Comments to "Band-limited stochastic processes in discrete and continuous time" by D.S.G. Pollock

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When analysing macroeconomic time series, it is often of interest to obtain estimates of the underlying business cycle. In his paper, Pollock illustrates for quarterly GDP in the UK that, when this cycle is estimated by fitting the popular AR(2) model to the deviations of seasonally adjusted observations from a deterministic trend, the estimated parameters are not able to represent the expected behaviour of a cycle. In particular, the estimated roots are real and too close to the unit cycle. Furthermore, even when the model is estimated by the modified Whittle estimator, the plug-in spectrum implied by the estimated parameters has a large spike that appears to be missrepresenting the periodogram. Pollock attributes these biases to the fact that the spectrum of an AR(2) model is non-zero for the entire range of frequencies while the observed cycle has zero valued spectral densities everywhere in the interval $(\pi/8, \pi]$ *. Consequently, he proposes a continuous-time band-limited process for the underlying cycle. He also shows that in order to estimate properly the cycle, the data should be filtered using an anti-aliasing filter to remove the spectral elements that fall outside the range of the underlying cycle. Finally, to avoid interference, the continuous process should be sampled at a rate corresponding to its highest frequency.*

Although the proposed methodology seems an attractive alternative to estimate cycles, the empirical example chosen to illustrate the conclusions of the paper seems rather contentious. There are alternative explanations about why the estimated roots of the AR(2) model fitted to the deviations of the seasonaly adjusted GDP from the deterministic trend are real and non-stationary. First, it is well known that seasonal adjustment of time series could lead to misspecified models, misleading inferences about the parameters and poor forecasts; see Plosser (1979) and Nerlove et al. (1979) for early references and Findley and Martin (2006) and Ooms and Hassler (1997) for results related with frequency domain analysis. Second, the deviations of the seasonaly adjusted GDP from the deterministic trend could be far from stationary suggesting that the trend could be stochastic rather than deterministic. In this case, the estimated cycle could absorb part of the non-stationarity no represented by the deterministic trend. Third, it could also be possible that the cycle is misspecified when fitting an AR(2) process because its dynamics are generated by alternative ARIMA models. Finally, the presence of outliers, level shifts, conditional heterocedasticity and other types of non-linearities could affect the results.

In this note, we focus on the second possible explanation of why it is possible to estimate an AR(2) model with real roots close to unity namely that the estimated cycle depends on the specification of the trend. For this purpose, we fit unobserved component models to series of seasonally adjusted GDP. The model considered is given by

$$
(1) \qquad \begin{aligned}
y_t &= \mu_t + \psi_t + \varepsilon_t \\
\mu_t &= \mu_{t\;\;1} + \beta_t + \eta_t \\
\beta_t &= \beta_{t\;\;1} + \xi_t \\
\psi_t &= \rho\left(\cos\lambda_c \psi_{t\;\;1} + \sin\lambda_c \psi^*_{t\;\;1}\right) + \kappa_t \\
\psi^*_t &= \rho\left(\quad \sin\lambda_c \psi_{t\;\;1} + \cos\lambda_c \psi^*_{t\;\;1}\right) + \kappa^*_t
\end{aligned}
$$

where μ_t *is the trend,* ψ_t *the cycle and* ε_t *the irregular component. All the noises,* ε_t, η_t, ξ_t, κ_t

and κ_t^* are assumed to be mutually uncorrelated white noise processes with variances σ_ε^2, σ_η^2, σ_ξ^2 and σ_κ^2 respectively. Note that the variances of κ_t and κ_t^* are assumed to be equal. These variances can be estimated by Quasi-Maximum Likelihood by maximizing the prediction error form of the Gaussian likelihood. Alternatively, they can also be estimated using the Whittle estimator which maximizes the frequency domain expression of the likelihood. The parameter λ_c is the period of the cycle and $\rho < 1$ is a dumping parameter. Both parameters can be estimated as additional parameters of the model. Once the parameters have been estimated, the Kalman filter allows to obtain one-step ahead estimates of the underlying components. Also, it is possible to use smoothing algorithms to obtain estimates of the components based on the whole sample available; see, for example, Harvey (1989) for an extensive treatment of model (1).

We first assume that the series has no cycle and that the trend is deterministic, i.e. $\sigma_\eta^2 = \sigma_\xi^2 = 0$. Figure 1 plots the series of GDP of UK[1] *observed quarterly from the first quarter of 1955 up to the first quarter of 2007 inclusive, together with the estimated deterministic trend and smotthed irregular component*[2]*. This figure suggests that the irregular is not stationary and, consequently, it is expected that the cycle resulting from this series may have some roots close to unity. Furthermore, Figure 2, that plots the correlogram of the residuals, is in concordance with this results. The pattern of the sample autocorrelations is not the one expected when looking at stationary cycles. Consider now that we add a cycle to the deterministic trend. In Figure 3, that plots the estimated components, it is possible to observed that the estimated cycle is very close to the estimated irregular component plotted in Figure 1.*

As mentioned before, the results above suggest that the trend of the GDP series cannot be adequately represented by a deterministic fucntion of time. Consequently, we also consider stochastic trends. First, we assume that the rate of growth of the trend is constant over time, i.e. $\sigma_\xi^2 = 0$. Figure 4 plots the estimated trend, cycle and transitory components in this case. Note that the estimated cycle has the expected shape which is rather different from the cycle estimated when the trend was assumed to be deterministic and plotted in Figure 3.

