Bidding with memory in the presence of synergies: a genetic algorithm implementation

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***Abstract*— a genetic algorithm has been developed to solve bidding strategies in a dynamic multi-unit auction: the Ausubel auction. The genetic algorithm aims to maximize each bidder's payoff. To this end, a memory system about past experiences has been implemented. An extensive set of experiments have been carried out where different parameters of the genetic algorithm have been used in order to make a robust test bed. The present model has been studied for several environments that involve the presence or absence of synergies. For each environment, the bidding strategies, along with their effects on revenue and efficiency, are analyzed.**

No theoretical predictions have been developed yet for this auction format when values involve synergies; therefore, the aim of this work is to study the auction outcome where theoretical predictions are unknown. The results obtained with the genetic algorithm developed in this research reveal that without synergies, bidders tend to bid sincerely. Nevertheless, in the presence of synergies, when bidders have memory about their past results, they tend to shade their bids.

I. Introduction

MULTI-UNIT auctions are widely used in different markets for selling goods such as Personal Communications Services (PCS) licenses, treasury bills, electricity, emission permits, etc. In multiple-object environments where individual bidders may demand more than one homogeneous item, the seller must choose among a wide variety of auction formats. In this paper the Ausubel auction in the presence and absence of synergies is analyzed. Without synergies, i.e., with substitute consumptions, where valuations are weakly decreasing, Ausubel demonstrated that sincere bidding by all bidders is an equilibrium, yielding to an efficient outcome [1]. Nevertheless, when this auction involves synergies or complementarities (when the value of multiple objects exceeds the sum of the objects' values separately), the theoretical equilibrium has not yet been developed. Therefore, the challenge of this research is to study this auction when super additive values are presented (synergies).

A recent method used for analyzing strategies on auctions with evolutionary games is by means of genetic algorithms (GAs). GAs were developed by [2] as a robust method of adaptive searching, learning and optimization in complex problem domains. The aim of this work is to find out whether dynamic strategies that maximize each bidder's payoff can be achieved automatically by means of GAs based on their past experiences. A great number of simulations have been run in order to study the bidders' behaviors and their effect on efficiency and revenues following the same research line as [3].

The methodology used in this computational experiment is quite similar to the one presented by Andreoni and Miller [4] for the analysis of different auction formats. These authors developed a GA model to capture the bidding patterns evident for the first and second price auctions. In the research presented in this work a GA has also been developed to study bidders' behavior. The main difference of the GA implemented is that it allows bidders to have memory about their past experiences in previous auctions. The results obtained reveal that bidders tend to bid sincerely in the Ausubel auction without synergies. However, in the presence of synergies, bidders learn in their past experience to underbid.

There are several markets where synergies can be found. Probably, the best known and recent one is the PCS license sales done in most countries (USA, UK, Germany, etc). Synergies among PCS licenses are classified according to [5] as local and global synergies: *"Local synergies are those gains in value that specifically arise from obtaining two or more geographically neighbouring licenses. Global synergies are those gains in value which accrue from obtaining increased numbers of licenses or markets: economies of scale or scope among multiple licenses which arise irrespective of their geographic locations."* Another market in which synergies play an important role is in procuring transportation services. Usually, it is better to have a group of continuous lanes or specific lanes that complement possible networks. Similar synergies can be found in many other markets, such as emission permits or oil lease auctions.

The presence of synergies can lead to aggressive bidding among participants (overbidding) as they are interested in acquiring more lots in order to get their super additive

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values. However, this strategy can involve an important risk. If bidders only win some of the demanded lots (instead of all of them) they can earn negative profits. According to the potential loss, bidders might keep away from overbidding to avoid exposure to such loss, i.e., to avoid the "exposure problem".

There is some previous work done in auctions with synergies in presence of the exposure problem [6-10]. Nevertheless, none of them include the use of GAs nor have tested the Ausubel auction format.

