WAVELET PACKET MODULATION
A NEW EQUALIZATION SCHEME

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Abstract

Multicarrier modulation systems have experienced a tremendous growth in complexity and possibilities during the past decade, allowing data transmission rates never before imagined, leading to the commercial appearing of high-definition video on demand on mobile scenarios, faster Ethernet connections, increased number of digital television channels, etc. This tremendous change came up with the application of OFDM in various branches related to information interchange: ADSL and VDSL networks, DVB, WLAN, digital radio, or even the 4th generation standard for mobile communications (LTE). Therefore, OFDM is a cornerstone for communication exchange, but at the same time there are some aspects that might be improved, especially its flexibility and spectral efficiency. Wavelet-Based communication systems have some advantages like lower transmitted signal Sidelobe Level (SLL), higher spectral efficiency as well as higher flexibility with respect to OFDM technology. However, there is an important lack for WPM systems: its structure is complex and there is a lack of specific equalization schemes for it. In this thesis, we will go in deep of the Wavelet Packet System and try to find an equivalent and more efficient model, that leads to a proper specific equalization to improve the characteristics of these systems. Then, after introducing Wavelet theory and the equalization methods, we will present the concept of a post-processing equalization scheme, whose main components will be described in detail. Afterwards the new system will be tested in different scenarios in order to verify and validate theoretical assumptions, to finally conclude this work with a summary of conclussions and related work.

Acknowledgements

I would like to thank M.Sc. Qinwei He for his constant support and assistance as supervisor of this thesis. I would also like to thank Prof. Dr.-Ing. Anke Schmeink for granting me this proyect for such an important institute as TI is.
Finally, I would like to dedicate this work to my family and friends for their continued support throughout my life.

Contents

Notation glossary

$\|\cdot\|_2$	Euclidean norm
$\lvert\cdot\rvert$	Absolute value
$n \in \mathbb{Z}$	Integer number
$a \in \mathbb{R}$	Real number, scalar
$>$	Less than
$<$	Bigger than
$L^2(\mathbf{R})$	space of all square integrable function with domain $\mathbf{R}$
a_k	element k in set a
x_i	transmitted i$^{\text{th}}$ symbol
$\hat{x}_i$	received i$^{\text{th}}$ symbol
$\subset$	Proper subset / strict subset
$\cup$	Union
$*$	Convolution
$\frac{\delta}{\delta e}$	Partial derivative with respect to e
R_{xy}	Cross-correlation matrix of vectors x and y
$\mathcal{ZT}$	Z transform
$\mathcal{ZT}^{-1}$	Inverse Z transform
$=$	Equal to
$\neq$	Non-equal to
$\lfloor\cdot\rfloor$	Round number to lower integer

1 Introduction

1.1 The Increasing Needs in Telecommunications Scenario.

Throughout history, humans have always been researching in order to find new suitable communication systems that may improve the already developed ones. From the Napoleonic semaphore in late 1700's where towers sent visual signals encoding information between two points, to the telecommunication systems used nowadays, decades of human effort have been put in order to improve the information exchange possibilities of society.

User preferences are pushing the boundaries of communications technologies towards a scenario where more users can access higher data rates in more complex situations, and these increasing demands must be constantly met through research.

Nowadays, the interchange of information between entities is becoming a central part in today's life. A simple text message, a video call, satellite's control, mobile connectivity, location, information interchange, etc. they are all involved in the world of wireless communications. Studies have shown that these services have become a cornerstone for today's society. This importance has been reflected in a high increase of demand both in number of users and bandwidth. While in early 2000s users required 64 kbps connections in fixed environments like a house or an office, nowadays more than 100 Mbps in mobile devices are required. This huge increase of demand can be easily checked in the forecast of Mobile data traffic by 2019 carried out by Cisco [18].

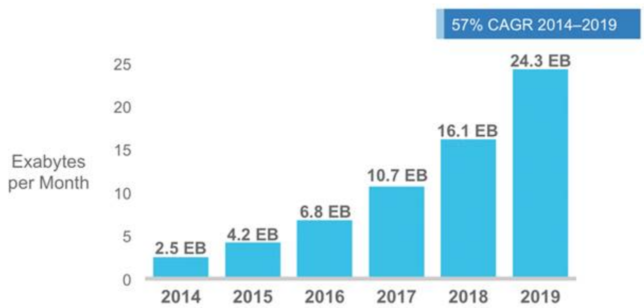

Figure 1.1: Cisco Mobile Data Traffic per Month forecasts until 2019

1.2 Multicarrier Modulations. Orthogonal Frequency Division Multiplexing.

The need to increase the available data rate for users of systems situated at a variety of complex locations caused the introduction of Multicarrier Modulations (MCM) to the commercial communication panorama. In these systems, several branches of parallel information are transmitted over a media together, with null (or almost) interference between them. The information is modulated in so-called 'basis-functions', that allow the joint propagation of different users streams over the channel with ideally no interference thanks to the orthogonality property between them.

Several modulation schemes take advantage of MCM theory properties. The most remarkable ones are introduced in the following sections.

1.2.1 Orthogonal Frequency Division Multiplexing.

Since first related work was published in the 60's, Orthogonal frequency division multiplexing (OFDM) [15] has experienced a tremendous growth and becomes the main benchmark of the wireless information interchange technique (or its wired variant Discrete Multi-Tone).

OFDM technology came up to cover the need of a communication system that could allow the parallel information interchange through a channel minimizing the interference between symbols (Inter-Symbol interference or ISI) as well as the interference between different carriers (Inter-Carrier Interference or ICI).

For that purpose, OFDM was developed as a system which, using sinusoidal waves as basis-functions, locates information of several users in different spectrum locations. This way, parallel data is transmitted through a bandlimited channel, allowing both ICI and ISI reduction.

From this point on, OFDM became a cornerstone for wireless commmunication systems, and researching efforts were focused on improving this promising new technology [7]. Major contributions to OFDM were the application of Inverse Discrete Fourier Transform (IDFT) and Discrete Fourier Transform (DFT) for modulation/demodulation stages, as well as time-domain raised-cosine windowing.

Moreover a guard band between adjacent channels was applied in order to reduce interference. This guard would be afterwards replaced by a cyclic extension of the corresponding OFDM symbol commonly known as Cyclic Prefix (CP). By introducing extra redundancy

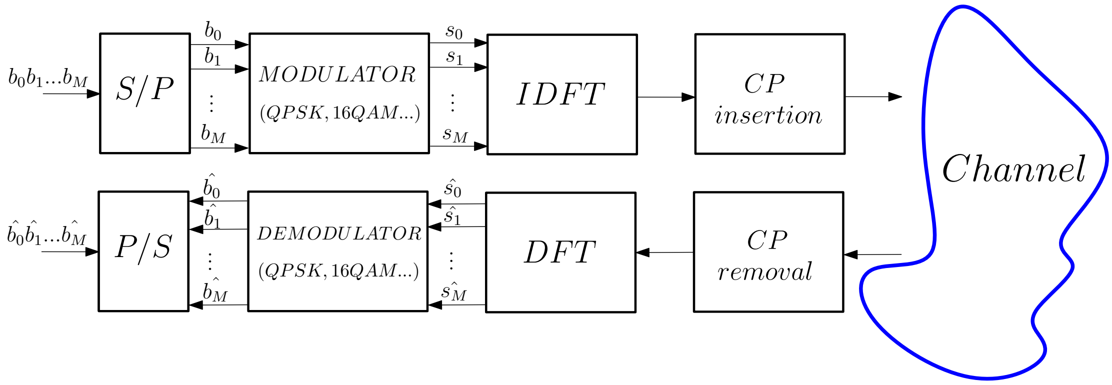

Figure 1.2: OFDM block diagram.

through CP insertion, both ICI and ISI interference are removed and system sensibility to time synchronization errors is reduced.

In figure 1.2 a system block of a traditional OFDM system is represented, where the DFT-IDFT block pair is usually substituted by a FFT-IFFT pair, which reduces the complexity of the implementation.

All previously mentioned OFDM characteristics conformed a system with a set of properties that make this communication system an optimum option when it is desired a high flux of information in parallel branches through a channel. Remarkable advantages of this system as simple time domain equalization, interference cancellation, as well as no need of frequency tuned receiver filters, converted this system in a highlighted technology for wireless communications systems.

Consequently, OFDM technology has been applied to a wide set of applications regarding information exchange. Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), Wireless LAN networks, WiMAX , etc. are some examples of standards that take advantage of OFDM technology properties.

It can be seen how this technology had a tremendous impact, specially on Wireless communications. Even so, OFDM is also applied in well-known wired system as, for example common ADSL systems present anywhere with internet access.

Moreover, Orthogonal Frequency Division Multiplexing is a key technology in recent and future communication systems as 4^{th} (LTE) and 4.5^{th} generation mobile services [17]. Nonetheless this system is not perfect, and has some concrete drawbacks that might be offset.

On one hand, OFDM systems suffer from a loss of spectral efficiency due to the redundancy introduced by previously mentioned Cyclic Prefix, although the CP insertion is necessary

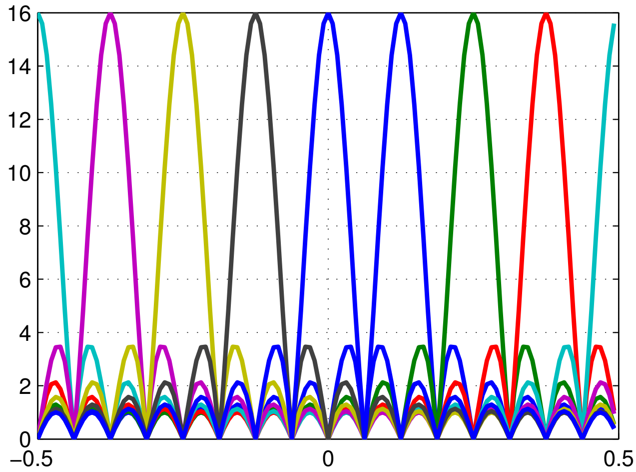

Figure 1.3: OFDM frequency spectrum.

in order to ensure maximum ISI reduction.

On the other hand, linear transmitter circuitry is required to minimize big Peak-to-average power ratio (PAPR) values in OFDM systems, which leads to low values of power efficiency.

Finally, another important drawback is its ridigity. Vital elements of these kind of systems as CP or FFT are determined depending on channel characteristics, number of users, etc. Therefore, it is difficult to adapt OFDM system on the fly to different circumstances that may vary during information interchange.

All these disadvantages are intended to be compensated by finding a specific equalizer for such a promising system as the wavelet-based one is.

1.2.2 Generalised Frequency Division Multiplexing.

GFDM [8] is a new more flexible scheme having similar concepts with respect to OFDM, that tries to improve OFDM spectral efficiency by introducing a pulse shaping filter that is applied on the different subchannels, controling signal radiation out of the desired band. This out of band radiation is a major drawback for OFDM and may cause interference with signals located closely in the spectral domain.

In GFDM, different techniques as CP insertion can also be applied in order to introduce redundancy to eliminate the ISI component due to channel disturbances.

Moreover a block structure integrating both symbols in different subcarriers and time intervals is created, allowing a faster and more flexible data processing of information.

1.2.3 Filter Bank Multicarrier.

Filter Bank Multicarrier (FBMC) [13] tries, from a different approach, to avoid both ISI and ICI even without cyclic prefix insertion, in order to increase spectral efficiency.

Inter Carrier Interference is avoided by introducing Offset Quadrature Amplitude Modulations (QAM), while FBMC gets rid of ISI by application of specific equalization on each of the different subcarriers.

Moreover, OFDM Side Lobe Level is decreased by application of pulse shaping filters generated by means of polyphase networks.

1.2.4 Wavelet based OFDM.

In 1807 a French mathematician named Joseph Fourier defined as 'wavelet' a brief oscillation in time. From that moment where the concept of a 'small wave' was proposed, wavelet theory has been widely studied and applied in different fields as discussed in section 1.3.2. Focussing on telecommunications field, information of different users is orthogonally multiplexed in frequency by means of previously mentioned wavelets, created through a filter bank structure.

Wavelet theory can be applied to traditional OFDM technology, allowing CP and FFT removal using instead Discrete Wavelet Transform (DWT). This way, spectral efficiency increases as CP redundancy dissapears, while transmission of power is reduced due to use of wavelet transform.

1.3 Wavelet Applications.

1.3.1 The Application of Wavelet Theory to Communications.

Since first applications in early 50's, Wavelet theory has been applied in important areas like signal representation, computer graphics, image and video processing, etc. Nonetheless, wavelet theory can also be applied to commnications panorama obtaining a system suitable for information sending. Using same orthogonality principles as the ones of traditional OFDM systems, parallel data streams representing different users can be sent to the channel. By this means, we can get rid of the afforementioned OFDM drawbacks as well as take advantage of Wavelet systems flexibility and high spectral efficiency.

Unfortunately, the application of Wavelet Theory to communications area is still a pending task. Researching is still necessary in order to find a proper solution for this technology that may improve today's communications systems based on wavelet theory.

Well known equalizers, used before in different communication systems have been applied in the Wavelet domain, in order to test it as an alternative to traditional OFDM systems. Widely-used equalizers as Minimum mean square error equalizer (MMSE), Decision Feesback equalizer (DFE) or Zero forcing equalizer (ZF) have been tested, showing good results. However these solutions were totally general and did not take into account the special charactaristics of the Wavelet scheme.

