How Much Competition is a Secondary Market? *

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Abstract

In this paper, we build a dynamic equilibrium model of durable goods oligopoly, in which consumers face lumpy costs of transacting in the secondary markets and to which they respond by buying and selling infrequently. We calibrate the model using aggregate data from the U.S. automobile industry and measure transaction costs and the substitutability between products. We use our estimates to directly quantify how much competition active secondary markets represent for durable-goods producers.

Keywords: Secondary Markets, Durable Goods, Oligopoly, Transaction Costs, Automobile Industry, Market Power

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1 Introduction

In recent years, especially due to the rapid rise of Internet retailing, a large number of liquid markets for a huge variety of used goods have sprung up. Everything is sold and resold on the Internet, from animals to toys to books, plants, clothing, appliances; even automobiles and housing units. One market observer notes:

> This evolution is beginning to redefine socially accepted norms for consumer buying and selling behavior. Specifically, we are beginning to embrace the notion of temporary ownership. We will soon live in a world where the norm is to sell our designer shoes after wearing them twice, where Verizon will automatically send us the newest, best, most high tech mobile phone every six months, and where we'll lease our Rolex watches instead of buying them. The "informed consumer" will soon choose the brand of her next handbag based on how much it will likely fetch on eBay next year – which corresponds to how much it will really cost her to own it up until then.[1]

Has this dramatic expansion in secondary markets (or "temporary ownership", to use the more colorful terminology above) helped or hurt new good producers? In this paper, we seek to quantify how the tradability of durable goods in secondary markets affects firms' behavior and profits by obtaining a direct measure of the effect of secondary markets on market power. We build a dynamic equilibrium model of durable goods oligopoly, in which consumers face lumpy costs of transacting in the secondary markets and to which they respond by buying and selling infrequently.

The durability of the product and the existence of a secondary market have important competitive implications for manufacturers which are absent in a static model. As pointed out by Coase (1972), the fact that the good is durable allows consumers to postpone their purchases. Consumers anticipate that if higher willingness to pay consumers purchase the product initially and exit the market, the firm will only be left with lower valuation consumers and will only be able to sell to them if it lowers prices. With the anticipation of such a price decrease, consumers delay their purchases and are only willing to buy immediately if the firm cuts back initial price. Therefore, in order to make a sale, the firm is forced to drop prices immediately, which weakens its market power and lowers its profits. A durable-goods monopolist would like to announce low levels of future production (or high prices) as a way to prevent consumers from delaying their purchases. Nonetheless, such

[1] From Nissanoff (2006).

price announcements are time inconsistent if the firm cannot commit to keeping prices high once the higher valuation consumers have purchased.

In this setting, secondary markets have an ambiguous effect on competition and firms' profits. The secondary market introduces, in the form of used products, a large number of imperfect substitutes to the new goods. This *substitution effect* tends to erode the market power and profitability of the durable-goods producers. However, this detrimental effect on market power is mitigated by two benefits of secondary markets: first, there is a *commitment benefit* because the existence of substitutes in the secondary market causes firms to hold back their production, thus giving them some indirect commitment ability. Second, there is a *sorting benefit*, because secondary markets improve the efficiency of the allocation of new and used products among the heterogeneous consumers and raise the willingness to pay of those purchasing from the primary market, who can now sell their past purchases to lower willingness to pay types. By recovering those who are willing to pay the most, firms can sell at higher prices and earn more profits. Thus, with secondary markets, the firm achieves indirect price discrimination.[2] At the same time, the firm still earns indirect revenue from the sale of used products to lower valuation consumers because the new product price capitalizes the future resale prices of the products in the secondary market.

Whether secondary markets help or hurt producers is ultimately an empirical question, which depends on specific features of product markets, such as market structure and consumer preferences. In this paper, we build a dynamic equilibrium model which allows us to examine the effects of secondary markets in oligopolistic industries with heterogeneous consumers. Consumers in our model incur lumpy costs of transacting in the secondary markets. With transaction costs, the timing of purchases is important on the demand side, with consumers forming expectations on future prices and buying and selling infrequently to avoid the lumpy costs.

The incorporation of consumer transactions costs in our model makes it well-suited to address the effects of secondary markets. Indeed, transactions costs measure the size of the secondary market and, by varying the magnitude of transactions costs, our model nests a number of models in the existing literature. At one extreme, when transactions costs become prohibitively large, the secondary market shuts down, and we have a model with durability but without secondary markets (similar to Coase's original setting, and Bond and Samuelson (1984)). At the other extreme, when transactions costs are reduced to zero,

[2] Although indirect, the final effect of price discrimination is as in Mussa and Rosen (1978) and Maskin and Riley (1984).

the secondary market becomes frictionless, and consumers optimally pursue a policy of "renting" their most preferred vintage each period (as in Rust (1985a), Hendel and Lizzeri (1999), Esteban and Shum (2007)).

As an empirical illustration, we calibrate its parameter values to match aggregate data from the American automobile industry. Using the calibrated parameter values, we run counterfactuals to illustrate how a change in transactions costs would affect the profitability of firms. We find that, in general, more active secondary markets lead to lower profits for producers, so that the substitution effect dominates. The benefits of secondary markets dominate only when the number of firms is reduced to 1 (ie. the monopoly case).

1.1 Existing literature

The competitive effects of secondary markets in durable goods industries is a long-standing question. A well-established theoretical literature, including Swan (1985), Rust (1985b), and Bulow (1986) have emphasized the substitution effect, and its detrimental effects on monopolist's market power. These papers also consider whether the monopolist may choose to reduce the durability of her product in order to avoid the competition from the secondary market.

However, a more recent strand of the literature, including papers by Liang (1999), Anderson and Ginsburgh (1994), Hendel and Lizzeri (1999) and Porter and Sattler (1999), have discussed the benefits of secondary markets for producers. Liang (1999) was the first to point out the commitment benefit of secondary markets. However, in her model new and used goods are perfect substitutes (as in Swan (1985)), so that secondary markets do not have an allocative role. On the other hand, Anderson and Ginsburgh (1994), Hendel and Lizzeri (1999) and Porter and Sattler (1999) consider models of durable goods oligopoly with secondary markets under the assumption that firms commit to a production sequence. These papers show the sorting benefits from the secondary markets, and imply that the firm may prefer the secondary market to be as frictionless as possible, because a frictionless secondary market maximizes the extent to which the firm can price discriminate to consumers with high tastes for quality. However, given their assumption that firms can commit to future production sequences, they do not face the Coasian problem, and hence secondary markets do not confer commitment benefits.

Our contribution in this paper is to construct a durable goods oligopoly model with secondary markets and consumer transactions costs, where new and used goods are imperfectly substitutable, and the producers are not able to commit. Hence, in our model, both the commitment and sorting benefits of the secondary market should play a role. Moreover, an attempt has been made to construct the model in a flexible fashion, so that it could be amenable for empirical work.[3]

Our modeling of transaction costs imply that we can disentangle the separate effects of durability from those of secondary markets. Some previous papers, including Liebowitz (1982), Benjamin and Kormendi (1974) and Rust (1985b), have used models in which the firms can affect the secondary market only by their choices of durability. Our approach allows us to separate the competitive pressure created by the trade of the product in secondary markets from the competitive pressure created by the durability of the product. Because of transactions costs, the consumer-level demand functions display "(S,s)"-type nonlinearities, relating to much of the macroeconomics literature on durable goods (including Eberly (1994), Attanasio (2000), Stolyarov (2002), and Adda and Cooper (2000)). More recently, the empirical Industrial Organization literature has also studied the demand side problem when consumers face an intertemporal decision, where they can either purchase or delay, with the expectation of a new model or a lower price.[4]

There is also a literature analyzing, as in our case, an oligopoly framework with sales of durable goods (c.f. Gul (1987), Carlton and Gertner (1989), Esteban (2002), Sobel (1984), Bulow (1986), and Driskill (2001)).[5] Esteban and Shum (2007) derive a dynamic time-consistent oligopoly model and estimate it using data for the Automobile industry. The tractability of their model is obtained from a linear-quadratic specification, which requires the assumptions of no transaction costs and limited product and consumer valuation heterogeneity. Nair (2004), also close to our work, estimates an equilibrium dynamic durable goods model for the console-video game market, in which consumers purchase or delay based on the expectation of future prices, and exit the market after purchase. In his model, as in our present model, both consumers and firms are forward-looking, and solve dynamic programming problems, but there is no secondary market for used goods.

[3] Huang, Yang, and Anderson (2001) consider a model of finitely durable-goods sales and lease monopoly with used goods and transaction costs. As in our model, they also consider Markov perfect equilibrium.

[4] This literature is quite large, and includes Erdem, Imai, and Keane (2003), Hendel and Nevo (2006a), Hendel and Nevo (2006b), Melnikov (2000), Gowrisankaran and Rysman (2006), Gordon (2006), Hartmann (2006), Iizuka (2007), Chevalier and Goolsbee (2005), Carranza (2007),Schiraldi (2006), and Copeland (2006).

