PERFORMANCE OF MIMO SYSTEMS

Patricia Grande González
Thomas Gros Vidal

Department of Information Technology
Lund University

Advisor: Göran Lindell

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Abstract

Demand in high data rate communications, driven by internet and cellular mobile, have increased, specially in wireless local area networks, emerging home audio visual networks and multimedia services in general. The limitation of the available radio spectrum makes it impossible for the data rate needs to be accomplished by an increase in the bandwidth. The deployment of multiple antennas in the transmitter and the receiver, multiple input multiple output (MIMO), a cost effective technology, makes it feasible to meet the high data rate demands.

In this work, several scenarios such as the transmission under Rayleigh and Rice channel conditions are analyzed. Different transmission schemes are used, using different numbers of transmit and receive antennas. The focus of the project is an investigation of the fundamental performance tradeoff between bit error probability and bit rate in these systems, related to the number of antennas deployed and the SNR.

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1. Introduction

By the end of the last century, in accordance with market needs, cellular mobile networks and next generation wireless systems in general faced some challenges. In order to offer high bit rate data services such as video calls or web browsing, the next generation systems had to improve their coverage, quality and their power and bandwidth efficiency. Remote units had to remain simple, in order to be accepted by the market. In general, a large improvement in SNR is needed under Rayleigh fading conditions to reduce the BER, and due to the requirements of the systems, the improvement in SNR could not be based on an increase of the transmit power.
At that time, there were some ways of combating multipath fading. Transmitter power control and predistortion of the signal to overcome the effect of the channel should be effective, but presented some problems. Power transmit control implies a dynamic range for the remote unit, and in some cases, that could exceed the radiation power limitations [14]. Predistortion of the signal implies knowledge of the channel by the transmitter. That is possible only by the means of feedback or if the information from base to remote unit (downlink) and from remote unit to base (uplink) is transmitted in the same channel. Using multiple antennas in the receiver and combining or switching the signals to improve the quality of the received signal would also be an effective way of combating fading. The problem in this case was the increase of the size, cost and power of the mobile units. So this technique could be applied only in base stations, improving their reception, and improving the overall performance of the system. That was more economical than adding antennas to all the remote units.

Some ideas using diversity in transmission were starting to appear, when Alamouti came up with a simple transmit diversity scheme,

[5], with a very simple decoding algorithm, that made MIMO communications emerge. The idea behind MIMO is that the received signals can be combined to improve the quality or the data rate, increasing the quality of service or the operator’s revenues. And this is done meeting the restrictions, without a bandwidth or a transmit power increase. Few years after the appearance of this technology, it has penetrated large scale standard driven commercial wireless networks and products.

The goal of this project is to analyze and simulate different MIMO schemes, focused on minimizing the bit error probability or maximizing the bit rate, and to understand the tradeoff between these two parameters, under different channel conditions. Chapter 2 presents a brief theoretical background. Chapter 3 analyzes in depth Alamouti’s scheme. In chapter 4, OSTBC, orthogonal space time block codes are introduced, which are a generalization of Alamouti’s scheme for N transmit antennas. In chapter 5, the spatial multiplexing scheme, focused on maximizing the bit rate, is investigated. In these chapters, the results for the simulations are included. Chapter 6 explains some details about the computer simulation program used to do the simulations, and chapter 7 presents the conclusions of the project.

2. Theoretical background

The communication links can be classified depending on the number of antennas used to transmit and to receive. The different schemes may be valuable, or feasible, in different scenarios, depending on the application they are used for.

2.1 SISO: single input single output

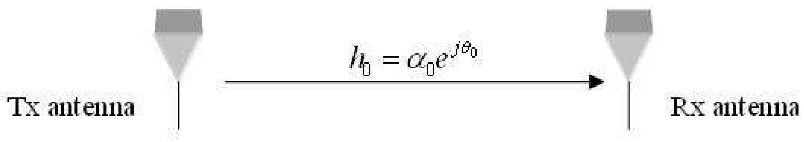


Fig1.- SISO diagram

In an ideal communication link, one antenna (single input) transmits the information, or symbol, and another one (single output) receives the information and takes the decision about which was the symbol sent. This situation is reliable when the channel is only contaminated by noise. The channel coefficient is in general a complex number, defined by a module and a phase.

To introduce some concepts, the project started with the simulation of a SISO system, using a BPSK modulation, which means that there were only two possible symbols to be sent: A= 1 and A= –1. BPSK is also known as 2-PAM.

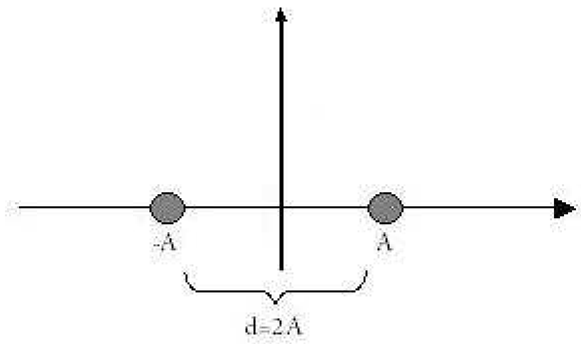


Fig 2.- BPSK constellation

Where $d=d_{min}$ is the minimum distance between any pair of symbols in the constellation. In the BPSK case, the minimum distance is 2A. In all the constellations, it was assumed that A=1 for the sake of simplicity. And also for this reason, the channel coefficient was considered, only in this system to be 1.

Q(x) is a function that gives the area of a tail of a Gaussian function, or in other words, gives the probability that a Gaussian function takes a value bigger or equal to x. The definition of Q(x) is [15]:

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \qquad (1)$$

Q(x) is strongly related to another useful function used to calculate the BER of systems, erfc(x). Their relation is:

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \qquad (2)$$

The goal was to find the BER (Bit error ratio) or bit error probability for several values of the SNR (signal to noise ratio). The bit error probability is the average number of information bit errors per detected information bit. The definition used for the SNR was:

$$SNR = 10\log_{10}\frac{E_b}{N_0} \quad (3)$$

Where E_b is the average energy per bit received, and $N_0/2$ is the variance of the noise in the receiver.
The system was simulated under the influence of AWGN (additive white Gaussian noise) [16]. Although this model doesn't adjust to real communication situations, because it doesn't consider impairments such as fading, or frequency selectivity, it gives a simple mathematical model that helps simulate the best possible situation in a communication link, before taking into account the other phenomena. It models fluctuations in the signal due to natural sources, like for example thermal vibrations of the atoms in the antennas. The power spectral density of AWGN is flat, or has the same value for each of the frequencies. The white light contains all the wavelengths, so this kind of noise is named in analogy with it, due to the fact that it has the same power in all the frequencies. For this system in particular, it would not be necessary to do a simulation to find the approximate value of the BER, because it is relatively easy to find an exact expression of it.
The symbol error probability is [1]:

$$P_s = Q\left(\sqrt{d_{\min}^2 \frac{1}{2N_0}}\right) \quad (4)$$

$$P_s = Q\left(\sqrt{4A^2 \frac{1}{2N_0}}\right) \quad (5)$$

$$E_b = A^2 \quad (6)$$

$$P_s = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{7}$$

P_s is the symbol error probability. Then, BER= P_s/q, where q is the number of bits per symbol. In this modulation, there is only one bit per symbol, hence, q=1, so $P_s=P_b$.
The way of obtaining the bit error probability through the simulation was to simulate the sending of many bits, adding AWGN, and waiting for the receiver to commit n errors, a large number of them (between 400 and 800), so the results are statistically more reliable. When all the errors have been committed, the way of approximating the BER= number of bit errors/total number of sent bits.

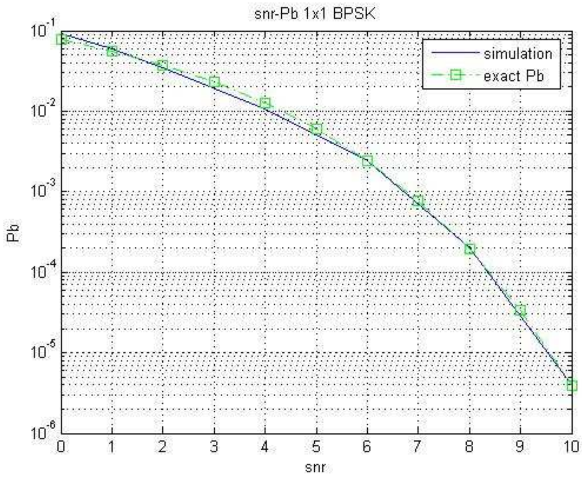


Fig.3- Simulation of the SISO BPSK AWGN system and the exact BER of the system

As seen in the comparative graph, the results of the simulation differ very little from the exact P_b, only for low values of SNR. These differences could be reduced by simulating the system until more errors were committed.

2.2 SIMO: single input multiple output

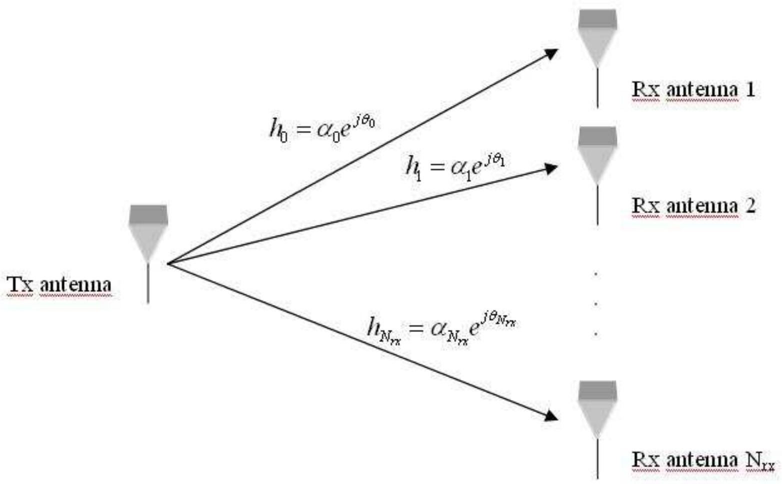


Fig4.- SIMO diagram

In this situation, the transmit antenna sends the information to multiple receive antennas, which jointly take a decision about which was the symbol that was sent, combining previously the received signals. From now on, the notation for this kind of systems will be 1 x N_r. There is space diversity in this scheme, due to the fact that the signal reaches the antennas through different paths, and therefore, some of them may have better quality than others, and that can make the receiver make a more confident decision concerning the symbols sent. Diversity could be defined as the situation in which the receiver improves its decision by taking into account more (redundant) copies of the same information. The diversity is achieved in reception. An example of this scheme would be MRRC (maximal-ratio receive combining) [17], an effective method to combat fading patented in 1992.

2.3 MISO: multiple input single output

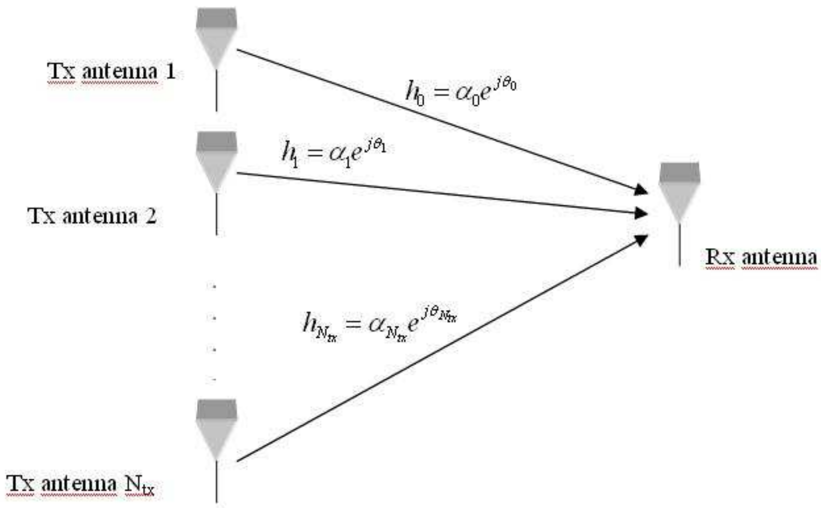


Fig 5.- MISO diagram

In this scheme, the diversity is achieved in transmission, where several transmit antennas send the same information to only one receive antenna. The receiver combines the received signal in order to take a decision about the sent symbol. This scheme is broadly used in many applications today, as for instance, mobile telephones, due to the fact that it is economic to deploy more than one antenna in the base station, but not in the mobile phone. From now on, these schemes will be notated as N_t x 1.

2.4 MIMO- multiple input multiple output

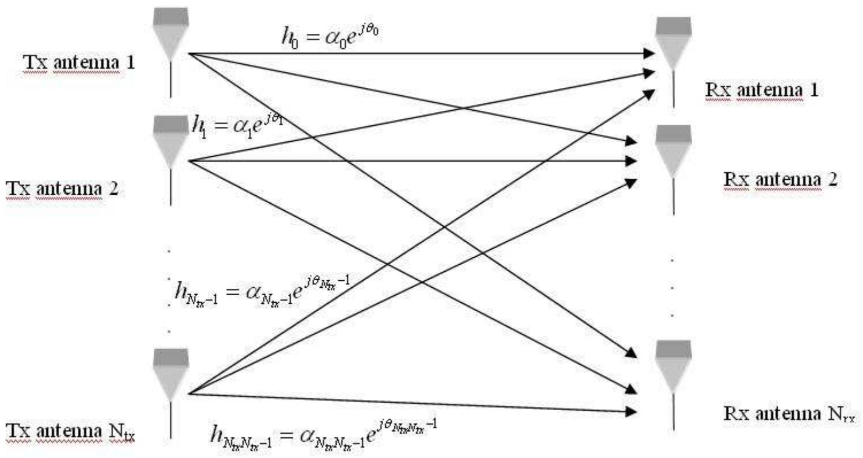


Fig 6.- MIMO diagram

In this scheme, the diversity is achieved both in transmission and reception. It exceeds the performance of all of the above schemes. Many MIMO configurations are possible. In general, an N x M configuration means that N antennas will be used to transmit and M antennas will be used to receive. This scheme is able to transform multipath propagation into an advantage.

All of the above schemes, except the SISO, have the advantage that they are able to improve the performance of the system, in BER and/or bit rate, without the need of an increase in bandwidth used or power sent, two of the most important restrictions for wireless applications.

2.5 Channel modeling

In this project, several channel models were used to simulate the different communication systems. The models used for the channel were the deterministic, the Rayleigh fading channel and the Rician fading channel, which is a generalization of the Rayleigh fading. Obviously, the model of the channel is completed by the addition of AWGN (Additive White Gaussian Noise).

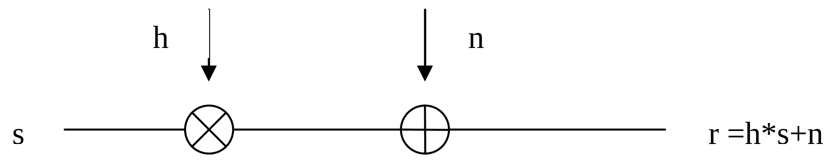


Fig 7.- classical channel fading model

This model fits the SISO or point to point communication system. In a MIMO system, N_t x N_r, there are N_t x N_r point to point communication links. The above model is only suitable to model the link between any arbitrary transmit antenna and any arbitrary receive antenna. To model in a compact way the whole communication link, vectorial notation must be used. It could be defined by [2]:

$$r = Hs + n \tag{8}$$

Where r contains the received signal by each of the antennas in any instant of time, so it is a column vector with N_r rows. It becomes a matrix with P columns if information is received along P time instants. H is a matrix that contains the channel coefficient that defines the path from every transmit to every receive antenna, so its dimensions are N_t x N_r. The vector s contains the symbols that are sent in any arbirary instant of time, and n contains the AWGN that contamines every signal that arrives to the receivers. The vector s has one column and N_t rows and n has the same dimensions of r.

The Rayleigh fading channel models the fluctuations in the magnitude of a radio signal caused by the propagation environment, specially by multipath reception. Multipath is the situation in which the signals reach the antenna following different paths, caused mainly by reflecting obstacles. The model is suitable when many objects are situated in the LOS (line of sight) and scatter the radio signal, so there is NLOS (no line of sight)[19]. A city center, with many buildings that refract, reflect, diffract and attenuate the signal is suitable for the model to be used, as well as an indoor office. Furthermore, the receiver and the transmitter can introduce changes in the scenario when they move. But even if they don't move, changes in the environmental conditions lead to changes in the propagation conditions. Rayleigh fading also fits tropospheric and ionospheric propagation, because the large quantity of particles in those atmospheric layers act as scatterers [18]. The modelling of the Rayleigh fading implies that in the simulation programs, the channel coefficients will have a real and an imaginary part, and each of them will be zero-mean Gaussian processes, independent and identically distributed. In the simulations, flat fading is always assumed, attending to the fact that bandwidth considerations are not the goal of the project, so the fading affects equally all the possible frequencies, or the frequency in which the simulation is supposed to be made. In Rayleigh fading conditions, the received versions of the signal combine in either a constructive or destructive way. In the second case, the signal may become masked by noise, and if the system under study is a SISO system, the communication could be interrupted.

