Worst-Case Modelling for Management Decisions under Incomplete Information, with Application to Electricity Spot Markets

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Summary. Many economic sectors often collect significantly less data than would be required to analyze related standard decision problems. This is because the demand for some data can be intrusive to the participants of the economy in terms of time and sensitivity. The problem of modelling and solving decision models when relevant empirical information is incomplete is addressed. First, a procedure is presented for adjusting the parameters of a model which is robust against the worst-case values of unobserved data. Second, a scenario tree approach is considered to deal with the randomness of the dynamic economic model and equilibria is computed using an interior-point algorithm. This methodology is implemented in the Australian deregulated electricity market. Although a simplified model of the market and limited information on the production side are considered, the results are very encouraging since the pattern of equilibrium prices is forecasted.

Key words: Economic modelling, equilibrium, worst-case, scenario tree, interior-point methods, electricity spot market

1 Introduction

Decision makers need to build and solve stochastic dynamic decision models to make planning decisions accurately. Three steps are involved. The first is the specification of the structure of the stochastic dynamic decision model reflecting the essential economic considerations. The second step is the calibration of the parameters of the model. The final step is the computation of the model's outcome for forecasting and/or simulating economic problems.

In the first part of this paper, we propose an integrated approach to address this problem. The first task, the specification of the model, involves a trade-off between complexity and realism. A more realistic model is usually a multistage stochastic problem that will become increasingly impractical as the problem size increases. In general, any multistage stochastic problem is characterized by an underlying exogenous random process whose realizations are data trajectories in a probability space. The decision variables of the model are measurable functions of these realizations. A discrete scenario approximation of the underlying random process is needed for any application of the stochastic problem. This field of research has became very popular due to the large number of finance and engineering applications. For example, (Bounder, 1997), (Kouwenberg, 2001) and (Høyland and Wallace, 2001) developed and employed scenarios trees for a stochastic multistage asset-allocation problem. (Escudero, Fuente, García and Prieto, 1996), among others, considered scenarios trees for planning the production of hydropower systems. We obtain a discrete approximation of the stochastic dynamic problem using the simulation and randomized clustering approach proposed by (Gülpinar, Rustem and Settergren, 2004). In particular, we consider a scenario tree approach to approximate the stochastic random shocks process that affects the market demand.

On the other hand, firms make decisions on production, advertisement, etc. within the constraints of their technological knowledge and financial contracts. In many actual production processes, these constraints contain parameters, often unknown even when they have physical meaning. Decision makers do not usually observe all data required to estimate accurately the parameters of the model. For example, decision makers often lack enough information on the specifications of competitors. In these circumstances, standard econometric techniques cannot help to estimate the parameters of an economic model and still, decision makers require a full specification of the market to design optimal strategies that optimize their returns.

We propose a robust methodology to calibrate the parameters of a model using limited information. The robustness in the calibration of the model is achieved by a worst-case approach. Worst-case modelling essentially consists of designing the model that best fits the available data in view of the worst-case outcome of unobserved decision variables. This is a robust procedure for adjusting parameters with insurance against unknown data.

In the economic context, this approach turns out especially interesting to study situations in which a structural change takes place, for example when there are changes in the technologies of firms or a new firm enters the economy. As a consequence of these exogenous perturbations, the empirical data generating process is modified and classical estimations cannot be made. In this context, a model in which decision makers assess the worst-case effect of the unobserved data is a valuable tool for the decision maker against a risk in future decisions. Worst-case techniques has been applied in n-person games to study decision making in real-world conflict situations (see for example Rosen, 1965). In a worst-case strategy, decision makers seek to minimize the maximum damage their competitor can do. When the competitor can be interpreted as nature, the worst-case strategy seek optimal responses in the worst-case value of uncertainty. Choosing the worst-case parameters requires the solution of a min-max continuous problem. Pioneering contributions to the study of this problem have been made by (Danskin, 1967) and (Bram, 1966), while computational methods are discussed in (Rustem and Howe, 2002).

The third and final task is the computation of the equilibrium values (decisions and prices) for each scenario. We consider a variant of the interior-point method presented in (Esteban-Bravo, 2004) to compute equilibria of stochastic dynamic models.

In the second part of the paper, we consider the deregulated electricity market in NSW Australia to illustrate the applicability of this methodology. In recent years, the theoretical and empirical study of the electricity market has attracted considerable attention. In particular, the ongoing liberalization process in the electricity markets has created a significant interest in the development of economic models that may represent the behaviour of these markets (a detailed review on this literature can be found in Schweppe, Carmanis, Tabors and Bohn 1988, Kahn 1998, Green 2000, and Boucher and Smeers 2001). One of the key characteristics of these markets is that their databases often collect significantly less variables than necessary for building useful economic models. This is because the demand for some data can be intrusive to the firm in terms of time and sensitivity.

We consider a model that focuses on the effect we hope to study in detail: the process of spot prices. Similar selective approaches are adopted for the decision analysis of dispatchers (Sheblé, 1999), the financial system as a hedge against risk (e.g. Bessembinder and Lemmon, 2002), the externalities given by network effects (e.g. Hobbs 1986 and Jing-Yuan and Smeers 1999).

First, the model developed forecasts daily electricity demand. We assume that the demand is affected by exogenous factors and by an underlying stochastic random process. The discrete outcomes for this random process is generated using the simulation and randomized clustering approach proposed by (Gülpinar, Rustem and Settergren, 2004).

Our model for generators is a simplification of the standard models in the literature. We do not attempt to provide a realistic description of the underlying engineering problems in electricity markets. The literature in this area is extensive (e.g. McCalley and Sheblé 1994). Our aim is to forecast the process of spot prices using limited information on the production side. The knowledge of these prices is the basic descriptive and predictive tool for designing optimal strategies that tackle competition. Some authors have studied spot markets assuming a known probability distribution for spot prices (see, e.g., Neame, Philpott and Pritchard 2003), or considering spot prices as nonstationary stochastic processes (see Valenzuela and Mazumdar 2001, Pritchard and Zakeri 2003, and the references therein). We consider economic equilibrium models to this end. We simplify the effects of the transmission constraints dictated by Kirchoff's laws ((Schweppe, Carmanis, Tabors and Bohn, 1988) and (Hsu, 1997) also consider a simplified model of transmission network). This may be acceptable as we consider managing decisions using limited information. In any case, the approach presented here can be applied to any other modelling choices which include other phases of the electricity trading and other models of competition (as those presented in Day, Hobbs and Pang 2002).