This is the initial point of this Master Thesis. In the following pages, we will try to study the Wavelet technology, present its parts, check new equalization possibilities that may lead to a better behaviour, and try to improve OFDM characteristic performance.

1.3.2 Other Wavelet Applications.

On the other hand, apart from communications scenario, there are many more fields where Wavelet theory has been applied succesfully, taking advantage of both its spatial and time reolution, as well as its orthogonality property between different branches.

Some of the most important applications for Wavelet systems in a scope different from communications are [12]: data compression, medical applications (heart rate and blood pressure measurement ...), image treatment, signal processing, speech recognition, DNA analysis, computer graphics, fingerprint verification, etc.

As we can see, wavelet theory is already well stablished and its concepts have been applied in a wide set of fields with succesful results. It is our purpose along this master thesis to improve wavelet implementation for communications to make this system a viable alternative to well-stablished OFDM systems.

1.4 Report Guideline.

The document is organised as follows: first, Wavelet Theory will be introduced, presenting main concepts as signal representation through wavelets, wavelet series and discrete transform as well as wavelet packet modulator.

Afterwards we will briefly discuss about previous equalization schemes, differentiating between pre/post processing equalization, to make then a brief recap of well known equalizers and their main characteristics.

In Chapter 3 we will show a simpler equivalent system for the well-known WPM architecture, much simpler than traditional one, explaining its main components, while in Chapter 4 we present the Post-processing equalization scheme developed for this thesis, representing its parts, their function as well as the whole equalization process.

To conclude, chapter 6 will show the differences between previous equalization schemes and the one developed for this thesis by simulation, showing how it does affect the variation of important parameters of the system, while last chapter will show some discussions about possible related future work and final conclussions for this Master Thesis.

2 Wavelet Theory

2.1 Representing Signals. Scaling Function.

In this section our aim is to introduce the concept of scaling function. For that purpose, let us suppose we have a function $s(t)$, whose energy is limited. This implies:

$$||s(t)||^2 = \int_{-\infty}^{\infty} |s(t)|^2 \, \mathrm{d}t < \infty. \tag{2.1.1}$$

Then, we can express this function s(t) as:

$$s(t) = \sum a_k \phi_k(t) \;\; k \in Z, t \in \mathbf{R}. \tag{2.1.2}$$

Where a_k are the expansion coefficients and $\phi_k(t)$ is the scaling function. Consequently, following the expression showed above, we can span the whole space of all square integrable functions with domain $\mathbf{R}$, $(L(\mathbf{R}^2))$, with $\mathbf{R}$ the set of all real numbers.

Now, with respect to $\phi_{r,s}(t)$, this is based on shifted and scaled versions of a simpler scale function, directly related to the parameters r and s, where r is the scaling parameter and s the shift parameter. By this method, we can express the scaling function as:

$$\phi_{r,s}(t) = \mu^{r/2} \phi(\mu^r t - \beta s). \tag{2.1.3}$$

with

$$\mu = \text{fixed dilation step.}$$

$$\beta = \text{shift factor.}$$

Now we will try to show what does this scaling function relationship mean in a graphical way. First consider a simple scenario with a Haar scaling function, whose representation can be found in figure 2.1.

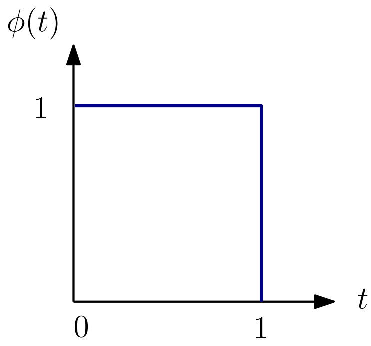

Figure 2.1: Haar scaling function

Then, by using equation (2.1.3) we can get the scaling function for zero shift and different scaling coefficients, being:

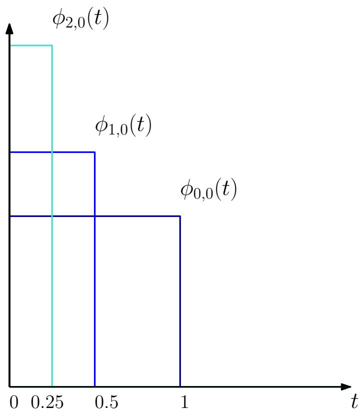

Figure 2.2: Scaling functions for zero shift and different scaling factors (based on Haar scaling function)

$$
\begin{aligned}
\phi_{0,0} &= \phi(t). \\
\phi_{1,0} &= 2^{1/2}\phi(2t). \\
\phi_{2,0} &= 2\phi(4t).
\end{aligned}
\tag{2.1.4}
$$

In figure 2.2 it is easy to check how $\phi_{0,0}$ can be expressed as combinations of shifted and scaled versions of $\phi_{1,0}$. Same reasoning can be applied to $\phi_{1,0}$ with respect to $\phi_{2,0}$. Therefore, we would have:

$$\phi_{0,0} = \frac{1}{\sqrt{2}}[\phi_{1,0} + \phi_{1,1}].$$
$$\phi_{1,0} = \frac{\sqrt{2}}{2}[\phi_{2,0} + \phi_{2,1}]. \tag{2.1.5}$$

As it can be checked, the thiner the trace of the scaling function, the more subspace we can span with that scaling function. This relation has a mayor importance in the next subsection: The multiresolution analysis.

2.2 Multiresolution Analysis.

Let us now consider one fixed scaling factor r_0. As shown in previous section, for this scaling factor the resulting scaling function spans a concrete subspace of $L(\mathbf{R}^2)$ called V_0. But, if we consider the scaling function with a higher order scaling factor, the space it will be able to span will increase, and this space V_1 will also include V_0. Therefore, the higher the order of r, the bigger the space covered, and the thinner the resolution, which implies:

$$V_{-\infty} \subset ... \subset V_0 \subset V_1 \subset V_2 \subset ... \subset V_\infty \tag{2.2.1}$$

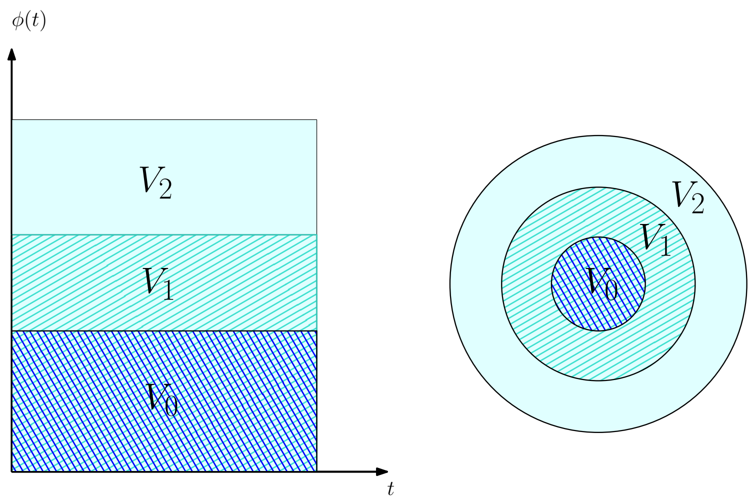

Figure 2.3: Space of $L(\mathbf{R}^2)$) spanned depending on scaling coefficient

If we consider we have a signal $s(t)$ we would like to represent, and it belongs to subspace V_n, then it could be approximated by an scaling function with scaling factor r_n, but also with factors r_{n+1}, r_{n+2} ... since the subspaces spanned by these factors (namely V_{n+1} V_{n+2}) do also contain the subspace V_n. Therefore, we can conclude that any low-order scaling function can be described by a higher order one as:

$$\phi(t) = \sum_n c_\phi(n)\sqrt{2}\phi(2t - n). \tag{2.2.2}$$

Where c_ϕ are the scaling function coefficients and n the shift parameter.

2.3 Wavelet Functions.

As showed in previous section, a low order scaling function can be approximated by a higher order scaling function, with shifted and scaled versions of it. But there is another possible approach. Let us denote D_n as the subspace difference between scaling function of order n and that of order $n+1$. Then, it is obvious that we can span the space V_{n+1} by combining space V_n and the subspaces D_n. Obviously, the subspace V_n can also be spanned by the combination of V_{n-1} and D_{n-1}. Then, by following the series, we can span any scaling function by combining the lowest order (coarser) scaling function with the area spanned by the difference of subspaces between adjacent scaling functions.

$$V_{n+1} = V_n \cup D_n = V_{n-1} \cup D_{n-1} \cup D_n = ... = V_0 \cup D_0 \cup D_1 \cup ... \cup D_n \tag{2.3.1}$$

We already know that V_0 is spanned by scaling functions, but we need another function to deal with the subspaces difference. Using same reasoning as in previous subsection, the subspace will only be able to be spanned by a scaling function of higher order, as it contains the subspace of interest (it is not possible to get D_n from V_n, it is required, at least, V_{n+1}). This implies that a wavelet function [16] will be represented by scaled and shifted versions of higher order scaling functions:

$$\varphi(t) = \sum_n c_\varphi(n)\sqrt{2}\phi(2t - n). \tag{2.3.2}$$

Where $\varphi(t)$ is the wavelet function, $c_\varphi(n)$ are the wavelet function coefficients and n the shift parameter.

2.4 Wavelet Series and Discrete Transform.

Wavelet series will provide us the tools to get the coefficients of the series that approximate the function we are interested in ($s(t) \in L(\mathbf{R}^2)$) as a sum of vector spaces and subspaces (scalar and wavelet functions).

Thus, our signal $s(t)$ can be expressed as:

$$s(t) = \sum_{s} a_{r_0,s}\phi_{r_0,s}(t) + \sum_{r=r_0}\sum_{s} b_{r,s}\varphi_{r,s}(t). \tag{2.4.1}$$

where $\phi_{r_0,s}(t)$ is the scalar function with fixed scaling factor r_0, $\varphi_{r,s}(t)$ is the wavelet function, $a_{r_0,s}$ is the scaling function series coefficient and $b_{r,s}$ is the wavelet function series coefficient. If we look at section 2.4 we can check how the first terms contributes to the coarser part of $L(\mathbf{R}^2)$ space, while the wavelet coefficients are related to the subspaces that conform the signal. That implies $r > r_0$ always. In equation (2.4.2) we have the expressions for the mentioned coefficients. In a communication scheme, if we wanted to send a representation of a signal through a channel, we would directly send the coefficients showed above, while in the receiver side it would be possible to recover the signal by applying the inverse Wavelet series (IWS). Therefore, the problem of representing a signal as sums of spaces and differences of subspaces reduces to finding the above mentioned coefficients:

$$\begin{aligned} a_{r_0,s} &= \int s(t)\phi_{r_0,s}(t)dt. \\ b_{r,s} &= \int s(t)\varphi_{r,s}(t)dt. \end{aligned} \tag{2.4.2}$$

Nonetheless, communication schemes are based on discrete signals instead of continous ones. Due to this fact, it is common to use the discrete wavelet transform [2] [3], whose coefficients are determined in a similar way to the continous case:

$$\begin{aligned} V_{j_0,k} &= \frac{1}{\sqrt{M}}\sum_{n} s[n]\phi_{j0,k}(n). \\ W_{j,k} &= \frac{1}{\sqrt{M}}\sum_{n} s[n]\varphi_{j,k}(n). \quad j > j_0. \end{aligned} \tag{2.4.3}$$

where $V_{j_0,k}$ and $W_{j,k}$ are the scaling and wavelet coefficients respectively and $M = 2^J$ with J equals number of levels of Wavelet transform. Once again, the receiver of these coefficients can easily recover the signal by applying the inverse wavelet transform:

$$s[n] = \frac{1}{\sqrt{M}} \sum_K V_{j_0,k} \phi_{j0,k}(n) + \frac{1}{\sqrt{M}} \sum_{j=j0} \sum_K W_{j,k} \varphi_{j,k}(n). \tag{2.4.4}$$

Moreover, it is possible to relate both wavelet and scaling coefficients of level $j+1$ with those of previous level j as [4]:

$$\begin{aligned} W_{j,k} &= \sum_v h_\varphi(v-2k) V_{j+1,v}. \\ V_{j,k} &= \sum_v h_\phi(v-2k) V_{j+1,v}. \end{aligned} \tag{2.4.5}$$

As we can see, coefficients from different levels are related by the responses h_φ and h_ϕ. We will discuss about this relation in next section.

2.5 Quadrature Mirror Filters.

As seen in subsection 2.4 we can obtain wavelet and scale coefficients at level j from scaling coefficients of level $j+1$. Moreover we can say that they are related through a convolution with a low pass filter (h_ϕ) and a highpass filter (h_φ).

Therefore, we can synthesize this expression as a mirror filter pair. These filters may be in analysis or synthesis configuration, depending on which coefficients are we trying to obtain (From level j to level $j+1$ or viceversa).

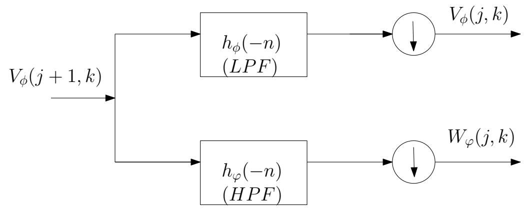

Figure 2.4: Coefficients relation between consecutive levels

This pair of mirror filters together with a downsampling stage in analysis section and upsampling stage in the synthesis part establishes a quadrature mirror filter pair configuration (QMF).

