This document is published in:

Proc. IEEE International Conference on Communications (ICC) (2014) pp. 5735 - 5740

DOI: 10.1109/ICC.2014.6884236

On the Choice of Blind Interference Alignment Strategy for Cellular Systems with Data Sharing

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***Abstract*—A cooperative blind interference alignment (BIA) strategy is considered for the downlink of cellular systems. The aim is to reduce intercell interference in order to protect users, especially at the cell edge. The strategy consists of appropriately splitting the available bandwidth and is shown to be well-suited to scenarios where the number of cell-edge users is considerable. For a system comprising two cells each with a base station of N_t antennas, it is shown that, compared to a previous augmented code approach where transmission to all users occurs in the same frequency band, the proposed strategy leads to better rates over a wide range of signal-to-noise ratios when the number of cell-edge users in both cells exceeds $2N_t - 1$.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems have emerged as a means to achieve high-capacity communication. Recently, there has been growing interest in increasing the Degrees of Freedom (DoF) of systems by exploiting the properties of interference rather than avoiding it. Coordinating transmission using knowledge of the Channel State Information at the Transmitter (CSIT) has led to techniques such as Linear Zero Forcing Beamforming (LZFB) and Interference Alignment (IA). In some scenarios, such techniques can achieve the maximum multiplexing gain. However, in order to employ them in a cellular network, high-capacity backhaul links between base stations (BSs) are typically required. Moreover, even at the cell level, accurate and instantaneous feedback between users and BSs is necessary [1]. This consumes a large amount of resources; consequently, attaining optimal multiplexing gains is challenging in a cellular system [2].

Recently, Blind Interference Alignment (BIA) was proposed as a means of achieving a growth in DoF without the need for CSIT [3]. A typical BIA scheme can employ reconfigurable antennas that can switch their radiation pattern among a fixed number of preset modes [4]. In [3] it is demonstrated that BIA achieves $\frac{N_t K_T}{N_t+K_T-1}$ DoF in the MISO downlink, with N_t transmit antennas and K_T active single-antenna users, which are the maximum DoF achievable in the absence of CSIT. However, if the standard BIA scheme of [3] is applied directly to each BS in a cellular scenario, it mitigates intracell but not intercell interference. The performance of BIA in cellular systems is analyzed in [5] for different code structures. With the aim to handle intercell interference, a Frequency Reuse (FR) and a cluster-based scheme are proposed in [6]. In [7] a cooperative BIA scheme is devised to mitigate the interference at the cell edge in a two base station scenario. Assuming that the data sent to cell-edge users are shared between the BSs, it is possible to formulate an augmented BIA code that achieves proper alignment of all interference to which the cell-edge users are subject. Hence, intercell interference only affects users away from the cell edge, which are characterized by high signal-to-interference ratio. Compared to the schemes in [6], this approach eliminates intercell interference for cell-edge users at the cost of more symbol extensions. Moreover, as will be explained in more detail in Section III, although cell-edge users benefit from a diversity gain, the method does not attain the maximum DoF available from the cooperation of the BSs.

In this paper we present a cooperative BIA scheme for cellular scenarios that is based on flexible bandwidth (FBW). Part of the bandwidth is allocated to transmission to users near the BSs (cell-center users) that is carried out independently by each BS. In the remaining bandwidth, BSs cooperate to send data to cell-edge users, exploiting all the transmit antennas of the network. It is demonstrated that the achievable rate of BIA based on FBW exceeds the rate of augmented code in a wide range of SNR $\in (\mathsf{SNR}_{min}, \infty)$. Moreover, the required coherence time is reduced when applying FBW.

The remainder of this paper is structured as follows. In Section II the problem formulation is stated. Section III is devoted to a brief review of BIA schemes that have been applied to cellular environments. In Section IV, the proposed flexible bandwidth allocation scheme is presented, its performance is analyzed and is compared with the scheme of [7]. The performance of the scheme is evaluated in Section V. Finally, concluding remarks are given in Section VI.

II. Problem Formulation

A MU-MISO cellular scenario is considered that comprises N_{BS} base stations B_i $i = 1, \dots, N_{BS}$ and K_T active users in each

[*] This work has been partially funded by research projects COMONSENS (CSD2008-00010) and GRE3N(TEC2011-29006-C03-02).
[†] This research work was partly carried out at the ESAT Laboratory of KU Leuven in the frame of the Belgian Programme on Interuniversity Attractive Poles Programme initiated by the Belgian Science Policy Office: IUAP P7/23 'Belgian network on stochastic modeling analysis design and optimization of communication systems' (BESTCOM) 2012-2017.
[‡] The work of D. Toumpakaris was supported by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework through the Research Funding Program Thales - Investing in knowledge society through the European Social Fund.

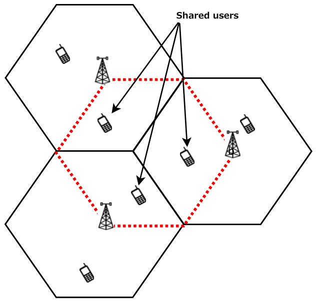


Fig. 1. 3-cell scenario with $K=1$ private user in each cell. $K_{sh}=3$ shared users are located within the edge area bounded by red dashed lines.

cell. Each BS is equipped with N_t transmit antennas, whereas each user has one reconfigurable antenna. The antenna can switch among M preset modes that modify its radiation pattern, and therefore, the received signal.

