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# TESIS DOCTORAL

# Essays on Price Regulation

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# ESSAYS ON PRICE REGULATION


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## ABSTRACT

The thesis consists of three chapters. In Chapter I ("Price Caps with Capacity Precommitment"), we study the effectiveness of price cap regulation in a monopolistic setting under demand uncertainty. In our model, a monopolist facing an uncertain demand. In the absence of capacity precommitment, price caps remain an effective regulatory instrument, just as they are when demand is deterministic. Price caps are also an effective instrument to regulate a monopoly that makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon observing the realization of demand. In this scenario, however, the optimal price cap must trade off the incentives for capacity investment and capacity withholding, is well above the unit cost of capacity and, when the unit cost of capacity is low, is below the price cap that maximizes capacity. Moreover, a price cap alone cannot eliminate inefficiencies. Under standard regularity assumptions on the demand distribution, the comparative static properties of price caps above the optimal price cap are analogous to those they have in the absence of capacity precommitment.

The Chapter II and Chapter III are aimed at analyzing the competitive consequences of imposition of the Arm's Length Principle (ALP, henceforth) requirements for international transfer pricing.

In order to discourage tax shifting activities by multinational firms, most countries follow taxation policies that are based on the OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with ALP (i.e., that transfer prices between companies of multinational enterprises for tax purposes be established on a market value basis, thus comparable to transactions between independent, unrelated, parties). Moreover, ALP puts associated and independent enterprises on a more equal footing for tax purposes, it avoids the creation of tax advantages that would otherwise distort the relative competitive positions of either type of entity. The failure to comply with the ALP may result in a penalty.

The OECD's recommendation that transfer prices between parent firms and their subsidiaries be consistent with the ALP for tax purposes does not restrict internal pricing poli-

cies. In Chapter II ("Strategic Incentives for Keeping One Set of Books under the Arm's Length Principle"), we show that under imperfect competition parent firms' accounting policies determine the properties of market outcomes: if parents keep one set of books (i.e., their internal transfer prices are consistent with the ALP), then competition in the external (home) market softens (intensifies) relative to an equilibrium where parent firms and subsidiaries are integrated. In contrast, if firms keep two sets of books (i.e., their internal transfer prices differ from those used for tax purposes) or maintain asymmetric accounting policies, then competition intensifies in both markets. Keeping one set of books turns out to be an equilibrium for most of the parameter space.

In Chapter III ("The Non-Neutrality of the Arm's Length Principle with Imperfect Competition"), we show that under imperfect competition the Arm's Length Principle is non-neutral: a strict (lax) application of the ALP softens competition among subsidiaries (parents). Thus, under imperfect competition regulating transfer pricing optimally requires trading off its impact on market outcomes and tax revenue.

## CHAPTER 1. PRICE CAPS WITH CAPACITY PRECOMMITMENT

### 1.1. Introduction

Since Littlechild (1983)'s report, when precise information about cost is available, price cap regulation is regarded as an effective instrument to mitigate market power, foster cost minimization and ultimately enhance surplus: when the demand is known with certainty, the introduction of a binding price cap rises firms' marginal revenue near the equilibrium output and leads to an increase of the equilibrium output and surplus, and to a decrease of the market price. Moreover, under broad regularity conditions on the demand and cost functions, both output and surplus decrease (and the market price increases) with the price cap above marginal cost. Further, in the most favorable conditions (e.g., when firms produce the good with constant returns to scale), a price cap equal to marginal cost is able to eliminate inefficiencies.[1]

We study the effectiveness of price cap regulation under demand uncertainty.[2] In order to avoid some potential conundrums that arise in dynamic oligopolistic settings, which are distractions from the issue under scrutiny (the impact of price cap regulation), we focus on the monopolistic case.

We show that in the absence of capacity precommitment, i.e., if the monopolist can produce instantly upon the realization of demand or has slack capacity, the effects of price caps remain exactly the same as when the demand is deterministic. The results obtained in this static setting can be easily extended to oligopolistic industries. There are important markets in which firms have slack capacity and can respond instantly to demand conditions, e.g., in electricity markets in which firms' bids are short lived. In these markets price caps provide an effective regulatory instrument.

[1] In contrast, rate-of-return regulation, used for most of the 20th century to regulate public utilities, distorts incentives for cost minimization – see, e.g., Joskow (1972) – or cost reduction – see, e.g., Cabral and Riordan (1989).

[2] Demand uncertainty may be interpreted also as variations of demand over time, as is common in electricity markets – see, e.g., Green and Newbery (1992).

These results naturally raise the question of how price caps affect capacity decisions. In order to tackle this issue, we consider a more interesting setting in which a monopolist makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon the realization of demand. (Thus, the monopolist may withhold capacity if it finds it beneficial to do so.) In this setting, inefficiencies arise both because the monopolist installs a low level of capacity in order to precommit to high prices, and because the monopolist withholds capacity for low demand realizations in order to avoid prices to fall too low. Capacity withholding is common in markets such as sport events, hotel accommodation, agricultural products (in which farmer associations sometimes destroy part of the output), etc. Capacity withholding has been observed also in electricity markets, in which generators may declare their capacity to be unavailable.[3]

The effect of price cap regulation with capacity precommitment (and withholding) is more subtle. We show that, much as in the absence of capacity precommitment, the introduction of a binding price cap raises the firms' marginal return to capacity investment near the equilibrium capacity and leads to an increase of the equilibrium capacity, the expected output and the expected total surplus, and to a decrease of the expected market price. However, price caps near the unit cost of capacity are suboptimal because they reduce the return to capacity investment below its cost, and lead the monopolist to install no capacity. The optimal price cap (i.e., the price cap that maximizes surplus) must trade off appropriately the incentives for capacity investment and capacity withholding, and tends to be well above the unit cost of capacity. When the unit cost of capacity is high the effect on capacity investment is dominant, and the optimal price cap maximizes capacity investment. When the unit cost of capacity is low, reducing the price cap below the level that maximizes capacity investment increases expected surplus. Thus, maximizing capacity investment does not warrant maximizing expected surplus. In either case, a price cap alone, although effective, is unable to provide the appropriate incentives for capacity investment

[3] Data for the California electricity market during the time period May 2000-December 2001 show that at the price cap some generators did not supply all of their uncommitted capacity – see Cramton (2003) and Joskow and Kahn (2002).

and simultaneously eliminate the inefficiencies arising from capacity withholding.

The comparative static properties of price caps are under capacity precommitment are more complex than in the (static) setting in which the monopolist can produce an arbitrary output upon the realization demand. Under standard regularity assumptions on the demand distribution, the effects of changes in the price cap on expected output and surplus depend on the magnitude of its effects on capacity investment and capacity withholding, which have opposite signs. Capacity investment is maximal for a binding price cap $r^*$, which is well above the unit cost of capacity. Further, capacity investment increases (decreases) with the price cap below (above) $r^*$. When the unit cost of capacity is large the signs of the effects of changes in the price cap on the expected output, expected surplus, and capacity investment coincide. Interestingly, when the unit cost of capacity is small the expected output and surplus decrease with the price cap above and around $r^*$, and thus the optimal price cap is below $r^*$. Also, a price cap affects the market price directly, but also indirectly via its impact on the level of capacity. Thus, an increase of the price cap increases the expected price above and around $r^*$, but has an ambiguous effect below $r^*$.

Earle et al. (2007) studies an oligopolistic model in which firms make output decisions ex-ante, and then supply their output inelastically and unconditionally upon the realization of demand.[4] In this setting, it shows that the output is suboptimally low and may increase with the price cap for price caps near marginal cost.[5]. Moreover, the comparative static properties of price caps when the demand is deterministic fail for a generic demand schedule.[6] In addition, Early et al. (2007) claim that versions of these results extend to the case where firms can freely dispose of their output upon observing the demand, thus choosing how much of their output to supply, which effectively yields a model equivalent to that we study

[4] In this setting, Reynolds and Rietzke (2012) study entry, and Zoettl (2011) studies firms' technological choice.

[5] Depending on the distribution of demand, production may be shut down altogether.

[6] Specifically, Earle at al. (2007) show that for any demand distribution such that output decreases with the price cap at a given binding price cap $\bar{p}$, it is possible to perturb the demand distribution on an arbitrarily small interval around $\bar{p}$ (by shifting the probability on the interval to the endpoints, creating two atoms) in such a way that with this new demand distribution output increases with the price cap near $\bar{p}$.

in the present paper. These results lead Earle et al. (2007) to conclude that the "standard arguments supporting the imposition of price caps break down in the presence of demand uncertainty."

This sweeping conclusion of Earle et al. (2007) is unfounded. As we show, in the absence of capacity precommitment the properties of price caps remain intact when demand is uncertain. Further, with output (or capacity) precommitment, price caps near marginal cost may lead to an expected marginal revenue close, or even below, marginal cost. Thus, the incentives for output investment may be poor, and may improve if the price cap constrained is relaxed. When the demand is deterministic these effect do not arise, and the output jumps up from zero to its optimal level when the price cap approaches the marginal cost from below. When demand is uncertain (and well behaved), the output (capacity) becomes eventually positive and increasing with the price cap as the price caps increases from below the marginal cost. Price cap regulation is still effective, but must be designed taking into account the firms' incentives to invest in output. Moreover, Earle et al. (2007)'s Theorem 4, which is to be expected, does not provide a basis for such conclusion.[7] Indeed, as Grimm and Zoettl (2010) show, under certain regularity conditions on the distribution of demand, price cap regulation remains effective, and their comparative static properties (relative to the price cap that maximizes capacity) are recovered.

