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IMPLEMENTATION WITH STATE DEPENDENT FEASIBLE SETS AND PREFERENCES: A RENEGOTIATION APPROACH*

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Abstract

In this paper we present a model of implementation based on the idea that agents renegotiate unfeasible allocations. We characterize the maximal set of Social Choice Correspondences that can be implemented in Nash Equilibrium with a class of renegotiation functions that do not reward agents for unfeasibilities. This result is used to study the possibility of implementing the Walrasian Correspondence in exchange economies and several axiomatic solutions to problems of bargaining and bankruptcy.

JEL CLASSIFICATION: C72, D60, D78

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1. Introduction

Since the classic papers of Hurwicz in the early seventies, a great deal of attention has been devoted to the problem of implementing social choice rules when preferences are state dependent (see, e.g. Jackson [2000] for a survey). In contrast, very few contributions have dealt with the problem of implementing social choice rules when the set of feasible outcomes is state dependent. The problem is that, in this case, some messages yield unfeasible allocations. Thus, we have to describe how to deal with unfeasible allocations. The standard approach is to design a state dependent mechanism in which the planner can ex-post verify if players are exaggerating endowments or technological capabilities (i.e. by asking them to put endowments on the table). If infeasibility occurs, players expect a serious punishment (Hurwicz et al., [1995], Tian [1993], Tian and Li [1995], Hong [1995], [1996], [1998], Serrano and Vohra [1997] and Dagan et al. [1999]).

We have several reservations about this approach: The assumptions of ex-post verification of exaggeration only, and a serious punishment if infeasibility arise are rather extreme. Moreover, it is not clear how to proceed without them. This approach also produces a curious asymmetry between mechanisms coping with state dependent preferences ("demand") and mechanisms coping with state dependent endowments ("supply"). The former are state independent but the latter are state dependent. Finally, the implementing mechanisms are hard to describe so it may be costly to use them.

In this paper we present a model based on the idea that unfeasible allocations are *renegotiated*. We model the social process which transforms unfeasible allocations into feasible ones by means of a *reversion function*. This concept originates in Maskin and Moore (1999) and has been developed by Jackson and Palfrey (2001). In these papers the reversion function formalized the process of renegotiation by means of which agents trade goods allocated by the mechanism or veto some feasible allocations. In our case, the reversion function represents the way in which society reacts to unfeasible allocations.[1] Consequently, the properties that we impose on the reversion function are very different from those assumed by the earlier literature.

In this paper we assume complete information. This is a clean scenario which looks to be a good candidate for a first trial of our ideas. Thus we concentrate on Nash implementation and assume that agents know the reversion function. Therefore the reversion function induces new preferences, to be called *reverted preferences* (this is the "translation principle" in Maskin and Moore [1999]). Reverted preferences are state dependent even if preferences are not. Hence, implementation when the feasible set is state dependent reduces to the case of implementation when only preferences are state dependent. However as remarked by Maskin and Moore, "results from the standard literature are too abstract to give a clear indication of how serious a constraint renegotiation is...".

We focus our attention in a class of reversion function in which, should an infeasibility arise at least one agent is made worse off. We call this a *non-rewarding* reversion function. Reversion functions considered before do not fall into this class because they assume that agents are made better off by renegotiating. The difference is explained by the fact that in their case, renegotiation comes from the inability of the mechanism to stop agents from reaching mutually beneficial trades. In our case renegotiation arises from the physical impossibility of carrying out the intended plans so that somebody has to make a sacrifice in order to achieve feasibility. An extreme case of a non-rewarding reversion function is when, should an infeasibility arise, all agents are punished so that they prefer any allocation without punishment to the situation in which they are pun-

[1] Renegotiation may be channelled by institutions or may be totally free. A striking example is that of a legal system. Once infractions are detected there are institutions designed to punish transgressors and to restore feasibility. In our case we can think of the feasible set including not only the properly feasible allocations, but also all punishments and additional devices that can be administrated by the designed institutions, as well as the delays that may occur.

ished. This strong form of punishment -which we will call *generalized severe*- resembles the one implicitly assumed in the previous literature, but in our case it only serves an instrumental role: we show that in the class of non-rewarding reversion functions, the generalized severe reversion function implements the largest class of social choice rules (Proposition 1).

An easy adaptation of the classic result shows that monotonicity, when reverted preferences are given by the generalized severe reversion function, is a necessary and almost sufficient condition of implementation in Nash equilibrium (Remark 1). Thus, our first task is to characterize monotonicity. We show that it is equivalent to a weak form of unanimity and a generalized form of contraction consistency (Proposition 2). The former property is satisfied by most social choice rules and the latter is similar to Nash's independence of irrelevant alternatives.

Next, we apply the previous result to several frameworks and compare our findings with the earlier literature. In the case of exchange economies, weak unanimity is trivially satisfied by any individually rational social choice rule. We show that the Constrained Walrasian rule satisfies generalized contraction consistency and thus is implementable (Proposition 3). However the individual rationality requirement which in Hurwicz et al. (1995) is necessary and sufficient for feasible implementation, is not sufficient for implementation in our framework. The reason is that they not only assumed that players never exaggerate their endowments. Since a mechanism is designed for each state of the world, it is implicitly assumed that players never use messages designed for a different state of the world even if such messages lead to an outcome which is feasible in the current state of the world. We turn our attention to bargaining problems. We show that if the disagreement point is not state dependent the Nash Bargaining solution is implementable with a non-rewarding reversion function (Proposition 4). This agrees with the findings of Serrano (1997) and Naeve (1999). We also show that the Kalai-Smorodinski solution is not implementable. Finally we consider the taxation problem in which the mechanism has to collect a given amount of taxes. We find a negative result, namely that a taxation method is implementable if and only if it is a serial dictatorship, i.e. agents are arranged so that any agent pays the minimal tax compatible with the following agents to be able to complete the required amount (Proposition 5). This negative result contrasts with the permissive results obtained by Dagan et al. (1999). The difference between our approaches is that in their case the report sent by agents matters for the renegotiation and in our case it does not. Our result serves to highlight the negative consequences of disregarding reports (i.e. a fiscal amnesty) even if all agents show endowments.

The rest of the paper goes as follows: Section 2 spells out the model. Section 3 introduces reversion functions. Sections 4 and 5 study implementation under the assumption that the reversion function is non rewarding. Section 6 concludes.

2. The model

In this section we provide the main definitions. Let us first describe the environment.

