Hochschule Ravensburg-Weingarten

Master thesis

Modelling the wheels of the Robot MAX2D and surfacing

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Abstract

Energy is a scalar physical quantity that describes the amount of work that can be performed by a force, an attribute of objects and systems that is subject to a conservation law. Different forms of energy include kinetic, potential, thermal, gravitational, sound, light, elastic, and electromagnetic energy. The forms of energy are often named after a related force.

Any form of energy can be transformed into another form, but the total energy always remains the same. There is no absolute measure of energy, because energy is defined as the work that one system does (or can do) on another. Thus, only the transition of a system from one state into another can be defined and thus measured. Here we will work on mechanical energy manifest in many forms, but it can be broadly classified into elastic potential energy and kinetic energy which is a function of its movement and electric energy.

Saving energy may result in increase of financial capital, environmental value, national security, personal security, and human comfort. Individuals and organizations that are direct consumers of energy may want to conserve energy in order to reduce energy costs and promote economic security. Industrial and commercial users may want to increase efficiency and thus maximize profit.

In this report we can see the transformation of the energy in a Robot where the input Energy is an Electrical Energy, and the Robot transfers this energy to the Mechanical Energy. Following in this project we can see the wasted energy in the Robot during this transformation. Wasted energy in this Robot is mostly found on the mechanical devices especially on the wheels. Wheels are in contact with the ground and according to the weight and acceleration of the Robot we will find the friction forces that are wasting the energy. The goal of this project is to define the value of the wasted energy in different kinds of movements.

To reach the goal of this thesis it is necessary to have a program to simulate the movement of the Robot by taking care of the affection of the mechanical forces. The program that is used in this project is Matlab. This program is one of the most powerful tools in simulation and calculation in engineering. For programming we will use the dimension of the Robot and the Pacejka parameters that are characteristic of the tire of the Robot. We will give the necessary parameters to the program and according to the movement, velocity, contact surface and etc, the Matlab Program will simulate the behaviour of the energies and the forces of the Robot and its tires.

Nomenclature

a_c	Centrifugal acceleration [m/s^2]
C	Gravity centre [m,m]
C_{rr}	Rotation friction coefficient
E_e	Electrical energy [J]
E_f	Friction energy [N]
E_k	Kinetic energy [J]
E_{kr}	Kinetic rotation energy [J]
F	Total force [N]
F_c	Centrifugal force [N]
F_f	Friction force [N]
F_z	Force in z direction [kN]
G	Gravity [m/s^2]
h_{GC}	Height of the gravity centre [m]
I	Current [A]
I_{xx}, I_{xy}…	Inertial moment of the Robot when it is rotating in different axis [kg·m^2]
m	Mass [kg]
r	Distance [m]
R	Radius of rotation [m]
s	Slope
v	Linear velocity [m/s]
v_M	Linear velocity of the wheels from the mobile reference system [m/s]
v_o	Linear velocity of the gravity centre [m/s]
V	Voltage [V]
α	Drift angle [grad]
γ	Fall angle [grad]
Δ	Increase
μ	Friction coefficient
ω	Angular velocity around the gravity centre [r.p.m.]
Ω	Angular velocity around the mobile reference system [r.p.m.]
(x, y)	Point of the trajectory in global coordinate system
(ξ, ζ)	Point of the trajectory in mobile coordinate system
i	In each point
‘, ‘’	First and second derivatives

Contents

1. Introduction

Energy is a scarce commodity. With the decline of non-renewable energy sources humans have tried to conserve non-renewable energy sources and are also working on improving the infrastructures and devices for a better use of the energy.

On the other hand, in the industry, machines and the use of Robots are helping and improving production, in addition to saving the time, space and human work. At first the advancement in this technology was incompatible with necessary energy saving. This energy saving is interesting not only because of the energetic problem, but also because the more energy that is using, the price that's pay for it increases. In other words, we can see this energy saving problem as a monetary travel.

Is tested out that in wheeled vehicles the most wasted energy is dissipated through the friction between the wheels and ground. The rest of the wasted energy is dissipated for warming the electronic devices, as friction in the brakes and etc, and is really difficult to manage. As a result in this report we will consider the energy saving problem as a friction problem between the wheels and ground.

This report is focused on one of these industrial Robots, in particular the new MAX2D (*figure 1.1*). This is a new special Robot with a high dynamic chassis that enables high mobility and freedom in a 2D dimension. The principal benefit of this high dynamic chassis is the saving of manoeuvring spaces and the rotation of the Robot around itself.

This MAX2D Robot has an input current of 3 A and an internal battery which assists the movement of the Robot when the wheels speed is high enough. This Robot controlled by the Matlab Program that allows us to introduce the angular velocity of the wheels besides the direction of the movement of each wheel.

Figure 1.1:MAX2D.

What makes this Robot so special is that in each of its four wheels we have a separate motor and a brake. This allows the wheels to rotate independently of each other and it gives us the high freedom that was explained above. For the revolution of each wheel the brake needs to be open. With an active brake there can be no vertical revolution, therefore there is forward movement. However when the brake is open the wheel rotates around the attachment point of the Robot and the Robot remains stationary. In *figure 1.2* we can see one detail of MAX2D wheel. *Figure 1.3* is an example of how the wheels are positioned at one of the movements, concretely in a rotation movement around itself.

Figure 1.2:Detail of the wheels of MAX2D.

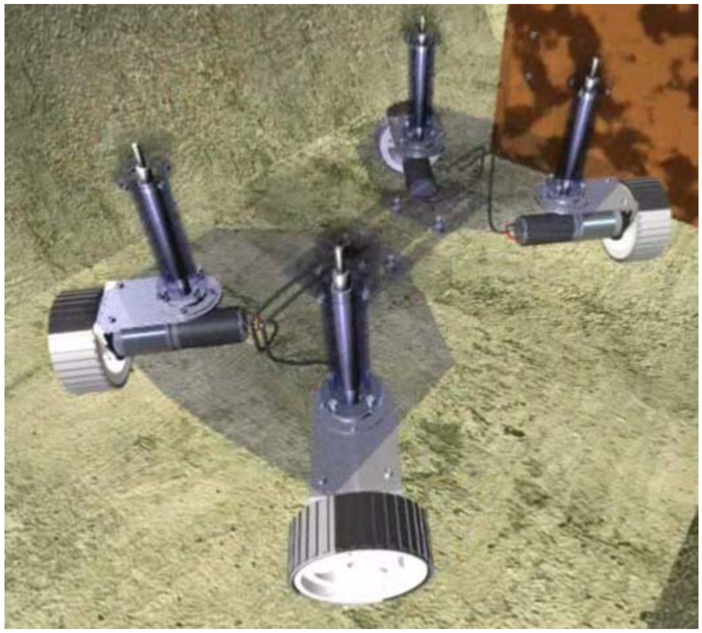

Figure 1.3:Movement of the wheels.

As we can see in *figure 1.3* the wheels are not rigid, so if the Robot is in a curve motion the tires of the wheels are deforming. This tire deformation is caused by forces that appear on the tire. Sometimes these forces are in the same direction as the friction forces, and sometimes these forces are in the opposite direction, hence forces that dissipate energy in the wheels are not only due to the weight but also due to the tire the force is supporting.

To carry out the study of the friction forces, we have divided this project into two main blocks:

1. In the first block we will study different trajectories. The objective is to determine theoretically how the wasted energy and the forces on the tires are. After finishing this study and comparison between the different obtained results, we will be able to determine in which point of the trajectories the forces are higher and in which points they are lower.

2. The second block is the practical part. With the Robot moving in the straight line and also applying a pure rotation of the Robot around itself we will compare the theoretical results with the experimental measures that we would take. The objective is to determine the percentage of energy that is wasted through the friction between the wheels and the ground.

For all of the parts that explained above, we will create some programs in Matlab that allow us to vary the different parameters and functions.

The application of this study is the energy saving of the Robot MAX2D. Friction forces and energy wasted depend on the Robot acceleration at each point of the trajectory. In future investigations, this report could be used to determine the optimum and less expensive trajectory between two points.

2. Literature review

2.1. Rotation Centre

Rotation centre can be defined as the point in the plane around which all the other points of the figure are rotating. This point is stationary at all times.

To determine the rotation centre we have to consider that all the points of the mobile body are rotating on it. Therefore the velocity of each point is perpendicular to the connection line between this point and the instant centre of rotation. In the *figure 2.1* centre of rotation is obtained geometrically from the velocity of two points of the mobile solid.

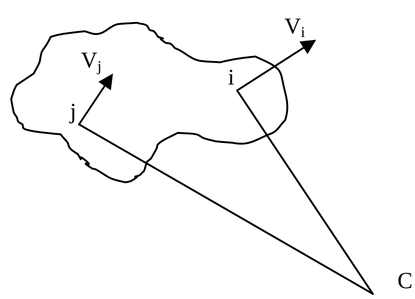

Figure 2.1:Centre of rotation from two known velocities.

In vehicles, wheels are rotating round the centre of rotation. If the directions of the velocities on each wheel are known, centre of rotation could be easily defined in a geometrical way and it is following the explanation that mentioned above. In the *figure 2.2* this centre of rotation is acquired just when the car is turning the front wheels.

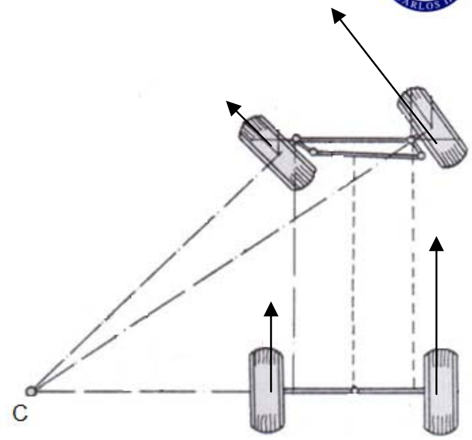

Figure 2.2:Centre of rotation in a vehicle.

However, in this report studies a four wheeled Robot with high freedom on each wheel that can rotate independently. The velocities of the wheels are not known, neither the angle of the wheels; nevertheless the trajectory of this vehicle is defined. In this case, the *formulas 2.1* and *2.2* for radius of rotation and instant centre of rotation can be applied [3]:

[2.1] $$R = \left| \frac{\left(1 + y'^2\right)^{3/2}}{y''} \right|$$

[2.2] $$C(x, y) = \left(x - \frac{y'\left(1 + y'^2\right)}{y''}, y + \frac{1 + y'^2}{y''} \right)$$

R: Radius of rotation [m].
y=y(x): trajectory that the Robot is following.
C: Centre of rotation [m, m].

2.2. Curve motion

To start with, we consider a circular movement with radius and fixed centre of rotation. The linear velocity of the body is known, so angular velocity is easily obtained:

[2.3] $$\omega = \frac{v}{R}$$

ω: angular velocity [rad/s].
v: linear velocity [m/s].
R: Instant radius of rotation [m].

In curve motion an instant circular motion could be applied at each point. The instant centre of rotation and radius of rotation above should be fixed for each instant. On the other hand we can not forget that the distance between the gravity centre and the instant centre of rotation is not the same as the distance between another point of the body and the centre of rotation, consequently; the linear velocity is different in each point of the Robot.

By simplifying the calculations we can find out a mobile reference system is located in the gravity centre of the Robot. The static reference system is considered at one point of the surroundings. According to applying solids kinematic on the Robot, the velocity of each point of the Robot in an instant is [3]:

[2.4] $$\Omega_i = v_{oi} \times R_i$$

[2.5] $$v_{ai} = v_M + v_{oi} + \left(R_{ai} \times \Omega_i\right)$$

Ω: Angular velocity of the robot from its gravity centre [rad/s].

v_o: Linear velocity of the gravity centre of the Robot from the global coordinate system [m/s].

R: Instant radius of rotation [m]

v_a: Linear velocity in one point of the Robot from the global coordinate system [m/s].

v_M: Linear velocity in one point of the Robot from the gravity centre of the Robot (mobile coordinate system) [m/s].

In a body that is moving in a curve motion there is also a centrifugal radial force. This centrifugal force is actuating in outward radial direction, and applying Newton's second law [3] its module could be acquired by:

[2.6] $$F_c = m \cdot a_c$$

[2.7] $$F_c = m \cdot \frac{v^2}{R}$$

F_c: Centrifugal force [N].

m: Mass of the body [kg].

v: Linear velocity of the gravity centre of the body [m/s].

a_c: Centrifugal acceleration [m/s^2].

R: Radius of rotation [m].

2.3. Friction forces

Friction is defined as the resisting force between two solids in contact. There are two types of friction forces [4]:

1. Static friction force: between solids that are not moving relative to each other.

2. Dynamic friction: between solids that are moving relative to each other.

[2.8] $$F_f = \mu \cdot m \cdot g$$

F_f: Friction force in slide motion [N].
μ: Slide friction coefficient.
M: mass of the solid [kg].
g: gravity. g=9.81m/s^2.

Both of these frictions are for sliding movement. The wheels of the Robot in this project are not sliding wheels. Therefore these two friction forces do not have any effect on the tires of the Robot. The force that allows the wheel to roll is the rolling resistance. Its formula is similar than the dynamic and static friction forces, but the rolling resistance coefficient is much smaller than the coefficient for sliding friction forces.

[2.9] $$F_f = C_{rr} \cdot m \cdot g$$

F_f: Friction force in roll motion [N].
C_{rr}: Rolling resistance.
m: mass of the solid [kg].
g: gravity. g=9.81m/s^2.

2.4. Tires

The tire is the element of the wheel that is in contact with the ground. Its function is supporting the mass of the body, this mass generates a deformation on the tire. The contact between tire and ground is not just one point and it can be appreciated in *figure 2.3*.

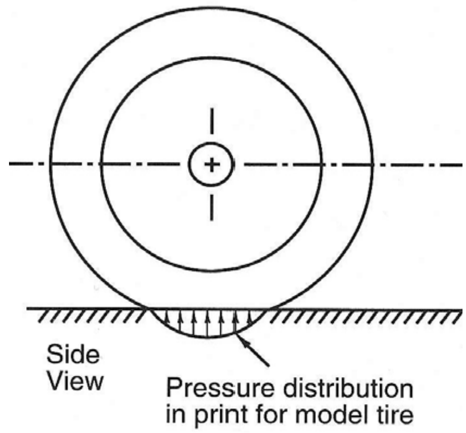

Figure 2.3:Tire deformation. [2]

As a result, supporting force is distributed in this contact surface and there are some forces acting on the tire:

1. Lateral force. Is operating on the contact surface, perpendicular to the direction of movement. Its function is to change the direction of the wheel in curve motion.

2. Slip angle is due to the lateral force and is defined by the angle between the longitudinal axis of the wheel and the direction of movement.

3. Aligning moment. Aligns the wheel with the direction of the movement.

4. Longitudinal force. Due to Newton's second law there can not be acceleration (tangential acceleration in curve motion) without an applied force.

These forces are depending on:

1. Type of tire.

2. Derive rigidity.

3. Camber angle. Is the angle between the tire and the vertical plane (*figure 2.4*). Usually this angle is taken as zero.

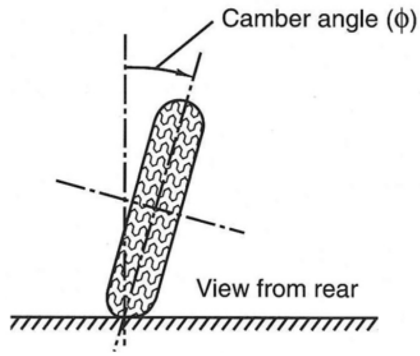

Figure 2.4:Camber angle. [2]

4. Friction coefficient.

5. Supported mass for the wheel.

6. Slip coefficient, can be defined as a comparison (in percentage) between the angular velocity of the wheel when the vehicle is moving and the angular velocity of the same wheel in free rolling. As we can see in *figure 2.5*, longitudinal force is increasing with the slip coefficient until maximum value is reached, the Normalized Traction Force then decreases slowly but remains at the Onset of spinning.

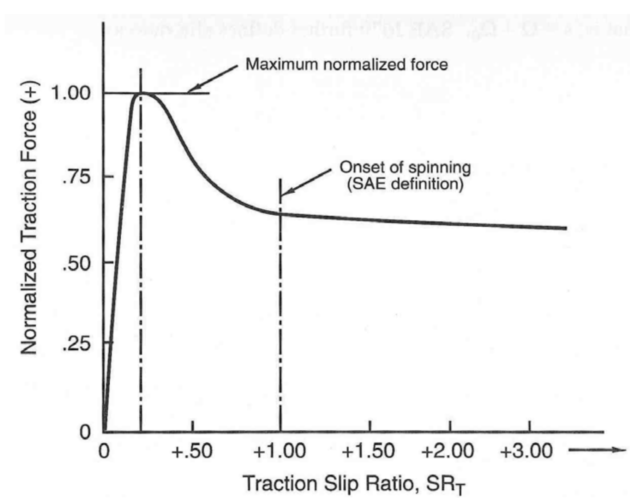

Figure 2.5:Traction force versus slip angle. [2]

7. Other factors like temperature and pressure of the wheel and speed of the vehicle, contact surfaces and etc.

There are some models that can give an approximation of the forces that are acting in the tire. Below there is a short description of three of them: brush model, finite element model and Pacejka formula.

2.4.1. Brush model

Since the tire is elastic, this model switches the contact area of the tire by a spring brush. There are two clear defined areas, the static area and the slide area as we can see in *figure 2.6.* Through some simplified formulas, this model allow us to get the longitudinal and transversal forces of the tire.