We apply a worst-case approach to provide indicative values of the parameters in the model using the information available. The worst-case criteria ensures robustness to calibrate these parameters. Robustness is ensured as the best parameter choice is determined simultaneously with the worst-case outcome of unobserved data.

Finally, we compute the expected value of future equilibrium prices and we see that the model captures the essential features of the prices' behaviour. From the analysis of the results, we can conclude that this approach is able to forecast the pattern of equilibrium prices using limited information on the production side.

2 The Methodology

The design of an economic model describing the main features of a certain managerial problem is an essential step for decision makers. The model should allow the practitioner to forecast and design economic policies that reduce, for example, the production cost and market prices. The dynamic stochastic framework has been extensively used in economics to model almost any problem involving sequential decision-making over time and under uncertainty.

Consumers are the agents making consumption plans. Market demand reflects the consumer's decisions as the demand curve shows the quantity of a product demanded in the market over a specified time period and state of nature, at each possible price. Demand could be influenced by income, tastes and the prices of all other goods. The study of demand pattern is one of the key steps in managerial problems.

Firms make decisions on production, advertisement, etc. within the constraints of their technological knowledge and financial contracts. In particular, firms should maximize their expected profits subject to technological and risk constraints. In many actual production processes, these constraints contain parameters, often unknown even when they have physical meaning. Prices could be decision variables as in Cournot models, or could be considered as parameters as in perfect competition models.

Market equilibrium y is a vector of decision variables of agents (consumers and firms) and prices that makes all decisions compatible with one another (i.e. y clears the market in competitive models or y satisfies Nash equilibrium in strategic models). In general, an equilibrium y can be characterized by a system of nonlinear equations $H(\theta, y, x) = 0$, where θ is a vector of parameters, and x is a vector of exogenous variables that affects agents' decisions through technologies and tastes.

To obtain predictive models for decision makers, we face the problem of having to estimate several parameters θ. The optimal determination of these parameters is essential for building economic models that can address a large class of questions. Although some of the parameters can be calibrated easily using the available data, others remain uncertain due to the lack of empirical information. We propose a worst-case strategy to adjust or calibrate these parameters to the model using limited empirical data.

2.1 Worst-Case Modelling

Some of the variables (y, x) can be empirically determined (observed data). Let z be the vector of non-observable variables, r be the vector of observable variables, and let $H(\theta, z, r) = 0$ denote the system of nonlinear equations that characterize an equilibrium of the economy, where θ is a vector of parameters. The aim of the worst-case modelling is essentially to fit the best model (the best choice of parameters θ) to available data in view of the worst-case unobservable decision z. When designing economic models, the worst-case design problem is a continuous minimax problem of the form

$$\min_{\theta\in\Theta, r\in R} \max_{z\in Z} \|r - \widehat{r}\|_2^2 \qquad \textit{subject to} \;\; H(\theta, z, r) = 0, \tag{1}$$

where $\Theta \subset \mathbb{R}^n$ is the feasible set of parameters, $R \subset \mathbb{R}^m$ is the feasible set of observable variables, $Z \subset \mathbb{R}^l$ is the feasible set of non-observable variables and $\widehat{r}$ is a data sample of r. In other words, our aim is to minimize the maximum deviation for the worst-scenario of realizable decisions. Thus, the optimal solution θ^* to this problem defines a robust optimal specification of the economic model. This criterion for choosing parameters typically can be applied to engineering, economics and finance frameworks.

For solving continuous minimax problems we use the global optimisation algorithm developed by (Žaković and Rustem, 2003). They consider an algorithm for solving semi-infinite programming problem since any continuous minimax problem of the form

$$\min_{\theta\in\Theta} \max_{z\in Z} \{f(\theta, z) : g(\theta, z) = 0,\} \tag{2}$$

can be written as a semi-infinite programming problem. Note that the above problem is equivalent to

$$\min_{\theta\in\Theta, \rho} \left\{\rho : \max_{z\in Z} \{f(\theta, z) \leq \rho : g(\theta, z) = 0\}\right\}, \tag{3}$$

and since $\max_{z\in Z} f(\theta, z) \leq \rho$ if and only if $f(\theta, z) \leq \rho$, for all $z \in Z$, we can solve the alternative semi-infinite problem:

$$\begin{array}{ll} \min_{\theta\in\Theta,\rho} & \rho \\ \textit{subject to} & f(\theta, z) \leq \rho, \;\; \forall \;\; z \in Z, \\ & g(\theta, z) = 0, \;\; \forall \;\; z \in Z. \end{array} \tag{4}$$

Žaković and Rustem's algorithm involves the use of global optimisation to compute the global worst-case. The global optimisation approach is essential to guarantee the robustness property of the solution of the minimax problems. This is because a crucial step to solve the semi-infinite problem is to find $\theta \in \Theta$, $f(\theta, z) \leq \rho$, $g(\theta, z) = 0$, for all $z \in Z$. To reduce the cost of computing global optima, it is recommended to restrict the domains Θ and Z as much as possible given the information available. The monograph edited by (Pardalos and Resende, 2002) reviews the global optimisation literature (see Chap. 6).

2.2 Modelling the Uncertainty

As discussed in the introduction, the importance of considering uncertainty via scenarios is well known in finance and engineering applications. In this section, we extend the scenario tree methodology to the computation of equilibria in stochastic dynamic economic models. In such models, agents (consumers and firms) face a problem involving sequential decision making over time and under uncertainty. Given the parameters $\theta \in \Theta$ calibrated using the available information, assume that each agent face the decision problem:

$$\max_{x_t} \sum_{t=0}^{T} E\left[U_\theta\left(x_t, a_t, t\right)\right] \quad subject \;\; to \;\; g_\theta\left(x_t, a_t, t\right) \leq 0 \;\; a.e., \tag{5}$$

where $\{x_t\}$ are the decision variables, $\{a_t\}$ are observable Markovian random variables with a continuous distribution function, $U_\theta\left(x_t, a_t, t\right)$ represents the agents' preferences and a.e. denotes "almost everywhere". This decision model will be characterized by the information available at each period of time, among other things. We assume that this information is the same for all agents. Let σ_t be the σ-algebra generated by $\{a_s : 0 \leq s \leq t\}$ and let $\{\sigma_t\}$ be the complete specification of the revelation of information through time, called filtration.