Let $I = \{1, ..., n\}$ be the set of agents. Let ω_i be type of i and Ω_i be agent i's type set. Let $\Omega \subset \prod_{i=1}^{n} \Omega_i$ be the set of all possible states of the world. Each $\omega \in \Omega$ is characterized by a list of individual outcome sets $(X_1(\omega), ..., X_n(\omega))$, a feasible set $A(\omega) \subset \prod_{i=1}^{n} X_i(\omega) \equiv X(\omega)$ and a preference profile $R(\omega) = (R_1(\omega), ..., R_n(\omega))$. The outcome set of i might include sanctions that can be charged to i and other constraints such as individual rationality, etc. $A(\omega)$ contains all feasible allocations including punishments that arise in state ω. Set $A \equiv \bigcup_{\omega \in \Omega} A(\omega)$. Let $a = (a_1, ..., a_n) \in A$ be an allocation also written (a_i, a_{-i}). Let $A_i(\omega) = \{a_i \in X_i(\omega) : \exists\ a_{-i} \text{ such that } (a_i, a_{-i}) \in A(\omega)\}$ be agent i's feasibility constraint. Observe that $A(\omega)$ can be written too as $\bigcap_{i=1}^{n} \{a : a_i \in A_i(\omega)\}$. $R_i(\omega)$ is a preference relation, a complete, reflexive and transitive binary relation on $X(\omega)$. $P_i(\omega)$ denotes the corresponding strict preference relation. Let $L_i(a, \omega) = \{x \in$

$A(\omega) : aR_i(\omega)x\}$ be agent i's lower contour set of a_i. Let $\Re_i \equiv \cup_{\omega_i \in \Omega_i} R_i(\omega_i)$ be the set of i's admissible preferences relations. Set $\Re = \prod_{i=1}^{n} \Re_i$. With abuse of notation, for every profile of preferences $R \in \Re$ we will also denote by R the preference profile that R induces on A.

A correspondence $F : \Omega \twoheadrightarrow A$ such that $F(\omega) \subset A(\omega)$ for all $\omega \in \Omega$ will be called a *Social Choice Rule* (*SCR* for brevity).

A *mechanism* is a pair (M, g) where $M \equiv \prod_1^n M_i$ is the *message space* and $g : M \rightarrow A$ is the outcome function. M_i denotes agent i's *message space.* Let $m = (m_1, .., m_n) \in M$, be a list of messages also written (m_i, m_{-i}). Given $\omega \in \Omega$, a mechanism (M, g) induces a game $(M, g, R(\omega))$.

A message profile $m^* \in M$ is a *Nash equilibrium for* $(M, g, R(\omega))$ if, for all $i \in I$ $g(m^*)R_i(\omega)g(m^*_{-i}, m_i)$ for all $m_i \in M_i$.

$NE(M, g, R(\omega))$ will denote the set of allocations that are yielded by all Nash equilibria for $(M, g, R(\omega))$.

The mechanism (M, g) *implements* F *in Nash equilibrium* if, for all $\omega \in \Omega$ $NE(M, g, R(\omega)) = F(\omega)$.

3. Reversion functions

Since outcomes that are feasible in some states may be unfeasible in others, we have to describe how society deals with unfeasible allocations. We assume that if an allocation is unfeasible it is transformed into a feasible one by a process that might involve delays (because renegotiation takes time), penalties to some individuals, etc. This systematic way in which the reallocation process takes place will be called a reversion function.[2] This reallocation may correspond to a "free-market renegotiation" or to a process where the planner applies some kind of punishment or a bankruptcy rule. Formally:

Definition 1. *A reversion function is a map $h : A \times \Omega \rightarrow A$ such that for each $\omega \in \Omega$, i) $h(a, \omega) \in A(\omega)$ $\forall a \in A$ and ii) If $a \in A(\omega)$, $h(a, \omega) = a$.*

A reversion function always yields feasible allocations (condition i) above) and is such that feasible allocations are not renegotiated (condition ii) above). The latter condition is made in order to separate the issue of infeasibility from the issue of pure renegotiation.[3]

If the reversion function can be chosen by the planner, under weak conditions, any single valued SCR can be implemented as the next example shows.

Example 1. *Assume that there is a state of the world, say ω', in which the feasible set is larger than in any other state, i.e. $A(\omega) \subset A(\omega')$, any $\omega \neq \omega'$. Then, any single-valued SCR such that $F(\omega') \in A(\omega') \setminus \bigcup_{\omega' \neq \omega} A(\omega)$ can be implemented for some reversion function: Take a constant mechanism $g(m) = F(\omega')$, $\forall m \in M$ and a reversion function $h(F(\omega'), \omega) \equiv F(\omega)$, $\forall \omega \in \Omega$. h fulfils the conditions for a reversion function since $F(\omega')$ is unfeasible at any state different than ω' and by definition it provides the desired allocations. Notice that implementation occurs in dominant strategies.*

This example implies that in order to obtain meaningful results it is better not to allow the designer of the mechanism to also design the reversion function. This is also intuitively agreeable because it seems reasonable that there are aspects of renegotiation

[2] See Amorós (2004) for a model with several renegotiation functions.

[3] A tautological interpretation of the latter condition is that $A(\omega)$ is the set of allocations that are not renegotiated at ω.

that are beyond the control of the designer. Thus, in the rest of the paper we will assume that the reversion function is exogenously given.

To explain the next step, consider the simplest possible case: At states of the world ω and ω' the preference profile, say R, is the same. Let a, b and c be three allocations that are feasible at state ω. Assume that aP_ibP_ic for some agent i. In state ω', a is not feasible and is renegotiated to c and b is feasible. So, even if the underlying preferences are the same in both states, player i prefers a to b at state ω and b to a at ω'. To formalize and extend this idea we give the following definition.

Definition 2. *Given $\omega \in \Omega$ and a reversion function h, the reversion of $R(\omega)$ on $A(\omega)$, denoted by $R^h(\omega)$ is*

$$aR_i^h(\omega)b \Leftrightarrow h(a,\omega)R_i(\omega)h(b,\omega),\ \forall\ a,b \in A,\ i \in I.$$

$L_i^h(a,\omega) = \{b \in A : h(a,\omega)R_i(\omega)h(b,\omega)\}$ will be called the lower contour set of a at ω with respect to $R^h(\omega)$.