Most of these research works present a model where only some bidders have synergies for two items. The environment developed in this work is more complicated as all bidders present synergies for all the items in which they are interested. This assumption makes both the development of the model and the description of truthful bidding in the Ausubel auction quite complex.

The remainder of this article is structured in the following manner. The auction model is described in section 2. Section 3 deals with the experimental environments, the dynamic bidding strategies and the fundamentals of the bidding algorithm. Section 4 evaluates the experimental results by studying the bidders' strategy and the auction revenues and efficiency in the presence and absence of synergies. Finally, in section 5 the main conclusions and future work are presented.

II. Theoretical Framework

This paper focuses on an alternative ascending-bid auction developed by Ausubel [1]. In this auction, the auctioneer employs a price that starts at zero and increases continuously. For each price, p^l, each bidder i simultaneously indicates the quantity $q_i^l(p)$ he desires (demands are non-incremental in price). Bidders choose at what price to drop out of the bidding, with dropping out being irrevocable. When the price p^* is reached, such that aggregate demand no longer exceeds supply (or until the ending time is reached), the auction is over. When the auction is over each bidder i is assigned the quantity $q_i(p^*)$ and is charged the standing prices at which the respective objects were "clinched": " With m object for sale at a price p^l, bidder i clinches an object when the aggregate demand of all other bidders drops, at least, from m to $m-1$ but bidder i still demands two units or more, that is, when the demand of all bidders except him is smaller than the total supply but the total demand exceeds the total supply. In this situation bidder i is guaranteed at least one object no matter how the auction proceeds". In this way the auction sequentially implements the Vickrey rule that says each bidder pays the amount of the k^{th} highest rejected bid, other than his own, for the k^{th} object won.

In this paper, the model is presented for an independent-private-value (IPV) framework and bidders' value for each unit by itself is $v_{i,1}$ which is drawn independently and identically distributed from a uniform distribution with support [0, V], being V=100. However, earning multiple units might generate synergies. Thus a bidders' valuation function has been extended for multiple items from the one presented in [6] model (for two items). The value for bidder i of winning k units is equal to:

$$v_{i,k} = kv_{i,1} + \sum_{z=1}^{k-1} \alpha v_{i,z} \quad \text{for } \alpha \geq 0 \quad (1)$$

being α the synergy parameter. Therefore, the value for bidder 1 of winning the first and the second unit at the same time is equal to $v_{1,2} = 2v_{1,1} + \alpha\, v_{1,1}$. This function is very versatile and a good approach to the real world among firms and markets as it establishes, in an exponential way, the dependence level of earning multiple items, and allows having different synergy values.

In the presence of synergies, for the Ausubel auction format, the truthful bidding strategy has not been defined in previous works. The problem associated with super additive values is to decide how many items to bid on. This strategy has been proposed in the present work in the following manner. Assume that a bidder values 10 earning 1 unit, 30 earning 2 units and 60 earning 3 units. Then the situations described in Table I could be found. When the auction starts, bidders must decide how many units to ask for at each price. If bidders bid sincerely, they compare the standing price with their personal average values, including the synergy values. The average value for each bidder will differ depending on the number of items considered. The average value if bidder i earn k units is equal to:

$$v^a{}_{i,k} = \frac{v_{i,k}}{k} \quad (2)$$

If bidder i bids sincerely, he will demand k units until the price (p^l) equals the average value ($v^a{}_{i,k}$). In order to have a better understanding of the Ausubel auction when synergies are presented (an Appendix is included with an example where bidders bid sincerely with $\alpha = 0.4$).

TABLE I
BIDDING STRATEGIES IN AN AUSUBEL AUCTION THAT INVOLVES SYNERGIES

Possible bids for a bidder who values 10 earning 1 unit, 30 earning 2 units and 60 earning 3 units.