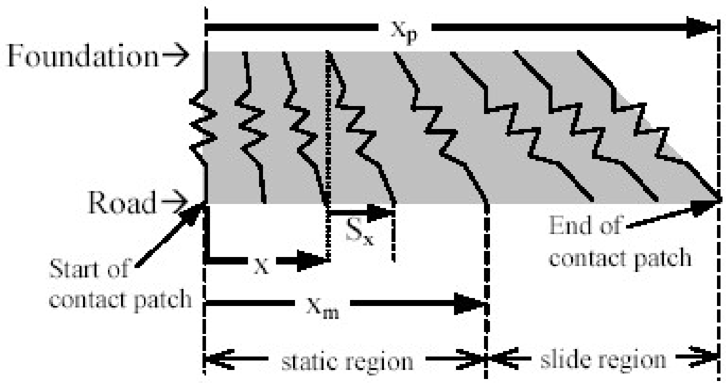

Figure 2.6:Brush model. [2]

2.4.2. Finite element method

Is a computational method that solves the problem of force. It consists of dividing the tire in small easy geometrical shapes or elements and then the behaviour of each element with a finite number of parameters is studied. The behaviour of each element depends on the elements which are near to it. In *figure 2.7* the finite elements grid of the tire is shown.

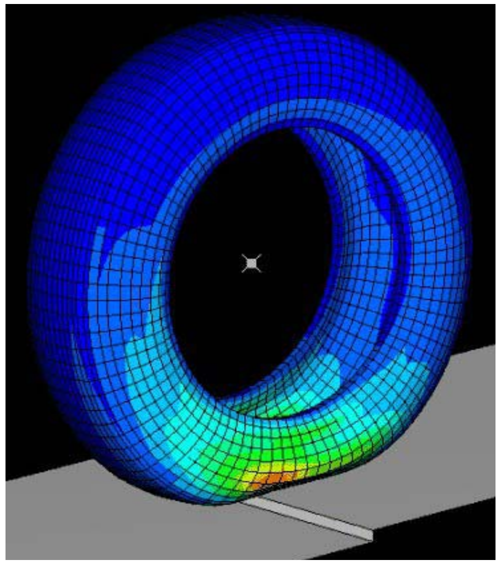

Figure 2.7:Discretized wheel by finite elements. [2]

2.4.3. Pacejka formula

This formula is known as and acts like a magic formula. It is estimation through equations and some experimental coefficients that are characteristic of the tire. Because the equations are easy, every simulation program can find the solution for the forces and the moment of the tire. The shape for these forces depends on the drift angle using Pacejka formulas which presented in *figure 2.8.*

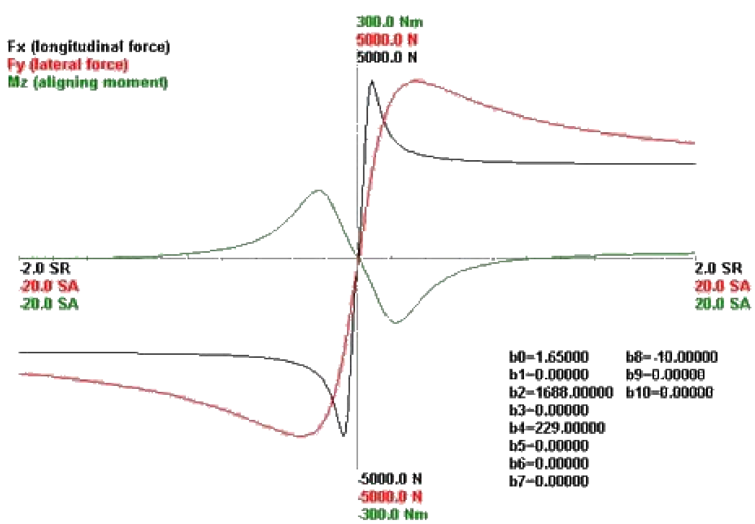

Figure 2.8:Forces and moments using the magic formula. [2]

If the tire is rotating fast enough (around $\alpha = 6^{\circ}-15^{\circ}$), there is a loss of adhesion in the contact surfaces and both the longitudinal and transversal forces are decreasing.

Pacejka formula is a good estimation of the efforts of the tire. However it is only valid when longitudinal and transversal forces are acting in an independent way. Nevertheless, when longitudinal and transversal deformation forces are acting at the same time these forces need to be slightly modified by the combined efforts.

COMBINED EFFORTS

Combined efforts happen when deformation forces in longitudinal and radial direction are acting at the same time by using the friction circle that we can see in the *figure 2.9*.

The friction circle means that when both forces (longitudinal and transversal) are combined, the result can not exceed as a maximum value that is the radius of the friction circle.

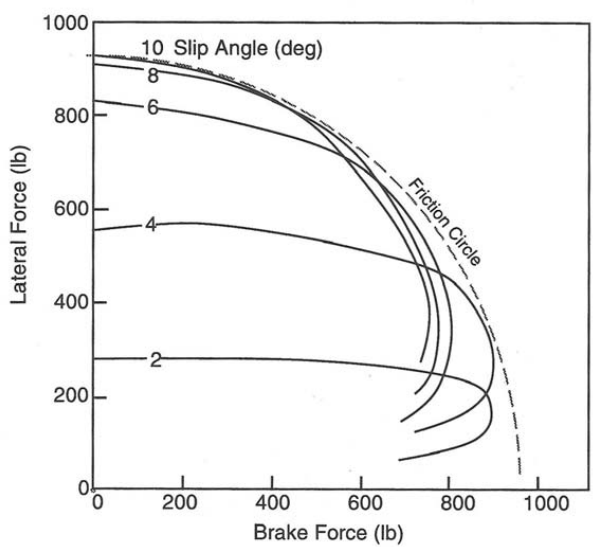

Figure 2.9:Friction circle. [2]

One simple method to combine longitudinal and transversal force is the linear approximation [6]. To reach to the combined deformation forces we need to follow both steps below:

1. Through Pacejka formula longitudinal and transversal deformation forces, F_x and F_y, are obtained.

2. Decreasing the transversal force until the vector (F_x, F_y) satisfies the *equation 2.10* and do not exceed the radius in the friction circle.

[2.10] $$F_y = F_{y0}\sqrt{1-\left(\frac{F_x}{F_{x0}}\right)^2}$$

F_y: Combined transversal force.
F_x: Combined longitudinal force.
F_{y0}: Transversal force obtained through magic formula.
F_{x0}: Maximum longitudinal force ($D+S_v$ in Pacejka formula).

This method gives priority to longitudinal forces instead transversal forces.

2.4.4. Choosing the best model

In the *table 2.1* we can see a comparison between the tire's models that were shortly explained above:

MODEL	ADVANTAGES	DISADVANTAGES
Brush Model	Simple	High computational charge
Finite element method	Flexible. Personal for each tire.	High computational charge. Difficult.
Pacejka	Standard model. Comun in automotion. Precise.	Experimental tests. Not too many dates available.

Table 2.1:Comparison for three different tire models. [2]

Pacejka model has chosen because it is a standard model and easy to use to study the behaviour of the tire. The most important reason for this is that all the tire providers are using this method for testing their tires, therefore it is easier to obtain the needed parameters. Because it is complicated to programme the friction circle and it also takes a high computational charge, Pacejka simple model is used instead of complex model with combined efforts. Furthermore, the difference between this method and the simple Pacejka formula is not really significant.

The Pacejka equations are summed in the *table 2.2*:

	PARAMETERS	MAGIC FORMULA
Transversal force	$C = 1.30$ $D = a_1 F_z^2 + a_2 F_z$ $BCD = a_3 \sin\left(a_4 \tan^{-1}\left(a_5 F_z\right)\right)$ $B = \frac{BCD}{CD}$ $E = a_6 F_z^2 + a_7 F_z + a_8$ $S_h = a_9 \gamma$ $S_v = \left(a_{10} F_z^2 + a_{11} F_z\right)\gamma$ $\Delta B = -a_{12} \lvert\gamma\rvert B$	$\phi = (1-E)(\alpha + S_h) + \frac{E}{B}\tan^{-1}\left(B(\alpha + S_h\right.$ $F_y = D\sin\left(C\tan^{-1}(B\phi)\right) + S_v$
Longitudinal force	$C = 1.65$ $D = b_1 F_z^2 + b_2 F_z$ $BCD = \frac{b_3 F_z^2 + b_4 F_z}{e^{b_5 F_z}}$ $B = \frac{BCD}{CD}$ $E = b_6 F_z^2 + b_7 F_z + b_8$	$\phi = (1-E)\sigma + \frac{E}{B}\tan^{-1}(B\sigma)$ $F_x = D\sin\left(C\tan^{-1}(B\phi)\right)$
Aligning moment	$C = 2.2812$ $D = \left(c_1 F_z^2 + c_2 F_z\right)\cdot\frac{r}{F_z}$ $B = c_5 + \left(c_4 F_z\right) + \left(c_3 F_z^2\right)$ $E = \left(c_6 F_z^2 + c_7 F_z + c_8\right)\cdot\left(1 + c_{12}\right)$ $S_h = c_9 \gamma$ $S_v = \left(c_{10} F_z^2 + c_{11} F_z\right)\gamma$	$\phi = B(\alpha + S_h) - E\begin{pmatrix} B(\alpha + S_h) - \\ \tan^{-1}\left(B(\alpha + S_h)\right)\end{pmatrix}$ $M_z = -D\cos(\phi)\cdot\cos\alpha + S_v$

Table 2.2:Pacejka magic formula and its coefficients. [8]

F_z: Vertical load in the tire [kN]
F_x: Longitudinal deformation force [N].
F_y: Transversal deformation force [N].
M_z: Aligning moment [Nm].
a_i, b_i, c_i: Pacejka coefficients.
α: Slip angle [grad].
σ: Slip coefficient ratio.
γ: Camber angle [grad].

As can be seen in *table 2.2* there are some unknown coefficients (a_i, b_i, c_i). These coefficients are characteristics of the tire and only the producer can give these parameters.

2.5. Energy

The final goal of the present study is the comparison between the energy that is given to the Robot and the energy that is wasted through the friction of the wheels. The Robot is receiving energy from an electronic power supply, so it is electric energy. This energy can be easily calculated by using the basics of electronics:

[2.11] $$E_e = V \cdot I$$

E_e: Electric energy [J].
V: Voltage [V].
I: Current [A].

On the other hand, the wheels are dissipating energy because of the friction forces and transversal forces in the tires (in curve motion). This energy is mechanical energy and can be founded by using *formula 2.14*.

[2.12] $$E = F \cdot dr$$

[2.13] $$v = \frac{dr}{dt}$$

[2.14] $$E = F \cdot v \cdot \cos\alpha$$

E: Displacement energy [J].
F: Applied force [N].
r: Distance [m].
v: Velocity of the body [m/s].
t: time [s].
α: Angle between the applied force and the displacement direction [rad].

Finally the Robot has kinetic energy because it is moving. This kinetic energy can be divided in translation kinetic energy and rotation kinetic energy (because of the rotation around its gravity centre).

[2.15] $$E_k = \frac{1}{2} m \cdot v^2$$

E_k: kinetic energy [J].
m: mass of the robot [kg].
v: velocity of the robot [m/s].

[2.16] $$E_{kr} = \frac{1}{2} \vec{\omega}\left(I\vec{\omega}\right)$$

E_{kr}: Rotation kinetic energy [J].
ω: Rotation velocity in vectorial form [rad/s].
I: Inertia matrix of the robot (3x3) [kg·m^2].

The difference between the total energy and the friction and kinetic energy is the wasted energy in the components of the Robot (breaks, charging the battery, components efficiency…)

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3. Procedure

For the first step and before starting any calculation, the parameters of the trajectory should be defined. The *figure 3.1* shows the Robot through the points a, b, c and d. The mobile reference system (ξ, ζ) is located at the centre of the mass of the Robot but the centre of rotation (x, y) has another reference system form each angle and we should measure these distances as well. In the *figure 3.1* nomenclatures that will be used in the calculations and the programs can also be found.

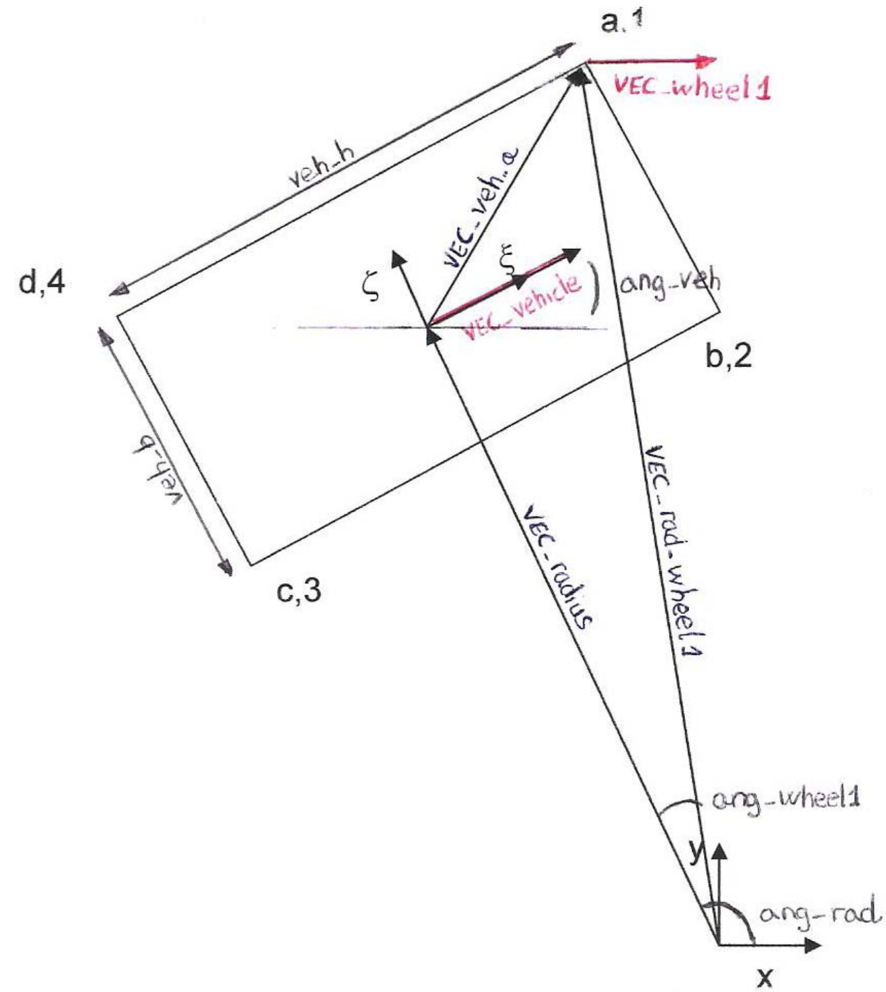

Figure 3.1:Robot geometry.

Moreover, the dimensions of the Robot are needed and *figure 3.2* demonstrates a picture of the Robot with the dimensions that were measured. These parameters can be found at the *table 3.1*.

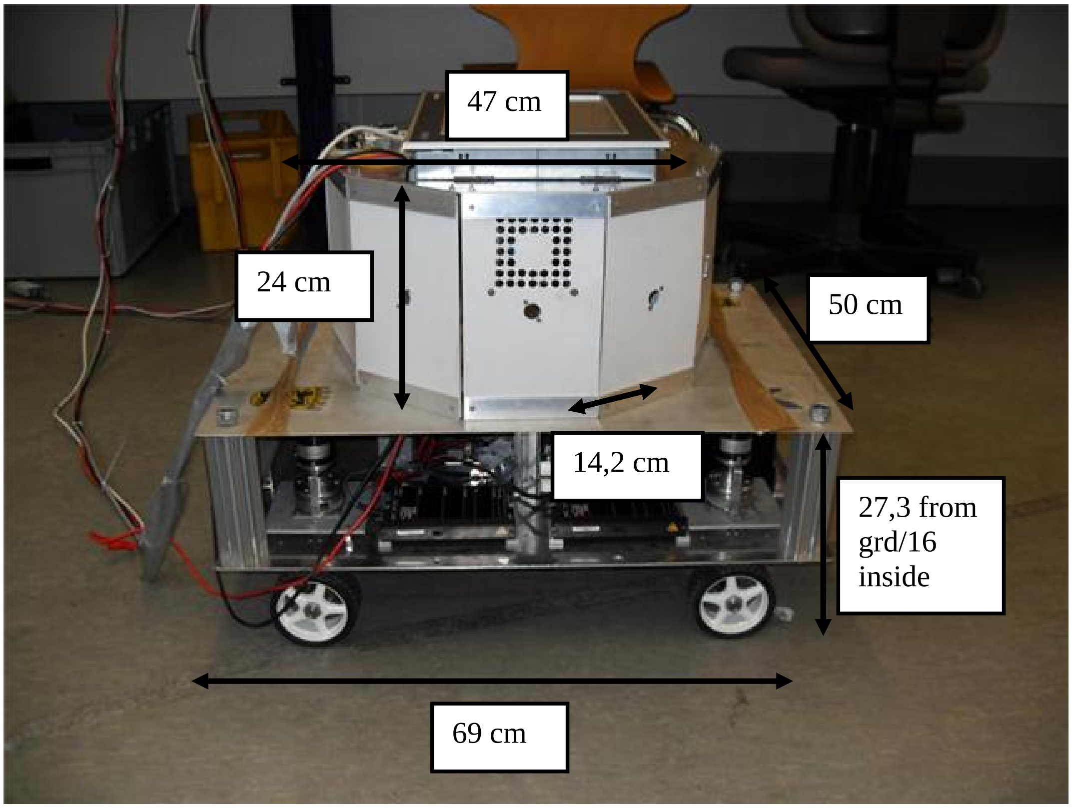

Figure 3.2:Robot MAX2D and its measures.

VEH_B	0.5 M
veh_h	0.69 m
veh_gc	0.1259 m
wheel_h	0.1 m
m	31 kg

Table 3.1:Robot geometry.

The parameter of *wheel_h* is the radius of the wheel and *veh_gc* is the height of the gravity centre of the Robot. It was obtained through the geometry of the Robot [3]:

[3.1] $$m_{screen} = 1\,kg$$

[3.2] $$h_{GC} = \frac{\sum_i m_i h_i}{\sum_i m_i}$$

m_i: mass each component of the robot [kg].
h_i: Altitude of each component of the robot [m].