With respect to the pair of filters, there are some constraints that must be satisfied in order to avoid aliasing due to the decimation stages. Then, in time domain, the coefficients of the high pass and low pass filters for analysis stage must satisfy:

$$h_0[L-1-n] = (-1)^n h_1[n]. \tag{2.5.1}$$

where $h_0[n]$ and $h_1[n]$ are, the high and low pass filters of QMF respectively, while for synthesis stage:

$$f_0[L-1-n] = (-1)^n f_1[n]. \tag{2.5.2}$$

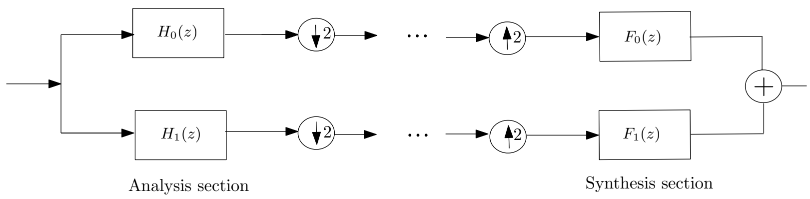

Figure 2.5: Quadrature Mirror filters in frequency domain.

If we transfer these restrictions to frequency domain, we obtain the following relations:

$$\begin{aligned} H_0(w) &= H(w). \\ H_1(w) &= H(w-\pi). \\ F_0(w) &= 2H(w). \\ F_1(w) &= -2H(w-\pi). \end{aligned} \tag{2.5.3}$$

Where H_0 and H_1 are, respectively the High-pass and low-pass filter frequency responses of the analysis QMF, while F_0 and F_1 correspond to the synthesis part.

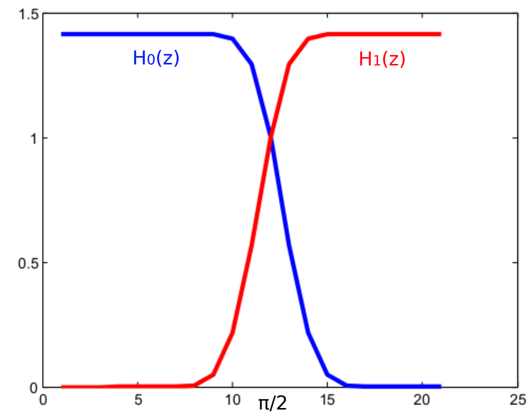

Figure 2.6: Analysis filters frequency response.

2.6 Wavelet Packet System.

As it can be seen in figure 2.6, the QMF pair will create two different frequency allocations along the spectrum.

Let us consider we want to generate thiner equally spaced frequency slots in the spectrum, where user symbols will be allocated for transmission. Then, it is necessary to iterate both branches of the QMF pair as many times as the frequency resolution of our system requires (this is also applied for DWT, but iteration is performed only in low-pass branch).

Then, the spectrum would be composed of frequency slots as the ones in figure 2.7

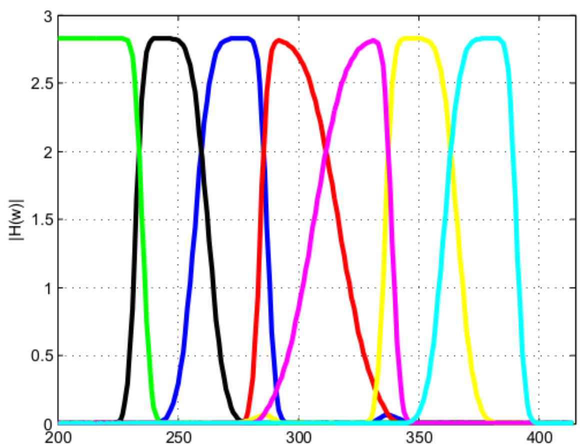

Figure 2.7: 3-levels QMF bank frequency responses.

This is the principle applied by Wavelet Packet Modulation (WPM) systems. QMF pairs are iterated J times along the branches. Therefore, factor J will determine the number of levels (iterations) of our system.

There is also a relation between the number of levels of the system and the total amount of frequency allocations (users), and is given by $M = 2^J$, which obviously corresponds to the total number of branches of the system.

QMF pairs iterations shape what is usually known as quadrature mirror filter bank (QMFB bank).

A WPM system can be seen in figure 2.8 where it is easy to check the concatenation of filters through the QMF bank configuration. In this case, since its a 3-levels WPM, QMF bank is composed of 3 iterations of QMF pair, leading to 8 branches both at input and output.

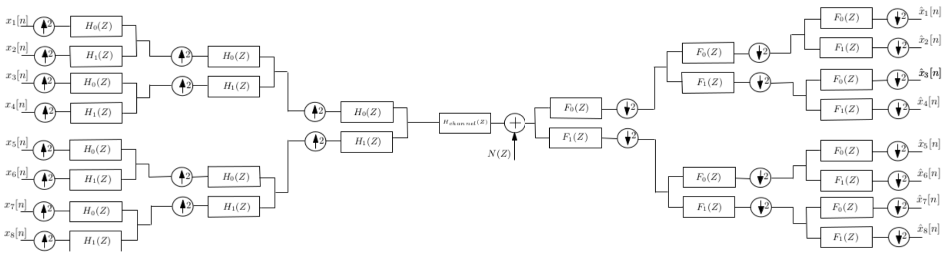

Figure 2.8: 3-levels Wavelet system.

3 Previous Equalization Schemes

It is important to point out what is the WPM system equalization panorama to give an idea about what we are trying to obtain. In general, in these systems, equalization has not been explored deepthly, falling into a lack of custom solutions to compensate the ISI and ICI that may occur in the wavelet domain. General solutions have been applied to these communication scenarios, with good results in general.

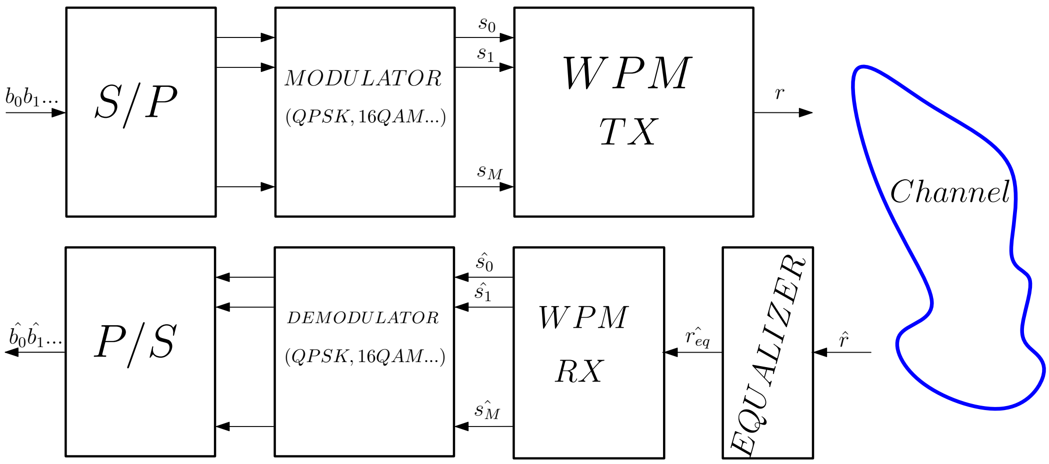

Figure 3.1: Block diagram of a complete WPM communications system

In figure 3.1 reader may find a classical schematic representation of a complete Wavelet-based communication system, composed of the following elements:

- Serial-to-parallel converter: takes as inputs the bits to be transmitted and converts them into M parallel branches, corresponding to the number of users of the WPM system.
- Modulator: for any of the M branches converts the inputs bits to output modulated symbols (QPSK,16QAM...)
- WPM transmitter: from input symbols to WPM transmitter output (1 real-valued stream signal)
- Channel: characterized by its time response, it introduces multipath propagation and

attenuation, leading into ISI and ICI which should be compensated by the equalizer. It also adds white gaussian noise.

- Equalizer: compensates channel distortion.
- WPM receiver: back from one signal to M output branches.
- Demodulator: from input symbols to output bits.
- Parallel-to-Serial converter: recovers the original input bit stream from the M branches of bits.

We could have also added a codification stage as first block in order to add redundance against the channel or to increase security, but this block was not taken into account for this thesis for sake of simplicity.

Regarding equalization [5], an easy way to do implement it, and also, the closest one to equalizing methods in other communication schemes is to place it at the beginning of the receiver stage. This way, we just have to get back the signal at the input of the Wavelet receiver by applying well-known equalization methods as Zero forcing equalization or Minimum mean square error. As we will see when comparing this method with the one developed throughout this thesis, the performance of these kind of pre-processing equalizers is quite correct, but it is slightly complex, as we have to equalize a very long signal, which can be highly time and power demanding. Moreover, this technique is completely general and does not focus or take advantage from the wavelet properties and how these may affect the equalization process.

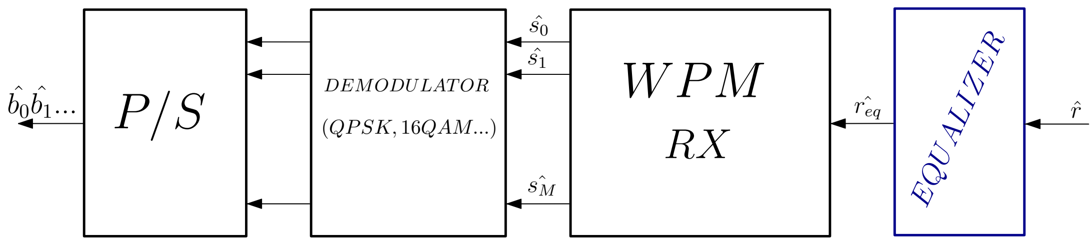

Figure 3.2: Pre-processing equalization block diagram.

Therefore, it could be much more efficient to move to a post-processing equalization scheme as the one in figure 3.3. In this case, the complexity should be much lower, since we are dealing with the output symbols in any of the branches, which are much smaller in size than the received signal. This could lead to simple equalizers at the different branches of the demodulator, easy to implement and less time and power demanding.

Consequently, this thesis will focus on finding a proper post-processing equalization for such an specific communication system as the WPM is.

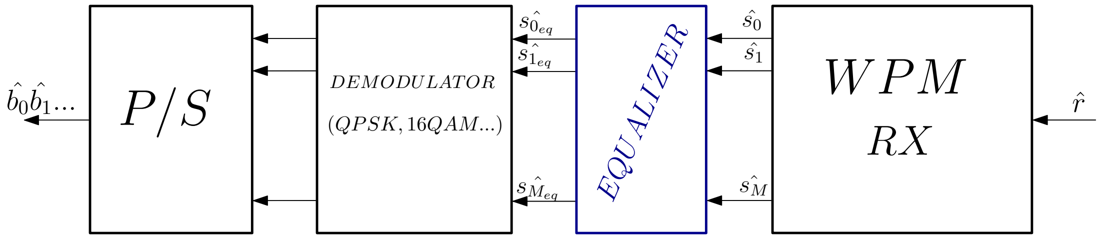

Figure 3.3: Post-processing equalization block diagram.

3.1 Equalizers.

In this section the main equalizers tested in wavelet based systems will be mentioned and briefly discussed. In this manner reader may check different equalization possibilities, their advantages and drawbacks as well as the possibility of its application in a post-processing equalization situation.

3.1.1 Zero forcing Equalizer.

Zero Forcing (ZF) equalization [6] [21] has as objective the removal of any interference caused by the channel. For that purpose, time domain equalization coefficients are derived based on the time domain response of the correspondant channel. Then, the convolution of the disturbed signal with the correspondant equalizer results in a removal of channel interference.

This implies that if we want to recover information s from vector r:

$$r = s * h. \tag{3.1.1}$$

suppossing h is the channel response, we must pass the signal through an equalizer with response:

$$e = h^{-1}. \tag{3.1.2}$$

so that we get the output:

$$z = r * e = (s * h) * e = (s * h) * h^{-1} = s. \tag{3.1.3}$$

Then, the equalizer response must correspond with that of the inverse of the channel. Therefore this equalization technique supposses prior knowledge of channel response, which can be found by means of channel estimation techniques as Least squares channel estimation (LSE) [14].

There is an important drawback regarding ZF equalizers: an amplification of noise power that may lead to a degradation of BER performance.

3.1.2 Minimum Mean Square Error Equalizer.

Minimum Mean Square Error equalizers (MMSE) [11] do not focus on minimizing channel distortion due to multipath propagation, but to overcome the AWGN noise added by the channel itself. Therefore, these equalizers addapt iteratively their coefficients in order to reduce the mean square error between the equalizer output symbol x_i and the expected output symbol $\hat{x_i}$. Consequently, filter coefficients are addapted until

$$\mathbb{E}(x_i - \hat{x_i}) = 0. \tag{3.1.4}$$

When (3.1.4) is satisfied, channel noise factor has been removed from system's output.

Therefore, we must find coefficients e such that:

$$\frac{\delta}{\delta e}\mathbb{E}(x_i - \hat{x_i}) = 0. \tag{3.1.5}$$

is satisfied. After developing (3.1.5), we can get a closed expression for equalizer coefficients:

$$e = R_{yy}^{-1} R_{xy}. \tag{3.1.6}$$

with R_{yy}^{-1} the auto-correlation of the received signal and R_{xy} the cross-correlation between input signal and received signal.