To reduce the cost of computing optima, we approximate the process $\{a_t\}$ and the associated information set $\{\sigma_t\}$ by a discrete process $\{a_{t,s}\}_{s=1}^{S_t}$ of possible outcomes for each t, and a discrete information structure $\{\mathcal{F}_t\}_{t=1}^{T}$. A discrete information structure is formally defined as follows: Given a finite sample space $\Omega = \{\omega_1, \ldots, \omega_M\}$ that represents the states of world, a discrete information structure is a sequence of σ-algebras $\{\mathcal{F}_t\}_{t=1}^{T}$ such that: 1) $\mathcal{F}_1 = \{\Omega, \emptyset\}$, 2) $\mathcal{F}_T = 2^{\Omega}$, 3) $\mathcal{F}_{t+1}$ is finer than $\mathcal{F}_t$, $\forall t = 1, \ldots, T-1$. The *scenario tree* associated with the discrete information structure $\{\mathcal{F}_t\}_{t=1}^{T}$ is defined as $\Im = \bigcup_{t \in \mathbb{T},\, s \in \mathbb{S}_t} (t, s)$, where $\mathbb{T} = \{0, \ldots, T\}$ and $\mathbb{S}_t = \{1, \ldots, S_t\}$. Each (t, s) is called a *tree node* or *scenario*. For each scenario tree we can define a preorder relation $\succ$ such that $(t, s) \succ (t', s')$ if and only if the node (t', s') comes after (t, s) in the tree, that is, if $t' > t$ and $s' \subset s$.

Two main approaches to generate discrete scenario trees have been considered to date. The first one is known as the optimisation approach. This method considers the relevant statistical properties of the random variable such as the first four moments of the marginal distributions. Then a nonlinear optimisation problem is formulated where the objective is to minimize the square distance between the statistical properties of the constructed tree and the actual specifications. The second approach is called the simulation approach and only uses the sample from the fitted cumulative distribution function. In this paper, we generate discrete scenario trees using the simulation and randomized clustering approach proposed by (Gülpinar, Rustem and Settergren, 2004). This method is a simulation-based approach that clusters scenarios randomly.

2.3 Computing Stochastic Dynamic Equilibria

Once the uncertainty of the problem is represented by a discrete scenario tree, the stochastic dynamic decision problem of each agent can be written as follows:

$$\max_{x_{t,s}} \sum_{t=0}^{T} \sum_{s=0}^{S_t} \beta_{ts} \, U_\theta \left(x_{t,s}, a_{t,s}, t\right) \quad subject \;\; to \;\; g_\theta \left(x_{t,s}, a_{t,s}, t\right) \leq 0, \forall \left(t, s\right), \tag{6}$$

where $\{\beta_{ts}\}$ are the conditional probabilities associated to state s at each period t, with $\beta_0 = 1$, and $\lambda \geq 0$ denotes the Lagrange's multipliers associated with the inequality constraints. Under appropriate convexity assumptions, equilibria are characterized by the first-order conditions of the agents' problems and the market clearing conditions that define the economic model. These optimality conditions can be seen as a special class of problems known as nonlinear complementarity (complementarity conditions stem from complementarity slackness in the first-order optimality conditions). Mathematically, these problems are stated as follows: find $p^T = (x^T, \lambda^T) \geq 0$ such that $F(p) \geq 0$ and $p^T F(p) = 0$. A nonlinear complementarity problem can be reformulated as a standard system of equations $H(z) = 0$, where

$$H(z) = \begin{pmatrix} p^T F(p) \\ F(p) - s \end{pmatrix}, \tag{7}$$

s are *slack variables* and $z^T = (x^T, \lambda^T, s^T) \geq 0$. Often, the decision variables, the Lagrange's multipliers and the slack variables may take any value within a certain range bounded by positive finite lower and upper bounds, $l \leq z \leq u$. A brief summary of standard approaches for solving these problems can be found in (Esteban-Bravo, 2004). In Chap. 13, (Pardalos and Resende, 2002) provide an excellent introduction to complementarity and related problems.

The final stage of the methodology is the computation of equilibria for the stochastic economic model using the generated scenario tree. In this paper, we consider a version of the interior point method given in (Esteban-Bravo, 2004). This algorithm can find accurate solutions with little computational cost, what it is a desirable property as the scenario tree can be expanded to arbitrarily large sizes as the temporal horizon increases. The main idea of the algorithm is the application of the Gauss-Newton method to solve the following perturbed system of nonlinear equations,

$$\begin{aligned} J(z_k)^T H(z_k) - w_k^1 + w_k^2 &= 0, \\ (Z_k - L)\, W_k^1 - \mu &= 0, \\ (U - Z_k)\, W_k^2 - \mu &= 0, \\ w_k^1, w_k^2 &> 0, \end{aligned}$$

where $Z_k = diag(z_k)$, $L = diag\,(l)$, $U = diag\,(u)$ and $J(z_k)$ denote the Jacobian matrix of H. Note that when $\mu \to 0$, we compute the original problem. Following the Gauss-Newton approach, the Hessian of the perturbed system is approximated by its first term. As a consequence, this algorithm has the very desirable property that it finds accurate solutions with little computational cost.

3 Modelling the NSW Spot Electricity Market

In this section, we focus on an application of the robust modelling methodology for the deregulated electricity market in NSW, Australia. The deregulated electricity market should be modelled as a sequential trade for goods and assets. A model of sequential markets is a system of reopening spot markets, which is a market for immediate delivery. In other words, a seller and a buyer agree upon a price for a certain amount of electric power (MWs) to be delivered at the current period (in case of electricity markets, in the near future). This agreement is monitored by an independent contract administrator who matches the bids of buyers and sellers.

We consider an economy with three generators that face the NSW Electricity System. In NSW electricity markets, the role of a financial contract is small and, as a consequence, we only focus on spot markets that trade most of the local electricity.

To meet electricity demand and for the spot electricity market to operate efficiently, a reliable forecast of daily electricity demand is required. Typically, the electricity demand is affected by several exogenous variables such as air temperature, and varies seasonally (the total demand will generally be lower over weekend days than weekdays, and higher in summer or winter than in fall or spring). Electricity forecasting process must therefore consider both aspects. The time spans involved in electricity forecasts may range from half an hour to the next few days. The technique described in this paper considers the day-to-day forecast as the aim is to guide decisions on capacity, cost and availability to meet the demand or the necessity to purchase from other producers. In the very short-term electricity market, the demand varies little in response to price changes so we can say that the demand is not affected by prices, i.e. it is inelastic within the observed range of prices variation.