The symbols transmitted by BS B_i at time instant t can be written as $\mathbf{x}_i[t] = [x_{i,1},\ldots,x_{i,N_t}]^T$. The received average power, including path loss and shadowing, coming from BS B_j to user k in cell i is denoted as $\gamma_j^{[ik]}$. Hence, in the considered cellular system, the Signal-to-Interference Ratio (SIR) of user k in cell i because of interference from BS B_j equals $1/\alpha_j^{[ik]} = \gamma_i^{[ik]}/\gamma_j^{[ik]}$. Hence, the normalized received signal at user k in cell i is

$$y^{[ik]}[t] = \mathbf{h}_i^{[ik]}(m[t])^T \mathbf{x}_i[t] + \sum_{j=1, j\neq i}^{N_{BS}} \sqrt{\alpha_j^{[ik]}}\mathbf{h}_j^{[ik]}(m[t])^T \mathbf{x}_j[t] + z^{[ik]}, \tag{1}$$

where $\mathbf{h}_j^{[ik]}(m[t]) \in \mathbb{C}^{N_t\times 1}$ is the channel vector between BS B_j and user k in cell i corresponding to the m-th preset mode ($m = 1,\ldots,M$) at time t and $z^{[ik]}$ is complex circularly symmetric Gaussian noise with unit variance. The entries of $\mathbf{h}_j^{[ik]}(m[t])$ are i.i.d complex Gaussian random variables of zero mean. From now on, for the sake of an easy exposition, the temporal index will be omitted.

If BIA is applied to each cell, intracell interference can be eliminated. In order to also mitigate intercell interference, the BSs share the task of transmitting the data of the cell-edge users. Thus, only a portion of the overall information is conveyed through backhaul links, bringing down the cost compared to traditional optimal multiplexing gain schemes, where, besides CSIT, full coordination is required. To simplify the presentation, a symmetric scenario is assumed in this work; in each cell there are K cell-center users, which will be called *private* from now on, and K_{sh}/N_{BS} cell-edge users. Therefore, the total number of users in a cell is $K_T = K + K_{sh}/N_{BS}$. Moreover, the transmit power of all base stations is P.

III. Blind Interference Alignment schemes for cellular systems and the proposed scheme

We begin by a brief overview of the BIA scheme of [3]. We then review schemes that apply BIA to cellular scenarios before presenting the proposed technique.

A. Blind Interference Alignment over the MISO downlink

Without needing CSIT, BIA enables interference cancellation in a downlink MU-MISO system where each user has with $N_r = 1$ reconfigurable antenna. Assuming a switching pattern among the N_t preset modes of the reconfigurable antenna, which provides N_t independent values of $\mathbf{h}_i^{[ik]}(m)$, it is possible to remove the interference from transmissions to the rest of users. It can be shown that BIA allows to transmit N_t symbols to each user over $N_t + K_T - 1$ symbol extensions. Thus, BIA achieves $\frac{N_t K_T}{N_t+K_T-1}$ DoF in a MU-MISO system with N_t transmit antennas serving K_T users [3].

Some examples where the transmitter is equipped with N_t antennas are given in [5], [6], [7]. After zero forcing interference cancellation, the received signal at user k is

$$\begin{aligned} \widetilde{\mathbf{y}}^{[k]} &= \mathbf{H}^{[k]}\mathbf{u}^{[k]} + \widetilde{\mathbf{z}}^{[k]}, \text{ where} \\ \mathbf{H}^{[k]} &= \left[\mathbf{h}^{[k]}(1),\ldots,\mathbf{h}^{[k]}(N_t)\right]^T \in \mathbb{C}^{N_t\times N_t}, \end{aligned} \tag{2}$$

$\widetilde{\mathbf{y}}^{[k]}$ is a $N_t \times 1$ vector that contains the N_t data symbols and $\widetilde{\mathbf{z}}^{[k]}$ is the noise vector after zero-forcing cancellation. The number of preset modes equals N_t.

If constant transmit power is assumed [6], the noise after zero forcing cancellation is circularly symmetric complex Gaussian with zero mean and covariance matrix

$$\mathbf{R}_{\tilde{z}} = \begin{bmatrix} (2K_T-1)\mathbf{I}_{N_t-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}. \tag{3}$$

Therefore, the achievable sum rate is given by [3, Theorem 2]

$$R_{BIA} = \sum_{k=1}^{K_T} \frac{1}{N_t+K_T-1}\mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{P}{N_t}\mathbf{H}^{[k]}\mathbf{H}^{[k]^H}\mathbf{R}_{\tilde{z}}^{-1}\right)\right], \tag{4}$$

where P is the power that is transmitted by the BS and $\mathbf{A}^H$ is the Hermitian conjugate of $\mathbf{A}$.