3.1.3 Decision Feedback Equalizer.

In contrast with afforementioned equalizers, Decision Feedback (DFB)[9] can be categorized as non-linear equalizers. Its main principle is the following: once we have determined the transmitted input from the received symbol we can remove the ISI component of the channel to future transmitted symbols. This property makes the system non-linear, since after characterizing the ISI contribution, this must be removed from the system through a feedback configuration composed of a feedback filter, whose taps converge iteratively through Least Mean Squares process (LMS).

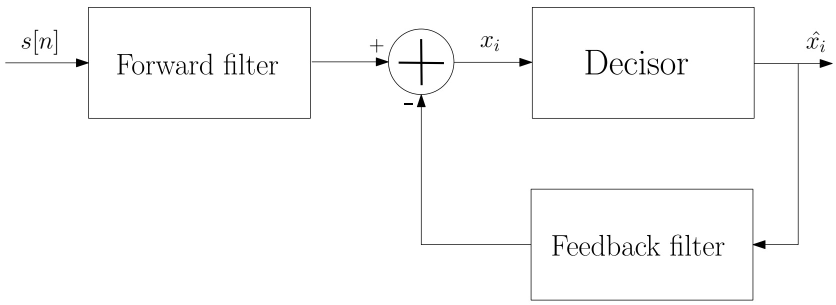

Figure 3.4: DFB equalizer block diagram.

This feedback configuration makes Decision Feedback equalizers more complex. On the other hand these systems are specially useful in highly frequency selective channels, moreover when they contain deep spectral nulls.

4 Equivalent WPM System

As we mentioned in previous chapters, our main goal is to find a specific post processing equalizer that somehow improves previous schemes in performance, time cost, simplicity, and so on. It is obvious that the wavelet transmitter and receiver complexity grows exponentially when we are in a communication scenario with a big number of users taking part, which implies an increase in number of levels (J) of the system.

Therefore, it would be appropiate to simplify our system under study in order to ease the deduction of the equalizer coefficients.

For that purpose, let us consider the Z-transform of the filter response $h[n]$ as:

$$H(Z) = \sum_{n=1}^{N} h[n] z^{-n}. \tag{4.0.1}$$

Taking previous relation into account, in [20] it can be found an equivalent structure for Wavelet system in figure 4.1.

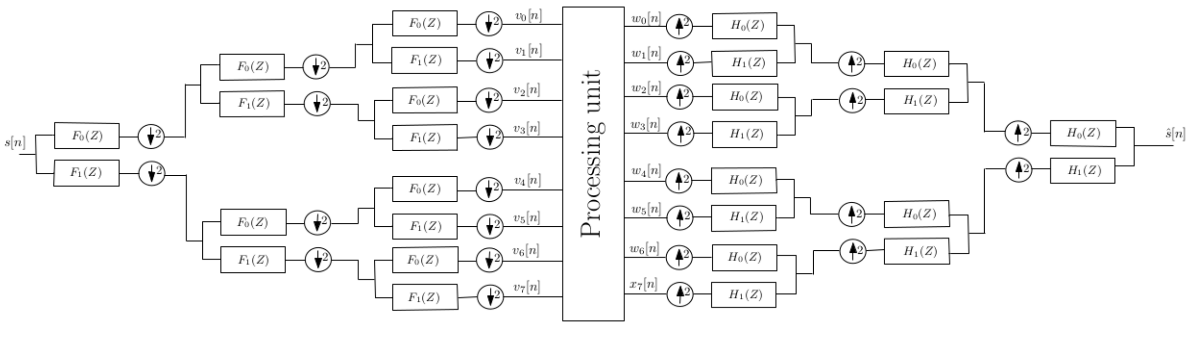

Figure 4.1: 3-levels Wavelet system in spectral decomposition configuration.

This Wavelet system is an example of application of Wavelet Theory to a field which differs from the information-sending purpose of communication WPM systems. In this case, a signal $s[n]$ is decomposed by means of the QMF bank into different spectral components going into the processing unit, to finally recover signal $\hat{s}[n]$ after receiver stage. For this system structure, the equivalent system derived in [20] is shown in figure 4.2

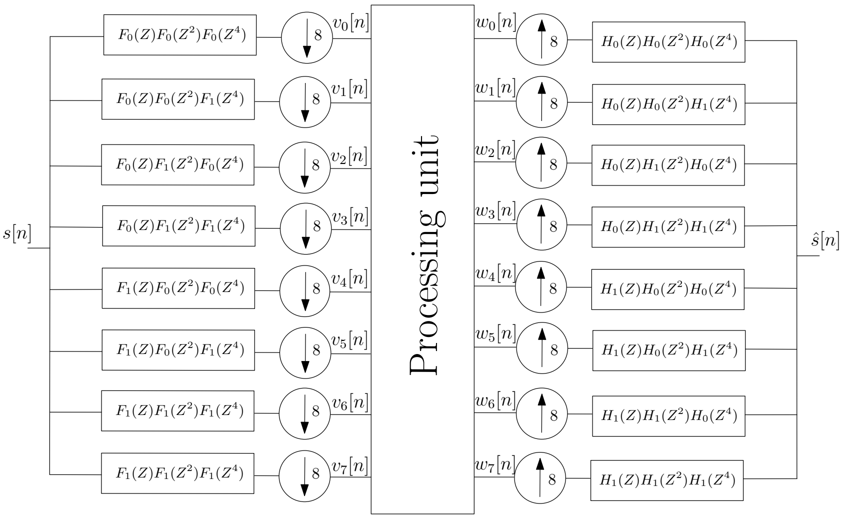

Figure 4.2: 3-levels Wavelet equivalent system in spectral decomposition configuration.

Nonetheless, our study is focused in a Wavelet system configuration that allows the exchanging of symbols $x_i[n]$ between both sides of a communication channel. For that purpose, the configuration of the WPM system must correspond to that of figure 4.3.

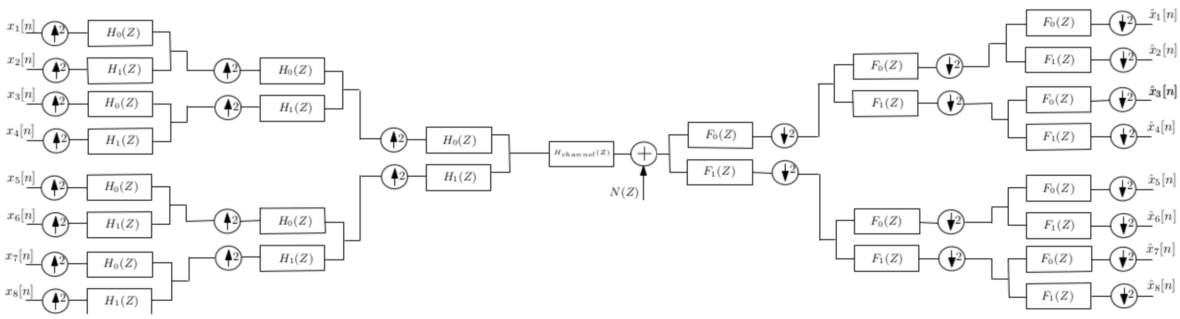

Figure 4.3: 3-levels Wavelet system.

It can be checked how in this case the structures of transmitter and receiver are interchanged, having the transmitter as inputs the symbols x_i to be sent, which after going through the QMF bank, pass through the channel and are recovered by means of the WPM receiver. This scheme does correspond with a typical commmunication scenario in contrast with that of figure 4.1.

Following the relation between figure 4.1 and figure 4.2 and what stated in [20], we can obtain an equivalent 3-levels WPM system for a communications scenario, being represented in figure 4.4.

As we can see, for this new structure upsampling and downsampling stages are joined at

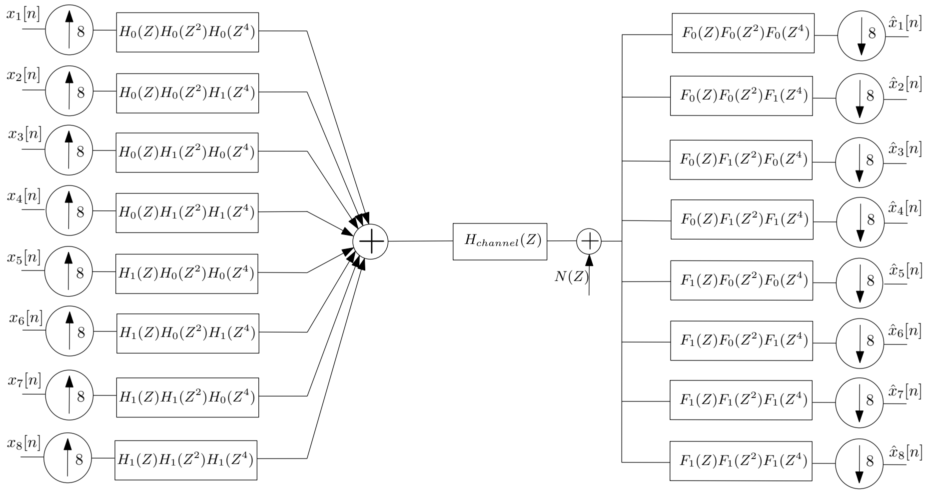

Figure 4.4: 3-levels Wavelet equivalent system.

the very beginning and end of the system, with a decimation/upsampling factor that is equal to the number of users (2^J) of the system (8 in this case).

Moreover, we have moved from a situation with a large amount of filters, to a new structure composed of just 8 equivalent filters at the transmitter and receiver stage. All these modifications carry an important improvement as circuit simplicity refers.

Taking as starting point these derivations, we can get a similar system for which instead of taking into account a $J = 3$ levels system, any possible number of levels will be considered, for which becomes even more important to reduce complexity.

Therefore, the resulting Wavelet equivalent transmitter and receiver can be checked in figure 4.5

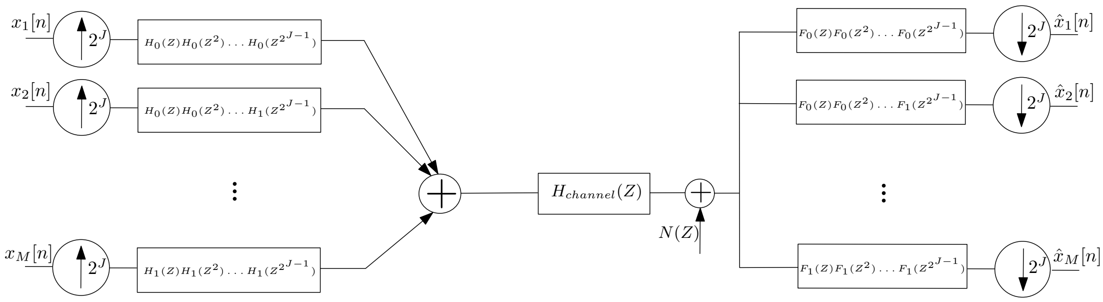

Figure 4.5: J-levels Wavelet equivalent system.

Moreover, we can compress the equivalent filters expressions as:

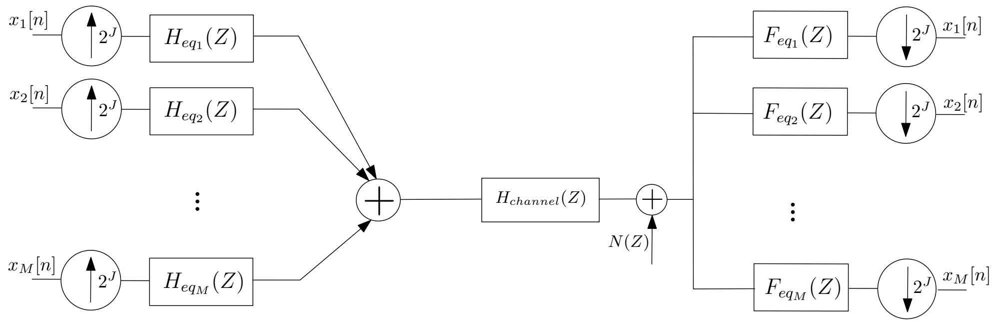

Figure 4.6: J-levels equivalent system.

$$
\begin{aligned}
H_{eq_1} &= \left\{ H_0(Z)H_0(Z^2)...H_0(Z^{2^{J-1}}) \right\}. \\
&\vdots \\
H_{eq_M} &= \left\{ H_1(Z)H_1(Z^2)...H_1(Z^{2^{J-1}}) \right\}.
\end{aligned} \tag{4.0.2}
$$

$$
\begin{aligned}
F_{eq_1} &= \left\{ F_0(Z)F_0(Z^2)...F_0(Z^{2^{J-1}}) \right\}. \\
&\vdots \\
F_{eq_M} &= \left\{ F_1(Z)F_1(Z^2)...F_1(Z^{2^{J-1}}) \right\}.
\end{aligned} \tag{4.0.3}
$$

obtaining a final equivalent system showed in figure 4.6

As we can see, the changes are significant, and this new equivalent system might be very useful in order to find a new equalization method for wavelet packet based communication systems.

4.1 Test.

In order to prove that the new scheme is fully functional, we will compare the outputs of a pair of WPM systems: a conventional one and another developed with the principles aforementioned. If the outputs of both results are the same, then the system will be totally equivalent.

For that purpose, both systems were implemented with J=4 (16 users). First, the outputs of both transmitters can be checked at figure 4.7 figure 4.8, where real and imaginary part have been separated in order to show in in a more intuitive way the equivalence of both systems.