[5] Gavazza (2007) builds a model of used aircraft transactions where lessors, who centralize the exchange, can reduce frictions in trade.

In the empirical IO literature, there has also been a large literature (including Bresnahan (1981), Berry, Levinsohn, and Pakes (1995), Goldberg (1995), Petrin (2002)) which estimates demand-supply models for the automobile industry. In these papers, firms solve a static problem, and do not recognize the intertemporal linkages between the new and used car markets. In such a setting, secondary markets unambiguously hurt producers' profits, and transactions costs could aid firms by reducing the stock in secondary markets (and keeping used car prices high). As discussed above, these effects could be reversed in the dynamic setting.

The model that we present is complicated because both the consumer and firm sides are dynamic programming models. Hence, structural estimation of the model appears more complicated than the types of models considered in the recent literature on the estimation of dynamic games (eg. Bajari, Benkard, and Levin (2007), Aguirregabiria and Mira (2007), Bajari and Hong (2005)). In current work (Chen, Esteban, Shum, and Tanaka (2007)), we are exploring the estimation of the model, focusing on an application to real estate markets in Japan.

The next section presents the model. Section 3 presents the calibration exercise and results. Section 4 presents the equilibrium and the steady state in the calibrated version of the model. Section 5 presents the results regarding the effects of secondary markets, obtained by varying the size of consumer transactions costs. Section 6 concludes.

2 Model

In this section, we describe our model of a durable goods oligopoly with secondary markets in which consumers are heterogeneous and have transactions costs. An attempt has been made to specify a model which is potentially amenable to empirical work, and we take some precedence from existing empirical specifications. Hence, our consumer demand model resembles the "dynamic logit" specifications of dynamic discrete-choice models which started with Rust (1987). Our supply-side resembles Esteban and Shum (2007).

There are J different types of cars in the market (including both new and used), which differ in model and vintage and which are heterogeneous in quality (product characteristics). We index all available car types (new and used) by $j = 0, 1, \ldots, J$, where $j = 0$ is the outside option of no car and the first F cars, cars $j = 1, \ldots, F$, with $F \leq J$, are new. For each car j, we let $\alpha_j \geq 0$ denote its quality-product characteristics index. We assume, for simplicity, that each firm produces only one car model. Thus, with $f = j = 1, \ldots, F$, we can index firms as well as the different new car models produced.

We define the depreciation of car models as follows. We let $d(j) \in \{F+1, \ldots, J\}$ denote the next period's index of a car which is currently indexed j (for $j \neq 0$), with the convention that $d(0) = 0$. Also, we let $a(j) \in \{1, \ldots, J\}$ denote the previous period's index of a car which currently has index j. Thus, $d(a(j)) = j$.

2.1 Consumers' problem

There is a continuum of consumers of size M, and we denote a generic consumer by i. Consumers are differentiated in two dimensions. On the one hand, consumers differ in their marginal utility of money, γ, of which there are $L < \infty$ distinct types in proportions $(\pi_1, \ldots, \pi_L)$ with $\sum_l \pi_l = 1$. We let l_i denote i's type and γ_i his marginal utility of money. On the other hand, consumers are also heterogeneous in their valuation of goods, which is modeled as an i.i.d shock that perturbs the choice of product by the consumer. Let $\vec{\epsilon}_{it} \equiv (\epsilon_{i0t}, \epsilon_{i1t}, \ldots, \epsilon_{iJt})$ be the vector of idiosyncratic shocks of consumer i for period t, which are i.i.d. across (i, j, t).

We let r_{it} $(= 0, F+1, \ldots, J)$ denote the index of the car owned by a consumer i at the beginning of period t. Thus, $r_{it} > F$ or $r_{it} = 0$ if i does not own a car at the beginning of t (or his car has just died). We let $K^l_{jt} \equiv \#\{i : r_{it} = j,\ l_i = l\}$ denote the fraction of consumers in the population who are type l and own car j at the beginning of period t. Notice that, by our timing convention, $K^l_{jt} = 0$ for $j = 1, \ldots, F$ (because no consumers own new goods at the beginning of any period).

A consumer i derives the following one-period utilities for each of the possible consumption/sell choices at t. If the consumer i keeps in period t the car she already owns – the car with index r_{it} – she gets utility

$$\alpha_{r_{it}} + \epsilon_{ir_{it}t}.$$

If the consumer sells and purchases a car with index j instead (where $j \neq r_{it}$), she gets utility

$$\alpha_j + \gamma_i \cdot (p_{r_{it}t} - p_{jt} - k_0) + \epsilon_{ijt},$$

where γ_i measures consumer i's marginal utility of money, which varies across consumers, and k_0 is the transaction cost she incurs when selling a car. We let $k_0 = 0$ if $r_{it} = 0$ and let $k_0 = k$ otherwise. If, instead, the consumer sells a car and does not purchase a replacement, she gets utility

$$\gamma_i \cdot (p_{r_{it}} - p_{0t} - k_0) + \epsilon_{i0t}.$$

We let p_{jt} denote the price of product j in period t. We set $p_{0t} = 0$ for all t. Let $\vec{p}_t = (p_{0t}, p_{1t}, \ldots, p_{Jt})$ be the price vector at time t. In this specification, γ captures vertical differentiation among the new and used cars in consumers' preferences, while the ϵ's capture horizontal differentiation.[6] We assume that ϵ_{ijt} is independently and identically distributed across consumers i, car types j, and periods t. Define $\vec{\epsilon}_{it} \equiv \{\epsilon_{i0t}, \ldots, \epsilon_{iJt}\}$ as the vector of ϵ's specific to consumer i in period t.

Define the vector $\vec{K}_t = (K^1_{0t}, \ldots, K^1_{Jt}, K^2_{0t}, \ldots, K^2_{Jt}, \ldots, K^L_{0t}, \ldots, K^L_{Jt})'$ to be the vector of all $j = 0, 1, \ldots, J$ vehicle holdings at the beginning of period t by each of the $l = 1, \ldots, L$ groups of consumers. Similarly, let $\vec{B}_t$ denote the vector of used car stocks in period t, with elements K^l_{jt} for $j \in \{F+1, \ldots, J\}$ and $l \in \{1, \ldots, L\}$. Thus, $\vec{B}_t$ only differs from $\vec{K}_t$ in that it removes the entries for the outside option and for all new cars $f = 1, \ldots, F$ (recall that the entries for new cars are zeros as the state for the consumer can never be to start t with an undepreciated new car, which would correspond to cars $f = 1, \ldots, F$).

To write the consumer's problem in a dynamic programming framework, we define the individual and aggregate states as follows. For each consumer i, the individual state is $\omega_{it} = (\vec{B}_t, r_{it}, \vec{\epsilon}_{it})$. It will be convenient to separate $\tilde{\omega}_{it} = (\vec{B}_t, r_{it})$, which are the state variables which exhibit persistence over time. Similarly, the aggregate state is $\vec{B}_t$. We assume that consumers take the transition function $\vec{B}_{t+1} = H(\vec{B}_t)$ as given. They also take as given the mapping between the aggregate state $\vec{B}_t$ and prices, which we denote by $\vec{p}_t = G(\vec{B}_t)$. In equilibrium, functions $H(\cdot)$ and $G(\cdot)$ will be consistent with consumers' and firms' optimal decisions, and will be described below.

Let $s_{it} \in \{0, \ldots, J\}$ denote i's optimal consumption choice in t. Then the current utility in period t is:

$$\begin{aligned} u(s_{it}, \omega_{it}; \gamma_i) &= \alpha_{s_{it}} + \mathbf{1}_{s_{it} \neq r_{it}} \cdot \gamma_i \cdot (p_{r_{it}} - p_{s_{it}} - k_0) + \epsilon_{i s_{it} t} \\ &\equiv \tilde{u}(s_{it}, \tilde{\omega}_{it}; \gamma_i) + \epsilon_{i s_{it} t}. \end{aligned}$$

[6] Our assumptions on heterogeneity can be contrasted with those in Porter and Sattler (1999). The model in that paper is a vertically differentiated model, where the distribution of γ is continuous and uniform, and there are no ϵ errors. It is similar to the "pure" vertical model in Berry and Pakes (1999). However, the pure vertical differentiation model is not so convenient for computational reasons, as the market shares may be discontinuous functions of the model parameters. In Esteban and Shum (2007), a pure one-dimensional vertical differentiation model of auto demand is estimated.

The second term in the first equation above represents the disutility incurred when the consumer sells her r_{it}-indexed car and buys the s_{it}-indexed car, which includes the transaction cost term k_0.

Given the functions $H(\cdot)$ and $G(\cdot)$, the consumer's Bellman equation is:

$$V\left(\omega_{it};\gamma_i\right) = \max_{s_{it}} \left[u(s_{it},\omega_{it};\gamma_i) + \beta E_{\vec{\epsilon}_{it+1}} V\left(\omega_{it+1};\gamma_i\right)\right], \tag{1}$$

where β is the discount factor. Given the depreciation schedule, the used car vehicle holdings evolve according to

$$r_{it+1} = d(s_{it}),$$

which, together with function H, determines ω_{it+1}. Then the consumer's optimal policy function associated with Bellman equation (1) can be written as:

$$s_{it} = s^*(\omega_{it};\gamma_i).$$

Note that, given positive transaction costs, the optimal car choice is state-dependent, because the choice of s_{it} depends explicitly on r_{it}, which is the index of the currently-owned car. In other words, consumers are not indifferent between keeping a car and selling/repurchasing it from the secondary market since they incur transaction costs in the later. This is a significant difference between this model and previous models (e.g. Esteban and Shum (2007)) which do not allow for transactions costs.