Generators make decisions about the amount of electricity to produce within the constraints of their technological knowledge. Modelling the technologies of a generation company requires special attention. Generators can produce electricity by means of hydro, thermal and pumped storage plants. A pumped storage hydro plant is designed to save fuel costs by serving the peak load (a high fuel-cost load) with hydro energy and then pumping the water back up into the reservoir at light load periods (a lower cost load). Moreover, generators face uncertainty because of the inflows in the case of hydro generation and the price of fuel in the case of thermal and pumped storage. As the generation system in NSW is overwhelmingly thermally based, we just consider this kind of technology.

3.1 The Demand

The problem of modelling the pattern of the electricity demand has previously been studied in the literature; see e.g. (Rhys, 1984), (Harvey and Koopman, 1993), (Henley and Peirson, 1997), (Valenzuela and Mazumdar, 2000), among others. In this paper, we assume that the daily electricity demand is affected exogenously by air temperature. In addition, we take into account its daily pattern. Note that the total electricity demand is generally lower over weekend days than weekdays, and higher in summer or winter than in fall or spring. Also in the short term we can assume that the aggregate demand for electricity is inelastic, as the quantity of power purchased varies little in response to price changes.

We consider electricity demand data for each day in New South Wales, Australia, between 1999 and 2002 (see http://www.nemmco.com.au/data/). The data sequence starts at January 1, 1999 and ends at April 30, 2002. All values are in MW and according to Eastern Standard Time. The result is a sequence of 1247 values (see Fig. 3 in Appendix). Let x_j denote the electricity demand at the j-th day. The data for temperature are drawn from the file AUCNBERA.txt given in the website http://www.engr.udayton.edu/weather. This dataset contains information on the daily average temperatures for Canberra. Let f_j denote the average temperature (^{o}F) at the j-th day.

With a daily temporal frequency, there are patterns repeated over a stretch of observations. In particular, we observe two seasonal effects: weekly (such as weekdays and weekends) and monthly (such summer and winter). Assuming that these effects follow a deterministic pattern, we consider stational dummy variables. Let $d_{jt} = 1$, if the t-th observation is a j-th day and $d_{jt} = 0$, otherwise, defining the weekly periodic effects, with $j = 1$ for Mondays, $j = 2$ for Tuesdays, and so on; and $\delta_{jt} = 1$, if the t-th observation is a j-th month and $\delta_{jt} = 0$, otherwise, defining the monthly periodic effects, with

$j = 1$ for January, $j = 2$ for February and so on. Thus, given $n = 1247$ pairs of observations (x_j, f_j), with $j = 1, \ldots, n$, we consider the following regression model:

$$X_t = \mu + c_1 f_j + c_2 f_j^2 + \sum_{j=1}^{6} \gamma_j (d_{jt} - d_{7t}) + \sum_{j=1}^{6} \gamma_j' (d_{jt} - d_{7t}) f_j$$
$$+ \sum_{j=1}^{11} \beta_j (\delta_{jt} - \delta_{12t}) + \sum_{j=1}^{11} \beta_j' (\delta_{jt} - \delta_{12t}) f_j$$
$$+ \sum_{j=1}^{6} \gamma_j'' (d_{jt} - d_{7t}) f_j^2 + \sum_{j=1}^{11} \beta_j'' (\delta_{jt} - \delta_{12t}) f_j^2 + \varepsilon_t, \tag{8}$$

where $\{\varepsilon_j\}_j$ is a Gaussian and second order stationary process with zero mean and covariance function, $\gamma(s) = E[\varepsilon_t \varepsilon_{t-s}]$. This specification avoids the multicollinearity problems derived from the fact that $\sum_{j=1}^{7} d_{jt} = 1$ and $\sum_{j=1}^{12} \delta_{jt} = 1$. In particular, we consider a linear regression model using all the dummies variables and assuming that $\sum_{j=1}^{7} \gamma_j = 0$, and $\sum_{j=1}^{12} \beta_j = 0$. For an introduction to the estimation of this type of models see e.g. (Brockwell and Davis, 1987).

The regression parameters were estimated by the ordinary least squares (OLS) method using STATA (see http://www.stata.com/). Least-square regression estimations can be found in Appendix. The degree of explanation of this model is quite significant, as its R-Squared and adjusted R-Squared values are 0.7793 and 0.7695, respectively.

The study of the plots of residual autocorrelation and partial autocorrelation estimates (see below Fig. 4 in Appendix) suggests an autoregressive AR(1) model for the perturbation ε_t. Model (2) considers an autoregressive AR(1) specification for the process $\{\varepsilon_j\}_j$, $\varepsilon_j = \tau \varepsilon_{j-1} + a_j$, where $|\tau| < 1$ and $\{a_j\}_j$ are independent identically distributed disturbances with zero mean and constant variance, σ_a^2. Using STATA to estimate Model (2) by the OLS method, its coefficient estimation is $\widehat{\tau} = 0.60069$, with $\sigma_a^2 = 1.5402e$, and its R-Squared and adjusted R-Squared values are 0.3687 and 0.3682, respectively. Simple and partial autocorrelations of its residuals, shown in Fig. 5 in Appendix, reveal that Model (1) and Model (2) fit data.

The Markovian process $\{a_t\}_t$ is approximated by a discrete scenario tree $\{a_{ts}\}$ as presented in Sect. 2.2. Following the simulation and randomized clustering approach proposed in (Gülpinar, Rustem and Settergren, 2004) and given the covariance matrix σ_a^2, we construct a tree with a planning horizon of $T+1$ days (today and T future periods of time) and a branching structure of $1-2-4-6$. This means that the tree has an initial node at day 0, 2 at day 1,... The scenario tree provides information about the probabilities β_{ts} associated with the different states s at each period t, with $\beta_0 = 1$, and the

$AR(1)$ stochastic process of error terms a_{ts} at each state s and period t. The values of these elements can be found in Appendix.

3.2 The Problem of the Electricity Generators

As we mentioned before, in the very short-term electricity market, the demand varies little in response to price changes. In this applications where the planning time horizon is assumed to be three days or periods of time, we should consider a pure competitive behaviour of generators rather than oligopolistic strategies. Therefore, in a deregulated environment, the purpose of the short-term generator is to maximize its expected profit on its technological constraints over a time period of length $T+1$, today and the planning time horizon. This means that each generator collects its revenue from selling electricity at spot prices in the spot market. There are network capacity constraints affecting generators and therefore the total amount of electricity that these generators can produce will be bound by the network externalities. The notation used to present the problem of the electricity generators is the following, at each period $t = 0, 1, \ldots, T$:

Decision variables:

p_t, spot price,

y_{jt}, spot electricity production of generator j,

w_{jt}, input of generator j.

