

## **EXISTENCE AND EQUIVALENCE OF COMPETITIVE AND CORE ALLOCATIONS IN LARGE EXCHANGE ECONOMIES WITH DIFFERENTIAL INFORMATION\*\***

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### **Abstract**

We study the equivalence of the core and competitive allocations in exchange economies with a continuum of traders and differential information. We show that if the economy is “irreducible”, then a competitive equilibrium (in the sense of Radner) exists. Moreover, the set of competitive equilibrium allocations coincides with the “private core”. We also show that the “weak fine core” of an economy coincides with the set of competitive allocations of an associated symmetric information economy in which the traders information is the “joint information” of all the traders in the original economy.

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**Keywords:** Atomless exchange economy, Differential information, Radner-competitive equilibrium, Private core, Weak fine core.

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# 1 Introduction

It is well known that the core and competitive allocations become equivalent in perfectly competitive economies (that is, when no individual can affect the overall outcome) with complete information—see, e.g., Aumann (1964) and Bewley (1973). The main purpose of this work is to study the equivalence of core and competitive allocations in exchange economies with a continuum of traders and differential information. Radner (1968 and 1982) introduced a model of an exchange economy with differential information in which every trader is characterized by a state dependent utility function, a random vector of initial endowments, an information partition (i.e., a partition of the space of states of nature), and a prior belief. In these works Radner extended the notion of Arrow-Debreu competitive equilibrium to exchange economies with differential information. The notion of competitive equilibrium which we use in the present paper is a straightforward extension of that of Radner (1982) to economies with a continuum of traders. The existence of competitive equilibrium in economies with a continuum of traders and complete information was studied in Aumann (1966) and Hildenbrand (1970). In this paper, by using general results from Hildenbrand (1974) we derive the existence of competitive equilibrium for economies with a continuum of traders and differential information.

In a seminal paper, Wilson (1978) examines the core of an exchange economy with differential information. Wilson focuses on two special cases: the *coarse core*, defined by the condition that the information for all traders in a blocking coalition is that they have in common, and the *fine core*, defined by giving every member of a blocking coalition the joint information of the members of the coalition. Wilson then showed that the coarse core is non-empty, and that the fine core may be empty. Since Wilson's article, several works on cooperative solution concepts for an economy with differential information appeared in the literature; see, for example, Kobayashi (1980), Yannelis (1991), Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), and Allen (1991, 1995, 1997). Yannelis (1991) introduced the notion of *private core*, defined by the condition that in a blocking coalition the net trade of each member of the coalition is measurable with respect to his information partition. Yannelis (1991)

showed that under appropriate assumptions the private core of an economy is non-empty. Allen (1991) and Koutsougeras and Yannelis (1993) introduced the notion of *weak fine core*, a version of Wilson's fine core, and showed that this core is non-empty. In the definition of weak fine core all net trades are measurable with respect to the "joint partition" of all the traders (i.e., the smallest partition which refines the information of all the traders), and as in the fine core, blocking net trades are measurable with respect to the "joint partition" of all the members of the coalition.

In this paper we study the equivalence of the core and competitive allocations in exchange economies with a continuum of traders and differential information. We consider two period Radner-type economies with a finite number of states of nature and a continuum of traders. In these economies consumption takes place in the second period. In the first period there is uncertainty about the state of nature; in this period traders arrange contracts that may be contingent on the realized state of nature in the second period.

We first examine the existence of competitive equilibria in these economies. We show that if an economy is *irreducible*, then an equilibrium exists under mild conditions (continuity and weak monotonicity of traders utilities). The irreducibility condition was introduced by McKenzie (1959) for exchange economies with a finite number of traders, and it has been extended to economies with a continuum of traders by Hildenbrand (1974). It expresses the idea that the endowment of every coalition, if added to the allocation of the complementary coalition, can be used to improve the welfare of every member of the complementary coalition. We show that this condition is satisfied if, for example, for every state of nature the initial endowment of every trader is in the interior of the commodity space. We also show that if the economy is irreducible, then the set of private core allocations of the economy coincides with the set of Radner competitive equilibrium allocations. We provide simple examples which show that without irreducibility these results may not hold.