Now define $\tilde{V}(\tilde{\omega}_{it};\gamma_i) \equiv E_{\vec{\epsilon}} V(\omega_{it};\gamma_i)$ to be the expected value function, before the shock is observed. It is given by

$$\tilde{V}(\tilde{\omega}_{it};\gamma_i) = E_{\vec{\epsilon}} \left\{ \max_{s_{it}} \left[\tilde{u}(s_{it},\tilde{\omega}_{it};\gamma_i) + \epsilon_{is_it} + \beta\tilde{V}(\tilde{\omega}_{it+1};\gamma_i)\right] \right\}. \tag{2}$$

We further assume that each ϵ_{ijt} is distributed type 1 extreme-value, independent across consumers, goods, and time. Eq. (2) can then be written as

$$\tilde{V}(\tilde{\omega}_{it};\gamma_l) = \log \left\{ \sum_{j=0}^{J} \exp\left(\tilde{u}(j,\vec{B}_t,r_{it};\gamma_l) + \beta\tilde{V}(\vec{B}_{t+1},d(j);\gamma_l)\right) \right\}. \tag{3}$$

We will iterate over this functional equation to solve for the expected value function.

2.1.1 Deriving aggregate demand functions

In what follows, we derive the demand for car j in period t, assuming that the current price vector is $\vec{p}_t$, and that consumers anticipate the next-period vector of used car stocks to be

$\vec{B}_{t+1}$. Consider the consumers i who owns a car ranked j' and are of type l. Among such consumers, those who choose car j are given by:

$$\int \mathbf{1}_{s^*(\vec{B}_t, r_{it}=j', \epsilon_{it}; \gamma_l)=j} dF(\vec{\epsilon}_{it}).$$

With the type 1 extreme-value assumption on the error terms, the sales of product j to the consumers i who own a car j' and are of type γ_l are

$$Q_j(\vec{p}_t, \vec{B}_{t+1}, j'; \gamma_l) = \frac{\exp\left(\alpha_j + \mathbf{1}_{j \neq j'} \cdot \gamma_i (p_{j't} - p_{jt} - k_0) + \beta E\tilde{V}(\vec{B}_{t+1}, d(j); \gamma_l)\right)}{\sum_{k=0}^{J} \exp\left(\alpha_k + \mathbf{1}_{k \neq j'} \cdot \gamma_i (p_{j't} - p_{kt} - k_0) + \beta E\tilde{V}(\vec{B}_{t+1}, d(j); \gamma_l)\right)}. \tag{4}$$

The total demand for car j in period t is

$$D_j\left(\vec{p}_t, \vec{B}_{t+1}\right) = M \cdot \sum_{l} \sum_{j' \neq j, j' < 1, j' > F} K^l_{j't} \cdot Q_j(\vec{p}_t, \vec{B}_{t+1}, j'; \gamma_l), \tag{5}$$

where M is the population size. Note that the demand for car j does not include the consumers who currently own car j and thus decide to keep their car and not sell it.

Also define

$$Y_j\left(\vec{p}_t, \vec{B}_{t+1}\right) = M \cdot \sum_{l} K^l_{jt} \cdot Q_j(\vec{p}_t, \vec{B}_{t+1}, j; \gamma_l), \; j = 0, F+1, \dots, J, \tag{6}$$

the consumers for whom $r_{it} = j > F$, and who keep their car in period t instead of switching to another one.

Finally, define the supply for each used car in period t as

$$S_j(\vec{p}_t, \vec{B}_{t+1}) = B_{jt} - Y_j\left(\vec{p}_t, \vec{B}_{t+1}\right), \text{ for } j = F+1, \dots, J,$$

which are the number of consumers who own car j at the beginning of period t, who choose another car $j' \neq j$, where $B_{jt} = \sum_l K^l_{jt}$ for $j = F+1, \dots, J$.

2.2 Firms' problem

We assume that firms choose quantities. To formulate the firm's problem, we must introduce some new notation. Let $q^*_j(\vec{B}_t)$ denote the equilibrium production policy function for firm j. To formulate the firm problem, we focus without loss of generality on firm 1. Given the current state $\vec{B}_t$, if firm 1 produces q_{1t} while all the other firms produce according to their equilibrium strategies $q_{jt} = q_j^*(\vec{B}_t)$, for $j \neq 1$, the current prices $\vec{p}_t$ and next-period state $\vec{B}_{t+1}$ are jointly determined by the system of equations

$$\begin{cases} K_{jt+1}^l = \sum_{j\prime} K_{j\prime t}^l Q_{a(j)}(\vec{p}_t, \vec{B}_{t+1}, j\prime; \gamma_l), \ j > F, \ l = 1, \ldots, L \\ D_1(\vec{p}_t, \vec{B}_{t+1}) = q_{1t} \\ D_j(\vec{p}_t, \vec{B}_{t+1}) = q_j^*(\vec{B}_t), \ j = 2, \ldots, F \\ D_j(\vec{p}_t, \vec{B}_{t+1}) = S_j(\vec{p}_t, \vec{B}_{t+1}), \ j > F. \end{cases} \tag{7}$$

This system of $(J-F)L+J$ equations defines the $(J-F)L+J$ unknowns in $\vec{B}_{t+1}$ and $\vec{p}_t$, as a function of $\vec{B}_t$, q_{1t}, and $\vec{q}_{-1t}(\vec{B}_t) \equiv \left(q_j^*(\vec{B}_t), \ j \neq 1\right)$. Let $\widehat{p}_1(\vec{B}_t, q_{1t})$ denote the solution for p_{1t}, firm 1's price, from this system of equations. Then firm 1's production problem can be written as

$$\max_{q_{1t}} \Pi_1(\vec{B}_t, q_{1t}) = q_{1t} \cdot (\widehat{p}_{1t}(\vec{B}_t, q_{1t}) - c_1), \tag{8}$$

where c_1 is firm 1's marginal cost of production. The Bellman equation that characterizes firm 1's value function under the presumption that all other firms and all consumers behave according to the MPE is

$$W_1(\vec{B}_t) = \max_{q_{1t}} \left[\Pi_1(\vec{B}_t, q_{1t}) + \beta W_1(\vec{B}_{t+1})\right]. \tag{9}$$

where next period's state $\vec{B}_{t+1}$ is derived from the system of equations (7).

2.3 Equilibrium

Markov-perfect equilibrium in the model consists of the following functions: price function $G(\vec{B})$, aggregate state transition function $H(\vec{B})$, firm policy functions $q_j^*(\vec{B})$, firm value functions $W_j(\vec{B})$, and consumer expected value functions $\tilde{V}(r, \vec{B}; \gamma_l)$, such that

1. Given $q_j^*(\vec{B})$ for all firms and $\tilde{V}(r, \vec{B}; \gamma_l)$ for all consumers, $G(\vec{B})$ and $H(\vec{B})$ solve the system of equations in (7).

2. Given $q_{-j}^*(\vec{B})$ for all other firms, $W_j(\vec{B})$ applied to the next period, and $\tilde{V}(r, \vec{B}; \gamma_l)$ for all consumers, $q_j^*(\vec{B})$ is the solution to firm j's maximization problem in the Bellman equation (9).

3. Given $G(\vec{B})$, $H(\vec{B})$, and $q_j^*(\vec{B})$ for all firms, $W_j(\vec{B})$ satisfies each firm's Bellman equation (i.e., Eq. (9) for firm 1, and analogous equations for the other firms).

4. Given $G(\vec{B})$ and $H(\vec{B})$, $\tilde{V}(r, \vec{B}; \gamma_l)$ satisfies the functional equation (3), for all consumer types l.

Note that both the firm and consumer problems are dynamic-programming problems, characterized by (respectively) the Bellman equations (9) and (2). Hence, this model is more complicated than existing dynamic game model in the empirical IO literature (two examples are the quality-ladder investment game of Ericson and Pakes (1995), and the dynamic entry games in Pesendorfer and Schmidt-Dengler (2003) and Aguirregabiria and Mira (2007)), which are dynamic only for the firms, but not for consumers.

We employ the collocation method to solve for the equilibrium. We approximate the above functions using tensor product bases of univariate Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002). For example, if there are two types of consumers and one type of used cars (so that $\vec{B}_t = (K^1_{2t}, K^2_{2t})$, where K^1_{2t} and K^2_{2t} are the used car stocks owned by the two types of consumers, respectively), firm policy function $q^*_j(.)$ is expressed as

$$q_j(K^1_{2t}, K^2_{2t}) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} \lambda_{ij} T_i(K^1_{2t}) T_j(K^2_{2t}),$$

where $T_i(K^l_{2t})$ is an ith-order Chebyshev polynomial in K^l_{2t}, $\lambda = (\lambda_{ij})$ is a vector of $(n+1)^2$ unknown coefficients, and n is the order of the approximation. The expressions for $G(.)$, $H(.)$, $W_j(.)$, and $\tilde{V}(.)$ are obtained analogously. With the collocation method, the above functions are evaluated at the pre-specified collocation points to check for the equilibrium conditions.

