/oL7,G2}$ for the first BPS and $T_{L7,G1}$ for the second BPS is not justifiable (since solution 2A is obviously better).

The option of using $T_{B,Tw/oL7}$ for the first BPS is possible, since $T_{L7,BG}$ can be selected for the second BPS and σ is 1.39 (smaller than 2). The main advantage of this option is the speed improvement for the most probable topology, in comparison with using $T_{B,Tw/oL7,G2}$ or $T_{B,Tw/oL7,G1}$ for the first BPS. The main drawback of this option is the protection slowness for T_{L7}, in comparison with the topologies related to other transmission contingencies. This solution is named 2B.

A different solution with two BPS is described after the explaining of solutions with three BPS, for the sake of clearness.

Table V. Optimal results with the 3rd set of pickup currents and EI curves, considering only the L7 outage and the generation contingencies in the GT.

Case	T_{L7}	$T_{L7,G1}$	$T_{L7,G2}$	$T_{L7,BG}$
TM_1	0.556	0.556	0.556	0.670
TM_2	0.050	0.050	0.050	0.050
TM_3	0.050	0.050	0.050	0.050
TM_4	0.067	0.067	0.067	0.070
TM_5	0.130	0.130	0.130	0.146
TM_6	0.756	0.756	0.756	0.820
TM_7	-	0.560	0.050	0.560
TM_8	0.721	0.721	0.721	0.784
TM_9	0.127	0.127	0.127	0.146
TM_{10}	0.099	0.099	0.099	0.104
TM_{11}	0.058	0.058	0.058	0.059
TM_{12}	0.050	0.050	0.050	0.050
TM_{13}	0.558	0.558	0.558	0.670
TM_{14}	-	0.050	0.560	0.560
t_σ	0.218	0.255	0.252	0.326
σ	1	1.13	1.11	1.39

$T_{L7,[G]}$: The topology with L7 outage and the generation contingency [G] are in the GT; [G] can be none, only G1, only G2, or both generation contingencies (BG).

6.2. Solutions with three BPS

The previous solutions with two BPS guarantee selectivity for all the analyzed topologies. Thus, the availability of an additional BPS is an opportunity to obtain an improvement in the protection speed. An evident way to take advantage of the additional BPS is by the assignment of the first BPS (n_{BPS}=1) to the optimal solution for the base case (already shown in Table IV), because it is the most probable topology.

The second GT should include transmission contingencies because their probability of occurrence is much higher than those of generation contingencies. The topology with L3 outage was selected as topology of reference because it has the lowest optimal value of $t_{avg,m,c}$ (t_{REF}=0.115s), among the topologies related to transmission contingencies. The second GT cannot include all the transmission contingencies without obtaining σ greater than 2. In order to obtain σ smaller than 2, the best option is the inclusion of all the transmission contingencies except L7 outage ($T_{Tw/oL7}$). Table VI shows the detailed results for that case and for all the combinations of that case with the inclusion of generation contingencies. The results for the case of including all the transmission contingencies in the GT (T_T) is also shown in Table VI. Thus, an option is the selection of $T_{Tw/oL7}$ for n_{BPS}=2 and $T_{L7,BG}$ for n_{BPS}=3. This option is named 3A.

Considering that σ is practically in the limit for $T_{B,Tw/oL7,G1}$, the selection of $T_{Tw/oL7,G1}$ for n_{BPS}=2 and $T_{L7,G2}$ for n_{BPS}=3 could be considered acceptable. This option is named 3B.

Table VI. Optimal results with the 3rd set of pickup currents and EI curves, without considering the base case in the GT.

