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INTEGRATION VERSUS SEGMENTATION IN A DEALER MARKET [1]

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Abstract

This paper compares two trading mechanisms in a dealer market with several securities, asymmetric information and imperfect competition. These two market structures differ in the information received by market makers. While in the first of them they observe the order flows of all assets when setting prices, in the second setting market makers are assumed to observe the order flow corresponding to one security. In order to make this comparison, we analyze several market indicators such as the volatility and the informativeness of equilibrium prices and the unconditional expected profits of insiders under both regimes.

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1. Introduction

One of the most surprising phenomena in asset markets over the past decade has been the proliferation of new markets. This growth leads to the competition between them and, hence, it is natural to wonder which market designs will survive.

In this paper, we turn our attention to a specific issue in market performance, which is transparency. O'Hara (1995) defines it as the ability of market participants to observe the information in the trading process. She emphasizes that, despite of the simplicity of this definition, this issue is complex. One difficulty is to make concrete what information is observable. A second difficulty is to determine who can observe the information.

Building on this insight, we compare some properties of two trading mechanisms in the context of a static, imperfectly competitive multi-asset market. More precisely, we contrast two distinct multi-security extensions of the Kyle's (1985) framework purposed by Caballé and Krishnan (1994) and Bossaerts (1993). These two papers differ in the information observable by market makers. Whereas in the first work they observe the order flows of all the assets when they set prices, in the second one market makers can only observe the order flow of the security that they price. Notice that Caballé and Krishnan present an integrated financial market, while Bossaerts models a segmented market. This terminology is adopted since one can interpret that in the first paper a unique market for all the securities is considered, whereas in the second one there is one market for each security.

In order to compare the two trading mechanisms, we contrast some market indicators such as the volatility and the informativeness of equilibrium prices and the unconditional expected profits of insiders under both regimes. Concerning the volatility of prices, we show that the variances of the equilibrium prices of each security under both regimes are equal. In relation to the informational content of prices, we prove that the equilibrium price vector corresponding to the segmented trading mechanism is more informative about the payoff of an asset than the corresponding to the integrated one. With regard to the unconditional expected profits of insiders, we show that they are smaller in the integrated trading mechanism than in the segmented one.

Previous studies have examined how differences in information about the market itself influence the performance of the market. Madhavan (1992) analyzes how transparency affects to market behavior and viability. Biais (1993) examines how the transparency of the quotes affects spreads when there is no private information. Pagano and Röell (1996) compare the price formation in several trading systems, which differ in the degree of transparency. They analyze how transparency affects the trading costs for uninformed traders. Other papers (see, for instance, Röell (1990), Admati and Pfleiderer (1991) and Fishman and Longstaff (1992)) focus on the notion of transparency referred to the degree to which agents can trade anonymously. Our concept of transparency is related to market orders rather than to identities of agents. The present paper differs from all the previous ones in the fact that we examine a multi-asset framework.

The remainder of this paper is organized as follows. Section 2 outlines the notation and the hypotheses, which are common for both settings. Section 3 introduces the characteristics of the integrated mechanism. It states the unique linear equilibrium and derives some market indicators associated with it. Section 4 performs the same analysis for the segmented setup. Section 5 compares the market indicators of both mechanisms. Concluding comments are presented in Section 6. Finally, an extensive proof is included in the Appendix.

2. The common framework

We establish the following notation: if D is a $N \times N$ matrix, then $(D)_n$ will be its nth row and $(D)_{n,n'}$ will denote its (n,n') element, for any $n,n' \in \{1,\ldots,N\}$. If D is a symmetric positive definite matrix, then $D^{1/2}$ will mean the unique symmetric positive definite square root of D.[1] The superscript T either on a vector or on a matrix will denote its transpose.