The Rician fading channel is a generalization of the Rayleigh fading channel. It is applied when the environment has plenty of scatterers, just like in the Rayleigh situation, but in this case, there is also a direct LOS between the transmit and the receive antenna [20]. That means that during the whole transmission, considering a static receiver and not a mobile one, one part of the signal is not reflected and doesn't have any obstacle to reach the receiver, so there is a path that remains constant, or in other words, a part of the channel coefficient doesn't vary. A possible interpretation of this situation is to think it is just like the Rayleigh case, but the coefficients don't have zero mean. Another one would be to think that each channel coefficient has two components, that must be added, one that is fixed or deterministic and doesn't vary during the whole transmission, and one that varies periodically and is a Rayleigh coefficient. This way, the channel coefficient matrix will be the addition of two matrices, one modelling the LOS path and the other one modelling the NLOS path The Rician factor, K, measures the relative strength of the LOS (specular) and the NLOS (scattered) paths to the receive antennas, and it is defined as [3]:

$$K = \frac{\left\| H_{los} \right\|^2}{E\left\{ \left\| H_{nlos} \right\|^2 \right\}} \qquad (9)$$

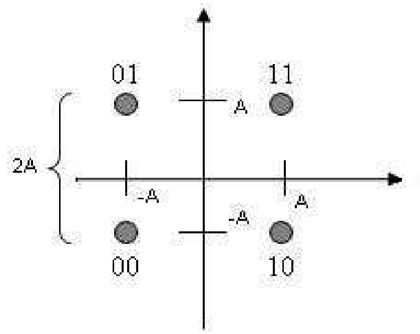
So here, it becomes clear that the Rician case is a generalization of the Rayleigh case. If k=0, which happens when the LOS component has no strength, or the coefficients of the H_{los} matrix are zero, or in other words, there is no Line of Signal in the communication link, the Rician case becomes the Rayleigh one.
Where the squared norm of a matrix, or squared Frobenius norm is defined as the sum of all the modules of its components squared. In this case [4]:

$$\left\| H_{los} \right\|^2 = \sum_{i=1}^{N_{tx} N_{rx}} \left| h_{ilos} \right|^2 \qquad (10)$$

2.6 Modulations

Various modulations [21] were used throughout the project to study the advantages or disadvantages they presented depending on the scheme that was under study. In order to calculate the average E_b (bit energy), first the average E_s (symbol energy) must be calculated. The energy of a symbol is calculated as its square module, and all symbols in all constellations are considered equally probable. The calculation of the E_b is a key point in order to have control over the SNR in the simulations. All of the symbols used throughout the simulations were coded using a Gray code. Using this code, symbols that are adjacent in the constellation differ in only one bit. When the SNR is high enough, it can be assumed that a decision error in a symbol is due to an error in one bit, and no more. Here is a representation of the constellations used, with the Gray coding used for each symbol and the calculation of their energy per bit.

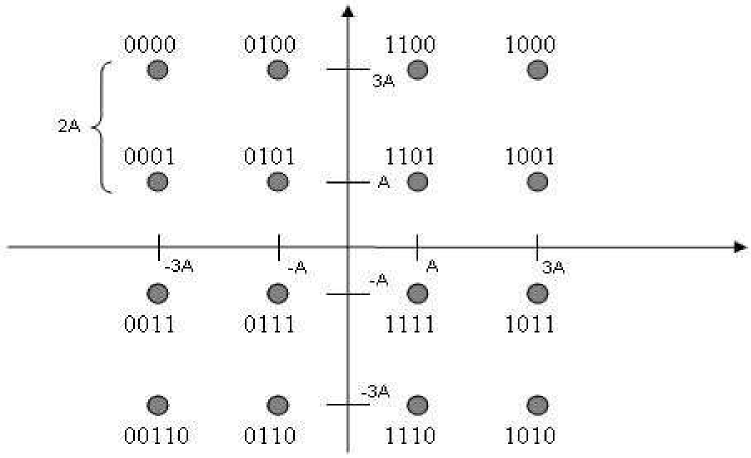
2.6.1 QPSK



$$E_s = \frac{2A^2 + 2A^2 + 2A^2 + 2A^2}{4} = \frac{8A^2}{4} = 2A^2$$

$$E_b = \frac{E_s}{q} = \frac{2A^2}{2} = A^2$$

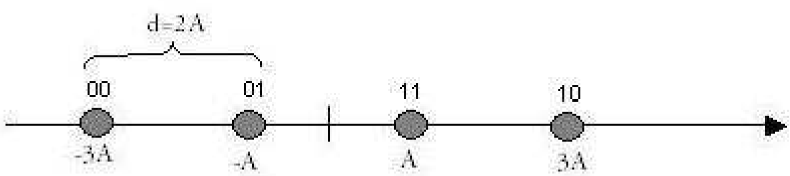
2.6.2 16-QAM



$$E_s = \frac{4\left(2A^2\right)+4\left(18A^2\right)+8\left(10A^2\right)}{16} = \frac{160A^2}{16} = 10A^2$$

$$E_b = \frac{E_s}{q} = \frac{10A^2}{4} = \frac{5A^2}{2}$$

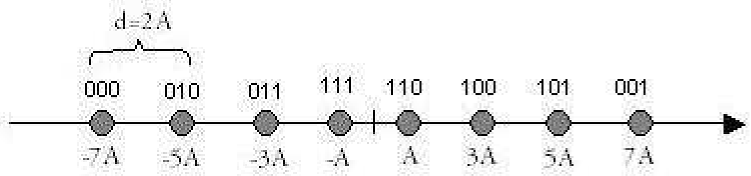
2.6.3 4-PAM



$$E_s = \frac{2(A^2) + 2(9A^2)}{4} = \frac{20A^2}{4} = 5A^2$$

$$E_b = \frac{E_s}{q} = \frac{5A^2}{2}$$

2.6.4 8-PAM



$$E_s = \frac{2(A^2) + 2(9A^2) + 2(25A^2) + 2(49A^2)}{8} = \frac{168A^2}{8} = 21A^2$$

$$E_b = \frac{E_s}{q} = \frac{21A^2}{3} = 7A^2$$

3. Alamouti Scheme

The biggest issue in making wireless transmission reliable is time-varying multipath fading, due basically to reflection, refraction and scattering. The bit error probability accomplished by a SISO (single input single output) system over a Rayleigh fading channel is not low enough to consider it a reliable wireless communication system. Alamouti came up with a simple transmit diversity scheme which improved the signal quality at the receiver in one side of the link by simple processing in two transmit antennas on the opposite side, obviously using a MISO system (multiple input single output). The idea behind using a transmit diversity scheme is that maybe some of the redundant sent signals can arrive in a better state to the receiver than others, and by exploiting them all together, the result should be better. The Alamouti scheme can also be easily generalized to two transmit antennas and M receive antennas, in a MIMO system (multiple input multiple output). One of the biggest advantages is that the scheme requires no bandwidth increase, because redundancy is applied in space and time across multiple antennas. It doesn’t require higher transmit power either. These restrictions are the most important for wireless communications systems. The new scheme is able to improve error performance, data rate or capacity of wireless systems without increasing bandwidth or transmit power. The smaller sensitivity to fading permits the system to use a higher level modulation (a modulation that transmits more bits per symbol) to increase the bit rate or a smaller reuse factor to increase the capacity.

The scheme is defined by two functions:

- the encoding and transmission sequence of information symbols at the transmitter
- the decision rule for maximum likelihood detection

Most of the information of this chapter was taken from [5].

3.1 The MISO Alamouti scheme

In this example, two transmit antennas and one receive antenna are used.

In a symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna zero will be denoted as s_0 and from antenna 1 as s_1. In the next symbol period, ($-s_1^*$) will be transmitted from antenna zero, and s_0^* will be transmitted from antenna one. The sign * denotes transposed and conjugated throughout the report.

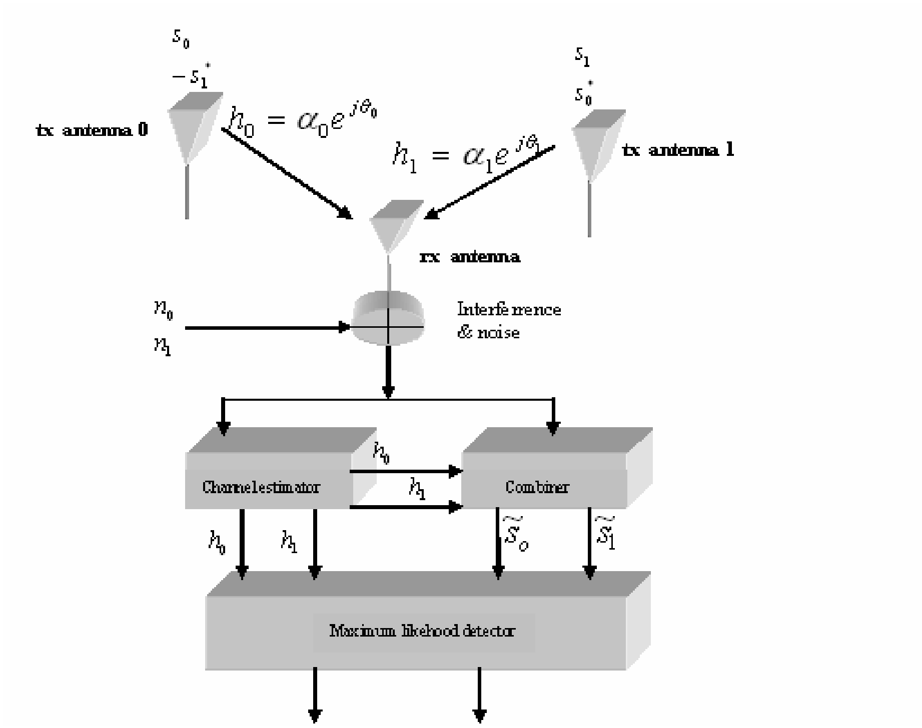


Fig 8. Diagram of the Alamouti scheme for two transmit and one receive antenna

	Tx 0	Tx 1
Time t	s_0	s_1
Time t+T	$-s_1^*$	s_0^*

Table 1.Transmission sequence for the scheme with two transmitting antennas

The channel is modeled by a complex multiplicative coefficient for both antennas, and it is assumed that fading is constant across two consecutive symbols, or in other words, the channel remains constant during two symbol periods, but will vary in the next two periods.

$$h_0(t) = h_0(t+T) = h_0 = \alpha_0 e^{j\theta_0} \quad (11)$$

$$h_1(t) = h_1(t+T) = h_1 = \alpha_0 e^{j\theta_1} \quad (12)$$

The received signal in each of the two consecutive symbol periods is:

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0 \quad (13)$$

$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \quad (14)$$

Where n_0 and n_1 are complex Gaussian random variables representing noise and interference. The combined signals, which are sent to the maximum likelihood decoder, are a simple addition of the signals received by the antennas multiplied by the coefficients of the channel, that means, it is assumed that the receiver has perfect knowledge of the channel, or perfect CSI (channel state information). The combiner shown in the figure builds the following signals and sends them to the maximum likelihood detector:

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* \quad (15)$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* \quad (16)$$

Expanding equations (15) and (16) using (11), (12), (13) and (14), the signals sent to the maximum likelihood decoder expressed as a function of the channel, the sent symbol and the noise are:

$$\tilde{s}_0 = (\alpha_0^2 + \alpha_1^2)s_0 + h_0^* n_0 + h_1 n_1^* \qquad (17)$$

$$\tilde{s}_1 = (\alpha_0^2 + \alpha_1^2)s_1 - h_0 n_1^* + h_1^* n_0 \qquad (18)$$

The maximum likelihood detector rule is to choose s_i if and only if:

$$d^2(\tilde{s}_0, s_i) \le d^2(\tilde{s}_0, s_k), \forall i \ne k \qquad (19)$$

The decision for $\tilde{s}_1$ is taken in the same way.
Equations (15), (16) and (19) are used in the program in order to do the simulation of the BER.

It is important to note that the Alamouti scheme doesn’t affect the bit rate, that is, it is still full rate or the rate is equal to one. In a 1x1 system, one symbol is transmitted in one symbol period, while in a MIMO system using the Alamouti scheme, two symbols are transmitted in two periods. The code rate is the measure of how many symbols are transmitted during one symbol period on average.

$$R = \frac{k}{P} \qquad (20)$$

Where k is the number of information symbols sent and P is the number of symbol periods during which they are sent. In Alamouti’s scheme, q bits are sent in every transmission instant, on average. It is also possible to express the rate in bits, where T denotes a symbol period:

$$R = \frac{2q}{2T} = \frac{q}{T} \qquad (21)$$

3.2 The MIMO Alamouti scheme

Two transmit antennas and two receive antennas are used in this example, but it is easy to generalize to M receive antennas. In this case, the encoding and transmission sequence of the symbols is exactly the same as in the case of a single receiver.

	Rx antenna 0	**Rx antenna 1**
Time t	r_0	r_2
Time t+T	r_1	r_3

Table 2. Notation for the received signals in each of the receive antennas

	Rx antenna 0	**Rx antenna 1**
Tx antenna 0	h_0	h_2
Tx antenna 1	h_1	h_3

Table 3. Definition of the channels between transmit and receive antennas

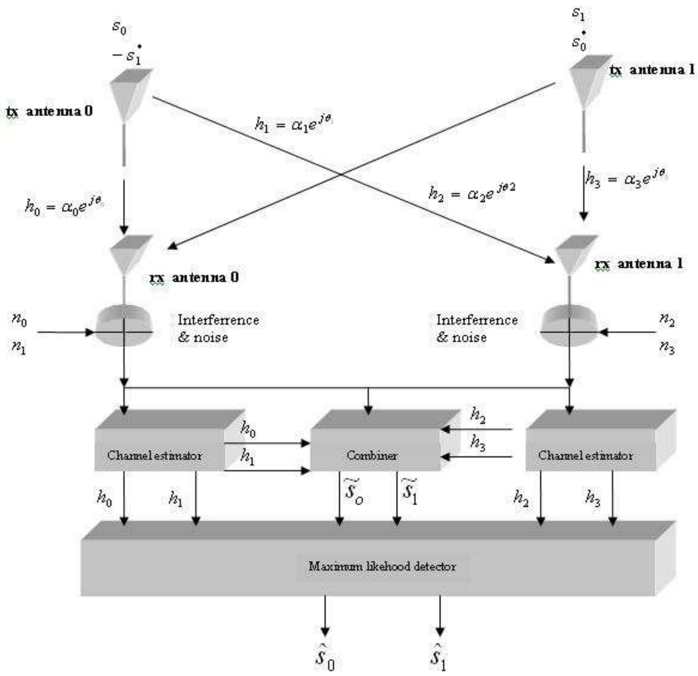


Fig 9. Diagram of the Alamouti scheme for two transmit and two receive antennas

The received signals for each of the antennas are:

$$r_0 = h_0 s_0 + h_1 s_1 + n_0 \tag{22}$$
$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1 \tag{23}$$
$$r_2 = h_2 s_0 + h_3 s_1 + n_2 \tag{24}$$
$$r_3 = -h_2 s_1^* + h_3 s_0^* + n_3 \tag{25}$$

With n_i complex Gaussian random variables.

The combiner builds these signals after the two symbol periods:

$$\tilde{s}_0 = h_0^* r_0 + h_1^* r_1 + h_2^* r_2 + h_3^* r_3 \tag{26}$$
$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^* \tag{27}$$

Expanding these signals:

$$\tilde{s}_0 = (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2) s_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^* \tag{28}$$
$$\tilde{s}_1 = (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2) s_1 - h_0 n_1^* + h_1^* n_0 - h_2 n_3^* + h_3^* n_2 \tag{29}$$

And the decision rule for the maximum likelihood detector is to decide s_i if and only if:

$$\begin{aligned}(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2 - 1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \le \\ (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2 - 1)|s_k|^2 + d^2(\tilde{s}_0, s_k), \forall i \ne k\end{aligned} \tag{30}$$

Which becomes:

$$d^2(\tilde{s}_0, s_i) \le d^2(\tilde{s}_0, s_k), \forall i \ne k \tag{31}$$

If all the symbols in the constellation have the same energy.

The decision of $\tilde{s}_1$ is taken in the same way.