Finally, we assume that the rate of growth of the trend is not constant over time. In this case, we assume a smooth trend evolution, i.e. $\sigma_\eta^2 = 0$. The corresponding estimated components have been plotted in Figure 5. Note that the variation in the slope is much smoother than before while the cycle is very similar to the one estimated when the trend was assumed to be deterministic.

The results above suggest that the estimated cycle depends cruzially on the assumptions made about the other components in which a time series is decomposed. In this sense, it could be also of interest to analyse how these results can be affected by the fact that the series considered by Pollock and in this note has been previously seasonally adjusted. As mentioned above, seasonal filters could have missleading effects on the dynamics of the filtered series. It is of interest to see how the estimates of the cycle change when adding a seasonal component to model (1).

Finally, I would like to point out that in order to evaluate the adequacy of the estimated models some diagnostics should be of interest. Furthermore, some of the arguments put forward by Pollock are based on the visual comparison of the sample periodogram and the plug-in parametric spectrum implied by the estimated parameters. Some kind of measure of the distance between these two functions should be useful to further analyse wheter a given estimated model is able to represent the observed sample properties of the considered series.

[1] The series has been obtained from the EcoWin data base and it is seasonally adjusted.

[2] All the estimates have been obtained using the STAMP 6.0 program of Koopman et al. (2000).

Figure 1. Estimated components of deterministic trend model

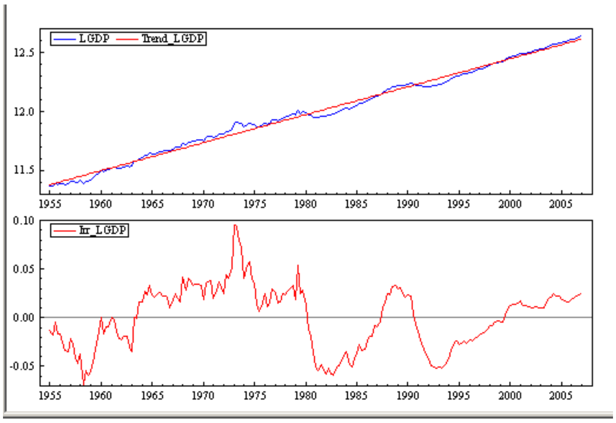


Figure 2. Correlogram of residuals of deterministic trend model.

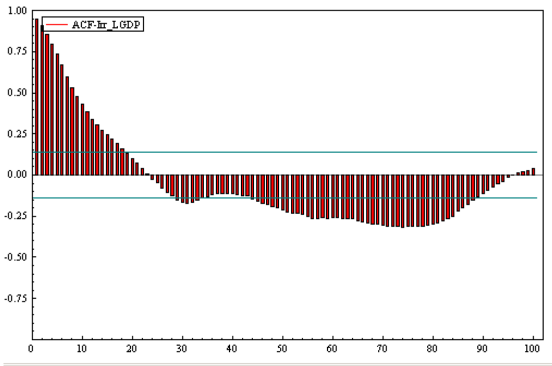


Figure 3. Estimated components of deterministic trend plus cycle model

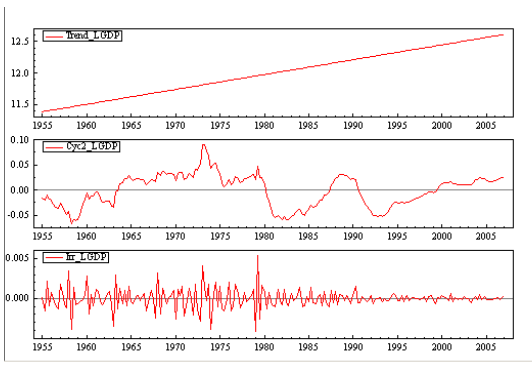


Figure 4. Estimated components of stochastic trend with fixed slope plus cycle model

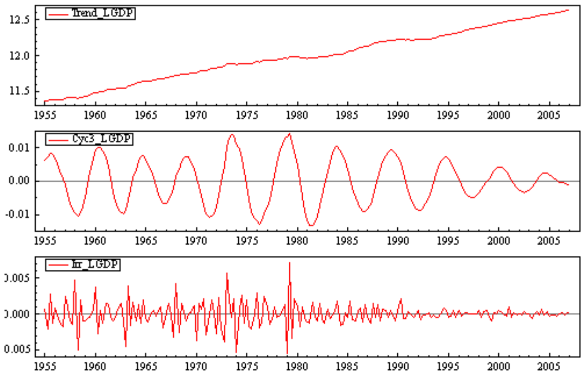
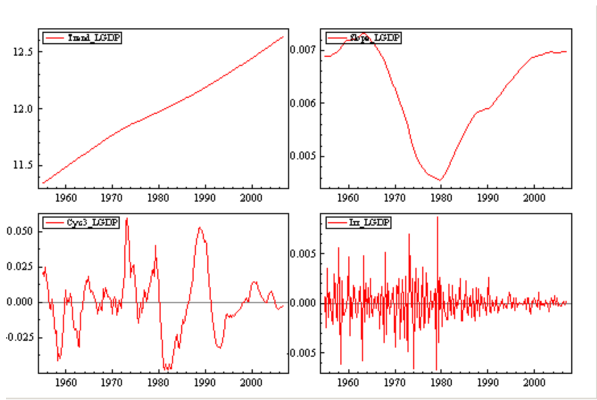


Figure 5. Estimated components of smooth trend plus cycle model



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