Then, when the reversion function is h, we can interpret that agents' preferences are the reverted preferences, i.e. they only care about reverted allocations. The next definition is a straightforward adaptation of the standard notion of implementation in Nash equilibrium.

Definition 3. *A social choice rule F is h-implementable in Nash Equilibrium if there exists a game form (M, g) such that for all $\omega \in \Omega$*

$$F(\omega) = h(NE(M, g, R^h(\omega)))$$

In words, F is h-implementable in Nash equilibrium if and only if it is implementable in Nash equilibrium when for each $\omega \in \Omega$ the correspondent preference profile is $R^h(\omega)$. In other words, once we consider that agents' preferences are those induced by the reversion function, we can deal with h-implementation exactly in the same way as done in the classical implementation problem.

In the study of the restrictions that a state dependent feasible set imposes on implementation, we concentrate on monotonicity (or Maskin-monotonicity). As observed by Jackson (2001), monotonicity is the most important obstacle to implementation in Nash equilibrium. For instance, it is not generally satisfied by the Walrasian social choice rule. Monotonicity is a necessary and almost sufficient condition for a SCR to be implementable in Nash equilibrium (see Maskin (1999) or Repullo (1987)). Thus it is the first condition to deal with.

A SCR satisfies monotonicity whenever an alternative is chosen at a state of the world and it rises in each agent's preference ranking at another state of the world, then it must be chosen also at this state. Now we restate the definition of monotonicity in terms of reverted preferences. Let h be a reversion function.

Definition 4. *A social choice rule F is h-monotonic if for any $\omega, \omega' \in \Omega$ and a such that $h(a, \omega) \in F(\omega)$ such that $L_i^h(a, \omega) \subset L_i^h(a, \omega')$ for all agents i then $h(a, \omega') \in F(\omega')$.*

The importance of h-monotonicity is highlighted by the following remark whose proof is a straightforward adaptation of an standard result mentioned before and, therefore, is omitted:

Remark 1. *If a social choice rule is h-implementable in Nash equilibrium, it is h-monotonic. Moreover in economic environments with $\#I > 2$ if a social choice rule is h-monotonic, it is h-implementable in Nash equilibrium.*[4]

[4] An economic environment is one in which no two agents agree on the top allocation of their preference rankings.

4. Non-Rewarding Reversion Functions: Basic Results

In this section, we restrict our attention to a class of reversion functions where renegotiation is not advantageous for all players. We will call them *Non-Rewarding.* We will show that inside this class a particular reversion function -that we will call *Generalized Severe*- implements the maximal set of SCR. Then, we will characterize the SCR that can be implemented under generalized severe reversion functions.

Let us start by defining the following class of reversion functions:

Definition 5. *A reversion function is non-rewarding if for $a \in A(\omega)$ either there exists $i \in I$ and $c \in A(\omega')$ such that $aR_i(\omega)h(c,\omega)$ with $cP_i(\omega')h(a,\omega')$ or $L_j^h(a,\omega) \subset L_j^h(a,\omega')$ for all j.*

Consider the case where the feasible set is constant and only preferences change. In this case the first condition in the definition postulates the existence of a pair of allocations for which there is a preference reversal.[5] In the case where preferences are fixed and the feasible set varies, the idea is that when agents renegotiate, something bad happens -delays, punishments engineered from the designer, etc.- and this is what causes the inversion of the ranking of a and c in the reverted preferences. The second condition in the definition considers the case where a has improved in everybody´s ranking when passing from ω to ω'. It takes care of the case where $A(\omega') \subset A(\omega)$ because if $a \in A(\omega')$ no feasible reversal around a can take place.

Consider now a specific reversion function which belongs to the class of non-rewarding ones. Suppose that, should an infeasibility arise, players are redirected to what they consider to be the worst possible allocation. This reversion function resembles the assumption made in previous papers that agents do not choose unfeasible messages because the planner detects infeasibility and imposes a punishment in such a way that

[5] This condition was emphsized by Maskin and Moore (1991): "The other problem that renegotiation poses is that it interferes with "preference reversal".

agents prefer any other feasible allocation to this punishment. However our interest in this particular reversion function arises from the fact that it allows finding the maximal set of SCR that can be implemented under non-rewarding renegotiation (see Proposition 1 below).

Let $G \in A(\omega)$, $\forall \omega \in \Omega$ be such that for all i, $aP_i(\omega)G$ with $a \neq G$ and $a \in A(\omega)$. G will be called the "generalized punishment point". The reversion function with $h(a, \omega) = G$ if $a \notin A(\omega)$ will be called *generalized severe* and the induced preferences $R^h(\omega)$ will be called the *saturation of* $R(\omega)$ on $A(\omega)$ defined by the following properties.

For all $i \in I$:

(1) If $a, b \in A(\omega)$ then $aR_i^h(\omega)b$ if and only if $aR_i(\omega)b$

(2) If $a \in A(\omega)$ and $b \notin A(\omega)$ then $aP_i^h(\omega)b$

(3) If $a, b \notin A(\omega)$ then $aI_i^h(\omega)b$

We show that generalized severe reversion implements the largest set of social choice rules among the class of non-rewarding reversion functions.

Proposition 1. *Let F be a SCR which is $h-$implementable in Nash Equilibrium with a non rewarding reversion function. Then F is implementable in Nash Equilibrium with a generalized severe reversion function.*

Proof Let (M, g) implementing F with reversion function h.

Let $a \in F(\omega)$. Let $m(\omega, a)$ be a Nash equilibrium of $\left\{M, g; R^h(\omega)\right\}$ such that $g(m(\omega, a)) = a$.

Let $B_i(\omega, a) = g(M_i \times \{m_{-i}(\omega, a)\})$ be the attainable set of i.

Set $B_i^h(\omega, a) = \bigcup_{\omega' \in \Omega} \{c \in A(\omega') : aR_i(\omega)h(c, \omega),\ cP_i(\omega')h(a, \omega')$ if for some $b \in B_i(\omega, a)$, $aR_i(\omega)h(b, \omega)$, $h(b, \omega')P_i(\omega')h(a, \omega')\}$

Set $B^h = \operatorname{Im} g \cup (\bigcup_{\substack{\omega, \omega' \in \Omega \\ i \in I}} \{c \in A(\omega') : aR_i(\omega)h(c, \omega),\ aP_i(\omega')h(c, \omega')$ if for some $b \in B_i(\omega, a)$, $aR_i(\omega)h(b, \omega)$, $h(b, \omega')P_i(\omega')h(a, \omega')\})$.