We restrict attention to symmetric MPE, and use an iterative algorithm to compute the equilibrium. The algorithm takes firm policy function $q^{*0}_j(\vec{B})$, firm value function $W^0_j(\vec{B})$, and consumer expected value function $\tilde{V}^0(r, \vec{B}; \gamma_l)$ as its input and generates updated functions $q^{*1}_j(\vec{B})$, $W^1_j(\vec{B})$, and $\tilde{V}^1(r, \vec{B}; \gamma_l)$ as its output. Each iteration proceeds as follows. We first obtain $q^{*1}_j(\vec{B})$ by solving the maximization problem on the r.h.s. of (9), taking $W^0_j(\vec{B})$ and $\tilde{V}^0(r, \vec{B}; \gamma_l)$ as given and assuming all other firms follow $q^{*0}_j(\vec{B})$. This step also produces price function $G^1(\vec{B})$ and aggregate state transition function $H^1(\vec{B})$. We next obtain $W^1_j(\vec{B})$ according to (9), taking $q^{*1}_j(\vec{B})$, $G^1(\vec{B})$, and $H^1(\vec{B})$ as given. We then solve for $\tilde{V}^1(r, \vec{B}; \gamma_l)$ by iterating over the functional equation in (3), taking $q^{*1}_j(\vec{B})$, $G^1(\vec{B})$, and $H^1(\vec{B})$ as given. The iteration is completed by assigning $q^{*1}_j(\vec{B})$ to $q^{*0}_j(\vec{B})$, $W^1_j(\vec{B})$ to $W^0_j(\vec{B})$, and $\tilde{V}^1(r, \vec{B}; \gamma_l)$ to $\tilde{V}^0(r, \vec{B}; \gamma_l)$. The iterative algorithm terminates once the relative changes in the policy and value functions from one iteration to the next are below a pre-specified tolerance. The equilibrium $G(\vec{B})$ and $H(\vec{B})$ are then obtained by solving the system of equations in (7) once more, taking the equilibrium $q_j^*(\vec{B})$ and $\tilde{V}(r, \vec{B}; \gamma_l)$ as given.

3 Calibration

In this section we present calibration of the model. Some of the parameter values are set a priori based on data or recent empirical studies, and the remaining are obtained by finding the parameterization that best matches the steady-state quantities and prices in the model to the average values for the American automobile industry over the 1994–2003 period.

3.1 Model with stochastic death of used cars

According to the 2001 National Household Travel Survey (NHTS), the average age of cars in the U.S. was 9 years. Therefore, at any point of time the number of used cars in existence is many times larger than that of new cars. That presents a modeling difficulty in the dynamic framework. If we model cars as living for many periods, then the state space is huge and the heavy computational burden makes the model intractable. If we model cars as living for a small number of periods, then the number of used cars in existence is only a few times that of new cars, which is vastly different from reality and makes calibration impossible.

To overcome this difficulty, we assume that the life of a car consists of 2 stages, new and used, and that used cars die stochastically. In particular, while we continue to assume that after one period new cars depreciate into used cars (i.e. $d(1) = 2$) with probability one, we assume that in each period used cars die with probability $\delta \in (0, 1)$:

$$d(2) = \begin{cases} 0 & \text{with probability } \delta \\ 2 & \text{with probability } 1 - \delta. \end{cases}$$

This assumption allows us to model a large stock of used cars (the expected lifetime of each car is $1 + \sum_{t=0}^{\infty}(1-\delta)^t = (1+\delta)/\delta$ periods) without having to deal with a huge state space. The restriction is the implicit assumption that the quality of a used car remains constant, regardless of how long it lives.[7] Essentially, we assume that cars follow Swan's (1985) specification of depreciation, at every age except when new.

[7]Because $d(\cdot)$ is now stochastic, the $a(\cdot)$ mapping is no longer well-defined.

3.2 Model calibration

To keep our study tractable, we assume two consumer types (ie. $L = 2$), with each type constituting half of the consumer population. Also, we consider an oligopoly consisting of three firms, each producing identical new cars. Assume that in each period, the vector of idiosyncratic shocks for any consumer consists of three elements, one for new cars, one for used cars, and one for the outside option.

In this two-vintage, two-type case, the aggregate state $\vec{B}_t$ is equal to (K^1_{2t}, K^2_{2t}), where K^1_{2t} and K^2_{2t} are the used car stocks owned by the two types of consumers, respectively. With the stochastic depreciation assumption, the aggregate state transition is given by

$$K^l_{2t+1} = \sum_{j=0,2} K^l_{jt} Q_1\left(\vec{p}_t, \vec{B}_{t+1}, j; \gamma_l\right) + (1-\delta) \cdot \left[\sum_{j=0,2} K^l_{jt} Q_2\left(\vec{p}_t, \vec{B}_{t+1}, j; \gamma_l\right)\right]. \qquad (10)$$

In the above equation, the second term on the right-hand side is a new term arising from the stochastic depreciation assumption.

Table 1 summarizes the values of the parameters that are fixed in our calibration based on data or recent empirical studies. We assume the interest rate to be 4%, which is common for consumers and firms. That gives a discount factor $\beta = 1/1.04$. We assume there are two distinct ex-ante types, type I and type II, in equal proportions. In empirical terms, Type I consumers are identified as households with income above the US median, and Type II consumers are households with below-median income.

The depreciation parameter δ was chosen to match the average age of cars in the U.S. data. The 2001 National Household Travel Survey (NHTS) reports that the average automobile age in the United States was 9 years. In our model, that translates into a depreciate rate $\delta = 0.11$.

We chose c, the marginal costs of production, to equal the lower bound of marginal costs (after deflating it) in Copeland, Dunn, and Hall (2005) (page 28). There, a marginal cost of $17,693 (in 2000 dollars) is reported, which corresponds to $18,905 in 2003 dollars.[8]

The remaining parameters are: α_1, new car utility; α_2, used car utility; γ_1, type I consumers' marginal utility of money; γ_2, type II consumers' marginal utility of money; and k, the

[8] An alternative would be to use the marginal cost estimates in Berry, Levinsohn, and Pakes (1995) (pg 882), but recent estimates are significantly lower reflecting the reduction in marginal costs of production in the industry over more recent years.

transactions cost. These five are treated as free parameters for we could find no estimate for them from data or recent empirical studies. As stated previously, values for these parameters are obtained by finding the parameterization that best matches the steady-state quantities and prices in the model to the average values for the American automobile industry over the 1994–2003 period.

A standard approach in calibration is to do a grid search (e.g. Kydland and Prescott, 1982). However, because of the complexity of the model due to dynamics on both sides of the market, the computational burden of this approach is heavy.[9] Instead, we use the MPEC (Mathematical Programming with Equilibrium Constraints; see Luo, Pang, and Ralph, 1996) approach, recently advocated by Su and Judd (2006). The main benefit of this procedure is that we avoid computing the dynamic equilibrium of the model for every candidate set of parameter values. Instead, a large-scale constrained optimization approach is used, where the constraints are given by the equilibrium conditions of the consumers' and producers' dynamic optimization problems. With this approach, we reduce the computational burden in calibration dramatically. Using the SNOPT solver in the TOMLAB optimization environment, we obtain the solution to the constrained optimization problem in 82 hours. Details on the MPEC approach for calibration are contained in the Appendix.

Using the MPEC approach, we obtain the calibrated values for the free parameters by minimizing the sum of squared relative distances between the model steady state values and the U.S. averages of six variables, as presented in the right-hand column of Table 3. These six variables are: the percentages of Type I and Type II consumers who purchase new or used cars in any year; and average new and used car prices. These US averages are obtained from the owned vehicle component of the Consumer Expenditure Survey, for the years 1994–2003. As mentioned above, the Type I and Type II consumers are identified with, respectively, households with above- and below-median income. All prices are deflated to 2003 dollars.

Table 2 present the values of the free parameters which yield the best fit, and Table 3 presents the simulated steady-state values for the model, at the calibrated parameters. The calibrated parameters presented in Table 2 show that new cars yield a utility ($\alpha_1 = 2.10$)

[9]For a given parameterization, solving for the equilibrium takes, on average, about one hour using MATLAB 7.4 on a desktop computer. So if we try five values for each of the parameters, it will take 5^4 hours = 625 hours = 26 days to complete the search. And if we do a finer grid search around the parameterization chosen by the rough grid search, that will take another 26 days.

that is around 71% higher than the utility of used cars ($\alpha_2 = 1.23$). Type I consumers have a higher taste for quality and have a lower price sensitivity (γ) equal to 1.89, while type II consumers have a lower taste for quality, and a higher price sensitivity coefficient of 2.42.