Generalizing for the case of two transmit antennas and M receive antennas:

	Rx 0	Rx 1	…	Rx i	…	Rx M-1
Tx 0	h_0	h_2	…	H_{2i}	…	$h_{2(M-1)}$
Tx 1	h_1	h_3	…	H_{2i+1}	…	$h_{2(M-1)+1}$

Table 4. Definition of the channels between transmit and receive antennas

	Rx 0	Rx 1	…	Rx i	…	Rx M-1
Time t	r_0	r_2	…	R_{2i}	…	$r_{2(M-1)}$
Time t+T	r_1	r_3	…	r_{2i+1}	…	$r_{2(M-1)+1}$

Table 5. Notation for the received signals in the antennas for both symbol periods

The transmission sequence is the same as in both cases above exposed.
Analyzing the received signals for the case of two transmit antennas and two receive antennas, it is possible to generalize for the case of M receive antennas.

$$r_0 = h_0 s_0 + h_1 s_1 + n_0 \quad (32)$$
$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1 \quad (33)$$
$$r_2 = h_2 s_0 + h_3 s_1 + n_2 \quad (34)$$
$$r_3 = -h_2 s_1^* + h_3 s_0^* + n_3 \quad (35)$$
$$r_{2i} = h_{2i} s_0 + h_{2i+1} s_1 + n_{2i} \quad (36)$$
$$r_{2i+1} = -h_{2i} s_1^* + h_{2i+1} s_0^* + n_{2i+1} \quad (37)$$
$$r_{M-2} = h_{M-2} s_0 + h_{M-1} s_1 + n_{M-2} \quad (38)$$
$$r_{M-1} = -h_{M-2} s_1^* + h_{M-1} s_0^* + n_{M-1} \quad (39)$$

Below it is shown in vectorial notation, which will be used from now on.

$$S = \begin{pmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{pmatrix} = \left(s[1] \quad s[2]\right) \quad (40)$$

$$H = \begin{pmatrix} h_o & h_1 \\ h_2 & h_3 \\ h_4 & h_5 \\ \vdots & \vdots \\ h_{2i} & h_{2i+1} \\ \vdots & \vdots \\ \vdots & \vdots \\ h_{2M-2} & h_{2M-1} \end{pmatrix} = \begin{pmatrix} h_a & h_b \end{pmatrix} \qquad (41)$$

$$w = \begin{pmatrix} n_0 & n_1 \\ n_2 & n_3 \\ n_4 & n_5 \\ \vdots & \vdots \\ n_{2i} & n_{2i+1} \\ \vdots & \vdots \\ \vdots & \vdots \\ n_{2M-2} & n_{2M-1} \end{pmatrix} = \begin{pmatrix} w[1] & w[2] \end{pmatrix} \qquad (42)$$

$$r = HS + w = \begin{pmatrix} r_0 & r_1 \\ r_2 & r_3 \\ r_4 & r_5 \\ \vdots & \vdots \\ r_{2i} & r_{2i+1} \\ \vdots & \vdots \\ \vdots & \vdots \\ r_{2M-2} & r_{2M-1} \end{pmatrix} \qquad (43)$$

The combined symbol is a linear combination of the received signals and the channel coefficients, and that fact makes the receiver of the Alamouti scheme very simple to design, independently of how many receive antennas are deployed.

$$\tilde{s}_0 = h_a^* r[1] + r^*[2] h_b \tag{44}$$

$$\tilde{s}_1 = h_b^* r[1] - r^*[2] h_a \tag{45}$$

$$\tilde{s}_0 = s_0 \sum_{i=0}^{2M-1} \alpha_i^2 + \sum_{i=0}^{M-1} (h_{2i}^* n_{2i} + h_{2i+1} n_{2i+1}^*) \tag{46}$$

$$\tilde{s}_1 = s_1 \sum_{i=0}^{2M-1} \alpha_i^2 + \sum_{i=0}^{M-1} (-h_{2i} n_{2i+1}^* + h_{2i+1}^* n_{2i}) \tag{47}$$

The maximum likelihood detector, similarly to the case of two receiving antennas, will decide s_i if and only if:

$$\left(\sum_{i=0}^{2M-1} \alpha_i^2 - 1\right)|s_i|^2 + d^2(\tilde{s}_0, s_i) \le \left(\sum_{i=0}^{2M-1} \alpha_i^2 - 1\right)|s_k|^2 + d^2(\tilde{s}_0, s_k), \forall i \ne k \tag{48}$$

$$\left(\sum_{i=0}^{2M-1} \alpha_i^2 - 1\right)|s_i|^2 + d^2(\tilde{s}_1, s_i) \le \left(\sum_{i=0}^{2M-1} \alpha_i^2 - 1\right)|s_k|^2 + d^2(\tilde{s}_1, s_k), \forall i \ne k \tag{49}$$

Equation (43) has great importance in orthogonal space time block codes. It is valid independently of the number of transmit or receive antennas, although the dimensions of the matrices do vary, and even if the scheme used is not Alamouti's. If the symbol matrix, or transmission sequence and the channel coefficient matrix are defined in a certain manner, the received signals are obtained in a matrix just by adding the noise. In general, if there are N_{tx} transmit antennas and N_{rx} receive antennas, the channel matrix can be defined as:

$$H = \begin{pmatrix} h_1 & \cdots & \cdots & h_{N_{tx}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ h_{N_{tx}N_{rx}-N_{tx}+1} & \cdots & \cdots & h_{N_{tx}N_{rx}} \end{pmatrix} \tag{50}$$

Each column contains all the coefficients that define the channel from a transmit antenna to all the other receive antennas. Every row contains the channel coefficients from every transmit antenna to one of the receive antennas. The noise matrix is defined as:

$$W = \begin{pmatrix} n_1 & \cdots & \cdots & n_{N_p} \\ n_{N_p(N_{rx}-1)+1} & n_{N_p(N_{rx}-1)+2} & \cdots & n_{N_p(N_{rx}-1)+N_p} \\ \vdots & \vdots & \vdots & \vdots \\ n_{N_pN_{rx}-N_p+1} & n_{N_pN_{rx}-N_{px}+2} & \cdots & n_{N_pN_{rx}} \end{pmatrix} = \left(w[1] \ \cdots \ \cdots \ w[N_p] \right) \tag{51}$$

Where N_p is the number of periods during which an information block is sent. In Alamouti's scheme, the number of periods is 2. And the received signal matrix is:

$$r = HS + W = \begin{pmatrix} r_1 & \cdots & \cdots & r_{N_p} \\ r_{N_p(N_{rx}-1)+1} & r_{N_p(N_{rx}-1)+2} & \cdots & r_{N_p(N_{rx}-1)+N_p} \\ \vdots & \vdots & \vdots & \vdots \\ r_{N_pN_{rx}-N_p+1} & r_{N_pN_{rx}-N_p+2} & \cdots & r_{N_{tx}N_{rx}} \end{pmatrix} = \left(r[1] \ \cdots \ \cdots \ r[N_p] \right) \tag{52}$$

Where every column i of the matrix contains the signals received by all the antennas in symbol period i. And every row contains the signal received by one antenna in the consecutive symbols. The so called coding matrix, s, can't be defined generically. The first column contains all the information symbols that are ought to be sent during the first symbol period, but the other columns must be all orthogonal. The only restriction is that the s matrix must have as the number of rows, the number of columns of the H matrix. As will be seen in the spatial multiplexing chapter, the coding matrix can be very variable, depending on which is the goal of the communication link.

In Alamouti's paper, his combining scheme is just stated. It is unclear how he came up with the idea of combining the received signals with the specific channel coefficients. It is well known, in the other hand, that the ML receiver is the optimal receiver, in the sense that the result of applying that receiver will give the best possible result in bit error probability. What Alamouti did was apply the ML decision criterion to the received signal in order to make, theoretically, a joint decision of which were the two symbols that were sent. But defining carefully some vectors involved in the calculation, it comes out that the decision of each symbol can be taken separately.

If H is defined, where h_a and h_b are column vectors that contain the channel coefficients from transmit antenna 1 and 2 respectively to all of the receiving antennas.

$$H = \begin{pmatrix} h_a & h_b \end{pmatrix} \tag{53}$$

During the first of the symbol periods, the total sent signal to the downlink is:

$$H\begin{pmatrix} s_0 \\ s_1 \end{pmatrix} = s_0 h_a + s_1 h_b = V_1 \tag{54}$$

During the second period:

$$H\begin{pmatrix} -s_1^* \\ s_0^* \end{pmatrix} = -s_1^* h_a + s_0^* h_b = V_2 \qquad (55)$$

If the following vectors are defined:

$$X_1 = \begin{pmatrix} s_0 h_a \\ -s_1^* h_a \end{pmatrix} \qquad (56)$$

$$X_2 = \begin{pmatrix} s_1 h_b \\ s_0^* h_b \end{pmatrix} \qquad (57)$$

Note that the scalar product between these two vectors is 0, or in other words, they are orthogonal.

$$X_\alpha^* X_\gamma = 0 \qquad \mathit{for} \quad \alpha \neq \gamma \qquad (58)$$

Defining the following vectors:

$$R = \begin{pmatrix} r[1] \\ r[2] \end{pmatrix} \qquad (59)$$

$$Z = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = X_1 + X_2 \qquad (60)$$

Where r[1] and r[2] are column vectors that contain all the signals received by the antennas in period symbols 1 and 2 respectively.

The ML decision criterion to take a joint decision for the two symbols sent would be:

$$ML\ :\min_{s_0 s_1}\ \left\| R - Z \right\|^2 = \min_{s_0 s_1}\ \left(R - Z \right)^* \left(R - Z \right) =$$
$$= \min_{s_0 s_1}\ \left(- \left(Z^* R + R^* Z \right) + \left\| Z \right\|^2 \right) \qquad (61)$$

Where the term $\|R\|^2$ is not taken into account, because the minimization is respect to the symbols sent, and that term is just a fixed number the receiving antennas got from the transmission, so it would not be influenced by s_0 and s_1.
If the term $\|Z\|^2$ is expanded:

$$\left\|Z\right\|^2 = \left\|X_1 + X_2\right\|^2 = \left\|X_1\right\|^2 + \left\|X_2\right\|^2 = \left(\left|s_0\right|^2 + \left|s_1\right|^2\right)\left(\left\|h_a\right\|^2 + \left\|h_b\right\|^2\right) =$$
$$= \left|s_0\right|^2 \left\|H\right\|^2 + \left|s_1\right|^2 \left\|H\right\|^2 \qquad (62)$$

Where the crossed terms are 0, due to the orthogonality of X_1 and X_2. Hence, $\|Z\|^2$ is separable in two different equations, one that depends on s_0 and one that depends on s_1.

Now, if Z is rewritten as:

$$Z = \begin{pmatrix} s_0 h_a \\ s_0^* h_b \end{pmatrix} + \begin{pmatrix} s_1 h_b \\ -s_1^* h_a \end{pmatrix} \qquad (63)$$

Then:

$$Z^* R = s_0^* h_a^* r[1] + s_0 h_b^* r[2] + s_1^* h_b^* r[1] - s_1 h_a^* r[2] \qquad (64)$$
$$R^* Z = s_0 h_a r^*[1] + s_0^* h_b r^*[2] + s_1 h_b r^*[1] - s_1^* h_a r^*[2] \qquad (65)$$

Putting equations (64) and (65) together:

$$
\begin{aligned}
& Z^{*}R + R^{*}Z = \\
& s_0^{*}\left(h_a^{*} r[1] + h_b r^{*}[2]\right) + \\
& s_0\left(h_b^{*} r[2] + h_a r^{*}[1]\right) + \\
& s_1^{*}\left(h_b^{*} r[1] - h_a r^{*}[2]\right) + \\
& s_1\left(- h_a^{*} r[2] + h_b r^{*}[1]\right) = \\
& s_0^{*}\tilde{s}_0 + s_0\tilde{s}_0^{*} + s_1^{*}\tilde{s}_1 + s_1\tilde{s}_1^{*} = \\
& s_n^{*}\tilde{s}_n + s_n\tilde{s}_n^{*}
\end{aligned}
\qquad (66)
$$

Where the equation depends on s_0 and on s_1. The equation has to be minimized. But it can be separated in two separate equations, one that depends only on s_0 and one that depends only on s1, and minimizing each of those equations separately will give the same result than minimizing the whole equation. That, and the fact that $\|Z\|^2$ is also separable, means that the maximum likelihood decision can actually be taken separately for s_0 and for s_1.

And finally, the ML criterion results to be:

$$
ML : \min_{s_\alpha}\left(-\left(s_\alpha^{*}\tilde{s}_i + s_\alpha\tilde{s}_i^{*}\right) + |s_\alpha|^2\|H\|^2\right) \quad \alpha = 0 \ldots n \quad i = 0,1 \qquad (67)
$$

Which matches exactly with the criterion used by Alamouti in his paper. Where $\tilde{s}_i$ is the combined symbol that Alamouti used to take a decision, and n represents the different number of possible symbols of the modulation used. It would be 1 for BPSK, 3 for QPSK and 15 for 16-QAM. If all the symbols in the modulation have the same energy, like in BPSK or QPSK, the second part of the equation can be removed, because the same term will appear in both sides of the equation, independently of the symbol that is being tested. And the criterion comes out to be to decide s_i iff:

$$
d^2(\tilde{s}_0, s_i) \le d^2(\tilde{s}_0, s_k), \forall i \ne k \qquad (68)
$$

3.2.1 Simulation assumptions

The systems were simulated to find the bit error probability. The systems simulated were 1x1, 2x1, 2x2 and 2x4, using both QPSK and 16-QAM. The 1x1 systems simulated are obviously not using the Alamouti diversity scheme, but they were simulated to have a reference, and to show that a SISO communication system over a Rayleigh channel doesn't accomplish the necessary BER to be considered reliable. The amplitudes of fading from each transmit to each receive antenna, or channel coefficients, are assumed to be mutually uncorrelated complex and Rayleigh distributed, and the average powers at the receive antenna from each transmit antenna are the same. The mean of the channel coefficients used was 0 and the variance was 1 for each of the real and imaginary components of the channel. The definition of SNR used was:

$$SNR = 10\log_{10}\frac{E_b}{N_0} \tag{69}$$

The E_b is calculated in a different way than it is in 1x1 systems, taking into account the number of transmit and receive antennas:

$$E_b = N_{tx}N_{rx}2\sigma^2\frac{E_S}{q} \tag{70}$$

Where N_{tx} is the number of transmit antennas, N_{rx} is the number of receive antennas, E_s is the average symbol energy and q is the number of bits per symbol. Using these two equations, it is possible to find the value of N_0 in terms of the value of SNR. In every simulation $N_0/2$ was used as the value of the variance of the noise for the real and the imaginary components.

3.2.2 Eb derivations

In order to calculate the average received Energy per bit, first, the received total energy in one symbol period should be calculated.

$$r = Hs + w \tag{71}$$

As stated before, this expression is the model for the received signal in every MIMO system under study in this project. The first part of the equation represents the useful signal, and the second part represents the contamination of the noise. If Z is defined as follows:

$$\begin{pmatrix} z_1 \\ z_2 \\ . \\ . \\ z_{Nr} \end{pmatrix} = Z = Hs \qquad (72)$$

Where s is not the entire matrix of all the symbols sent during one complete transmission, but only the first column vector, or the symbols sent during the first symbol period, and z_i represents each of the received signals in the receive antennas. The total Energy received during one period should be calculated as:

$$E\left\{\|Hs\|^2\right\} = E\left\{(Hs)^* Hs\right\} = E\left\{|z_1|^2 + |z_2|^2 + \ldots.. + |z_{Nr}|^2\right\} \qquad (73)$$

$$|z_i|^2 = z_i z_i^* \qquad (74)$$

$$z_i = \sum_{j=1}^{Nt} h_{ij} s_j \qquad (75)$$

$$\overline{E}_{tot} = \sum_{i=1}^{Nr} E\left\{|z_i|^2\right\} = \sum_{i=1}^{Nr} E\left\{\sum_{j=1}^{Nt} h_{ij} s_j \sum_{m=1}^{Nt} h_{im}^* s_m^*\right\} = \sum_{i=1}^{Nr}\sum_{j=1}^{Nt}\sum_{m=1}^{Nt} E\left\{h_{ij} h_{im}^* s_j s_m^*\right\} \qquad (76)$$

Which comes from expanding $|z|^2$ and using the property that the expected value is a linear operator.

Taking into account the fact that the channel coefficients and the sent symbols are statistically independent:

$$\sum_{i=1}^{Nr}\sum_{j=1}^{Nt}\sum_{m=1}^{Nt} E\left\{h_{ij}h_{im}^{*}\right\}E\left\{s_j s_m^{*}\right\} \qquad (77)$$

And using:

$$E\left\{s_i s_j^{*}\right\} = 0 \qquad if \quad i \neq j \qquad (78)$$

Due to the fact that different sent symbols are independent and their mean is 0, because in the employed modulations, the symbols are symmetrically arranged around the coordinate origin in the constellation:

$$\sum_{i=1}^{Nr}\sum_{j=1}^{Nt} E\left\{\left|h_{ij}\right|^2\right\}E\left\{\left|s_j\right|^2\right\} \qquad (79)$$

In general, in any random variable:

$$E\left\{\left|x\right|^2\right\} = E\left\{\left|x - m_x\right|^2\right\} + m_x^2 \qquad (80)$$

If the random variable is Rayleigh distributed, the mean is 0, and the power is the same as the variance. In the case of the channel coefficients, used in the simulation, which are complex, and therefore have a real and an imaginary part:

$$E\left\{\left|h_{ij}\right|^2\right\} = E\left\{(h_R + jh_I)(h_R - jh_I)\right\} = E\left\{h_R^2\right\} + E\left\{h_I^2\right\} = 2\sigma^2 \qquad (81)$$

So finally, the total received energy in one symbol period in a Rayleigh fading channel is:

$$E_s\sum_{i=1}^{Nr}\sum_{j=1}^{Nt} 2\sigma^2 = E_s N_t N_r 2\sigma^2 \qquad (82)$$

To have the total received energy, the above result should be multiplied by the number of periods during which a block of information is sent. Note that in Alamouti's scheme, the number of transmitting antennas is exactly the same as the number of periods throughout which a block of symbols is sent. And to get the final result for the average bit energy received, the total energy received during the block should be divided by the number of information bits decoded in each block.

$$N_p = N_t \tag{83}$$

$$\frac{N_p N_t N_r 2\sigma^2 E_s}{N_t q} = \frac{E_{tot}}{N_t q} \tag{84}$$

And the final result is the one stated in (67).

3.2.3 Simulation Results

The 1x1 QPSK system is obviously not a reliable communication system, even in 15 dB SNR conditions. The 2x1 Alamouti system clearly improves the performance of the 1x1 classical system under Rayleigh fading conditions, although it could not be used in some communication applications that require a low BER. The systems become reliable when the number of receive antennas is increased. Note that the slope of the curves increases with the number of receive antennas. That is due to the increased diversity order, which depends on the number of antennas deployed.