In the more interesting setting studied in the present paper, in which capacity decisions are made ex-ante and output decisions are made ex-post, we show that under standard regularity assumptions on the demand distribution a price cap is an effective regulatory instrument to provide incentives for capacity investment and discouraging capacity withholding. Moreover, the comparative static properties of price caps, although more subtle, are analogous to those arising when demand is deterministic or when capacity has no precommitment value. (Unlike in these settings, however, a price cap alone cannot eliminate inefficiencies and must trade off the incentives for capacity investment and capacity withholding.) Further, we show in Appendix B that a crucial step in the proof of Early et al.

[7] By the Banach-Mazurkiewicz Theorem, in the real vector space of all real-valued continuous functions on [0,1] with the supremum norm, even nowhere differentiability is a generic property.

(2007)'s Theorem 4 fails when the monopolist can withhold capacity.[8]

Grimm and Zoettl (2010) also offer an analysis of the impact of price caps in an oligopolistic setting in which firms can dispose freely of their output, which is effectively equivalent to that of the present paper. However, the reduced form analysis of this dynamic setting provided by both Earle et al. (2007) and Grimm and Zoettl (2010) raises some questions. For example, it is unclear what is the appropriate mode of competition to consider at the ex-post stage. Moreover, there are well known difficulties therein to guarantee existence, uniqueness and symmetry of equilibrium – see, e.g., Reynolds and Wilson (2000), Gabszewicz and Poddar (1997). By focusing on the monopolistic case, we did not let ourselves get sidetracked by these issues.

Also, our results differ from those obtained by Grimm and Zoettl (2010), which conclude that the properties of price caps are virtually the same both with full capacity utilization and with capacity withholding. In particular, Grimm and Zoettl (2010) mistakenly conclude that maximizing the expected surplus amounts to maximizing capacity. (We show that when the cost of capacity is small maximizing surplus entails a lower price cap than the price cap that maximizes capacity.) Apparently, the calculation of the marginal revenue in Grimm and Zoettl (2010)'s equation (5) is incorrect in region $A$ – see Section 1. 3.

Other authors have studied price cap regulation in the presence of exogenous technological progress – in our setting the unit cost of capacity and production are constant over the regulatory period. Biglaiser and Riordan (2000), for example, study the incentive properties of price cap to produce optimal capacity investment and replacement. In their setting, they find that price caps provide better incentives than rate-of-return regulation, although in their setting (as in ours) optimal price caps must deal with a trade off involving the incentives for capacity investment and replacement.

In an oligopolistic industry, Roques and Savva (2009) study the effect of price caps on the timing of investments when demand is uncertain, and find that as in our setting a low

[8] In fact, in our setting when the cost of capacity is low the perturbation of the demand distribution used in the proof of Earle et al. (2007)'s Theorem 4 has an effect akin to that it has a flat spot of a deterministic demand: changes in the price cap on this flat spot have no impact on the level of output.

price cap may be suboptimal as it may disincentivize investment. Dobbs (2004) studies the effect intertemporal price cap regulation when a monopolist facing demand uncertainty has to decide the size and timing of its investments, and shows that optimal price caps lead to under investment and quantity rationing. Dixit (1991) studies a competitive market in which demand is uncertain and firms make ex-ante irreversible investments, and shows that introducing price ceilings lead to delay investments and higher prices over time.

The paper is organized as follows. We describe the monopoly in Section 1. 2. In Section 1. 3 we derive the monopoly equilibrium when a regulator imposes a price cap. We study the comparative static properties of price caps in Section 1. 4. In Section 1. 5 we study optimal price caps. We discuss an example in Section 1. 6, and we conclude in Section 1. 7. Appendix A contains technical proofs. In Appendix B we present an exercise showing that Earle et al. (2007)'s Theorem 4 fails in our setting. Appendix C studies a version of our model assuming full capacity utilization, and discusses the differing results obtained in that setting.

### 1.2. Price Caps without Capacity Precommitment

Consider a monopoly that produces a good with constant returns to scale and unit cost $b \in \mathbb{R}_+$. For simplicity let the market demand be given by $D(X, p) = \max\{X - p, 0\}$. If the demand is deterministic, i.e., if the maximum willingness to pay for the good $X$ is known to the firm, then the effect of a price cap $r \in \mathbb{R}_+$ on the monopoly equilibrium is well known. In order to avoid the trivial case in which the monopolist produces no output, assume that $X > b$. In the absence of a price cap, the monopoly equilibrium is $q^* = (X - b)/2$ and $p^* = (X + b)/2$. A low price cap $r < b$ leads the monopoly to serve no output, i.e., $q(r) = 0$, whereas a high (non-binding) price cap $r \geq p^*$ has no effect on the monopoly equilibrium, i.e., $q(r) = q^*$ and $p(r) = p^*$. An intermediate price cap $r \in [b, p^*)$, however, increases the monopolist's marginal revenue around $q^*$, and leads to an increase of the monopolist's output to $q(r) = X - r > q^*$, and a decrease of the market price to $p(r) = r < p^*$. See Figure 1.1. Thus, the output $q(r)$ (respectively, the price $p(r)$) decreases (increases) linearly with the price cap $r$ on $(b, p^*)$, and the surplus, as well as the consumer surplus, decrease with $r$ on this interval. Hence, setting a price cap $r$ equal to the unit cost of production $b$ maximizes the output as well as the surplus.[9] Figure 1.3 below provides a graph of the function $q(r)$.

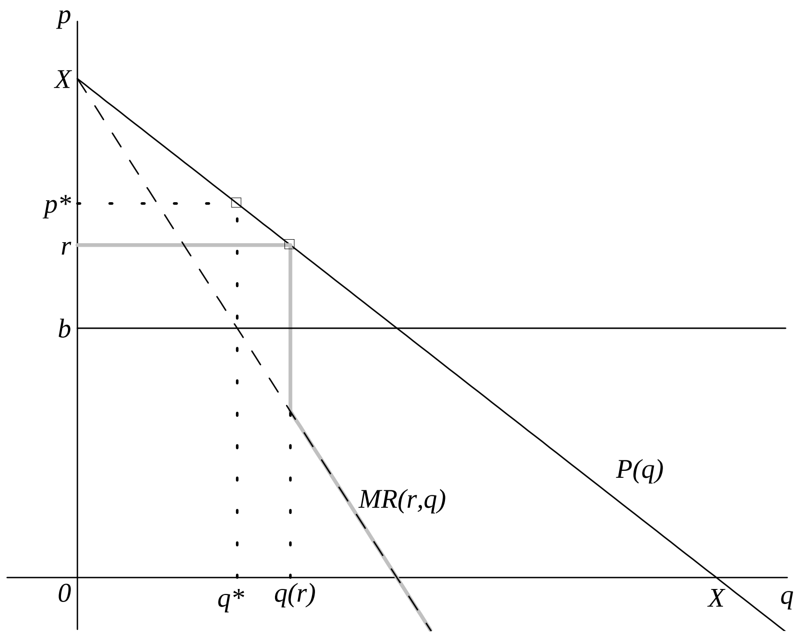


Figure 1.1. The Effect of a Price Cap with a Deterministic Demand

Let us consider know consider the case of demand uncertainty. (As noted above, demand uncertainty may be interpreted also as variations of demand over time – see, e.g., Green and Newbery (1992).) Assume that $X$ is a random variable with $p.d.f.$ $f$. Let us assume that the support of $X$ is a bounded interval $[\alpha, \beta] \subset \mathbb{R}_+$ such that $\beta > b$. Studying the impact of a price cap under demand uncertainty requires to specify the timing of decisions. Let us consider a simple model in which the monopolist decides its output upon observing the realization of demand.

In the absence of a price cap, for each demand realization $x \in [\alpha, \beta]$ the monopoly equilibrium is given by $q^*(x) = 0$ and $p(x) \geq x$ if $x < b$, and by $q^*(x) = (x - b)/2$ and $p(x) = (x+b)/2$ if $x > b$. Write $P^* = \max_{x \in [b,\beta]} p(x) = (\beta+b)/2$. The introduction of a price

[9] These properties extend to symmetric oligopolistic markets – see, e.g., Theorem 1 in Earle et al. (2007).

cap $r \in \mathbb{R}_+$ has a simple effect on the monopoly equilibrium: a low price cap $r < b$ leads the monopoly to serve no output regardless of the realization of demand, i.e., $Q(r, \cdot) = 0$. A high (non-binding) price cap $r \geq P^*$, results in an output $Q(r, \cdot) = q^*(\cdot)$. Intermediate price caps $r \in [b, P^*)$, however, have more complex effects on the monopolist output: it is easy to see that for low demand realizations $x \in [0, 2r - b)$, the price cap is non-binding and the monopolist output is $Q(r, x) = q^*(x)$, whereas for high demand realizations $x \geq 2r - b$ the monopolist serves the demand at the price cap, i.e., $Q(r, x) = x - r$. See Figure 1.2.