B. BIA in a cellular system

As was shown in Section III-A, use of BIA leads to cancellation of intracell interference. However, intercell interference may still be present and reduce rates. This issue was examined in detail in [5]. A main conclusion is that intercell interference can be reduced considerably by synchronous aligned code reuse. Neighboring cells employ the same BIA code and their symbol extensions are synchronized. This way, a given user in a cell is subject only to interference from the signals sent to one user in each neighboring cell, since all other signals from all other BSs are sent to its interference space. Therefore, the achievable sum rate of the users of cell i can be written as

$$\begin{aligned} R_{i,cell} &= \\ &\sum_{k=1}^{K_T} \frac{1}{N_t+K_T-1}\mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{P}{N_t}\mathbf{H}_i^{[ik]}\mathbf{H}_i^{[ik]^H}\left(\mathbf{R}_{int}^{[ik]}\right)^{-1}\right)\right], \end{aligned} \tag{5}$$

where $\mathbf{R}_{int}^{[ik]}$ is the covariance matrix of the sum of interference and noise of user k

$$\mathbf{R}_{int}^{[ik]} = \mathbf{R}_z + \sum_{j=1, j\neq i}^{N_{BS}} \frac{\alpha_j^{[ik]}P}{N_t}\mathbb{E}\left[\mathbf{H}_j^{[ik]}\mathbf{H}_j^{[ik]^H}\right]. \tag{6}$$

Notice, that for this scheme, the supersymbol comprises $(N_t-1)^{K_T} + K_T(N_t-1)^{K_T-1}$ symbol extensions.

C. Data sharing based on augmented code

As can be seen from (6), although synchronous aligned code reuse reduces intercell inferference significantly, if the entries of matrix $\mathbf{H}_j^{[ik]}$ are large, the rate of cell-edge users may be small. In [7] a cooperative scheme is proposed to address this issue in a two-cell system. The users of each cell are categorized into K *private* users near the BS that have large SIR, and $K_{sh}/2$ users located near the cell edge with small SIR. The scheme is called augmented code and is a modification of the original BIA scheme. As in [5], private users are served by employing the original BIA scheme. Although by applying this scheme intercell interference remains, its effect is small because of the distance of the private users from the BS of the neighboring cell. However, the antennas of the BSs of both cells are now used to transmit the same data symbols to the cell-edge users of both cells who are therefore *shared*. Hence, the data of cell-edge users need to be communicated between the BSs over a backhaul link or obtained directly from the network. Intercell interference for shared users is eliminated, and their rate increases compared to [5]. On the other hand, in spite of employing a coordinated scheme, shared users decode N_t symbols although the maximum multiplexing gain is $M = N_{BS}N_t = 2N_t$ in a two-cell deployment.

For shared users, the resulting channel matrix is given by the sum of the matrices from both base stations

$$\widetilde{\mathbf{H}}^{[k]} = \mathbf{H}_i^{[k]} + \sqrt{\alpha_j^{[ik]}}\mathbf{H}_j^{[k]} \in \mathbb{C}^{N_t \times N_t}. \tag{7}$$

with $i, j \in \{1,2\}$ and $j \neq i$. Note that the cell index i is not used in $\widetilde{\mathbf{H}}^{[k]}$, since shared user k can now be thought of as belonging to both cells. The augmented code scheme is based on duplicating each shared user, so that each BS serve all shared users in both cells. Therefore, augmented code requires $N_t + K + K_{sh} - 1$ symbol extensions per N_t DoF per user instead of $N_t + K_T - 1 = N_t + K + K_{sh}/2 - 1$ if each cell were applying synchronous aligned code reuse as in [5]. Thus, the achievable sum rate of the shared users can be expressed as

$$R_{sh_{AU}} = \sum_{k=1}^{K_{sh}} \frac{1}{N_t + K_T' - 1} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{P}{N_t}\widetilde{\mathbf{H}}^{[k]}\widetilde{\mathbf{H}}^{[k]^H}\left(\mathbf{R}_{\widetilde{z}_{AU}}\right)^{-1}\right)\right], \tag{8}$$

with $K_T' = K + K_{sh}$ and

$$\mathbf{R}_{\widetilde{z}_{AU}} = \begin{bmatrix} (2(K+K_{sh})-1)\mathbf{I}_{N_t-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}. \tag{9}$$

The sum rate of the private users of cell i is given by the same expression as in [5], with the difference that the larger size of the supersymbol needs to be taken into account

$$R_{i,private_{AU}} = \sum_{k=1}^{K} \frac{1}{N_t + K_T' - 1} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{P}{N_t}\mathbf{H}_i^{[ik]}\mathbf{H}_i^{[ik]^H}\left(\mathbf{R}_{\widetilde{int}_{AU}}^{[k]}\right)^{-1}\right)\right] \tag{10}$$

where $\mathbf{R}_{\widetilde{int}_{AU}}^{[k]} = \mathbf{R}_{\widetilde{z}_{AU}} + \frac{\alpha_j^{[ik]} P}{N_t}\mathbb{E}\left[\mathbf{H}_j^{[ik]}\mathbf{H}_j^{[ik]^H}\right]$ and $i, j \in \{1,2\}$ with $j \neq i$. Due to the duplication of cell-edge users, note that the supersymbol length is now $(N_t - 1)^{K_T'} + K_T'(N_t - 1)^{K_T' - 1}$.

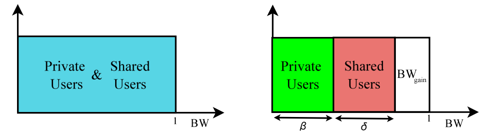


Fig. 2. Flexible Bandwidth scheme (FBW) compared to previous approaches.