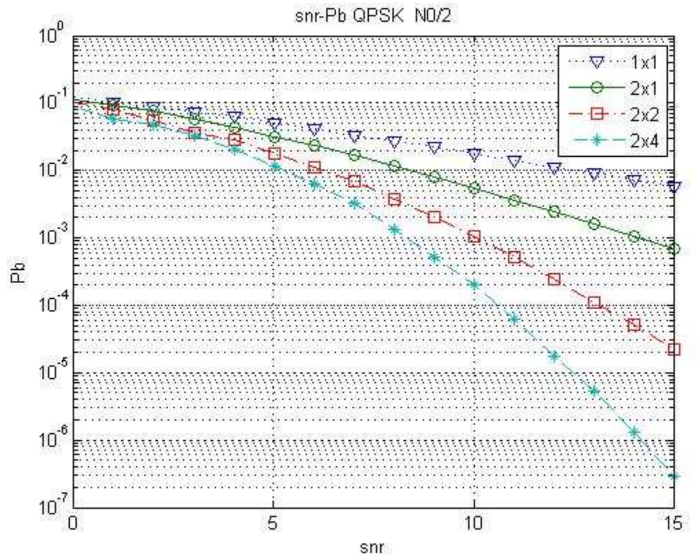


Fig 10.- BER under different SNR conditions for 1x1 and Alamouti 2x1, 2x2, 2x4 systems under Rayleigh channel conditions using QPSK modulation

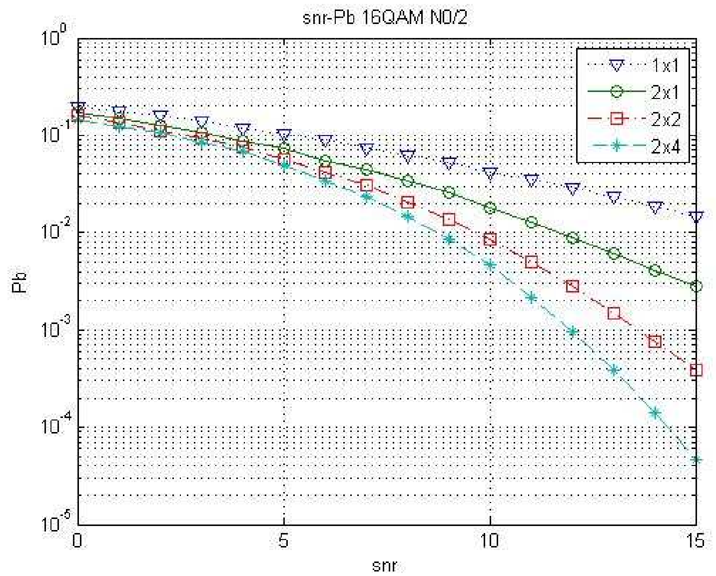


Fig 11.- BER under different SNR conditions for 1x1 and Alamouti 2x1, 2x2, 2x4 systems under Rayleigh channel conditions using 16-QAM modulation

In the 16 QAM simulations, the same conclusions as in QPSK can be taken. Note that the BER performance using the 16-QAM is not as good as the QPSK, but in compensation, the bit rate for 16-QAM doubles the one for QPSK.

In the comparative simulation that follows, it is clear that varying the bit rate leads to a payback in BER values and that increasing the number of receive antennas increases the reliability of the system. Note also that the slope of the simulation curves doesn't depend on the modulation used, and only on the number of antennas deployed. Both 2x4 simulations have the same slope when the SNR increases, just like the 2x2 simulations.

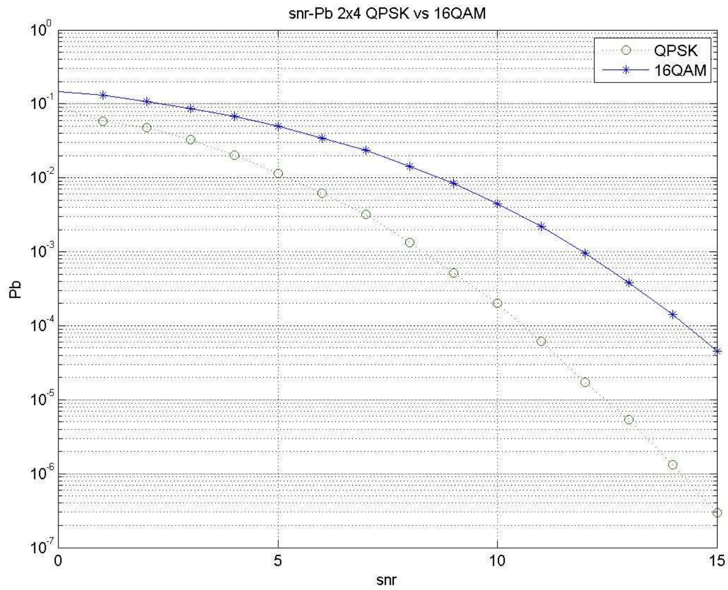


Fig 12.- Comparison between the 2x4 Alamouti scheme using 16-QAM and QPSK

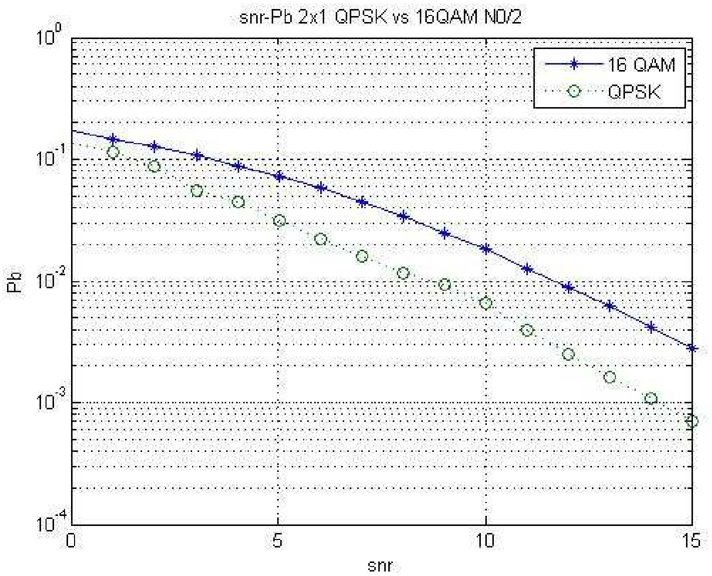


Fig 13. Comparison between the 2x1 Alamouti scheme using 16-QAM and QPSK

3.3 Bounds

In order to verify the validity of the program and its simulation results, specially using the QAM modulation, the graphs were compared with lower and upper bounds of the bit error probability. As a lower bound, the results of the symbol error probability of a single-input single-output stationary AWGN system was used, while as an upper bound, the result for the symbol error probability for M-ary QAM in presence of AWGN channel was used.

3.3.1 Lower bound (M-QAM)

Any system can perform, at the most, ideally, having the same BER of a SISO system in presence of AWGN. The well known result for the symbol error probability in a SISO system in presence of AWGN is:

$$P_s = 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{d_{\min}^2\frac{E_b}{N_0}}\right) \quad (85)$$

$$d_{\min}^2 = \frac{3\log_2(M)}{M-1} \quad (86)$$

In particular, for 16-QAM

$$P_s = 4\left(\frac{3}{4}\right)Q\left(\sqrt{\frac{12}{15}\frac{E_b}{N_0}}\right) \quad (87)$$

$$Q(x) = \frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right) \quad (88)$$

$$P_s = 3\frac{1}{2}erfc\left(\sqrt{\frac{12}{15}\frac{E_b}{N_0}\frac{1}{2}}\right) = \frac{3}{2}erfc\left(\sqrt{\frac{6}{15}\frac{E_b}{N_0}}\right) \quad (89)$$

3.3.2 Upper bound

In this bound, it was considered, just like in the simulations, that the 16 symbols are equally probable. Hence, the decision boundaries are exactly in between the middle of the symbols in the constellation. And the result comes out to be:

$$P_b \le P_s \le E\left\{4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right)\right\} \qquad Q(x) \le \frac{1}{2}e^{-\frac{x^2}{2}} \qquad \forall x \ge 0 \quad (90)$$

$$D_{\min}^2 = 4A^2\|H\|^2 \quad (91)$$

Where the minimum distance between symbols depends on the channel coefficients.

$$D_{\min}^{2} = 4A^{2}\sum_{i=1}^{N_r}\sum_{j=1}^{N_t}\left|h_{ij}\right|^{2} \qquad \left|h_{ij}\right|^{2} = \left(h_{ij}^{\mathrm{Re}}\right)^{2} + \left(h_{ij}^{\mathrm{Im}}\right)^{2} \qquad (92)$$

Expanding (90) using (93):

$$P_s \le 2\left(1-\frac{1}{\sqrt{M}}\right)E\left\{e^{-\frac{D_{\min}^{2}}{4N_0}}\right\} = 2\left(1-\frac{1}{\sqrt{M}}\right)E\left\{e^{-\frac{A^{2}\sum_{i=1}^{N_r}\sum_{j=1}^{N_t}\left|h_{ij}\right|^{2}}{N_0}}\right\} =$$

And because the real and imaginary part of the channel coefficient are independent:

$$= 2\left(1-\frac{1}{\sqrt{M}}\right)\left[E\left\{e^{-\frac{A^{2}}{N_0}\left(h_{11}^{\mathrm{Re}}\right)^{2}}\right\}E\left\{e^{-\frac{A^{2}}{N_0}\left(h_{11}^{\mathrm{Im}}\right)^{2}}\right\}\cdots\cdots\right] \qquad (93)$$

Using the expected value theorem:

$$E\left\{e^{-\frac{A^{2}}{N_0}\left(h_{ij}^{RE}\right)^{2}}\right\} = \int_{-\infty}^{\infty} e^{-\frac{A^{2}}{N_0}\left(h_{ij}^{\mathrm{Re}}\right)^{2}} \frac{1}{\sigma_{ij}^{\mathrm{Re}}\sqrt{2\pi}} e^{-\frac{\left(h_{ij}^{\mathrm{Re}}-m_{ij}^{\mathrm{Re}}\right)}{2\left(\sigma_{ij}^{\mathrm{Re}}\right)^{2}}} dh_{ij} = \cdots$$

$$= \frac{e^{-\frac{A^{2}\left(m_{ij}^{\mathrm{Re}}\right)^{2}}{N_0\left(1+\frac{A^{2}}{N_0}2\left(\sigma_{ij}^{\mathrm{Re}}\right)^{2}\right)}}}{\sqrt{1+\frac{A^{2}}{N_0}2\left(\sigma_{ij}^{\mathrm{Re}}\right)^{2}}} \qquad (94)$$

Taking into account that in pure Rayleigh fading, the mean is 0 and the variance was taken as 1 in the simulations:

$$m_{ij}^{\mathrm{Re,Im}} = 0, \quad \sigma_{ij}^{\mathrm{Re,Im}} = 1$$

$$E\left\{ e^{-\frac{A^2}{N_0}\left(h_{ij}^{\mathrm{Re,\ Im}}\right)^2} \right\} = \frac{1}{\sqrt{1 + \frac{2A^2}{N_0}}} \tag{95}$$

$$P_s \le 2\left(1 - \frac{1}{\sqrt{M}}\right)\left(\frac{1}{\sqrt{1 + \frac{2A^2}{N_0}}}\right)^{2N_{tx}N_{rx}} \tag{96}$$

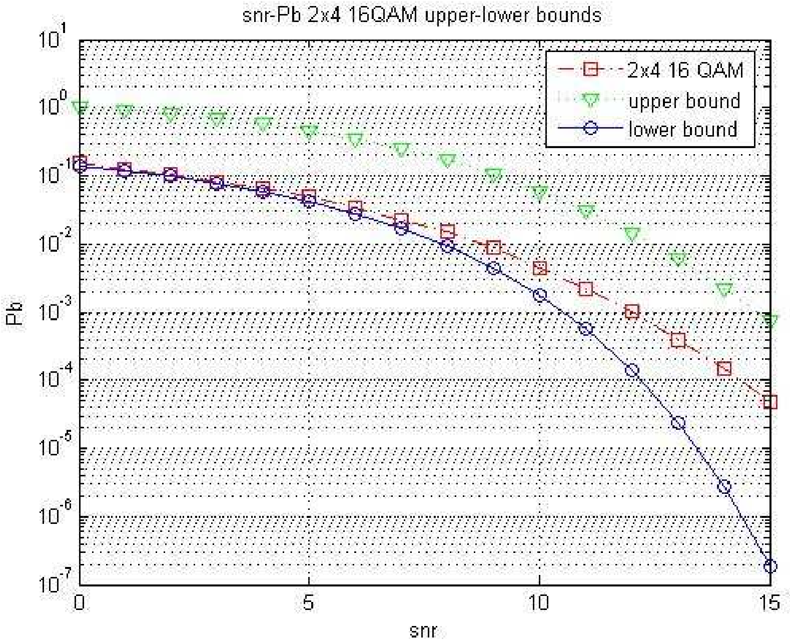


Fig 14.- 2x4 16-QAM Alamouti scheme and upper and lower bounds

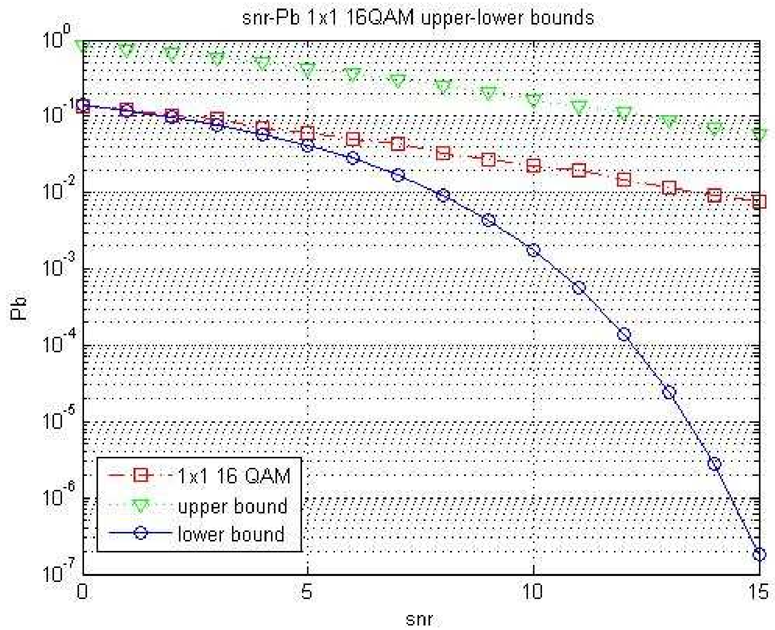


Fig 15.- 1x1 16 QAM Alamouti scheme and upper and lower bounds

Gray coding was used in the program, so it can be considered that a symbol error is caused by only one bit error, and not more. This means that the errors are committed only in between symbols which are next to each other in the constellation. And using a 16-QAM modulation, with 4 bits per symbol, it can be estimated that the BER is approximately equal to $P_s/4$. So the upper bound plotted in the figures corresponds to $P_s/4$.
An important aspect that is provided by the Alamouti scheme is the diversity order. It is defined as the slope of the log P_b, and it is a measure of how fast the error probability drops as the E_b/N_0 is increased. In order to be more specific, in this project, the 16-qam system is analyzed. Taking into account that when E_b/N_0 increases, the slope of the upper bound and the simulation are the same, the slope of the upper bound will be analyzed, for which we have a definite expression.

$$P_b = \frac{3}{2}\left(\frac{1}{\sqrt{1+\frac{2}{10}\frac{E_b}{N_0}}}\right)^{2N_{tx}N_{rx}} \tag{97}$$

$$\lim_{\frac{E_b}{N_0}\to\infty} \frac{3}{2}\left(\frac{1}{\sqrt{1+\frac{2E_b}{10N_0}}}\right)^{2N_{tx}N_{rx}} = \frac{3}{2}\left(\frac{1}{\frac{2E_b}{10N_0}}\right)^{N_{tx}N_{rx}} \tag{98}$$

$$\log P_b = \log\frac{3}{2} - N_{tx}N_{rx}\log\frac{2}{10}\frac{E_b}{N_0} \tag{99}$$

$$\log P_b = \log\frac{3}{2} - N_{tx}N_{rx}\log\frac{2}{10} - N_{tx}N_{rx}\log\frac{E_b}{N_0} \tag{100}$$

The first two terms are constant, and don't influence the slope of log E_b/N_0. The minus is due to the fact that the P_b obviously decreases when the SNR increases. So it is considered that the diversity order provided by the Alamouti scheme is N_tN_r. The bigger the diversity order is, the closer the communication system gets to the performance of the SISO in presence of stationary AWGN, but it should have a diversity of infinity to actually reach the performance of the SISO AWGN system

3.4 Comparison

Different applications over wireless communications systems require different minimum bit rates and bit error rates. Depending on the application, the constraints can be more or less restrictive. As an example, voice applications require from 8 to 32 kbps and a maximum BER of 10^{-3}, while database access and file transfers can require up to 1Mbps and a maximum BER of 10^{-7}. The GSM system requires a BER from 10^{-3} to 10^{-5} after channel coding[12]. Hence, the number of transmitting or receiving antennas and the modulation used must be chosen depending on the application they are going to be used for and its constraints. The constraint in the number of antennas used is basically because of the space limitations. For cellular phone communications, for instance, it can be possible, today, to deploy more than one transmitting antenna in one side of the link, in the base station, but not in the other, the cellular phone, although many advances are being done in this field. The use of a higher level modulation involves the transmission of more bits per symbol period, which means that there will be an increase in bit rate, but there will be a trade-off in bit error probability. If the BPSK (2 possible symbols) modulation is used, 1 bit per symbol period will be sent. If the modulation used is QPSK (four possible symbols), the number of bits sent per symbol period will be 2, while in 16-QAM (16 possible symbols), 4 bits will be sent during every symbol period.

To analyze and compare the different communication systems simulated, 10^{-4} was chosen as a reference in bit error probability. The different quality of the communication links needed by the different schemes to achieve that constraint is shown in the following table. In what concerns the bit rate, it would be more difficult to choose a reference, and the duration of the symbol period would be needed, which depends on the bandwidth used. These aspects are not part of the goal of the project, and although they are important in the design of a communication system, the table only gives a reference in what concerns bit error probability.

N_{tx} * N_{rx}	SNR (dB) QPSK	SNR (dB) 16-QAM
2x4	10.5	14.2
2x2	13.1	16.8
2x1	19.1	22.2
1x1	32.8	37

Table 6.- SNR needed to achieve a BER of 10^{-4}

The SNR needed to achieve the constraint in bit error probability is always bigger using the 16-QAM constellation, but in the other hand, the bit rate using 16-QAM, doubles the bit rate of the situation when QPSK is used.