Finally, the transactions cost k is calibrated to around $2,200. Data from the Kelley Blue Book indicates that the difference between the trade-in value (seller price for consumers) and the suggested retail value (buyer price for consumers) of a used car is typically between $2,000 and $3,000 in 2007 dollars. Hence, our calibrated value for k seems quite reasonable.

3.3 Equilibrium and steady state in the calibrated model

For the calibrated model, Figure 1 presents equilibrium production function and new car price function, and Figure 2 presents firm value function and used car volume of trade function. Not surprisingly, new car production, new car price, and firm value are all decreasing in the state variables K_2^1 and K_2^2, which measure the percentage of consumers who have used cars at the beginning of the period. Used car volume of trade is the highest when roughly half of the consumers have used cars.

Figure 3 presents the vector field, indicating the direction and speed of movement of the aggregate state in the state space. The steady state is reached when 70.9% of type I consumers and 68.8% of type II consumers have used cars at the beginning of the period. The movement of the aggregate state shows that the steady state is stable.

Using our calibrated parameter estimates, we were able to simulate the effects of opening the secondary market, by varying the transactions cost k. Graphical evidence on these effects is presented in Figures 4, 5, and 6. Figure 3 shows that as the transactions cost is reduced from a prohibitively large number, $100,000, to the estimated value of $2,200, the size of the secondary market, as measured by the volume of trade, increases quickly, but profits per firm decreases. The latter result is quite interesting, given the existing theory. As we discussed before, the opening of the secondary market has two countervailing effects on firms' profits: the substitution effect lower profits, but the sorting effect increases profits. The simulation results here show that, for our calibrated parameters, the substitution effect dominates. As we will see later, this arises in large part because, in an oligopoly, the sorting effect becomes diluted, relative to the monopoly case (which is the case considered in most of the existing literature).

The effects of opening the secondary market on new car production and prices is presented in Figure 5. There, we see that, as we would expect, new car production decreases as the secondary market becomes more active, but that new car prices do not change by very much (first falling slightly, and then rising slightly). Finally, in Figure 6, we present evidence on the effects of opening the secondary market on car purchase behavior. There is some evidence of the sorting effect, in new car purchase behavior. The top graph shows that, as the secondary market expands, the less price-sensitive Type I consumers' probability of buying a new car does not change much, while the more price-sensitive Type II consumers' probability falls in half, from 10% to 5%. This is evidence of the sorting effect. At the same time, however, the bottom graph indicates that there is little difference between the two groups in the increase in the probability of purchasing a used car, as the secondary market expands.

In Table 4, we present some implications of the calibrated model, evaluated at the steady state. First, for the producer, the estimated mark-up equals 0.17, or $4,000 (evaluated at the steady-state new car price of $23,000). This is in the same ballpark as the markup figures reported in Berry, Levinsohn, and Pakes (1995) (pg. 882) which were for an earlier time period (the 1970's and 1980's). The remaining rows on Table 4 show important behavioral differences between the two types of consumers. Unconditionally, the high-type I consumers are more likely to purchase new cars, while the low-type II consumers are more likely to hold on to their used car.

4 Does secondary market help or hurt new good producers?

Given the calibrated parameter values, we proceed to address the questions posed at the beginning of the paper: what is the effect of a more active secondary market on primary producers' profits? Before going to the counterfactuals, we consider some counteracting effects that the secondary market may have on profitability.

An important determinant of the primary producers' profitability is the demand for new cars. In our setting, where consumers are heterogeneous in their taste for quality, a more active secondary market makes it easier for consumers who value quality highly to sell their used cars. This "sorting" effect improves the profitability of the new good producers, as it allows them to sell new cars to consumers who value quality more.

On the other hand, the used cars in the secondary market also substitute against new cars, so that a larger secondary market may make the new car demand curve more elastic and, therefore, erode the profitability of the new car producers. This second effect we call the "substitution" effect. The overall impact of enlarging the secondary market on the new car producers will depend on the relative magnitude of these two effects, and which effect dominates is a question we now address in the counterfactuals.

In the top panel of Table 5, we present the steady-state outcomes for the "baseline" case, where we allow the size of the transactions cost to vary from \$100,000 to zero, but hold all other parameters fixed at the calibrated values in Table 2. The results indicate that a reduction in the transactions cost from \$100,000 to zero reduces profits from 0.14 to 0.10, a reduction of 26%. These results were plotted earlier, in Figure 4. Hence, it appears that for our calibrated parameter values, the net effect of a more active secondary market is to reduce the new car producers' profitability.

In the remaining panels of Table 5, we consider counterfactuals which increase or decrease the extent of consumer heterogeneity. Since the sorting effect occurs only when consumers are heterogeneous, we expect that the opening of the secondary market will have more beneficial effects for the new good producers when consumers are more heterogeneous. Indeed, we find this to be the case. In the second panel, we increase consumer heterogeneity by decreasing γ_1, but increasing γ_2 (thus increasing the willingness-to-pay for the high type consumers, and decreasing the willingness-to-pay for the low type consumers). For this case, we see that opening secondary markets increasing the producers' profits from 0.008 to 0.018, a sizeable 136%. In contrast, when we eliminate consumer heterogeneity, as is done for the results reported in the bottom panel of Table 5, we find that profits decrease from 0.017 to 0.011, a reduction of 36%. In these counterfactuals, we have focused on isolating the sorting effect by varying the degree of consumer heterogeneity, and we find that the sorting effect can have substantial effects on firms' profits.

Table 6 contains results from counterfactuals where we consider changes in the degree of quality differentiation between new and used cars. These counterfactuals are informative regarding the substitution effect of secondary markets. We should expect that as new and used cars are less differentiated, the substitution effect should become stronger, so that an expanded secondary market would hurt the new car producers more.

The bottom two panels from this table confirm this intuition. Throughout, we hold α_1, the quality of a new car, fixed at the baseline value of 2.10. When we reduce α_2 to 0.60, thus making new and used cars less substitutable, opening the secondary market increases firms' profits by 4%. However, when we increase α_2 to 1.80, so that new and used cars are closer in quality, the opening of the secondary market reduces profits by 42%. The differences in these two sets of counterfactuals highlight the importance of the substitution effect.

In a third set of counterfactuals, reported in Table 7, we consider changes in car durability, depending on the value of δ, the per-period death probability of a used car. Because consumers are forward-looking and consider the stream of utility flows emanating from a car when they decide to purchase, a more durable car will tend to make consumers perceive less differentiation between new and used cars, because the utility in the first-period – which is the only period in which new and used cars differ in quality – will constitute a smaller part of the lifetime utility from owning the car. Hence, more durable cars may enhance the substitution effect, thus hurting firms' profits.

The results in the bottom two panels of Table 7 confirm this intuition. In the second panel, durability is increased by reducing the death probability to $\delta = 0.05$. In that case, opening the secondary market reduces firms' profits by 48%. However, when durability is reduced, by increasing δ to 0.25, firm's profits increase by 24%.

In the final set of counterfactuals, reported in Table 8, we consider how the benefits of the secondary market vary depending on market structure. In the baseline case, we assume a Cournot triopoly (three firms), and find that when transactions costs move from \$100,000 to zero, profits per firm are reduced by 26%. The counterfactuals in Table 8 show that as the market structure becomes more concentrated, the benefits to producers from the opening of the secondary market emerge. In the second panel, we see that in the duopoly case, opening the secondary market reduces the per-firm profits by only 16%; in the bottom panel, we see that a monopolist would actually benefit from the opening of the secondary market, and his profit would *increase* 2%. These results also suggest that, in practice, the sorting benefits are likely to be most prominent in monopolistic industries.

5 Conclusion

To investigate how the tradability of durable goods in secondary markets affects firms' behavior and profits, we develop a dynamic equilibrium model of durable goods oligopoly, in which consumers face lumpy costs of transacting in the secondary markets and to which they respond by buying and selling infrequently. Both sides of the market, firms and consumers, are forward-looking. We calibrate the model to match aggregate data from the American automobile industry. The fit is very good.

A crucial element in the model that allows us to analyze the effects of the tradability of used products is the transaction costs in secondary markets. Using the calibrated version of the model, we run counterfactuals by varying the transaction costs, and show that when transaction costs are reduced and the secondary market becomes more active, firms are forced to decrease their production and charge a lower price, indicating that in this case the secondary market constitutes strong competition to the primary market and reduces demand for new products and hence firms' profits.

We then conduct such counterfactuals under various parameterizations. We find that opening the secondary market by reducing the transaction costs is more detrimental (less beneficial) to new good producers (1) the smaller is the heterogeneity among consumers, (2) the smaller is the differential between the new good utility and the used good utility, (3) the more durable is the used good, or (4) the less concentrated is the primary market.

Our study suggests that opening the secondary market has two effects on the demand for the new good: the sorting effect, which increases the demand, and the substitution effect, which decreases the demand. *Ceteris paribus*, a larger heterogeneity among consumers makes the sorting effect more prominent, hence opening the secondary market is more beneficial to new good producers. On the other hand, a greater substitutability of used products for new products makes the substitution effect more prominent, hence opening the secondary market is more detrimental to new good producers.