Case	$T_{Tw/oL7}$	$T_{Tw/oL7,G1}$	$T_{Tw/oL7,G2}$	$T_{Tw/oL7,BG}$	T_T
TM_1	0.192	0.595	0.222	1.121	0.762
TM_2	0.088	0.088	0.089	0.091	0.090
TM_3	0.081	0.081	0.083	0.088	0.086
TM_4	0.050	0.051	0.059	0.077	0.069
TM_5	0.055	0.077	0.122	0.233	0.140
TM_6	0.569	0.795	0.586	1.090	0.889
TM_7	0.454	0.903	0.457	0.933	0.461
TM_8	0.511	0.522	0.751	1.000	0.851
TM_9	0.055	0.126	0.080	0.245	0.137
TM_{10}	0.073	0.090	0.079	0.118	0.101
TM_{11}	0.105	0.109	0.106	0.116	0.112
TM_{12}	0.093	0.094	0.093	0.097	0.095
TM_{13}	0.197	0.216	0.625	1.069	0.790
TM_{14}	0.468	0.473	0.949	0.981	0.476
t_σ	0.184	0.232	0.237	0.346	0.255
σ	1.60	2.02	2.05	3.00	2.21

$T_{Tw/oL7,[G]}$: all the transmission contingencies except L7 outage, and the generation contingency [G] are in the GT; [G] can be none, only G1, only G2, or both generation contingencies (BG).

T_T: all the transmission contingencies are in the GT.

The base case can be grouped with a subset of transmission contingencies for the first BPS. For example, $T_{B,L2-5}$ includes the base case and topologies with L2, L3, L4 and L5 outages, and it is the selection for $n_{BPS}=1$. Table VII shows the optimal results for $T_{B,L2-5}$ and for two options for the remaining transmission contingencies (with or without L7 outage; $T_{L1,6,7}$ and $T_{L1,6}$, respectively). The optimal solution including both generation contingencies in a GT (T_{BG}) is also shown in Table VII. The topologies with lowest optimal value of $t_{avg,m,c}$ in each GT are selected as topologies of reference. A solution is the selection of $T_{B,L2-5}$ for $n_{BPS}=1$, $T_{L1,6,7}$ for $n_{BPS}=2$ and T_{BG} for $n_{BPS}=3$. This solution is named 3C.

Table VII. Optimal results with the 3rd set of pickup currents and EI curves, considering only generation outages in the GT.

Case	$T_{B,L2-5}$	$T_{L1,6,7}$	$T_{L1,6}$	T_{BG}	T_{L2-5}
TM_1	0.192	0.556	0.050	0.670	0.192
TM_2	0.088	0.050	0.050	0.050	0.088
TM_3	0.081	0.050	0.050	0.050	0.081
TM_4	0.050	0.067	0.050	0.050	0.050
TM_5	0.050	0.130	0.050	0.146	0.050
TM_6	0.569	0.756	0.050	0.586	0.569
TM_7	0.290	0.346	0.346	0.560	0.290
TM_8	0.510	0.724	0.050	0.560	0.510
TM_9	0.050	0.127	0.050	0.146	0.050
TM_{10}	0.050	0.099	0.071	0.056	0.050
TM_{11}	0.099	0.058	0.052	0.050	0.099
TM_{12}	0.091	0.051	0.050	0.050	0.091
TM_{13}	0.195	0.563	0.050	0.670	0.195
TM_{14}	0.294	0.350	0.347	0.560	0.294
t_σ	0.172	0.198	0.149	0.910	0.144
t_{REF}	0.116	0.148	0.148	0.451	0.115
σ	1.48	1.34	1.00	2.02	1.24

Topologies in the GT: base case and L2, L3, L4 and L5 outages ($T_{B,L2-5}$); L1, L6 and L7 outages ($T_{L1,6,7}$); L1 and L6 outages ($T_{L1,6}$); G1 and G2 outages (T_{BG}); L2, L3, L4 and L5 outages (T_{L2-5}).

6.3. Assessment tool for the options

If all the topologies are included in the optimal results for selecting the BPS, the selectivity is guaranteed and the options can be assessed using the objective function definition with γ_c, in per unit, replacing λ_c as weighting factor (i.e., with all the topologies, instead of a subgroup of them). Table VIII shows this assessment for the previous solutions. In case of using two BPS, solution 2A is worse than 2B due to the inclusion of a generation contingency in the same GT of the base case. In case of using three BPS, solution 3C is worse than 3A and 3B due to the inclusion of transmission contingencies in the same GT of the base case. Comparison between 3A and 3B is not evident; this tool is useful to see that the average of tripping times of main protections, taking the occurrence probability of each topology as weighting factor, is slightly smaller in 3A.