Considering that the mass of the Robot is divided between the screen and the electronic components (we assume as zero the mass of the top cover and the metal parts so as the mass of the wheels):

$$[3.3] \qquad h_{GC} = \frac{((m_{total} - m_{screen})(h_{platform} - h_{wheel})) + (m_{screen} \cdot h_{total})}{(m_{total})}$$

$$[3.4] \qquad h_{GC} = \frac{((30-1)kg \cdot (27.3-16)m) + (1kg \cdot 27.3m)}{31kg} = 0.1259\,m$$

To reach to this point we can divide it to two main parts:

1. Theoretical wasted energy in two different trajectories. Two trajectories would be studied at this first step. The first one will be $y=x^2$ and the second one is $y=x^3$. The wasted energy on the wheels and forces can be compared in both trajectories. For all of these a Matlab Program will be implemented (*Appendix 11.1*).

2. Friction in MAX2D. At the second part of the study, different velocities will be studied in the trajectory y=x (movement on the straight line) and also in simple rotation of the Robot around its centre of the mass. Both theoretical results (using also the Matlab Program attached in *Appendix 11.1* and *Appendix 11.2*) will be compared with experimental measures.

3.1. Theoretical wasted energy in two different trajectories

3.1.1. Geometry

The first step is defining the geometry and positioning of the different rotation centres, on each point of trajectory, radius of rotation, position of the wheels, direction of the velocities of the wheels and etc.

Radius and centre of rotation will be obtained by applying the *formulas 2.1* and *2.2*. Notice that if $y''(x) = 0$ for any point of the trajectory (x, y); both formulas tend to be infinity. This means that at this point that we have a relative maximum, minimum or inflexion point, so the Robot is not rotating at this point. We consider $y''(x)$ is small enough when $y''(x) < 10^{-6}$. For instance this value is multiplying by 10^{-6} and the problem will be solved. Logically this is not an exact value and the obtained forces will have an error around 10^{-6} N, that is small enough to be an acceptable error.

Two supporting programs are created:

1. The first one is a program that is called ***ang_xy.m*** (*Appendix 11.5*) that is returning the angles between the vectors that is defined for two points; the x-axis and its complementary angle. This is used to obtain the *ang_rad* and the *ang_veh* when the gravity centre (located at one point of the trajectory) and the rotation's centre (x, y) are known.

2. The second one is ***createvector_la.m,*** (*Appendix 11.6)* used to define a vector that is used for the complex numbers if the length and the angle of the x-axis are known.

All the calculations for the geometry of the Robot will be done by using the complex numbers.

To sum up, the *figure 3.3* displays a diagram displaying how the calculations were completed for the geometry of the Robot and the chosen trajectory. The used parameters can be checked in *figure 3.1*.

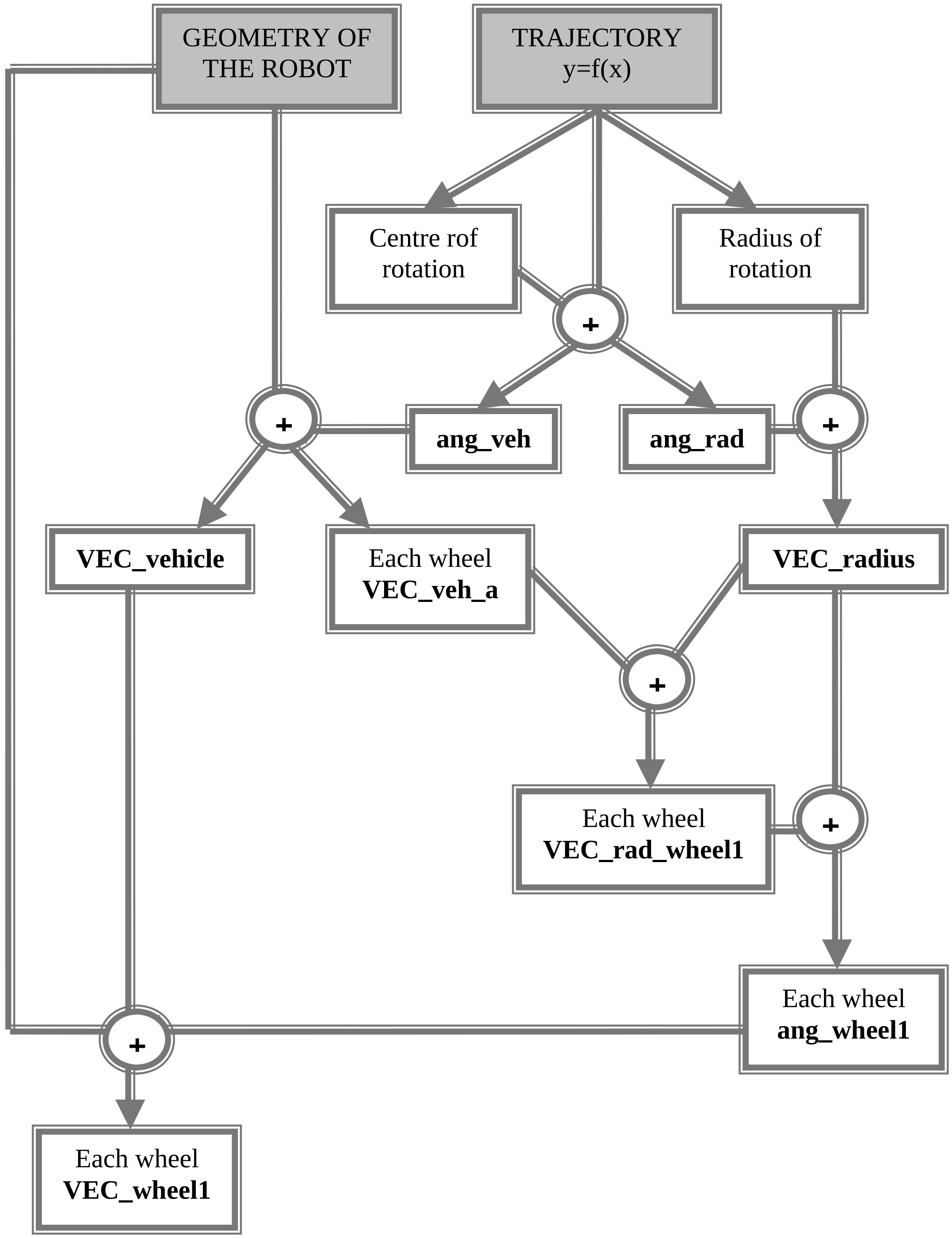

Figure 3.3:Calculation diagram of the geometry.

Finally we can see that the representation of the geometry is complete. In *figures 3.4* and *3.5* it is possible to see the differences between the centres of the rotations and the radius of the rotations at each point of the trajectory for the different studies of the functions.

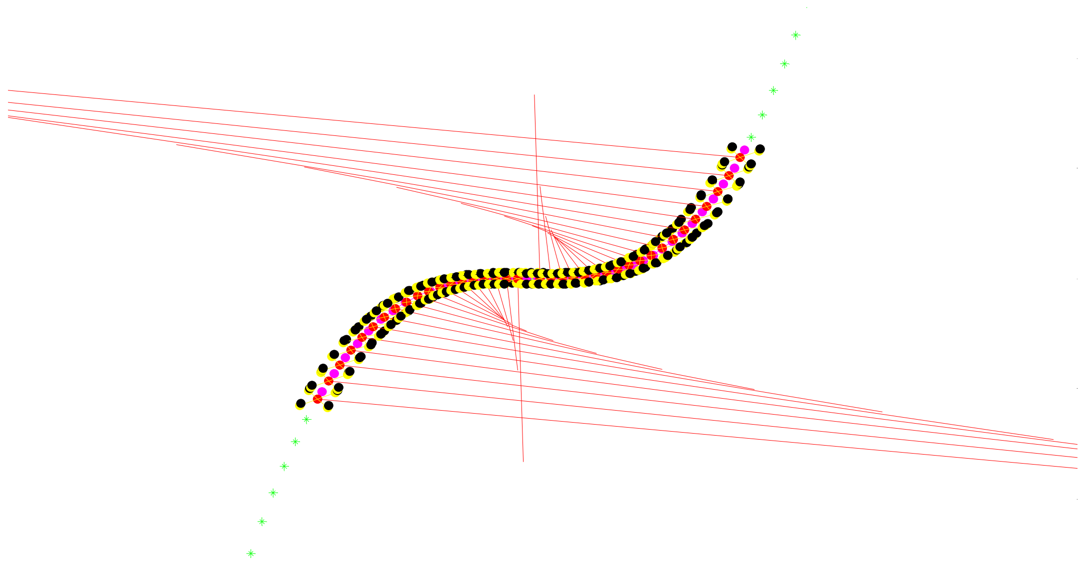

Figure 3.4:Function $y = x^3$

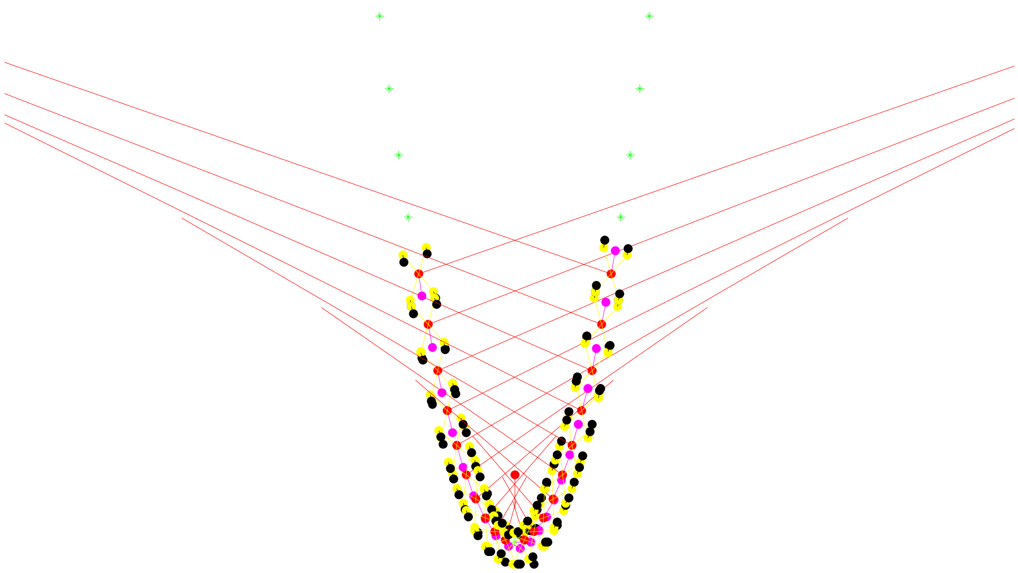

Figure 3.5:Function $y = x^2$

3.1.2. Velocities of the wheels

The linear velocity of the gravity centre of the Robot is known, so angular velocity is easily obtained by using *equation 3.4* this is the same at all the points of the Robot. On the other hand, the direction of this linear velocity is the same as the

VEC_vehicle vector direction and to get this linear velocity we need to apply the *equation 3.5*:

[3.5] $$v_o \left[m/s \right] = v \cdot VEC\_vehicle|_{unit}$$

v_o: Linear velocity of the gravity centre of the robot in vectorial form (v_x, v_y).

v: Module of the linear velocity of the gravity centre of the robot [m/s].

$VEC\_vehicle_{unit}$: Vector which module is 1 that has the same direction as the movement of the robot.

Two supporting programs are created to get this point:

1. The ***crossvecinv.m*** (*Annex 11.7*): If the result of a cross vector operation is known and the vectors of 'a' and 'b' are perpendicular (*formula 3.5*) this program gives us the module of the vector 'b'.

[3.6] $$|a \times b| = \begin{bmatrix} i & j & k \\ a_1 & a_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} = |a||b|$$

The program is used to find the angular velocity of the Robot when the radius of rotation and the linear velocity at the vectors form are known as itis shown in *formula 3.6.*

2. The ***crossvec.m*** (*Appendix 11.8*): Gives us the cross product between a complex number and the module of a vector perpendicular to the first one, in the other words the values a_1, a_2, b_3 of the *formula 3.6* are known and this program returns the cross product $a \times b$ at the vector form.

Applying *equation 2.5* the velocity of the wheels (*vel_a* of the first wheel) can be obtained. The independent movement of the wheels is considered as a pure rotation from around itself, so linear velocity of the wheel from the mobile reference system is zero ($V_M = 0$).

Also angular velocities of the wheels around centre of rotation will be obtained. To reach to these values, *equation 2.5* needs to be used, but in this case the velocity of the wheel is *vel_a* (for the first wheel) and this is not the same as the linear velocity of the gravity centre of the Robot. The radius of the rotation is the distance between the wheel and the centre of rotation *VEC_rad_wheel1*.

3.1.3. Friction forces

Once the geometry has been completed the forces can be obtained. Centrifugal force on the curve motion can be applied in the gravity centre of the Robot. Since the radius of rotation is known the module of this force can be founded from the *formula 2.7*, this force has applied on the outward radial direction (same direction than the radius of rotation vector).

For the acquirement of the friction forces, mass distribution is needed (*formula 2.9).* For a vehicle that is not moving or is moving linear with constant velocity, mass is equally distributed on the wheels. On a curve motion, there are two wheels of the Robot that they are out of the curve and two are inside the curve and their radiuses of the rotation are different (For example, in *figure 2.1* radius of rotation at 'a' and 'd' is different form the radius of rotation at 'b' and 'c'). Centrifugal force was calculated previously, so we will focus on the coordinate system that is fixed on the vehicle. According to this coordinate system the vehicle is not turning over any of its wheels, moment equilibrium on the Robot can be applied. In *figure 3.6* all the forces are represented, so the mass of the wheels can be easily obtained:

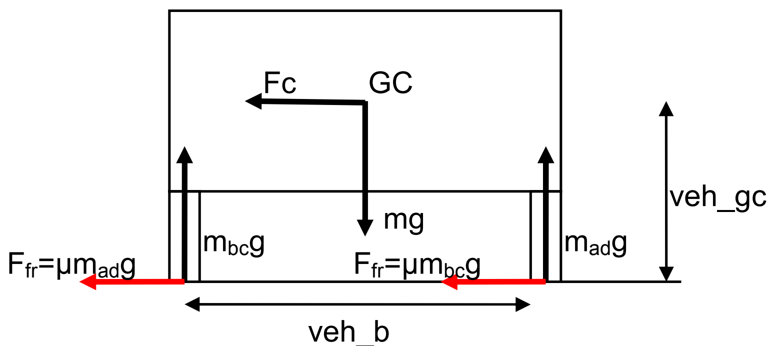

Figure 3.6:Forces in a curve motion.

Moment equilibrium (left wheel):

[3.7] $$\sum M = 0$$

[3.8] $$mg \cdot \frac{(veh\_b)}{2} - F_c \cdot (veh\_gc) - m_{ad}\, g \cdot (veh\_b) = 0$$

[3.9] $$m = m_{ad} + m_{bc}$$

By using *formulas 3.7, 3.8* and *3.9*, mass distribution is clearly defined:

[3.10] $$m_{ad} = \frac{mg \cdot \frac{(veh\_b)}{2} - F_c \cdot (veh\_gc)}{g \cdot (veh\_b)}$$

[3.11] $$m_{bc} = m - m_{ad}$$

m: Mass of the robot [kg].
g: Gravity g=9.81m/s^2.
F_c: Centrifugal force [N].
veh_gc, veh_b: dimensions of the robot [m].
m_{ad}: Mass supported by the wheels a and b [kg].
m_{bc}: Mass supported by the wheels b and c [kg].

Emphasize of these masses, are for two wheels, so mass of each wheel is half of the obtained value.

Friction coefficient μ is a constant that depends of the contact surfaces. In this case, materials are considered as rubber and dry concrete, so for the rolling resistance coefficient we will use C_{rr}=0.02 [7]. Applying *equation 2.9,* friction forces on each wheel are acquired.

3.1.4. Pacejka magic formula application

Pacejka method returns the forces that are making deformations on the tires with the slip angle and the vertical applied force on the wheel as inputs. The vertical applied force is coming of the supporting mass of each tire. The slip angle is the difference between the longitudinal direction of the wheel and the direction of the movement of the wheel. This angle is the difference between angle of *vel_a* (for the first wheel) and the angle of *VEC_vehicle*. Furthermore the characteristic parameters of the wheels need to be implemented in the program.

In Pacejka magic formula we have to take care of the units, because forces are given in kN and angles are in grades [2] [6]. Additionally we have to pay attention the reference systems, because in the Pacejka formula these forces are given iny in module and we need the vector form.

The program ***pacejka.m*** (*Appendix 11.3*) is a direct application of the formulas that are summed in *table 1.2.* Camber angle is assumed like as zero and the maximum slip angle is assumed as 75º.

Tables 3.2, 3.3 and *3.4* represent a diagram about how Pacejka formula is working. We can obtain the different coefficients by multiplying below parameters F_z

and F_z^2 by b_i. For example, to find the E_x coefficient '$E = b_6 F_z^2 + b_7 F_z + b_8$'. These coefficients are directly used in *formulas 3.12, 3.13* and *3.14* and finally transversal and longitudinal forces and aligning moment are obtained.

Fz	**Fz²**	**COEFFICIENT**		
		$+a_0$	C_y	Shape factor
a_1		$+a_2$	nup_y	
nup_y			D_y	Peak factor
		$a_3 \sin(a_4 \arctan(a_5 F_z))$	BCD_y	Slip stiffness
		$-a_{12}\lvert\gamma\rvert$	ΔB_y	
		$\frac{BCD_y}{C_y D_y}(1+\Delta B_y)$	B_y	
a_7	**a_6**	$+a_8$	E_y	Curvature factor
		$a_9\gamma$	Sh_y	Horizontal shift
a_{11}	**a_{10}**	$\cdot\gamma$	Sv_y	Vertical shift

Table 3.2:Coefficients for transversal force.