A The MPEC Approach to Calibration

In calibrating the model, some of the parameter values are set a priori based on data or recent empirical studies (summarized in Table 1), and the remaining are obtained by finding the parameterization that best matches the steady-state in the model to the average values in the American automobile industry over the 1994–2003 period. We use the MPEC (Mathematical Programming with Equilibrium Constraints; see Luo, Pang, and Ralph, 1996) approach, recently advocated by Su and Judd (2006).

Consider the two-vintage, two-type case presented in Section 3. Let $(D_1^{1ss}, D_1^{2ss}, D_2^{1ss}, D_2^{2ss}, p_1^{ss}, p_2^{ss})$ and $(D_1^{1US}, D_1^{2US}, D_2^{1US}, D_2^{2US}, p_1^{US}, p_2^{US})$ be the model steady state and the U.S. averages, respectively, where D_j^l is the percentage of type l consumers who purchase car j, for $l = 1, 2$ and $j = 1, 2$, p_1 is new car price, and p_2 is used car price. Let $G(\vec{B})$, $H(\vec{B})$, $q^*(\vec{B})$, $W(\vec{B})$, and $\tilde{V}(\vec{B}, r_i; \gamma_l)$ be the equilibrium price function, aggregate state transition function, firm policy function, firm value function, and consumer expected value function, respectively. Here the aggregate state is $\vec{B} = (K_2^1, K_2^2)$, where K_2^1 and K_2^2 are the used car stocks owned by the two types of consumers, respectively. Let $\Omega = [0, \pi_1] \times [0, \pi_2]$ be the aggregate state space.

Let $Z(\vec{p}, \vec{B}'; \vec{B}, W, \tilde{V}, q_1, q_{-1}) = 0$ denote the system of equations (7) in Section 2 that consists of the aggregate state transition functions and the market-clearing conditions, where q_1 denotes firm 1's production and q_{-1} denotes each of firm 1's rivals' production. That is, for any $(\vec{B}, W, \tilde{V})$, if firm 1 produces q_1 and each of firm 1's rivals produces q_{-1}, $Z(.) = 0$ says that the pair $(\vec{p}, \vec{B}')$ satisfies the aggregate state transition functions and the market-clearing conditions.

Let $\vec{B}^{ss} = (K_2^{1ss}, K_2^{2ss})$ denote the aggregate state in the steady state. Let $\theta = (\alpha_1, \alpha_2, \gamma_1, \gamma_2, k)$ denote the set of parameters that we want to calibrate using the MPEC approach, and let $(N, \beta, \pi_1, \pi_2, \delta, c) = (3, 1/1.04, 0.5, 0.5, 0.11, 1.9)$ be the set of fixed parameters. The calibration solves the following minimization problem:

$$
\begin{aligned}
\min_\theta \quad & \left(\frac{D_1^{1ss}-D_1^{1US}}{D_1^{1US}}\right)^2+\left(\frac{D_1^{2ss}-D_1^{2US}}{D_1^{2US}}\right)^2+\left(\frac{D_2^{1ss}-D_2^{1US}}{D_2^{1US}}\right)^2+\left(\frac{D_2^{2ss}-D_2^{2US}}{D_2^{2US}}\right)^2 \\
& +\left(\frac{p_1^{ss}-p_1^{US}}{p_1^{US}}\right)^2+\left(\frac{p_2^{ss}-p_2^{US}}{p_2^{US}}\right)^2 \\
\text{subject to} \quad & Z(G(\vec{B}),H(\vec{B});\vec{B},W,\tilde{V},q^*(\vec{B}),q^*(\vec{B}))=0,\ \forall\vec{B}\in\Omega & (11)\\
& q^*(\vec{B})=\arg\max_{q_1\in[0,1]} q_1(p_1-c)+\beta W(\vec{B}'),\ \forall\vec{B}\in\Omega & (12)\\
& \quad \text{such that } Z(\vec{p},\vec{B}';\vec{B},W,\tilde{V},q_1,q^*(\vec{B}))=0 \text{ and } p_1 \text{ is the first element in } \vec{p} \\
& W(\vec{B})=q^*(\vec{B})(p_1-c)+\beta W(H(\vec{B})),\ \forall\vec{B}\in\Omega & (13)\\
& \quad \text{where } p_1 \text{ is the first element in } G(\vec{B}) \\
& \tilde{V}(\vec{B},r_i;\gamma_l)=\log[\exp(\tilde{u}(r_i,1,G(\vec{B});\gamma_l)+\beta\tilde{V}(H(\vec{B}),d(1);\gamma_l)) & (14)\\
& \qquad +\exp(\tilde{u}(r_i,2,G(\vec{B});\gamma_l)+\beta E_{d(2)}\tilde{V}(H(\vec{B}),d(2);\gamma_l)) \\
& \qquad +\exp(\tilde{u}(r_i,0,G(\vec{B});\gamma_l)+\beta\tilde{V}(H(\vec{B}),d(0);\gamma_l))], \\
& \qquad \forall\vec{B}\in\Omega,\ r_i=0,2,\ l=1,2 \\
& (p_1^{ss},p_2^{ss})'=G(\vec{B}^{ss}) \\
& \vec{B}^{ss}=H(\vec{B}^{ss}) & (15)\\
& D_1^{1ss}=K_2^{1ss}Q_1(G(\vec{B}^{ss}),H(\vec{B}^{ss}),2;\gamma_1)+(1-K_2^{1ss})Q_1(G(\vec{B}^{ss}),H(\vec{B}^{ss}),0;\gamma_1) \\
& D_1^{2ss}=K_2^{2ss}Q_1(G(\vec{B}^{ss}),H(\vec{B}^{ss}),2;\gamma_2)+(1-K_2^{2ss})Q_1(G(\vec{B}^{ss}),H(\vec{B}^{ss}),0;\gamma_2) \\
& D_2^{1ss}=(1-K_2^{1ss})Q_2(G(\vec{B}^{ss}),H(\vec{B}^{ss}),0;\gamma_1) \\
& D_2^{2ss}=(1-K_2^{2ss})Q_2(G(\vec{B}^{ss}),H(\vec{B}^{ss}),0;\gamma_2) \\
& \gamma_2>\gamma_1
\end{aligned}
$$

Note that constraints (11)∼(14) correspond to the equilibrium conditions specified in Section 2, and that constraint (15) gives the steady state condition on the aggregate state.

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Table 1: Fixed Parameters

Discount factor (β)	1/1.04
# of distinct ex-ante consumer types	2
% of type I consumers	50%
% of type II consumers	50%
Probability of used car quantity depreciation (δ)	0.11
Marginal cost (c)	$19,000
# of firms	3

Table 2: Calibrated parameters

New car utility (α_1)	2.10
Used car utility (α_2)	1.23
Type I consumers' marginal utility of money (γ_1)	1.89
Type II consumers' marginal utility of money (γ_2)	2.42
Transactions cost k	$2,200

Table 3: Steady-state values, at calibrated parameters

	Model steady state values	U.S. data averages (1994–2003)[a]
% of Type 1 consumers[b]:		
who purchase new cars	10.3	9.8
who purchase used cars	17.9	18.7
% of Type 2 consumers[c]:		
who purchase new cars	5.1	4.2
who purchase used cars	19.9	18.6
New vehicle price	\$23,000	\$23,000
Used vehicle price	\$9,100	\$9,000

[a]From Consumer Expenditure Survey, owned vehicle module.
[b]Households with above-median income
[c]Households with below-median income

Table 4: Steady-state results, at calibrated parameter values

Firm markup	0.17	
Consumers' Transition probabilities $P(r_{t+1}\|r_t)$		
	Type I	Type II
$P(1\|2)$	0.09	0.04
$P(2\|2)$	0.71	0.75
$P(0\|2)$	0.20	0.20
$P(1\|0)$	0.12	0.06
$P(2\|0)$	0.61	0.64
$P(0\|0)$	0.26	0.30

Table 5: Effects of opening secondary market: more vs. less consumer heterogeneity

Variable	*Transactions cost ($10,000)*			
	10	2	0.22	0
Baseline: $\gamma_1 = 1.89$, $\gamma_2 = 2.42$				
New car production per firm	0.030	0.030	0.026	0.025
Used car transactions	0.00	0.04	0.19	0.22
New car prices	2.35	2.32	2.30	2.31
Used car prices	—	2.01	0.91	0.77
Profits per firm	0.014	0.012	0.010	0.010 (-26%)
More heterogeneity: $\gamma_1 = 1.00$, $\gamma_2 = 4.00$				
New car production per firm	0.029	0.029	0.027	0.026
Used car transactions	0.00	0.08	0.18	0.21
New car prices	2.16	2.42	2.58	2.60
Used car prices	—	1.51	0.43	0.26
Profits per firm	0.008	0.015	0.018	0.018 (+136%)
Less heterogeneity: $\gamma_1 = 2.00$, $\gamma_2 = 2.00$				
New car production per firm	0.030	0.030	0.026	0.025
Used car transactions	0.00	0.05	0.19	0.22
New car prices	2.46	2.36	2.32	2.34
Used car prices	—	1.99	0.85	0.72
Profits per firm	0.017	0.014	0.011	0.011 (-36%)