IV. Data sharing Using Flexible Bandwidth

Although the augmented code solution leads to a diversity gain, the same symbols are sent to shared users by both BSs, and therefore the maximum multiplexing gain is not achieved. The scheme has the advantage of keeping the number of data streams to each user equal to N_t; consequently, the amplification of the noise because of zero forcing is limited, and the complexity of the reconfigurable antenna is not too high. On the other hand, for a N_{BS} cell scenario, more DoF can be attained when BIA over $N_{BS}N_t$ antennas, and therefore reconfigurable antennas with $M = N_{BS}N_t$ preset modes, is employed to serve the shared users. [1]

Motivated by the provision of flexible bandwidth allocation in latest-generation mobile communications standards such as LTE or LTE-A, and the development of reconfigurable antennas [4], we propose a frequency division scheme suitable for N_{BS} cells, to which we refer as FBW in the following. This scheme employs different parts of the available spectrum for transmission to private and shared users as is shown in Fig. 2. Assuming that bandwidth $\beta + \delta$ suffices to attain the same performance as BIA with augmented code, $BW_{gain} > 0$ is the amount of bandwidth that can be used to improve the user rates. In other words, BW_{gain} corresponds to the bandwidth efficiency improvement achieved by FBW transmission. Because there is no bandwidth sharing for shared and private users the number of symbol extensions of BIA is reduced, and the efficiency of BIA in each part of the spectrum improves. Moreover, assuming reconfigurable antennas with enough preset modes, $N_{BS}N_t$ antennas can be used for BIA transmission to shared users instead of N_t. In contrast to the BIA scheme with augmented code, in FBW each BS transmits N_t different symbols to a shared user instead of sending the same symbols as the other BS. Nevertheless, a penalty is expected in FBW because of orthogonal transmission. Furthermore, because $N_{BS}N_t$ antennas are used for transmission to the shared users, the power of the noise after interference subtraction may be larger than [7].

To begin with the performance analysis of FBW, δ is defined as the portion of the total bandwidth that is allocated to the shared users. Because BIA with $N_{BS}N_t$ antennas is employed, the achievable sum rate can be written as

$$R_{sh_{FBW}} = \delta \sum_{k=1}^{K_{sh}} \frac{1}{M + K_{sh} - 1} \mathbb{E}\left[\log\det\left(\mathbf{I} + \frac{P}{N_t}\widetilde{\mathbf{H}}_{sh}^{[k]}\widetilde{\mathbf{H}}_{sh}^{[k]^H}\mathbf{R}_{\widetilde{z}_{sh}}^{-1}\right)\right] \tag{11}$$

where the channel matrix is now $\widetilde{\mathbf{H}}_{sh}^{[k]} =$

[1] In practice, in a network with user mobility, each user should be able to switch among M preset modes, since it may transition from being private to being shared and vice versa.

$\left[\mathbf{H}_i^{[ik]};\ldots;\sqrt{\alpha_{(i-1)}^{[k]}}\mathbf{H}_{(i-1)}^{[ik]};\sqrt{\alpha_{(i+1)}^{[ik]}}\mathbf{H}_{(i+1)}^{[ik]};\ldots;\sqrt{\alpha_{N_{BS}}^{[ik]}}\mathbf{H}_{N_{BS}}^{[ik]}\right] \in \mathbb{C}^{M\times N_{BS}N_t}$ with $M = N_{BS}N_t$, i.e., the reconfigurable antennas of the shared users now need $N_{BS}N_t$ preset modes. Because BIA is applied to K_{sh} users over $M+K_{sh}-1$ time slots per M DoF per user, the noise matrix after interference cancellation is

$$\mathbf{R}_{\tilde{z}_{sh}} = \begin{bmatrix}(2K_{sh}-1)\mathbf{I}_{M-1} & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}. \tag{12}$$

Similarly, if β is the portion of the total bandwidth allocated to the private users, the achievable sum rate for the private users of cell i is given by

$$R_{i,private_{FBW}} = \beta\sum_{k=1}^{K}\frac{1}{N_t+K-1}\mathbb{E}\left[\log\det\left(\mathbf{I}+\frac{P}{N_t}\mathbf{H}_i^{[ik]}\mathbf{H}_i^{[ik]^H}\left(\mathbf{R}_{\tilde{z}_{private}}^{[ik]}\right)^{-1}\right)\right], \tag{13}$$

where

$$\mathbf{R}_{\tilde{z}_{private}}^{[ik]} = \mathbf{R}_{\tilde{z}} + \sum_{j=1,j\neq i}^{N_{BS}}\frac{\alpha_j^{[ik]}P}{N_t}\mathbb{E}\left[\mathbf{H}_j^{[ik]}\mathbf{H}_j^{[ik]^H}\right] \tag{14}$$

and

$$\mathbf{R}_{\tilde{z}}^{[k]} = \begin{bmatrix}(2K-1)\mathbf{I}_{N_t-1} & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}. \tag{15}$$

Note that, when the private users are far from the BS of the neighboring cell, the rates are limited by the SNR rather than the SIR.

Since the bandwidth is divided into a BIA code using N_t antennas and a BIA code using $M = N_{BS}N_t$ antennas for K and K_{sh} users, respectively, the required coherence time is given by the maximum between the supersymbol lengths $(N_t-1)^K + K(N_t-1)^{K-1}$ and $(M-1)^{K_{sh}} + K_{sh}(M-1)^{K_{sh}-1}$.