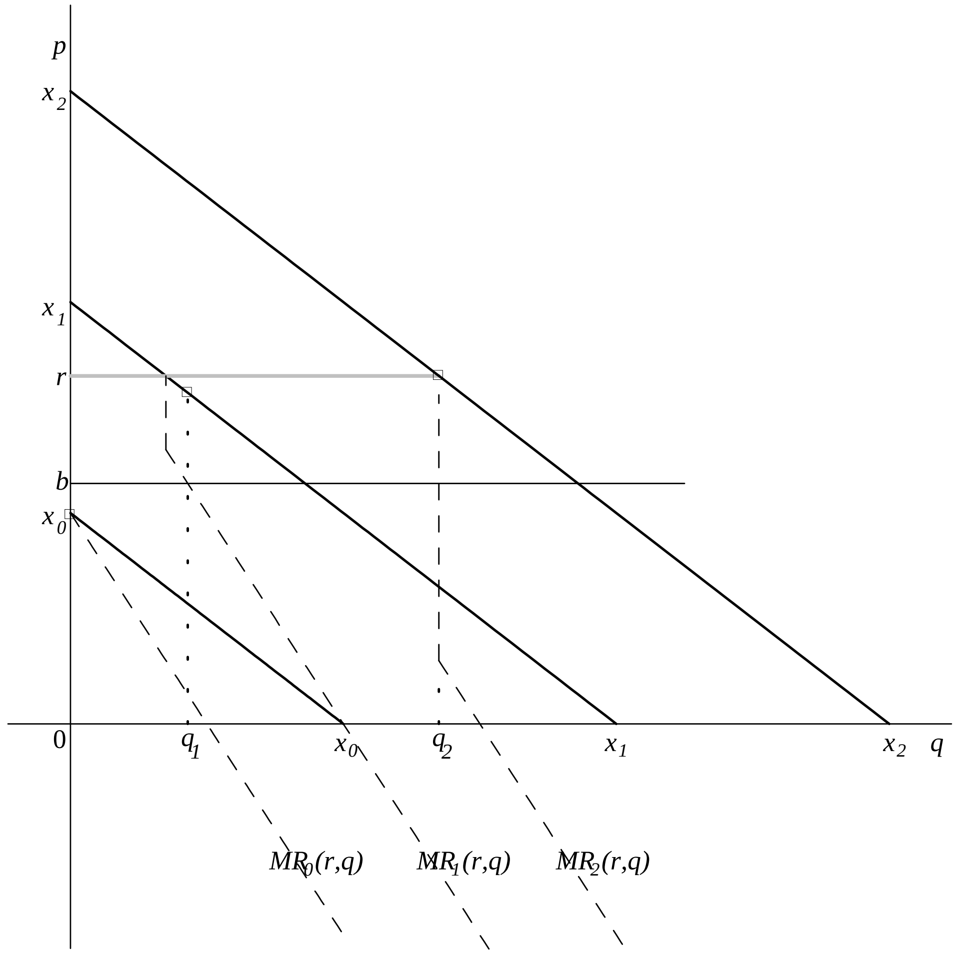


Figure 1.2. The Effect of a Price Cap with Demand Uncertainty

Hence the expected output $E(Q(r,X))$ is given for $r \in (b, P^*)$ by

$$E(Q(r,X)) = \frac{1}{2}\int_b^{2r-b} (x-b)\, f(x)dx + \int_{2r-b}^{\beta} (x-r)\, f(x)dx.$$

Differentiating this expression, and noting that $\beta > 2r - b$ for $r < P^*$, yields

$$\frac{dE(Q(r,X))}{dr} = -\int_{2r-b}^{\beta} f(x)dx < 0.$$

Thus, as in the case of demand certainty the expected output and the expected surplus decrease with the price cap on $(b, P^*)$. The expected price is not well defined since for $x < b$ the monopolist supplies no output. However, decreasing the price cap decreases the market price for demand realizations $x > 2r - b$, and has no effect on the market price for demand realizations $x \in (b, 2r - b)$, and therefore unambiguously decreases the expected price over the realizations in which there is trade. Thus, as in the case of a deterministic demand, when demand is stochastic setting a price cap $r$ equal to the unit cost of production $b$ maximizes the expected output as well as the expected surplus. We summarize these results in Proposition 1.1.

**Proposition 1.1.** *Consider a monopolist facing an uncertain demand, and assume that it is not capacity constrained (i.e., may produce the good instantly upon the realization of demand). Then a binding price cap leads to an increase of the equilibrium output and surplus, and a decrease of the expected price. Moreover, the output and surplus (expected price) decrease (increases) with the price cap price for binding price caps above marginal cost. Further, a price cap equal to marginal cost maximizes surplus, and leads to an efficient outcome.*

Thus, whether the demand is deterministic or stochastic, price cap regulation is an effective instrument to mitigate market power and foster efficiency. Figure 1.3 illustrates these conclusions – the functions $q(r)$ and $E(Q(r,X))$ are calculated assuming that $X = 1/2$ and $X$ is distributed uniformly on $[0,1]$, respectively. Proposition 1.1 can be easily extended to a Cournot oligopolistic setting.

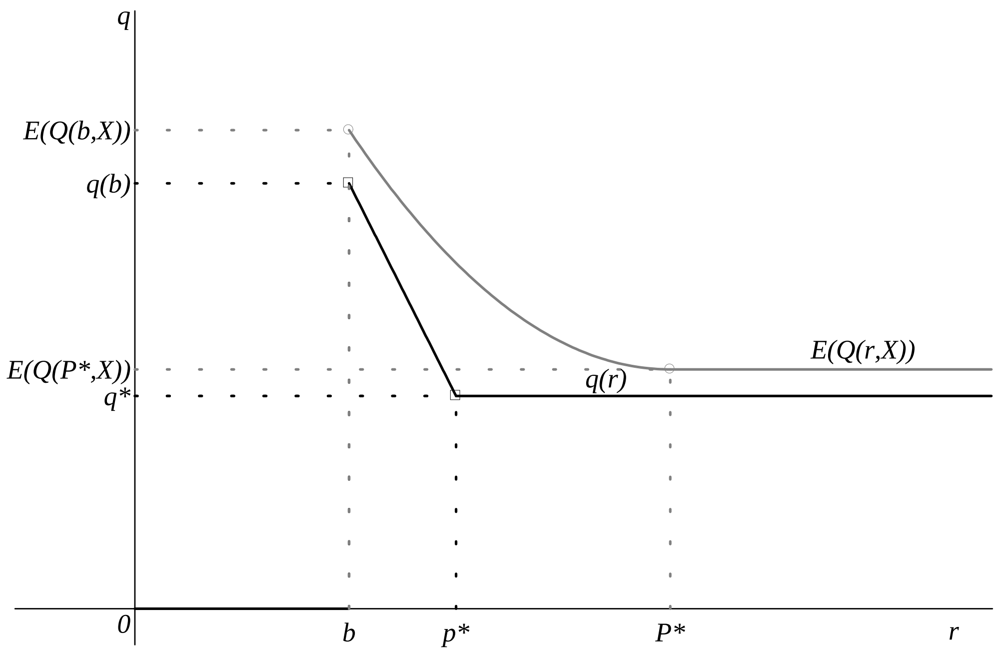


Figure 1.3. Output with Demand Certainty and Demand Uncertainty.

This analysis is useful when firms are not capacity constrained and produce instantly upon the realization of demand. Relevant examples are the Spanish or California electricity markets, in which (at least in recent times) firms have excess capacity and their bids are short lived (i.e., firms compete to serve the demand for short periods of time, e.g., an hourly or half hourly periods). Of course, price cap regulation has an impact on firms' capacity investments, which are long run decisions made prior the realization of demand. Thus, endogenizing firms' capacity investment decisions seems a natural next step to take.

In what follows we study the impact of price caps in a model in which the monopolist makes ex-ante capacity investment decisions and then, upon observing the realization of demand, decides how much to produce, and may withhold capacity if doing so is beneficial. In this setting the level of capacity is a long run decision, whereas the level of output is a short run decision. One may also interpret this setting as if the monopolist decides its output before demand is realized, but once demand is realized the monopolist decides how much to supply, and may supply less than its total output. Relevant examples include the electricity markets mentioned above, markets for agricultural products (in which producers may destroy part of their output if doing so is beneficial), sport events, hotel accommodation, etc.

Earle et al. (2007) and Grimm and Zoettl (2010) study an arguable less interesting model in which firms decide their output ex-ante and supply it inelastically whatever the realization of demand. Such a model may be of interest in, e.g., electricity markets in which firms bids are long lived (i.e., a firm must commit to supply their capacity during the entire day). Appendix C provides an analysis in our setting of the effect of price caps in this model of full capacity utilization. This analysis allows for a comparison of the effects of price caps with and without capacity withholding. We discuss this issue in the concluding session.

Also, both Earle et al. (2007) and Grimm and Zoettl (2010) claim to have results for the model of capacity investment and withholding we study in the present paper. However, it is unclear whether their reduced form analysis is correct in this dynamic setting. Moreover, some of their conclusions are incorrect. Specifically, we show in Appendix B that the proof of Theorem 6 in Earle et al. (2007) is incorrect. Also a mistake in the calculations of Grimm and Zoettl (2010) leads to wrong conclusions the effects of price caps – we comment on this issue below.