Parameters:

T, maximum number of periods,

J, number of generators,

$0 \leq r$, discount rate for generators,

q_{jt}, unit generation cost (input's price) of generator j,

A_j, a_j, parameters associated with the technology of generator j,

N, maximum capacity of the network,

M, rate limit to generation over two periods,

l_j, u_j, minimum and maximum of generation capacity of generator j, respectively.

The generation constraints are:

Cobb-Douglas type technological constraint: $y_{jt} \leq A_j w_{jt}^{a_j}$.

Network capacity constraint: $\sum_{j' \neq j} y_{j't} + y_{jt} \leq N$.

Rate limit to generation over two consecutive periods: $y_{jt} - y_{jt-1} \leq M$.

We consider a market with three generators, $j = 1, \ldots, J$ with $J = 3$, that aim to maximize the expected revenues and minimize the expected costs:

$$\sum_{t=0}^{T} \left(\frac{1}{1+r} \right)^t \left[p_t \cdot y_{jt} - q_{jt} \cdot w_{jt} \right]. \tag{9}$$

Thus, the decision problem of each generator is given by

$$
\begin{array}{ll}
\max_{y_j,w_j} & \sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t [p_t\cdot y_{jt}-q_{jt}\cdot w_{jt}] \\
subject\ \ to & y_{jt}\le A_j w_{jt}^{a_j},\forall t, \\
& \sum_{j'\neq j}^{T} y_{j't}+y_{jt}\le N,\forall t, \\
& y_{jt}-y_{jt-1}\le M,\forall t, \\
& l_j\le y_{jt}\le u_j,\forall t, \\
& 0\le w_j.
\end{array}
\tag{10}
$$

Most of studies on the generator's problem have been based on a cost function (i.e. the minimum cost of producing a given level of output from a specific set of inputs), even though this formulation is equivalent to the one that uses technology constraints (this is proved by dual arguments, see e.g. Varian, 1992 and Mas-Colell, Whinston and Green, 1995). But, in case of having limited information, the formulation with technological constraints is more recommendable as the specification of cost functions requires detailed information on the labor costs, inputs costs, and buildings and machinery amortization, among others. In particular, we choose a Cobb-Douglas technology as this specification is characterized by a ready capability to adapt to new, different, or changing requirements. For example, they can exhibit increasing, decreasing or constant return to scale depending on the values of their parameters. Furthermore, we can readily derive the analytical form of its associated cost function.

On the other hand, the modelling considered here has strong simplifications on the transmission side although these simplifications could have effects for the analysis. This is because we lack sufficient information to calibrate this externality.

Next, we introduce the concept of equilibrium, the basic descriptive and predictive tool for economists. The equilibrium of this economy is a vector prices p^* and an allocation $\left(y_j^*,w_j^*\right)$ for all $j=1,\ldots,J$, that satisfies:

- For each $j=1,\ldots,J$, $\left(y_j^*,w_j^*\right)$ is the solution of Problem (10).
- Generators fulfil market demand, i.e.

$$
\sum_j y_{jt}^* = \widehat{X}_t,\forall t, \tag{11}
$$

where $\widehat{X}$ is the estimation of the market demand determined by Model (8).

Then, under appropriate convexity assumptions, equilibria can be characterized by the first order conditions of all generators' problems (10) and the market clearing conditions (11). In other words, the vector (p^*,y^*,w^*) is an equilibrium if, for all $j = 1, \dots, J$, there exist Lagrange multiplier vectors $\gamma_j^1, \gamma_j^2, \gamma_j^3 \geq 0$, such that:

$$\begin{aligned}
&\left(\tfrac{1}{1+r}\right)^t p_t^* - \gamma_{jt}^1 - \gamma_{jt}^2 + \gamma_{jt}^3 = 0, \forall t,\\
&-\left(\tfrac{1}{1+r}\right)^t q_{jt} + \gamma_{jt}^1 A_j a_j w_{jt}^{*a_j - 1} = 0, \forall t,\\
&y_{jt}^* - A_j w_{jt}^{*a_j} + h_{jt}^1 = 0, \forall t,\\
&\gamma_{jt}^1 h_{jt}^1 = 0, \forall t,\\
&\textstyle\sum_j y_{jt}^* + h_{jt}^2 - N = 0, \forall t,\\
&\gamma_{jt}^2 h_{jt}^2 = 0, \forall t,\\
&y_{jt}^* - y_{jt-1}^* + h_{jt}^3 - M = 0, \forall t,\\
&\gamma_{jt}^3 h_{jt}^3 = 0, \forall t,\\
&\textstyle\sum_j y_{jt}^* = \widehat{X}_t, \forall t,\\
&l_j \leq y_{jt}^* \leq u_j,\\
&0 \leq w_j^*,
\end{aligned} \tag{12}$$

where h^1, h^2 and $h^3 \geq 0$ are slack variables.

3.3 Worst-Case Calibration

To obtain predictive decision models for generators, we are faced with the problem of having to estimate several parameters. The optimal calibration of these parameters is the aim of this section.

As we mentioned before, some of the parameters can be calibrated easily. Given that the planning horizon of electricity generators considered is short, $T = 3$, the impact of the discount factor parameter is small. In this model, we set the discount factor as $r = 0.05$ for all generators.

Fuel prices are subject to a substantial margin of error. However, in the case of coal, prices are determined in a world market and the data can be found in http://www.world bank.org/prospects/pinksheets0. In this model we assume that fuel prices for each generator are given as $q_{1t} = 25.6$, $q_{2t} = 26$, $q_{3t} = 15$, for all $t = 0, 1, \dots, T$.

One of the most important parameters in the management of the electricity generation is the maximum capacity of the network N. As $\max |X_t| = 455670$, where X_t is the observed electricity demand, estimates of this parameter can be specified as $N = 456000$. The rate limit to generation over two periods M plays also an important role in the generation of electricity. As $\max |X_t - X_{t-1}| = 84622.3$, we set $M = 85000$. Generation capacity is also constrained by lower and upper bounds: $l_j = 0$ for all $j = 1, 2, 3$ and $u_1 = 350000$, $u_2 = 220000$, $u_3 = 280000$.

To calibrate the parameters that remain uncertain, we will consider the worst-case modelling presented in Sect. 2.1. In this context, the parameters should satisfy the optimality conditions (12), the available information is the daily average price p^o and daily observed demand X^o, and the worst-case unobservable decision corresponds to the input's decision variable w.