The proposed scheme is developed for a N_{BS} BSs scenario. However, in order to compare to the solution based on augmented BIA, from now on we will focus on the two-cell scenario. In the following, using a particularization of above expressions with $N_{BS} = 2$, we show that, in a 2-cell deployment, FBW attains more DoF compared to the augmented code approach when the number of shared users over both cells $K_{sh} > 2N_t - 1$. Moreover, we also demonstrate that, if $K_{sh} > 2N_t - 1$, the achievable rates are larger than those of [7] as long as the SNR of the cell-edge users exceeds a threshold.

Theorem 1. *In the two-cell scenario with K private users per cell and K_{sh} cell-edge users in both cells, when SNR $\to\infty$, FBW achieves larger sum rate than data sharing with augmented code if $K_{sh} \geq 2N_t - 1$.*

Proof: Because the variance of the noise is finite, SNR $\to \infty$ corresponds to $P\to\infty$. The achievable sum rate for the shared users in a two-cell scenario can be written in terms of the DoF metric [8], because interference is canceled. From (8) and (11),

$$R_{sh_{AU}}(P\to\infty) = \frac{N_tK_{sh}}{N_t+K+K_{sh}-1}\log(P) + o(\log(P)) \tag{16}$$

$$R_{shFBW}(P\to\infty) = \delta\frac{2N_tK_{sh}}{2N_t+K_{sh}-1}\log(P) + o(\log(P)). \tag{17}$$

The term $o(\log(P))$ corresponds to some function $f(P)$ that satisfies $\lim_{P\to\infty}\frac{f(P)}{\log(P)} = 0$. Therefore, the two approaches achieve the same sum DoF for the shared users when

$$\delta = \frac{2N_t+K_{sh}-1}{2(N_t+K+K_{sh}-1)}. \tag{18}$$

Although the intercell interference to which the private users are subject is small, at the limit transmission becomes interference-limited. Letting $P\to\infty$ while keeping the SIR of private users fixed to α_{priv}, if $\check{\mathbf{R}}^{[k]} = \mathbb{E}\left[\mathbf{H}_j^{[ik]}\mathbf{H}_j^{[ik]^H}\right]$ the sum rate of the private users in cell i can be written as

$$R_{i,private_{AU}}(P\to\infty) = \frac{K}{N_t+K+K_{sh}-1}\mathbb{E}\left[\log\det\left(\mathbf{I}+\frac{1}{\alpha_{priv}}\mathbf{H}_i^{[ik]}\mathbf{H}_i^{[ik]^H}\left(\check{\mathbf{R}}^{[k]}\right)^{-1}\right)\right] \tag{19}$$

$$R_{i,private_{FBW}}(P\to\infty) = \beta\frac{K}{N_t+K-1}\mathbb{E}\left[\log\det\left(\mathbf{I}+\frac{1}{\alpha_{priv}}\mathbf{H}_i^{[ik]}\mathbf{H}_i^{[ik]^H}\left(\check{\mathbf{R}}^{[k]}\right)^{-1}\right)\right]. \tag{20}$$

Thus, the same sum rate is achieved for private users by both methods when they are assigned the following portion of the bandwidth

$$\beta = \frac{N_t+K-1}{N_t+K+K_{sh}-1}. \tag{21}$$

If the bandwidth gain is defined as $BW_{gain} = 1-(\beta+\delta)$, the use of FBW is favorable compared to the augmented code if

$$\beta+\delta<1 \Rightarrow \frac{2N_t+K_{sh}-1}{2(N_t+K+K_{sh}-1)} + \frac{N_t+K-1}{N_t+K+K_{sh}-1} < 1 \Rightarrow \frac{4N_t+K_{sh}+2K-3}{2(N_t+K+K_{sh}-1)} < 1 \Rightarrow K_{sh} > 2N_t-1, \tag{22}$$

which concludes the proof. ■

More generally, it can be shown that the rates that are achieved with the FBW scheme remain better than those obtained using the augmented code as long as the SNR exceeds a certain threshold.

Theorem 2. *For the two-cell scenario, assuming that the power received by the shared users is large enough so that $\log(1+SNR)\approx\log(SNR)$, if $K_{sh} > 2N_t-1$, FBW achieves a larger sum rate than BIA with augmented code if*

$$\text{SNR} > N_t\frac{1+\alpha_{sh}}{\alpha_{sh}}\left(\frac{2K_{sh}-1}{2(K+K_{sh})-1}\right)^{\frac{N_t-1}{N_t}}\frac{2K_{sh}-1}{e^{\frac{1}{N_t}\sum_{l=0}^{N_t-1}\psi(2N_t-1)}}, \tag{23}$$

where α_{sh} is the SIR of the shared users (assumed equal for all) and $\psi(\cdot)$ is the Euler digamma function.

Proof: For private users it is easy to see that if $\beta = \frac{N_t+K-1}{N_t+K+K_{sh}-1}$ as in (21), it suffices to compare the expectation terms in (10) and the evaluation of (13) at $N_{BS}=2$. The only difference between the terms is the noise covariance matrix, which is larger in (10) because the augmented code involves $K+K_{sh}$ users, whereas in FBW the noise is only proportional to K. Therefore, if $K_{sh} > 2N_t-1$, meaning that $BW_{gain} > 0$ still holds when (21) and (18) are satisfied, FBW achieves a larger sum rate at any SNR value for the private users of each cell.