### 1.3. Capacity Precommitment and Withholding

Consider a monopolist facing an uncertain demand that must decide how much capacity to install before the demand is realized. We assume that the cost of installing a unit of capacity is a positive constant $c$. Once capacity is installed the good can be produced with constant returns to scale up to capacity. We assume without loss of generality that the production cost is zero. As in Section 1. 2, the market demand is $D(X, p) = \max\{X - p, 0\}$, where $X$ is a random variable. The monopolist decides its output upon observing the realization of the demand parameter $X$. In order to reduce notation, we assume that the support of $X$ is the interval $[0,1]$.[10] Also we denote by $f$ and $F$ the *p.d.f.* and *c.d.f.* of $X$, respectively, and assume that $E(X) > c$ in order to rule out the trivial cases in which the monopolist installs no capacity.

Assume that a regulatory agency imposes a price cap $r \in [0,1]$. Since the cost of capacity is sunk and the cost of production cost (up to capacity) is zero, then at the stage of output choice the monopolist maximizes revenue. If the monopolist had an unlimited capacity, then the equilibrium output is that calculated in Section 1. 2 for $b = 0$; i.e., for $x \in [0,1]$, $Q(r,x) = x - r \leq 1 - r$ if $r < x/2$, and $Q(r,x) = x/2 \leq 1/2$ if $r \geq x/2$. Hence levels of capacity $k > \max\{1-r, 1/2\}$ are suboptimal since the monopolist would always have idling capacity, and therefore may increase its profit by installing less capacity since $c > 0$. Thus, we restrict attention to price cap-capacity pairs $(r,k) \in [0,1]^2$ such that $k \leq \max\{1-r, 1/2\}$.

Figure 1.4 describes a partition of this set of price cap-capacity pairs into three regions, $A = \{(r,k) \in [0,1]^2 \mid r \leq k \leq 1-r\}$, $B = \{(r,k) \in [0,1]^2 \mid k < \min\{1-r, r\}\}$, and $C = \{(r,k) \in [0,1]^2 \mid 1-r \leq k \leq 1/2\}$. We calculate the equilibrium price $P(r,k,x)$ and output $Q(r,k,x)$ in these regions for each realization $x$ of the demand parameter $X$.

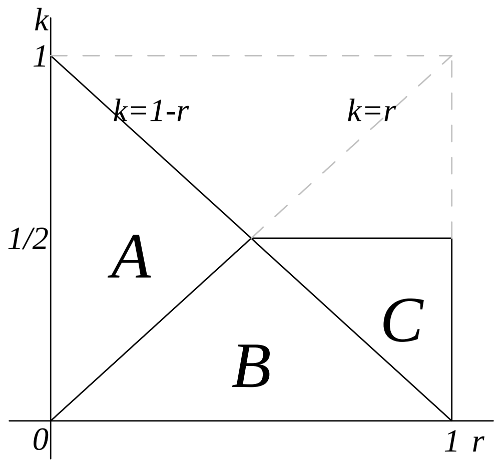


Figure 1.4. Relevant Price Cap-Capacity Pairs.

[10] This assumption facilitates the presentation and the interpretation of our results, but entails a small loss of generality because the cost of production given capacity and the lower bound of the support of $X$ coincide.

Table 1.1A describes the prices and output for $(r, k) \in A$.

| $X$ | $[0, 2r)$ | $[2r, r+k)$ | $[r+k, 1]$ |
|---|---|---|---|
| $P(r, k, x)$ | $x/2$ | $r$ | $r$ |
| $Q(r, k, x)$ | $x/2$ | $x - r$ | $k$ |

Table 1.1A: Equilibrium output and price for $(r, k) \in A$.

Figure 1.5 illustrates the results in Table 1. 1A. For low demand realizations $x < 2r$ marginal revenue remains positive for levels of output greater than the demand at the price cap, $q = x - r$; therefore neither the price cap nor the level of capacity are binding; hence the outcome is the unconstrained monopoly equilibrium, i.e., $q = p = x/2$. For intermediate demand realizations $x \in [2r, r+k)$ marginal revenue for levels of output greater than $q = x - r$ is negative, and therefore the price cap is binding; the monopolist serves the demand at the price cap, and withholds capacity. (Hence for low and intermediate demand realizations a marginal decrease of the price cap leads to an increase of output, much as in the models of Section 1. 2.) For high demand realizations $x \geq r + k$ marginal revenue equals the price cap up to the level of capacity, and hence the monopolist supplies its entire capacity, the price cap remains binding, and the demand is rationed. Note that for price cap-capacity pairs in this region the market price $P(r, k, x)$ is independent of the level of installed capacity $k$.

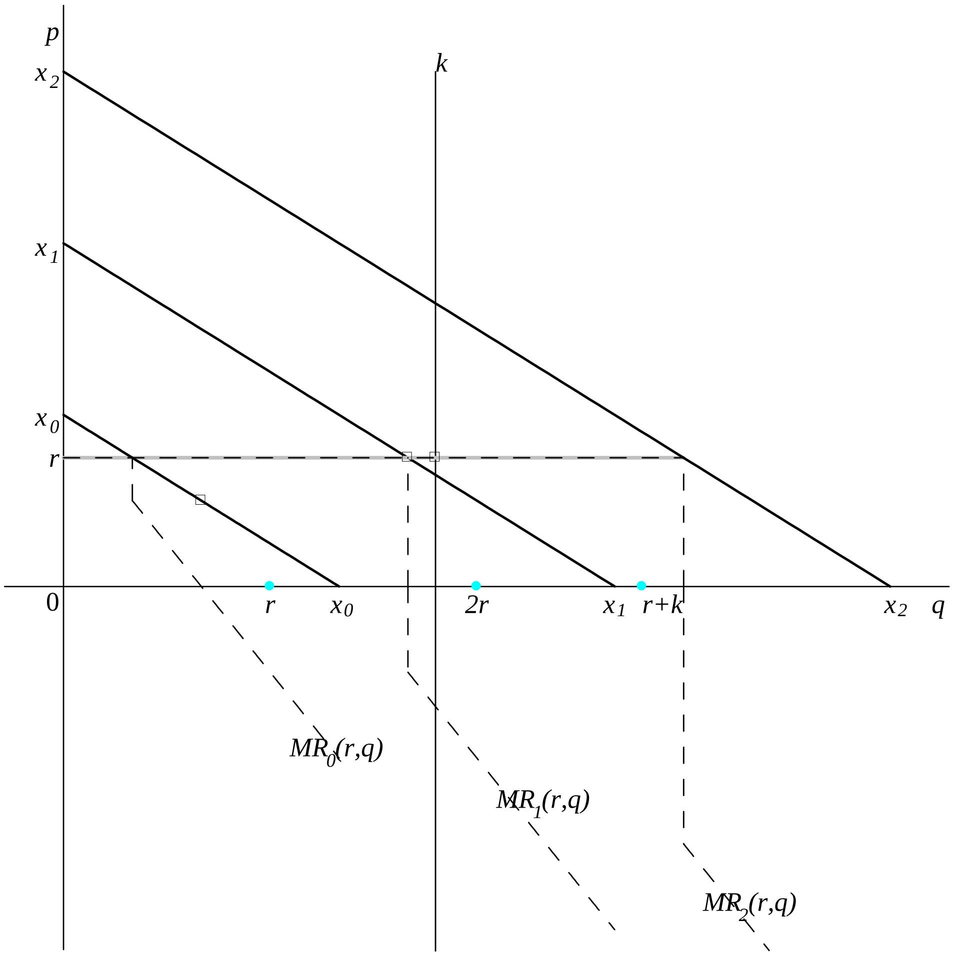


Figure 1.5. The Effect of a Price Cap when $(r, k) \in A$.

Table 1. 1B describes the prices and output for $(r, k) \in B$.

| $X$ | $[0, 2k)$ | $[2k, r + k)$ | $[r + k, 1]$ |
|---|---|---|---|
| $P(r, k, x)$ | $x/2$ | $x - k$ | $r$ |
| $Q(r, k, x)$ | $x/2$ | $k$ | $k$ |

Table 1.1B: Equilibrium output and price for $(r, k) \in B$.