As we can determine parameters N and M, we will not consider their associated constraints in the calibration analysis. In addition, it is predictable that the variable $h^1 = 0$ (generators are willing to generate the maximum amount of electricity) which implies $y_{jt} = A_j w_{jt}^{a_j}$ for all t and j. Therefore, the optimal conditions (12) can be simplified as follows:

$$p_t A_j a_j w_{jt}^{a_j - 1} - q_{jt} = 0, \forall t,$$
$$\sum_j A_j w_{jt}^{a_j} - X_t^o = 0, \forall t,$$

where X_t^o is the daily observed demand at each day t. Let $C(w, p, A, a)$ denote this system of nonlinear equations.

Thus, we define the best choice of parameters $\{A_j\}$, $\{a_j\}$ in view of the worst-case unobservable decisions p, $\{w_j\}$ as the solution of the minimax problem:

$$\begin{array}{ll} \min_{A_j, a_j, p \geq 0} \max_w & \|p - p^o\| \\ subject\ \ to & C(w, p, A, a) = 0, \end{array} \tag{13}$$

given the observed demand X_t^o and the average price p_t^o at each day t. In particular, we consider the following observed data:

day	28/4/2002	29/4/2002	30/4/2002
X_t^o	334443.7	382222.6	389739.8
p_t^o	23.53	32.67	25.15

(14)

As recommended before to guarantee little computational cost, we suggest to restrict the interval of the variables $\{A_j\}$, $\{a_j\}$, $\{w_j\}$, p given the information available. In the context of the Australian electricity market, the bounds should be:

	A_1	A_2	A_3	a_1	a_2	a_3	$\{w_{jt}\}$	p
lower bound	16500	16500	18000	0.1	0.1	0.1	10000	15
upper bound	18000	18000	20000	1.0	1.0	1.0	17000	70

(15)

Therefore, the solution to Problem (13) is

	$j = 1$	$j = 2$	$j = 3$
A_j	18000	18000	20000
a_j	0.194774	0.195836	0.152826.

(16)

3.4 Computing Equilibrium

Given the scenario tree computed in Sect. 3.1, the stochastic version of the generators' problem (10) is defined as:

$$\begin{aligned}
\max_{y_j,w_j} \quad & \textstyle\sum_{t=0}^{T}\sum_{s=0}^{S_t}\left(\frac{1}{1+r}\right)^t \beta^{t,s}\ \left[p_{t,s}\cdot y_{jt,s} - q_{jt,s}\cdot w_{jt,s}\right] \\
\textit{subject to} \quad & \\
& y_{jt,s} - A_j w_{jt,s}^{a_j} \le 0,\ \forall t,s, \\
& \textstyle\sum_{j'\neq j} y_{j't,s} + y_{jt,s} \le N,\ \forall t,s, \\
& y_{jt,s} - y_{jt-1,s(t-1)} \le M,\ \forall t,s, \\
& l_j \le y_{jt,s} \le u_j,\ \forall t,s, \\
& 0 \le w_{jt,s},\ \forall t,s,
\end{aligned} \tag{17}$$

where $s\,(t-1)$ is the predecessor state. Problem (17) can be transformed into an equality constrained problem by introducing slack variables h_j^1, h_j^2, $h_j^3 \ge 0$, and a barrier function that penalizes the infeasibility of the inequality constraints in the slack h_j^1, h_j^2, $h_j^3 \ge 0$ and decision variables $0 \le w_j$ and $l_j \le y_j \le u_j$. Thus, the transformed problem is defined as follows:

$$\begin{aligned}
\max_{y_j,w_j} \quad & \textstyle\sum_{t=0}^{T}\sum_{s=0}^{S_t}\left(\frac{1}{1+r}\right)^t \beta^{t,s}\ \left[p_{t,s}\cdot y_{jt,s} - q_{jt,s}\cdot w_{jt,s}\right] \\
& -\mu\ \textstyle\sum_{t=0}^{T}\sum_{s=0}^{S_t}[\log\left(u_j - y_{jt,s}\right) + \log\left(y_{jt,s} - l_j\right) + \log\left(w_{jt,s}\right) \\
& +\textstyle\sum_{m=1}^{3}\log\left(h_{jt,s}^m\right)] \\
\textit{subject to} \quad & \\
& y_{jt,s} - A_j w_{jt,s}^{a_j} + h_{jt,s}^1 = 0, \forall t,\ \forall s, \\
& \textstyle\sum_{j'\neq j} y_{j't,s} + y_{jt,s} + h_{jt,s}^2 - N = 0, \forall t,\ \forall s, \\
& y_{jt,s} - y_{jt-1,s} + h_{jt,s(t-1)}^3 - M = 0, \forall t,\ \forall s.
\end{aligned} \tag{18}$$

Under appropriate convexity assumptions, the vector (y_j, w_j, h_j) is said to satisfy the necessary and sufficient conditions of optimality for Problem (18) if there exist Lagrange multiplier vectors $\gamma_j^1, \gamma_j^2, \gamma_j^3 \ge 0$ such that for all j, all t, all s :

$$\begin{aligned}
& \left(\tfrac{1}{1+r}\right)^t\ \beta^{t,s} p_{t,s} + \mu\left(u_j - y_{jt,s}\right)^{-1} - \mu\left(y_{jt,s} - l_j\right)^{-1} - \gamma_{jt,s}^1 - \gamma_{jt,s}^2 + \gamma_{jt,s}^3 = 0, \\
& -\left(\tfrac{1}{1+r}\right)^t\ \beta^{t,s} q_{jt,s} - \mu w_{jt,s}^{-1} + \gamma_{jt,s}^1\ A_j a_j w_{jt,s}^{a_j-1} = 0, \\
& -\mu\left(h_{jt,s}^m\right)^{-1} - \gamma_{jt,s}^m = 0,\ \forall m = 1,2,3, \\
& y_{jt,s} - A_j w_{jt,s}^{a_j} + h_{jt,s}^1 = 0, \\
& \textstyle\sum_j y_{jt,s} + h_{jt,s}^2 - N = 0, \\
& y_{jt,s} - y_{jt-1,s} + h_{jt,s(t-1)}^3 - M = 0.
\end{aligned} \tag{19}$$