Consider a financial market with N securities. Let $\tilde{v}$ be the payoff vector, $\tilde{v} = (\tilde{v}_1,\ldots,\tilde{v}_N)^T$, which has a multivariate normally *(MN)* distribution with mean vector $\bar{v}$ and a nonsingular variance matrix Σ_v. Three kinds of agents participate in the market: noise traders, informed investors and market makers. Noise traders demand a vector of random, inelastic quantities, not based on maximizing behavior, denoted by $\tilde{z} = (\tilde{z}_1,\ldots,\tilde{z}_N)^T$, which is $MN(\bar{z},\Sigma_z)$, where Σ_z is nonsingular.

There are K risk-neutral informed investors, indexed by k. These agents possess private information about the payoff vector. Let $\tilde{s}_k$ represent the vector of signals received by insider k and let $\tilde{\xi}_k$ denote his informational advantage, defined as $\tilde{\xi}_k = E(\tilde{v}|\tilde{s}_k) - \bar{v}$, which is $MN(\vec{0},\Sigma_\xi)$, where $\vec{0}$ denotes the zero vector and the variance matrix Σ_ξ is nonsingular.[2] In addition, we assume that for any $k,j \in \{1,\ldots,K\}$, with $k \neq j$,

[1] $D^{1/2} = E\Lambda^{1/2}E^T$, where $\Lambda^{1/2}$ is a diagonal matrix with the positive square roots of the eigenvalues of D in the diagonal, and E is an orthogonal matrix, whose columns are eigenvectors of D [see Bellman (1970)].

[2] As in Caballé and Krishnan (1994), and in contrast to Bossaerts (1993), instead of using the signals received by insiders, we perform the analysis with the informational advantages, since in this way we allow the dimension of the space of signals not to coincide with the number of securities of the market.

$\mathrm{cov}\left(\tilde{\xi}_k, \tilde{\xi}_j\right) = \Sigma_c$, where Σ_c is symmetric and positive definite. The random vectors $\tilde{v}$, $\tilde{\xi}_1, \ldots, \tilde{\xi}_K$ are assumed to be independent of $\tilde{z}$.

Market makers are risk-neutral and set the prices of the securities they trade, after observing their information set. We assume competition among market makers, so that this forces them to choose prices such that they earn zero expected profits.

3. The integrated mechanism

The framework of this section is the one proposed by Caballé and Krishnan (1994). Let $\tilde{x}_k = x_k\left(\tilde{\xi}_k\right)$ be the demand of insider *k*, which is a *N*-dimensional random vector. In this setup we suppose that market makers are able to observe the order flows of all the assets. Thus, the zero expected profits condition implies that the price vector $\tilde{p}$ satisfies

$$\tilde{p} = p(\tilde{\omega}) = E\left(\tilde{v} \middle| \tilde{\omega}\right), \text{ a.s.,} \qquad [1]$$

where $\tilde{\omega} = \sum_{k=1}^{K} \tilde{x}_k + \tilde{z}$ is the vector of order flows.

Definition 3.1. An equilibrium is a vector of strategies $x = \left(\left(x_1\left(\tilde{\xi}_1\right)\right)^T, \ldots, \left(x_K\left(\tilde{\xi}_K\right)\right)^T\right)^T$ and a price vector $\tilde{p} = p(\tilde{\omega})$ such that

1) for any $k \in \{1, \ldots, K\}$ and for any alternative vector of strategies x' differing from x only in the *k*th component, it holds that

$$E\left[\left(\tilde{v} - p\left(\sum_{j=1}^{K} x_j\left(\tilde{\xi}_j\right) + \tilde{z}\right)\right)^T x_k\left(\tilde{\xi}_k\right)\right] \geq E\left[\left(\tilde{v} - p\left(\sum_{j \neq k} x_j\left(\tilde{\xi}_j\right) + x'_k\left(\tilde{\xi}_k\right) + \tilde{z}\right)\right)^T x'_k\left(\tilde{\xi}_k\right)\right], \text{ and}$$

2) $\tilde{p}$ satisfies [1].