Figure 1.6 illustrates the results in Table 1. 1B. For low demand realizations $x < 2k$ the expected marginal revenue is negative for output levels equal to $k$ and $x < r$, and therefore neither the price cap nor the level of capacity are binding; hence the outcome is the unconstrained monopoly equilibrium, i.e., $q = p = x/2$. For intermediate demand realizations $x \in [2k, r+k)$ marginal revenue is positive for output levels greater than $k$, and therefore the monopolist supplies its full capacity, i.e., $q = k$; the price cap is non-binding since $p = x - k < r + k - k = r$. (Thus, for these realizations changes in the price cap have affects neither the level of output nor the market price.) For high demand realizations $x > r + k$ the monopolist continues supplying its entire capacity, i.e., $q = k$, but the price cap becomes binding, i.e., $p = r$, and the demand is rationed, i.e., $x - p = x - r > q$. In this region the market price $P(r, k, x)$ depends on the level of capacity

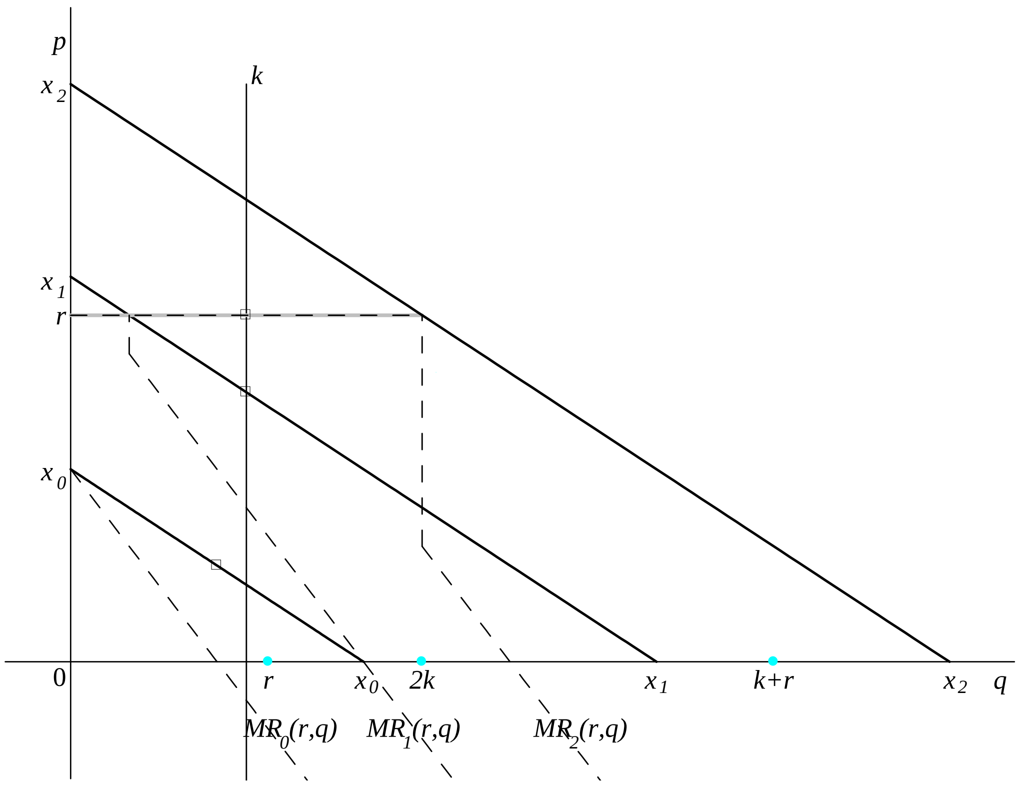


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Figure 1.6. The Effect of a Price Cap when $(r, k) \in B$.

Table 1.1C describes the prices and output for $(r,k) \in C$.

| $X$ | $[0, 2k)$ | $[2k, 1]$ |
|---|---|---|
| $P(r,k,x)$ | $x/2$ | $x-k$ |
| $Q(r,k,x)$ | $x/2$ | $k$ |

Table 1.1C: Equilibrium output and price for $(r,k) \in C$.

In region $C$, the price cap is never binding. The monopolist withholds capacity only for low demand realizations $x < 2k$, and supplies its entire capacity otherwise. Demand is never rationed. The market price $P(r,k,x)$ depends on the level of capacity.

Note an important feature of equilibrium that stands in contrast to the case where the monopolist is not capacity constrained: when both capacity and the price cap are binding, demand is rationed.

The monopolist's revenue is

$$R(r,k,x) = P(k,r,x)Q(r,k,x),$$

and its expected profit is

$$\bar{\Pi}(r,k) = E\left(R(r,k,X) - ck\right) = E\left(R(r,k,X)\right) - ck,$$

Clearly $\bar{\Pi}$ is continuous on $A \cup B \cup C$.

In equilibrium, the monopolist's capacity maximizes $\bar{\Pi}(r,\cdot)$. Thus, in an interior equilibrium the capacity $k^*$ is such that the monopolist's expected marginal revenue from installing an additional infinitesimal unit of capacity $\overline{MR}(r,k)$, where

$$\overline{MR}(r,k) := \frac{\partial E\left(R(r,k,X)\right)}{\partial k},$$

is equal to the marginal cost of capacity $c$; i.e., $k^*$ solves

$$\overline{MR}(r,k) = c. \tag{1}$$

In addition, the second order condition

$$\frac{\partial \overline{MR}(r,k)}{\partial k} < 0 \tag{2}$$

holds at $k^*$.

Using the results described in tables 1.1A, 1.1B and 1.1C we readily calculate the monopolist's expected revenue

$$E\left(R(r,k,X)\right) = \int_0^1 P(k,r,x)Q(r,k,x)f(x)dx$$

for $(r,k)$ in either $A$, $B$ or $C$. Differentiating this expression we obtain the expected marginal revenue, which is

$$\overline{MR}(r,k) = \int_{r+k}^1 rf(x)dx \tag{3}$$

for $(r,k) \in A$,

$$\overline{MR}(r,k) = \int_{2k}^{r+k} (x-2k)\, f(x)dx + \int_{r+k}^1 rf(x)dx \tag{4}$$

for $(r,k) \in B$, and

$$\overline{MR}(r,k) = \int_{2k}^1 (x-2k)f(x)dx \tag{5}$$

for $(r,k) \in C$. Since (3) and (4) coincide for $k = r$, and (4) and (5) coincide for $r > 1/2$ and $k = 1-r$, then $\overline{MR}$ in continuous on $A \cup B \cup C$.

In region $A$, increasing marginally capacity affects the revenue only for high demand realizations $x > r+k$ for which the monopolist supplies its entire capacity. For these demand realizations the price cap $r$ is binding. Thus, the expected revenue increases by $r$ times the probability that the additional marginal unit of capacity is supplied, i.e.,

$$\overline{MR}(r,k) = r[1 - F(r+k)],$$

which is a version of equation (3). In region $B$, a marginal increase of capacity increases revenue not only for demand realizations $x > r+k$, but also for intermediate demand realizations $2k < x < r+k$, in which the price cap is non-binding and the monopolists supplies its full capacity; therefore the marginal revenue is independent of the price cap. In region $C$, a marginal increase of capacity affects the revenue only when the demand at the price cap exceeds capacity, i.e., when $x > r+k$.

Differentiating $\overline{MR}$ we get

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -rf(r+k) < 0 \tag{6}$$

for $(r,k) \in A$,

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -kf\,(r+k) - 2\,[F(r+k) - F(2k)] < 0 \tag{7}$$

for $(r,k) \in B$, and

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -2\,[1 - F(2k)] < 0 \tag{8}$$

for $(r,k) \in C$. Hence the expected marginal revenue function $\overline{MR}$ is decreasing, and therefore the inequality (2) holds on $A \cup B \cup C$. Moreover, since (6) and (7) coincide for $k = r$, then $\overline{MR}$ is differentiable on $A \cup B \cup C$, except perhaps in the boundary of $B$ and $C$.

Thus, for all $r \in [0,1]$ the monopolist's equilibrium capacity $k^*(r)$ is the unique solution of the equation (1). Moreover, the Maximum Theorem implies that $k^*$ is a continuous function. We summarize these results in Proposition 1.2.

**Proposition 1.2**. *The monopoly equilibrium capacity $k^*$ is a well defined continuous function of the price cap all $r \in [0,1]$.*

Calculating the equilibrium capacity is somewhat involved. Obviously, the equilibrium capacity is zero for price caps below the unit cost of capacity $c$. Moreover, it is easy to see that the equilibrium capacity is also zero for price caps $r$ above but near the unit cost of capacity: because the probability of demand realizations $x < c$ is positive, for $r$ above but near $c$ the expected marginal revenue is below $c$ even for $k = 0$. Therefore installing capacity entails losses. Thus, the equilibrium capacity is zero unless the price cap is sufficiently high that expected marginal revenue for levels of capacity near zero is greater than $c$, i.e., $r \geq \underline{r}(c)$, where $\underline{r}$ is defined by the equation $\overline{MR}(r,0) = c$. Hence, unlike in the setting in which the monopolist makes output decisions ex-post, price caps near the unit cost of capacity are suboptimal.[11]

[11] If the lower bound of the support of $X$ is $\alpha > c$ (instead of zero as we have assumed), then for $r = c$ the expected marginal revenue is $c$ and profits are zero for $k \in [0, \alpha - c]$, whereas profits are negative for

As in the setting in which the monopolist makes output decisions ex-post, sufficiently large price caps are non-binding. The upper bound on the interval of binding price caps is determined by the distribution of the demand parameter $X$; specifically this bound $\bar{r}(c)$ is defined by the equation $c = \overline{MR}(r, 1-r)$.

Intermediate price caps $r \in [\underline{r}(c), \bar{r}(c))$ affect the equilibrium capacity in more complex ways. We are able to identify the level of capacity assuming that the hazard rate of $X$ is increasing. In particular, as we shall see in the next section, unlike in the setting in which the monopolist makes output decisions ex-post, the equilibrium capacity is not monotonically decreasing with the price cap in this interval.

Proposition 1.3 makes these results precise. Write $M^*$ for the maximum value of $M(r) := \overline{MR}(r,r)$ on $(0,1/2)$. If $c < M^*$, then the equation $M(r) = c$ has two solutions $r_-(c), r_+(c)$, which satisfy $\underline{r}(c) < r_-(c) < r_+(c) < \bar{r}(c) <1$. If $c \geq M^*$, then $c \geq \overline{MR}(r,r)$ for all $r \in [0,1/2]$. The proof of Proposition 1.3, which is given in Appendix A, establishes these properties.