The 1x1 systems under Rayleigh fading channel are included in the comparison, to show their unreliability under Rayleigh fading. Notice the more the approximate improvement of 14 dB in any of the modulations when the Alamouti diversity scheme is used.

4. OSTBC: orthogonal space time block codes

In space-time block codes, multiple copies of a data stream are transmitted over several antennas, and with simple processing in the receiving antennas, the different received versions of the signal are used to improve the reliability of the communication link. The theory of orthogonal designs is used to design analogs of Alamouti's scheme, which are named space-time block codes when more than two transmit antennas are used. In orthogonal space time block codes, any two pairs of columns of the coding matrix result to be orthogonal. The coding matrix is the matrix that defines from which antenna and in which instant the symbols are sent. The orthogonal structure of the code enables the receiver to use a simple maximum likelihood decoding algorithm and take decisions for the symbols separately instead of having to take a joint decision, as was shown in (67) for Alamouti's scheme.

It is possible to design an orthogonal space-time block code for any arbitrary number of transmit antennas, using either real or complex constellations. However, these codes only reach the maximum or full transmission rate when the constellation is real, such as PAM. With the use of a complex constellation, like QAM, the block codes achieve a rate of ½ , independent from the number of transmit antennas. For two, three, and four transmit antennas, it is possible to design a STBC that achieves ¾ of the maximum rate with a complex constellation. The focus of this project is on the full- rate OSTBC, that is, using real constellations, specifically, 2, 4,and 8-PAM, and using always four transmit antennas and analyzing two different numbers of receive antennas, one and four. In general, these cases were studied under the influence of a Rician channel.

In OSTBC, the way of achieving diversity, which is the most practical form of combating severe Rayleigh fading, is through space and time, or what is the same, without the need of increasing the bandwidth used, which is in general restricted. Hence, the diversity is obtained with the deployment of several antennas at the transmitter and/or the receiver. Moreover, in most of the applications, the remote station is required to be small, so it is not possible, today, to deploy multiple receive antennas, and the diversity must be achieved in transmission. That is the reason why the case of four transmit antennas and one receive is also studied.

4.1 The MIMO 4xNr OSTBC scheme

H is defined similarly to how it was done in the Alamouti scheme, but now with four transmit antennas. The ith column of the matrix defines the channel coefficients between the ith antenna and the N_{rx} receive antennas.

$$H = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ h_5 & h_6 & h_7 & h_8 \\ \vdots & \vdots & \vdots & \vdots \\ h_{4N_{rx}-3} & h_{4N_{rx}-2} & h_{4N_{rx}-1} & h_{4N_{rx}} \end{pmatrix} = \begin{pmatrix} h_a & h_b & h_c & h_d \end{pmatrix} \qquad (100)$$

The coding matrix is defined as [6]:

$$S = \begin{pmatrix} s_0 & -s_1 & -s_2 & -s_3 \\ s_1 & s_0 & s_3 & -s_2 \\ s_2 & -s_3 & s_0 & s_1 \\ s_3 & s_2 & -s_1 & s_0 \end{pmatrix} = \begin{pmatrix} s[1] & s[2] & s[3] & s[4] \end{pmatrix} \qquad (101)$$

Note that it applies only for real constellations, due to the fact that any pair of columns of S are orthogonal if and only if the symbols of the modulation are real. If the matrix is analyzed, some interesting features can be recognized. The row of the matrix indicates which of the transmit antennas is going to send the symbols. The first row of symbols will be sent by the first antenna, the second row will be sent by the second antenna, and so on. Hence, the number of rows indicates both the number of transmit antennas being used and the number of effective information symbols that are going to be sent during the transmission. The column of the matrix indicates in which symbol period the symbols are ought to be sent. The first column of symbols will be sent by the four transmit antennas simultaneously during the first symbol period, the second column will be sent during the second symbol period, and so forth. So taking a look at the equation written before:

$$R = \frac{k}{P} \tag{102}$$

With the knowledge of the dimensions of the S matrix, it is possible to know the rate of the transmission scheme. k is the number of symbols sent, or number of rows in the matrix, and P is the number of symbols during which they are sent, or the number of columns. A square matrix means that the scheme reaches the full rate, while a matrix with more columns than rows doesn’t reach the full rate.

Each of the signals that reach the receive antennas is contaminated by noise:

$$W = \begin{pmatrix} n_1 & n_2 & n_3 & n_4 \\ n_5 & n_6 & n_7 & n_8 \\ \vdots & \vdots & \vdots & \vdots \\ n_{4N_{rx}-3} & n_{4N_{rx}-2} & n_{4N_{rx}-1} & n_{4N_{rx}} \end{pmatrix} = \begin{pmatrix} w[1] & w[2] & w[3] & w[4] \end{pmatrix} \tag{103}$$

And the received signals in the receive antennas in each symbol period are:

$$r = HS + W = \begin{pmatrix} r_1 & r_2 & r_3 & r_4 \\ r_5 & r_6 & r_7 & r_8 \\ \vdots & \vdots & \vdots & \vdots \\ r_{4N_{rx}-3} & r_{4N_{rx}-2} & r_{4N_{rx}-1} & r_{4N_{rx}} \end{pmatrix} = \begin{pmatrix} r[1] & r[2] & r[3] & r[4] \end{pmatrix} \quad (104)$$

Where r_i can be expressed as a function of the channel coefficients, the sent symbols and the noise.

$$\begin{aligned} r_1 &= h_1 s_0 + h_2 s_1 + h_3 s_2 + h_4 s_3 + n_1 \\ r_2 &= -h_1 s_1 + h_2 s_0 - h_3 s_3 + h_4 s_2 + n_2 \\ &\vdots \\ r_{4N_{rx}} &= -h_{4N_{rx}-3} s_3 - h_{4N_{rx}-2} s_2 + h_{4N_{rx}-1} s_1 + h_{4N_{rx}} s_0 + n_{4N_{rx}} \end{aligned} \quad (105)$$

Note that the above definitions are expressed depending on N_{rx}, so they are valid for the cases with four and with one receive antennas. In the case of having only one receive antenna, the H, W and r matrices would only have one row.

4.2 Maximum likelihood decoding

In Alamouti's scheme, the received signals were combined in the receiver, and the decision was taken with the built combined signals. As it was done in (44) and (45), it was easy to generalize Alamouti's scheme for more than two receive antennas, through careful observation of (28) and (29), taking into consideration that in the combined symbol, the sent symbol ended up multiplied by the squared modules of the coefficients of the channel. In the case of 4 transmit antennas, it is not trivial to build the combined signal, and make the sent symbol be multiplied by the square modules of the channel coefficients. In order to obtain the minimum BER, the maximum likelihood criterion should be applied, very similarly to how it was done in the Alamouti chapter.

If H is defined, where h_a, h_b, h_c and h_d are column vectors that contain the channel coefficients from transmit antenna 1, 2, 3 and 4 respectively to all of the receiving antennas, so H comes out to be a N_r x 4 matrix.

$$H = \begin{pmatrix} h_a & h_b & h_c & h_d \end{pmatrix} \tag{106}$$

During the first of the symbol periods of the transmission, the total signal transmitted to the receiver is:

$$H\begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = s_0 h_a + s_1 h_b + s_2 h_c + s_3 h_d = V_1 \tag{107}$$

In the consecutive symbol periods, the sent signal is:

$$H\begin{pmatrix} -s_1 \\ s_0 \\ -s_3 \\ s_2 \end{pmatrix} = -s_1 h_a + s_0 h_b - s_3 h_c + s_2 h_d = V_2 \tag{108}$$

$$H\begin{pmatrix} -s_2 \\ s_3 \\ s_0 \\ -s_1 \end{pmatrix} = -s_2 h_a + s_3 h_b + s_0 h_c - s_1 h_d = V_3 \tag{109}$$

$$H\begin{pmatrix} -s_3 \\ -s_2 \\ s_1 \\ s_0 \end{pmatrix} = -s_3 h_a - s_2 h_b + s_1 h_c + s_0 h_d = V_4 \qquad (110)$$

Similarly to how it was done in (54) and (55) but taking into account that now there are 4 transmit antennas.
Defining the vectors:

$$X_1 = \begin{pmatrix} s_0 h_a \\ -s_1 h_a \\ -s_2 h_a \\ -s_3 h_a \end{pmatrix} \qquad (111)$$

$$X_2 = \begin{pmatrix} s_1 h_b \\ s_0 h_b \\ s_3 h_b \\ -s_2 h_b \end{pmatrix} \qquad (112)$$

$$X_3 = \begin{pmatrix} s_2 h_c \\ -s_3 h_c \\ s_0 h_c \\ s_1 h_c \end{pmatrix} \qquad (113)$$

$$X_4 = \begin{pmatrix} s_3 h_d \\ s_2 h_d \\ -s_1 h_d \\ s_0 h_d \end{pmatrix} \tag{114}$$

$$X_\alpha^* X_\gamma = 0 \qquad for \quad \alpha \neq \gamma \tag{115}$$

These four vectors are orthogonal, and that fact is of extreme importance in order to be able to make a separate decision for each symbol in the receiver, because it will enable the receiver to minimize four separate equations, each of them depending on only one symbol. It will not be necessary to minimize one equation that depends on four symbols.
Due to the fact that there are now 4 transmit antennas, defining:

$$R = \begin{pmatrix} r[1] \\ r[2] \\ r[3] \\ r[4] \end{pmatrix} \tag{116}$$

$$Z = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = X_1 + X_2 + X_3 + X_4 \tag{117}$$

Where r[i] is a column vector containing all the signals received by the antennas in period i.

Now, applying the ML criterion, just as it was done in (73):

$$ML : \min_{s_0 s_1 s_2 s_3} \|R - Z\|^2 = \min_{s_0 s_1 s_2 s_3} (R - Z)^* (R - Z)$$
$$= \min_{s_0 s_1 s_2 s_3} \left(-\left(Z^* R + R^* Z \right) + \|Z\|^2 \right) \qquad (118)$$

With the only difference that now there are four symbols sent during four symbol periods.

If the $\|Z\|^2$ is expanded:

$$\|Z\|^2 = \|X_1 + X_2 + X_3 + X_4\|^2 = \|X_1\|^2 + \|X_2\|^2 + \|X_3\|^2 + \|X_4\|^2 =$$
$$= \left(|s_0|^2 + |s_1|^2 + |s_2|^2 + |s_3|^2\right)\left(\|h_a\|^2 + \|h_b\|^2 + \|h_c\|^2 + \|h_d\|^2\right)$$
$$= |s_0|^2 \|H\|^2 + |s_1|^2 \|H\|^2 + |s_2|^2 \|H\|^2 + |s_3|^2 \|H\|^2 \qquad (119)$$

Due to the orthogonality of X_i with X_j.

Rewriting Z:

$$Z = \begin{pmatrix} s_0 h_a \\ s_0 h_b \\ s_0 h_c \\ s_0 h_d \end{pmatrix} + \begin{pmatrix} s_1 h_b \\ -s_1 h_a \\ -s_1 h_d \\ s_1 h_c \end{pmatrix} + \begin{pmatrix} s_2 h_c \\ s_2 h_d \\ -s_2 h_a \\ -s_2 h_b \end{pmatrix} + \begin{pmatrix} s_3 h_d \\ -s_3 h_c \\ s_3 h_b \\ -s_3 h_a \end{pmatrix} \qquad (120)$$

Then:

$$Z^* R = s_0^* \left(h_a^* r[1] + h_b^* r[2] + h_c^* r[3] + h_d^* r[4] \right) + s_1^* \left(h_b^* r[1] - h_a^* r[2] - h_d^* r[3] + h_c^* r[4] \right) +$$
$$s_2^* \left(h_c^* r[1] + h_d^* r[2] - h_a^* r[3] - h_b^* r[4] \right) + s_3^* \left(h_d^* r[1] - h_c^* r[2] + h_b^* r[3] - h_a^* r[4] \right) \qquad (121)$$

$$R^{*}Z = s_0\left(r^{*}[1]h_a + r^{*}[2]h_b + r^{*}[3]h_c + r^{*}[4]h_d\right) + s_1\tilde{s}_1^{*} + s_2\tilde{s}_2^{*} + s_3\tilde{s}_3^{*} \quad (122)$$

$$\tilde{s}_0 = h_a^{*}r[1] + h_b^{*}r[2] + h_c^{*}r[3] + h_d^{*}r[4] \quad (123)$$

Where the consecutive $\tilde{s}$ are defined as above.

Putting together the three terms of the ML equation:

$$ML:\min_{s_\alpha}\left(-\left(s_\alpha^{*}\tilde{s}_i + s_\alpha\tilde{s}_i^{*}\right) + |s_\alpha|^2\|H\|^2\right) \quad \alpha = 0,\ldots,n \qquad i = 0,1,2,3 \quad (124)$$

As a result of the separability of the equation in four parts, each of them depending on one of the sent symbols and where n depends on the number of symbols of the modulation used. The equation has exactly the same aspect as (67) in the Alamouti case, but note that the definition of $\tilde{s}$ has changed.
In this project, the simulations used with four transmitting antennas are always real, so it is possible to extract the common factor:

$$ML:\min_{s_\alpha}\left(-\left(s_\alpha(\tilde{s}_i + \tilde{s}_i^{*})\right) + |s_\alpha|^2\|H\|^2\right) \quad \alpha = 0,\ldots,n \qquad i = 0,1,2,3 \quad (125)$$

Alamouti used a combined symbol, $\tilde{s}$, in order to take the decision. In his combined symbol, shown in (66), the sent symbol was multiplied by the squared modules of all the channel coefficients, $\|H\|^2$, the channel coefficients multiplied by noises, and the crossed terms of coefficients and sent symbols were cancelled as seen in (28) and (29). Now, the situation is not the same. If $\tilde{s}$ is expanded, the sent symbol is multiplied by some squared module coefficients, but some crossed terms of channel coefficients and sent symbols are not cancelled. So using $\tilde{s}$ as a combined symbol to make a decision would not give the same results as in Alamouti's scheme. Although if $\tilde{s}$ and $\tilde{s}^{*}$ are added, the result is similar to Alamouti's. The outcome is the sent symbol multiplied by twice the addition of all the squared modules of the channel ($2\left\|H\right\|^2$) in addition to the channel coefficients multiplied by the noises, exactly like in Alamouti's result. So that combination could be used just like Alamouti used his combined symbol. It becomes clear analyzing equation (125) and taking the 2-PAM modulation for simplicity. Using this modulation, the term $\left|s_\alpha\right|^2\left\|H\right\|^2$ disappears, due to the fact that both possible symbols have the same energy. If it is assumed that a 1 is sent, if the noise is not too big, and the 1 is multiplied by $2\left\|H\right\|^2$, the new combined symbol will be bigger than 0. If in equation (125) we assign s_α =1, the overall equation will be negative. If we assign s_α =-1, the overall equation comes out to be positive. And comparing both of the possible assumptions, the decision would be taken in favour of s_α =1, because the overall equation comes out to be smaller. Hence, the decision will be taken correctly, with no error, the new combined symbol can be taken as a reference, and the new decision rule could be to decide that the sent symbol was a 1 if $\tilde{s} + \tilde{s}^* > 0$ and to decide the sent symbol was a –1 otherwise. It is possible to use the new combined symbol when the rest of the modulations are used also, but the different energy of the symbols must be taken into account.

4.3 Simulation assumptions

The systems were simulated to find the bit error probability. The systems simulated were the 4x1 (MISO) and the 4x4 (MIMO), using modulations 2-PAM, 4-PAM and 8-PAM. In this case, the amplitudes of fading from each transmit to each receive antenna are considered to be Rician. The BER was studied for several values of the Rician factor (K). More in detail, the systems were simulated for K=0 (Rayleigh fading), K=0.5, K=2, and K=10, to study the influence of the LOS component in the communication link.
The channel matrix is now [7]:

$$H = H_{los} + H_{nlos} \tag{126}$$

Where the matrix H_{los} remains constant during the whole transmission, while H_{nlos} varies in every transmission block, which in this case lasts for four symbol periods. The variance of the elements of H_{los} or the Rayleigh components, have zero mean and variance 1.
The Rician factor was calculated as:

$$K = \frac{\left\| H_{los} \right\|^2}{E\left\{ \left\| H_{nlos} \right\|^2 \right\}} = \frac{\left\| H_{los} \right\|^2}{N_{tx} N_{rx} 2\sigma^2} \qquad (127)$$

Where:

$$E\left\{ \left\| H_{nlos} \right\|^2 \right\} = E\left\{ \sum_{i=0}^{N_{tx}N_{rx}-1} \left| h_{nlos} \right|^2 \right\} = \sum_{i=0}^{N_{tx}N_{rx}-1} E\left\{ \left| h_{nlos} \right|^2 \right\} \qquad (128)$$

$$E\left\{ \left| h_{inlos} \right|^2 \right\} = E\left\{ (h_R + jh_I)(h_R - jh_I) \right\} = E\left\{ h_R^2 \right\} + E\left\{ h_I^2 \right\} = 2\sigma^2 \qquad (129)$$

Because of the fact that the real and imaginary coefficients of the channel are independent.
And using (127) in (126):

$$E\left\{ \left\| H_{nlos} \right\|^2 \right\} = N_{tx} N_{rx} 2\sigma^2 \qquad (130)$$

Each of the components of the noise, the real and the imaginary, have $N_0/2$ as the value of the variance of the noise.