Let A_{AU} and A_{FBW} denote the value of the determinants in (8) and (11), respectively. Using the assumption $\log(1+\mathsf{SNR}) \approx \log(\mathsf{SNR})$,

$$R_{sh_{AU}} \approx \kappa \mathbb{E}\left[\log A_{AU}\right] = \kappa \mathbb{E}\left[\log\det\left(\frac{P}{N_t}\widetilde{\mathbf{H}}^{[k]}\widetilde{\mathbf{H}}^{[k]^H}\mathbf{R}_{\widetilde{z}_{AU}}^{-1}\right)\right]. \tag{24}$$

where $\kappa > 0$ equals a strictly positive constant. Since $\widetilde{\mathbf{H}}^{[k]}$ and $\mathbf{R}_{\widetilde{z}_{AU}}$ are $N_t \times N_t$ matrices, and the entries of $\widetilde{\mathbf{H}}^{[k]}$ are i.i.d. Gaussian with zero mean and variance $(1+\alpha_{sh})g^{[k]}$, where $g^{[k]}$ is the channel gain at user k,

$$A_{AU} = \left(\frac{P}{N_t}\right)^{N_t} \det\left(\mathbf{R}_{\widetilde{z}_{AU}}^{-1}\right)\det\left(\mathbf{\Sigma}\mathbf{H}^{[k]}\mathbf{\Phi}\mathbf{H}^{[k]^H}\right), \tag{25}$$

where $\mathbf{H}^{[k]} \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$, $\mathbf{\Sigma} = g^{[k]}(1+\alpha_{sh})\mathbf{I}_{N_t}$, $\mathbf{\Phi} = \mathbf{I}_{N_t}$ and $\alpha_{sh} = \alpha_j^{[ik]}$ with $i, j \in \{1,2\}$, $j \neq i$ and k referring a shared use k at any BS i. Thus, since $\boldsymbol{W} = \mathbf{H}^{[k]}\mathbf{H}^{[k]^H}$ is a Wishart matrix $\boldsymbol{W} \sim W_{N_t}(N_t, \mathbf{I})$, applying [9, Theorem 2.11]

$$\mathbb{E}\left[\log\det\left(\mathbf{H}^{[k]}\mathbf{H}^{[k]^H}\right)\right] = \sum_{l=0}^{N_t-1}\psi(N_t - l) \tag{26}$$

where $\psi(\cdot)$ is the Euler digamma function. Finally, since $\mathbf{R}_{\widetilde{z}_{AU}}$ is a diagonal matrix,

$$\begin{aligned}
&\mathbb{E}\left[\log A_{AU}\right] = \\
&\log\left(\left(\frac{P}{N_t}\right)^{N_t}\left(\frac{1}{2(K+K_{sh})-1}\right)^{N_t-1}\left(g^{[k]}(1+\alpha_{sh})\right)^{N_t}\right) \\
&+\mathbb{E}\left[\log_e\det\left(\mathbf{H}^{[k]}\mathbf{H}^{[k]^H}\right)\right] = \\
&\log\left(\left(\frac{P}{N_t}\right)^{N_t}\left(\frac{1}{2(K+K_{sh})-1}\right)^{N_t-1}\left(g^{[k]}(1+\alpha_{sh})\right)^{N_t}\right) \\
&+\sum_{l=0}^{N_t-1}\psi(N_t-l).
\end{aligned} \tag{27}$$

Similarly, the achievable sum rate of FBW can be approximated as

$$R_{sh_{FBW}} \approx \kappa \mathbb{E}\left[\log A_{FBW}\right] = \kappa \mathbb{E}\left[\log\det\left(\frac{P}{N_t}\widetilde{\mathbf{H}}_{sh}^{[jk]}\widetilde{\mathbf{H}}_{sh}^{[jk]^H}\mathbf{R}_{\widetilde{z}_{sh}}^{-1}\right)\right]. \tag{28}$$

Because BIA over the antennas of both BSs is used for FBW, the size of the matrices $\widetilde{\mathbf{H}}_{sh}^{[k]}$ and $\mathbf{R}_{\widetilde{z}_{sh}}$ is $2N_t \times 2N_t$. It is possible to rewrite A_{FBW} as

$$A_{FBW} = \left(\frac{P}{N_t}\right)^{2N_t}\det\left(\mathbf{R}_{\widetilde{z}_{sh}}^{-1}\right)\det\left(\mathbf{\Sigma}'\mathbf{H}_{sh}^{[k]}\mathbf{\Phi}\mathbf{H}_{sh}^{[k]^H}\right), \tag{29}$$

where $\mathbf{H}_{sh}^{[k]} \sim \mathcal{CN}(0, \mathbf{I}_{2N_t})$, $\mathbf{\Sigma}' = g^{[k]}\begin{bmatrix}\mathbf{I}_{N_t} & \mathbf{0} \\ \mathbf{0} & \alpha_{sh}\mathbf{I}_{N_t}\end{bmatrix}$ and $\mathbf{\Phi} = \mathbf{I}_{2N_t}$. Thus, since $\boldsymbol{W}_{sh} = \mathbf{H}_{sh}^{[k]}\mathbf{H}_{sh}^{[k]^H}$ is a Wishart Matrix $\boldsymbol{W} \sim W_{2N_t}(2N_t, \mathbf{I})$, applying, again, [9, Theorem 2.11].