Table VIII. Use of the objective function to assess different options.

Option	2A	2B	3A	3B	3C
$\sum \gamma_c t_{avg,m,c}$	0.224	0.186	0.122	0.124	0.174

6.4. Alternative solutions with two BPS

The advantage of using the optimal solution of the base case for the first BPS is attractive, due to the high probability of occurrence of this topology. If the base case is selected for $n_{BPS}=1$ and only two BPS are available, some topologies must be out of the second GT in order to reach an optimal solution with $\sigma < 2$. That is, the improvement of speed for the base case is preferred in this option, despite the lack of selectivity for some topologies. If the solution for case $T_{Tw/oL7,G2}$ is selected for $n_{BPS}=2$, two topologies were not included in any optimal solution. Thus, each one of these two topologies must be assigned to one BPS, with the consequent lack of selectivity.

The effects of using each BPS in a remaining topology can be analyzed in order to make the selection. For example, for the topology with L7 outage: a) the value of $t_{avg,m,L7}$ is 0.150s using the first BPS, and 0.262s using the second BPS; b) the number of unsatisfied selectivity constraints is 8 using the first BPS and 6 using the second BPS. Thus, the first BPS could be assigned to the topology correspondent to L7 outage because this option is considerably faster and the lack of selectivity is not considerably greater, but there is not an evident selection. Other features could be included in the analysis; e.g., counting the number of cases with a smaller S_M, where the selectivity is probable although is not guaranteed.

6.5. Solutions with more BPS

Availability of more BPS is an opportunity to improve the protection speed. For example, in case of four BPS, a solution is the use of base case for $n_{BPS}=1$, $T_{L2\text{-}5}$ for $n_{BPS}=2$, $T_{L1,6,7}$ for $n_{BPS}=3$ and T_{BG} for $n_{BPS}=4$. $T_{L2\text{-}5}$ considers L2, L3, L4 and L5 outages in the GT, and the solution is shown in Table VII. The corresponding values of σ are 1, 1.24, 1.34 and 2.02.

7. Conclusion

A novel method for the optimal coordination of directional overcurrent protections considering the occurrence probability of different configurations and the effect of grouping cases was shown. The objective function is the average of main protection tripping times, taking the occurrence probability of each topology as weighting factor. A slowness index σ was defined for the solution of a subgroup of topologies, regarding the optimal solution of a topology taken as reference. The slowness index σ was applied as a way to assess how much loss of speed is being accepted to obtain selectivity in the other possible system topologies. An iterative algorithm to obtain the optimal settings for that subset of configurations, without an inadmissible increment of delays for main protections, was shown. Some topologies could be out of the solved optimization problems if the number of available BPS is low, and some ways to solve the trade-off between selectivity and speed for them are shown. Numerical results for a system taken as an example are shown, as an efficient way to explain the analyzed problems and solutions.

The proposed method is suitable for optimal coordination of DOCP considering contingencies in interconnected power systems, or considering different topologies in microgrids or in systems with distributed generators. The need of numeric values for the occurrence probability of each topology could be seen as a possible drawback of this method; however, even the use of approximated practical values could probably yield useful results. The influence of this topic can be analyzed in the future research, as well as the possible application of different clustering techniques and/or ways of considering the topologies' occurrence probability in the problem formulation. Furthermore, the application of the proposed method in larger systems is interesting for future research, especially if several configurations with high probability of occurrence could imply important difficulties to obtain a reasonably low number of subgroups of topologies.

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Nomenclature

DOCP: directional overcurrent protections

t_{ik}, t_{jk}: tripping time of DOCP at main and backup relays for a given fault k, respectively.

S_M: safety margin to guarantee selectivity.

TM: time multiplier of a DOCP

TM_{MINi}, TM_{MAXi}: minimum and maximum values of TM for the DOCP i, respectively (range of feasible settings).