4.4 Simulation results

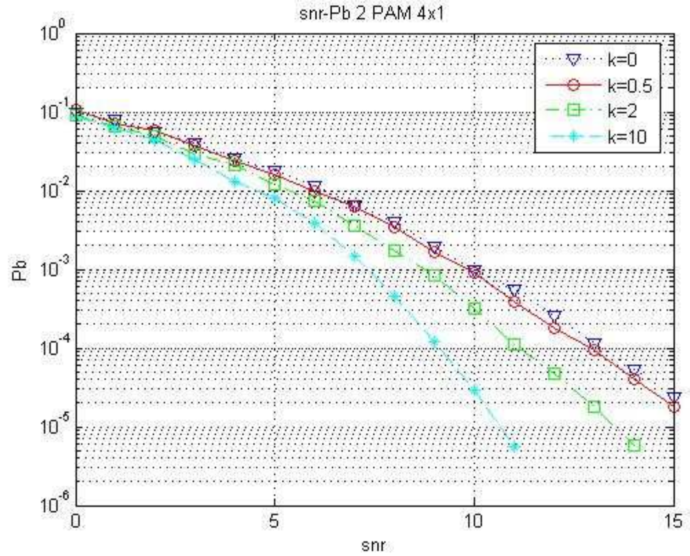


Fig 16.- simulation of the 4x1 for several k values using 2-PAM
From now on, k in the legends stand for K

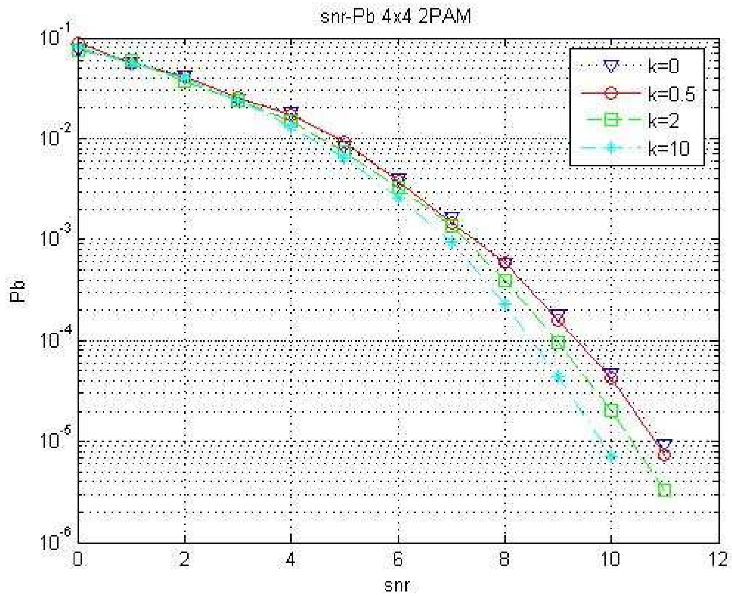


Fig 17.- simulation of the 4x4 for several k values using 2-PAM

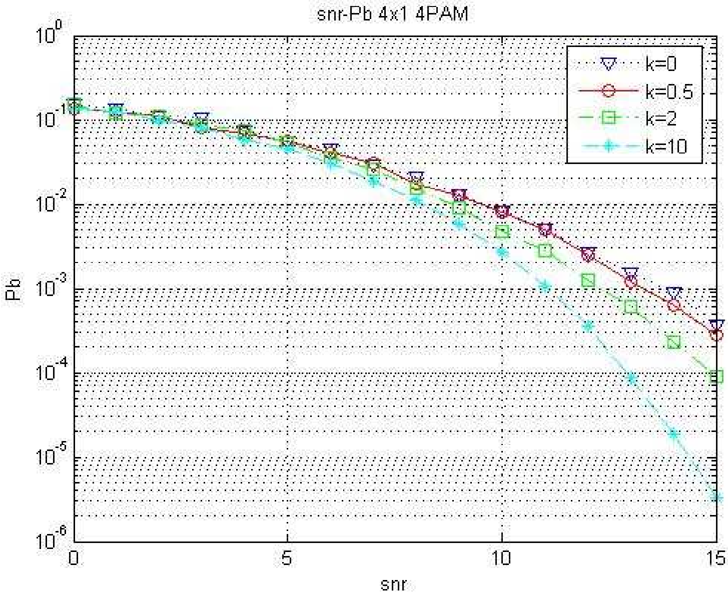


Fig 18.- simulation of the 4x1 for several k values using 4-PAM

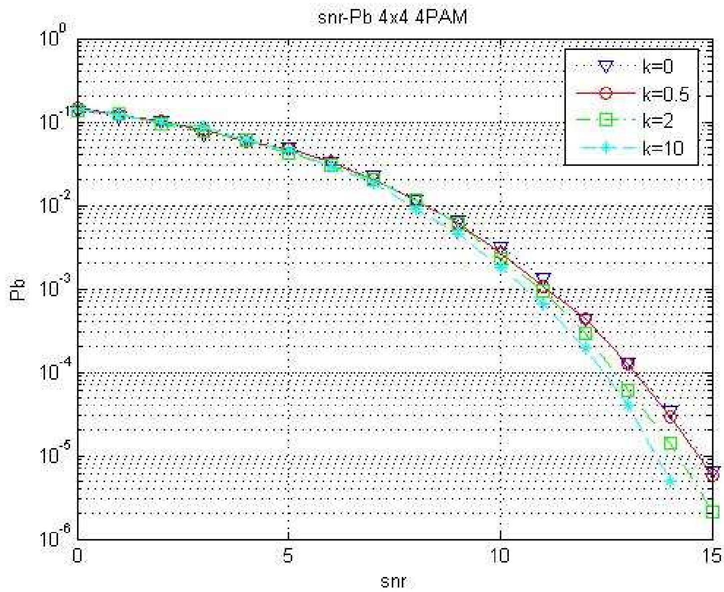


Fig 19.- simulation of the 4x4 for several k values using 4-PAM

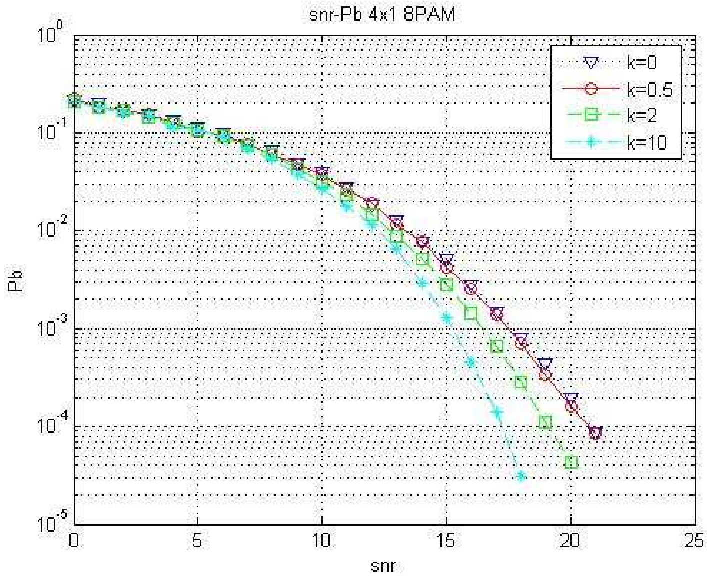


Fig 20.- simulation of the 4x1 for several k values using 8-PAM

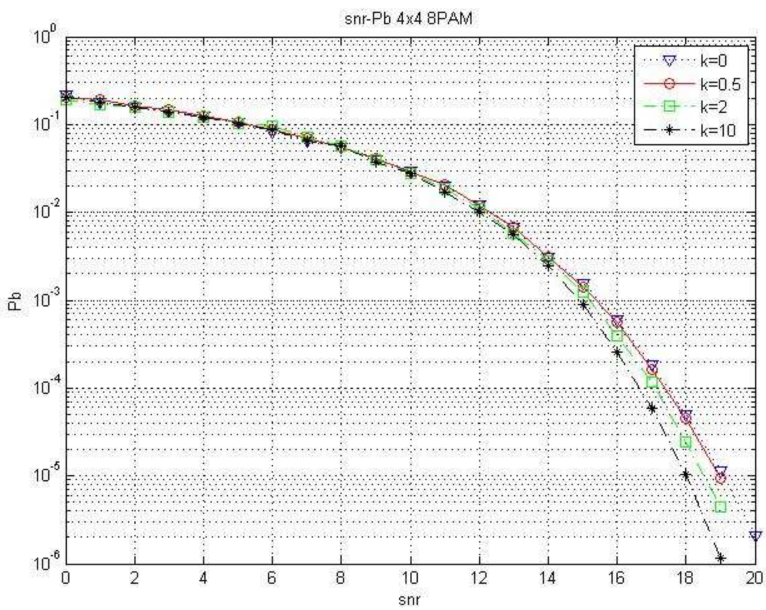


Fig 21.- simulation of the 4x4 for several k values using 8-PAM

4.5 Comparison

The same way it was done in the Alamouti chapter, the different scenarios were compared, to see which were the required SNR conditions to achieve a bit error probability of 10^{-4}. This was done for the 4x1 and the 4x4 system, and for several values of the Rician factor.

4.5.1 4x1

K	SNR (dB) 2PAM	SNR (dB) 4PAM	SNR (dB) 8PAM
0	13.2	16.9	20.9
0.5	13	16.7	20.9
2	11.2	14.9	19.1
10	10.7	12.9	17.2

Table 7.- SNR needed to achieve a BER of 10^{-4} in 4x1 OSTBC

4.5.2 4x4

K	SNR (dB) 2PAM	SNR (dB) 4PAM	SNR (dB) 8PAM
0	9.4	13.2	17.4
0.5	9.3	13.2	17.3
2	8.9	12.6	17.1
10	8.5	12.5	16.6

Table 8.- SNR needed to achieve a BER of 10^{-4} in 4x1 OSTBC

As it is seen in the comparative tables, there is a tradeoff between bit error probability and bit rate. If the bit rate is increased, or in other words, the modulation requires more bits per symbol, the system needs a higher SNR to reach the same BER results. So, for a given SNR value, a lower order modulation will present a lower bit error probability. In general, using the same K factor and the same number of antennas, the next order modulation needs approximately four more dB’s of SNR to reach the BER target.

The influence of the Rician factor can also be observed. The increase of K, or the strength of the LOS component of the channel, improves the performance of the link. The same system requires a smaller value of SNR to reach a fixed BER value if the LOS component of the channel has more strength. In the 4x1 systems, the SNR needed to achieve the BER target is approximately four dB’s smaller for any modulation when K=10 than when the channel behaves like a pure Rayleigh fading channel. In the 4x4 systems, the system also improves with the increase of K, but the improvement is not so noticeable. The Rician influence over the channel can be interpreted as a deterministic value of the channel throughout the transmission, a value that doesn’t change or fade, a value that can be easily known. The MIMO scheme is thought to combat the Rayleigh fading conditions, and the LOS component of the channel is not something that has to be fought against, it’s effects don’t have to be mitigated, they are in fact, positive for the link, under the assumptions in this project. A 4x4 system has a diversity order of 16, and a 4x1 system has a diversity order of 4, so the 4x4 combats fading much better than the 4x1 does. The effects of the increase of K are more noticeable in the 4x1 case because the fading is not so well combated in that system, and the influence of a constant signal path gives a lot of benefits. In the 4x4 system, the Rayleigh fading is very mitigated, and the conditions are good, so the influence of a larger K doesn’t give such a big improvement, the system, somehow, doesn’t have the margin to improve that much.

Two systems with different diversity order were simulated using OSTBC. One has a diversity order of 4, and the other one has a diversity of 16. In Alamouti’s scheme, the 2x2 system had a diversity order of 4, so it can be compared with the OSTBC scheme with the same diversity. The comparison should be between systems with the same diversity order , under the sole influence of Rayleigh fading and with the same number of bits per symbol in the modulation, so that the bit rate is also the same. While the 2x2 Alamouti system using QPSK needs 13.1 dB to reach a P_b of 10^{-4}, the 4x1 OSTBC 4-PAM system needs a SNR of 16.9 dB. This approximately 3.8 dB difference is basically due to the fact that the

QPSK modulation has a better performance than the 4-PAM modulation. Even under the influence of only AWGN, a QPSK modulation needs a SNR of 9 dB to reach a 10^{-4} BER, while a 4-PAM modulation needs 12.5 dB of SNR to reach the same BER.

5. Spatial Multiplexing

The main focus of the Alamouti and OSTBC schemes were to combat effectively channel impairments, or in other words, improve the bit error probability of the systems in comparison with the SISO system, which was not reliable under Rayleigh fading conditions. These schemes achieved their goals without sacrificing bit rate, and achieving the maximum available diversity. In the cases studied in this project, the full bit rate was accomplished, because n symbols were sent during n symbol periods. The bit error probability is key, for a communication to be reliable, but in some applications, though, it may be interesting or possible, depending on the channel conditions, to have a very high bit rate. The focus of the spatial multiplexing scheme, also known as uncoded, is to achieve the highest bit-rate, sacrificing some of the available diversity [22].
The systems simulated were 4x1, 4x2, 4x3 and 4x4 due to the special interest that this later system has. The modulations used were BPSK, QPSK. The channel was modelled as Rician under several values of K.

5.1 The scheme

When 4 transmit antennas are deployed, the matrix channel is defined by:

$$H = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ h_5 & h_6 & h_7 & h_8 \\ \vdots & \vdots & \vdots & \vdots \\ h_{4N_{rx}-3} & h_{4N_{rx}-2} & h_{4N_{rx}-1} & h_{4N_{rx}} \end{pmatrix} = \begin{pmatrix} h_a & h_b & h_c & h_d \end{pmatrix} \qquad (131)$$

The technique to achieve a higher transmission rate than in the previous schemes is to modify the coding matrix. Here, instead of transmitting an information block during N_{tx} symbol periods, the information block is transmitted all at a time, using only one symbol period. N_{tx} independent data symbols are transmitted every symbol period. Hence, spatial multiplexing sacrifices the diversity achieved in the other schemes throughout time in order to increase the bit rate significantly. The maximum diversity order achieved by this scheme is N_{rx}. The only diversity the system has is due to the fact that the decision is taken with the help of N_r receive antennas. Any information bit is transmitted only from one transmit antenna and received by N_r receive antennas. The encoding matrix is now transformed into an encoding vector:

$$S = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = (s[1]) \qquad (132)$$

The noise vector is defined as follows:

$$W = \begin{pmatrix} w_1 \\ \vdots \\ \vdots \\ w_{N_{rx}} \end{pmatrix} = (w[1]) \qquad (133)$$

Where n_i is complex AWGN.

The received signal vector for each of the symbol periods will then be:

$$r = HS + W = \begin{pmatrix} r_1 \\ \vdots \\ \vdots \\ r_{N_{rx}} \end{pmatrix} = (r[1]) \qquad (134)$$

Where the received signals are:

$$\begin{aligned} r_1 &= h_1 s_1 + h_2 s_2 + h_3 s_3 + h_4 s_4 + n_1 \\ r_2 &= h_5 s_1 + h_6 s_2 + h_7 s_3 + h_8 s_4 + n_2 \\ \vdots \quad & \quad \vdots \qquad \vdots \qquad\quad \vdots \qquad \vdots \qquad \vdots \\ r_{N_{rx}} &= -h_{4N_{rx}-3} s_1 - h_{4N_{rx}-2} s_2 + h_{4N_{rx}-1} s_3 + h_{4N_{rx}} s_4 + n_{N_{rx}} \end{aligned} \qquad (135)$$

The decoding is done in a very different way than in the other schemes. There is no possible combiner, attending to the fact that the whole information block is sent in one symbol period. There are no copies of the symbols sent in later periods that could be combined with the first ones in order to take a more confident decision. So in order to take the optimum decision, the one that will give the minimum bit error probability, the ML criterion should be applied [8]:

$$ML : \min_{\hat{s}} \left\| r - H\hat{s} \right\| \qquad (136)$$

The vector $\hat{s}$ contains one of the combinations of the four symbols sent. In order to apply the criterion, all the possible combinations of the symbols sent must be checked. Expanding the equation:

$$\min_{\hat{s}} (H\hat{s})^* r + r^* H\hat{s} + (H\hat{s})^* H\hat{s} \qquad (137)$$

It is not possible to separate the equation in four different ones, each depending on only one of the symbols, and therefore, the decision

must be taken jointly. The decisions on the four symbols are taken at the same time. The vector s contains the four symbols. The complexity of the ML decoding is high, and it increases with the number of antennas and high order modulations. When the modulation used is BPSK, with one bit per symbol, the ML decoding algorithm must check 16 (2^4) possible vectors in order to take the decision. When QPSK is used, which needs two bits per symbol, the algorithm must check 256 (2^8) possible vectors. And if 16-QAM is used, with 4 bits per symbol, 65536 (2^{16}) possibilities must be checked in order to take a decision for only four symbols. Therefore, in spatial multiplexing, the number of calculations needed to achieve optimal decoding become prohibitive in many cases. It should take much longer for the 16-QAM to reach the target, taking into account the compromise between BER and SNR. In the simulations, ML decoding was used, but there are some alternative suboptimal decoding strategies to reduce the number of calculations needed.
An alternative, for example, is trying to invert the channel matrix. As it is well known by now:

$$r = Hs + w \tag{138}$$

By inverting the channel matrix, H, the result would be:

$$Y = H^{-1}r = s + H^{-1}w = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix} \tag{139}$$

And using this result, the decisions could be taken separately for every symbol, and the number of calculations would be considerably reduced, although the bit error probability would not be the minimum.