**Proposition 1.3**. *(1.3.1) The equilibrium capacity is $k^*(r) = 0$ whenever $r \in [0, \underline{r}(c))$, and is $k^*(r) = k_C$, where $k_C$ solves the equation*

$$\int_{2k}^{1} (x - 2k) f(x) dx = c,$$

*whenever $r \in [\bar{r}(c), 1]$.*

*(1.3.2) Assume that the hazard rate of $X$ is increasing. If $c \in (0, M^*)$, then the equilibrium capacity is $k^*(r) = k_A(r)$, where*

$$k_A(r) = F^{-1}(1 - \frac{c}{r}) - r$$

*whenever $r \in [r_-(c), r_+(c)]$, and is $k^*(r) = k_B(r)$, where $k_B$ solves the equation*

$$\int_{2k}^{r+k} (x - 2k)\, f(x) dx + \int_{r+k}^{1} r f(x) dx = c,$$

$k > \alpha - c$. Hence the equilibrium capacity may be positive, and may increase or decrease with $r$ near the unit cost of capacity depending of the distribution of demand. See Grimm and Zoettl (2010)'s Section 4 for a discussion of this issue.

*whenever* $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. *If* $c \in (M^*, E(X))$, *then the equilibrium capacity is* $k^*(r) = k_B(r)$ *for all* $r \in [\underline{r}(c), \bar{r}(c))$.

Using the results in tables 1.1A, 1.1B and 1.1C, and the description on the equilibrium capacity given in Proposition 1.3, one can calculate the expected output and market price as well as the expected (consumer and total) surplus, thus providing a complete description of the monopoly equilibrium. We study in the next section the effect of changes in the price cap on these values.

### 1.4. Comparative Statics

In this section we study the comparative static properties of price caps. We show that if the hazard rate of $X$ is increasing and its p.d.f. $f$ is continuously differentiable, then there is a price cap that maximizes the equilibrium capacity $r^*(c) \in (\underline{r}(c), \bar{r}(c))$. Moreover, we show that the equilibrium capacity increases with the price cap on the interval $(\underline{r}(c), r^*(c))$, and decreases with the price cap on the interval $(r^*(c), \bar{r}(c))$. Thus, relative to the capacity maximizing price cap $r^*(c)$ the effects of price caps on capacity (and welfare, as we shall see in the next section) are analogous, although more subtle, than when the monopolist is not capacity constrained. Two important differences are worth noticing: For low price caps, i.e., price caps above but near $\underline{r}(c) > c$, capacity increases with the price cap, and therefore the price cap that maximizes capacity is above the marginal cost. Moreover, when the unit cost of capacity is small, the price cap that maximizes expected output is below $r^*(c)$, but the above the unit cost of capacity. (Recall that for a capacity unconstrained monopoly setting a price cap equal to marginal cost maximizes output – see Proposition 1.1.)

Let $r \in (\underline{r}(c), \bar{r}(c))$. Since the expected marginal revenue $\overline{MR}(r, k)$ is differentiable in regions $A \cup B$, we can differentiate equation (1) to get

$$\frac{\partial \overline{MR}(r, k)}{\partial k} dk + \frac{\partial \overline{MR}(r, k)}{\partial r} dr = 0.$$

And since $\overline{MR}$ is decreasing, i.e.,

$$\frac{\partial \overline{MR}(r, k)}{\partial k} < 0,$$

then

$$\frac{dk^*}{dr} = -\frac{\partial \overline{MR}(r,k)}{\partial r}\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1},$$

and

$$\frac{dk^*}{dr} \gtreqqless 0 \Leftrightarrow \frac{\partial \overline{MR}(r,k)}{\partial r} \gtreqqless 0.$$

Assume that $f$ (the *c.d.f.* of $X$) is continuously differentiable. Then $\overline{MR}$ is twice continuously differentiable, and

$$\begin{aligned}
\frac{d^2k^*}{dr^2} &= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1}\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) \\
&\quad +\frac{\partial \overline{MR}(r,k)}{\partial r}\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-2}\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k}\right) \\
&= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1}\left(\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right)+\frac{dk^*}{dr}\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k}\right)\right).
\end{aligned}$$

Hence, for $r$ such that $dk^*/dr = 0$, we have

$$\frac{d^2k^*}{dr^2} \gtreqqless 0 \Leftrightarrow \frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) \gtreqqless 0.$$

Denote by $h$ the hazard rate of $X$, i.e., $h(x) = f(x)/[1-F(x)]$ for all $x \in (0,1)$. If $(r,k^*(r)) \in A$, then differentiating $\overline{MR}$ given in (3) yields

$$\frac{\partial \overline{MR}(r,k)}{\partial r} = 1 - F(r+k) - rf(r+k) = (1-F(r+k))(1-rh(r+k)),$$

and

$$\begin{aligned}
\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k)\left(1+\frac{dk_A}{dr}\right)(1-rh(r+k)) \\
&\quad -(1-F(r+k))\big(h(r+k)+rh'(r+k)\big)\left(1+\frac{dk_A}{dr}\right).
\end{aligned}$$

Assume that $dk_A/dr = 0$. Then $1 - rh(r+k^*(r)) = 0$, and

$$\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) = -(1-F(r+k^*(r)))\big(h(r+k^*(r))+rh'(r+k^*(r))\big).$$

If the hazard rate is increasing (i.e., $h' > 0$), then we have

$$\frac{d^2k_A}{dr^2} < 0,$$

and therefore every critical point of $k_A$ is a local maximum.

If $(r, k_B(r)) \in B$, then differentiating $\overline{MR}$ given in (4) yields

$$\frac{\partial \overline{MR}(r,k)}{\partial r} = 1 - F(r+k) - kf(r+k) = (1 - F(r+k))\,(1 - kh(r+k))\,,$$

and

$$\begin{aligned}\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k^*(r))\,(1-k^*(r)h(r+k^*(r)))\left(1+\frac{dk_B}{dr}\right)\\ &\quad -(1-F(r+k^*(r)))\,k^*(r)h'(r+k^*(r))\left(1+\frac{dk_B}{dr}\right)\\ &\quad -(1-F(r+k^*(r)))\,h(r+k^*(r))\frac{dk_B}{dr}.\end{aligned}$$

Assume that $dk_B/dr = 0$. Then $1 - k^*(r)h\,(r + k^*(r)) = 0$, and

$$\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) = -\,(1-F(r+k^*(r)))\,k^*(r)h'(r+k^*(r)).$$

If the hazard rate is increasing (i.e., $h' > 0$) we have

$$\frac{d^2k_B}{dr^2} < 0,$$

and therefore every critical point of $k_B$ is a local maximum.

Thus, for $r \in (\underline{r}(c), \bar{r}(c))$, $d^2k^*(r)/dr^2 < 0$ whenever $dk^*(r)/dr = 0$. Moreover, since $k_B(\bar{r}(c)) = 1 - \bar{r}(c)$, and

$$\begin{aligned}\left.\frac{\partial \overline{MR}(r,1-r)}{\partial r}\right|_{r=\bar{r}(c)} &= 1-F\left(\bar{r}(c)+(1-\bar{r}(c))\right)-(1-\bar{r}(c))\,f\left(\bar{r}(c)+(1-\bar{r}(c))\right)\\ &= -\,(1-\bar{r}(c))\,f(1)\\ &< 0,\end{aligned}$$

then $dk_B(\bar{r}(c))/dr < 0$. And since $k_B(\underline{r}(c)) = 0$, and

$$\left.\frac{\partial \overline{MR}(r,0)}{\partial r}\right|_{r=\underline{r}(c)} = 1 - F(\underline{r}(c)) > 0,$$

then $dk_B(\underline{r}(c))/dr > 0$. Hence $k^*$ has a global maximum at some $r^*(c) \in (\underline{r}(c), \bar{r}(c))$, and satisfies $dk^*/dr > 0$ on $(\underline{r}(c), r^*(c))$ and $dk^*/dr < 0$ on $(r^*(c), \bar{r}(c))$ – see Lemma 1 in

Appendix A. Since $k^*$ is continuous on $[0,1]$, is equal to zero on $[0, \underline{r}(c)))$ and is equal to $k_C$ on $[\bar{r}(c), 1)$, this implies that $k^*$ is quasi-concave, i.e., single peak, on $[0,1]$.

We state these results in Proposition 1.4.

**Proposition 1.4**. *Assume that the hazard rate of $X$ is increasing and its p.d.f. $f$ is continuously differentiable. Then $k^*$ is quasi-concave and has a global maximum at some $r^*(c) \in (\underline{r}(c), \bar{r}(c))$. Moreover, $dk^*(r)/dr$ is positive on $(\underline{r}(c), r^*(c))$, and is negative on $(r^*(c), \bar{r}(c))$.*

It is also useful to calculate the expected output and the expected price using the results described in tables 1.1A, 1.1B and 1.1C, and to examine how they are affected by changes of the price cap. The expected output is

$$E(Q(r, k^*(r), X) = \int_0^{2r} \frac{x}{2} f(x)dx + \int_{2r}^{r+k^*(r)} (x-r)f(x)dx + \int_{r+k^*(r)}^1 k^*(r)f(x)dx,$$

for $r \in [r_-(c), r_+(c)]$, and

$$E(Q(r, k^*(r), X) = \int_0^{2k^*(r)} \frac{x}{2} f(x)dx + \int_{2k^*(r)}^1 k^*(r)f(x)dx$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Thus,

$$\frac{dE(Q(r, k^*(r), X)}{dr} = -[F(r+k^*(r)) - F(2r)] + \frac{dk^*}{dr}(1 - F(r+k^*(r)))$$

for $r \in [r_-(c), r_+(c)]$, and

$$\frac{dE(Q(r, k^*(r), X)}{dr} = \frac{dk^*}{dr}(1 - F(2k^*(r)))$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$.