Price	Demand (q_i^l)	Outcome
Up to 10	3 units and then 0	Profits of earning 3 items for a price of 10 or less is ≥30
		Profits of earning 2 items for a price of 10 or less is ≥10
		Profits of earning 1 items for a price of 10 or less is ≥0
		This is the safer bid because the bidder will never have losses. Nevertheless this bid does not reflect the real value of the bidder including the synergies. Following this strategy means that the bidder has less probability of obtaining the super additive value of earning the 3 items.
Up to 15	3 units and then 0	Profits of earning 3 items for a price of 15 or less is ≥15
		Profits of earning 2 items for a price of 15 or less is ≥0
		Profits of earning 1 items for a price of 15 or less is ≥-5
		With this strategy the bidder includes the synergy value for the second unit and therefore has a higher probability of earning the 3 items. Nevertheless, he will be affected by the exposure problem if he only earns one item
Up to 20	3 units and then 0	Profits of earning 3 items for a price of 20 or less is ≥0
		Profits of earning 2 items for a price of 20 or less is ≥-10
		Profits of earning 1 items for a price of 20 or less is ≥-10
		Demanding 3 units up to a price 20 implies that the bidder bids according his true value including the synergies for all the items and he has the highest probability of earning the three items. However, he faces the exposure problem if he only earns 1 or 2 items.

The sincere bidding strategy: this strategy is the only one that includes in the bid the real value incorporating the synergies. Therefore, this is the strategy considered sincere bidding for this work. This strategy implies bidding for the highest quantity until the price reaches the average value of earning all the units including the synergies (60 / 3 = 20). Once this price is reached, the bidder will make his minimum possible bid.

The overbidding strategy: this strategy assumes that the bidder will continue bidding for 3 items even after reaching the price of 20. The problem with this strategy is that the bidder will have more probabilities to incur losses.

The underbidding strategy: with this strategy the bidder will demand less than 3 items before reaching the price of 20. The benefits of this strategy are that he has a higher probability of not having losses, but the bidder also reduces the chances of winning all the items.

In each auction the seller offers m number of indivisible units of a homogeneous good to n number of bidders. Each bidder i obtains a total value of $v_{i,k}$ for the k^{th} units earned, for k=1,..,m. Thus if bidder i gets q^*_i units for a total payment of P^*_i, he obtains the following payoff:

$$v_{i,q_i^*} - P^*_i \text{, for } i = 1,.., n, \text{ and } q^*_i = 1,..,m, \quad (3)$$

where $v_{i,q_i^*} = q_i^* v^a_{i,q_i^*}$ and P^*_i is defined below in (8).

For any round l, the aggregate demand by all bidders is:

$$Q^l = \sum_i q_i^l$$

According to [1] definitions, the vector of cumulative clinches C_i^l of bidder i at prices up to p^l is defined by:

$$C_i^l = \max\left\{0, M - \sum_{j \neq i} q_j^l\right\} \quad (4)$$

for l= 1, ..., L-1 and i =1,...,n., and,

$$C_i^L = q_i^* \quad (5)$$

where L is the last auction round and q^*_i is the final quantity assigned to bidder i.

Moreover, the vector of current clinches at round l, c^l_i, is the difference between the cumulative clinches at time l and the cumulative clinches at time l-1, i.e.,

$$c_i^l = C_i^l - C_i^{l-1} \quad (6)$$

for l= 1, …, L and $c_i^0 = C_i^0$, for all i =1, … ,n.

The auction outcome associated with any final history l=L is defined in (7) and (8).

$$\text{Allocation: } q_i^* = C_i^L \text{, for } i = 1, \dots, n \text{, and,} \quad (7)$$

$$\text{Payment: } P_i^* = \sum_{l=0}^{L} p^l c_i^l \text{, for } i = 1, \dots, n \quad (8)$$

In the same way, the revenue per auction of the seller is defined as the total payments made by all bidders (9).

$$\text{Revenues: } R^* = \sum_{i=1}^{n} P_i^* \quad (9)$$

As the price goes up and demanded quantities goes down, it is possible that, for a certain increase in price, the supply might not be covered at the final price (Q^L<M). Under these circumstances a rationing rule is introduced. In this paper the proportional rationing rule is considered. Nevertheless, it will not be possible to remove units that were already clinched in previous rounds.