For the sake of tractability, we will focus on linear equilibria. Caballé and Krishnan (1994) provide the explicit characterization of the unique linear equilibrium in this setup, which is stated in the following result:

Proposition 3.1. There exists a unique linear equilibrium defined as follows:

$$p(\tilde{\omega}) = \bar{v} + \frac{\sqrt{K}}{2} A(\tilde{\omega} - \bar{z}) \text{ and } x_k\left(\tilde{\xi}_k\right) = B\tilde{\xi}_k \text{, for all } k,$$

where

$$A = \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2} \text{ and } B = \frac{1}{\sqrt{K}} A^{-1}\left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1} \text{, with}$$

$$M = \Sigma_z^{1/2} G \Sigma_z^{1/2} \text{ and}$$

$$G = \left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1} \Sigma_\xi \left(I + \frac{(K-1)}{2}\Sigma_\xi^{-1} \Sigma_c\right)^{-1}. \qquad [2]$$

Next, we are interested in analyzing some indicators of the performance of the financial market such as the volatility of the price vector and the revelation of information through it. The following result provides a property of the equilibrium price vector, which facilitates the computation of the mentioned market indicators.

Corollary 3.2. $\mathrm{var}(\tilde{p}) = \mathrm{cov}(\tilde{v}, \tilde{p})$.

Proof. It is easy to see that $\mathrm{cov}(\tilde{v}, \tilde{p}) = \mathrm{cov}(\tilde{v} - \tilde{p}, \tilde{p}) + \mathrm{var}(\tilde{p})$. Hence, all we need to prove is that $\mathrm{cov}(\tilde{v} - \tilde{p}, \tilde{p}) = 0$. However, notice that [1] and the projection theorem [see, for instance, Gourieroux and Monfort (1989)] provide the last equality. ■

Corollary 3.3. The volatility of $\tilde{p}$, measured by its variance matrix, is given by

$$K\left(2\Sigma_{\xi}^{-1}+(K-1)\Sigma_{\xi}^{-1}\Sigma_{c}\Sigma_{\xi}^{-1}\right)^{-1}.$$

Proof. Using corollary 3.2, the expression of $\tilde{p}$ stated in proposition 3.1 and doing some algebra, the desired result is obtained. ■

Corollary 3.4. The informativeness of the price vector I_p, measured by $\Sigma_v - \mathrm{var}(\tilde{v}|\tilde{p})$, is

$$K\left(2\Sigma_{\xi}^{-1}+(K-1)\Sigma_{\xi}^{-1}\Sigma_{c}\Sigma_{\xi}^{-1}\right)^{-1}.$$

Proof. From the normality assumption, we know that $I_p = \mathrm{cov}(\tilde{v},\tilde{p})(\mathrm{var}(\tilde{p}))^{-1}\mathrm{cov}(\tilde{p},\tilde{v})$, and using corollary 3.2, we get

$$I_p = \mathrm{var}(\tilde{p}). \qquad [3]$$

Finally, applying corollary 3.3, the desired result is obtained. ■

4. The segmented mechanism

The setting of this section is related to the one chosen by Bossaerts (1993). The superscript S refers to the segmented mechanism. In this framework market makers' quotes can depend only on the order flow of their own market. Using the zero expected profits condition, we obtain that, for any $n \in \{1,\ldots,N\}$, the selected price of the nth security satisfies

$$\tilde{p}_n^S = p_n^S(\tilde{\omega}_n^S) = E(\tilde{v}_n|\tilde{\omega}_n^S), \text{ a.s.,} \qquad [4]$$

where $\tilde{\omega}_n^S = \sum_{k=1}^{K}\tilde{x}_{k,n}^S + \tilde{z}_n$ is the order flow corresponding to the nth security.