[3.12] $$F_y = D_y \sin\left(C_y \tan^{-1}\left(B_y\left((1-E_y)(\alpha + S_{hy}) + \frac{E_y}{B_y}\tan^{-1}\left(B_y(\alpha + S_{hy})\right)\right)\right)\right) + S_{vy}$$

Fz	**Fz²**	**COEFFICIENT**		
		$+b_0$	C_x	Shape factor
b_1		$+b_2$	nup_x	
nup_x			D_x	Peak factor
b_4	**b_3**	$\cdot e^{-b_5 F_z}$	BCD_x	Slip stiffness
		$\frac{BCD_x}{C_x D_x}$	B_x	
b_7	**b_6**	$+b_8$	E_x	Curvature factor
b_9		$+b_{10}$	Sh_x	Horizontal shift

Table 3.3:Coefficients for longitudinal force.

[3.13] $$F_x = D_x \sin\left(C_x \tan^{-1}\left(B_x\left((1-E_x)\sigma + \frac{E_x}{B_x}\tan^{-1}(B_x\sigma)\right)\right)\right) + S_{hx}$$

F_Z	F_Z^2	COEFFICIENT		
		$+c_0$	C_m	Shape factor
c_2	c_1	$+\frac{r}{F_z}$	D_m	Peak factor
c_4	c_3	C5	B_m	Stiffness factor
c_7	c_6	c_8	ΔE_m	
		$\Delta E_m \cdot (1 + c_{12})$	E_m	Curvature factor
		$c_9 \gamma$	Sh_m	Horizontal shift
c_{11}	c_{10}	$\cdot \gamma$	Sv_m	Vertical shift

Table 3.4:Coefficients for aligning moment.

[3.14] $$M_z = -D\cos\left(B(\alpha + S_h) - E\left(B(\alpha + S_h) - \tan^{-1}\left(B(\alpha + S_h)\right)\right)\right) \cdot \cos\alpha + S_v$$

F_z: Vertical load in the tire [kN]
F_x: Longitudinal deformation force [N].
F_y: Transversal deformation force [N].
M_z: Aligning moment [Nm].
a_i, b_i, c_i: Pacejka coefficients.
α: Slip angle [grad].
σ: Slip coefficient ratio.
γ: Camber angle [grad].

3.1.5. Deformation forces on tires

The total force acting on the tire is the friction force plus the deformation forces that are obtained from Pacejka formula. Friction forces are measured from a static coordinate system and deformation forces are measured from the mobile coordinate system. This coordinate system is located at the gravity centre of the Robot. For operating the forces, they need to be referred to the same coordinate system.

Mobile coordinate system is rotated around the static coordinate system. *Figure 3.7* is a representation of the action of both reference systems.

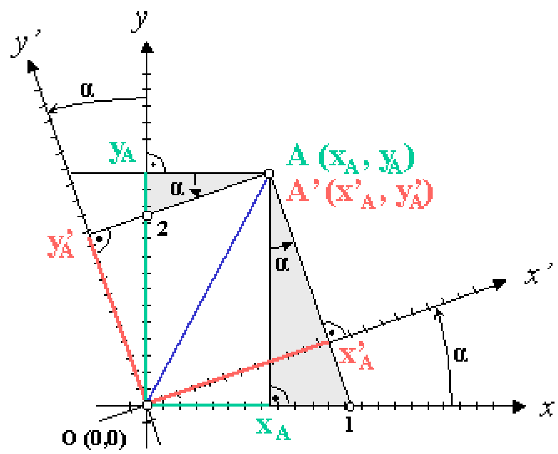

Figure 3.7:Rotation of coordinate systems [3].

Point A could be defined from the (x, y) coordinate system as [3]:

[3.15] $$A = x_a i + y_a j$$

x_a, y_a: Coordinates of the point A in the fixed reference system.

If point A is needed in (x', y') reference system, *formula 3.16* [3] can be used to get its coordinates.

[3.16] $$A = (x_a \cos\alpha + y_a \sin\alpha)i + (-x_a \sin\alpha + y_a \cos\alpha)j$$

α: Rotation angle of the rotated coordinate system [rad].

To simplify this transformation, an equation system can be used. The objective is to obtain the forces that are given in the rotated coordinate system in an orthogonal coordinate system. The angle that the Robot is rotated is the same as the angle of *VEC_vehicle.* Transformation matrix should be the inverse of the used in *formula 3.17* that is used, because now forces are given in (x', y') system.

[3.17] $$M_{trans} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

β: Angle between the linear trajectory that the Robot is following and the x-axis in the fixed reference system [rad].

The problem could be solved with by following system:

[3.18] $$F_{GLOBAL} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} M_{trans}$$

F_{GLOBAL}: Force in fixed reference system [N].

F_x, F_y: Force in mobile coordinate sysyem (located in the gravity centre of the robot) [N].

M_{trans}: Transformation matrix.

3.1.6. Energy

The final objective of the study is to calculate the energy that is wasted through the friction of the wheels. By applying *equation 3.190,* this energy could be measured. The force that we have to use in this formula should be constant, because it is variable and the forces are changing, therefore average of the force values could be accepted. The velocity that is in the *formula 2.5* is the velocity of the wheels and should be also constant, so as in the force case the average would be used. Finally we can get the *formula 3.20.*

[3.19] $$E = \frac{F_i + F_{i+1}}{2} \cdot \frac{v_i + v_{i+1}}{2} \cdot \cos\alpha$$

F_i: Force in a point i of the trajectory [N].

v_i: Velocity in a point i of the trajectory [m/s].

α: Angle between the applied force and the direction of the movement [rad].

Total energy that is wasted on the Robot is the sum of the wasted energy on each wheel.

3.2. Friction in MAX2D

3.2.1. Straight motion

1. PRACTICAL METHOD

The energy that we used for the Robot needs to be measured. The input energy of our Robot is getting from an Electronic power supply source and an internal battery. The overall used current for the movement of the Robot is the input current minus the current that is used for the screen that is located at the top the Robot (*figure 3.8*) and minus the used current for the battery. Observe that the screen uses always the same current whether the Robot is standing or moving.

Figure 3.8:Screen situated at the top of the Robot.

For the first step we need to move the Robot on a straight line, the front wheels and the back wheels must be parallel to each other. To obtain this, we used some mechanical fixation parts for the wheels. The second step is applying different velocities by the program that is controlling the Robot and measure the main input current of the Robot with an Ampermeter for different velocities. We know the current values of the screen, the battery and the main current that the Robot needs to move therefore, we can find the current value of the motors and so the energy can be easily calculated.

To reach to this point we started to do the first measurement, and we found the first problem: the internal battery sometimes is charging and discharging, therefore the input current has high variations so the values that we took were not correct. The battery was needed to run the Robot because the main power supply could support maximum 3 A which was not enough to run the Robot on high speeds. To avoid this problem, we removed the internal battery and connected another power supply in parallel to the first one. Then we had 3 A maximum from each power supply (6 A in total) and a fixed input current. The overall used current is the input measured current minus the current used for the screen.

2. THEORICAL METHOD

The Robot is moving on a straight line, so to apply this movement in the Matlab Program we use the function $y = x$. Notice that in this function, the Robot is not on the curve motion at any point, so Pacejka forces will be constant and equal to zero for all the studied velocities.

3.2.2. Turning motion

1. PRACTICAL METHOD

As we can see at the previous case, the wheels need to be fixed mechanically. For this movement, wheels must have an angle of 45º. The top screen is also working during this movement, and the current that is needed for it is the same as that we measured above. First we must apply different velocities in the program to run the Robot, and then measure the input current with an Ampermeter.

In this occasion the battery was removed from the beginning, so measures did not need to be taken twice. The overall used current in the anterior case is the input measured current minus the current of the screen.

2. THEORICAL METHOD

In this second case the Robot is rotating around a vertical axis through the gravity centre. This function could not be directly implemented in the program that was created; indeed this Matlab Program needs to be slightly modified. The points that need to be changed are described on below:

1. The geometrical calculations with the centre of rotation, radius of rotation and direction of all the wheels have to be removed.

2. Linear velocities of the wheels are known and its trajectory is circular around the gravity centre. The trajectory of the wheels can be calculated through a circular function, where the radius (*formula 3.20*) is the distance between the wheel and the gravity centre.

[3.20] $$x^2 + y^2 = R^2$$

[3.21] $$R = \sqrt{\left(\frac{veh\_h}{2}\right)^2 + \left(\frac{veh\_b}{2}\right)^2}$$

[3.22] $$y = \sqrt{\left(\frac{veh\_h}{2}\right)^2 + \left(\frac{veh\_b}{2}\right)^2 - x^2}$$

(x,y): Point of the circular trajectory.
R: Radius of the circular trajectory [m].
veh_h, veh_b: dimensions of the robot [m].

3. The slip angle of the wheels has always kept the same value and, as an approximation, could be obtained like the angle between the straight line that has connected two consecutive points of the trajectory (i, i+1) and the derivative function at the point where the wheel is (*figure 3.9*).

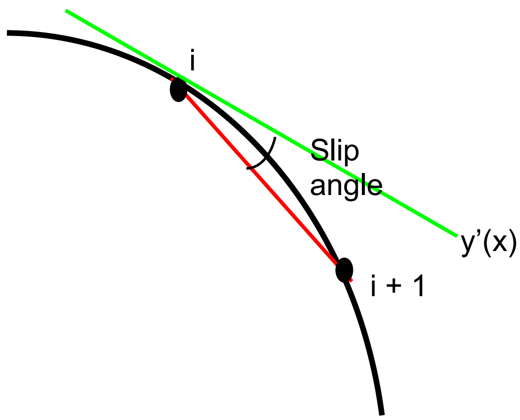

Figure 3.9:Movement and derivative function between two points of the trajectory. [3]

The next step is the obtaining of the angle of the wheels. With this angle the position of the wheels and the direction where these wheels are moving can be settled by using complex numbers.

Velocity of the wheels, friction forces and Pacejka forces are obtained by using the same formulas that we had in the first Matlab Program (***friction_pacejka_energy.m***). The only variation that we have now is; the equality of the mass distributed on the wheels.

Pacejka forces need to be changed also to a global static coordinate system. By using the same correlations, we can easily obtain these forces.

Total forces on tires and energy calculation are finding the same as in the anterior case.

4. Measures and results

4.1. Problem and fix values

The first important problem was to find the parameters that are needed for using the Pacejka formula, for the calculation of the deformation forces of the tire of the Robot. These parameters are the characteristic of each tire and only the producer can provide them.

The tire of the Robot, is P7 LMT 44-27/80, with a radius r=0.1m. By asking some important tire manufactures such as Michelin, Dunlop and Continental we tried to get necessary coefficients for the tire of the Robot. These manufactures have only the Pacejka parameters for the tires on the car that are different from the tires that we used for the Robot. Because the forces that we will obtain are not exactly the same as the forces that deform the tire therefore we can not reach to the correct conclusion. Nevertheless this study can give us an idea about how the deformation forces are changing with the trajectory and where the critical points are so as where the maximum and minimum forces are.

The parameters that have been chosen for the study are given from Dunlop and specific for MF-TYRE 5.0, with radius 0.2159 m and width of 0.1905 m [9]:

$a_0 = 1.3$	$b_0 = 1.65$	$c_0 = 2.2812$
$a_1 = -1.0815$	$b_1 = 0$	$c_1 = 0.042392$
$a_2 = 0.34037$	$b_2 = 1$	$c_2 = 0.24857$
$a_3 = -11.799$	$b_3 = 0$	$c_3 = 2.0413$
$a_4 = 2$	$b_4 = 15$	$c_4 = 1.5386$
$a_5 = 0.91533$	$b_5 = 0.2$	$c_5 = -7.3244$
$a_6 = 0$	$b_6 = b_7 = b_8 = b_9 = b_{10} = 0$	$c_6 = 0.010808$
$a_7 = -2.934$		$c_7 = -0.0029542$
$a_8 = a_9 = a_{10} = a_{11} = a_{12} = 0$		$c_8 = 1.3854$
		$c_9 = c_{10} = c_{11} = 0$
		$c_{12} = 0.0021268$

We consider also that the maximum slide angle as given by Dunlop is $\alpha_{\max} = 0.18014^{\circ}$.

The last concern that we had was that the wheels are rolling without sliding along the ground. This means that the friction coefficient and the force that make the wheel moves satisfy the equation [3]:

[4.1] $$\mu_e \geq \frac{F}{3mg}\left|1-\frac{2r}{R}\right|$$

μ_e= friction coefficient.
F= Applied force in the gravity centre of the wheel [N].
R= radius of the wheel [m].
r=radius with the wheel with applied load [m].

4.2. Forces in two different trajectories

In this part of the study we will obtain the friction and the deformation forces for two different motions. In the first case we will study these deformation and friction forces when the Robot is moving through the curve $y = C \cdot x^3$ for different values of C and in the second case the curve $y = C \cdot x^2$ will be studied for different values of C.

The values for the linear velocity and the module of the rolling resistance factor that where used in the program are $|v| = 1 \, {}^{m}\!/\!{}_{s}$ and $C_{rr} = 0.02$. Furthermore fixed parameters of the Robot same as its geometry, mass and etc should be used in this program (these values can be found in *table 3.1*).

4.2.1. Curve y=Cx^3

Six different curves will be studied for six different values of C, C=1, 2, 4… 20. In the *figure 4.1* all these trajectories are represented. Notice that when C is increasing, the curve will be sharper, so if we anticipate the results, we can say that the forces that are acting on the tire will increase also.

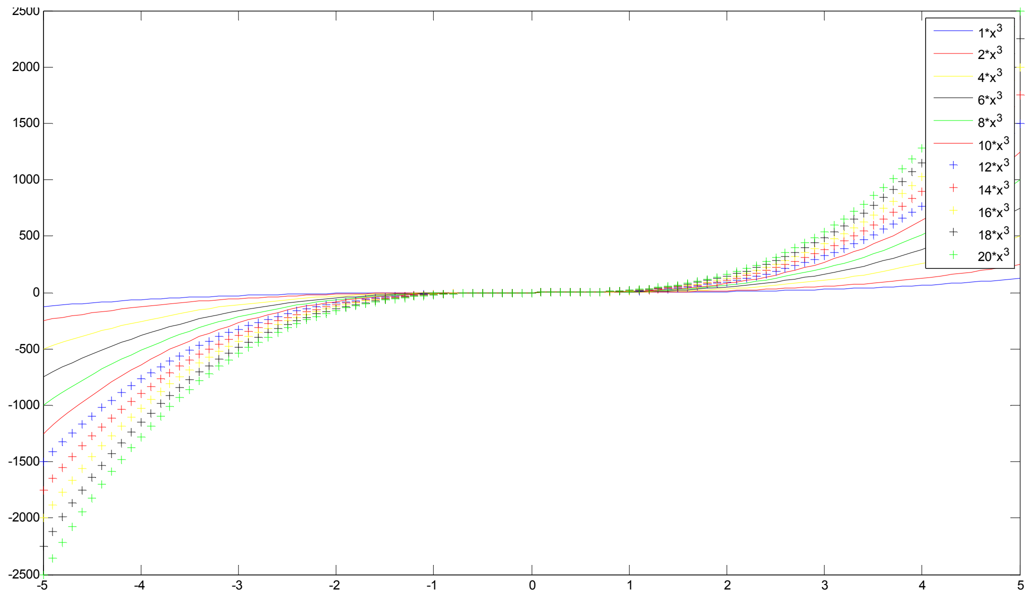

Figure 4.1:Function $y = C \cdot x^3$.

First of all is interesting to know that how the friction forces and the deformation forces in both axis (x and y) are varying. The shape of the curves are almost the same for each C value, so in *figure 4.2* and *figure 4.3* only the C=1 value is showed because for the other values would be the same.

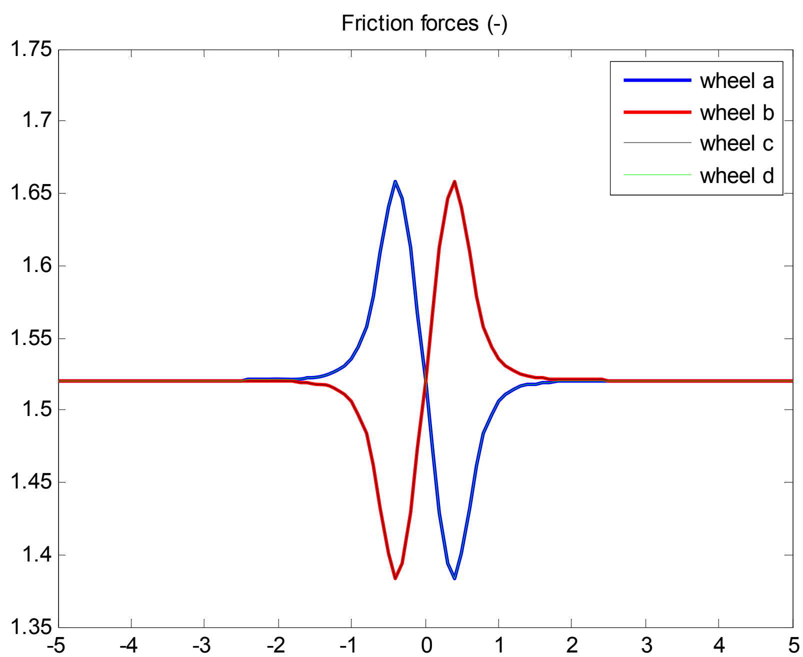

Figure 4.2:Friction forces in curve $y = x^3$.

In *figure 4.2* we can see how the friction forces are varying on each wheel. The maximum and minimum values are obtained at the point of the trajectory that has the minimum instant radius of rotation. This point is also varying when C coefficient is changing, but it is not a goal of the study to determine that where this point is.