In the experiments done, bidders are informed about the four last auctions results, so they use this information in order to decide their future bids.

III. THE GA THAT MAXIMIZE THE BIDDERS' PAYOFF

GAs are powerful heuristic search strategies based upon a model of organic evolution, [11]. In this work, all bidders bid according to the GA against their computer rivals learning from one auction to another. Each individual plays the strategy determined by the GA; and after an auction simulation each one receives a score that represents the payoff of the auction (evaluation function). However, we observed that real-life bidders base their strategies on their experience, so we decided to include memory in the evaluation function (evaluation as an arithmetic mean of the last payoffs). Thus, successful present and past strategies have more probability of appearing in the offspring (selection) and players who fared well in the four previous rounds will transmit the information to the other participants playing in the following. Those who fared poorly in the last rounds will observe which bidding strategies succeeded better for the winners and will develop alternatives based on them.

Here, as in many aspects of game theory, there is not one global goal for all the bidders of the system to operate in. Instead players have their own goal: to have their own greatest payoff, see equation (3). Therefore, the final result of such an evolution is not the final convergence in one optimal solution; instead GAs are used to study the bidder's behavior and the bidding tendencies under all the different possible situations. The study of these results is quite motivating from two different points of view:

-- From the point of view of the bidders: to know which strategy yields the highest expected profit, and to have information concerning the risk of playing different alternatives.

-- From the point of view of the auctioneer: to know whether the auction rules used to sell their goods are appropriate in terms of revenues and efficiency.

A. *The experimental environments*

Experiments have been designed with fifteen lots auctioned (m=15) and four bidders (n=4) who are risk neutral and have no budget constraints (all variables are considered discrete).

All bidders have independent-private-values (IPV) that are privately observed by the respective bidders, making this an incomplete information game. As was described in section 2, each bidder's value for each unit by itself is $v_{i,1}$ which is drawn independently and identically distributed from a uniform distribution with support [0, V], being V=100. The value of earning multiple units is defined in (1). In the absence of synergies, the synergy parameter is equal to zero. In the presence of synergies, several parameters have been tested: α=0.2, 0.4, 0.6, 0.8 and 1. The bidder' values are selected randomly but they keep constant in all the 10,000 auctions run in each experiment and among all the environments. This is done in pursuit of equity in the comparison made with the results.

B. *Bidding strategy*

In each auction participants must decide how many items to demand at the standing price. Each bidder privately knows his personal value for earning one object ($v_{i,1}$) or multiple items ($v_{i,k}$) so he will choose among three global strategies: underbid, overbid or bid sincerely. In order to allow the GA to test a wide range of candidate solutions, a bidding function has been defined. This function allows bidders to underbid or overbid more or less aggressively, or, to bid sincerely modifying a single parameter (a). With such a wide range of strategies, this function can reflects the complex performance of bidders. The bidding function considered is defined as follows:

$$b_{i,k} = av_{i,k} + \frac{1}{2}(1-a)V \text{ , for } 0 \le a \le 1.5 \quad (10)$$

which is symmetric around ½ V. In the experimental environment tested, the parameter a ranges from 0 to 1.5 with intervals of 0.1. This range value allows bidders to test the three global strategies with different intensities for a certain value of $v_{i,k}$. The GA searches for the best value of the parameter a which maximizes each bidder's payoff of the last four auctions independently. Nevertheless, $b_{i,k}$ is the total bidding value for k units but bidders must decide the number of units to be demanded at each price. To this end the average bidding value is calculated:

$$b^a{}_{i,k} = \frac{b_{i,k}}{k} \quad (11)$$

Bidders will demand k units until the price in each round (p^l) reaches the average bidding value ($b^a{}_{i,k}$). All these strategies have an upper bound that is the lowest of either the number of units being auctioned or, alternatively, the units demanded in the previous round (as demand is required to be non increasing). The lower bound is the number of units that the participant has already clinched. Some strategies could possibly lead to non-integer numbers. In these circumstances the GA rounds up.