For the weak fine core, we show (without assuming irreducibility) that if the traders utility functions are continuous and strictly increasing, and if for every trader there is a state of nature such that his initial endowment in this state of nature is

non-zero, then the weak fine core coincides with the set of competitive allocations of an associated economy with symmetric information which is identical to the original economy, except for the traders information, which is taken to be the “joint information” of all the traders in the original economy.

The paper is organized as follows. In Section 2 we describe the model. In Section 3 we discuss the existence of competitive equilibrium (in the sense of Radner). In Section 4 we prove the equivalence between competitive and private core allocations. Finally, in Section 5 we establish the equivalence of the weak fine core and the set of competitive allocations of the associated symmetric information economy.

## 2 The Model

We consider a Radner-type exchange economy  $\mathcal{E}$  with differential information (e.g., Radner (1968 and 1982)). Our commodity space is  $\mathbb{R}_+^l$ . The space of traders is a measure space  $(T, \Sigma, \mu)$ , where  $T$  is a set (the set of *traders*),  $\Sigma$  is a  $\sigma$ -field of subsets of  $T$  (the set of *coalitions*), and  $\mu$  is a non-atomic measure on  $\Sigma$ . The economy extends over two time periods,  $\tau = 0, 1$ . Consumption takes place at  $\tau = 1$ . At  $\tau = 0$  there is uncertainty over the state of nature; in this period traders arrange contracts that may be contingent on the realized state of nature at  $\tau = 1$ . There is a finite space of states of nature, denoted by  $\Omega$ . At  $\tau = 1$  traders do not necessarily know which state of nature  $\omega \in \Omega$  actually occurred, although they know their own endowments, and may also have some additional information about the state of nature. We assume that the information of a trader  $t \in T$  is described by a partition  $\Pi_t$  of  $\Omega$ . We denote by  $\mathcal{F}_t$  the field generated by  $\Pi_t$ . If  $\omega_o$  is the true state of the economy at  $\tau = 1$ , trader  $t$  observes the member of  $\Pi_t$  which contains  $\omega_o$ . Every trader  $t \in T$  has a probability measure  $q_t$  on  $\Omega$  which represents his *prior beliefs*. The preferences of a trader  $t \in T$  are represented by a *random utility function*,  $u_t : \Omega \times \mathbb{R}_+^l \rightarrow \mathbb{R}_+$  such that for every  $(t, x) \in \Omega \times \mathbb{R}_+^l$ , the mapping  $(t, x) \rightarrow u_t(\omega, x)$  is  $\Sigma \times \mathcal{B}$  measurable, where  $\omega$  is a fixed member of  $\Omega$ , and  $\mathcal{B}$  is the  $\sigma$ -field of Borel subsets of  $\mathbb{R}_+^l$ . We assume also that for every  $x \in \mathbb{R}_+^l$  the function  $u_t(\cdot, x)$  is  $\mathcal{F}_t$ -measurable. If  $x$  is a function from  $\Omega$  to

$\mathbb{R}_+^l$  we denote by  $h_t(x)$  the *expected utility with respect to  $x$*  of trader  $t \in T$ . That is

$$h_t(x) = \sum_{\omega \in \Omega} q_t(\omega) u_t(\omega, x(\omega)).$$

An *assignment* is a function  $\mathbf{x} : T \times \Omega \rightarrow \mathbb{R}_+^l$  such that for every  $\omega \in \Omega$  the function  $\mathbf{x}(\cdot, \omega)$  is  $\mu$ -integrable on  $T$ . There is a fixed *initial assignment*  $\mathbf{e}$ ;  $\mathbf{e}(t, \omega)$  represents the *initial endowment density* of trader  $t \in T$  in the state of nature  $\omega \in \Omega$ . We assume that for almost every  $t \in T$  the function  $\mathbf{e}(t, \cdot)$  is  $\mathcal{F}_t$ -measurable.

Throughout the paper we use the following notations. For two vectors  $x = (x_1, \dots, x_l)$  and  $y = (y_1, \dots, y_l)$  in  $\mathbb{R}^l$  we write  $x \geq y$  when  $x_k \geq y_k$  for all  $1 \leq k \leq l$ ,  $x > y$  when  $x \geq y$  and  $x \neq y$ , and  $x \gg y$  when  $x_k > y_k$  for all  $1 \leq k \leq l$ .