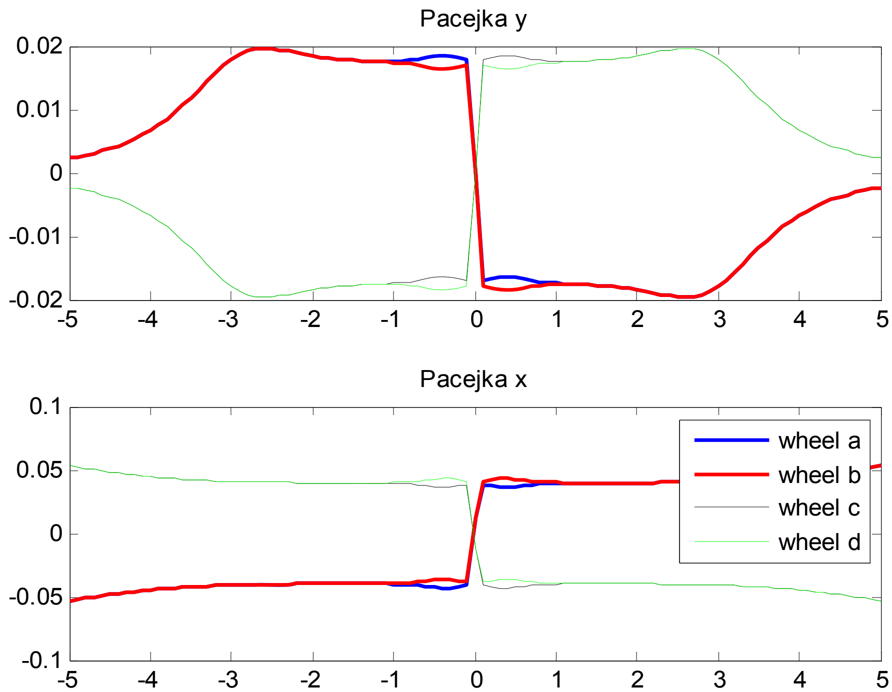

Figure 4.3:Deformation forces in curve $y = x^3$.

In the *figure 4.3* deformation forces are represented. We can see how Pacejka forces in y axis are near to 0 when the vehicle is moving on a straight line (for negative or positive values). For the force on longitudinal direction (x axis) the maximum value is obtained at the extremes of the trajectory. We can see how the tires that are in the same axis have almost the same value from these forces; however when the instant radius of rotation is minimum, Pacejka forces at the tires that are in the same axis begin to show differences.

The values that we can see in *table 4.1* are the values that are obtained from the Matlab Program. Observe that the Pacejka forces that are noted in the table are the maximum values. It is important to know how high the force on the tire is.

C	1	2	4	6	8	10
F_f maximum [N]	1.6577	1.7118	1.7949	1.8207	1.8648	1.9318
F_f minimum [N]	1.3803	1.3262	1.2431	1.2173	1.1732	1.1062
Maximum Pacejka F_x [N]	0.0537	0.0750	0.0760	0.0760	0.0760	0.0759
Maximum Pacejka F_y [N]	0.0196	0.0196	0.0196	0.0196	0.0199	0.0203

C	12	14	16	18	20
Fr maximum [N]	1.9887	2.0344	2.0696	2.0949	2.1111
Fr minimum [N]	1.0524	1.0067	0.9715	0.9462	0.9300
Maximum Pacejka F_x [N]	0.0758	0.0760	0.0759	0.0759	0.0760
Maximum Pacejka F_y [N]	0.0206	0.0209	0.0211	0.0212	0.0213

Table 4.1:Forces in the tires for the curve $y = C \cdot x^3$

4.2.2. Curve y= Cx^2

In the first case, six different curves will be studied for six different values of C (C=1, 2, 4, 6… 20). In the *figure 4.4* there is a representation of all these curves and as in the anterior case $y = C \cdot x^3$ the curve is sharper when the coefficient C is increasing. Deformation forces on the tire should increase also with C, but at this point it is easy to notice that the maximum values will be obtained at the point of the trajectory (x, y)=(0, 0).

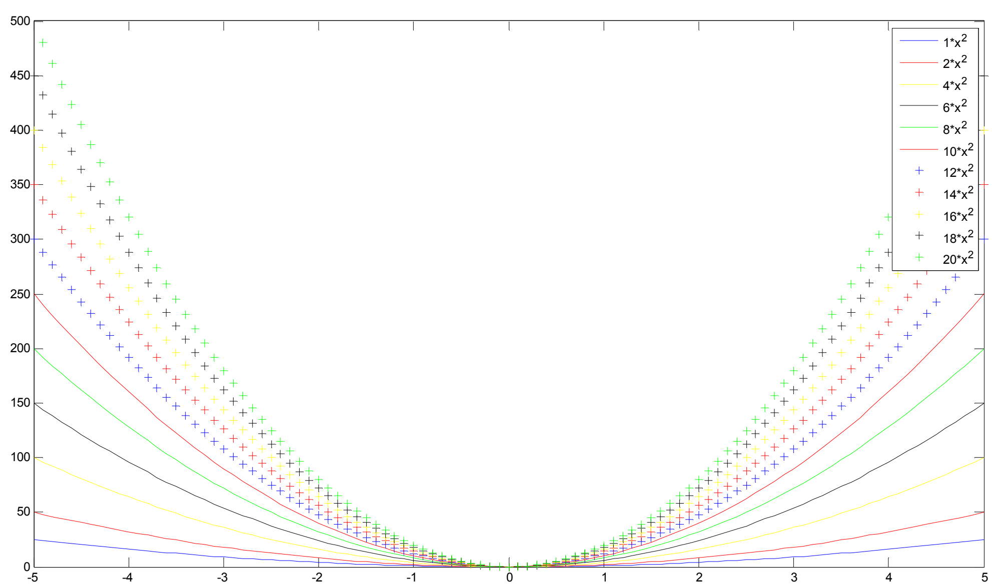

Figure 4.4:Function $y = C \cdot x^2$.

In *figures 4.5* and *4.6* the friction and Pacejka forces will be presented for C=1 and we can see how the forces vary. The shapes of the curves are almost the same for the different values of C; obviously the maximum values for the friction and Pacejka forces are different.

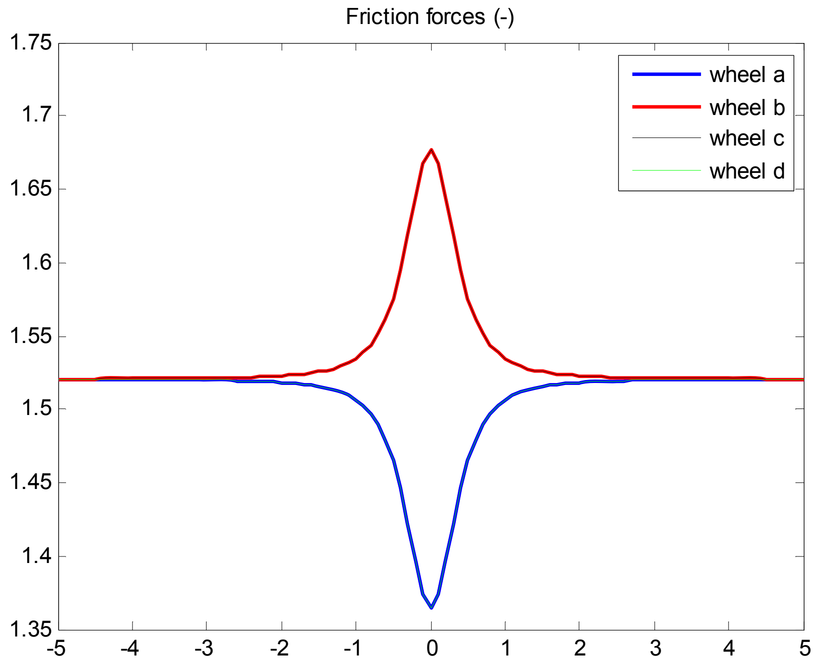

Figure 4.5:Friction forces in curve $y = C \cdot x^2$.

As we predicted before, the maximum values in the friction forces are obtained for the point (x, y) = (0, 0) that has the minimum instant radius of rotation. This critical point is unchanged for all the trajectories.

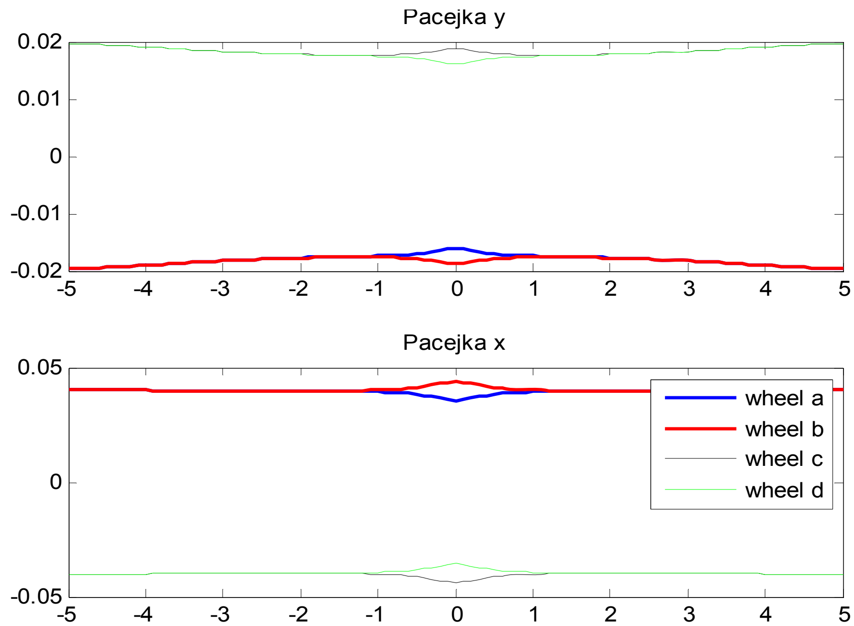

Figure 4.6:Deformation forces in curve $y = C \cdot x^2$.

Figure 4.6 represents Pacejka forces in radial and longitudinal directions. Because the curve that the vehicle is following is always in the same direction, these forces do not change the sign. The maximum value for x axis now is at the point with the minimum radius of rotation. In y axis we have the maximum value when the trajectory is almost straight. In this case the tires that are in the same axis have almost the same value for the deformation forces; and these forces at the points that are close to the minimum radius of rotation start to be different.

For our study is interesting to know that how the maximum values are varying. In the *table 4.2* we can see the different C coefficients and the forces at the tires.

C	**1**	**2**	**4**	**6**	**8**	**10**
F_f maximum [N]	**1.6765**	1.8325	2.1445	2.4565	2.7685	3.0805
F_f minimum [N]	**1.3646**	1.2086	0.8966	0.5846	0.2726	0.0394
Pacejka F_x [N]	**0.0403**	0.0419	0.0480	0.0571	0.0667	0.0738
Pacejka F_y [N]	**0.0196**	0.0196	0.0196	0.0196	0.0196	0.0196

C	**12**	**14**	**16**	**18**	**20**
Fr maximum [N]	3.3939	3.7062	4.0184	4.3306	4.6429
Fr minimum [N]	0.3528	0.6651	0.9773	1.2895	1.4760
Pacejka Fx [N]	0.0760	0.0760	0.0760	0.0760	0.0760
Pacejka Fy [N]	0.0057	0.0112	0.0171	0.0236	0.0305

Table 4.2:Forces in the tires for the curve $y = C \cdot x^2$.

4.3. Friction in MAX2D

4.3.1. Straight motion

1. PRACTICAL METHOD

The first step is to measure the main input current, battery's current and the current of the screen when the Robot is not moving. These values (*table 4.3*) will be used for the calculations of the energy used by the Robot.

DEVICE	CURRENT [A]
Screen	0.62
Input main current	1.09
Battery current	0.005

Table 4.3:Measures in standing Robot for straight motion with battery supply.

The program that is controlling the Robot can change the angular velocity of the wheels. *Formula 2.3* is used to find the linear velocity from this studied angular velocity. In *table 4.4* there are the values that were used in the program to find the friction forces in the tires.

R.P.M.	VELOCITY [M/S]
200	0.087267
400	0.174533
600	0.2618
800	0.349067
1000	0.436333
1200	0.5236
1400	0.610867
1600	0.698133
1800	0.7854
2000	0.872667
2500	1.090833
3000	1.309
3500	1.527167
4000	1.745333

Table 4.4:Calculation of the linear velocity.

In the first measurements, the battery was used during the running of the Robot; we believed that the best option was measuring the main current input and current of the battery. The final current used by the Robot can be acquired with *formula 4.2.* Input current was almost fixed for each velocity but there was no fixed value for the current of the battery. It is possible that these measurements are not correct. Therefore, we can not conclude the study with these measures. Nevertheless we continued using the calculations from the first test.

[4.2] $$I_{used}\left[A\right] = I_{total} - I_{screen} + I_{batery}$$

I_{used}: Used current by the robot [A].
I_{total}: Input current [A].
I_{screen}: Used current by the screen [A].
I_{batery}: Used current by the battery [A].

R.P.M.	TOTAL INPUT [A]	BATERY CURRENT [A]	USED CURRENT [A]
200	1.9	0.26	1.54
400	1.9	0.17	1.45
600	1.9	0.2	1.48
800	1.9	0.25	1.53
1000	1.9	0.32	1.6
1200	1.9	0.46	1.74
1400	1.9	0.55	1.83
1600	1.9	0.7	1.98
1800	1.9	0.84	2.12
2000	1.9	1.05	2.33
2500	1.9	1.5	2.78
3000	1.9	2.15	3.43
3500	1.9	3.5	4.78
4000	1.9	5	6.28

Table 4.5:Measured and obtained current for straight motion with battery supply.

Figure 4.7 represents the relationship between the used current for running the Robot and the linear velocity of the Robot.

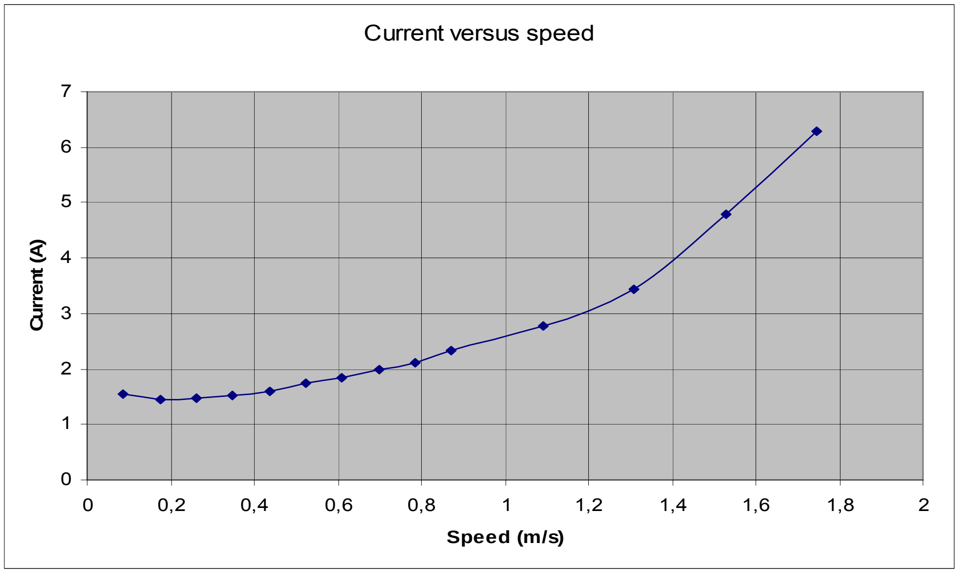

Figure 4.7:Current versus speed for straight motion with battery supply.

According to the *formula 2.14,* the current that the Robot uses for movement (on the straight line while the velocity is constant) should be linearly dependent on the speed. Is it evident that our test this is not accurate; in *figure 4.7* we have a curve that is a direct representation of the measurements we have completed. The reason for the inaccurate measurements is that the battery is charging or discharging; therefore in these moments the battery is using or giving the current. The battery was needed to support the high current needed for high speeds and to start of the Robot. Due to the problem mentioned above, we decided to remove the battery and repeat all the measurements; we used another power supply in parallel to the initial power supply that is necessary to give enough current to the Robot without using a battery.

SOLVING THE PROBLEM: THE BATERY REMOVAL

To measure the accurate current we need to remove the battery and measure the current of the screen and main input current of the Robot. We hope that the screen current is the same as the value that was previously measured, because this current is not variable. In *table 4.6* we can see the new values that we will use for our calculations:

DEVICE	CURRENT [A]
Screen	0.62
Main current	1.1

Table 4.6:First measures for standing Robot.

Obviously is not necessary to repeat the calculations for the linear velocity for each angular velocity, instead we can check all the values in *table 4.4.* In this new measurement (without battery), we don't have any device that is consuming power from the main power supply, hence we can obtain the current used for the Robot through *formula 4.3.* The results for these calculations are shown in *table 4.7.*

$$[4.3] \qquad I_{used}\left[A\right] = I_{total} - I_{screen}$$

I_{used}: Used current by the robot [A].
I_{total}: Input current [A].
I_{screen}: Used current by the screen [A].

R.P.M.	TOTAL INPUT CURRENT [A]	USED CURRENT [A]
200	1.6	0.98
400	1.7	1.08
600	1.8	1.18
800	2	1.38
1000	2.2	1.58
1200	2.3	1.68
1400	2.4	1.78
1600	2.6	1.98
1800	2.9	2.28
2000	3.2	2.58
2500	3.4	2.78
3000	3.6	2.98
3500	4.3	3.68
4000	5.1	4.48

Table 4.7:Measured and used current for straight motion.

In order to find out that whether these values are accurate, the current used should be represented, current versus speed, as we did in *figure 4.8.* We concluded that this dependence is almost linear, this means the values that were measured with the Amperemeter are accurate and we can complete the energy calculation.