Let $Z^1 = \mu (U-Y)^{-1}$, $Z^2 = \mu (Y-L)^{-1}$, $Z^3 = \mu W^{-1}$ and $Z^{4m} = \mu (H^m)^{-1}$,where $Y = diag(y)$, $L = diag(l)$, $U = diag(u)$, $W = diag(w)$, and $H^m = diag(h^m)$ for all $m = 1,2,3$. Then, the above conditions can be written as:

$$\begin{aligned}
&\left(\tfrac{1}{1+r}\right)^t \beta^{t,s} p_{t,s} + z^1_{jt,s} - z^2_{jt,s} - \gamma^1_{jt,s} - \gamma^2_{jt,s} + \gamma^3_{jt,s} = 0,\\
&-\left(\tfrac{1}{1+r}\right)^t \beta^{t,s} q_{jt,s} - z^3_{jt,s} + \gamma^1_{jt,s} A_j a_j w_{jt,s}^{a_j-1} = 0,\\
&-z^{4m}_{jt,s} - \gamma^m_{jt,s} = 0, \forall m = 1,2,3,\\
&y_{jt,s} - A_j w_{jt,s}^{a_j} + h^1_{jt,s} = 0,\\
&\textstyle\sum_j y_{jt,s} + h^2_{jt,s} - N = 0,\\
&y_{jt,s} - y_{jt-1,s(t-1)} + h^3_{jt,s} - M = 0,\\
&(U_j - Y_j)\, Z^1_j e - \mu e = 0,\\
&(Y_j - L_j)\, Z^2_j e - \mu e = 0,\\
&W_j Z^3_j e - \mu e = 0,\\
&H^m_j Z^{4m}_j e - \mu e = 0, \forall m = 1,2,3,
\end{aligned} \tag{20}$$

for all $j = 1,2,3$, all $s = \{1, \dots S_t\}$, where $S_t = 2$, and all $t = 0,1,2,3$.

Assume that we aim to forecast the prices, inputs and electricity outputs in equilibrium for the days May 1, May 2 and May 3, 2002, given the temperature data $f_1 = 45.5$, $f_2 = 41.4$, $f_3 = 42.0$. Using the initial point $\xi_0 = 1^T$, the interior-point algorithm converges to the equilibrium given in Appendix.

To show the accuracy of the computed equilibrium, we consider the expected value of the computed equilibrium prices. Given the probabilities β_{ts} associated with the different states s at each period t, by Bayes' rule, we calculate the marginal probability $\pi_{ts} = \prod_{(t,s)\succ(t',s')} \beta_{t's'}$ (see Appendix). Then, the expected value of the computed equilibrium prices $E[p_t] = \sum_{s=1}^{S_t} \pi_{ts} p_{ts}$ and the actual prices for $t = 1,2,3$ (which can be found in http://www. nemmco.com.au/data/) are shown in Fig. 1, what reveals that the model captures the essential features of the price's behaviour.

Let now assume that we aim to forecast the prices, inputs and outputs in equilibrium for the days October 1, October 2 and October 3, 2002, given the temperature data $f_1 = 58.8$, $f_2 = 49$, $f_3 = 48.0$. The actual and forecast equilibrium prices are shown in Fig. 2.

Note that the accuracy of the prediction depends on the data used to calibrate the parameters of the model. A structural change in the market can affect the prices and productions in equilibrium, and in that case an updated calibration of the model should be considered using the new information.

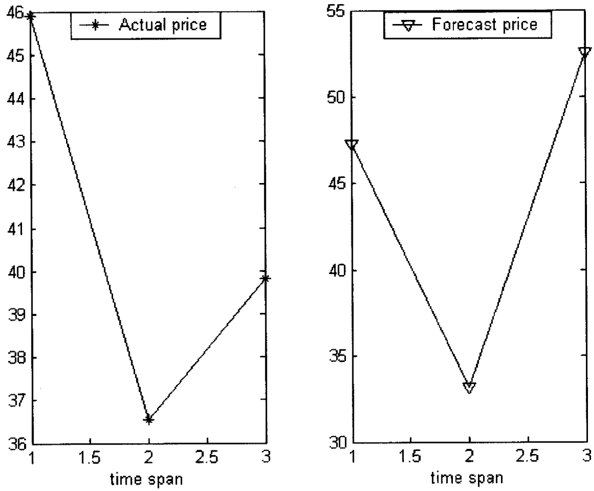

Fig. 1. Actual and computed prices for May 1st, 2nd, 3rd, 2002

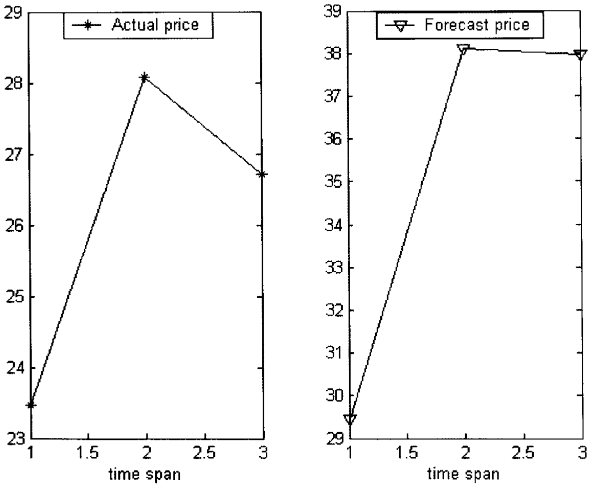

Fig. 2. Actual and computed prices for October 1st, 2nd, 3rd, 2002

4 Summary and Conclusions

This paper presents a methodology to build and solve stochastic dynamic economic models using limited data information. The approach has the potential for application in many economics sectors as practitioners often face the problem of having significantly less data than necessary for analyzing standard decision problems.

Decision-makers require the use of a stochastic dynamic complex model to approximate economic problems in a realistic way, and they often lack sufficient information to estimate the parameters involved in the model accurately. In this paper, we present a robust procedure for calibrating the parameters of model that best fits to the available data. The robust calibration of the model is achieved by a worst-case approach, involving the computation of a minimax problem. Also, we consider a scenario tree approach to model the underlying randomness of the demand. We generate scenarios using the simulation and randomized clustering approach and then, we compute equilibria by means of the interior-point approach. This algorithm can find accurate solutions incurring little computational cost.

We illustrate the performance of the method considering the NSW Australian deregulated electricity market. From the analysis of the results, we can conclude that this approach is able to forecast the pattern of equilibrium prices using limited information on the production side.