Observe that for all $\omega \in \Omega$, $a \in F(\omega)$, and i, $B_i^h(\omega, a) \subset B^h$. Thus:

-$B_i^h(\omega,a)\cap A(\omega)\subset B_i(\omega,a)\cap A(\omega)$ for all ω

-$aR_i^h(\omega)x$ for all $x\in B_i(\omega,a)$ and for all $i\in I$

-if $aR_i(\omega')x$ for all $i\in I$, $x\in B_i^h(\omega,a)\cap A(\omega')$ then $a\in F(\omega')$. Otherwise there would exist $j\in I$ and $b\in B_j(\omega,a)$ such that $h(b,\omega')P_i(\omega')h(a,\omega')$. h is non-rewarding so there exists $c\in A(\omega')$ such that $aR_j(\omega)h(c,\omega)$, $cP_j(\omega')h(a,\omega')$. By definition c belongs to $B_j^h(\omega,a)$.

Using the assumption that h is non-rewarding we can prove, exactly as above:

-if $b\in B_i^h(\omega,a)$ for some $i\in I$ is such that for some $\omega'\in\Omega$, $bR_i(\omega')x$ for all $x\in B_i^h(\omega,a)\cap A(\omega')$, and if $bR_j^h(\omega')x$ for all $x\in B^h\cap A(\omega')$ for each $j\neq i$ then $b\in F(\omega')$.

-if $b\in B^h$ is such that for some $\omega'\in\Omega$, $bR_i^h(\omega')x\ \forall\ x\in B$ for all i, then $b\in F(\omega')$

For all i set $M_i'=\{(\omega,a);\ a\in F(\omega)\}\times B^h\times\mathbf{N}$, where $\mathbf{N}$ is the set of integers. Let $M'=\prod_1^N M_i'$, and $g':M'\longrightarrow A$ such that:

a) $g'(m)=a$ if $m_i=(\omega,a,b,n)\ \forall i$.

b) If there exists a unique i such that for all $j\neq i$ $m_j=(\omega,a,b,n)$ and $m_i=(\omega_i,a_i,b_i,n_i)$ is such that $(\omega,a,b,n)\neq(\omega_i,a_i,b_i,n_i)$ then set $g'(m)=b$ if $b_i\in B_i^h(\omega,a)$, otherwise set $g'(m)=a$.

c) Otherwise set $g'(m)=b_i$ where $i=\min\arg\max_j\{n_j;m_j=(\omega_j,a_j,b_j,n_j)\}$.

This is the canonical mechanism in Nash implementation. It is proved immediately that(M',g') implements F by generalized severe punishment. ■

In words, generalized severe reversion implements any SCR implementable with non-rewarding reversion functions.[6, 7] The rest of this section will be devoted to study the former. According to Remark 1 this leads us to study h-monotonicity under saturated preferences.

We now introduce two properties that are necessary and sufficient for h-monotonicity under generalized severe reversion.

Definition 6. *A SCR F satisfies Weak Unanimity (WU) if for all $\omega, \omega' \in \Omega$ such that $A(\omega') \subset A(\omega)$ and for all $a \in A(\omega) \backslash A(\omega')$ such that $L_i(a, \omega) \cap A(\omega') \subset L_i(a, \omega')$ for all $i \in I$, $a \notin F(\omega)$.*

When preferences are fixed, WU says that if all alternatives available at ω' are also available at ω, the SCR will not select at ω an alternative which is available at ω but not at ω' if all players prefer any allocation available at ω' to it. If this condition is not satisfied, when the actual state is ω all agents have incentives to underrepresent the economy and implement the decision intended for state ω'. Notice that WU is equivalent to the following condition: if $A(\omega') \subset A(\omega)$ and $a \in F(\omega) \backslash A(\omega')$ then there exists $b \in A(\omega'), b \neq G$ such that aR_ib for some $i \in I$.

[6] Proposition 1 can be proved under the following assumption that generalizes that of a non-rewarding reversion function: whenever there exists $i \in I$ with $aR_i(\omega)h(b, \omega)$ and $h(b, \omega')P_i(\omega')h(a, \omega')$ $a \in A(\omega)$ then there exists j and $c \in A(\omega')$ such that: i) $aR_j(\omega)h(b, \omega)$ and $h(b, \omega')P_j(\omega')h(a, \omega')$ and ii) $aR_j(\omega)h(c, \omega)$ with $cP_j(\omega')h(a, \omega')$. This says that at least one agent suffers as a consequence of unfeasibility in a way that could have been done through a feasible allocation. For instance, the agent who is deemed responsible for the unfeasibility is punished and there is an agent who does not get the bundle she consumed at the other state.

[7] The assumption of non rewarding is necessary for Proposition 1 to hold. Let $\Omega = \{\omega, \omega'\}$, $A(\omega) = \{a, b, c, G\}$ and $A(\omega') = \{a, b, G\}$. Let $n = 2$ and $R_i(\omega) = R_i(\omega') = R$ for $i = 1, 2$ where $bPaPc$. Let $F(\omega) = a$ and $F(\omega') = b$. Let $h(c, \omega') = b$. h does not satisfy the non rewarding assumption at c. F is h-implementable in NE by the simple mechanism which leaves agent 1 to choose among a and c. But it cannot be implemented by severe generalized punishment because $F(\ )$ is not monotonic with respect to saturated preferences.

Definition 7. *A SCR F satisfies Generalized Contraction Consistency (GCC) if, for $\omega, \omega' \in \Omega$, and for $a \in F(\omega) \cap A(\omega')$ such that $L_i(a,\omega) \cap A(\omega) \cap A(\omega') \subset L_i(a,\omega')$ and $A(\omega') \backslash A(\omega) \subset L_i(a,\omega')$.for all $i \in I$, $a \in F(\omega')$.*

When preferences are fixed and $A(\omega') \subset A(\omega)$, $A(\omega') \backslash A(\omega) = \varnothing \subset L_i(a,\omega)$ for all i. In such a case GCC prescribes choosing at state ω' any feasible allocation we have chosen at ω. Thus GCC is a weak version of Nash Independence of Irrelevant Alternatives (see Roemer [1996], p. 55). In the general case, GCC says that if a is selected at state ω, is feasible at ω' and no better alternatives are available in $A(\omega') \backslash A(\omega)$, then a must be selected also at ω'.