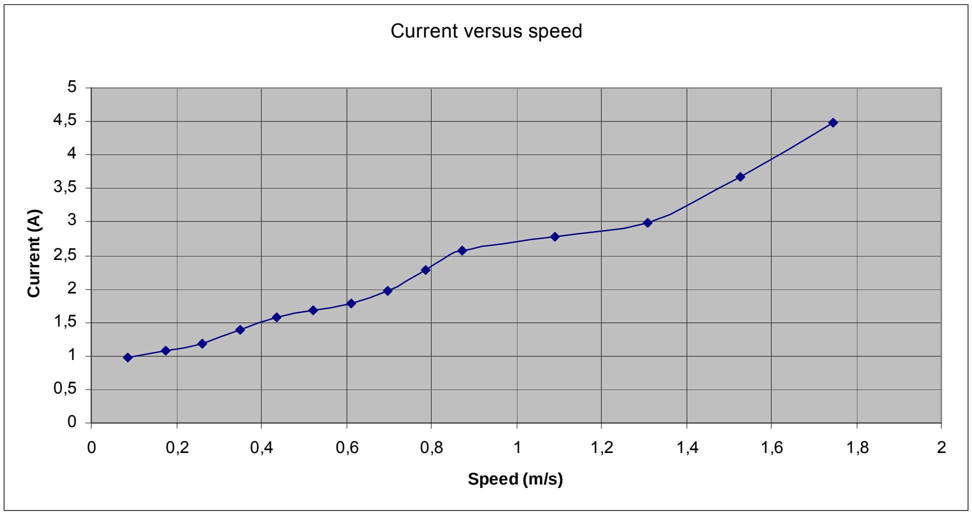

Figure 4.8:Current versus speed for straight motion.

We have a little disturbance around $v = 1 m/s$, although the slope of the function after and before this point is almost the same we still consider the values as trustable enough for following analysis.

After we collect all the currents of the Robot in different velocities we will obtain the energy of the Robot that it used in each velocity. This energy could be calculated from electrical power as it is shown in *formula 2.14*. The voltage in the Robot is always constant, 24 V, hence the used energy for each velocity is:

VELOCITY [M/S]	CURRENT [A]	ELECTRIC POWER [W]= [J/S]
0.087267	0.98	23.52
0.174533	1.08	25.92
0.2618	1.18	28.32
0.349067	1.38	33.12
0.436333	1.58	37.92
0.5236	1.68	40.32
0.610867	1.78	42.72
0.698133	1.98	47.52
0.7854	2.28	54.72
0.872667	2.58	61.92
1.090833	2.78	66.72
1.309	2.98	71.52
1.527167	3.68	88.32
1.745333	4.48	107.52

Table 4.8:Electrical energy calculation for straight motion.

2. THEORETICAL METHOD

In this theoretical method we will make a comparison between the energy that is given to the Robot by the main power supply and the energy that is wasted through the friction in the wheels. In straight motion Pacejka forces are zero; due to the forces that are depending on the mass distribution on the tires (which is fixed because there is no acceleration) and the drift angle (that is zero for straight motion). Kinetic energy needs to be obtained also.

Friction forces are constant for all the points of trajectory. For each velocity that we calculated in *Table 4.4*, the friction forces that the program ***friction_and_pacejka.m*** returns are:

VELOCITY [M/S]	FRICTION FORCE IN EACH TIRE (FROM MATLAB) [N]
0.087267	1.5206
0.174533	1.5206
0.2618	1.5206
0.349067	1.5206
0.436333	1.5206
0.5236	1.5206
0.610867	1.5206
0.698133	1.5206
0.7854	1.5206
0.872667	1.5206
1.090833	1.5206
1.309	1.5206
1.527167	1.5206
1.745333	1.5206

Table 4.9:Friction forces for the Robot for straight motion.

By viewing *formulas 2.14* and *2.15* it is possible to find the energy that is wasted due to tire friction we can also obtain the kinetic energy of the Robot. Observe that when the Robot is moving in a straight line, angular velocity around its gravity centre is zero, thus there is no rotation energy. In the *table 4.10* we can see the wasted friction energy on the Robot (this means that this value is the wasted energy on all the tires) and kinetic energy of the Robot for each velocity.

VELOCITY [M/S]	ENERGY WASTED THROUGH FRICTION [J/S]	KINETIC ENERGY [J]
0.087267	0.530791	0.11804
0.174533	1.061582	0.472159
0.2618	1.592372	1.062358
0.349067	2.123163	1.888637
0.436333	2.653954	2.950995
0.5236	3.184745	4.249433
0.610867	3.715535	5.78395
0.698133	4.246326	7.554547
0.7854	4.777117	9.561224
0.872667	5.307908	11.80398
1.090833	6.634885	18.44372
1.309	7.961862	26.55896
1.527167	9.288839	36.14969
1.745333	10.61582	47.21592

Table 4.10:Energy calculation for straight motion.

4.3.2. Turning motion

1. PRACTICAL METHOD

For second experiment, battery was removed from the beginning, we have correct measurements and it was not necessary to measure twice. Screen and main input currents were measured again when the Robot is standing. Both values should be the same and in *table 4.11* you can see the values taken, however input current is a little bit higher. These values will also be used in the calculations of the current used by the Robot.

DEVICE	VOLTAGE [A]
Computer	0.62
Main current	1.5

Table 4.11:Second measures for standing Robot.

For the linear velocities we can go back to the *table 4.11.* For the straight motion we could measure currents until to an angular velocity with 4000 r.p.m. but in this measurement we could measure only until 3500 r.p.m., Due to the sliding of the wheels of the Robot we can not move the Robot faster. The current that the Robot is using for its movement can be obtained through *formula 4.3.*

R.P.M.	INPUT CURRENT [A]	CURRENT [A]
200	1.7	1.08
400	1.8	1.18
600	2.05	1.43
800	2.2	1.58
1000	2.4	1.78
1200	2.6	1.98
1400	2.75	2.13
1600	2.9	2.28
1800	3.1	2.48
2000	3.3	2.68
2500	3.7	3.08
3000	4.3	3.68
3500	5	4.38

Table 4.12:Measured and used current for turning motion.

A representation of the current versus the linear speed of the Robot is necessary to have an idea about the credibility of the values.

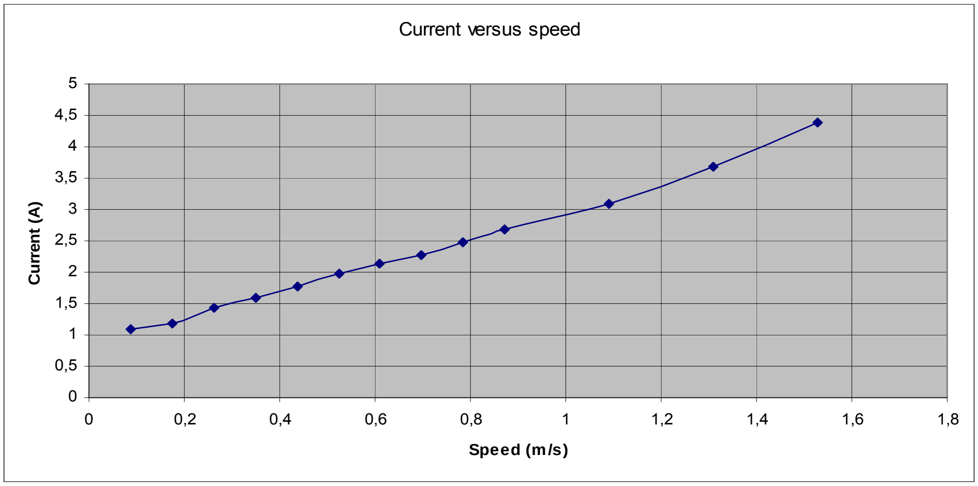

Figure 4.9:Current versus speed for turning motion.

In *figure 4.9* we can see how the main current of the Robot is increasing while the velocity is increasing and following with an almost perfect linear tendency. This demonstrates that the measurements are correct.

Used energies by the Robot for each velocity were finally obtained through the *formula 2.14* and they are collected in the following table:

VELOCITY [M/S]	CURRENT [A]	ELECTRIC POWER [W]=[J/S]
0.087267	1.08	25.92
0.174533	1.18	28.32
0.2618	1.43	34.32
0.349067	1.58	37.92
0.436333	1.78	42.72
0.5236	1.98	47.52
0.610867	2.13	51.12
0.698133	2.28	54.72
0.7854	2.48	59.52
0.872667	2.68	64.32
1.090833	3.08	73.92
1.309	3.68	88.32
1.527167	4.38	105.12

Table 4.13:Electrical energy calculation for turning motion.

2. THEORICAL METHOD

The Robot is turning around its gravity centre so mass distribution and friction forces will be the same on each wheel. On the other hand, Pacejka forces depend on the drift angle and the weight the tire is supporting. This drift angle is constant because the wheels are fixed and is the same for each wheel. Pacejka forces and friction forces do not depend on the velocity of this motion. The values for the forces that were given by the program are:

PACEJKA FORCE IN X AXIS (FROM MATLAB) [N]	PACEJKA FORCE IN Y AXIS (FROM MATLAB) [N]	FRICTION FORCE IN EACH TIRE (FROM MATLAB) [N]
0.039729	-0.01752	1.5206

Table 4.14:Friction and Pacejka forces at the Robot for turning motion.

By reapplying the *formula 2.14*, wasted energy on the tires will be acquired. In this case kinetic velocity is not needed because the Robot has no forward movement. However, this turning motion is associated with a rotation kinetic energy that is given by the *equation 2.15,* angular velocity is also needed and we can obtain it from the program ***pacejka_dunlop.m*** Pacejka forces are deformation forces, so we can associate them with deformation energy on the tires that can not be calculated.

Because we do not have the inertia matrix that is necessary in the energy calculation, we can not compare it with the other energies that were obtained. Nevertheless we can find this rotation energy by the *formula 4.5:*

[4.4]
$$E = \frac{1}{2}\begin{pmatrix} 0 & 0 & \omega \end{pmatrix}\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} = \frac{1}{2}\omega I_{zz}\omega$$

E: Rotation kinetic energy [J].

ω: Angular velocity of the turning motion from the gravity centre of the robot [rad/s].

I_{xx}, I_{yy}, I_{zz}: Intertia moment in rotations over the x, y and z axis.

I_{xy},I_{xz},I_{yz}: Inertia matrix elements.

Where I_{zz} is the inertia of the Robot and when the Robot is rotating directly over its gravity centre in z-direction. This inertia is constant and depends only on the geometry of the Robot, so we can use the formula:

[4.5]
$$E/I_{zz} = \frac{1}{2}\omega^2$$

VELOCITY [M/S]	ANGULAR VELOCITY [S-1]	ENERGY WASTED THROUGH FRICTION [J/S]	ROTATION ENERGY [J/IZZ]
0.087267	0.102412	1.5206	0.005244
0.174533	0.204824	1.5206	0.020976
0.2618	0.307236	1.5206	0.047197
0.349067	0.409647	1.5206	0.083905
0.436333	0.512059	1.5206	0.131102
0.5236	0.614471	1.5206	0.188787
0.610867	0.716883	1.5206	0.256961
0.698133	0.819295	1.5206	0.335622
0.7854	0.921707	1.5206	0.424771
0.872667	1.024118	1.5206	0.524409
1.090833	1.280148	1.5206	0.819389
1.309	1.536178	1.5206	1.179921
1.527167	1.792207	1.5206	1.606003

Table 4.15:Energy calculation for turning motion.

5. Discussion of the results

5.1. Functions y=C·x^3 and y=C·x^2

5.1.1. Friction forces

The *figure 5.1* shows the maximum values of the friction forces for each C coefficient. This maximum force is given by the wheels conforming to the *figures 4.2* and *4.5*.

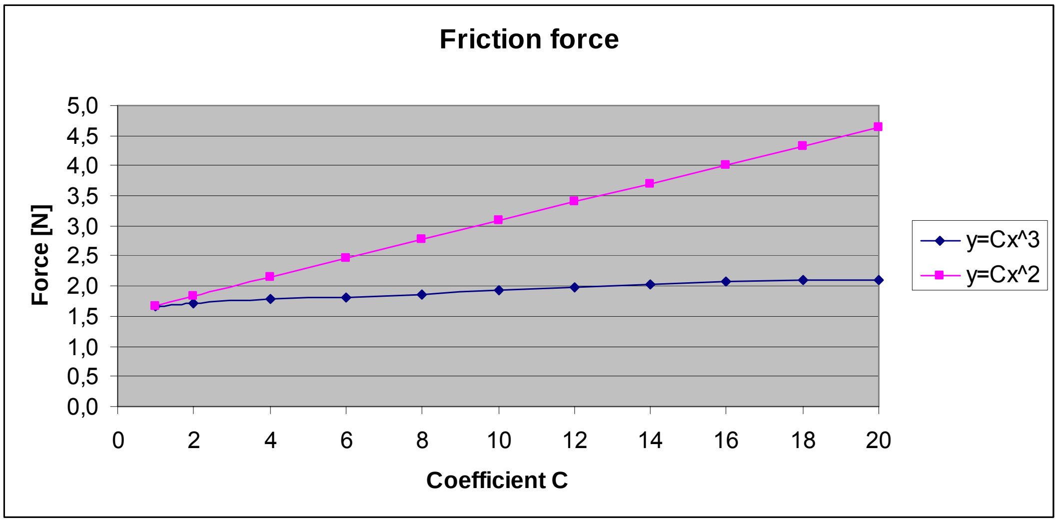

Figure 5.1:Friction force versus C coefficient.

In both cases maximum friction forces are following a linear function. For the obtained representation of the function $y = Cx^3$ the slope is lower because the trajectory the Robot is following has a softer curve.

Because friction forces depend on the trajectory that the Robot is following, we can obtain this friction force as a function of the C coefficient.

$$y = Cx^3$$
$$s = \frac{\Delta y}{\Delta x} = \frac{1.9887 - 1.7949}{12 - 4} = 0.024225$$
$$Ff_{\max} = 0.024225C + 1.698$$

$$y = Cx^2$$
$$s = \frac{\Delta y}{\Delta x} = \frac{3.3939 - 2.1445}{12 - 4} = 0.15617$$
$$Ff_{\max} = 0.15617C + 1.5198$$

(x, y): Point of the trajectory [m,m].
s: Slope of the curve.
F_{fmax}: Maximum friction force [N].

On the other hand, we can say that the radius of rotation where these maximum forces occur is also varying according to the coefficient C used in the function. With *equation 2.1*, radius of rotation can be found. If the point of maximum friction force and the coefficient C is known (for example, in function $y = Cx^2$ this point is always the coordinate origin) maximum friction forces can be easily acquired.

$$y = Cx^3 \qquad R = \left| \frac{\left(1 + \left(3Cx^2\right)^2\right)^{3/2}}{\left(3 \cdot 2 \cdot Cx\right)} \right|$$

$$y = Cx^2 \qquad R = \left| \frac{\left(1 + \left(2Cx\right)^2\right)^{3/2}}{\left(2C\right)} \right|$$

(x, y): Point of the trajectory [m,m].
R: Radius of rotation [m].

5.1.2. Pacejka forces

Figure 5.2 displays the maximum longitudinal deformation force at the tire. Notice that this maximum force is given in the extremes of the trajectory for the curve $y = Cx^3$ and near the critical point (0, 0) for the curve $y = Cx^2$.

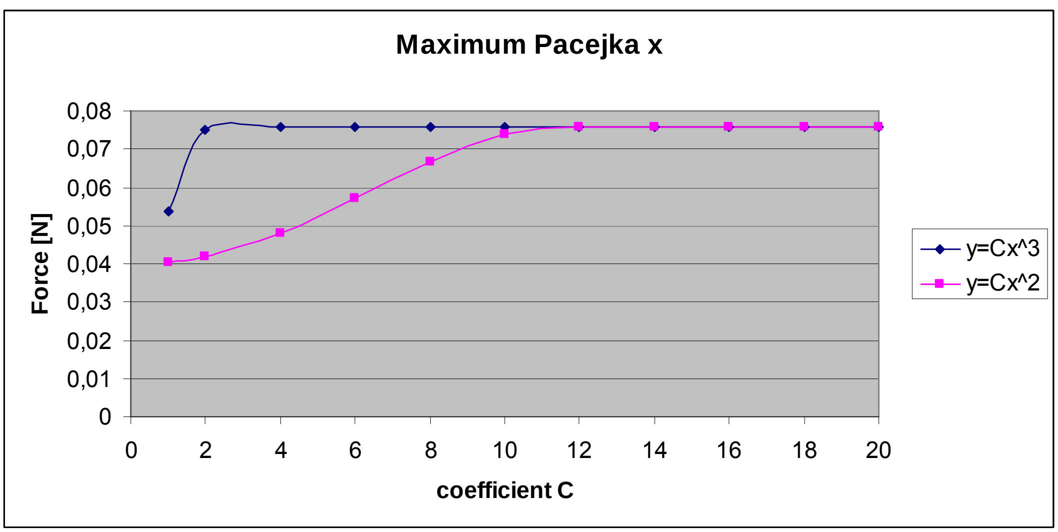

Figure 5.2:Pacejka longitudinal force versus C coefficient.

Forces in axis x for the functions that were studied are the same (F=0.0760 N) for the C values that are approximately 11 and above, this value is asymptotic. Nevertheless this value is growing faster for the function $y = Cx^3$ than $y = Cx^2$. In $y = Cx^2$ that we can see the curve is growing at a slower rate, so instantaneous radius of rotation will be higher in $y = Cx^3$ than in $y = Cx^2$ therefore longitudinal friction forces will be higher. When C is high enough, the curve will be "straight enough" for the maximum value for the longitudinal deformation of the tire to be reached. We can conclude that the force on the tire can not be higher than 0.0760 N and that this value is not depending on the trajectory the vehicle is following.

Finally, in *figure 5.3* maximum Pacejka force in radial direction is shown. We can see how there is a minimum value for the transversal deformation force but this minimum is not the same for both trajectories. We can also see that after reaching the minimum point both graphs grow as a linear function.