C. *The genetic algorithm mechanism*

The objective function that the GA tries to maximize is the payoff to each bidder according to (3). In order to achieve this aim, bidders have memory of their past experience so they will decide their bid taking into account the results of the last four auctions. Encoding the bidding strategies is a direct process. As the value of a ranges from 0 to 1.5, with precision 0.1, we have 16 possible actions for each bidder, which means encoding the genetic individuals with 4 bits.

The assessment of the GA is done running 10,000 auctions per experiment and 50 runs for each mutation rate: 0.01 and 0.05. The simulations were performed with 4 individuals, a roulette wheel selection mechanism, a single point crossover and a flip bit mutation operator. All auctions have the same bidder' values and 15 items to be auctioned. The parameters used in the experiments are reported in Table II.

The GA used in this research is the "simple" GA that Goldberg describes in his book, [12] using the GAlib library developed by Mathew Wall at the MIT (http://lancet.mit.edu/galib-2.4/). It uses non-overlapping populations, which means that in each generation the algorithm creates an entirely new population of individuals by selecting from the previous population and then mating to produce the new offspring for the new population. This process continues until the stopping criteria are met (number of auctions = 10,000). Elitism is on, meaning that the best individual from each generation is carried over to the next generation.

TABLE II
PARAMETERS OF THE GA USED FOR THE EXPERIMENTS

GA type	Simple GA
Population	4 individuals
Encoding	Vector of 4 bits
Selection	Roulette Wheel
Crossover	Positional single point random between [1..3]
Mutation	Flip bit Change each bit with probability: [0.01, 0.05]

Often the objective scores must be transformed in order to help the GA to maintain diversity or differentiate between very similar individuals. In this case, as the payoffs can be very similar, the linear scaling method is used to adapt values of the payoffs to the fitness values. Linear scaling transforms the objective score based on a linear relationship using the maximum and minimum scores in the population as the transformation metric, [12].

IV. EXPERIMENTAL RESULTS

A. Results without synergies (α =0)

In the absence of synergies, Ausubel demonstrated that sincere bidding by all bidders is equilibrium, yielding to an efficient outcome [1]. As it is reported in Table III, the results obtained in this research support this fact. For the 50 experiments done (each experiment of 10,000 auctions) for both mutation rates (0.01 and 0.05) bidders tend to do truthful bidding. Although they have 16 possible strategies, they bid sincerely, this is, a=1 in 87.4% and 55.6% of the times, for each mutation rate. Hence, it yields an outcome close to full efficiency (0.989 and 0.962) and the seller's revenues are much the same as if all bidders always bid sincerely (97.5% and 93.2%). When we compare the GA performance for both mutation rates, we find out that the GA behaves better for the lowest mutation factor. This is because higher mutation factor means that the bidding strategies change randomly with more frequency.

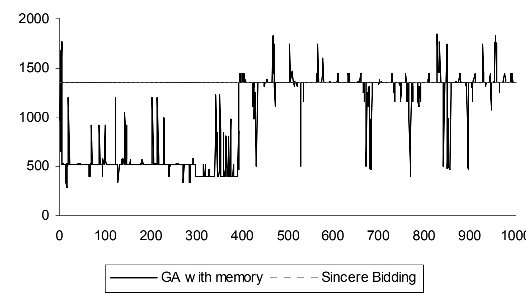

Fig.1. Evolution of the seller's revenue when all bidders follow the GA strategy compared to those obtained if all bidders bid sincerely (α = 0).
At the beginning the bidders are mainly underbidding, and therefore, the seller's revenue are lower to those obtained if all bidders bid sincerely. Nevertheless, bidders learn that they can increase their payoff bidding sincerely, which is the final stable strategy.