### 3 Competitive Equilibrium

In this section we extend Radner's (1982) definition of competitive equilibrium to our model (see Radner (1982), Section 3.4), and discuss conditions under which its existence can be guaranteed. Throughout the rest of the paper, an economy  $\mathcal{E}$  is an atomless economy with differential information as described in Section 2.

A *private allocation* for an economy  $\mathcal{E}$  is an assignment  $\mathbf{x}$  such that

(3.1) For almost all  $t \in T$  the function  $\mathbf{x}(t, \cdot)$  is  $\mathcal{F}_t$ -measurable, and

(3.2)  $\int_T \mathbf{x}(t, \omega) d\mu \leq \int_T \mathbf{e}(t, \omega) d\mu$  for all  $\omega \in \Omega$ .

A *price system* is a non-zero function  $p : \Omega \rightarrow \mathbb{R}_+^l$ . Let  $t \in T$  and let  $M_t$  be the set of all  $\mathcal{F}_t$ -measurable functions from  $\Omega$  to  $\mathbb{R}_+^l$ . For a price system  $p$ , define the *budget set* of  $t$  by

$$B_t(p) = \left\{ x \mid x \in M_t \text{ and } \sum_{\omega \in \Omega} p(\omega) \cdot x(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot \mathbf{e}(t, \omega) \right\}.$$

A *competitive equilibrium* (in the sense of Radner) for an economy  $\mathcal{E}$  is a pair  $(p, \mathbf{x})$  where  $p$  is a price system and  $\mathbf{x}$  is private allocation such that

(3.3) For almost all  $t \in T$  the function  $\mathbf{x}(t, \cdot)$  maximizes  $h_t$  on  $B_t(p)$ , and

(3.4)  $\sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{x}(t, \omega) d\mu = \sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{e}(t, \omega) d\mu$ .

A *competitive allocation* is a private allocation  $\mathbf{x}$  for which there exists a price system  $p$  such that  $(p, \mathbf{x})$  is a competitive equilibrium.

Usually in the literature, the inequality (3.2) in the definition of a private allocation is replaced with a strict equality; see, e.g., Radner 1968, Krasa and Yannelis (1994), Allen (1997). Here we follow Radner (1982) who noted that the total amount to be disposed of might not be measurable with respect to the information partition of a single agent. Einy and Shitovitz (1998) provided an example of an economy with differential information which has a competitive equilibrium, but if the inequality (3.2) in the definition of a private allocation is replaced with an equality, then the economy does not have a competitive equilibrium where all prices are non-negative—see Example 2.1 in Einy and Shitovitz (1998).

A function  $u : \mathfrak{R}_+^l \rightarrow \mathfrak{R}$  is *(strictly) increasing* if for all  $x, y \in \mathfrak{R}_+^l$ ,  $(x > y) \implies x \gg y$  implies  $u(x) > u(y)$ .

Throughout the paper we will often refer to the following conditions.

(A.1) For every  $\omega \in \Omega$  we have  $\int_T \mathbf{e}(t, \omega) d\mu \gg 0$ .

(A.2) For almost all  $t \in T$  and for every  $\omega \in \Omega$ , the function  $u_t(\omega, \cdot)$  is continuous and increasing on  $\mathfrak{R}_+^l$ .

(A.3) *Irreducibility*: for every private allocation  $\mathbf{x}$  and for every two disjoint coalitions  $T_1, T_2 \in \Sigma$  such that  $\mu(T_1) > 0$  and  $\mu(T_2) > 0$ , and  $T_1 \cup T_2 = T$ , there exists an assignment  $\mathbf{y}$  such that  $\mathbf{y}(t, \cdot) \in M_t$  for almost all  $t \in T_2$ , and such that

(A.3.1)  $h_t(\mathbf{y}(t, \cdot)) > h_t(\mathbf{x}(t, \cdot))$  for almost all  $t \in T_2$ , and

(A.3.2) for all  $\omega \in \Omega$ :  $\int_{T_1} \mathbf{e}(t, \omega) d\mu + \int_{T_2} \mathbf{x}(t, \omega) d\mu \geq \int_{T_2} \mathbf{y}(t, \omega) d\mu$ .

Irreducibility, Condition (A.3), was introduced in McKenzie (1959) for economies with a finite number of traders. This condition was extended for atomless economies by Hildenbrand (see Hildenbrand (1974), pages 143 and 214), and it expresses the idea that the endowment of every coalition is desired. Our definition is a variant of Hildenbrand's (1974).