Definition 4.1. An equilibrium is a vector of strategies $x^S = \left(\left(x_1^S(\tilde{\xi}_1)\right)^T, \ldots, \left(x_K^S(\tilde{\xi}_K)\right)^T\right)^T$ and a price vector $\tilde{p}^S = p^S(\tilde{\omega}^S)$ such that

1) for any $k \in \{1, \ldots, K\}$ and for any alternative vector of strategies x'^S differing from x^S only in the kth component, it holds that

$$E\left[\sum_{n=1}^{N}\left(\tilde{v}_n - p_n^S\left(\sum_{j=1}^{K} x_{j,n}^S(\tilde{\xi}_j) + \tilde{z}_n\right)\right) x_{k,n}^S(\tilde{\xi}_k)\right] \geq$$

$$\geq E\left[\sum_{n=1}^{N}\left(\tilde{v}_n - p_n^S\left(\sum_{j\neq k} x_{j,n}^S(\tilde{\xi}_j) + x'^S_{k,n}(\tilde{\xi}_k) + \tilde{z}_n\right)\right) x'^S_{k,n}(\tilde{\xi}_k)\right], \text{ and}$$

2) for all $n \in \{1, \ldots, N\}$, the nth component of $\tilde{p}^S$ satisfies [4].

Remark 4.1. Notice that the previous definition coincides with definition 3.1, except in the fact that now the price of an asset is contingent only in its own order flow.

The following result finds the unique linear equilibrium in closed form.

Proposition 4.1. There exists a unique linear equilibrium defined as follows:

$$p^S(\tilde{\omega}^S) = \bar{v} + \frac{\sqrt{K}}{2} A^S(\tilde{\omega}^S - \bar{z}) \text{ and } x_k^S(\tilde{\xi}_k) = B^S \tilde{\xi}_k, \text{ for all } k,$$

where A^S is a $N \times N$ diagonal matrix, with

$$(A^S)_{n,n} = \sqrt{\frac{(G)_{n,n}}{(\Sigma_z)_{n,n}}}, \text{ for all } n, \text{ and } B^S = \frac{1}{\sqrt{K}}(A^S)^{-1}\left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1}.$$

Proof. See Appendix. ■

It is important to point out that only the diagonal of Σ_Z is relevant in the equilibrium coefficients that we have just derived. This property follows from the fact that market makers cannot take into account the interactions between different components of the vector of demands of noise traders. Next, we derive the corresponding market indicators in the segmented mechanism.

Corollary 4.2. For any $n \in \{1,...,N\}$, $\mathrm{var}(\tilde{p}_n^S) = \mathrm{cov}(\tilde{v}_n, \tilde{p}_n^S)$.

Proof. It is left since it is very similar to the proof of corollary 3.2. ■

Corollary 4.3. The volatility of $\tilde{p}^S$, given by $\mathrm{var}(\tilde{p}^S)$, is the following matrix

$$\frac{K}{4}\left[\left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1}\left(\Sigma_\xi + (K-1)\Sigma_c\right)\left(I + \frac{(K-1)}{2}\Sigma_\xi^{-1}\Sigma_c\right)^{-1} + H\right],$$

where H is a $N \times N$ matrix with its element *(n,n')* given by the following expression:

$$(H)_{n,n'} = \sqrt{\frac{(G)_{n,n}}{(\Sigma_Z)_{n,n}}}(\Sigma_Z)_{n,n'}\sqrt{\frac{(G)_{n',n'}}{(\Sigma_Z)_{n',n'}}}, \text{ for any } n,n' \in \{1,...,N\}.$$

Proof. Using the expression of $\tilde{p}^S$ stated in proposition 4.1 and doing straightforward computations, the result is obtained. ■

Corollary 4.4. The informativeness of the price vector I_p^S is given by

$$K\left[\Sigma_\xi^{-1} + (K-1)\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1} + \left(\Sigma_\xi^{-1} + \frac{(K-1)}{2}\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1}\right)H\left(\Sigma_\xi^{-1} + \frac{(K-1)}{2}\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1}\right)\right]^{-1}.$$