The average received energy per bit, E_b, has changed in this case. The calculations remain the same until (70), which is the result for the total received energy in one symbol period. In the spatial multiplexing scheme, the transmission lasts for only one period, so the received energy in one symbol period is the total received energy. In order to calculate the average bit energy received:

$$E_b = \frac{E_s N_{tx} N_{rx} 2\sigma^2}{N_{tx} q} = \frac{E_s N_{rx} 2\sigma^2}{q} \qquad (140)$$

5.2 Simulation results

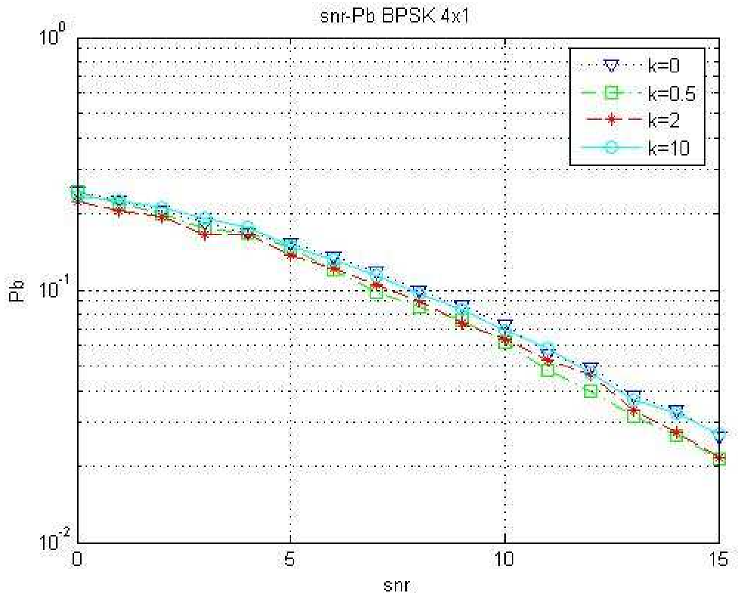


Fig 22.- Spatial multiplexing using 4x1 BPSK and several values of K

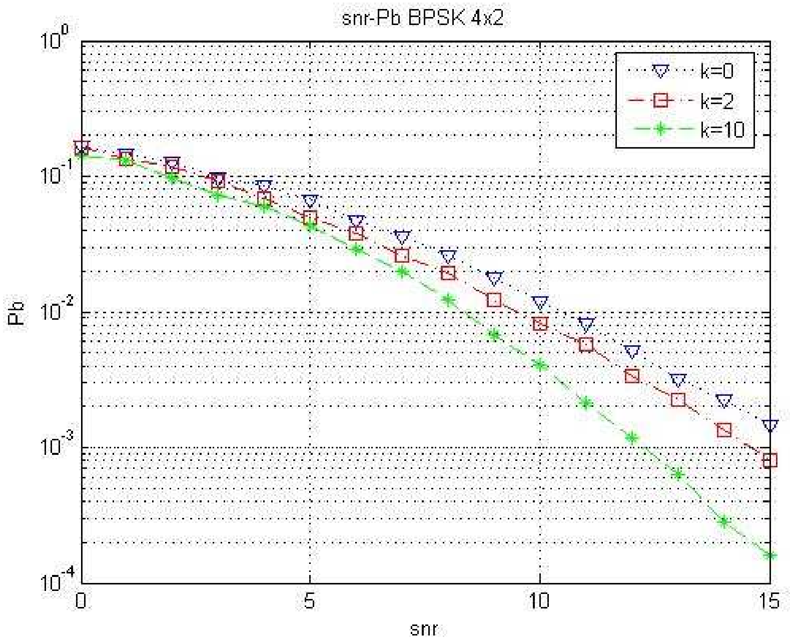


Fig 23.- Spatial multiplexing using 4x2 BPSK and several values of K

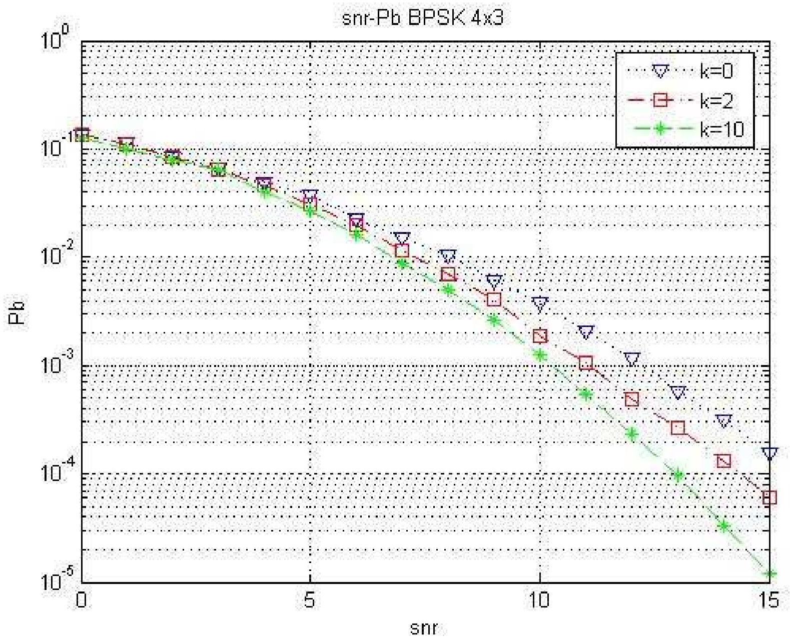


Fig 24.- Spatial multiplexing using 4x3 BPSK and several values of K

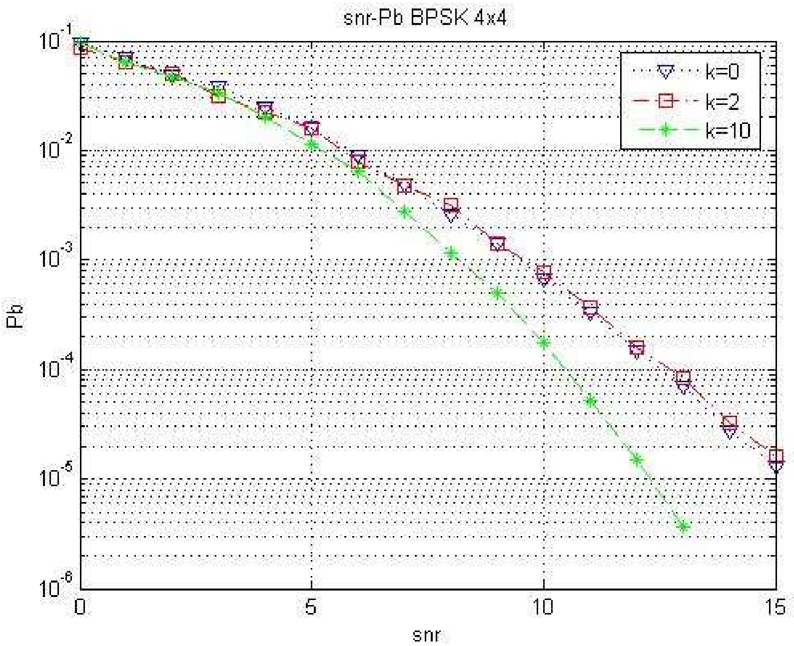


Fig 25.- Spatial multiplexing using 4x4 BPSK and several values of K

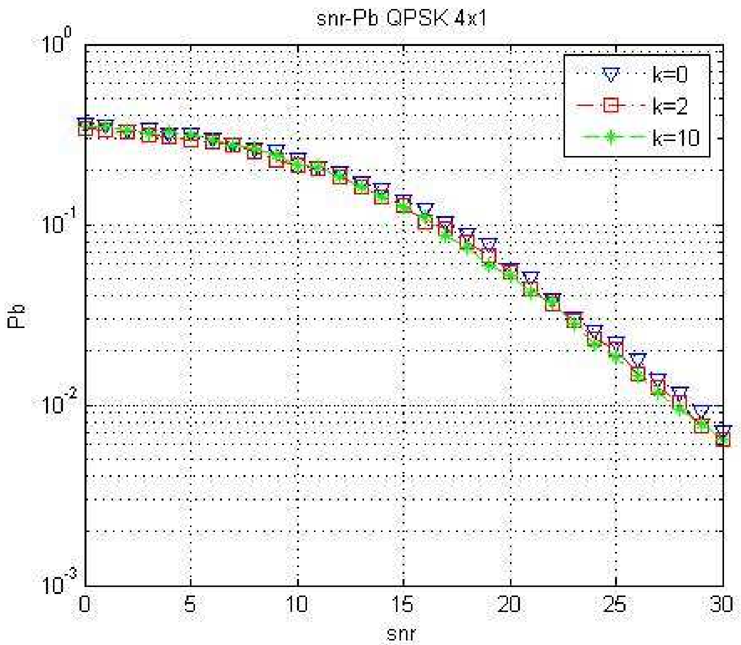


Fig 26.- Spatial multiplexing using 4x1 QPSK and several values of K

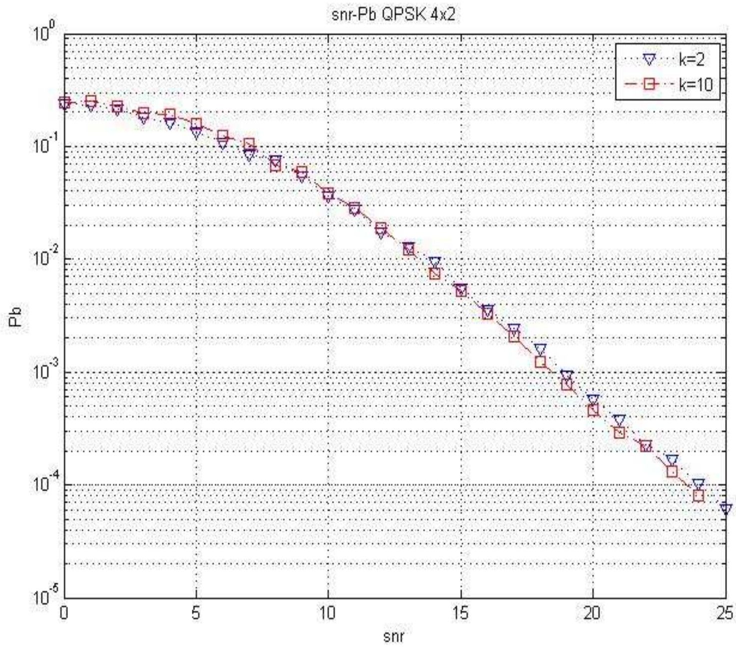


Fig 27.- Spatial multiplexing using 4x2 QPSK and several values of K

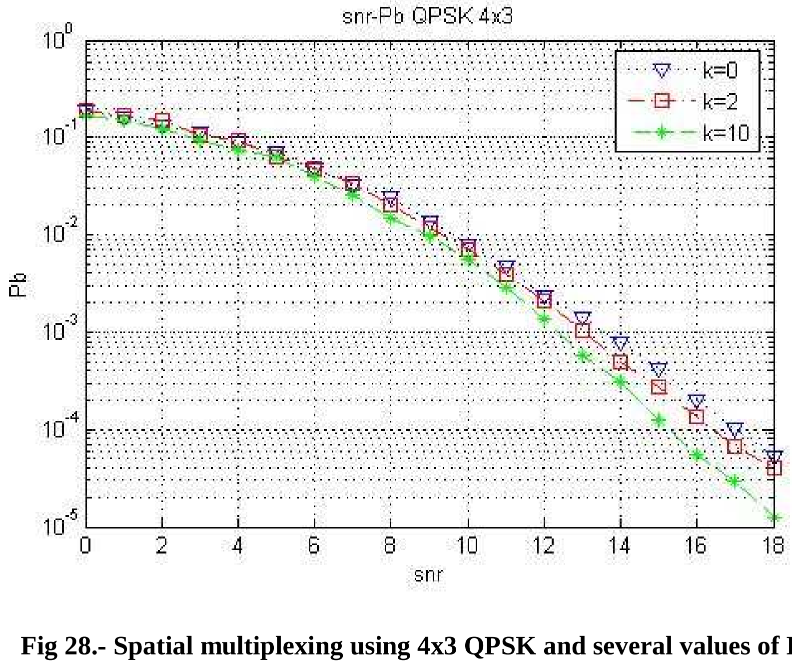


Fig 28.- Spatial multiplexing using 4x3 QPSK and several values of K

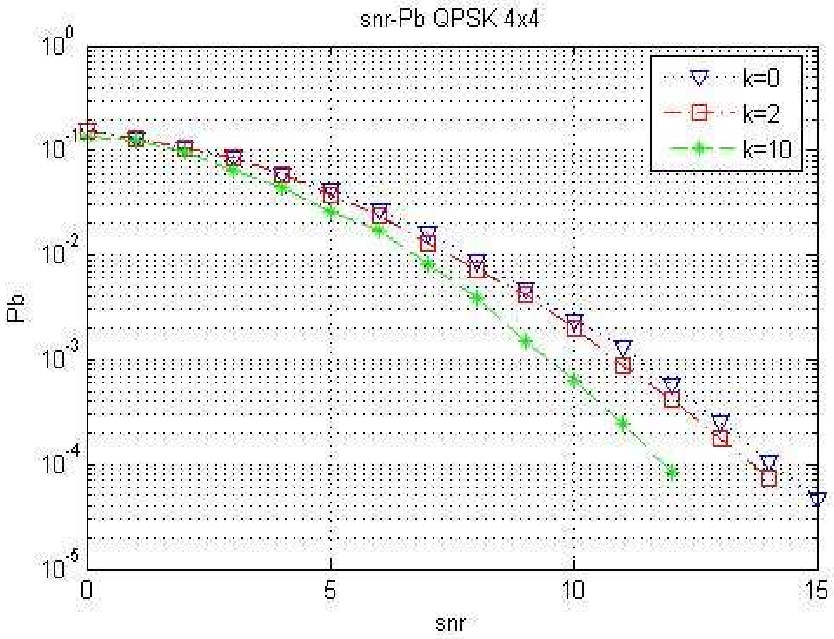


Fig 29.- Spatial multiplexing using 4x4 QPSK and several values of K

In the simulations above, the compromise between bit rate and bit error probability is shown once again. And similarly to how it happened in OSTBC, the performance of the system improves as the value of K increases. The values of the LOS matrix were generated randomly as usual.

In order to study if there is any influence of how the strength of the LOS matrix coefficients is distributed, in the following simulations, the LOS channel matrix coefficients were not generated randomly. Although there is still control over the K, all of the coefficients have the same value. Or in other words, the strength of the LOS matrix is distributed equally in all the coefficients.

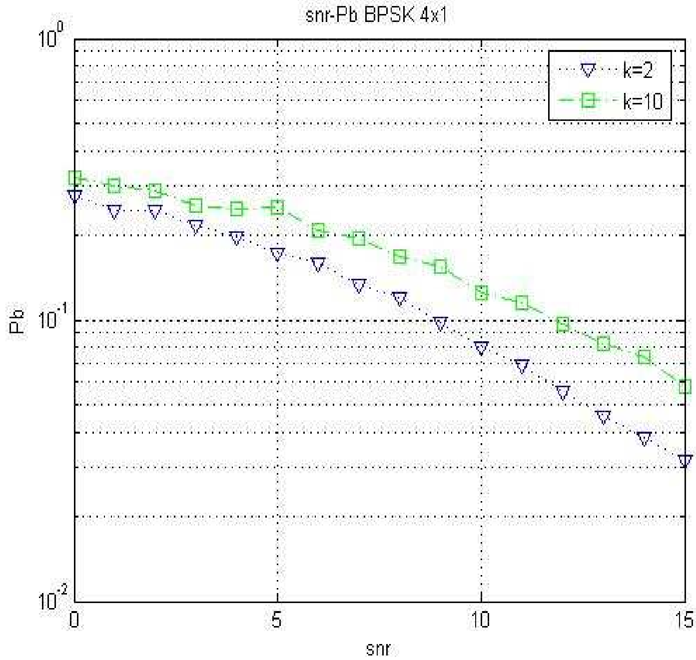


Fig 30.- spatial multiplexing with 4x1 BPSK, but all the coefficients in the LOS matrix are equal

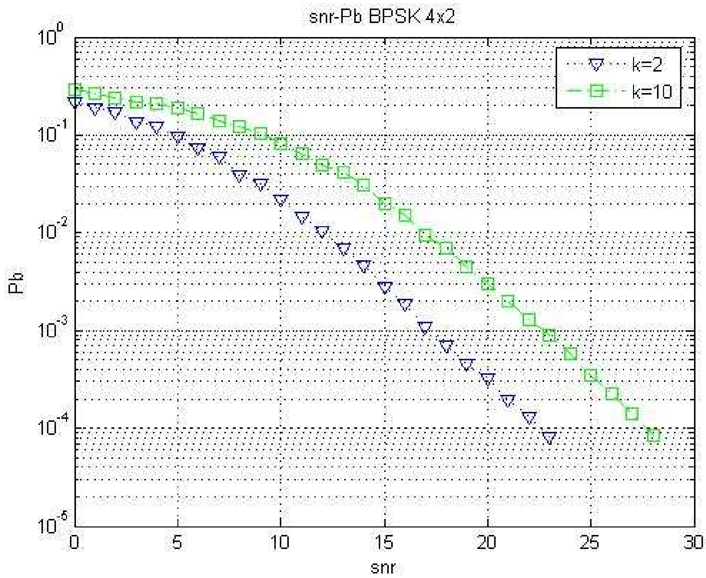


Fig 31.- spatial multiplexing with 4x2 BPSK, but all the coefficients in the LOS matrix are equal