Appendix

The least-square regression coefficients of Model (8) are:

$\widehat{\mu} = 420453.7$,

$\widehat{\gamma} = (-96356.6, -16410.4, 86594.03,\ 38223.82, 17360.08, 37401.59)^T$,

$\widehat{\beta} = (-481045.5, 12954.2, -25529.96,\ -27617.52, 265764.7, 136627.8,$
$37069.41, -55922.99, -35578.56, -679.53, 98605.83)^T$,

$\widehat{c}_1 = -1976.111$,

$\widehat{\gamma}' = (2243.55, 642.10, -2502.44,\ -997.01, -129.57, -1008.41)^T$,

$\widehat{\beta}' = (12235.02, -4899.86, -94.86, 63.39, -7934.55,$
$-2853.79, 1208.76, 5784.12, 2677.32, 93.16494, -4471.44)^T$,

$\widehat{c}_2 = 16.45$,

$\widehat{\gamma}'' = (-20.82, -4.14, 21.37,\ 10.16, 1.53, 8.87)$,

$\widehat{\beta}'' = (-75.68, 46.27, 8.56, 5.59,\ 59.65, 17.30, -22.62,$
$-82.25, -35.93, -3.36, 46.17)^T$.

The values of the probabilities β_{ts} associated with the different states s at each period t, with $\beta_0 = 1$, and the $AR(1)$ stochastic process of error terms a_{ts} at each state s and period t are:

for $t = 1$,

	1	2
β_s	0.26	0.74
a_s	0.73	0.59

(21)

for $t = 2$,

	1	2	3	4
β_s	0.79	0.20	0.79	0.20
a_s	0.67	0.51	0.60	0.80

(22)

for $t = 3$,

	1	2	3	4	5	6	7	8
β_s	0.47	0.52	0.62	0.37	0.40	0.59	0.6	0.4
a_s	0.54	0.72	0.68	0.54	0.75	0.55	0.57	073

(23)

The computed values of equilibrium are:
for $t = 0$, $p_0^* = 49.39$, and

	1	2	3
y_{j0}^*	1.51e5	1.53e5	1.05e5
w_{j0}^*	57.10e3	57.22e2	53.07e3

(24)

for $t = 1$,
$p_1^* = (47.29, 47.29)$, and

s=1	1	2	3	s=2	1	2	3
y_{js}^*	1.50e5	1.52e5	1.04e5	y_{js}^*	1.50e5	1.52e5	1.046e5
w_{js}^*	54.11e3	54.21e3	50.42e3	w_{js}^*	54.11e3	54.21e3	50.42e3

(25)

for $t = 2$,
$p_2^* = (33.19, 33.19, 33.19, 33.19)$, and

s=1	1	2	3	s=2	1	2	3
y_{js}^*	1.38e5	1.39e5	9.81e4	y_{js}^*	1.38e5	1.39e5	9.81e4
w_{js}^*	34.85e3	34.90e3	33.19e3	w_{js}^*	34.85e3	34.90e3	33.19e3

(26)

s=3	1	2	3	s=4	1	2	3
y_{js}^*	1.38e5	1.39e5	9.81e4	y_{js}^*	1.38e5	1.39e5	9.81e4
w_{js}^*	34.85e3	34.90e3	33.19e3	w_{js}^*	34.85e3	34.90e3	33.19e3

(27)

for $t = 3$,
$p_3^* = (52.53, 52.53, 52.53, 52.53, 52.53, 52.53, 52.53, 52.53)$, and

s=1	1	2	3	s=2	1	2	3
y_{js}^*	1.54e5	1.56e5	1.06e5	y_{js}^*	1.54e5	1.56e5	1.06e5
w_{js}^*	61.64e3	61.77e3	57.07e3	w_{js}^*	61.65e3	61.77e3	57.07e3

(28)

s=3	1	2	3	s=4	1	2	3
y_{js}^*	1.54e5	1.56e5	1.06e5	y_{js}^*	1.54e5	1.56e5	1.06e5
w_{js}^*	61.65e3	61.77e3	57.07e3	w_{js}^*	61.64e3	61.77e3	57.07e3

(29)

s=5	1	2	3	s=6	1	2	3
y_{js}^*	1.54e5	1.56e5	1.06e5	y_{js}^*	1.54e5	1.56e5	1.06e5
w_{js}^*	61.65e3	61.77e3	57.07e3	w_{js}^*	61.64e3	61.77e3	57.07e3

(30)

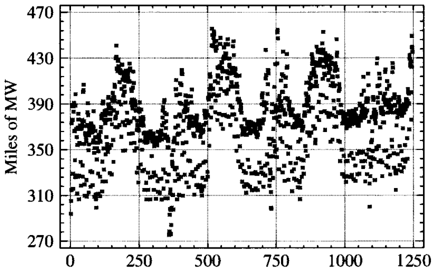

Fig. 3. Daily electricity demand in NSW, Australia

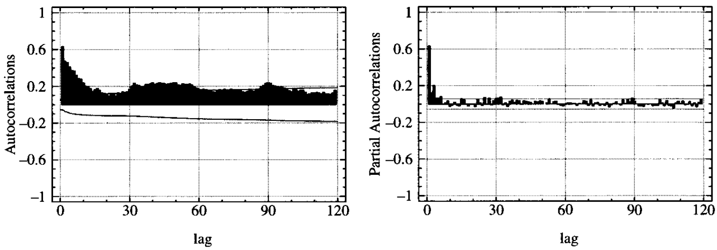

Fig. 4. Estimated autocorrelations and partial autocorrelations for residuals of Model (1)

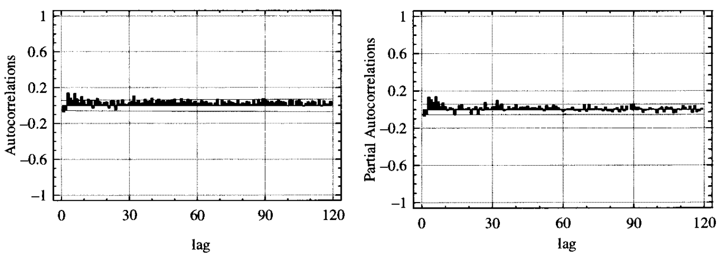

Fig. 5. Estimated autocorrelations and partial autocorrelations for residuals of Model (2)

s=7	1	2	3	s=8	1	2	3
y^*_{js}	1.54e5	1.563e5	1.06e5	y^*_{js}	1.54e5	1.56e5	1.06e5
w^*_{js}	61.65e3	61.77e3	57.07e3	w_{*js}	61.65e3	61.77e3	57.07e3

(31)

The marginal probabilities π_{ts} are:
for $t = 1$,

	1	2
π_{ts}	0.22	0.78

(32)

for $t = 2$,

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline \pi_{ts} & 0.132 & 0.088 & 0.632 & 0.148 \end{array} \tag{33}$$

for $t = 3$,

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline \pi_{ts} & 0.068 & 0.064 & 0.072 & 0.016 \end{array} \tag{34}$$

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