TABLE III
RESULTS OBTAINED WIHTOUT SYNERGIES (α = 0)

Mutation Rate	0.01	0.05
Average efficiency	0.989	0.962
Average seller's revenue	97.5%	93.2%
% sincere bidding (a=1)	87.4%	55.6%
% underbidding (for v> $^V/_2$) and a<1)	6.8%	22.5%
% overbidding (for v> $^V/_2$) and a<1)	5.8%	22.0%

The average seller's revenues (see equation (9)) are measured as a percentage of the seller's revenues if all bidders bid sincerely. Efficiency is defined as the sum of the values of the fifteen units sold in an auction as a percentage of the sum the fifteen highest values in that auction. If all bidders bid sincerely, full efficiency is guaranteed (efficiency =1).

TABLE IV
RESULTS OBTAINED WIHT SYNERGIES (α = 0.4)

Mutation Rate	0.01	0.05
Average efficiency	0.931	0.931
Average seller's revenue	26.6%	62.2%
% sincere bidding (a=1)	2.6%	20.3%
% underbidding (for v> $^V/_2$) and a<1)	96.9%	63.9%
% overbidding (for v> $^V/_2$) and a<1)	0.6%	15.8%

The average seller's revenues (see equation (9)) are measured as a percentage of the seller's revenues if all bidders bid sincerely. Efficiency is defined as the sum of the values of the fifteen units sold in an auction as a percentage of the sum the fifteen highest values in that auction. If all bidders bid sincerely, full efficiency is guaranteed (efficiency =1).

Figure 1 reports the bidding behavior for the first 1,000 auctions of an experiment without synergies and a mutation factor of 0.01. As it shows, bidders start underbidding but some of the bidders learn that they can increase their payoff by bidding sincerely. Therefore, bidders tend to do truthful bidding.

B. Results with synergies (α =0.4)

In the presence of synergies, no theoretical predictions have been done about the outcome in an Ausubel auction. The experimental results obtained in the 50 experiments done revel that bidders tend to underbid: 96.9% and 63.9% of the times for each mutation rate (see Table IV).

As bidders no longer bid sincerely, full efficiency is not guaranteed. This means that the bidders with the highest valuations would not necessarily be awarded with the items. In the experiments done, the average efficiency level is 0.931 for both mutation rates used. Another consequence of this underbidding strategy is the reduction of the seller's revenue. The demand reduction of the bidders implies that the items are clinched at lower prices, and therefore, the seller earns fewer revenues. In the results obtained in these experiments the seller only earns 26.6% and 62.2% of the revenues to those obtained if all bidders bid sincerely, per each mutation rate.

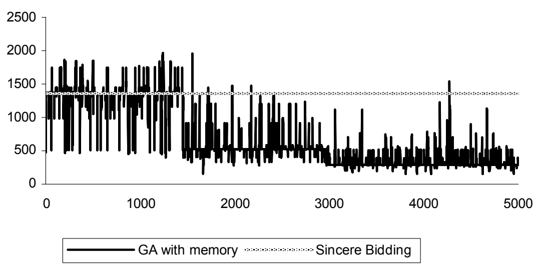

Fig.2. Evolution of the seller's revenue when all bidders follow the GA strategy compared to those obtained if all bidders bid sincerely (α = 0.4).

At the beginning the bidders become stable bidding sincerely, nevertheless, when they learn that with this strategy they can have negative profits (exposure problem) they decide to underbid. Therefore, at the final stable strategy, bidders are reducing their demands and the seller's revenues are reduced.

When super additive values are considered, bidders are affected by the exposure problem. This means that if they do not earn all the demanded items, they can have negative profits. In this computational experiment bidders learn, in their past experiences, that bidding sincerely can lead to potential losses. Hence, this is a risky strategy. Therefore, they prefer to avoid this risk by reducing their demands, this is, by underbidding. Once again, when we compare the results for both mutation factors, the GA outperforms for the lowest mutation factor.