Proposition 2. *A SCR is h-monotonic under generalized severe punishment if and only if it satisfies Generalized Contraction Consistency and Weak Unanimity.*

Proof Let h denote the generalized severe punishment reversion function.

We begin by proving the necessity of WU and GCC. Let F be h-monotonic.

Then F must satisfy WU. Let $\omega, \omega' \in \Omega$, let $A(\omega') \subset A(\omega)$ and let $a \in A(\omega) \backslash A(\omega')$ such that $L_i(a,\omega) \cap A(\omega') \subset L_i(a,\omega')$ and $i \in I$. By contradiction, let $a \in F(\omega)$. Then $L_i^h(a,\omega) = (L_i(a,\omega) \cap A(\omega)) \cup A \backslash A(\omega) = (L_i(a,\omega) \cap A(\omega')) \cup$ $(L_i(a,\omega) \cap A(\omega) \backslash A(\omega')) \cup A \backslash A(\omega) \subset (L_i(a,\omega') \cap A(\omega')) \cup A \backslash A(\omega') = L_i^h(a,\omega')$ for all $i \in I$. Then h-monotonicity implies that $a \in F(\omega')$, which is a contradiction as $F(\omega') \subset A(\omega')$.

Now we consider GCC. Let $a \in F(\omega) \cap A(\omega')$ such that $L_i(a,\omega) \cap A(\omega) \cap A(\omega') \subset L_i(a,\omega')$ and $A(\omega') \backslash A(\omega) \subset L_i(a,\omega')$ for all $i \in I$.

$L_i^h(a,\omega) = (L_i(a,\omega) \cap A(\omega) \cap A(\omega')) \cup (L_i(a,\omega) \cap A(\omega) \backslash A(\omega')) \cup$ $(A(\omega') \backslash A(\omega)) \cup ((A \backslash A(\omega)) \backslash A(\omega')) \subset (L_i(a,\omega') \cap A(\omega) \cap A(\omega')) \cup$ $(L_i(a,\omega') \cap A(\omega')) \cup A \backslash A(\omega') = L_i^h(a,\omega')$. h-monotonicity implies $a \in F(\omega')$.

We next show the sufficiency of WU and GCC for F to be h-monotonic.

Let $\omega, \omega' \in \Omega$, and let $a \in F(\omega)$ such that $L_i^h(a, \omega) \subset L_i^h(a, \omega')$ for all i. Consider the following three cases.

i) $A(\omega) \cap A(\omega') = \{G\}$

ii) $A(\omega) \cap A(\omega') \neq \{G\}$ and $a \notin A(\omega')$

iii) $A(\omega) \cap A(\omega') \neq \{G\}$ and $a \in A(\omega')$

i) It is not possible. In such a case there is no $i \in I$ $L_i^h(a, \omega) \subset L_i^h(a, \omega')$, since, from the definition of saturated preferences it follows that $aP_i^h(\omega)b$ and $bP_i^h(\omega')a$ for all $b \in A(\omega')$.

ii) It must be the case that $A(\omega') \subset A(\omega)$. Otherwise from the definition of saturated preferences for all $b \in A(\omega')\backslash A(\omega)$: $aP_i^h(\omega)b$ and $bP_i^h(\omega')a$ for all i. Then we must have $L_i(a, \omega) \cap A(\omega') \subset L_i(a, \omega')$ for all $i \in I$ Otherwise for some $i \in I$,. $b \in A(\omega')$: $aR_i(\omega)b$ and $bP_i(\omega')a$. But from WU it would follow that $a \notin F(\omega)$, a contradiction.

iii) It must be the case that $L_i(a, \omega) \cap A(\omega) \cap A(\omega') \subset L_i(a, \omega')$ and

$A(\omega')\backslash A(\omega) \subset L_i(a, \omega')$ for all $i \in I$. Otherwise either there exists $b \in A(\omega) \cap A(\omega')$ such that $aR_i(\omega)b$ and $bP_i(\omega')a$ for some $i \in I$, or there exists $b \in A(\omega')\backslash A(\omega)$ such that $aP_i^h(\omega)b$ and $bP_i(\omega')a$ for some $i \in I$. Then GCC implies that $a \in F(\omega')$.

So F is monotonic. ■.

5. Non-Rewarding Reversion Functions: Applications

In this section we apply the findings of previous sections to exchange economies and to bargaining and bankruptcy problems.

5.1. Exchange Economies: Withholding

First, we notice that in this environment with more than two agents, h-monotonicity is necessary and sufficient for F to be h-implementable in Nash equilibrium.

There are n agents and K goods. Let $X_i = \mathbf{R}_+^K$ be i's consumption set. We assume that agents' preferences and consumption sets do not vary but endowments do. Let u_i be a utility function that represents agent i's preferences. Let $\Omega_i \subset \mathbf{R}_+^K$ be the set of agent i's possible endowments. For $\omega = (\omega_1, ..., \omega_n) \in \Omega$ set $\overline{\omega} = \sum_{i=1}^n \omega_i$. We consider that the planner can only transfer goods among players. Then the allocation set contains the set of the balanced net transfers and the generalized punishment point, $A = \{x \in R^{K\times n} : \sum_{s=1}^K x_s = 0\} \cup \{G\}$. For all $\omega \in \Omega$ the feasible set is $A(\omega) = \{x \in A : x_i + \omega_i \geq 0 \text{ for } i = 1, ..., n\} \cup \{G\}$. Then $A(\omega') \subset A(\omega)$ if and only if $\omega' \leq \omega$. In order to describe preferences on net transfers, notice that the utility agent i gets from transfer x_i when her endowment is ω_i is $u_i(x_i + \omega_i)$. Thus, the utility function is state dependent even if preferences are not. For each ω and for all $x \in A(\omega)$ for all i set $u_i(x, \omega) \equiv u_i(x_i + \omega_i)$.

Saturated preferences can be represented by the following utility functions:

$$\begin{aligned} u_i^\omega(x) &= u_i(x, \omega) \; x \in A(\omega) \backslash \{G\} \\ u_i^\omega(G) &= u_i(0) - \varepsilon, \; \varepsilon > 0. \end{aligned}$$

Let us first consider WU. It is easily seen that it suffices to consider only endowments ω, ω' such that $\omega' \leq \omega$. Then WU amounts to the following condition:

Condition α : For all ω, $\omega' \in \Omega$ such that $\omega' \leq \omega$, if $a \in F(\omega) \backslash A(\omega')$ there exists i such that $u_i(\omega_i + a_i) \geq u_i(\omega_i - \omega_i')$.