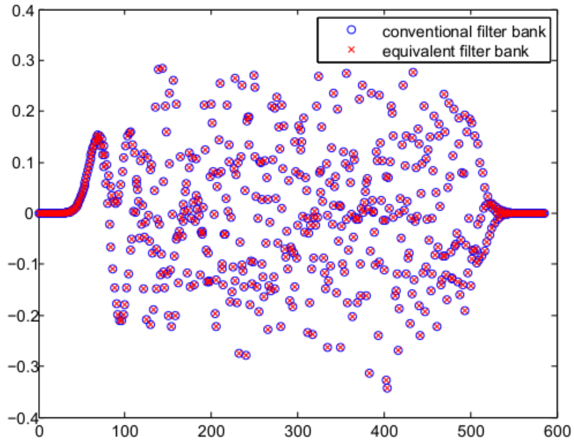

Figure 4.7: Transmitter output real part.

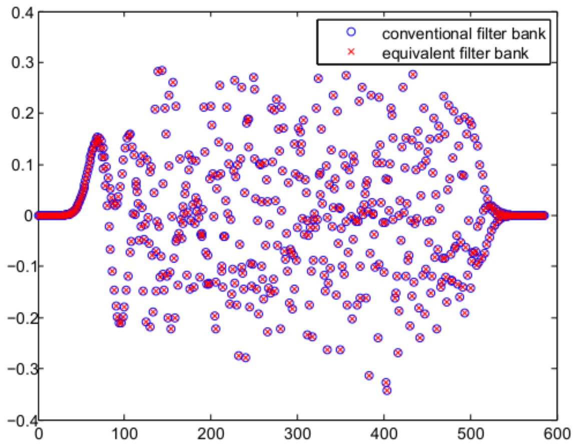

Figure 4.8: Transmitter output imaginary part.

As we can see, outputs are equal, therefore both WPM transmitter schemes behave in the same way.

To prove its complete equivalence, WPM receiver outputs must be also equal. For that purpose system outputs for first branch of both traditional and equivalent WPM receiver are compared :

As we can see in figure 4.9 figure 4.10 both imaginary and real part of first branch outputs

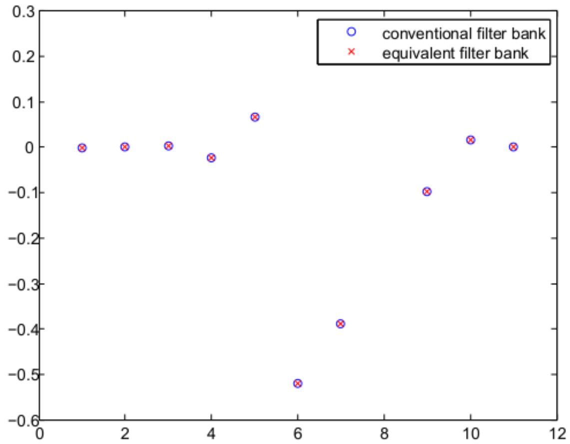

Figure 4.9: Receiver output for branch 1 real part.

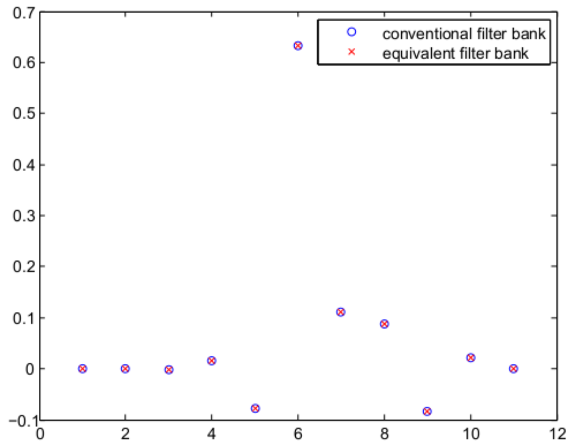

Figure 4.10: Receiver output for branch 1 imaginary part.

do coincide, having the transmitted symbol in sixth position. Therefore, WPM equivalent receiver behaves as traditional one, and consequently, we can assert both systems equivalence. Now, we can focus our efforts in finding a new equalization scheme having as reference the system in figure 4.6

5 New Equalization Scheme

5.1 Introduction.

It has been previously discussed about main properties of Wavelet Systems, as well as those of OFDM, trying to emphasize those points where it might be useful to find a new equalization scheme that improves the information-sending in a WPM scenario. Nevertheless, before going to the heart of the procedure to find such an equalizer it might be useful to stress the improved properties we want it to have.

We have already mentioned some advantages of Wavelet Packet Systems [11] as its flexibility as well as Side Lobe Level suppression and higher spectral efficiency with respect to that of OFDM. But the application of WPM to communications field in an effective manner requires a system specific equalization for this scenario. This new system should be able to, at least, match the performance in terms of free-error data rate sent with respect to the case of pre-detection equalization.

Moreover, this process should be done in a more efficient way, showing up this increased effectiveness either in equalization computing time, or in computation complexity, modelled as the number of operations necessary in order to perform the whole equalization process.

5.2 Zero Forcing Equalization for New WPM Scheme.

In order to find a specific equalizer for the new scheme derived in chapter 4 we should first get a mathematical approach to the signals present in different stages of the equivalent WPM system. For that purpose we will have as reference the equivalent model presented in figure 5.1.

Where

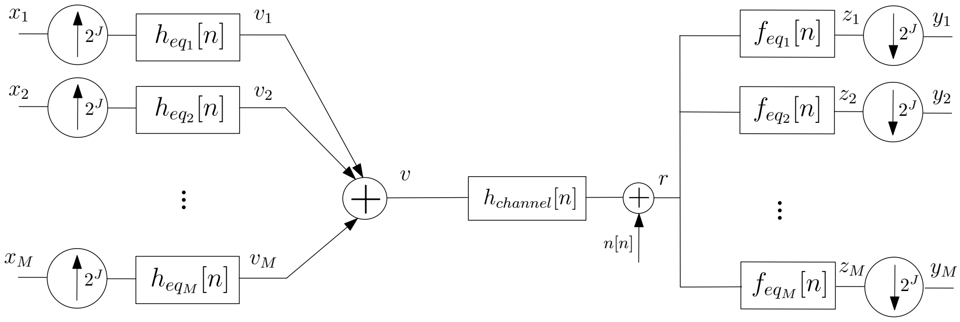

Figure 5.1: J-level time domain equivalent system.

$$h_{eq_0} = \mathcal{ZT}^{-1}\Big\{H_{eq_0}(Z)\Big\} = \mathcal{ZT}^{-1}\Big\{H_0(Z)H_0(Z^2)...H_0(Z^{2^{J-1}})\Big\}$$
$$\vdots$$
$$h_{eq_M} = \mathcal{ZT}^{-1}\Big\{H_{eq_M}(Z)\Big\} = \mathcal{ZT}^{-1}\Big\{H_1(Z)H_1(Z^2)...H_1(Z^{2^{J-1}})\Big\}$$

For this system we have:

$$v_i[n] = x_i[n/M] * h_{eq_i}[n]. \tag{5.2.1}$$

with $M = 2^J$ being both the number of users of the system as well as the upsampling/downsampling rate of the equivalent system.

Therefore, the total output signal of the transmitter stage will be given by:

$$v[n] = \sum_{i=1}^{M} v_i = \sum_{i=1}^{M} x_i[n/M] * h_{eq_i}[n]. \tag{5.2.2}$$

If we expand the convolution, the expression for $v[n]$ is:

$$v[n] = \sum_{i=1}^{M} \sum_{k_1=-\infty}^{\infty} x_i[k_1/M]h_{eq_i}[n-k_1]. \tag{5.2.3}$$

The signal will go through the channel, modelled as the combination of the coefficients

$h_{channel}[n] = \mathcal{ZT}^{-1}\Big\{H_{channel}(Z)\Big\}$ representing multipath propagation and attenuation, and additive noise $n[n] = \mathcal{ZT}^{-1}\Big\{N(Z)\Big\}$, obtaining at the input of the receiver:

$$\begin{aligned} r[n] = v[n] * h_{channel}[n] + n[n] &= \sum_{k_2=-\infty}^{\infty} v[k_2]h_{channel}[n-k_2] + n[n] \\ &= \sum_{i=1}^{M}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} x_i[k_1/M]h_{eq_i}[k_2-k_1]h_{channel}[n-k_2] + n[n]. \end{aligned} \tag{5.2.4}$$

As it can be seen, time-domain zero forcing equalization can be applied at receiver input to remove the interference caused by the channel [4]. This kind of equalization is known as Pre-detection equalization, and it has been proved to contribute positively to the performance of such a communication system.

On the other hand, we would like to find a new equalization scheme at the very end of the system (Post-detection equalization). For that purpose, we can check the signal right before the final downsampling stage at a concrete branch j:

$$\begin{aligned} z_j[n] = r[n] * f_{eq_j}[n] &= \sum_{k_3=-\infty}^{\infty} r[k_3]f_{eq_i}[n-k_3] \\ &= \sum_{i=1}^{M}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty}\sum_{k_3=-\infty}^{\infty} x_i[k_1/M]h_{eq_i}[k_2-k_1]h_{channel}[k_3-k_2]f_{eq_j}[n-k_3]. \end{aligned} \tag{5.2.5}$$

Which should be 0 for $i \neq j$, in case there were no ICI due to the wavelet packet filter orthogonality properties. As it can be seen, the noise term has been removed, since Zero Forcing equalization tries to eliminate the interference caused by the channel, although this may lead to noise power increasing, not taken into account in future derivations.

Consequently, using expression (5.2.5) we will try to find a more efficient post-processing equalization scheme that still satisfies some performance constraints.

5.3 New Post-Processing Equalization Scheme.

As it has been shown in section section 5.2 we can take advantage of the new WPM system we derived during this thesis to cancel channel distortion after the receiver equivalent filter

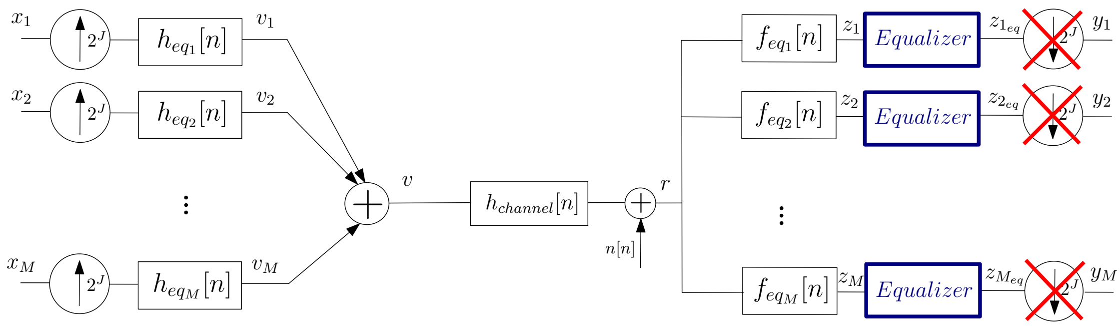

Figure 5.2: J-level time domain equivalent system with post-processing equalization.

of a concrete branch, instead of equalizing at receceiver's input, since the equalization will not be affected by the interjection of another filter as f_{eq_j}. By applying equalization at the output of any of the M branches of the system, we can focus on the specific information related to the user of the correspondant branch.

Therefore, a possible approach is to shrink the output signal of the equivalent receiver filter f_{eq_j} before this signal is convolved with the time response of the correspondent Zero Forcing equalizer. By shrinking the signal, reducing significantly its length with respect to the original one, the equalization would be easier and faster.

Moreover, the advantage of applying equalization not at the input of the receiver, but at the branch itself, is that we are not interested in the information of the M users any more, but only in the information regarding the user of the branch we are in. Therefore, we may be able to relate this shrinking to the system branch, and some factor of length reduction.

Nonetheless, as we will see in following subsections, this signal shortening will require the removal of final downsampling stage, turning this system into a real post-processing equalization scheme.

Therefore, the receiver would be conformed as it is shown in the scheme of figure figure 5.2

In subsection section 5.3.1 the blocks conforming the equalization are explained in detail.

5.3.1 Signal Shortening. Post-processing Equalization.

First, signal at filters f_{eq_j} output will go through a signal shortener. The function of this block will only be to shrink the signal before input it into the equalizer. There will be two main parameters at this step, which are the following:

- Center position: will represent the central point from which we must shorten the signal.

- Step: this factor stands for the number of signal symbols we will take both from the left and the right of the center Position.

The total number of samples taken from the original signal will be given by $2 \cdot step + 1$. It is easy to guess how the smaller the step, the smaller the signal at the output of this block, and therefore the simpler and faster the consequent equalization. Nonetheless, shrinking the signal excessively may lead to a loss of information that may cause an important performance degradation. This step size-performance trade-off will be tested in chapter 6. Moreover it is important to define which should be the value for the center position, since shortening the signal from one point or another may lead to get the information we are interested in, or just a bunch of useless symbols. The central position actually coincides with the length of the signal before going through the channel. This length can be derived from wavelet filter lengths and number of levels J, as [10]:

$$l = (M - 1)(L_{filter} - 1) + 1 \tag{5.3.1}$$

with $M = 2^J$ the total number of users of the system, directly related to the number of levels J and L_{filter} the wavelet filter length.

Therefore, the purpose of this block in the receiver part is to cut the input signal from $(l + step)$ to $(l - step)$ leading to an output signal of length $(2 \cdot step + 1)$.

In figure 5.3 a graphical representation of the signal treatment carried by this block is represented.

Once our signal has been shortened we must perform the equalization itself, which will be simpler due to the shrinking process.