Hence

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(Q(r, k^*(r), X)}{dr} < 0$$

for $r \in [r_-(c), r_+(c)]$, that is, the expected output decreases with the price cap beyond the price cap that maximizes capacity, and therefore the price cap that maximizes output is below $r^*(c)$. Moreover,

$$\frac{dE(Q(r, k^*(r), X)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk^*}{dr} \gtreqless 0.$$

for $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, that is, the expected output increases with the price cap for $r \in (\underline{r}(c), r^*(c))$, and decreases for $r \in (r^*(c), \bar{r}(c))$.

Likewise for $r \in [r_-(c), r_+(c)]$ the expected price is

$$E(P(r, k^*(r), X) = \int_0^{2r} \frac{x}{2} f(x)dx + \int_{2r}^1 rf(x)dx,$$

and for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$ it is

$$E(P(r, k^*(r), X) = \int_0^{2k^*(r)} \frac{x}{2} f(x)dx + \int_{2k^*(r)}^{r+k^*(r)} (x - k^*(r))f(x)dx + \int_{r+k^*(r)}^1 rf(x)dx.$$

Hence, for $r \in [r_-(c), r_+(c)]$

$$\frac{dE(P(r, k^*(r), X)}{dr} = 1 - F(2r) > 0,$$

that is, the expected price unambiguously increases with the price cap on $[r_-(c), r_+(c)]$. This result is easy to understand: for $(r, k) \in A$ the market price $P(r, k, X)$ is independent of $k$, and therefore a change in the price cap only has a direct (positive) effect on $P$. Hence the expected market price increases with the price cap regardless of its impact on capacity.

For $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$,

$$\frac{dE(P(r, k^*(r), X)}{dr} = -\frac{dk^*}{dr}[F(r + k^*(r)) - F(2k^*(r))] + [1 - F(r + k^*(r))],$$

and therefore

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(P(r, k^*(r), X)}{dr} > 0.$$

These results are also clear: for $(r, k) \in B$ the market price $P(r, k, X)$ depends on $k$, and therefore a change in the price cap has a direct (positive) effect on $P$, but also has an indirect effect on $P$ via its impact on the level of capacity. When this indirect effect is also positive, i.e., when $dk^*/dr < 0$, then the total effect is positive, but when the indirect effect is negative, the sign of the total effect is ambiguous.

Under the assumptions of Proposition 1.4, $dk^*/dr < 0$ on $(r^*(c), \bar{r}(c))$. Hence $dE(P(r, k^*(r), X)/dr > 0$ on $([r_-(c), r_+(c)] \cap [0, 1]) \cup [r^*(c), \bar{r}(c))$. Obviously, changes in the price cap have no effect on the expected price for $r \in [0, \underline{r}(c)] \cup [\bar{r}(c), 1]$. Otherwise the sign of $dE(P(r, k^*(r), X)/dr$ is ambiguous.

We summarize these results in Proposition 1.5.

**Proposition 1.5**. *Assume that the hazard rate of $X$ is increasing and its p.d.f. $f$ is continuously differentiable.*

*(1.5.1) If $r^*(c) \in (r_-(c), r_+(c))$, then the expected output decreases with the price cap above and around $r^*(c)$, and the expected price increases with the price cap on $[r_-(c), \bar{r}(c))$.*

*(1.5.2) If $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then the expected output increases with the price cap on $(\underline{r}(c), r^*(c))$ and decreases on $(r^*(c), \bar{r}(c))$, and the expected price increases with the price cap on $[r^*(c), \bar{r}(c))$.*

Thus, with capacity precommitment the comparative static properties of price caps are more subtle than in the absence of capacity precommitment: when $c$ is sufficiently small, the capacity maximizing price cap $r^*(c) \in (r_-(c), r_+(c))$ does not warrant maximizing the expected output: decreasing the price cap below $r^*(c)$ leads to an increase of the expected output even though installed capacity decreases. Of course, this fact has direct implications on the price cap that maximizes the expected surplus, as we shall see in the next section.

### 1.5. Optimal Price Caps

A regulator who wants to maximize the expected surplus using a price cap as its single instrument, and cannot force the monopolist to serve its full capacity, must trade off the incentives for capacity investment and capacity withholding, and must account for the cost of installing capacity (some of which may be seldom utilized). Thus, the optimal price cap may differ from the price cap that maximizes capacity investment $r^*(c)$. (In contrast, in the model of full capacity utilization studied by Earle et al. (2007) and Grimm and Zoettl (2010), maximizing the expected surplus simply amounts to maximizing capacity – see Appendix C.) Indeed, we show that when the unit cost of capacity is small this is the case: the optimal price cap is below $r^*(c)$. When the unit cost of capacity is high, however, providing appropriate incentives for capacity investment becomes the dominant objective, and thus the optimal price cap is $r^*(c)$.

Following the literature, we simplify somewhat the problem by assuming efficient rationing, i.e., when the price cap is binding the consumers with the largest willingness to pay receive priority to buy the good. Table 1.s 2A describes the surplus $S(r, k, X)$ for each realization of the demand parameter when $(r, k) \in A$.

| $X$ | $[0, 2r)$ | $[2r, r+k)$ | $[r+k, 1]$ |
|---|---|---|---|
| $S(r, k, x)$ | $\frac{3}{8}x^2$ | $\frac{1}{2}(x^2 - r^2)$ | $\frac{1}{2}(2x - k)\,k$ |

Table 1.2A: Social Surplus in Region $A$.

Recall that the monopolist withholds capacity for demand realizations $x \in [0, r+k)$. Hence the expected surplus depends directly on the price cap, as well as indirectly through its effect on the monopolist capacity decision. The expected surplus for $(r, k) \in A$ is

$$\begin{aligned} E(S(r,k,X)) &= \frac{3}{8}\int_0^{2r} x^2 f(x)dx + \frac{1}{2}\int_{2r}^{r+k}(x^2 - r^2)f(x)dx \\ &\quad + \frac{1}{2}\int_{r+k}^{1}(2x-k)kf(x)dx - ck. \end{aligned} \tag{9}$$

Table 1.2BC below describes the surplus $S(r, k, X)$ for each demand realization when $(r, k) \in B \cup C$.

| $X$ | $[0, 2k)$ | $[2k, 1]$ |
|---|---|---|
| $S(r, k, X)$ | $\frac{3}{8}x^2$ | $\frac{1}{2}(2x - k)\,k$ |

Table 1.2BC: Social Surplus in Regions $B$ and $C$.

In $B \cup C$ a price cap has no direct effect on the expected surplus, but only has an indirect effect via its influence on the monopolist capacity choice. (Of course, the price cap also determines the distribution of surplus.) The expected surplus for $(r, k) \in B \cup C$ is

$$E(S(r,k,X)) = \frac{3}{8}\int_0^{2k} x^2 f(x)dx + \frac{1}{2}\int_{2k}^{1}(2x-k)\,kf(x)dx - ck. \tag{10}$$

The optimal price cap maximizes the surplus $\bar{S}(r) := E(S(r, k^*(r), X)$.

For price caps $r \in [r_-(c), r_+(c)]$ the price cap-equilibrium capacity pair $(r, k^*(r))$ is in region $A$. Differentiating $\bar{S}$ given in (9) yields

$$\frac{d\bar{S}(r)}{dr} = -r[F(r + k^*(r)) - F(2r)] + \frac{dk^*(r)}{dr}\left(\int_{r+k^*(r)}^{1}(x - k^*(r))f(x)dx - c\right),$$

Recall that $r^*(c)$ is the capacity maximizing price cap identified in Proposition 1.4. If $r^*(c) \in [r_-(c), r_+(c)]$, then $dk^*(r^*(c))/dr = 0$ and $k^*(r^*(c)) = k_A(r^*(c)) > r^*(c)$ imply

$$\frac{d\bar{S}(r^*(c))}{dr} = -r^*(c)[F(r^*(c) + k^*(r^*(c))) - F(2r^*(c))] < 0. \tag{11}$$

Hence the expected surplus decreases with the price cap at $r^*(c)$. Even though decreasing the price cap below $r^*(c)$ decreases capacity, it discourages capacity withholding and increases surplus. Hence the optimal price cap is below $r^*(c)$.