TABLE V
RESULTS OBTAINED WIHT MUTATION RATE 0.01

Synergy value (α)	**0.2**	**0.6**	**0.8**	**1**
Average efficiency	0.956	0.972	0.964	0.978
Average seller's revenue	74.9%	77.2%	72.0%	63.7%
% sincere bidding (a=1)	4.5%	9.5%	4.1%	5.1%
% underbidding (for $v > {}^{V}/_{2}$) and a<1)	74.0%	64.5%	76.7%	71.9%
% overbidding (for $v > {}^{V}/_{2}$) and a<1)	21.5%	26.0%	19.2%	23.0%

The average seller's revenues (see equation (9)) are measured as a percentage of the seller's revenues if all bidders bid sincerely. Efficiency is defined as the sum of the values of the fifteen units sold in an auction as a percentage of the sum the fifteen highest values in that auction. If all bidders bid sincerely, full efficiency is guaranteed (efficiency =1).

Figure 2 reports an example of this learning behavior for a mutation factor of 0.01. In this experiment, at the beginning the main strategy is to bid sincerely, hence the seller's revenues are nearly the same to those obtained if all bidders bid sincerely. Nevertheless, with the experience, bidders learn that with this strategy they can have negative losses, so they decide to shave their bids. Hence, the seller earns fewer profits than with the previous strategy.

C. Results with synergies (α =0.2, 0.6, 0.8 and 1)

Finally, we have tested the GA for other synergies values: α = 0.2, 0.6, 0.8 and 1. As the GA performs better with less mutation, we have done 50 experiments for each synergy value for a mutation factor equal to 0.01. Table V reports the results obtained for these environments. As the results reveal, bidders tend to underbid for all the synergies values tested. Hence, the auction outcome no longer guarantees full efficiency and the seller's revenue is reduced.

V. CONCLUSIONS

This paper investigates a complex setting where all bidders in Ausubel auction have preferences that involve synergies for all the demanded items, and therefore, they are affected by the exposure problem. As far as we know there is no theoretical or experimental work done with synergies in multiple items for this auction format. Hence, the novelty and main contribution of this work is to study the bidders' behavior in this experimental setting, allowing bidders to have multiple strategies to bid on different quantities of items.

To explore this framework, an evolutionary computation technique has been used: a GA with memory about the past experience in the last auctions. This method allows simulation of the bidders' behavior in the auction and the study of how participants learn from their experience.

In order to study the bidding behavior of the participants, a bidding function has been defined. This function expands the possible range of strategies, allowing bidders to underbid or overbid more or less aggressively, or to bid sincerely, by modifying a single parameter. The GA searches the value of this parameter "a" for each bidder to maximize his payoff for the last four auctions. With such a wide range of strategies, this function reflects the complex performance of bidders and allows the study of their psychological reasons for such behaviors.

The results obtained in all the experiments conducted for the different mutation rates and different synergies values, reveal that bidders tend to bid sincerely in the absence of synergies (α=0), just as the theory predicts, see [1]. Nevertheless, in the presence of synergies (α>0), underbidding is the strategy that turns out to be the most frequent one. When bidders have super additive values, they are affected by the exposure problem, so bidding sincerely can yield potential losses. As bidders learn this fact in their past auctions, they finally prefer a less risky strategy; this is, to reduce their demands. As a consequence of the underbidding strategy that dominates among the bidders, the final allocation of the items is no longer efficient and the seller earns fewer revenues to those obtained if all bidders bid sincerely.