For that purpose, as we mentioned before, a time-domain Zero Forcing equalizer is concatenated at the output of the signal shortener. This equalizer will need as inputs the number of taps for the equalizer as well as the channel impulse response (assumed to be known).

Once the coefficients are found, input shortened signal will be convolved with these coefficients. Last step performed by this block will be to select the position p of the entire output signal that contains the information of the symbol we are interested in. This position is related to the number of taps of equalizer filter n_{taps}, $step$ size, length of channel $l_{channel}$ and number of levels as follows:

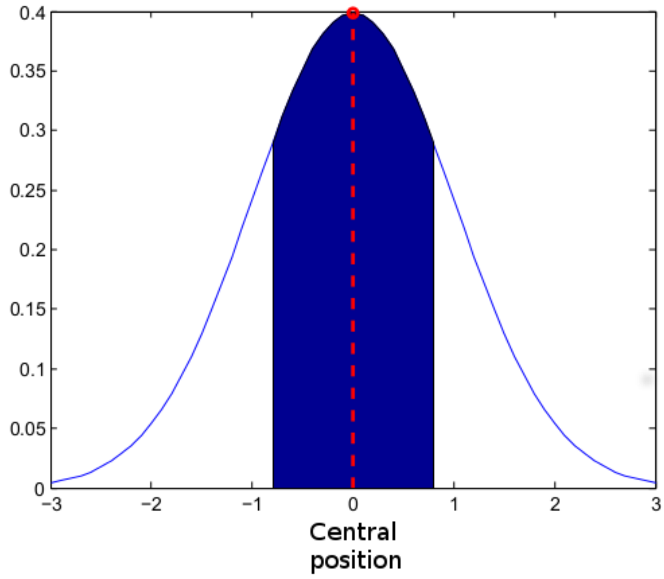

Figure 5.3: Signal shortening.

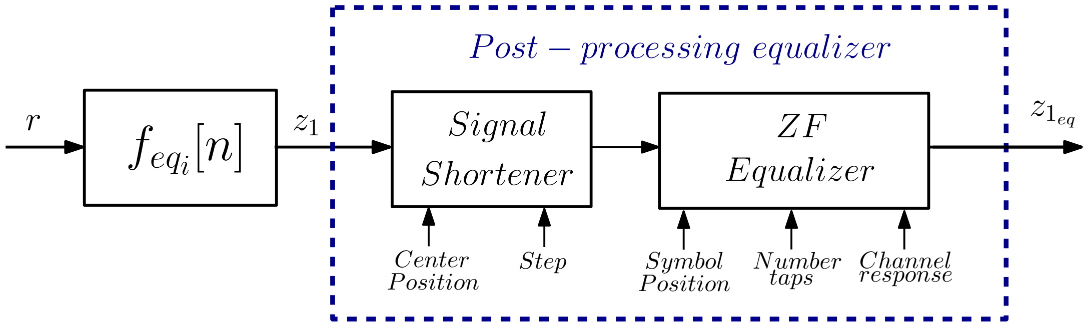

Figure 5.4: Receiver branch for user i with post-processing equalization.

$$p = step + \lfloor \frac{l_{channel}}{2} \rfloor + \lfloor \frac{n_{taps} - 1}{2} \rfloor + 1 \tag{5.3.2}$$

Once this symbol has been picked, last step would be to apply a decider to recover the most-probable sent symbol. This whole process can be checked in a schematic block diagram represented in figure figure 5.4

5.3.2 Recap.

It would be helpful for the reader of this Master Thesis to make a recap of all the steps that conform the new equalization scheme derived throughout this document.

First, final downsampling stage should not be applied. This step is necessary due to the shrinking process, and will simplify equalization.

The signal resulting from the convolution of the output of WPM transmitter with the channel (at the input of the receiver) will be treated branch per branch. At any of these, this signal will be cut, taking only $2 \cdot step + 1$ samples, $step$ samples both from right and left of position l.

Once the signal has been shortened, is convolved with the coefficients resulting from Zero Forcing equalization.

Last step is to select from the output signal of the equalizer the symbol at position p and decide which is the most-likely sent symbol.

6 Simulation Results

In this section we will show the benefits derived from the application of the post-detection equalization system. Our purpose is to prove the validity of the derived system for information exchange purposes.

As shown in previous chapters, main goal of new equalization scheme is to simplify the channel distortion cancellation by reducing signal size in any of the M branches. Therefore, our system will be helpful if, in addition to being simpler and faster, important parameters as BER remain close to traditional system performance. If we can obtain suitable error rates while equalization is simplified, the new scheme will be worth to be applied in future telecommunication scenarios.

6.1 Simulation Setup.

All simulations were carried out with a model developed in Matlab® software. Moreover, all simulations used a $J = 3$ levels equivalent system model derived in chapter 4, using as possible input symbols those belonging to a normalized 16-QAM constellation, being represented in figure 6.1.

In order to test the new wavelet system for different scenarios, proposed channels to simulate its behaviour will be two:

6.1.1 Channel 1.

First channel will be an impulsed channel proposed by Proakis in [19] whose coefficients are given by:

$$h[n] = [0.04 \quad -0.005 \quad 0.07 \quad -0.21 \quad -0.5 \quad 0.72 \quad 0.36 \quad 0 \quad 0.21 \quad 0.03 \quad 0.07] \tag{6.1.1}$$

Also, its time and frequency response can be checked at figure 6.2 figure 6.3 respectively.

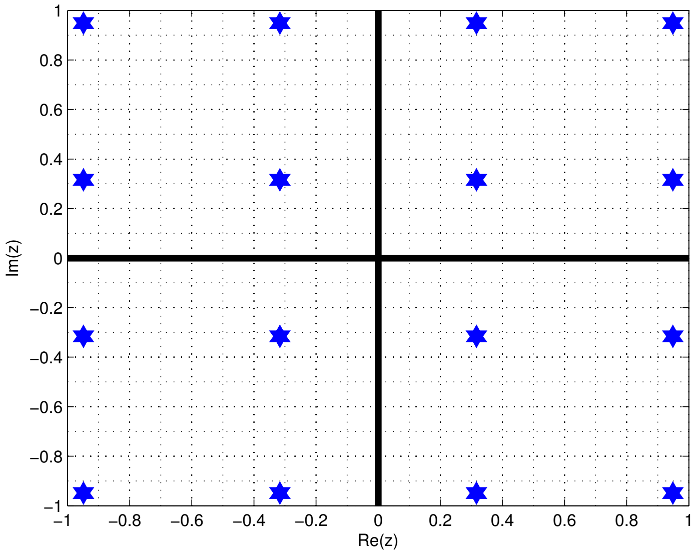

Figure 6.1: 16-QAM constellation.

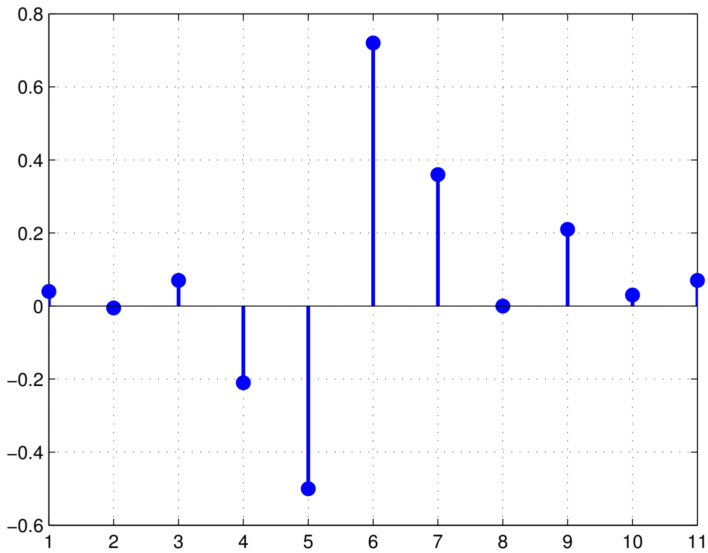

Figure 6.2: Channel 1 impulse response.

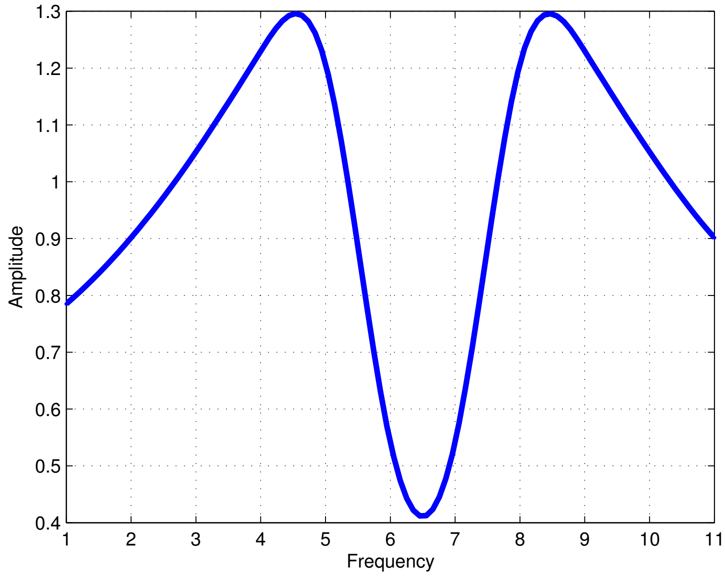

Figure 6.3: Channel 1 frequency response.

This 11-tap channel does not have high order frequency selectivity, and it has been chosen as test channel since some previous research regarding wavelet applications to communications have been carried out considering this Proakis-proposed channel, which makes easier the comparison between traditional and new equalization processes.

6.1.2 Channel 2.

On the other hand, our second channel to simulate will be a highly frequency selective one, modeled as an addition of both real and imaginary normal distributions. Its impulse and frequency response is present in figure 6.4 figure 6.5. Such a channel was chosen since it might be interesting to test the behaviour of the sytem in a high-frequency selectivity situation.

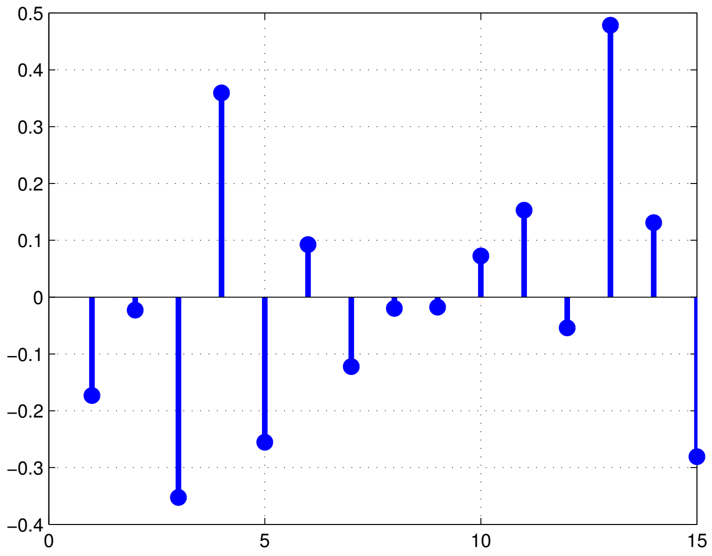

Figure 6.4: Channel 2 impulse response.

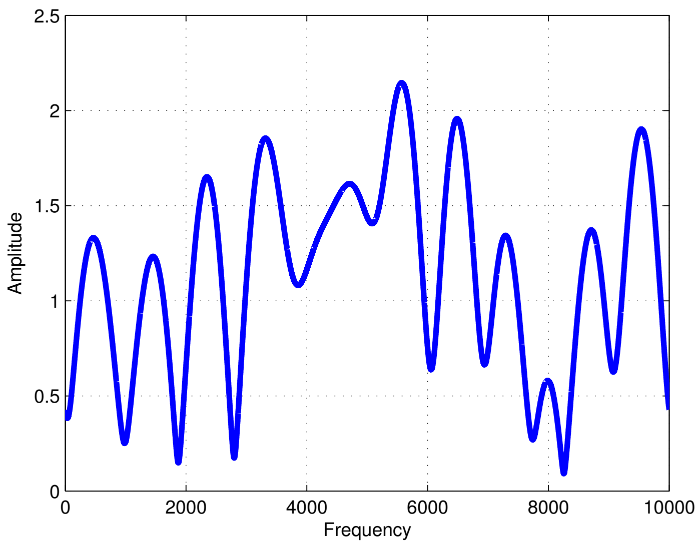

Figure 6.5: Channel 2 frequency response.

6.2 Simulation.

In following lines the reader may see the comparison between performance of both traditional wavelet system ZF equalization and post-processing ZF equalization in terms of Bit Error Rate (BER) versus Signal to noise Ratio (SNR), for different values of *step* parameter.

Then, new equalization scheme will be tested varying different parameters as wavelet filter family and number of equalizer taps, to finally evaluate time computation benefits derived from signal shrinking process, by testing equalization times for both scenarios.

6.2.1 Pre vs Post-Detection System Comparison.

This section will show the performance of the two different equalization procedures, measured as the amount of wrong bits received over the total for each of the different SNR values.

In order to compare post-detection equalization with traditional pre-detection one, these two are simulated, varying for the first one the *step* value, which will determine the length of the signal and, therefore the simplicity of the equalization process. This simulation is critical since will determine if the equalizer derived along this thesis is feasible even though is conceptually easier and functionally faster.