Observe that if $(0, \omega_i) \subset \Omega_i$ for all i then Condition α requires simply the SCR to be individually rational for at least one agent. It is a very weak requirement and it is obviously satisfied by many SCR, e.g., any Pareto efficient or any individually rational SCR.

Stronger requirements are imposed by GCC. Also in this case, it suffices to consider only endowments ω, $\omega' \in \Omega$ such that $\omega' \leq \omega$. GCC is satisfied if and only if the following condition holds:

Condition β. For all ω, $\omega' \in \Omega$ such that $\omega' \leq \omega$, if $a \in F(\omega) \cap A(\omega')$ and $a \notin F(\omega')$ there exists i and $x \in A(\omega')$ such that

$$\begin{aligned} u_i(\omega_i + a_i) &\geq u_i(\omega_i + x_i) \\ u_i\left(\omega_i' + a_i\right) &< u_i\left(\omega_i' + x_i\right) \end{aligned}$$

Let us compare our conditions with Hong (1998). She showed that a SCR is implementable by a collection of state dependent mechanisms if and only if the following condition is satisfied

$$u_i(\omega_i + f_i(\omega)) \geq u_i(\omega_i - \omega_i') \text{ for all } i \tag{H}$$

Our Condition (α) is weaker than condition (H): If $x \in A(\omega')$ then $u_i(\omega_i + x_i) \geq u_i(\omega_i - \omega_i')$ for all i as all u_i are increasing. Then if $f(\omega) \in A(\omega')$, $u_i(\omega_i + f_i(\omega)) \geq u_i(\omega_i - \omega_i')$ for all i. So for ω, ω' with $f(\omega) \in A(\omega')$ condition (H) holds. Notice that our condition depends on the fact that each agent cannot simply retain part of her endowment, but she has to make it compatible with other agents' messages. But our Condition (β) is not implied by Condition (H). Assume for instance that $f(\omega) \in A(\omega')$ then (H) imposes no restrictions on $f(\omega')$. If the translations by $\omega - \omega'$ of all agents' indifference curves through $\omega' + f(\omega)$ are strictly above all agents' indifferences curves through $\omega + f(\omega)$, then condition (β) implies $f(\omega) = f(\omega')$. Formally if for all $y \in \{y : u_i(\omega_i' + f_i(\omega))) = u_i(y_i)$ for all $i\}$ we have $u_i(y_i + \omega_i - \omega_i') > u_i(\omega_i + f_i(\omega))$ for all i, condition (β) imposes that $f(\omega) = f(\omega')$.

The difference between our conditions and Hong's is explained by the fact that her goal is to design one feasible mechanism $(M(\omega), g(\omega))$ for each possible endowments ω, in a way such that the larger the feasible set, the larger the message space. Two of her assumptions make our approaches different:

i) Hong assumes that players can not exaggerate their endowment and that they can be punished by the message they send, not only for the allocation they intend to obtain, if such an allocation is not feasible.

ii) Hong gives each player the power of retaining part of her endowments. In our framework we assume that players can collectively cheat the planner through the mechanism by asking for a feasible allocation in which some agents retain a part of their endowment.

Finally, let us analyze the implementation of the Constrained Walrasian SCR. a is a *Constrained Walrasian Allocation (CWA) at* ω iff there exists $p \in \mathbf{R}_{+}^{K}$ such that $\forall$ $i = 1, ..., n$, $a \in \arg\max \{u_i(\omega_i + x_i) : px_i \leq 0,\ x \in A(\omega)\}$. p is said to be an equilibrium price supporting a at ω. Let $CW(\omega)$ denote the set of CWA at ω.

Proposition 3. *Let utility functions be increasing, continuous and quasi-concave. Let* $\Omega_i = (0, \overline{\omega}_i)$ *for all* i *for some* $\overline{\omega}_i \in (0, \infty)$. *Then the Constrained Walrasian SCR is implementable in Nash Equilibrium by generalized severe punishment.*

Proof Under our assumptions $CW(\omega)$ is not empty for all $\omega \in \Omega$. To prove the claim it suffices to show that CW satisfies Condition β. Let $\omega' \leq \omega$, $a \in CW(\omega) \cap A(\omega')$ and $a \notin CW(\omega')$. Let p an equilibrium price at ω. Then there exists $x \in A(\omega')$ with $u_i(\omega' + x_i) > u_i(\omega' + a_i)$ and $px_i \leq 0$ some i. $A(\omega') \subset A(\omega)$ so $x \in \{px_i \leq 0, x \in A(\omega)\}$. From the definition of CW it follows $u_i(\omega_i + a_i) \geq u_i(\omega_i + x_i)$. Then CW satisfies Condition β.[8] ■

5.2. Bargaining with unknown utility possibility set

We now consider the non-cooperative implementation of cooperative solution concepts (Dagan and Serrano [1998] and Naeve [1999]).

A bargaining problem is a pair (U, v) where $U \subset \mathbf{R}_{+}^{n}$ is the utility possibility set and $v \in U$ is the disagreement point. We assume that U is convex, closed, with a non empty

[8] The Walrasian Correspondence WC defined by $WC(\omega) = \arg\max \{u_i(\omega_i + x_i) : px_i \leq 0\}$ is not implementable in Nash Equilibrium by generalized severe punishment. An example is available from the authors under request but intuitively it is clear that in our case preferences vary, so we are back to the classical framework where such a problem is well known.

interior and comprehensive (i.e. $u \in U$ and $u' \leq u$, $u' \in \mathbf{R}^n_+$ implies $u' \in U$). For each bargaining problem, (U,v) let $U_v = \{u \in U : u \geq v\}$ be bounded. The Nash Bargaining Solution (NBS) is defined as $NBS(U,v) = \arg\max_{u \in U_v} \prod_{i=1}^n (u_i - v_i)$. It is completely characterized by the following properties: strong efficiency, individual rationality, scale covariance, symmetry and independence of irrelevant alternatives. Let $NBS(U,v)_i$ be the utility received by i.