Proof. From the normality assumption, we get that

$$I_p^S = \mathrm{cov}(\tilde{v}, \tilde{p}^S)\left(\mathrm{var}(\tilde{p}^S)\right)^{-1}\mathrm{cov}(\tilde{p}^S, \tilde{v}).$$

Taking into the last expression, corollary 4.3 and the formula of $\tilde{p}^S$ given in proposition 4.1, and doing some algebra, the desired result follows. ■

Remark 4.2. In contrast to the integrated setting, in the segmented one we cannot ensure that I_p^S and $\text{var}(\tilde{p}^S)$ coincide. Observe that corollary 3.2 was crucial in the derivation of [3]. However, in the segmented setup we do not obtain such a strong result, since corollary 4.2 only guarantees that the diagonals of $\text{cov}(\tilde{v}, \tilde{p}^S)$ and $\text{var}(\tilde{p}^S)$ are equal.

5. Comparison

Before comparing the volatility and the informativeness of prices, we show the following result, which will be crucial to prove corollaries 5.2 and 5.3.

Corollary 5.1. $\text{cov}(\tilde{v}, \tilde{p}) = \text{cov}(\tilde{v}, \tilde{p}^S)$.

Proof. From propositions 3.1 and 4.1, we get that the parts of $\tilde{p}$ and $\tilde{p}^S$ which are correlated with $\tilde{v}$ coincide. Therefore, the result is obtained. ■

The next corollary states that the volatility of each price is the same in both settings.

Corollary 5.2. For any $n \in \{1, \ldots, N\}$, $\text{var}(\tilde{p}_n) = \text{var}(\tilde{p}_n^S)$.

Proof. It immediately follows from corollaries 3.2, 4.2 and 5.1. ■

Concerning the revelation of information about the payoff of an asset through a price, the next corollary says that the informativeness of a price about the payoff of a security is the same in both market structures.

Corollary 5.3. For any $n, n' \in \{1, \ldots, N\}$,

$$(\Sigma_v)_{n',n'} - \mathrm{var}(\tilde{v}_{n'}|\tilde{p}_n) = (\Sigma_v)_{n',n'} - \mathrm{var}(\tilde{v}_{n'}|\tilde{p}_n^S) = \frac{(\mathrm{cov}(\tilde{v}_{n'}, \tilde{p}_n))^2}{\mathrm{var}(\tilde{p}_n)}.$$

Proof. It is easy to see that the normality assumption and corollaries 5.1 and 5.2 provide the previous two equalities. ■

Next, we will compare the volatility and the informativeness of the two price vectors. Concerning the first market indicator, note that corollary 5.2 tells us that $\mathrm{var}(\tilde{p})$ and $\mathrm{var}(\tilde{p}^S)$ have the same diagonal. However, in general we cannot ensure that these two matrices are identical. For instance, observe that the formula of $\mathrm{var}(\tilde{p})$ is independent of Σ_z, while the expression of $\mathrm{var}(\tilde{p}^S)$ may depend on it. On the other hand, the next corollary shows that $\tilde{p}^S$ is more informative about the payoff of an asset than $\tilde{p}$. This result follows from the combination of: i) $\tilde{p}$ is as informative about $\tilde{v}_n$ as $\tilde{p}_n$, ii) Corollary 5.3 and, iii) the fact that the revelation of information through $\tilde{p}_n^S$ is smaller or equal than $\tilde{p}^S$.

Corollary 5.4. For any $n \in \{1,\ldots,N\}$, $(I_p^S)_{n,n} \geq (I_p)_{n,n}$.