Table 6: Effects of opening secondary market: more vs. less quality differentiation

Variable	*Transactions cost ($10,000)*			
	10	2	0.22	0
BASELINE: $\alpha_1 = 2.10$, $\alpha_2 = 1.23$				
New car production per firm	0.030	0.030	0.026	0.025
Used car transactions	0.00	0.04	0.19	0.22
New car prices	2.35	2.32	2.30	2.31
Used car prices	—	2.01	0.91	0.77
Profits per firm	0.014	0.012	0.010	0.010 (-26%)
MORE DIFFERENTIATION: $\alpha_1 = 2.10$, $\alpha_2 = 0.60$				
New car production per firm	0.027	0.025	0.022	0.021
Used car transactions	0.00	0.05	0.21	0.24
New car prices	2.18	2.19	2.24	2.26
Used car prices	—	1.52	0.56	0.45
Profits per firm	0.007	0.007	0.007	0.008 (+4%)
LESS DIFFERENTIATION: $\alpha_1 = 2.10$, $\alpha_2 = 1.80$				
New car production per firm	0.031	0.031	0.028	0.027
Used car transactions	0.00	0.04	0.16	0.19
New car prices	2.63	2.48	2.38	2.37
Used car prices	—	2.45	1.24	1.07
Profits per firm	0.022	0.018	0.014	0.013 (-42%)

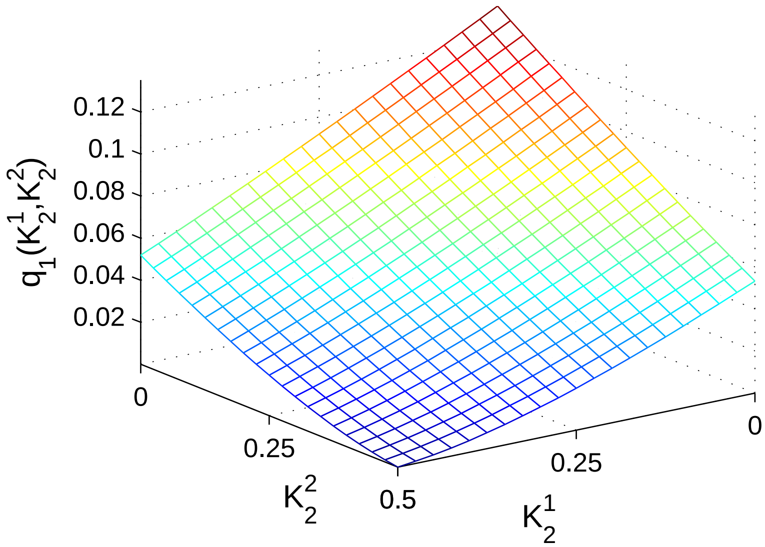
Table 7: Effects of opening secondary market: more vs. less durability

Variable	*Transactions cost ($10,000)*			
	10	2	0.22	0
BASELINE: $\delta = 0.11$				
New car production per firm	0.030	0.030	0.026	0.025
Used car transactions	0.00	0.04	0.19	0.22
New car prices	2.35	2.32	2.30	2.31
Used car prices	—	2.01	0.91	0.77
Profits per firm	0.014	0.012	0.010	0.010 (-26%)
MORE DURABILITY: $\delta = 0.05$				
New car production per firm	0.015	0.015	0.013	0.012
Used car transactions	0.00	0.03	0.16	0.20
New car prices	2.45	2.38	2.27	2.26
Used car prices	—	1.92	0.56	0.40
Profits per firm	0.008	0.007	0.005	0.004 (-48%)
LESS DURABILITY: $\delta = 0.25$				
New car production per firm	0.052	0.051	0.047	0.046
Used car transactions	0.00	0.05	0.22	0.25
New car prices	2.22	2.24	2.32	2.35
Used car prices	—	2.00	1.13	1.04
Profits per firm	0.017	0.017	0.020	0.021 (+24%)

Table 8: Effects of opening secondary market: changes in market structure

Variable	*Transactions cost ($10,000)*			
	10	2	0.22	0
BASELINE: $N = 3$ (TRIOPOLY)				
New car production per firm	0.030	0.030	0.026	0.025
Used car transactions	0.00	0.04	0.19	0.22
New car prices	2.35	2.32	2.30	2.31
Used car prices	—	2.01	0.91	0.77
Profits per firm	0.014	0.012	0.010	0.010 (-26%)
DUOPOLY: $N = 2$				
New car production per firm	0.045	0.044	0.038	0.036
Used car transactions	0.00	0.04	0.19	0.23
New car prices	2.62	2.60	2.62	2.65
Used car prices	—	2.26	1.19	1.08
Profits per firm	0.032	0.031	0.027	0.027 (-16%)
MONOPOLY: $N = 1$				
New car production per firm	0.081	0.078	0.063	0.060
Used car transactions	0.00	0.05	0.22	0.25
New car prices	3.90	3.91	4.42	4.67
Used car prices	—	3.36	2.79	2.89
Profits per firm	0.162	0.156	0.158	0.166 (+2%)

Figure 1: The calibrated model: new car production function and price function



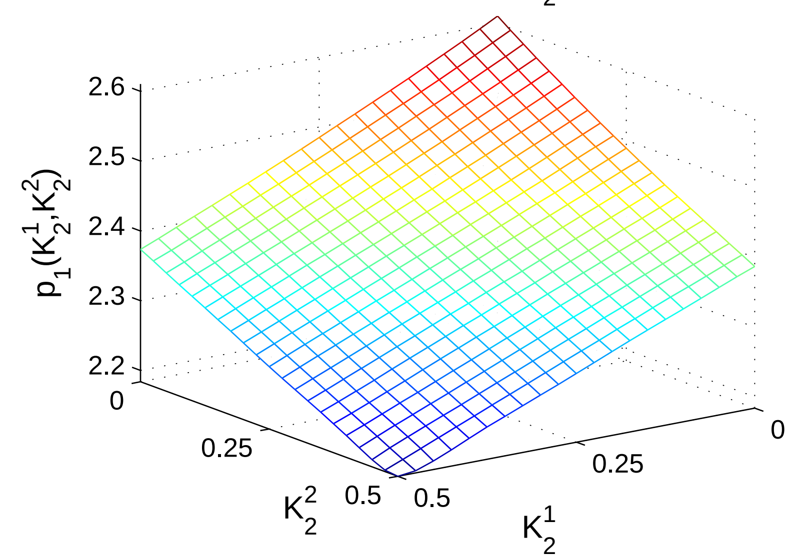


Figure 2: The calibrated model: firm value function and used car volume of trade function