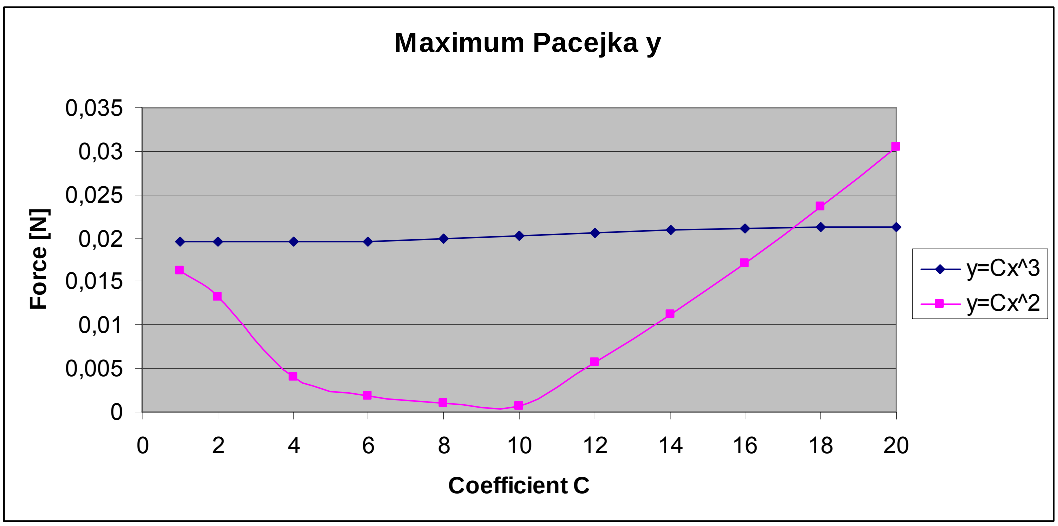

Figure 5.3:Pacejka transversal force versus C coefficient.

Remember that transversal Pacejka force occurs at point (0, 0) when the curve is $y = Cx^2$. Because the curve needs to be two times derivable (otherwise the *equation 2.1* can not be applied to obtain the radius of rotation) we completed an approximation in our program. This approximation was explained before in the point "*procedure: friction forces"* the results near to the critical point are not 100% accurate. In this case it only affects on the transversal deformation force when the trajectory is $y = Cx^2$.

Notice that when Pacejka forces in radial direction are minimum, Pacejka forces in longitudinal direction are at a maximum. Critical points below obtain the minimum radius of rotation:

$$y = 3x^3$$
$$y' = 9x^2$$
$$y'' = 18x$$

$$R = \left| \frac{\left(1 + \left(9x^2\right)^2\right)^{3/2}}{(18x)} \right|$$

$$R' = \frac{\frac{3}{2}\left(1 + \left(9x^2\right)^2\right)^{1/2} \cdot 2\left(9x^2\right) \cdot 18x \cdot (18x) - \left(1 + \left(9x^2\right)^2\right)^{3/2} \cdot 18}{(18x)^2} = 0$$

$$2 \cdot 3^5 x^4 - 1 - 9^2 x^4 = 0$$

$$x = \sqrt[4]{1/405} \quad y = 3\left(1/405\right)^{4/3}$$
$$R = 0.3276m$$

$$y = 10x^2$$
$$y' = 20x$$
$$y'' = 20$$

$$R = \left| \frac{\left(1 + (20x)^2\right)^{3/2}}{20} \right|$$

$$x = 0 \quad y = 0 \quad R = 0.05m$$

In the case $y = 3x^3$ radius is also extreme when (x, y) = (0, 0) (R'=0). In this case the curve that defines the trajectory is not derivable, so this value is not useful for the study. For the curve $y = 10x^2$ we do not need to calculate at which points the radius is maximum or minimum, because we know that it only occurs at point (0, 0). The radius where the Pacejka forces are critical (maximum values in longitudinal direction and minimum values in transversal direction) is higher in $y = 3x^3$ than in $y = 10x^2$.

5.2. Experimental results in MAX2D

5.2.1. Straight motion

In this case the results that are interesting for us are the energies that are given to the Robot, energy that is wasted through the wheels and kinetic energy of the Robot. In *figure 5.4* we can see how these energies are varying according to the velocity.

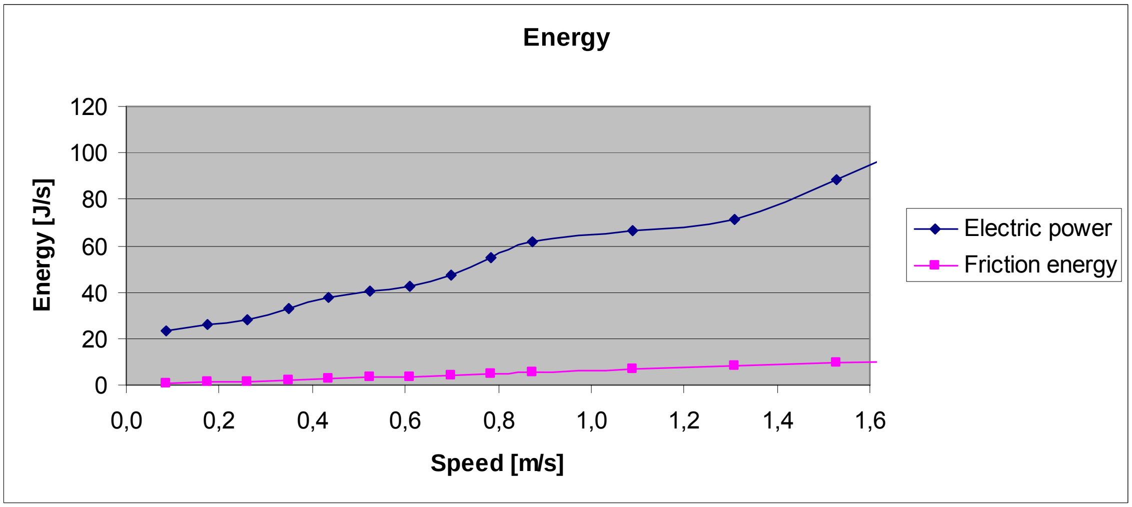

Figure 5.4:Energy versus speed in straight motion.

Observe that each movement can be divided into two simple movements: a translation or movement in a straight line and a rotation around its gravity centre. Translation movement is related to the kinetic energy (*formula 2.15*). This energy can be easily founded and represented because we have all the values of the formula. Rotation movement is related to the rotation energy that is defined through the *formula 2.16*; because we do not have inertial moment of the Robot this energy can not be found.

Friction energy follows a linear function according to the velocity. Kinetic energy, rotation energy and energy given by the main power supply depend on the velocity power two. It means that when the linear velocity of the Robot is increasing, the difference between the energy the Robot is using and the kinetic energy is increasing.

Through *figure 5.4* we can also see how the friction force is just a small percentage of the total energy. We can conclude that the main part of the energy is transformed into kinetic energy. The difference between electric power, friction and kinetic energy is shared between rotation energy and wasted energy by the electronic devices of the Robot.

5.2.2. Turning motion

The Robot is moving in turning motion around its gravity centre. This motion has not kinetic energy, because the Robot is not moving its gravity centre (there is no displacement). The energy that is used in this movement is called rotation kinetic energy, and due to not having the inertial moment of the Robot we can not acquire the values of kinetic rotation energy.

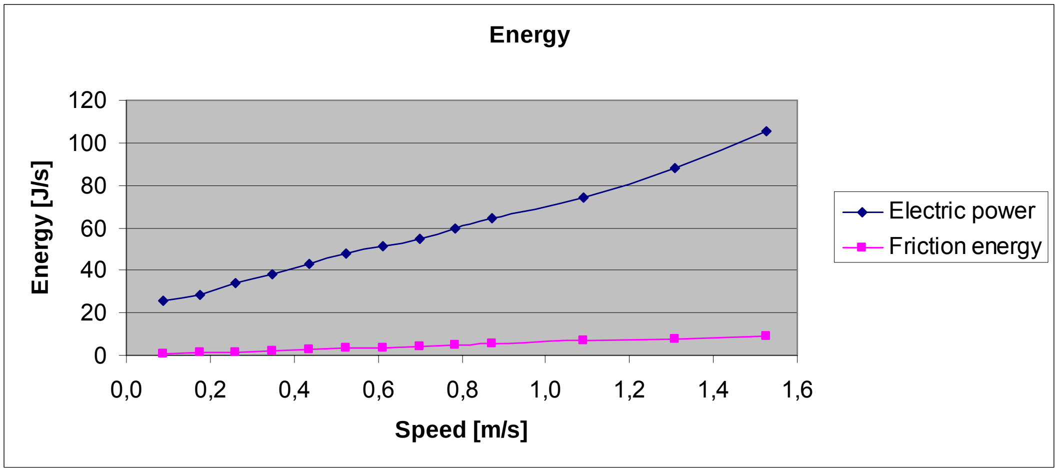

Figure 5.5:Energy versus speed in turning motion.

In *figure 5.5* energy given to the Robot and friction energy are represented. Friction forces in straight motion and in turning motion are exactly the same, so energy wasted through friction would be also the same. Rotation energy is following a quadratic function. In this case, electric power function is growing faster, so the difference between electric power and friction energy is greater. This difference is shared among rotation energy, wasted energy by the electronic devices and deformation energy of the tires. This deformation energy is not easily found; we will need the characteristics of the tire, material, pressure, temperature and etc.

Pacejka forces in turning motion are the same for each velocity. This means that the deformation in the tires is always the same so deformation energy is constant. If we suppose that the electronic devices energy does not depend on the Robot's movement, the rotation energy is much greater than the energy used in a straight motion.

5.2.3. Comparison between straight and turning motions.

The last step in this discussion of results will be the comparison between the input energy when the Robot is moving in a straight line and input energy when the Robot is turning over its gravity centre. *Figure 5.6* represents clearly the variation between the two energies.

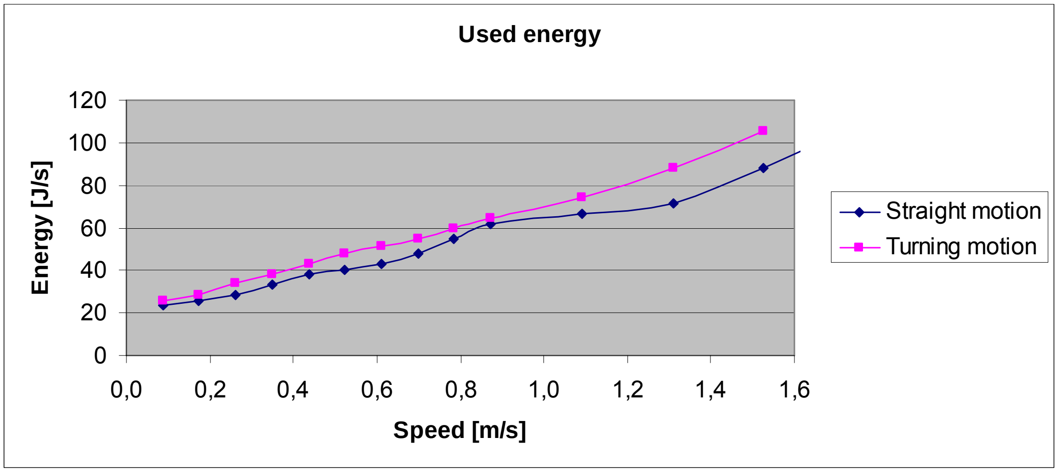

Figure 5.6:Used energy versus speed in straight and turning motion.

In straight motion there is kinetic energy, friction energy and wasted energy by the electronic devices. In turning motion we have turning energy, deformation energies on the tires and wasted energy through friction forces. These friction forces are the same in both motions. Turning motion energy is always greater than straight motion, nevertheless when the velocity is increasing the functions have a greater different. This means that turning energy is growing at a faster rate with velocity than kinetic energy.

6. Conclusion

The aim of this project is to study the friction and deformation forces in the tires of the robot MAX2D. We need to find these forces in the tires to discover how much energy is wasted during the movement of the Robot due to the weight and the accelerations that the tires should support.

To discover these forces in different points of trajectory we have had to simulate the movement of the Robot with the Matlab Program. This program shown us how friction forces and deformation forces in the tires vary along the trajectory of the Robot.

In this program we need to give the positioning and movement of the Robot, velocity, mass distribution in the wheels, deformation forces and friction forces in the tires.

To reach to this point we need the parameters of the shape of the Robot, the trajectory that the Robot is following and the Pacejka coefficients of the tires.

To get the shape parameters of the Robot we simply measured the dimensions of the Robot. The trajectory depends on the path the Robot follows. Pacejka coefficients are the characteristics of the tire according to the material, type of the wheels, pressure and temperature which varies between different tire manufactures.

We used the program to study two different movements; $y=x^2$ and $y=x^3$.

At last we have to define the energy that is given by power supply to run the Robot and the mechanical energy that is wasted in the tires. We will use these two energy values to get the optimum acceleration and path the Robot has to follow to save as much energy is possible.

According to the study of the two different movements mentioned above *figure 5.2* demonstrates the two different curves that show us the behaviour of the maximum longitudinal deformation of the tires. From the results of this test we can see that the deformation forces in the tires can not be higher than F=0.0760 N. This means that the tire can not become more deformed from acceleration after this rate. As we can see in *figure 5.2* the minimum value in transversal deformation of the tire has the maximum value in longitudinal deformation.

As we stated above we have electrical energy that is given to run the Robot, mechanical energy that is wasted through friction and deformation in the tires and also kinetic energy that is due to the velocity of the Robot.

In *figure 5.4* and *figure 5.5* we can see how the given energy and wasted energies are varying according to the different velocities. The higher the speed there is more energy needed to run the Robot and more energy will be wasted on the tires.

We can run the Robot in two different movements, straight and turning motion.

Figure 5.4 shows the energies of the Robot in straight motion. As the speed is constant in a straight movement there will not be any acceleration and deformation in the tires. Therefore Robot needs less energy in this movement.

But as we can see in *figure 5.5* as the speed is constant when the Robot has the turning movement, we will have centrifugal acceleration and deformation in the tires. Therefore Robot will need more energy for its movements because more energy will be wasted on the tires.

If we compare the two given energy curves in straight movement and turning movement of the Robot (*figure 5.6*) we can see that the Robot needs more energy to run in straight motion than in a turning motion. As mentioned above more energy would be wasted in the tires because of the acceleration and deformation in turning motion therefore we can see more energy is used.

7. Next steps

Due to the rapid progress of today's technologies it is possible that in the near future developments will be made that will allow us to upgrade and complete these kinds of appliances. It is for this reason that I have prepared some suggestions that could be completed in the next steps of the project.

In this project it was not necessary to consider different weights and accelerations of the Robot because the goal of the project was to obtain the friction forces, deformation of the tire and different energies. In fact all our calculations in this project are for the fixed parameters of the weight and acceleration of the Robot.

For the next step we want to see the behavior of the Robot by changing the load on the Robot and measuring the main current that the Robot uses with the same accelerations that we tried in the last assignment.

In this new step we will change the program and apply different accelerations to see the effects of different loads on the energy and mechanical parameters of the tires of the Robot. According to the different loads that we will try on the Robot and comparing them with the last tests, it could be possible that we see a change in the currents, friction and deformation forces on the tires of the Robot.

The goal of this step can be achieved by observing the behavior of the Robot in different conditions and defining satisfactory parameters of the Robot; managing energy, economy and safety.