**Proposition 3.1.** *Assume that an economy  $\mathcal{E}$  satisfies assumption (A.2). If for almost every  $t \in T$  and all  $\omega \in \Omega$  we have  $\mathbf{e}(t, \omega) \gg 0$ , then  $\mathcal{E}$  satisfies Irreducibility (A.3).*

**Proof:** Assume that  $\mathbf{e}(t, \omega) \gg 0$  for almost every  $t \in T$  and all  $\omega \in \Omega$ . Let  $\mathbf{x}$  be

a private allocation in  $\mathcal{E}$ , and let  $T_1, T_2 \in \Sigma$  be two disjoint coalitions such that such that  $\mu(T_1) > 0$  and  $\mu(T_2) > 0$ , and  $T_1 \cup T_2 = T$ . Then for all  $\omega \in \Omega$  we have

$$\int_{T_1} \mathbf{e}(t, \omega) d\mu \gg 0.$$

Let  $a \in \mathbb{R}_+^l$  be such that  $\mu(T_2)a \gg 0$ , and such that for all  $\omega \in \Omega$  we have

$$\int_{T_1} \mathbf{e}(t, \omega) d\mu \geq \mu(T_2)a.$$

Define  $\mathbf{y} : T \times \Omega \rightarrow \mathbb{R}_+^l$  by

$$\mathbf{y}(t, \omega) = \begin{cases} 0 & t \in T_1 \\ \mathbf{x}(t, \omega) + a & t \in T_2 \end{cases}.$$

Then for all  $t \in T_2$ ,  $\mathbf{y}(t, \cdot) \in M_t$ . Since for almost all  $t \in T$  and all  $\omega \in \Omega$ ,  $u_t(\omega, \cdot)$  is increasing, we have

$$h_t(\mathbf{y}(t, \cdot)) > h_t(\mathbf{x}(t, \cdot)),$$

for almost all  $t \in T_2$ . From the choice of  $a$  it is clear that (A.3.2) holds for  $\mathbf{x}$  and  $\mathbf{y}$ .

□

A *quasi equilibrium* for the economy  $\mathcal{E}$  is a pair  $(p, \mathbf{x})$ , where  $p$  is a price system and  $\mathbf{x}$  is a private allocation, such that

(3.5) For almost all  $t \in T$ , either  $\sum_{\omega \in \Omega} p(\omega) \cdot \mathbf{e}(t, \omega) = 0$ , or the function  $\mathbf{x}(t, \cdot)$  maximizes  $h_t$  on  $B_t(p)$ , and

$$(3.6) \sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{x}(t, \omega) d\mu = \sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{e}(t, \omega) d\mu.$$

**Proposition 3.2.** *If an economy  $\mathcal{E}$  satisfies conditions (A.1)– (A.3), then every quasi equilibrium of  $\mathcal{E}$  is a competitive equilibrium.*

**Proof:** Proposition 3.2 is a direct consequence of Proposition 1 in Hildenbrand (1974), page 214, when the consumption sets are  $M_t$  and the utilities functions  $h_t$ ,  $t \in T$ , and the production sets are  $(\mathbb{R}_-^l)^\Omega$ . □

**Theorem A.** *If an economy  $\mathcal{E}$  satisfies assumptions (A.1) – (A.3) then it has a competitive equilibrium.*

**Proof:** First note that our definition of quasi equilibrium is a special case of Hildenbrand (1970 and 1974) definition of quasi equilibrium for a coalition production economy where the consumption sets are  $M_t$  and the utilities functions  $h_t$ ,  $t \in T$ , and the production sets are  $(\mathbb{R}_-^L)^\Omega$  (see Hildenbrand (1970), Section 2, page 611). Therefore by Theorem 2 in Hildenbrand (1970), our economy has a quasi equilibrium  $(p, \mathbf{x})$ . By Proposition 3.2,  $(p, \mathbf{x})$  is a competitive equilibrium of  $\mathcal{E}$ .  $\square$

The following corollary is a direct consequence of Proposition 3.1 and Theorem A.