We consider here non rewarding reversion function more suited to the situation. We say that a reversion function is *Not Severe* if for all $a \in A$ and $\omega \in \Omega$ $h(a,\omega) \neq G$. We consider U_v as the feasible set of (U,v) and we assume that unfeasible allocations are renegotiated into the disagreement point. Let h to denote such reversion function. Clearly h is Non Rewarding. Agent i's reverted preferences at (U,v) are described by

$$u_i^h(u(U,v))) = u_i \text{ if } u \in U_v$$
$$u_i^h(u(U,v)) = v_i \text{ otherwise.}$$

If the disagreement point is not known by the planner, NBS fails to satisfy GCC: Let $n = 2$ and let $U = \{x \in \mathbf{R}^2_+ : x_1^2 + x_2^2 \leq 1\}$. Let $v = (0,0)$ and let $v' = \left((\frac{1}{2})^{\frac{1}{2}}, 0\right)$. Then $NBS(U,v) = \left((\frac{1}{2})^{\frac{1}{2}}, (\frac{1}{2})^{\frac{1}{2}}\right) \in U_{v'} \subset U_v$ but $NBS(U,v) \neq NBS(U,v')$. Thus, according to Proposition 1, NBS is not implementable in NE by any non-rewarding reversion function.[9]

The Kalai-Smorodinski solution does not satisfy GCC even with fixed disagreement point. Then Proposition 1 implies that it cannot be implemented in NE by any non rewarding reversion function.[10]

Instead, when the disagreement point is known, the NBS satisfies both GCC and WU as the reader can easily check. However, Proposition 2 cannot be used to conclude that

[9] This result agrees with the findings of Serrano (1997).

[10] A different interpretation of preferences on the utility possibility set may lead to more permissive results. One can interpret them as if they were a measure of agents' satisfaction with respect to the disagreement point. A representation consistent with this view is $u_i(u,(U,v)) = u_i - v_i$. Then the

the NBS is implementable by generalized severe punishment because Maskin Theorem requires at least three agents (see Remark 1).[11]

We prove the result directly by using the characterizations by Moore and Repullo (1990).

Proposition 4. *Let $n \geq 2$. The Nash Bargaining Solution is h-implementable in Nash Equilibrium with a Non-Severe h if the disagreement point v is known.*

Proof Let $x = NBS(U, v)$. Let $i \in I$ and let (U', v) be a bargaining problem. Let $u \in L_i^h(x, (U, v))$ such that, at (U', v) and with reverted preferences u is maximal for i in $L_i^h(x, (U', v))$ and u is maximal in $\mathbf{R}_+^n$ for all agents different from i. We first prove that $u = NBS(U', v)$. Observe that it must be the case that u is feasible at U' otherwise all agents different from i would prefer some point in the interior of U'_v and that $u_j = \max \left\{ u'_j : u' = (u'_j, u'_{-j}) \in U'_v \right\}$ for all $j \neq i$. In particular u lies on the boundary of U'. If $u \neq NBS(U', v)$ then $NBS(U', v)_i > u_i$. If $NBS(U', v) \notin U_v$ u is not maximal in $L_i^h(x, (U, v))$ for i when preference are reverted at (U', v), a contradiction. Finally consider the case $NBS(U', v) \in$

preferences that h induces in this case are

$$\begin{aligned} u_i^h(u, (U, v)) &= u_i - v_i \text{ if } u \in U_v \\ u_i^h(u, (U, v)) &= 0 \text{ otherwise} \end{aligned}$$

Observe that $u_i^h(u, (U, v)) = u_i^h(u - v, (U - v, 0))$. The reader can easily check that from the translation invariance property of the NBS the analysis of the problem with unknown endowments amounts to the previous situation with the endowment fixed and known at 0. In this case applying Proposition 4 below yields a positive result.

[11] NBS does not satisfy no-veto power either. Let $v = (0, 0, 0)$ and $U = \left\{ x \in \mathbf{R}_+^3 : \max\{x_1, x_2\} \leq 1, \max\{x_1 + x_3, x_2 + x_3\} \leq 1 \right\}$. Agent 1 and agent 2 prefer $u = (1, 1, 0) \in U$ to any other allocation, under saturated preferences but $NBS(U, v) = (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$.

U_v. $NBS(U',v) \neq NBS(U,v)$ and $NBS\ (U,v) \notin U'_{v'}$ otherwise u would not be maximal in $L_i^h(x,(U,v))$ for i under reverted preferences. Consider the segment joining $NBS(U',v)$ and u. Such a segment lies in U'_v because U'_v is convex and it intersects $\{u' \in U_v : NBS(U,v)_i \geq u'_i\ \}$ because U_v is convex and $NBS\ (U,v) \notin U'_v$. All along the segment the coordinate i increases from u'_i to $NBS(U',v)_i$. Then there exists a point in $\{u' \in U_v : NBS(U,v)_i \geq u'_i\ \}$ which has the i-th coordinate strictly greater than u_i, a contradiction. Let u be maximal in R^n_+ for all agents when preferences reverted at (U',v) then $u_j = \max\left\{u'_j : u' = (u'_j, u'_{-j}) \in U'_v\right\}$ for all j. From efficiency it follows that $u = NBS(U',v)$.

NBS satisfies Individual Rationality, Pareto efficiency and GCC, too. Then, when $n \geq 3$ the family of sets $\left\{L^h(x,(U,v))\right\}_{x=NBS(U,v)}$ satisfies condition μ in Moore and Repullo (1990). When $n = 2$ it satisfies condition $\mu 1$ in the same paper, because of the disagreement point. Then the application of Theorems 1 and 2 there, respectively leads to the claim. ■

5.3. Taxation

A *taxation problem*, is a pair $(x,T) \in R^n_+ \times R_+$ where x is the vector of taxable incomes and T is the total amount to be collected such that $\sum_{i=1}^n x_i \geq T$ (Dagan et alia [1999]). A *tax allocation* t is a vector of R^n_+ and it is feasible for the taxation problem (x,T) if $t \leq x$ and $\sum_{i=1}^n t_i = T$. A *taxation method* is a function f which associates a tax allocation to each taxation problem. We assume that the planner knows the amount to be collected, T, but she does not know the taxable vector x. Let $S^n(T) = \left\{t \in R^n_+ : \sum_{i=1}^n t_i = T\ \right\}$ be the set of tax allocations that collect T. Let $\Omega^n(T) = \left\{x \in R^n_+ : \sum_{i=1}^n x_i \geq T\ \right\}$ be the set of the states of the world. Let $T^n(x) = T^n(x,T) = \left\{t \in R^n_+ : 0 \leq t \leq x,\ \sum_{i=1}^n t_i = T\right\}$ be the set of feasible tax allocations at x. Each agent's preferences only depend on her after tax income and are strictly increasing. Then we can assume $u_i(t,x) = x_i - t_i$ for each $x \in \Omega^n(T)$ and for each $t \in T^n(x,T)$. Assume that only income exaggeration can be detected and punished. The reversion function is non rewarding. Therefore, the hypothesis of Proposition 1 are fulfilled.