8. Bibliography and references

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9. List of figures

10. List of tables

11. Appendix

11.1. Matlab Program: friction_and_pacejka.m

```
% This script give us friction forces in the tires, Pacejka forces in the
% tires and slip angle in each point of a trajectory.

% Size of the Robot
veh_h=.69;
veh_b=.5;
wheel_h=.1;
% Mass
m=31;
% Height of the GC
veh_gc=0.1259;

% Module of the linear velocity of the wheels
modv=1;

% Friction rolling coefficient
fr=0.02;

g=9.81;

% Trajectory of the gravity centre of the Robot
syms x;
y=12*x^2;

% First calculations with the trajectory. We need to consider that
% sometimes the first and the second derivative could be constant or cero,
% so we create the loops below to solve this problem.
[dy, d2y]=matfuc1(y);
x=-5:0.1:5;
yn=eval(y);
trajectory=complex(x,yn);
dyn=eval(dy);
d2yn=eval(d2y);

if (length(dyn)<length(trajectory))

    aux=dyn;
    clear dyn

    for (k=1:length(trajectory))
        dyn(k)=aux;
    end

end

clear k

if (length(d2yn)<length(trajectory))
```

```
    aux=d2yn;
    clear d2yn

    for (k=1:length(trajectory))
        d2yn(k)=aux;
    end

end

s=sign(d2yn);
s(s==0)=1;
index=find(abs(d2yn)<10^(-6));
d2yn(index)=s(index)*10^(-6);

% Radius and center of rotation
radius=abs(((1 + dyn.^2) .^(3/2) ) ./ d2yn);
center=complex(  x-(dyn.*(1+(dyn.^2))./(d2yn))  ,  yn+(1+(dyn.^2))./(d2yn)
);

for (i=1:length(trajectory))

     j=sqrt(-1);

     % Because the position of the wheels needs to be known, we need the
     % some geometrical calculations
     [ang_rad(i), ang_veh(i)]=ang_xy(trajectory(i),center(i));
     VEC_radius(i)=createvector_la(radius(i),ang_rad(i));
     VEC_vehicle(i)=createvector_la(veh_h/2,ang_veh(i));

VEC_veh_a(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+atan(
veh_b/veh_h));
     VEC_veh_b(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)-
atan(veh_b/veh_h));

VEC_veh_c(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+pi+at
an(veh_b/veh_h));

VEC_veh_d(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+pi-
atan(veh_b/veh_h));
     VEC_rad_wheel1(i)=VEC_radius(i)+VEC_veh_a(i);
     VEC_rad_wheel2(i)=VEC_radius(i)+VEC_veh_b(i);
     VEC_rad_wheel3(i)=VEC_radius(i)+VEC_veh_c(i);
     VEC_rad_wheel4(i)=VEC_radius(i)+VEC_veh_d(i);
     ang_wheel1(i)=angle(VEC_radius(i))-angle(VEC_rad_wheel1(i));
     ang_wheel2(i)=angle(VEC_radius(i))-angle(VEC_rad_wheel2(i));
     ang_wheel3(i)=angle(VEC_rad_wheel3(i))-angle(VEC_radius(i));
     ang_wheel4(i)=angle(VEC_rad_wheel4(i))-angle(VEC_radius(i));
     VEC_wheel1(i)=createvector_la(wheel_h,angle(VEC_vehicle(i))-
ang_wheel1(i));
     VEC_wheel2(i)=createvector_la(wheel_h,angle(VEC_vehicle(i))-
ang_wheel2(i));

VEC_wheel3(i)=createvector_la(wheel_h,angle(VEC_vehicle(i))+ang_wheel3(i));

VEC_wheel4(i)=createvector_la(wheel_h,angle(VEC_vehicle(i))+ang_wheel4(i));

     % Linear velocity of the gravity center (vo) and angular velocity of
the
```

```
    % gravity center (omega)
    vo1(i)=modv*(VEC_vehicle(i)./(abs(VEC_vehicle(i))));
    omega(i)=crossvecinv(vo1(i), VEC_radius(i), s(i));

    % Linear velocity of the wheels and module of this linear velocity.
    vel_a(i)=vo1(i)+crossvec(VEC_veh_a(i),omega(i));
    vel_b(i)=vo1(i)+crossvec(VEC_veh_b(i),omega(i));
    vel_c(i)=vo1(i)+crossvec(VEC_veh_c(i),omega(i));
    vel_d(i)=vo1(i)+crossvec(VEC_veh_d(i),omega(i));
    mod_a(i)=abs(vel_a(i));
    mod_b(i)=abs(vel_b(i));
    mod_c(i)=abs(vel_c(i));
    mod_d(i)=abs(vel_d(i));

    % Angular velocity of the wheels
    w_a(i)=crossvecinv(vel_a(i), VEC_rad_wheel1(i), s(i));
    w_b(i)=crossvecinv(vel_b(i), VEC_rad_wheel2(i), s(i));
    w_c(i)=crossvecinv(vel_c(i), VEC_rad_wheel3(i), s(i));
    w_d(i)=crossvecinv(vel_d(i), VEC_rad_wheel4(i), s(i));

    % Direction of the movement
    unit_a(i)=VEC_wheel1(i)./(abs(VEC_wheel1(i)));
    unit_b(i)=VEC_wheel2(i)./(abs(VEC_wheel2(i)));
    unit_c(i)=VEC_wheel3(i)./(abs(VEC_wheel3(i)));
    unit_d(i)=VEC_wheel4(i)./(abs(VEC_wheel4(i)));

    if (s(i)>0)
        unit_a(i)=-real(unit_a(i))+imag(unit_a(i))*j;
        unit_b(i)=-real(unit_b(i))+imag(unit_b(i))*j;
        unit_c(i)=-real(unit_c(i))+imag(unit_c(i))*j;
        unit_d(i)=-real(unit_d(i))+imag(unit_d(i))*j;
    end

    % Centrifugal force
    modFc(i)=m*((abs(vo1(i))^2)/(abs(VEC_radius(i))));
    Fc(i)=-(VEC_radius(i)./(abs(VEC_radius(i))))*modFc(i);

    % Calculation of the mass in the interior and exterior wheels
    m_ad(i)=(((m*g*veh_b/2)-(modFc(i)*veh_gc*s(i)))/(veh_b*g));
    m_bc(i)=m-m_ad(i);

    % Friction forces
    Fr_a(i)=-fr*(m_ad(i)/2)*g*unit_a(i);
    Fr_b(i)=-fr*(m_bc(i)/2)*g*unit_b(i);
    Fr_c(i)=-fr*(m_bc(i)/2)*g*unit_c(i);
    Fr_d(i)=-fr*(m_ad(i)/2)*g*unit_d(i);

    % Slip angle in each wheel in grades
    anga(i)=(angle(vel_a(i))-angle(VEC_vehicle(i)))*(180/pi);
    angb(i)=(angle(vel_b(i))-angle(VEC_vehicle(i)))*(180/pi);
    angc(i)=(angle(vel_c(i))-angle(VEC_vehicle(i)))*(180/pi);
    angd(i)=(angle(vel_d(i))-angle(VEC_vehicle(i)))*(180/pi);

    % Forces because of the tire deformation. These forces are FROM THE
    % GRAVITY CENTER
    [Fp_ax(i), Fp_ay(i), Mya(i)]=pacejka_dunlop((m_ad(i)/2)*g, anga(i),
0);
```

```
    [Fp_bx(i), Fp_by(i), Myb(i)]=pacejka_dunlop((m_bc(i)/2)*g, angb(i),
0);
    [Fp_cx(i), Fp_cy(i), Myc(i)]=pacejka_dunlop((m_bc(i)/2)*g, angc(i),
0);
    [Fp_dx(i), Fp_dy(i), Myd(i)]=pacejka_dunlop((m_ad(i)/2)*g, angd(i),
0);

    % Forces because of the tire deformation FROM THE STATIC COORDINATE
    % SYSTEM
    beta=angle(VEC_vehicle(i))*s(i);
    Mtrans=[cos(beta) -sin(beta); sin(beta) cos(beta)];
    F_pa(:,i)=[Fp_ax(i) Fp_ay(i)]*(Mtrans);
    F_pb(:,i)=[Fp_bx(i) Fp_by(i)]*(Mtrans);
    F_pc(:,i)=[Fp_cx(i) Fp_cy(i)]*(Mtrans);
    F_pd(:,i)=[Fp_dx(i) Fp_dy(i)]*(Mtrans);

    % TOTAL forces in tires
    F_a(i)=F_pa(1,i)+F_pa(2,i)*j-Fr_a(i);
    F_b(i)=F_pb(1,i)+F_pb(2,i)*j-Fr_b(i);
    F_c(i)=F_pc(1,i)+F_pc(2,i)*j-Fr_c(i);
    F_d(i)=F_pd(1,i)+F_pd(2,i)*j-Fr_d(i);

    % TOTAL forces in tires (direccion del movimiento en abs)
    F_ax(i)=Fp_ax(i)-abs(Fr_a(i));
    F_bx(i)=Fp_bx(i)-abs(Fr_b(i));
    F_cx(i)=Fp_cx(i)-abs(Fr_c(i));
    F_dx(i)=Fp_dx(i)-abs(Fr_d(i));

end

% Representation of the forces in the tires
subplot(2, 1, 1)
plot(x,abs(F_a),'b')
hold on
plot(x,abs(F_b),'r')
hold on
plot(x,abs(F_c),'k')
hold on
plot(x,abs(F_d),'g')
title 'Total forces in tires (-)'

subplot(2, 1, 2)
plot(x,abs(Fr_a),'b','LineWidth', 2)
hold on
plot(x,abs(Fr_b),'r','LineWidth', 2)
hold on
plot(x,abs(Fr_c),'k')
hold on
plot(x,abs(Fr_d),'g')
title 'Friction forces (-)'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'

% Representation of the Pacejka forces
figure(2)
subplot(2, 1, 1)
plot (x,(Fp_ay),'b','LineWidth', 2)
hold on
plot (x,(Fp_by),'r','LineWidth', 2)
```

```
hold on
plot (x,(Fp_cy),'k')
hold on
plot (x,(Fp_dy),'g')
title 'Pacejka transversal force'

subplot(2, 1, 2)
plot (x,(Fp_ax),'b','LineWidth', 2)
hold on
plot (x,(Fp_bx),'r','LineWidth', 2)
hold on
plot (x,(Fp_cx),'k')
hold on
plot (x,(Fp_dx),'g')
title 'Pacejka longitudinal force'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'

% Representation of the slip angle
figure(3)
plot(x, anga,'b','LineWidth', 2)
hold on
plot(x, angb,'r','LineWidth', 2)
hold on
plot(x, angc,'k')
hold on
plot(x, angd,'g')
title 'Slip angle'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'
```

11.2. Matlab Program: rotation.m

```
% This script give us friction forces in the tires, Pacejka forces in the
% tires and slip angle in each point of a turning motion.

% Size of the Robot
veh_h=.69;
veh_b=.5;
wheel_h=.1;
% Mass
m=31;
% Height of the GC
veh_gc=0.1258;

% Module of the linear velocity of the wheels
modv=0.174533333;

% Friction rolling coefficient
fr=0.02;

g=9.81;

% Trajectory of the wheels. Turning motion
syms x;
y=sqrt((sqrt((veh_h/2)^2+(veh_b/2)^2))-(x^2));
```

```
x=-
(sqrt((veh_h/2)^2+(veh_b/2)^2)):(((sqrt((veh_h/2)^2)+(veh_b/2)^2))/10):(sqr
t((veh_h/2)^2+(veh_b/2)^2));
yn=eval(y);
trajectory=complex(x,yn);

% Slip angle in each wheel. Is the same angle all the wheels
j=sqrt(-1);
anga=90-(angle((x(2)-x(1))+yn(2)*j)*(180/pi));
angb=90-(angle((x(2)-x(1))+yn(2)*j)*(180/pi));
angc=90-(angle((x(2)-x(1))+yn(2)*j)*(180/pi));
angd=90-(angle((x(2)-x(1))+yn(2)*j)*(180/pi));

for (i=1:length(trajectory))

     j=sqrt(-1);

     % Because the position of the wheels needs to be known, we need the
     % some geometrical calculations
     ang_veh(i)=atan(veh_b/veh_h)-atan(yn(i)/x(i));

VEC_veh_a(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+atan(
veh_b/veh_h));
     VEC_veh_b(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)-
atan(veh_b/veh_h));

VEC_veh_c(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+pi+at
an(veh_b/veh_h));

VEC_veh_d(i)=createvector_la(sqrt((veh_h/2)^2+(veh_b/2)^2),ang_veh(i)+pi-
atan(veh_b/veh_h));

     VEC_wheel1(i)=createvector_la(wheel_h,ang_veh(i)+angle(VEC_veh_a(i)));
     VEC_wheel2(i)=createvector_la(wheel_h,ang_veh(i)+angle(VEC_veh_b(i)));
     VEC_wheel3(i)=createvector_la(wheel_h,ang_veh(i)+angle(VEC_veh_c(i)));
     VEC_wheel4(i)=createvector_la(wheel_h,ang_veh(i)+angle(VEC_veh_d(i)));

     % Linear velocity of the wheels
     vel_a(i)=createvector_la(modv, ang_veh(i)+angle(VEC_veh_a(i)));
     vel_b(i)=createvector_la(modv, ang_veh(i)+angle(VEC_veh_b(i)));
     vel_c(i)=createvector_la(modv, ang_veh(i)+angle(VEC_veh_c(i)));
     vel_d(i)=createvector_la(modv, ang_veh(i)+angle(VEC_veh_d(i)));

     % Direction of the movement
     unit_a(i)=VEC_wheel1(i)./(abs(VEC_wheel1(i)));
     unit_b(i)=VEC_wheel2(i)./(abs(VEC_wheel2(i)));
     unit_c(i)=VEC_wheel3(i)./(abs(VEC_wheel3(i)));
     unit_d(i)=VEC_wheel4(i)./(abs(VEC_wheel4(i)));

     % Friction forces
     Fr_a(i)=-fr*(m/4)*g*unit_a(i);
     Fr_b(i)=-fr*(m/4)*g*unit_b(i);
     Fr_c(i)=-fr*(m/4)*g*unit_c(i);
     Fr_d(i)=-fr*(m/4)*g*unit_d(i);

     % Pacejka forces. These forces are FROM THE GRAVITY CENTER
     [Fp_ax(i), Fp_ay(i), Mya(i)]=pacejka_dunlop((m/4)*g, anga, 0);
```

```
    [Fp_bx(i), Fp_by(i), Myb(i)]=pacejka_dunlop((m/4)*g, angb, 0);
    [Fp_cx(i), Fp_cy(i), Myc(i)]=pacejka_dunlop((m/4)*g, angc, 0);
    [Fp_dx(i), Fp_dy(i), Myd(i)]=pacejka_dunlop((m/4)*g, angd, 0);

end

% Representation of the forces in the tires
plot(x,abs(Fr_a),'b','LineWidth', 2)
hold on
plot(x,abs(Fr_b),'r','LineWidth', 2)
hold on
plot(x,abs(Fr_c),'k')
hold on
plot(x,abs(Fr_d),'g')
title 'Friction forces (-)'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'

% Representation of the Pacejka forces
figure(2)
subplot(2, 1, 1)
plot (x,(Fp_ay),'b','LineWidth', 2)
hold on
plot (x,(Fp_by),'r','LineWidth', 2)
hold on
plot (x,(Fp_cy),'k')
hold on
plot (x,(Fp_dy),'g')
title 'Pacejka transversal force'

subplot(2, 1, 2)
plot (x,(Fp_ax),'b','LineWidth', 2)
hold on
plot (x,(Fp_bx),'r','LineWidth', 2)
hold on
plot (x,(Fp_cx),'k')
hold on
plot (x,(Fp_dx),'g')
title 'Pacejka longitudinal force'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'

% Representation of the slip angle
figure(3)
plot(x, anga,'*b','LineWidth', 2)
hold on
plot(x, angb,'*r','LineWidth', 2)
hold on
plot(x, angc,'*k')
hold on
plot(x, angd,'*g')
title 'Slip angle'
legend 'wheel a' 'wheel b' 'wheel c' 'wheel d'
```

11.3. Matlab Program: pacejka_dunlop.m

```
function [Fx0, Fy0, My]=pacejka_dunlop(Fz, alpha, gamma)
% Give us the longitudinal, transversal and aligning moment in a tire
% applying Pacejka's formulas.

% Parameters:
% alpha --> slip angle
% gamma --> camber angle
% Fz --> Vertical load at the tire
% s --> percent slip

% Vertical load. Should be in kN
Fz=Fz/1000;
% Radius of the tire
r=0.1;

% Angles should be in degrees.
alphamax=0.18014;
s=(alpha*100)/alphamax;

% Tire MF-TYRE 5.0 (Dunlop) coefficients.
a0=1.30;
a1=-1.0815;
a2=0.34037;
a3=-11.799;
a4=2;
a5=(1/1.0925);
a6=0;
a7=-2.934;
a8=0;
a9=0;
a10=0;
a11=0;
a12=0;
b0=1.65;
b1=0;
b2=1;
b3=0;
b4=15;
b5=0.2;
b6=0;
b7=0;
b8=0;
b9=0;
b10=0;
c0=2.2812;
c1=0.042392;
c2=0.24857;
c3=2.0413;
c4=1.5386;
c5=-7.3244;
c6=0.010808;
c7=-0.0029542;
c8=1.3854;
c9=0;
```

```
c10=0;
c11=0;
c12=0.0021268;


%% LONGITUDINAL FORCES
% Svx y Shx avoid values Fx=0 when s=0. Usually, Sv and Sh are zero.

Svx=0;

Cx=b0;

nupx=(b1*Fz)+b2;
Dx=nupx*Fz;

BCDx=((b3*(Fz^2))+(b4*Fz))*(exp(-b5*Fz));

Bx=BCDx/(Cx*Dx);

Ex=(b6*(Fz^2))+(b7*Fz)+b8;

Shx=(b9*Fz)+b10;

% Magic formula for longitudinal force
Fx0=Dx*sin(Cx*atan(Bx*(1-Ex)*(s+Shx)+(Ex*atan(Bx*(s+Shx)))))+Svx;


%% TRANSVERSAL FORCES

Cy=a0;

nupy=(a1*Fz)+a2;
Dy=nupy*Fz;

BCDy=a3*sin(a4*atan(a5*Fz));

By=BCDy/(Cy*Dy);

Ey=(a6*(Fz^2))+(a7*Fz)+a8;

Shy=a9*gamma;

Svy=((a10*(Fz^2))+(a11*Fz))*gamma;

ABy=-a12*abs(gamma)*By;

By=By+ABy;

% Magic formula for transversal force
Fy0=Dy*sin(Cy*atan(By*(1-Ey)*(alpha+Shy)+(Ey*atan(By*(alpha+Shy)))))+Svy;


%% ALIGNING MOMENT
% Is the moment exerted by the ground on the tire.

Cm=c0;

Dm=c1*(Fz^2)+(c2*Fz)*(r/Fz);
```

```
Bm=c5+(c4*Fz)+(c3*(Fz^2));

Em=((c6*(Fz^2))+(c7*Fz)+c8)*(1+c12);

Shm=(c9*gamma);

Svm=((c10*(Fz^2))+(c11*Fz))*gamma;

% Magic formula for aligning moment
My=(-Dm*(cos(Cm*(atan((Bm*(alpha+Shm))-(Em*((Bm*(alpha+Shm))-
atan(Bm*(alpha+Shm))))))))*cos(alpha)+Svm)*Fy0;
```

11.4. Matlab Program: matfuc1.m

```
% computing of matematicaly described function

function [dy, d2y]=matfuc1(fun)

dy=diff(fun);
d2y=diff(dy);
end
```

11.5. Matlab Program: ang_xy.m

```
function [ang1, ang2] = ang_xy(a,b)
% angle between vectors that are given by their imaginary numbers

syms zmx;
if((real(b)-real(a))~=0)
pr=((imag(b)-imag(a))/(real(b)-real(a)))*(zmx-real(a))+imag(a);
    if(real(a)<real(b))
        ang1=pi+atan(eval(diff(pr)));
    else
        ang1=atan(eval(diff(pr)));
    end
    if(imag(a)<imag(b))
        ang2=ang1+pi/2;
    else
        ang2=ang1-pi/2;
    end
else
ang1=pi/2;
ang2=0;
end
end
```

11.6. Matlab Program: createvector_la.m

```
function v = createvector_la(l, a)
% from [length, angle]  to complex

% This function create an imaginary number from the length of its vector
% and its angle

v = l * exp(i * a);
```

11.7. Matlab Program: crossvecinv.m

```
function escinv = crossvecinv(v1, v2, s)
% Give us the cross product between a complex number and the module of the
% second one that is in z-axis (mod(w))

modv1=sqrt((real(v1)^2)+(imag(v1)^2));
modv2=sqrt((real(v2)^2)+(imag(v2)^2));
if (s>0)
    escinv=modv1/modv2;
else
    escinv=-modv1/modv2;
end
end
```

11.8. Matlab Program: crossvec.m

```
function esc = crossvec(v1, w)
% Give us the cross product between a complex number and the module of the
% second one that is in z-axis (mod(w))

v1=[real(v1),imag(v1),0];
v2=[0,0,w];
pr=cross(v2,v1);
esc=pr(1)+pr(2)*i;
end
```