**Corollary 3.3.** *If an economy  $\mathcal{E}$  satisfies (A.1), (A.2) and in addition for every  $\omega \in \Omega$  and almost all  $t \in T$  we have  $\mathbf{e}(t, \omega) \gg 0$ , then  $\mathcal{E}$  has a competitive equilibrium.*

## 4 The Private Core

In this section we extend the definition of private core introduced in Yannelis (1991) to our economy, and show that under conditions (A.1) – (A.3) the set of competitive allocations of the economy coincides with the set of private core allocations.

An assignment  $\mathbf{x}$  is a *private core allocation* for the economy  $\mathcal{E}$  if

(4.1)  $\mathbf{x}$  is a private allocation, and

(4.2) there do not exist a coalition  $S \in \Sigma$  and an assignment  $\mathbf{y}$  such that

$$(4.2.1) \mu(S) > 0,$$

$$(4.2.2) \mathbf{y}(t, \cdot) \text{ is } \mathcal{F}_t\text{-measurable for all } t \in S,$$

$$(4.2.3) \int_S \mathbf{y}(t, \omega) d\mu \leq \int_S \mathbf{e}(t, \omega) \text{ for all } \omega \in \Omega, \text{ and}$$

$$(4.2.4) h_t(\mathbf{y}(t, \cdot)) > h_t(\mathbf{x}(t, \cdot)) \text{ for almost all } t \in S.$$

The *private core* of an economy  $\mathcal{E}$  is the set of all private core allocations of  $\mathcal{E}$ .

**Proposition 4.1.** *Every competitive allocation of an economy  $\mathcal{E}$  is a private core allocation of  $\mathcal{E}$ .*

**Proof:** Proposition 4.1 is a special case of Proposition 2, page 216 in Hildenbrand (1974).  $\square$



**Theorem B.** *Under assumptions (A.1) – (A.3) the set of competitive allocations of an economy  $\mathcal{E}$  coincides with the private core of  $\mathcal{E}$ .*

**Proof:** By Proposition (4.1) it suffices to show that every private core allocation in  $\mathcal{E}$  is a competitive allocation. Let  $\mathbf{x}$  be a private core allocation in  $\mathcal{E}$ . By Theorem 1, page 216 of Hildenbrand (1974), there is a price system  $p$  such that  $(p, \mathbf{x})$  is a quasi equilibrium for  $\mathcal{E}$ . By Proposition 3.2 we obtain that  $(p, \mathbf{x})$  is a competitive equilibrium for  $\mathcal{E}$ .  $\square$

The following corollary is a direct consequence of Proposition 3.1 and Theorem B.

**Corollary 4.2.** *If an economy  $\mathcal{E}$  satisfies (A.1), (A.2), and in addition for every  $\omega \in \Omega$  and almost all  $t \in T$  we have  $\mathbf{e}(t, \omega) \gg 0$ , then the set of competitive allocations of  $\mathcal{E}$  coincides with the private core of  $\mathcal{E}$ .*

We now give an example of an atomless economy with complete information which satisfies (A.1) and (A.2) but does not satisfies Irreducibility (Condition (A.3)), and which does not have a competitive equilibrium, although it has a non-empty core.

**Example 4.3.** Consider an atomless economy  $\mathcal{E}$  in which the space of traders is  $([0, 2], \mathcal{B}, \lambda)$ , where  $\mathcal{B}$  is the  $\sigma$ -field of Borel subsets of  $[0, 2]$  and  $\lambda$  is the Lebesgue measure. Traders have complete information, and the commodity space is  $\mathfrak{R}_+^2$ . Every trader in the interval  $T_1 = [0, 1]$  has an initial endowment  $e_1 = (1, 0)$  and utility function  $u_1(x, y) = x$ , whereas each trader in the interval  $T_2 = (1, 2]$  has initial endowment  $e_2 = (1, 1)$  and utility function  $u_2(x, y) = y$ . The core of the economy  $\mathcal{E}$  consists of all allocations  $\mathbf{x}$  such that

$$u_t(\mathbf{x}(t)) = \begin{cases} \alpha(t) & t \in T_1 \\ 1 & t \in T_2 \end{cases},$$

where  $\alpha : T_1 \rightarrow \mathfrak{R}_+$  is an integrable function such that  $\alpha(t) \geq 1$  for almost all  $t \in T_1$  and  $\int_{T_1} \alpha(t) d\lambda \leq 2$ . It is easy to see that every core allocation in  $\mathcal{E}$  is a quasi equilibrium allocation with price system  $p = (0, 1)$ . However, the economy  $\mathcal{E}$  does not

have a competitive equilibrium. It is worth noticing that the core of this economy contains allocations that do not have the Equal Treatment Property.