Proof. Fix $n \in \{1,\ldots,N\}$. From [3] we know that $(I_p)_{n,n} = \mathrm{var}(\tilde{p}_n)$. Combining corollaries 3.2 and 5.3, we can rewrite $(I_p)_{n,n} = (\Sigma_v)_{n,n} - \mathrm{var}(\tilde{v}_n|\tilde{p}_n^S)$. Furthermore, since $\mathrm{var}(\tilde{v}_n|\tilde{p}_n^S) \geq \mathrm{var}(\tilde{v}_n|\tilde{p}^S)$, we obtain that $(I_p)_{n,n} \leq (I_p^S)_{n,n}$ by the definition of I_p^S. ■

Finally, we compare the expected profits for an arbitrary informed investor obtained in both market structures, denoted by $E(\Pi)$ and $E(\Pi^S)$, respectively. The next result provides that insiders are better off in the segmented mechanism.

Corollary 5.5. $E(\Pi^S) \geq E(\Pi)$.

Proof. In order to show this result, it suffices to prove that $E\left[(\tilde{v}-\tilde{p}^S)^T \tilde{z}\right] \leq E\left[(\tilde{v}-\tilde{p})^T \tilde{z}\right]$ because we are in two zero-sum games and the market makers' expected profits are zero. Notice that since $E(\tilde{p}) = E(\tilde{p}^S) = \bar{v}$ and the fact that $\tilde{v}$ and $\tilde{z}$ are independent, the last inequality is equivalent to $-\sum_{n=1}^{N} \operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) \leq -\sum_{n=1}^{N} \operatorname{cov}(\tilde{p}_n, \tilde{z}_n)$, which will be claimed by proving that $\operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) \geq \operatorname{cov}(\tilde{p}_n, \tilde{z}_n)$, for any $n \in \{1,\ldots,N\}$. Note that, from the expression of $\tilde{p}^S$ given in proposition 4.1, it follows that

$$\operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) = \frac{\sqrt{K}}{2} (A^S)_{n,n} (\Sigma_Z)_{n,n},$$

which can be expressed as

$$\operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) = \frac{\sqrt{K}}{2} \sqrt{\operatorname{var}((A^S)_n \tilde{z})} \sqrt{\operatorname{var}(\tilde{z}_n)}. \qquad [5]$$

From propositions 3.1, 4.1 and corollary 5.2, it follows that $\operatorname{var}((A^S)_n \tilde{z}) = \operatorname{var}((A)_n \tilde{z})$. Hence, [5] provides that $\operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) = \frac{\sqrt{K}}{2} \sqrt{\operatorname{var}((A)_n \tilde{z})} \sqrt{\operatorname{var}(\tilde{z}_n)}$. On the other hand, from proposition 3.1, we have that $\operatorname{cov}(\tilde{p}_n, \tilde{z}_n) = \frac{\sqrt{K}}{2} \operatorname{cov}((A)_n \tilde{z}, \tilde{z}_n)$. Finally, comparing the last two expressions, one concludes that $\operatorname{cov}(\tilde{p}_n^S, \tilde{z}_n) \geq \operatorname{cov}(\tilde{p}_n, \tilde{z}_n)$. ■

Dennert (1993) argues that in relation to the welfare conclusion in a dealer market, one should be concerned to the welfare of noise traders, since these agents are the ones that do not have any private information and they trade for liquidity reasons. Thus, from corollary 5.5, we can ensure that if we only care about risk-neutral noise traders, then a stock exchange deciding to organize a dealer market should implement the mechanism that leads to an integrated financial market.

6. Conclusions

The objetive of this paper has been the comparison between two trading mechanisms in a dealer market, based in two distinct multi-security extensions of the Kyle's (1985) framework purposed by Caballé and Krishnan (1994) and Bossaerts (1993).

The main conclusion to which this analysis leads is that insiders can use the lack of transparency of the segmented mechanism to exploit their private information. On the one hand, this implies that insiders are better off in the segmented mechanism. On the other hand, this makes the vector of order flows (or, equivalently, the vector of prices) corresponding to the segmented trading mechanism to be more informative about the payoff of an asset than the corresponding to the integrated one.