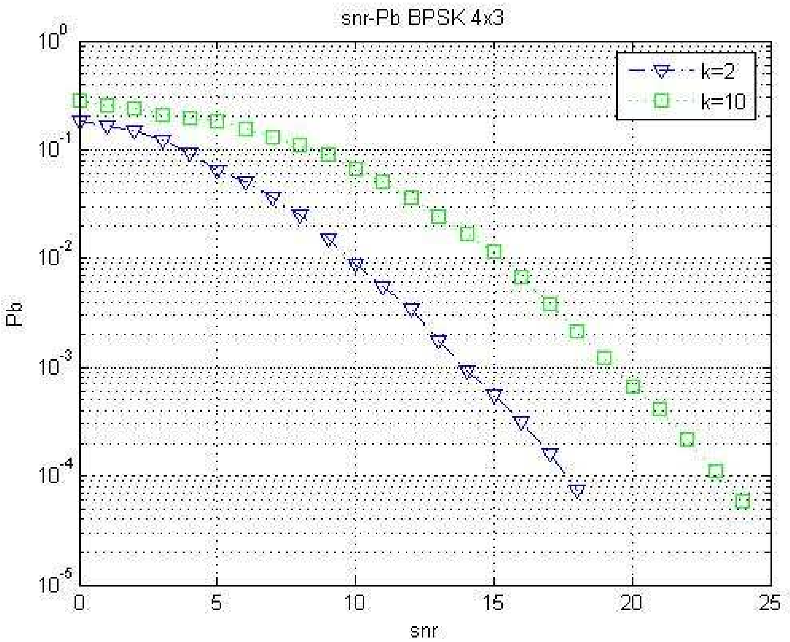


Fig 32.- spatial multiplexing with 4x3 BPSK, but all the coefficients in the LOS matrix are equal

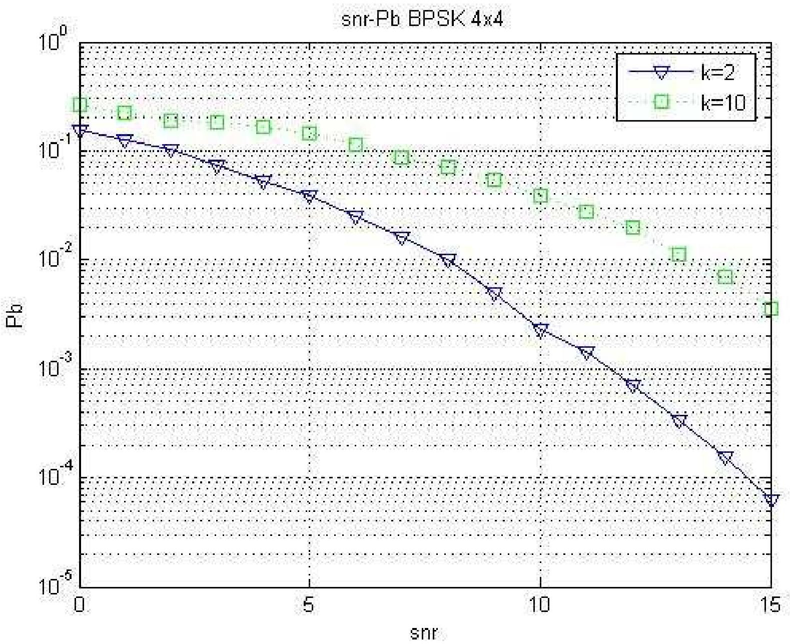


Fig 33.- spatial multiplexing with 4x4 BPSK, but all the coefficients in the LOS matrix are equal

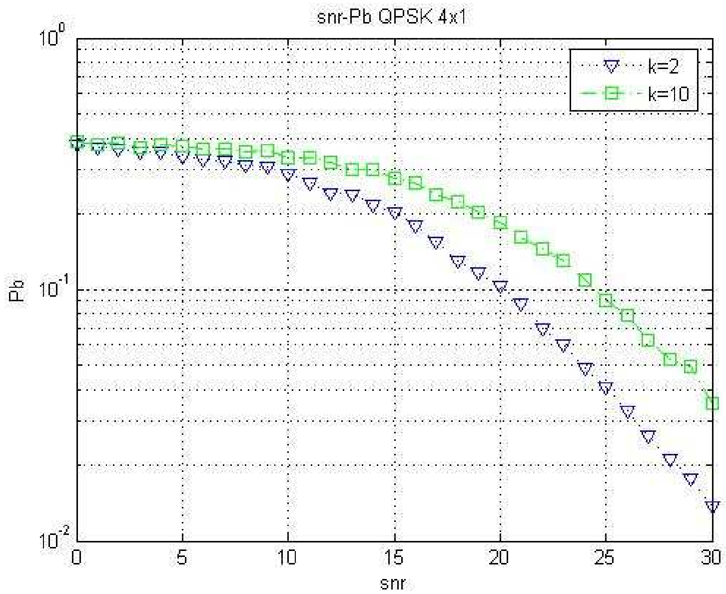


Fig 34.- spatial multiplexing with 4x1 QPSK, but all the coefficients in the LOS matrix are equal

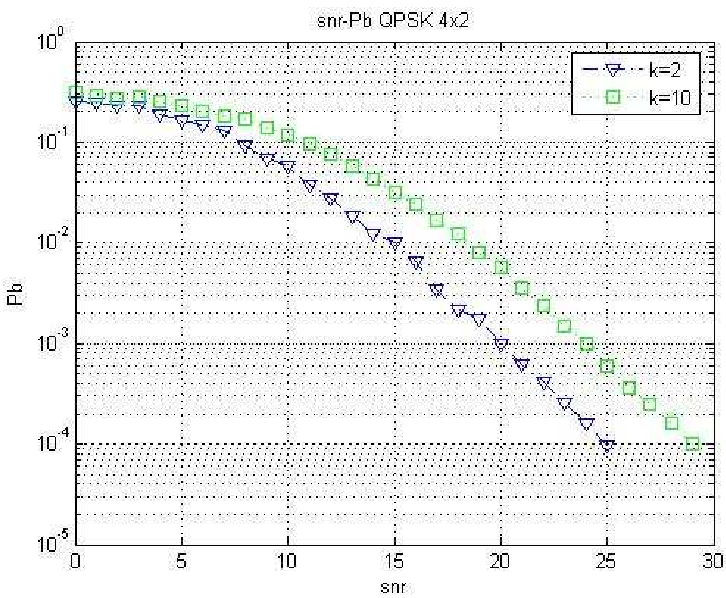


Fig 35.- spatial multiplexing with 4x2 QPSK, but all the coefficients in the LOS matrix are equal

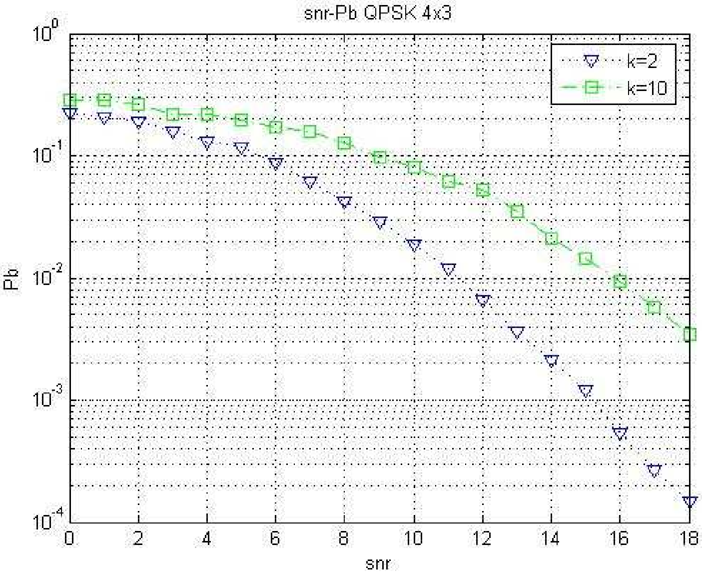


Fig 36.- spatial multiplexing with 4x3 QPSK, but all the coefficients in the LOS matrix are equal

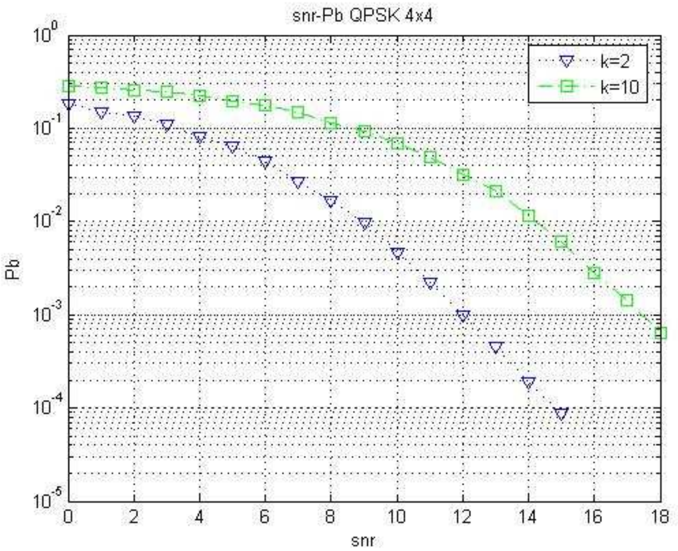


Fig 37.- spatial multiplexing with 4x4 QPSK, but all the coefficients in the LOS matrix are equal

In this case, when the LOS matrix coefficients are all equal, the performance of the system is worse than it is when the conditions are pure Rayleigh fading. This situation can be illustrated with an example. Suppose there are two transmit antennas and one receive antenna. The sent signal by the first antenna would be:

$$s_0 h_{los} + s_0 h_{1nlos} \tag{141}$$

The sent signal by the second antenna would be:

$$s_1 h_{los} + s_1 h_{2nlos} \tag{142}$$

And adding the noise in the receiver:

$$s_0 h_{los} + s_0 h_{1nlos} + s_1 h_{los} + s_1 h_{2nlos} + n \tag{143}$$

If the strength of the LOS components is supposed much bigger than the strength of the NLOS components, the elements of the NLOS matrix become negligible:

$$s_0 h_{los} + s_1 h_{los} + n \tag{144}$$

And if now it is supposed that the two symbols sent are the same but with an opposite sign ($s_0 = -s_1$), the received signal is only noise. And the decision has to be taken only with that value, so it is taken randomly, and that is why the results are much worse than when the LOS matrix is created randomly.

5.3 Comparison

As it was done in the previous chapters, the results are compared. The table shows the required SNR to reach a target BER of 10^{-4}. The differences between the case in which the coefficients of the LOS matrix are generated randomly and when they are not are shown in the tables.

5.3.1 K=2

N_{tx}*N_{rx}	BPSK	BPSK (equal coefficients)	QPSK	QPSK (equal coefficients)
4x2	19.9	21.5	24	24.9
4x3	14.3	17.6	16.4	18.5
4x4	12.9	14.5	13.7	14.9

Table 9: snr needed to achieve ber target of 10^{-4} with k=2

5.3.2 K=10

N_{tx}*N_{rx}	BPSK	BPSK (equal coefficients)	QPSK	QPSK (equal coefficients)
4x2	15.5	27.5	23.5	29
4x3	13	23.1	15.2	23.8
4x4	10.5	20.3	11.8	20.4

Table 10: snr needed to achieve ber target of 10^{-4} with k=10

With K=0 and the coefficients generated randomly, very good channel conditions were required to reach a target BER of 10^{-4}. A 4x4 QPSK system requires 14 dB of SNR with K=0. And the same system with 4x3 requires 17 dB. A 4x4 BPSK system requires nearly 13 dB of SNR to reach the target BER.

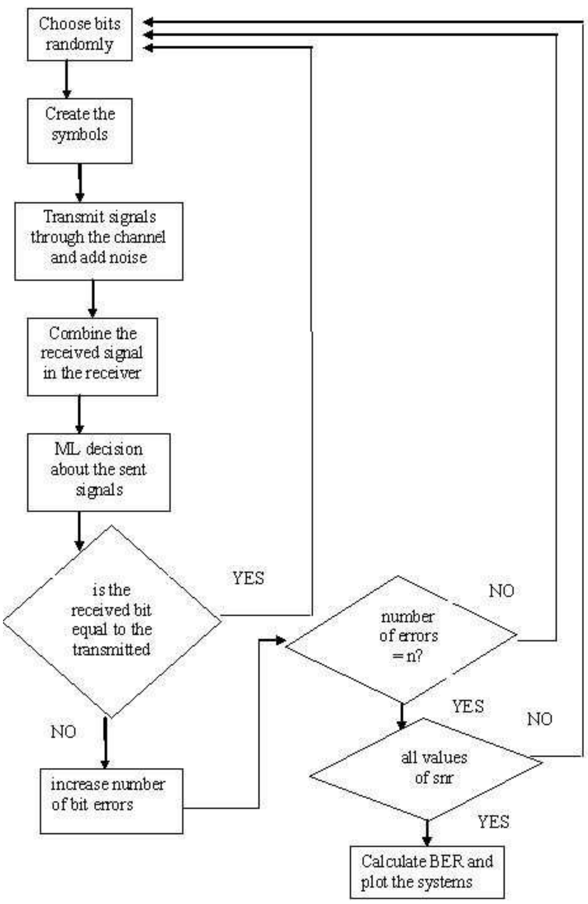
Some simulations were done using 16-QAM, but due to the high complexity of the calculations that needed to be done, only some isolated results were obtained, it was impossible to obtain a graph for several values of SNR. The 4x4 16-QAM system with K=0 needed 15 dB of SNR to reach the target BER and with K=2, the system required approximately 14dB. So the trend of improving the performance with the K, continues.

When the coefficients of the matrix are all the same, when they are not generated randomly, the higher the K gets, the worse the performance of the system is, and the more SNR it needs to reach a target BER.

6. Program

In this chapter, some of the most important aspects of the program will be explained.

6.1 Flow diagram



6.2 SNR

The systems are simulated for many values of SNR. In order to do the simulations, and get the correct graphs, it is important to have control over the SNR. This was done by varying the variance of the noise in accordance with the value of the SNR.
Using the definition of SNR:

$$SNR = 10\log_{10}\frac{E_b}{N_0} \tag{145}$$

And rearranging the equation to have the variance of the noise depending on the other factors:

$$10^{\frac{SNR}{10}} = \frac{E_b}{N_0} \qquad \Rightarrow \qquad N_0 = \frac{E_b}{10^{\frac{SNR}{10}}} \tag{146}$$

The E_b is calculated as was explained in the previous chapters, depending on the scheme, the number of antennas and the modulation that were used. The factor that varies in order to get the graphs is the SNR.

Using the matlab function randn, which returns a random value which is Gaussian distributed, with zero mean and variance equal to one, it is possible to vary the variance of the random variable. Using the results:

$$E\{y\} = E\{kx\} = kE\{x\}$$
$$\sigma_y^2 = E\left\{(y-\mu y)^2\right\} = E\left\{(kx-km_x)^2\right\} = E\left\{(k(x-m_x))^2\right\} = k^2\sigma_x^2 \tag{147}$$

So multiplying the correct constant, depending on the scheme used, by the randn function, the variance of the random variable is changed. The variance of the noise used in the simulations was $N_0/2$, for the real and the imaginary part, so the total variance is N_0.

In the Alamouti 2x2 QPSK scheme, for instance:

```
function[r0,r1,r2,r3]=add_noise(h0,h1,h2,h3,s0,s1,s2,s3
,snr)
a0=h0*s0+h1*s1;%this is the combining scheme
a2=h2*s0+h3*s1;
a1=h0*s2+h1*s3;
a3=h2*s2+h3*s3;
n0=complex((2/sqrt((10^(snr/10))))*randn,(2/sqrt((10^(s
nr/10))))*randn);
n1=complex((2/sqrt((10^(snr/10))))*randn,(2/sqrt((10^(s
nr/10))))*randn);
n2=complex((2/sqrt((10^(snr/10))))*randn,(2/sqrt((10^(s
nr/10))))*randn);
n3=complex((2/sqrt((10^(snr/10))))*randn,(2/sqrt((10^(s
nr/10))))*randn); %here the noises are generated,
taking into account the Eb of QPSK
r0=a0+n0;% this is the signal the receiver will get
r2=a2+n2;
r1=a1+n1;
r3=a3+n3;
```

6.3 Rician factor

The rician factor, k, defines the relative strength between the LOS and the NLOS components of the channel coefficients. In the simulations done for various values of k, it is important to have control over this factor. Using the definition of K:

$$K = \frac{\left\|H_{los}\right\|^2}{E\left\{\left\|H_{nlos}\right\|^2\right\}} = \frac{\left\|H_{los}\right\|^2}{N_{tx} N_{rx} 2\sigma^2} \qquad (148)$$

All the coefficients, for the LOS and for the NLOS were generated randomly, but the squared norm of the LOS matrix needed a definite value.

As an example, for an OSTBC, it was done as it follows:

```
function [ Hnlos ] = create_channel_4x4_2_pam( )
Hnlos=zeros(4);
for i=1:1:16
Hnlos(i)=complex(randn,randn);
end


function [ Hlos ] =
create_channel_los_4x4_2_pam(k,Nt,Nr )
desire_norm= k*Nt*Nr*2;
Hlos=zeros(4);
norm=0;
for i=1:1:16
    Hlos(i)=complex(randn,randn);
    norm=norm+(abs(Hlos(i)))^2;
end

Hlos= sqrt(desire_norm/norm)*Hlos;
```

7. Conclusions

This project presents an overview of the MIMO transmitting schemes, paying special attention to the Alamouti scheme, OSTBC and spatial multiplexing. The Alamouti scheme was thoroughly analyzed, generalizing it for N receive antennas, deriving the way in which Alamouti came up with the combining scheme and explaining the importance of the diversity order and the simple decoding algorithm. Some OSTBCs were analyzed and simulated, specifically the 4xNr schemes using real constellations, which maintain the full rate, under Rayleigh and Rice fading conditions, exposing and explaining the difference in the performance of the systems varying the channel parameters. The decoding algorithm for the receiver was also derived, and the differences with Alamouti's combiner, detailed. Like Alamouti's scheme, OSTBCs exploit all the possible diversity. The idea behind spatial multiplexing was explained, and several systems were simulated, under Rayleigh and Rice conditions. Sacrificing diversity, and hence, requiring a larger SNR to reach a BER specific value, spatial multiplexing is able to significantly increase the bit rate. All of the schemes were analyzed using different modulations. The compromise between bit rate and bit error probability was shown throughout this project, with independece of the scheme used, giving background in order to decide which modulation or which SNR would be needed, if a specific application had to be designed.

8. References

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