In the following example we consider an economy with asymmetric information in which the utility functions of the traders are strictly increasing and strictly concave. The economy does not satisfy Irreducibility (A.3), and it does not have a competitive equilibrium, although its private core is non-empty (it consists of the initial assignment).

**Example 4.4.** Consider an atomless economy  $\mathcal{E}$  in which the space of traders is  $([0, 2], \mathcal{B}, \lambda)$ , where  $\mathcal{B}$  is the  $\sigma$ -field of Borel subsets of  $[0, 2]$  and  $\lambda$  is the Lebesgue measure. The commodity space is  $\mathbb{R}_+^2$ , and the space of states of nature is  $\Omega = \{\omega_1, \omega_2\}$ . The information partition of every trader  $t$  in the interval  $T_1 = [0, 1]$  is  $\Pi_1 = \{\{\omega_1\}, \{\omega_2\}\}$ , his prior belief is  $q_1 = (\frac{1}{2}, \frac{1}{2})$ , his initial endowment is  $e_1$  where  $e_1(t, \omega_1) = e_1(\omega_1) = (1, 0)$  and  $e_1(t, \omega_2) = e_1(\omega_2) = (0, 1)$  for all  $t \in T_1$ , and his utility function  $u_1(\omega, (x, y)) = \sqrt{x} + \sqrt{y}$ , for all  $\omega \in \Omega$ . The information partition of every trader  $t$  in the interval  $T_2 = [1, 2]$  is  $\Pi_2 = \{\{\omega_1, \omega_2\}\}$ , his prior belief is  $q_2 = (\frac{1}{2}, \frac{1}{2})$ , his initial endowment is  $e_2$ , where  $e_2(t, \omega_1) = e_2(t, \omega_2) = (1, 1)$  for all  $t \in T_2$ , and his utility function is  $u_2(\omega, (x, y)) = \sqrt{x} + \sqrt{y}$ , for all  $\omega \in \Omega$ . It is easy to see that the economy does not have a competitive equilibrium. However, the unique private core allocation is the initial assignment  $\mathbf{e}$ . Note that  $(p, \mathbf{e})$ , where  $p(\omega_1) = (0, 1)$  and  $p(\omega_2) = (1, 0)$ , is a quasi equilibrium for  $\mathcal{E}$ .

## 5 The Weak Fine Core

In this section we extend to our model the definition of “weak fine core” introduced by Allen (1991) and Koutsougeras and Yannelis (1993), and we prove an equivalence theorem for this notion of core.

We first note that since  $\Omega$  is a finite set, there is a finite number of different information partitions. Let us be given an economy  $\mathcal{E}$ , and denote by  $\Pi_1, \dots, \Pi_n$  the  $n$  distinct information partitions of the traders. For every  $1 \leq i \leq n$ , let  $\mathcal{F}_i$  be the

field generated by  $\Pi_i$ , and let

$$T_i = \{t \in T \mid \mathcal{F}_t = \mathcal{F}_i\}.$$

We assume that for every  $1 \leq i \leq n$  the set  $T_i$  is measurable and  $\mu(T_i) > 0$ . If  $I \subset \{1, \dots, n\}$  is a non-empty set, we denote by  $\bigvee_{i \in I} \mathcal{F}_i$  the smallest field which contains each  $\mathcal{F}_i$ ,  $i \in I$ . If  $S \in \Sigma$  is a coalition with  $\mu(S) > 0$ , we denote

$$I(S) = \{i \in I \mid \mu(S \cap T_i) > 0\}.$$

An assignment  $\mathbf{x}$  for the economy  $\mathcal{E}$  is called a *weak fine core allocation* if

(5.1) For almost all  $t \in T$  the function  $\mathbf{x}(t, \cdot)$  is  $\bigvee_{i=1}^n \mathcal{F}_i$ -measurable;

(5.2) For every  $\omega \in \Omega$ ,  $\int_T \mathbf{x}(t, \omega) d\mu \leq \int_T \mathbf{e}(t, \omega) d\mu$ ;