Appendix

Proof of proposition 4.1. Notice that since we consider linear equilibria, we can write

$$\tilde{p}^S = A_0^S + A_1^S \tilde{\omega}^S \text{ and } \tilde{x}_k^S = C_k^S + B_k^S \tilde{\xi}_k, \text{ for all } k, \qquad [A1]$$

where A_0^S and C_k^S are N-dimensional vectors and A_1^S and B_k^S are $N \times N$ matrices, such that A_1^S is a diagonal matrix.

First, we consider the informed traders' decisions. By virtue of [A1], the quantities chosen by the kth insider $\tilde{x}_k^S$ solves the following optimization problem:

$$\max_{\tilde{x}_k^S} E\left[\left(\tilde{v} - A_0^S - A_1^S\left(\sum_{j\neq k}\left(C_j^S + B_j^S\tilde{\xi}_j\right) + \tilde{x}_k^S + \tilde{z}\right)\right)^T \tilde{x}_k^S \middle| \tilde{\xi}_k\right].$$

The first order condition of this problem is

$$E\left(\tilde{v}\middle|\tilde{\xi}_k\right) - A_0^S - A_1^S\bar{z} - A_1^S\sum_{j\neq k} C_j^S - A_1^S\sum_{j\neq k}\left[B_j^S E\left(\tilde{\xi}_j\middle|\tilde{\xi}_k\right)\right] = 2A_1^S\tilde{x}_k^S \qquad \text{[A2]}$$

and its second order condition implies the positive semidefiniteness of A_1^S. Using the assumptions of the informational advantages of insiders, we have that $E\left(\tilde{v}\middle|\tilde{\xi}_k\right) = \tilde{\xi}_k + \bar{v}$ and $E\left(\tilde{\xi}_j\middle|\tilde{\xi}_k\right) = \Sigma_c\,\Sigma_\xi^{-1}\,\tilde{\xi}_k$, for all $j \neq k$. Plugging the last two expressions into [A2], using the linearity assumption conformably with [A1] and operating, we get

$$A_1^S C_k^S = \bar{v} - A_0^S - A_1^S\bar{z} - A_1^S\sum_{h=1}^{K} C_h^S \text{ and} \qquad \text{[A3]}$$

$$I = A_1^S\left(2B_k^S + \sum_{j\neq k} B_j^S\,\Sigma_c\,\Sigma_\xi^{-1}\right). \qquad \text{[A4]}$$

In particular, observe that [A4] provides the nonsingularity of A_1^S.

Now, we will show that a linear equilibrium has to be symmetric in the strategies of insiders. Fix $k, k' \in \{1,\ldots,K\}$, with $k \neq k'$. Since [A3] and [A4] are satisfied for all $k \in \{1,\ldots,K\}$, we have

$$A_1^S C_{k'}^S = \bar{v} - A_0^S - A_1^S\bar{z} - A_1^S\sum_{h=1}^{K} C_h^S \text{ and} \qquad \text{[A3']}$$

$$I = A_1^S\left(2B_{k'}^S + \sum_{j\neq k'} B_j^S\,\Sigma_c\,\Sigma_\xi^{-1}\right). \qquad \text{[A4']}$$

Note that [A3], [A3'] and the nonsingularity of A_1^S imply that

$$C_k^S = C_{k'}^S. \qquad [A5]$$

On the other hand, using [A4], [A4'] and the nonsingularity of A_1^S, we get

$$2B_k^S + \sum_{j \neq k} B_j^S \Sigma_c \Sigma_\xi^{-1} = 2B_{k'}^S + \sum_{j \neq k'} B_j^S \Sigma_c \Sigma_\xi^{-1}.$$

Simplifying this expression and rearranging the resulting equation, we have

$$\left(B_k^S - B_{k'}^S\right)\left(2I - \Sigma_c \Sigma_\xi^{-1}\right) = 0. \qquad [A6]$$