$$\mathbb{E}\left[\log\det\left(\mathbf{H}_{sh}^{[k]}\mathbf{H}_{sh}^{[k]^H}\right)\right] = \sum_{l=0}^{2N_t-1}\psi(2N_t-l). \tag{30}$$

Finally,

$$\begin{aligned}
&\mathbb{E}\left[\log A_{FBW}\right] = \\
&\log\left(\left(\frac{P}{N_t}\right)^{2N_t}\left(\frac{1}{2K_{sh}-1}\right)^{2N_t-1}\left(g^{[k]}\right)^{2Nt}(\alpha_{sh})^{N_t}\right) + \\
&\mathbb{E}\left[\log\det\left(\mathbf{H}_{sh}^{[k]}\mathbf{H}_{sh}^{[k]^H}\right)\right] = \\
&\log\left(\left(\frac{P}{N_t}\right)^{2N_t}\left(\frac{1}{2K_{sh}-1}\right)^{2N_t-1}\left(g^{[k]}\right)^{2Nt}(\alpha_{sh})^{N_t}\right) \\
&+\sum_{l=0}^{2N_t-1}\psi(2N_t-l).
\end{aligned} \tag{31}$$

Hence, FBW leads to a larger sum rate for the shared users if $\mathbb{E}\left[\log A_{FBW}\right] > \mathbb{E}\left[\log A_{AU}\right]$, or

$$\begin{aligned}
&\log\left(\left(\frac{P}{N_t}\right)^{2N_t}\left(g^{[k]}\right)^{2Nt}\alpha_{sh}^{N_t}\left(\frac{1}{2K_{sh}-1}\right)^{2N_t-1}\right) + \\
&\sum_{l=0}^{2N_t-1}\psi(2N_t-l) \\
&> \log\left(\left(\frac{P}{N_t}\right)^{N_t}\left(g^{[k]}(1+\alpha_{sh})\right)^{N_t}\left(\frac{1}{2(K+K_{sh})-1}\right)^{N_t-1}\right) + \\
&\sum_{l=0}^{N_t-1}\psi(N_t-l) \Rightarrow \\
&\mathsf{SNR} > N_t\frac{1+\alpha_{sh}}{\alpha_{sh}}\left(\frac{2K_{sh}-1}{2(K+K_{sh})-1}\right)^{\frac{N_t-1}{N_t}}\frac{2K_{sh}-1}{e^{\frac{1}{N_t}\sum_{l=0}^{N_t-1}\psi(2N_t-l)}}.
\end{aligned} \tag{32}$$

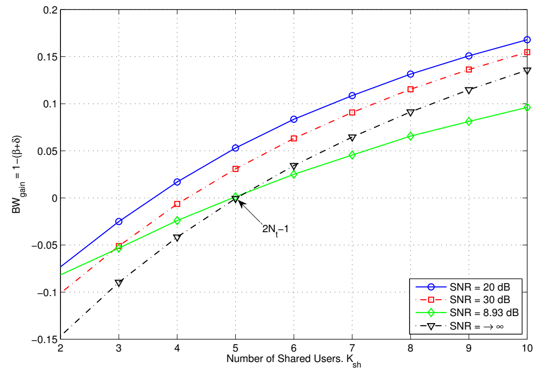
where $\mathsf{SNR} = Pg^{[k]}$. ■

V. Simulation Results

The rates attained by FBW are evaluated using simulations. The behavior predicted by the analysis of Section IV is confirmed, and the results are compared to the performance achieved by other BIA techniques.

Figure 3 shows the achievable bandwidth gain (BW_{gain}) when FBW is used instead of the BIA scheme with augmented code in a two-cell deployment. Each BS is equipped with $N_t = 3$ antennas serving a fixed number of $K = 6$ private users per cell. The average SIR α is assumed to be 10dB and 2dB for private and shared users, respectively. As can be seen, BW_{gain} grows as the number of shared users increases. Hence, the FBW approach is more suitable when many users are located near the cell edge. As the power increases to infinity, FBW starts to be superior to augmented code ($BW_{gain} > 0$) if $K_{sh} > 2N_t - 1 = 5$ in agreement with Theorem 1. On the other hand, for the same number of shared users, FBW achieves positive BW_{gain} for finite values of SNR such as 30 or 20 dB. The minimum SNR value that achieves $BW_{gain} > 0$ is given by Theorem 2 that specifies $\mathsf{SNR} > 8.93$ dB.

The achievable sum rates for shared and private users are plotted in Fig. 4, for a scenario where each BS is equipped with $N_t = 4$ antennas that serve $K = 8$ private users in each cell. The transmission power is fixed at 15dB and the average SIR is assumed to be 10dB and 2dB for private and shared



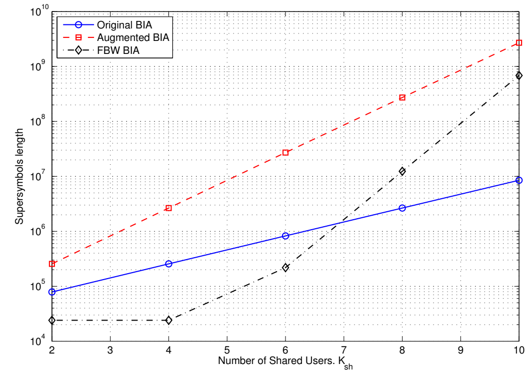


Fig. 3. Bandwidth gain of FBW compared to augmented code versus the number of shared users, K_{sh}. $\alpha_{priv} = 10$dB, $\alpha_{sh} = 2$ dB, $K = 6$, $N_t = 3$.