Let $\sigma : I \to I$ be a permutation or ranking on the agents. Set $(i) = \sigma^{-1}(i)$. Let $f^{(\sigma)}$ be the following feasible taxation method.

$$
\begin{aligned}
f^{\sigma}_{(1)}(x) &= \min\left\{t_{(1)} : t \in T^n(x,T)\right\} \\
f^{\sigma}_{(j)}(x) &= \min\left\{t_{(j)} : t \in T^{n-j+1}(x_{-\{(1),\ldots,(j-1)\}}, T - \sum_{i=1}^{j-1} f^{\sigma}_{(n-i)}(x))\right\} \quad j = 1,..,n
\end{aligned}
$$

The first agent (1) pays her last feasible amount. The second agent pays her last feasible amount given (1) payment and so on. Each agent is a dictator with respect to the following players. For this reason f^{σ} will be called the *σ-serial dictatorship.*[12]

An equivalent definition for f^{σ} is

$$
\begin{aligned}
f^{\sigma}_{(n)}(x) &= \min\left\{x_{(n)}, T\right\} \\
f^{\sigma}_{(n-j)}(x) &= \min\left\{x_{(n-j)}, T - \sum_{i=n}^{n-j+1} f^{\sigma}_{(n-i)}(x)\right\} \quad j = 1,..,n-1
\end{aligned}
$$

In words, player n pays the whole amount to be collected if she has enough income. Otherwise what is left is paid by player $n-1$ if she has enough income and so on.

Proposition 5. *Let h be a non rewarding reversion function and let f be a continuous feasible taxation method. If f is $h-$implementable in Nash Equilibrium then it is a serial dictatorship. If f is a serial dictatorship then it is h-implementable for any non rewarding h.*

[12] Serial dictatorship plays an important part in the characterization of SCF truthfully implementable in Dominant Strategies (Satterthwaite and Sonnenschein [1981]) and in the equilibrium in social systems in which property rights are weakly protected (Piccione and Rubinstein [1993]).

Proof For each x and $t \in T^n(x)$ set $I(t,x) = \{y : t \leq y \leq x\}$. We first prove that $f((I(f(x),x)) = \{f(x)\}$ for all x. In particular, if $I(f(x),x) \cap I(f(x'),x') \neq \varnothing$, then $f(x) = f(x')$. Let $x' \leq x$. In such a case $T(x') \subset T(x)$. Let $t = f(x) \in T(x')$, which is $f(x) \leq x'$ then $L_i(t,x) \cap T(x) \cap T(x') \subset L_i(t,x')$. If $x' \leq x$ and $f(x) \leq x'$ then GCC prescribes that $f(x') = f(x)$. In particular $f(x) = f(y)$ for all y such that $f(x) \leq y \leq x$.

By contradiction let f be not a serial dictatorship. Then exist x,i, j such that $0 < f_i(x) < x_i$ and $0 < f_j(x) < x_j$. Then $I(f(x),x)$ is at least 2 dimensional.

Let $y \geq x$. We show that $f(y) = f(x)$. On the contrary assume $f(y) \neq f(x)$. There is no loss of generality in assuming that $f(z) \neq f(x)$ for all z on the segment joining y and x. Otherwise, by continuity, we can substitute x with the point x', on the segment, having the largest coordinates. From the observation above it follows that for all such z, $f(z) \notin T(x)$. Let $z \to x$ on this segment then. By continuity $f(z) \to f(x)$. If $f(z)$ converges then $f(z) \to t^*$, where $t_i^* = 0$ or $t_j^* = 0$. Let $x^* = (T, ..., T)$. It follows that $f(y) = f(x)$ for all $y \geq f(x)$. If $y \ngeq f(x)$ then $f(y) = f^{\sigma}(y)$ for some σ, because otherwise $I(f(y),y) \cap I(f(x^*),x^*) \neq \varnothing$ and $f(x^*) = f(x)$ is not feasible at y. But in such a case f would not be continuous. A contradiction.

The proof of the second part of the claim is as in Proposition 4. ■

We end this section by noting that there are also discontinuous feasible taxation methods that are $h-$implementable in Nash equilibrium as the proof of the previous result suggests. Let $x^* = (T, ..., T)$, let $t \in \Omega(T)$ and let σ be a permutation on I.

$$
\begin{aligned}
f(x) &= (\frac{T}{n}, ..., \frac{T}{n}) \text{ for all } x \geq (\frac{T}{n}, ..., \frac{T}{n}) \\
f(x) &= f^{\sigma}(x) \text{ otherwise}
\end{aligned}
$$

It is not difficult to prove that f is implementable in NE through generalized severe punishment.

6. Conclusions

In this paper we have presented a new approach to deal with the implementation problem when feasible sets are state dependent. It is based on the idea that agents renegotiate unfeasible allocations into feasible ones. We have presented a class of reversion functions that are suited to our problem and we have found necessary and sufficient conditions for implementation when renegotiation takes this form. Finally we have used our characterization results to study the implementation in Nash equilibrium of social choice rules in exchange economies, bargaining problems and taxation methods, and we have compared our results with those obtained by the earlier literature.

A feature of the traditional approach of implementation when feasible sets are state dependent is that it requires a collection of state dependent mechanisms, contrary to the case when preferences are state dependent. This distinction contrast vividly with our intuition on how markets cope with unfeasible allocations, namely that the sign of excess demand entirely determines the adjustment irrespectively of the cause of infeasibility.[13] Thus our approach may offer a better understanding of market mechanisms than does the traditional one. But the traditional approach is better suited to deal with topics like withholding of endowments -in our case the state of the world, and thus endowments, is common knowledge- or tax evasion given the importance of reports in the renegotiation. Actually our approach can be generalized to deal with these cases by introducing uncertainty in the renegotiation process or the mechanism as an argument in the reversion function. These two extensions are easy to write, but require completely new analytical methods. Thus, they are left for future research.

[13] In fact, following the lead of Benassy (1986) many papers dealing with markets from the implementation point of view disregard the issue of individual feasibility.

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