However, it easy to see that $\text{var}\left(\widetilde{\xi}_k - \widetilde{\xi}_j\right) = 2\left(\Sigma_\xi - \Sigma_c\right)$, with $j \neq k$, which provides that $\Sigma_\xi - \Sigma_c$ is positive semidefinite. This implies that $2\Sigma_\xi - \Sigma_c$ is positive definite, and hence, we get that $2I - \Sigma_c \Sigma_\xi^{-1}$ is nonsingular. Consequently, from [A6], it follows that $B_k^S = B_{k'}^S$. Since both [A5] and the last equality hold for any $k, k' \in \{1, \ldots, K\}$, with $k \neq k'$, [A1] can be expressed as

$$\widetilde{p}^S = A_0^S + A_1^S \widetilde{\omega}^S \text{ and } \widetilde{x}_k^S = C^S + B^S \widetilde{\xi}_k, \text{ for all } k. \qquad [A7]$$

Hence, [A3] and [A4] can be written as

$$(K+1) A_1^S C^S = \overline{v} - A_0^S - A_1^S \overline{z}, \qquad [A8]$$

and $A_1^S B^S \left(2I + (K-1)\Sigma_c \Sigma_\xi^{-1}\right) = I$. Moreover, since A_1^S is nonsingular, we can solve the last equation for B^S as

$$\left(B^S\right)_n = \frac{1}{\left(A_1^S\right)_{n,n}} \left(\left(2I + (K-1)\Sigma_c \Sigma_\xi^{-1}\right)^{-1}\right)_n, \text{ for all } n. \qquad [A9]$$

We next determine the formula of $\tilde{p}_n^S$, for any $n \in \{1,\ldots,N\}$. Notice that $\tilde{\omega}_n^S = KC_n^S + (B^S)_n \sum_{j=1}^{K} \tilde{\xi}_j + \tilde{z}_n$, which is normally distributed. Computing $E(\tilde{v}_n | \tilde{\omega}_n^S)$, substituting the resulting formula into [4] and equating coefficients according to [A7], we get

$$\left(A_1^S\right)_{n,n} = \frac{K\left(\Sigma_\xi\right)_n \left(\left(B^S\right)_n\right)^T}{\left(B^S\right)_n \left(K\Sigma_\xi + K(K-1)\Sigma_c\right)\left(\left(B^S\right)_n\right)^T + \left(\Sigma_z\right)_{n,n}} \quad \text{and} \qquad [A10]$$

$$A_{0,n}^S = \bar{v}_n - \left(A_1^S\right)_{n,n}\left(KC_n^S + \bar{z}_n\right). \qquad [A11]$$

It remains to solve the system of equations formed by [A8]-[A11]. Plugging [A9] into [A10] and performing some algebra in the resulting equation, taking into account [2], we get that $(\Sigma_z)_{n,n}\left(\left(A_1^S\right)_{n,n}\right)^2 = \frac{K}{4}(G)_{n,n}$. The positive definiteness of Σ_z, G and A_1^S allows us to solve the last equality for $\left(A_1^S\right)_{n,n}$ as

$$\left(A_1^S\right)_{n,n} = \frac{\sqrt{K}}{2}\sqrt{\frac{(G)_{n,n}}{(\Sigma_z)_{n,n}}}. \qquad [A12]$$

Combining [A8] and [A11], we obtain that $\left(A_1^S\right)_{n,n} C_n^S = 0$. Therefore, $C_n^S = 0$. Hence, [A11] provides that $A_{0,n}^S = \bar{v}_n - \left(A_1^S\right)_{n,n}\bar{z}_n$. Taking into account that the last two equalities are satisfied for any $n \in \{1,\ldots,N\}$, [A7] provides that $\tilde{p}^S = \bar{v} + A_1^S\left(\tilde{\omega}^S - \bar{z}\right)$ and $\tilde{x}_k^S = B^S \tilde{\xi}_k$, for all *k*. Finally, using [A9], [A12] and the previous two formulae, we obtain the desired expressions. ■

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