GT: group of topologies to formulate one specific problem of optimal coordination of DOCP.

N_T: total number of analyzed topologies.

N_{GT}: number of topologies in a GT.

γ_c: the occurrence probability of each configuration c, considering the N_T topologies.

Γ_{GT}: The occurrence probability of one specific GT.

λ_c: the occurrence probability of the configuration c of one GT, in per unit of Γ_{GT}.

N_R: total number of analyzed relays.

$t_{i,m,c}$: tripping time of main DOCP i, for faults very near to it and for configuration c.

$N_{R,c}$: number of main relays which can trip for faults very near to them, for the system configuration c.

$t_{avg,m,c}$: average of $t_{i,m,c}$ for configuration c.

BPS: Bundle of Protection Settings for a GT; it is the set of adjustments of the N_R relays for that GT.

N_{BPS}: maximum number of BPS.

n_{BPS}: counter of the solved BPS in the algorithm to define the groups of topologies.

t_{REF}: optimal value of $t_{avg,m,c}$ for the topology taken as reference.

t_σ: optimal value of the objective function considering the different topologies of one GT.

SIGMA (or simply σ): Slowness Index of a Group of Multiple Arrangements.

σ_0: threshold for σ.

R_1 … R_{14}: relays of the system taken as an example.

NI, VI, EI: Normal Inverse, Very Inverse and Extremely Inverse, respectively.

$T_{B,T}$: case that considers the normal condition and all the transmission contingencies in the GT.

$T_{B,T,G1}$: case that considers the normal condition, all the transmission contingencies, and the contingency of generator G1 in the GT.

$T_{B,T,G2}$: case that considers the normal condition, all the transmission contingencies, and the contingency of generator G2 in the GT.

$T_{B,T,BG}$: case that considers the normal condition, all the transmission contingencies, and both generator contingencies in the GT.

$T_{B,Tw/oL7}$: case that considers the normal condition, all the transmission contingencies except L7 outage in the GT.

$T_{B,Tw/oL7,G1}$: case that considers the normal condition, all the transmission contingencies except L7 outage, and the contingency of generator G1 in the GT.

$T_{B,Tw/oL7,G2}$: case that considers the normal condition, all the transmission contingencies except L7 outage, and the contingency of generator G2 in the GT.

$T_{B,Tw/oL7,BG}$: case that considers the normal condition, all the transmission contingencies except L7 outage, and both generator contingencies in the GT.

T_{L7}: case that considers only the L7 outage in the GT.

$T_{L7,G1}$: case that considers the L7 outage and the contingency of generator G1 in the GT.

T_T: case that considers all the transmission contingencies in the GT.

$T_{Tw/oL7}$: case that considers all the transmission contingencies except L7 outage in the GT.

$T_{Tw/oL7,G1}$: case that considers all the transmission contingencies, except L7 outage, and the contingency of generator G1 in the GT.

$T_{Tw/oL7,G2}$: case that considers all the transmission contingencies, except L7 outage, and the contingency of generator G2 in the GT.

$T_{Tw/oL7,BG}$: case that considers all the transmission contingencies, except L7 outage, and both generator contingencies in the GT.

$T_{B,L2\text{-}5}$: case that considers the base case and topologies with L2, L3, L4 and L5 outages in the GT.

$T_{L1,6,7}$: case that considers topologies with L1, L6 and L7 outages in the GT.

$T_{L1,6}$: case that considers topologies with L1 and L6 outages in the GT.

T_{BG}: case that considers both generator contingencies in the GT.

Appendix: Data of the system taken as an example (data taken from [40]).

Each generator: 10kV, 150MVA, X=15%.

Each transformer: 10kV/150kV, 150MVA, X=4%.

Line impedances: L1 (0.4+j5)Ω; L2 (0.399+j4.998)Ω; L3 (0.4+j4.504)Ω; L4 (0.5+j4.5)Ω; L5 (0.495+j5.159)Ω; L6 (0.396+j4.5)Ω; L7 (0.5+j5)Ω.