APPENDIX

Example of an Ausubel auction that involves synergies (α= 0.4)

In this auction there are 15 items to sell (m=15) and 4 bidders (n=4) with a synergy parameter of α = 0.4 and the following private values:

Table A.1. Example of bidders' values with α =0.4

	1st unit	2nd unit	3rd unit	4th unit	5th unit	6th unit	7th unit
Bidder 1 $v_{1,1}$ = 25							
$v_{1,k}$	25	60	109	178	274	408	
$v^a_{1,k} = v_{1,k}/k$	25	30	36	44	55	68	
Bidder 2 $v_{2,1}$ = 10							
$v_{2,k}$	10	24	44	71	109	163	238
$v^a_{2,k} = v_{2,k}/k$	10	12	15	18	22	27	34
Bidder 3 $v_{3,1}$ = 40							
$v_{3,k}$	40	96	174	284	438		
$v^a_{3,k} = v_{3,k}/k$	40	48	58	71	88		
Bidder 4 $v_{4,1}$ = 5							
$v_{4,k}$	5	12	22	36	55	82	119
$v^a_{4,k} = v_{4,k}/k$	5	6	7	9	11	14	17

$v_{i,1}$: value for bidder *i* of each unit by itself; $v_{i,k}$: value for bidder *i* of earning *k* units; $v^a_{i,k}$: average value for bidder *i* earning *k* units

The Ausubel auction process for this example is reported in Table A.2.

Table A.2. Example of an Ausubel auction process with synergies

Price (p^l)	Quantities demanded by bidders (q_i^l)				Aggregate demand (Q_l)	Clinching of items
	i=1	i=2	i=3	i=4		
1	6	7	5	7	25	
18	6	7	5	0	18	1 "clinches" 3 items 2 "clinches" 4 items 3 "clinches" 2 items
56	6	4	5	0	15	1 "clinches" 3 items 3 "clinches" 3 items

This example assumes that each bidder is going to bid sincerely according to his real values. This means that bidders demand positive quantities until the standing price (p^l) equals the average value ($v^a_{i,k}$).

The auctioneer begins to increment the clock and price continuously, and bidders submit their demands. The first important change happens to the price when it reaches 17, as it equals $v^a_{4,7}$ and bidder 4 drops from bidding 7 items. Therefore at p^l =18 the aggregate demand at this price is 18 and the supply is 15. Bidder 1 has now mathematically guaranteed himself at least 3 objects as the aggregate demand of all competitors other than him has dropped to 12. In the same way, bidder 2 clinches 4 items and bidder 3 clinches 2 items. These bidders will have to pay 18 monetary units for each of these items.

Whenever a bidder clinches one unit, the maximum price to which he can bid is recalculated in the following way. For example, bidder 2 started asking for 7 units and he has already clinched 4 units at p^l =18, so he will have to pay a total amount of 72 monetary units for these 4 items. As his total value for winning 7 units is $v_{2,7}$ = 238, he still has a remaining utility of 166 ($v_{2,7}$ (238) – payment for the clinched units (72)), so he will try to win the 3 outstanding units. He can now pay up to (166/3) = 55 monetary units for each of the remaining items. Therefore, bidder 2 continues asking for 7 units up to p^l =55.

The auction ends when the price reaches 56 and bidder 2 makes his minimum possible bid (which he has already clinched) reducing the aggregate demand to just 15, thus equating demand with supply. With this last round the final outcome of the auction is that bidder 1 wins 6 objects, (3 for 18 and 3 for 56); bidder 2 wins 4 objects at 18 and bidder 3 earns 5 items, (2 at 18 and 3 at 56). The bidders' payoff (equation (3)) and the seller's revenues (equation (9)) for this example are calculated below:

Payoff Bidder 1 = $v_{1,6} - P^*_1 = q_1^* v^a_{1,6} - P^*_1$ = (6* 68) – (3*18 + 3*56) =186;
Payoff Bidder 2 = $v_{2,4} - P^*_2 = q_2^* v^a_{2,4} - P^*_1$ = (4*18) – (4*18) = 0;
Payoff Bidder 3 = $v_{3,5} - P^*_3 = q_3^* v^a_{3,5} - P^*_1$ = (5*88) – (2*18 + 3* 56) = 236;
Payoff Bidder 4 = 0;
Revenues = $P^*_1 + P^*_2 + P^*_3$ = 222+ 72 + 204 = 498

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