(5.3) there do not exist a coalition  $S \in \Sigma$  and an assignment  $\mathbf{y}$  such that

$$(5.3.1) \quad \mu(S) > 0,$$

$$(5.3.2) \quad \mathbf{y}(t, \cdot) \text{ is } \bigvee_{i \in I(S)} \mathcal{F}_i\text{-measurable for all } t \in S,$$

$$(5.3.3) \quad \int_S \mathbf{y}(t, \omega) d\mu \leq \int_S \mathbf{e}(t, \omega) d\mu \text{ for all } \omega \in \Omega, \text{ and}$$

$$(5.3.4) \quad h_t(\mathbf{y}(t, \cdot)) > h_t(\mathbf{x}(t, \cdot)) \text{ for almost all } t \in S.$$

The *weak fine core* of  $\mathcal{E}$  is defined as the set of all weak fine core allocations of  $\mathcal{E}$ .

We now introduce the following condition.

(A.4) If  $A \in \bigvee_{i=1}^n \mathcal{F}_i$  is non-empty, then  $q_t(A) > 0$  for almost all  $t \in T$ .

We denote by  $\mathcal{E}^*$  an economy identical to  $\mathcal{E}$  except for the information fields of the traders, which for all  $t \in T$  is taken to be  $\mathcal{F}_t^* = \bigvee_{i=1}^n \mathcal{F}_i$ . Note that the information in  $\mathcal{E}^*$  is symmetric.

In the proof of the following proposition we use a result of Vind (1972) which asserts that in Aumann (1964) atomless economy if an allocation is blocked, then the blocking coalition can be chosen with a measure which is arbitrarily close to the measure of the grand coalition.

**Proposition 5.1.** *Assume that an economy  $\mathcal{E}$  satisfies (A.1), (A.2) and (A.4), and in addition for almost all  $t \in T$  and for every  $\omega \in \Omega$  the function  $u_t(\omega, \cdot)$  is strictly increasing. Then the weak fine core of  $\mathcal{E}$  coincides with the private core of  $\mathcal{E}^*$ .*

**Proof:** It is clear that every private core allocation in  $\mathcal{E}^*$  is a weak fine allocation of  $\mathcal{E}$ . We prove the converse. Let  $\Pi = \bigvee_{i=1}^n \Pi_i$  (i.e.,  $\Pi$  is the smallest partition of  $\Omega$  that refines each  $\Pi_i$ ). Denote  $\Pi = \{A_1, \dots, A_k\}$ , and let  $X$  be the set of all members of  $(\mathbb{R}_+^l)^\Omega$  which are  $\bigvee_{i=1}^n \mathcal{F}_i$ -measurable. Then every member of  $X$  is constant on every  $A_j$ ,  $1 \leq j \leq k$ . Let the function  $\alpha : X \rightarrow \mathbb{R}_+^{kl}$  be defined by  $\alpha(x) = \hat{x}$ , where for  $1 \leq j \leq k$ ,  $\hat{x}_j = x(\omega_j)$  for some  $\omega_j \in A_j$ . Note that  $\alpha$  is a one to one mapping from  $X$  onto  $\mathbb{R}_+^{kl}$ . For every  $t \in T$  we define a function  $\hat{h}_t : \mathbb{R}_+^{kl} \rightarrow \mathbb{R}$  by  $\hat{h}_t(\hat{x}) = h_t(\alpha^{-1}(\hat{x}))$ . Then  $\hat{h}_t$  is continuous, and by (A.4) it is strictly increasing. Consider now the complete information atomless economy  $\hat{\mathcal{E}}$  in which the space of traders is  $(T, \Sigma, \mu)$ , the commodity space is  $\mathbb{R}_+^{kl}$ , the initial assignment is  $\hat{e}$ , where  $\hat{e}(t) = \alpha(e(t, \cdot))$  for all  $t \in T$ , and the utility function of trader  $t$  is  $\hat{h}_t$ . Let  $y$  be a weak fine core allocation of  $\mathcal{E}$ . Assume, contrary to our claim, that  $y$  is not a private core allocation of  $\mathcal{E}^*$ . For every  $t \in T$  let  $\hat{y}(t) = \alpha(y(t, \cdot))$ . Then  $\hat{y}$  is not in the core of the economy  $\hat{\mathcal{E}}$ . Therefore by the Theorem of Vind (1972), there exists a coalition  $S \in \Sigma$  and an assignment  $\hat{z}$  in  $\hat{\mathcal{E}}$  such that  $\mu(S) > \mu(T) - \min\{\mu(T_1), \dots, \mu(T_n)\}$ ,  $\int_S \hat{z}(t) d\mu \leq \int_S \hat{e}(t) d\mu$ , and  $\hat{h}_t(\hat{z}(t)) > \hat{h}_t(\hat{y}(t))$  for almost all  $t \in S$ . For every  $t \in T$  let  $z(t, \cdot) = \alpha^{-1}(\hat{z}(t))$ . Then for every  $\omega \in \Omega$  we have