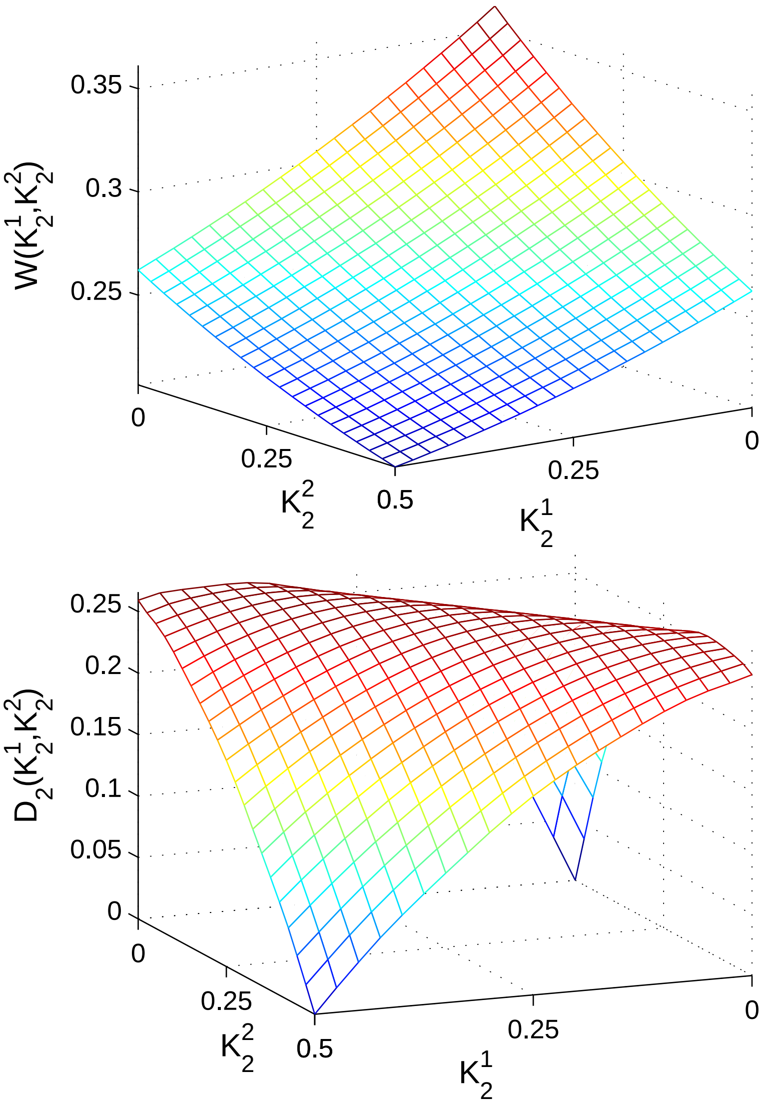


Figure 3: The calibrated model: vector field

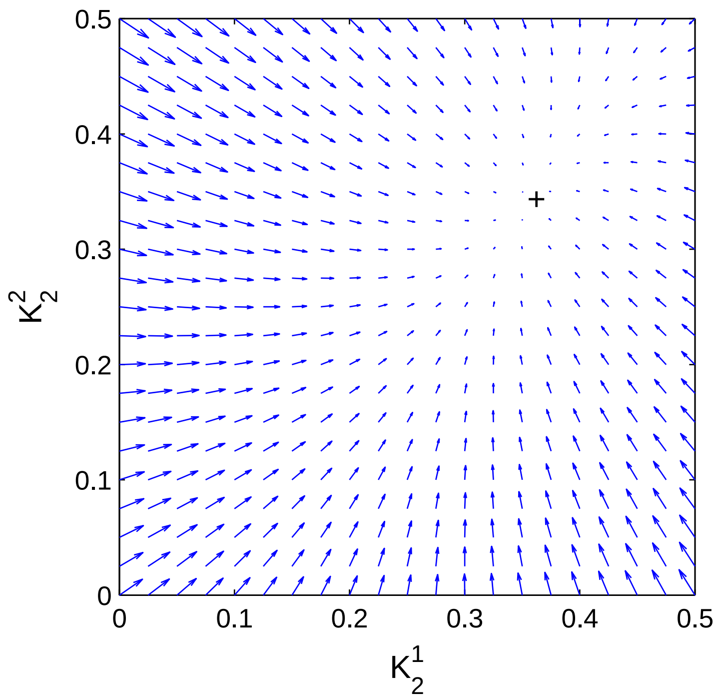


Figure 4: Opening secondary market in the calibrated model: size of secondary market and profits per firm

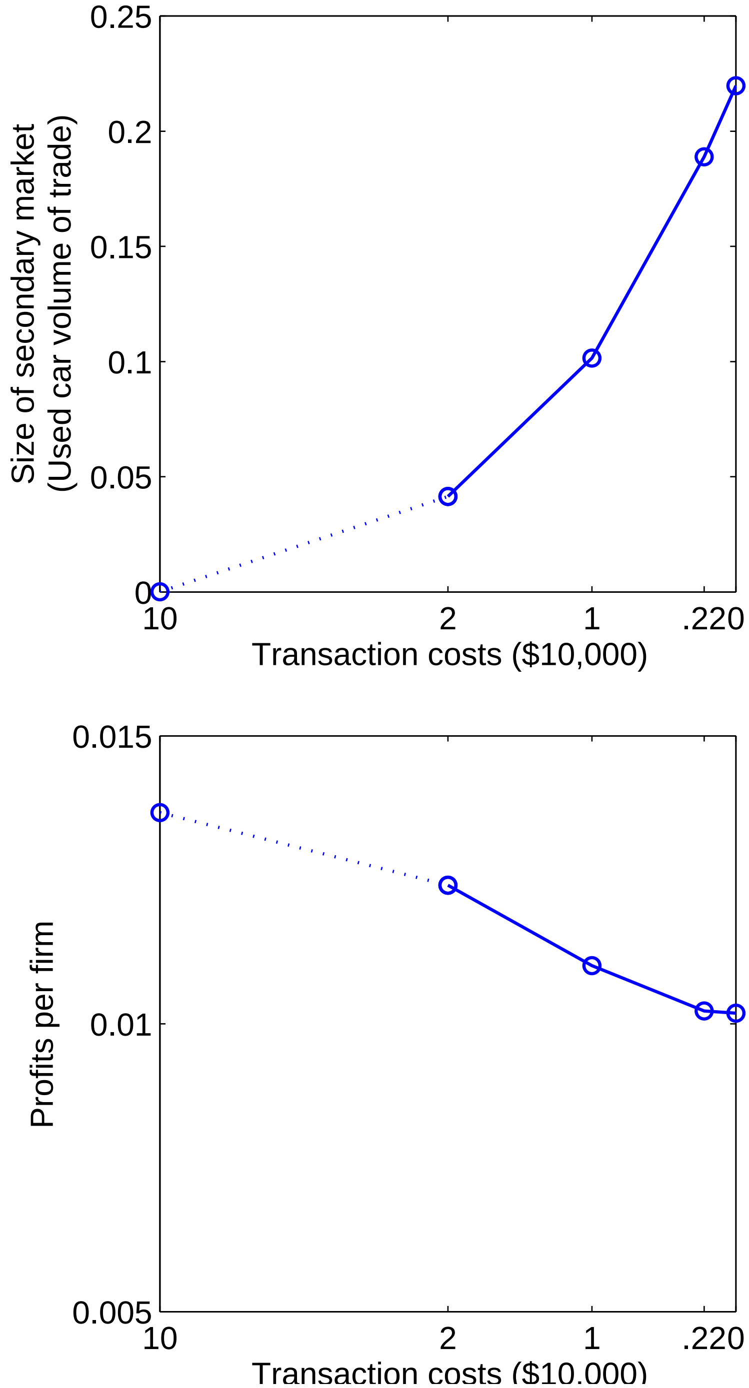


Figure 5: Opening secondary market in the calibrated model: new car production per firm and new car price

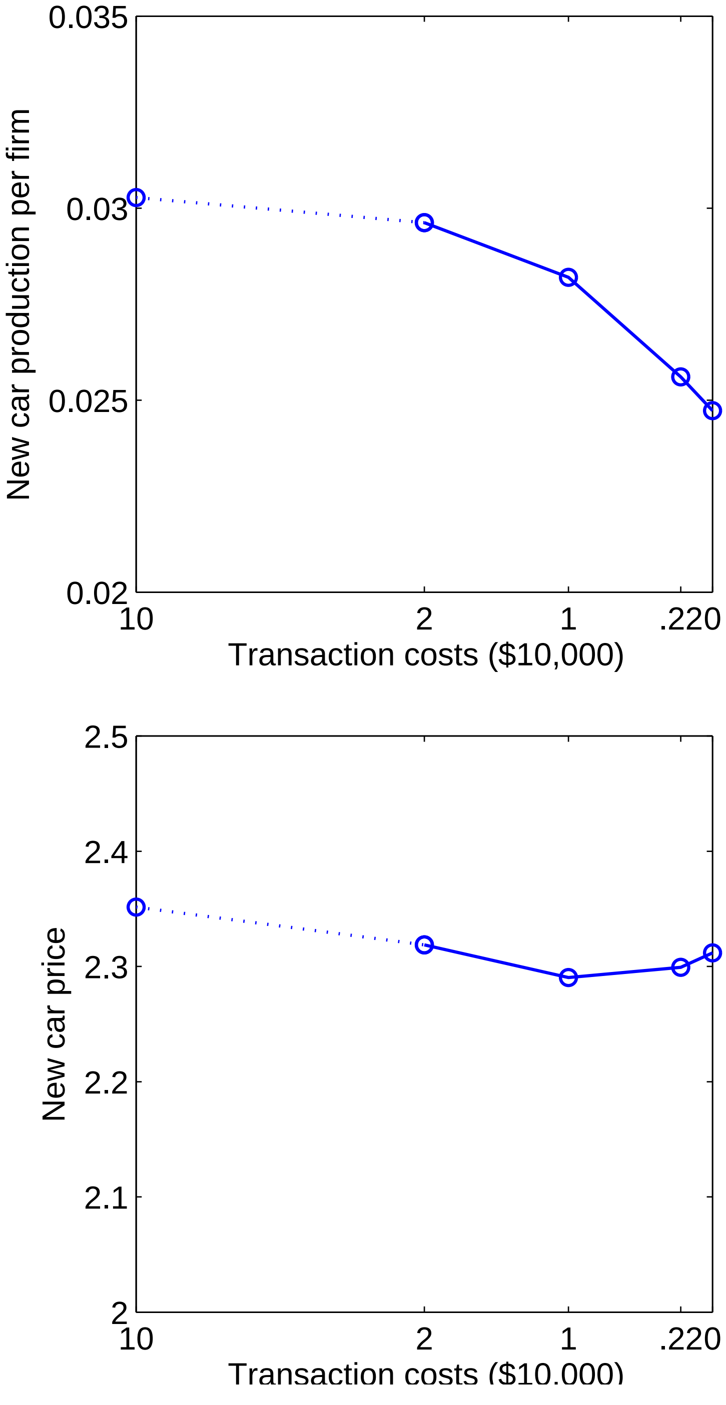


Figure 6: Opening secondary market in the calibrated model: new car and used car purchases by type