Simulations were carried out for J=3 levels WPM and Daubechies 20 as wavelet filter type. 30-taps equalization was applied for first channel, while for second channel the number of taps was increased to 60 in order to get a better shape for BER curve.

Channel 1.

In figure 6.6 a comparison of Bit Error Rates for pre-detection and post-detection equalization with different step sizes for first channel can be checked. As it can be seen, for very small step sizes (1 and 3) the signal recovered is not proper enough, while for bigger values as 5, the BER stabilizes, ending up in a curve with a performance close to the pre-detection case for a step size of 7.

Therefore, we can get approximately the same error we get if we equalize the long signal at the input of the receiver as if we pick just 15 samples of it $(2 \cdot step + 1)$ and equalize separately in any of the branches. We can consequently ensure that the new post-processing zero forcing equalization scheme developed for this thesis has still proper BER curves, close

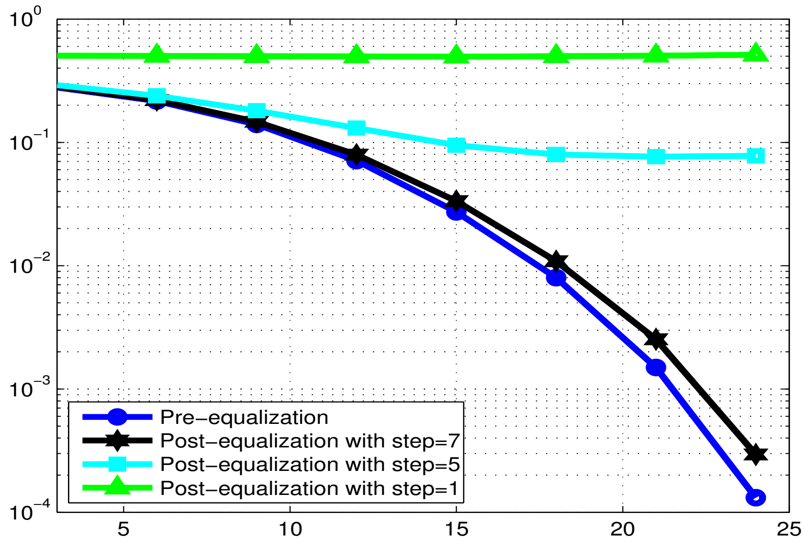

Figure 6.6: SNR vs BER for different step sizes for Channel 1.

to conventional WPM equalization, with *step* values small enough to simplify considerably the process of channel distortion cancellation.

Channel 2.

For second channel we obtained again a situation in which BER decreases as we increment the step size. In this case, by just selecting 61 taps, we can obtain a performance near to the pre-detection equalization case.

Nevertheless, for smaller step sizes like 25 and 20 we still get a good channel interference cancellation.

6.2.2 Wavelet Filters Comparison.

This section will compare the performance of post-processing equalization for different wavelet filters, with J=3 levels WPM system. In case of channel 1, *step* size will be seven and the equalizer will be 30 taps long.

On the other side, for second channel bigger *step* size and filter taps number will be used in order to obtain proper Bit Error Rate curves, being their values 20 and 60 respectively.

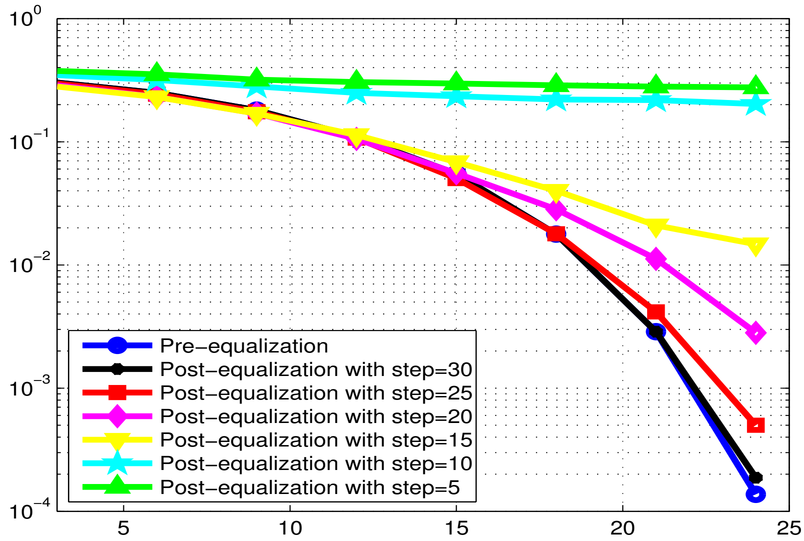

Figure 6.7: SNR vs BER for different step sizes for Channel 2.

Channel 1.

As it occurs in a conventional equalization for WPM systems [4] different performances are obtained depending on the filter type, just as expected. Daubechies 20, Symlet 20 and Coiflet 5 show proper results in error rate performance while Haar, since it is just a 2-tap filter, does not perform as well, specially for high Signal to Noise ratio values. Biorthogonal 3.5 and Reverse Biorthogonal 3.5 are not recommended for these communication systems, due to the lack of orthogonality between sub-carriers generated by WPM systems composed of these filters.

Channel 2.

Once again we can check in figure 6.9 what aforementioned applied, in for channel 2. In this case, Daubechies 20 highlights as the one with best performance. Symlet and Coiflet are one step behind as regards BER, while Biorthogonal and Reverse Biorthogonal show poor performance.

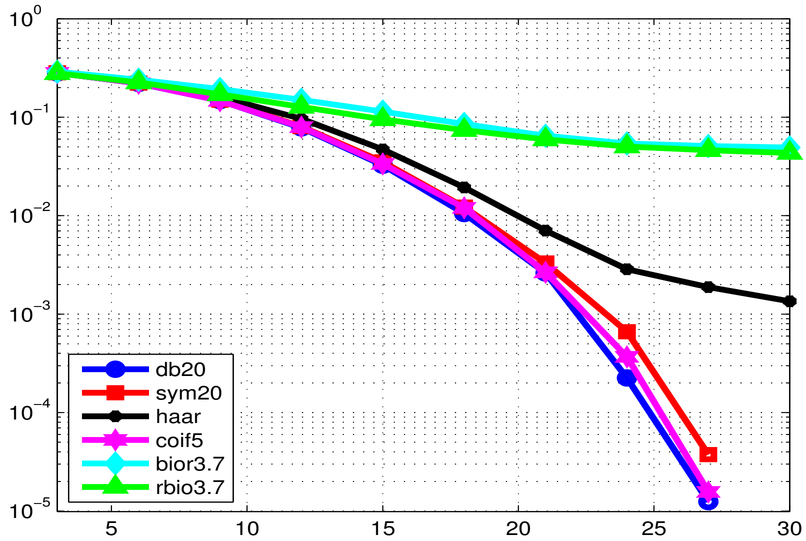

Figure 6.8: SNR vs BER for different wavelet filters for Channel 1.

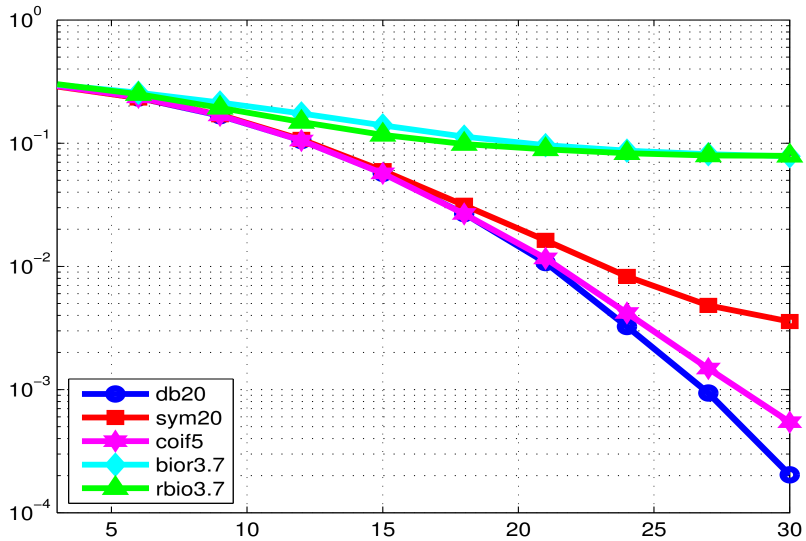

Figure 6.9: SNR vs BER for different wavelet filtersvfor Channel 2.

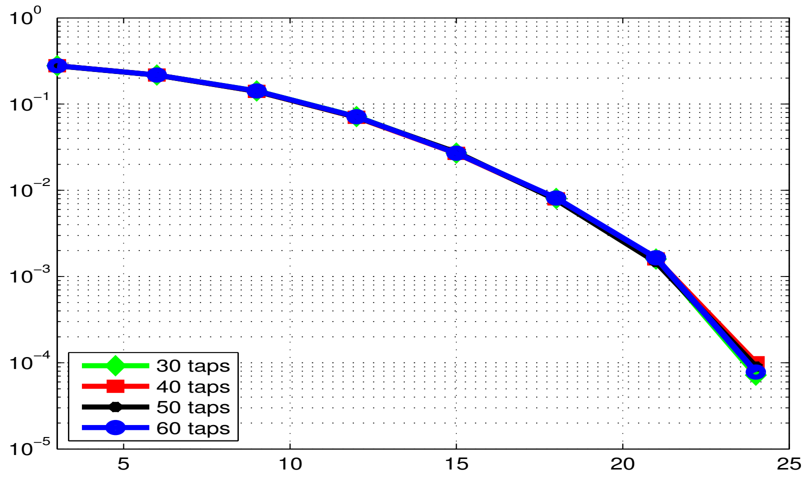

Figure 6.10: SNR vs BER for different number of equalizer taps for Channel 1.

6.2.3 Equalizer Taps Comparison.

In this section we will show the differences on system behaviour depending on number of taps for the equalizer filter.

Theoretically, the bigger the step size the better the equalization, since the equalizer filter will have more coefficients, resulting in a better channel distortion cancellation. As we will see, depending on channel characteristics we may just need a low number of equalizer taps in order to recover the transmitted symbols properly , and a increasing number of taps might not provide a performance boost that justifies the augmentation of this parameter.

Simulation conditions are the same as the ones defined in section 6.2.2 using Daubechies 20 families as wavelet filter family.

Channel 1.

For this channel we can check in figure 6.10 how the selection for the number of taps of the post-processing equalizer does not have a big impact in our system.

Therefore for this concrete situation, channel interference is already removed for a 30 taps equalization, and an increment of this parameter minimally affects Bit error rate.

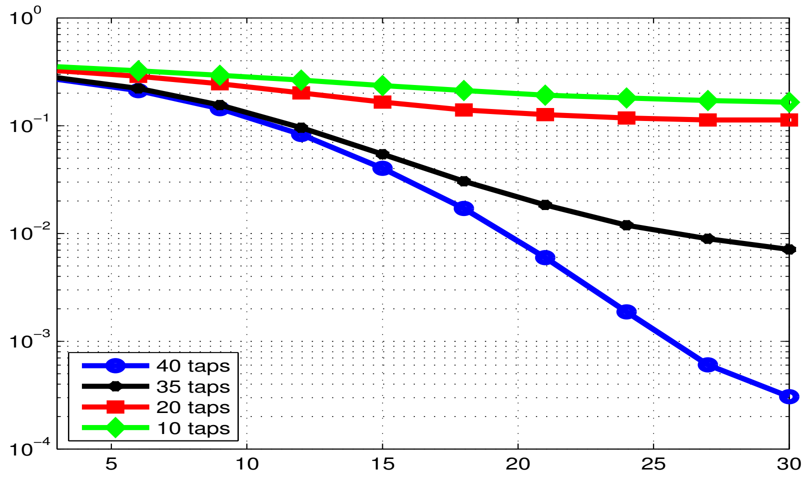

Figure 6.11: SNR vs BER for different number of equalizer taps for Channel 2.

Channel 2.

In this case we are in a situation where the number of taps of our equalizer becomes a crucial factor for the performance of our system. In figure 6.11 we can check how, the bigger the equalizer filter length, the better the performance of the system. This factor affects considerably BER, obtaining much different curves when the number of taps gets increased.

As it can be seen, 10 and 20 taps are not enough in order to equalize properly the interest signal. For 35 taps we actually have a curve that corresponds to a communication system with a correct behaviour.

But it would be necessary a better performance, especially for high Signal to noise ratio values. This can only be achieved when we use from 40-taps equalization onwards.

Therefore, here we have an example of a channel whose distortion will require en exhaustive equalization in order to be deleted.

6.2.4 Equalization Time Comparison.

One of the key points to test for the equalization scheme derived throughout this Master Thesis is not only to check that it actually satisfies the performance contraints, but also to test that by applying this new procedure we will actually have a time consumption or complexity reduction.

In this last section we will focus on the first one, trying to show that by shrinking the signal the time it takes to equalize the signal is actually much lower.

The total time, both for pre-detection equalization and post-detection equalization has been computed by an average over a certain number of iterations. For the comparison of these times, it has been taken as model a common mobile communications scenario, where every single user of the system gets the whole signal in his mobile phone, which implies that this signal the user gets is not already equalized. Therefore, every single user will do pre-detection equalization, even though in that signal the information about the rest of the users is also present.

Consequently, if the new equalization scheme shows some advantages with respect to the traditional one, the system will be worth to use (we don't have to add up the times of the M users and compare it with the time of one pre-detection equalization). Simulations were carried out for a 4-level WPM system with Daubechies 20 wavelet filters in using a high-frequency selective channel.