$$\int_S z(t, \omega) d\mu \leq \int_S e(t, \omega) d\mu.$$

Since  $\mu(S) > \mu(T) - \min\{\mu(T_1), \dots, \mu(T_n)\}$ , we have  $I(S) = \{1, 2, \dots, n\}$  and thus  $z$  is  $\bigvee_{i \in I(S)} \mathcal{F}_i$ -measurable.

For almost all  $t \in S$  we have

$$h_t(z(t, \cdot)) = \hat{h}_t(\hat{z}(t)) > \hat{h}_t(\hat{y}(t)) = h_t(y(t, \cdot)),$$

which contradicts the assumption that  $y$  is a weak fine core allocation of  $\mathcal{E}$ .  $\square$

**Lemma 5.2.** *Assume that an economy  $\mathcal{E}$  satisfies the assumptions of Proposition 5.1, and in addition for almost every  $t \in T$  there is  $\omega \in \Omega$  such that  $e(t, \omega) \neq 0$ . Then the economy  $\mathcal{E}^*$  is irreducible, i.e., it satisfies condition (A.3).*

**Proof:** Let  $x$  be a private allocation in  $\mathcal{E}^*$ , and let  $T_1, T_2$  be two disjoint coalitions

in  $\Sigma$  such that  $T = T_1 \cup T_2$ , and  $\mu(T_1) > 0$ ,  $\mu(T_2) > 0$ . For every  $(t, \omega) \in T \times \Omega$  let

$$\mathbf{y}(t, \omega) = \begin{cases} 0 & t \in T_1 \\ \mathbf{x}(t, \omega) + \frac{1}{\mu(T_2)} \int_{T_1} \mathbf{e}(t, \omega) d\mu & t \in T_2 \end{cases}.$$

Then for every  $t \in T$ ,  $\mathbf{y}(t, \cdot)$  is  $\bigvee_{i=1}^n \mathcal{F}_i$ -measurable. Since  $u_t(\omega, \cdot)$  is strictly increasing for almost all  $t \in T$  and all  $\omega \in \Omega$ , and  $\frac{1}{\mu(T_2)} \int_{T_1} \mathbf{e}(t, \omega) d\mu > 0$  for some  $\omega \in \Omega$ , it follows from (A.4) that for almost every  $t \in T_2$

$$h_t(\mathbf{y}(t, \cdot)) > h_t(\mathbf{x}(t, \cdot)).$$

Moreover, for all  $\omega \in \Omega$  we have

$$\int_{T_1} \mathbf{e}(t, \omega) d\mu + \int_{T_2} \mathbf{x}(t, \omega) d\mu = \int_{T_2} \mathbf{y}(t, \omega) d\mu.$$

Therefore  $\mathcal{E}^*$  is irreducible.  $\square$

**Theorem C.** *Assume that an economy  $\mathcal{E}$  satisfies the assumptions of Lemma 5.2. Then the weak fine core of  $\mathcal{E}$  coincides with the set of competitive allocations of  $\mathcal{E}^*$ .*

**Proof:** The proof follows directly from Proposition 5.1, Lemma 5.2 and Theorem B.  $\square$

We conclude with the following proposition.

**Proposition 5.3.** *If an economy  $\mathcal{E}$  satisfies the assumptions (A.1), (A.2), and in addition for every  $\omega \in \Omega$  and almost all  $t \in T$ ,  $\mathbf{e}(t, \omega) \gg 0$ , then the weak fine core of  $\mathcal{E}$  coincides with the set of competitive allocations of  $\mathcal{E}^*$ .*

**Proof:** The proof is the same as that of Theorem C, noticing that  $\mathcal{E}^*$  is irreducible and the theorem in Vind (1972) holds under the assumptions of Proposition 5.3.  $\square$

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