Fig. 5. Supersymbol length for FBW, augmented code, and original BIA. $K = 8$ and $N_t = 4$.

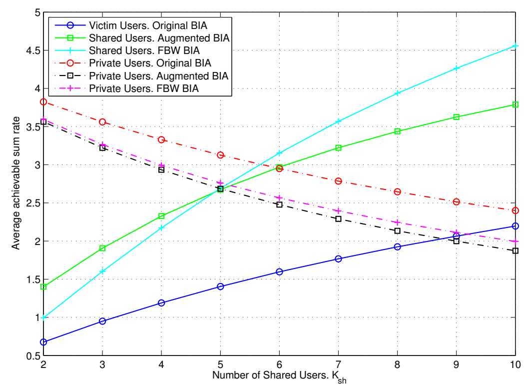


Fig. 4. Average achievable sum rates for shared and private users versus the number of shared users, K_{sh}. The SNR is fixed to 15 dB for all users, whereas the average SIR is 10 dB and 2 dB for private and cell-edge users, respectively. $K = 8$ and $N_t = 4$.

users, respectively. A heuristic approach is used to allocate the entire bandwidth: $\beta = \frac{K}{K_T}$ and $\delta = \frac{Ksh/2}{K_T}$. Note that for both augmented code and FBW, there is a penalty in the rates of private users as a result of being more fair to cell-edge users. Both approaches improve considerably the rates of the shared users compared to BIA transmission that does not deal explicitly with intercell interference to cell-edge users. As predicted from Theorem 1, the augmented code approach performs better than FBW for few cell-edge users, whereas the performance of FBW becomes better as K_{sh} grows.

The supersymbol length corresponding to the simulations of Fig. 4 is depicted in Fig. 5. As can be seen, for a small number of shared users FBW achieves even shorter supersymbol lengths than the original BIA scheme due the partitioning of the users. Taking into account the slope of the supersymbol length of FBW, it exceeds the length achieved by augmented code for large K_{sh}. However, this cross point corresponds to a length too large to consider in a real implementation ($> 10^8$). Assuming a 52 Mbps digital-to-analog converter and 8 samples per symbol, a coherence time greater that 33.6 msec is required to implement FBW for $K_{sh} = 6$ shared users plus $K = 8$ private users in each cell, while it corresponds to 4.1 sec and 127.1 msec for augmented code and original BIA, respectively.

VI. Conclusions

We studied a Blind Interference Alignment strategy that relies on flexible bandwidth allocation to separate transmission to cell-edge users from transmission to private users, which are characterized by a high SIR. It was shown that for the two-cell scenario the strategy can improve the rates of cell-edge users compared to previous approaches over a wide range of SNRs when their number exceeds a threshold. The method does not have any additional backhaul requirements other than the capacity that is needed to coordinate the transmission of data to the cell-edge users.

References

[1] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone and W. Yu, "Multi-Cell MIMO Cooperative Networks: A New Look at Interference," *IEEE Journ. on Select, Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[2] S. A. Ramprashad, G. Caire and H. C. Papadopoulos, "Cellular and Network MIMO architectures: MU-MIMO spectral efficiency and costs of channel state information," *Proc. of the Forty-Third Asilomar Conference on Signals, Systems and Computers*, pp. 1811–1818, Nov. 2009.

[3] T. Gou, C. Wang and S. A. Jafar, "Aiming Perfectly in the Dark-Blind Interference Alignment Through Staggered Antenna Switching," *IEEE Trans. Signal Processing*, vol. 59, no. 6, pp. 2734–2744, Jun. 2011.

[4] Qian Rongrong, M. Sellathurai, and D. Wilcox, "On the design of Blind Interference Alignment using ESPAR antenna," *Communications and Networking in China (CHINACOM), 2012 7th International ICST Conference on*, vol., no., pp.866,870, 8-10 Aug. 2012

[5] C. Wang, H. C. Papadopoulos, S. A. Ramprashad and G. Caire, "Design and operation of Blind Interference Alignment in cellular and cluster-based systems," *in Proc. Inf. Theory and Applications (ITA)*, Feb. 2011.

[6] C. Wang, H. C. Papadopoulos, S. A. Ramprashad and G. Caire, "Improved Blind Interference Alignment in a Cellular Environment Using Power Allocation and Cell-Based Clusters," *in Proc. 2011 IEEE Int. Conf. Commun. (ICC)*, Jun, 2011.

[7] S. Akoum, C. S. Chen, M. Debbah and R. W. Heath Jr, "Data Sharing Coordination and Blind Interference Alignment for Cellular Networks," *in Proc. of IEEE Global Telecommunications Conf.*, Dec, 2012.

[8] S. A. Jafar, "Interference Alignment: A New Look at Signal Dimensions in a Communication Network," *Foundations and Trends in Communications and Information Theory*

[9] A. M. Tulino and S. Verdú, "Random Matrix Theory and Wireless Communications", *Foundations and Trends In Communications and Information Theory*.

[10] S. A. Jafar, "Elements of Cellular Blind Interference Alignment - Aligned Frequency Reuse, Wireless Index Coding and Interference Diversity," *available at* http://arxiv.org/abs/1203.2384.