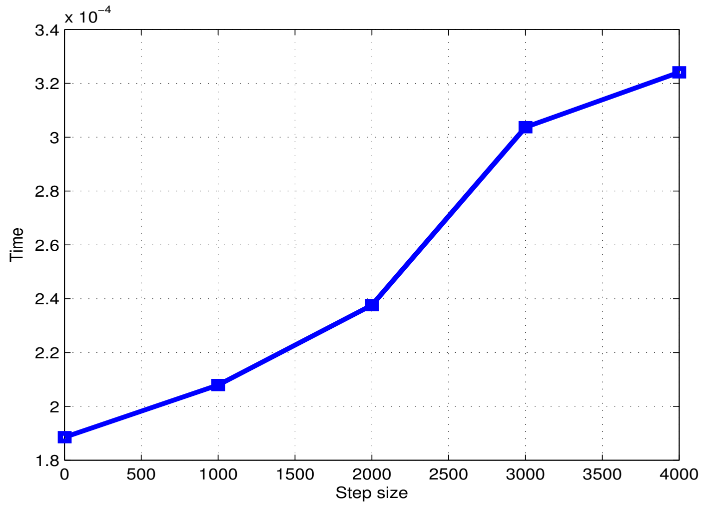

Figure 6.12: Equalization time with respect to the step size.

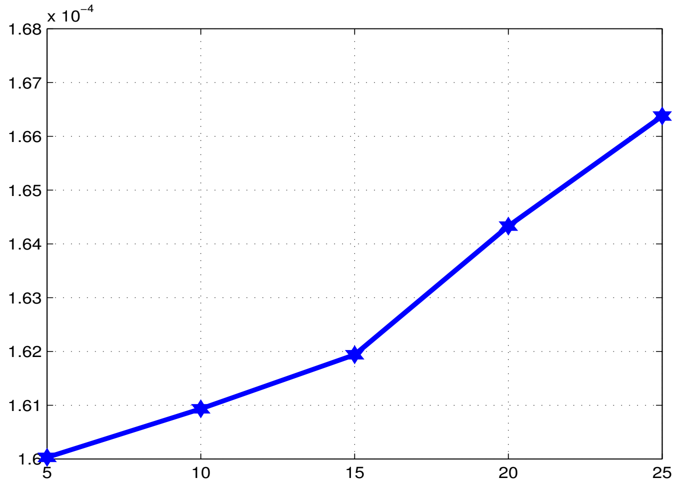

Figure 6.13: Equalization time with respect to the step size.

In figure 6.12 figure 6.13 the post-processing equalization times for different step sizes are compared.

In figure 6.12 we compare the times when the difference of *step* size is big, while for figure 6.13 this difference in *step* length is decreased in order to see in more detail how the length of this parameter affects the time complexity of the system. As expected, the bigger the step size, the bigger the signal we are equalizing and therefore, the bigger the time it takes to carry out the equalization. These differences in time are well shown on both figures.

On the other side it would be useful to compare these times with the ones corresponding to a a pre-detection equalization case. This equalization will be, on the other hand, dependant on the number of levels of the WPM system (J). The bigger this value, the bigger the signal at the input of the receiver and therefore the more time it will take to equalize it. Nonetheless, our new equalization scheme does not depend on this factor, since the signal is cut to a certain length related to the step size.

As we can see in figure 6.14 the bigger the step size the closer to the time we would get for pre-detection equalization (green line). In fact, for:

$$step = \frac{l}{2} - 1 \tag{6.2.1}$$

with l the length of the signal at the input of the receiver, the *step* size would be so big

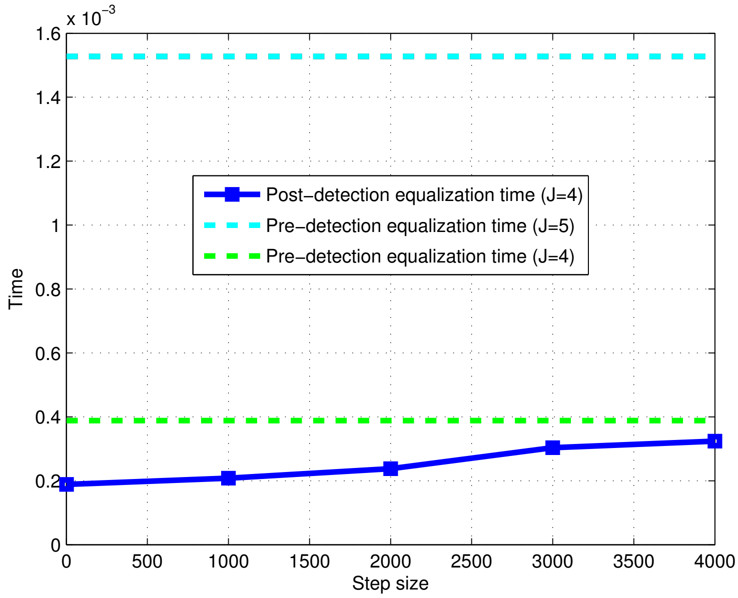

Figure 6.14: Equalization time for both pre-detection and post-detection equalization with respect to the step size.

that we would be actually picking the whole signal (no shortening) and therefore, the time should be approximately equal to the one of pre-detection equalization.

Moreover, for J=5 levels, the received signal is bigger and therefore the equalization time is even bigger (cyan line), so our equalizer will be more efficient.

By means of the previous graphs we have shown in this section how this new methodology can lead to a much more light-handed equalization regarding time computation.

7 Conclusions

7.1 Summary

Throughout this Master Thesis Wavelet Packet Modulations in a communications scenario have been deeply studied.

First, an introduction to show the context of this technology in telecommunications panorama was displayed, as well as the set of fields where wavelet systems have succesful applications. Afterwards, main characteristics of these kind of systems through the stating of is theory were specified to, then, derive a simpler and more useful new representation of WPM systems, together with the prove of its equivalence with respect to traditional WPM schemes.

Next, after a brief introduction to previously applied equalization models, a new equalization possibility, specific for these systems, was derived. Main components, of new WPM equalizer were carefully studied, emphasizing their purpose in the equalization process, involved parameters, characteristics and prior advantages, to finally recap the whole equalization process step by step.

Lastly in chapter 6 it was shown the behaviour of this new equalizer with respect to different adjustable parameters through simulation, emphasizing advantages of new WPM equalization scheme derived along this document.

7.2 Conclusions.

When a depth study of WPM systems was carried out, we intuited how challenging would be to find a new way to eliminate distortion introduced by the channel, also exploiting particular characteristics of this technology. Even so, it can be considered that this Master thesis has allowed to glimpse other forms of equalization for wavelet-based communication schemes, as well as some aspects we consider proven, such as:

- A simplified model for the well-known WPM system architecture is possible, where the filter bank structure has been removed, using instead equivalent filters for any of the M branches, being these two schemes functionally equivalent.

- Zero forcing equalization can be applied to this simplified model, both before and after the equivalent receiver filter with same results.

- Per-branch equalization can be achieved, with signal shortening leading to faster and easier channel distortion cancellation.

- It exists a trade-off between the shortening of the signal (given by factor *step*) and the performance in Bit Error Rate terms of the system. The bigger the *step* size, the shorter the signal and then the easier and simpler the equalization but nonetheless resulting in a performance degradation.
  Therefore, a proper selection of this step size might be critical for the characteristics of our communication system.

- The properties of this new equalization scheme may differ depending on different factors as wavelet filter family used, number of users and equalizer taps, coinciding with equalizations carried out in previous schemes.

7.3 Future Work

Along this Master Thesis we have studied wavelet technology deeply. Our main goal was to find a equalization opportunity from the scratch. Therefore, the channel distortion cancelation process indicated along this document is completely new, which implies that there are many aspects that, due to a lack of time and space, could not be studied and tested, although it would be appropiate to deepen this new equalizer.

Some of the aspects we consider may be interesting for future studies are :

- Extend this procedure to other well-known equalization techniques such as DFB and MMSE.

- Find mathematical relations for the central position of signal shortener and symbol of interest for equalizer output with respect to system parameters (number of levels, filter length, step size, ...)

- Using equivalent wavelet system derived throughout this thesis, find a different approach for equalization that takes even more advantage of such a simplification.

- Analyse the impact of signal shortening in other aspects appart from error rate at the output, for example Peak Average to Power Ratio (PAPR), spectral efficiency, equalization arithmetic complexity, optimum value for step size, etc.

- Study the possibility of introducing a Multiple-Input Multiple-Output (MIMO) technology, which by adding antenna redundancy allows Bit error rate reduction [1]. Check its behaviour and how it is affected by signal shortening Zero Forcing equalization in an equivalent wavelet system.

Bibliography

[1] Alamouti, S. M. A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications* (1998), 1451–1458.

[2] Asif, R., Hussaini, A., Abd-Alhameed, R., Jones, S., Noras, J., Elkhazmi, E., and Rodriguez, J. Performance of different wavelet families using dwt and dwpt-channel equalization using zf and mmse. In *Design and Test Symposium (IDT), 2013 8th International* (Dec 2013), pp. 1–6.

[3] Baig, S., ur Rehman, F., and Junaid Mughal, M. Performance comparison of dft, discrete wavelet packet and wavelet transforms, in an ofdm transceiver for multipath fading channel. In *9th International Multitopic Conference, IEEE INMIC 2005* (Dec 2005), pp. 1–6.

[4] Bajpai, A., Lakshmanan, M., and Nikookar, H. Channel equalization in wavelet packet modulation by minimization of peak distortion. In *Personal Indoor and Mobile Radio Communications (PIMRC), 2011 IEEE 22nd International Symposium on* (Sept 2011), pp. 152–156.

[5] Chen, B.-S., Chung, Y.-C., and Huang, D.-F. A wavelet time-scale deconvolution filter design for nonstationary signal transmission systems through a multipath fading channel. *Signal Processing, IEEE Transactions on 47*, 5 (May 1999), 1441–1446.

[6] Chen, S., Guangfa, D., and Yen, T. Zero-forcing equalization for ofdm systems over doubly-selective fading channels using frequency domain redundancy. *Consumer Electronics, IEEE Transactions on 50*, 4 (Nov 2004), 1004–1008.

[7] Edfors, O., Sandell, M., van de Beek, J.-J., Landström, D., and Sjöberg, F. An introduction to orthogonal frequency-division multiplexing. Tech. Rep. TULEA 1996:16, Luleå University of Technology, 1996.

[8] Faria da Rocha, C., and Bellanger, M. Sub-channel equalizer design based on geometric interpolation for fbmc/oqam systems. In *Circuits and Systems (ISCAS), 2011 IEEE International Symposium on* (May 2011), pp. 1279–1282.

[9] Ghobrial, A., and Adhami, R. Discrete wavelet transform domain adaptive decision feedback equalization. In *System Theory, 2002. Proceedings of the Thirty-Fourth Southeastern Symposium on* (2002), pp. 243–247.

[10] Jamin, A., and Mähönen, P. Wavelet packet modulation for wireless communications: Research articles. *Wirel. Commun. Mob. Comput. 5*, 2 (Mar. 2005), 123–137.

[11] Khan, U., Baig, S., and Mughal, M. Performance comparison of wavelet packet modulation and ofdm for multipath wireless channel. In *Computer, Control and Communication, 2009. IC4 2009. 2nd International Conference on* (Feb 2009), pp. 1–4.

[12] M. Sifuzzaman, M.R. Islam, M. A. Application of wavelet transform and its advantages compared to fourier transform. In *Journal of Physical Sciences, Vol. 13* (2009), pp. 121–134.

[13] Michailow, N., Gaspar, I., Krone, S., Lentmaier, M., and Fettweis, G. Generalized frequency division multiplexing: Analysis of an alternative multi-carrier technique for next generation cellular systems. In *Wireless Communication Systems (ISWCS), 2012 International Symposium on* (Aug 2012), pp. 171–175.

[14] Mohammadi, Z., Saadane, R., Wahbi, M., and Aboutajdine, D. Recovery of isi channels with wavelet packet modulation using linear equalization and channel estimation. In *I/V Communications and Mobile Network (ISVC), 2010 5th International Symposium on* (Sept 2010), pp. 1–4.

[15] Nick LaSorte, W. J. B., and Refai, H. H. The history of orthogonal frequency division multiplexing. In *SS01T2: History of Communications* (June 2009).

[16] Nikookar, H. *Wavelet Radio: Adaptive and Reconfigurable Wireless Systems Based on Wavelets*. EuMA High Frequency Technologies Series. Cambridge University Press, 2013.

[17] of Future Wireless Technologies Huawei European Research Center, D. E. S. D. Forward 2020, 5g. In *2015 Cambridge Wireless Radio Technology SIG & Small Cell SIG 5G A Practical Approach 1234* (May 2015).

[18] Paper, C. W. Cisco visual networking index: Global mobile data traffic forecast update, 2010-2015.

[19] Proakis, J. *Digital Communications*. McGraw-Hill Series in Electrical and Computer Engineering. Computer Engineering. McGraw-Hill, 2001.

[20] VAIDYANATHAN, P. P. *Multirate Systems and Filter Banks.* Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1993.

[21] VAZ, C., AND DAUT, D. Communications receivers employing wavelet-domain zero-forcing equalization of multipath fading channels. In *Vehicular Technology Conference (VTC Spring), 2012 IEEE 75th* (May 2012), pp. 1–5.