For price caps $r \in [0,1]\backslash[r_-(c), r_+(c)]$ we have $(r, k^*(r)) \in B \cup C$. Differentiating $\bar{S}$ given in (10) yields

$$\frac{d\bar{S}(r)}{dr} = \frac{dk^*(r)}{dr}\left(\int_{2k^*(r)}^{1}(x - k^*(r))f(x)dx - c\right). \tag{12}$$

For $r \in (\underline{r}(c), \bar{r}(c))\backslash[r_-(c), r_+(c)]$, we have $(r, k^*(r)) \in B$, $k^*(r) < r$, and

$$\overline{MR}(r, k^*(r)) = \int_{2k^*(r)}^{r+k^*(r)}(x - 2k^*(r))\,f(x)dx + \int_{r+k^*(r)}^{1} rf(x)dx = c.$$

Hence

$$\int_{2k^*(r)}^{1}(x - k^*(r))f(x)dx - c = \int_{2k^*(r)}^{r+k^*(r)} k^*(r)f(x)dx + \int_{r+k^*(r)}^{1}(x - k^*(r) - r)f(x)dx > 0,$$

and therefore

$$\frac{d\bar{S}(r)}{dr} = 0 \Leftrightarrow \frac{dk^*(r)}{dr} = 0.$$

Differentiating $d\bar{S}(r)/dr$ we get

$$\begin{aligned}\frac{d^2\bar{S}(r)}{dr^2} \quad &= \quad \frac{d^2k^*(r)}{dr^2}\left(\int_{2k^*(r)}^{1}(x - k^*(r))f(x)dx - c\right) \\ &\quad - \left(\frac{dk^*(r)}{dr}\right)^2[1 - F(2k^*(r))] - 2k^*(r)f(2k^*(r)).\end{aligned}$$

If $d\bar{S}(r)/dr = 0$, then $dk^*(r)/dr = 0$, which as shown above implies $d^2k^*(r)/dr^2 < 0$. Hence $d^2\bar{S}(r)/dr^2 < 0$. Thus, by Lemma 1.1 if $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $r^*(c)$ is the unique global maximizer of $\bar{S}$ on $(\underline{r}(c), \bar{r}(c))$.

Note that since in the boundary of regions $A$ and $B \cup C$ the equilibrium capacity is $k^*(r) = r$, then the expression for $d\bar{S}(r)/dr$ in equations (11) and (12) coincide, and therefore $\bar{S}$ is differentiable on $[0, 1]$. Proposition 1.6 summarizes these results.

**Proposition 1.6**. *Assume that hazard rate of $X$ is increasing and its p.d.f. $f$ is continuously differentiable, and let $r^*(c)$ be the capacity maximizing price cap identified in Proposition 1.4. If $r^*(c) \in [r_-(c), r_+(c)]$ then the expected surplus decreases with the price cap above and around $r^*(c)$, whereas if $r^*(c) \in [0, 1] \backslash (r_-(c), r_+(c))$, then $r^*(c)$ maximizes the expected surplus.*

In the absence of capacity precommitment an optimal price cap $r^*(c) = c$ eliminates all inefficiencies. With capacity precommitment, however, an optimal price cap has to trade off the incentives for capacity investment and capacity withholding. When the unit cost of capacity is sufficiently small that $r^*(c) \in [r_-(c), r_+(c)]$, it is socially optimal to set up a low price cap $r < r^*(c)$, even at the cost of reducing capacity. Moreover, a price cap alone is unable to eliminate inefficiencies, i.e., to provide the appropriate incentives to install the optimal level of capacity and discourage capacity withholding.

We show that whether the optimal price cap is $r^*(c)$ or it is below, the level of capacity installed by the monopolist, $k^*(r^*(c))$, is below the level that will be socially optimal if the entire capacity was served for each demand realization. Let us consider the artificial scenario in which a regulator chooses the level of capacity, and controls its use, in order to maximize surplus. In this scenario the surplus is realized when the level of capacity is $k \in [0, 1]$ is

$$S^*(k) = \frac{1}{2}\int_0^k x^2 f(x)dx + \frac{1}{2}\int_k^1 (2x - k)kf(x)dx - ck.$$

The socially optimal level of capacity $k^W$ maximizes $S^*(k)$. Differentiating $S^*$ yields

$$\frac{dS^*(k)}{dk} = \int_k^1 (x - k)\, f(x)dx - c,$$

and

$$\frac{d^2S^*(k)}{dk^2} = -[1 - F(k)] < 0.$$

Thus, $k^W$ solves the equation $dS^*(k)/dk = 0$.

It is easy to show that $k^W > k^*(r^*(c)) \geq k^*(r)$ for all $r \in [0,1]$. Let us fix $c$ and reduce notation by writing $k^*$ and $r^*$ for $k^*(r^*(c))$ and $r^*(c)$, respectively. If $r^* \in [r_-(c), r_+(c)]$, then $k^* \geq r^*$ and

$$\overline{MR}(r^*, k^*) = \int_{r^*+k^*}^{1} r^* f(x)dx = c$$

imply

$$\begin{aligned}
\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1} (x - k^*)\, f(x)dx - \int_{r^*+k^*}^{1} r^* f(x)dx \\
&= \int_{k^*}^{r^*+k^*} (x - k^*) f(x)dx + \int_{r^*+k^*}^{1} (x - r^* - k^*) f(x)dx \\
&> 0.
\end{aligned}$$

Hence $k^W > k^*$. If $r^* \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $k^* \leq r^*$ and

$$\overline{MR}(r^*, k^*) = \int_{2k^*}^{r^*+k^*} (x - 2k^*) f(x)dx + \int_{r^*+k^*}^{1} r^* f(x)dx = c$$

imply

$$\begin{aligned}
\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1} (x - k^*) f(x)dx - \left(\int_{2k^*}^{r^*+k^*} (x - 2k^*) f(x)dx + \int_{r^*+k^*}^{1} r f(x)dx\right) \\
&= \int_{k^*}^{2k^*} (x - k^*)\, f(x)dx \\
&\quad + \int_{2k^*}^{r^*+k^*} k^* f(x)dx + \int_{r^*+k^*}^{1} (x - r^* - k^*) f(x)dx \\
&> 0.
\end{aligned}$$

Hence $k^W > k^*$ as well.

Thus, a price cap alone cannot provide appropriate incentives to install the optimal level of capacity and simultaneously eliminate the inefficiencies arising from capacity withholding. (It is worth noticing that when the monopolist cannot withhold capacity a price cap is not able to induce the monopolist to install the optimal level of capacity either. Moreover, when the monopolist cannot withhold capacity, then both the surplus and the level of capacity installed with the optimal price cap are below $\bar{S}(r^*)$ and $k^*(r^*)$, respectively. See Figure 1.12 in Appendix C. Thus, if the only regulatory instrument available, in addition to imposing a price cap, is whether or not capacity withholding is permissible, then allowing capacity withholding is the best choice.)

### 1.6. An Example

Assume that $X$ is uniformly distributed on $[0,1]$, i.e., $f(x)=1$. Thus, $X$ has an increasing hazard rate $h(x)=(1-x)^{-1}$, and its *p.d.f.* $f$ is continuously differentiable. Since $E(X) = 1/2$, we consider values of the unit costs of capacity $c \in (0,1/2)$.

Let us calculate the equilibrium capacity in this setting. The function $k_A$ is given by

$$k_A(r) = F^{-1}(1-\frac{c}{r}) - r = 1 - \frac{c}{r} - r.$$

The marginal revenue given in (4) is

$$\overline{MR}(r,k) = \frac{k^2}{2} + \frac{r}{2}[2\,(1-2k) - r].$$

Solving equation (1) yields

$$k_B(r) = 2r - \sqrt{2c - r\,(2-5r)}.$$

The marginal revenue given in (5) is

$$\overline{MR}(r,k) = \frac{1}{2}\,(1-2k)^2\,.$$

Solving equation (1) yields

$$k_C = \frac{1-\sqrt{2c}}{2}.$$

Let us calculate the functions $\underline{r}$, $r_-$, $r_+$ and $\bar{r}$. The function $\underline{r}$ is the solution to the equation

$$c = \overline{MR}(r,0) = \int_0^r xf(x)dx + r\,(1-F(r)) = \frac{r\,(2-r)}{2},$$

i.e.,

$$\underline{r}(c) = 1 - \sqrt{1-2c}.$$

The function $M$ is given by

$$M(r) = \overline{MR}(r,r) = r\,(1 - F(2r)) = r(1-2r).$$

The functions $r_-$ and $r_+$ are the smaller and larger solutions to the equation

$$c = M(r),$$

are readily calculated as

$$r_-(c) = \frac{1}{4}\left(1 - \sqrt{1-8c}\right),\ r_+(c) = \frac{1}{4}\left(1 + \sqrt{1-8c}\right).$$

These functions are well defined for $c \in (0, 1/8)$, where $M^* = 1/8$ is the maximum value the $M$. For $c > 1/8$ the above equation has no solution on $[0,1]$, i.e., the interval $[r_-(c), r_+(c)]$ is empty. The function $\bar{r}$ solves the equation

$$c = \overline{MR}(r, 1-r) = \int_{2(1-r)}^{1} xf(x)dx - 2\,(1-r)\,[1 - F(2\,(1-r))] = \frac{(1-2r)^2}{2},$$

i.e.,

$$\bar{r}(c) = \frac{1+\sqrt{2c}}{2}.$$

It is easy to check that for $c \in (0, 1/2)$ we have

$$c < \underline{r}(c) < \frac{1}{2} < \bar{r}(c) < 1.$$

Further, for $c \in (0, 1/8)$ we have

$$\underline{r}(c) < r_-(c) < r_+(c) < \frac{1}{2} < \bar{r}(c).$$

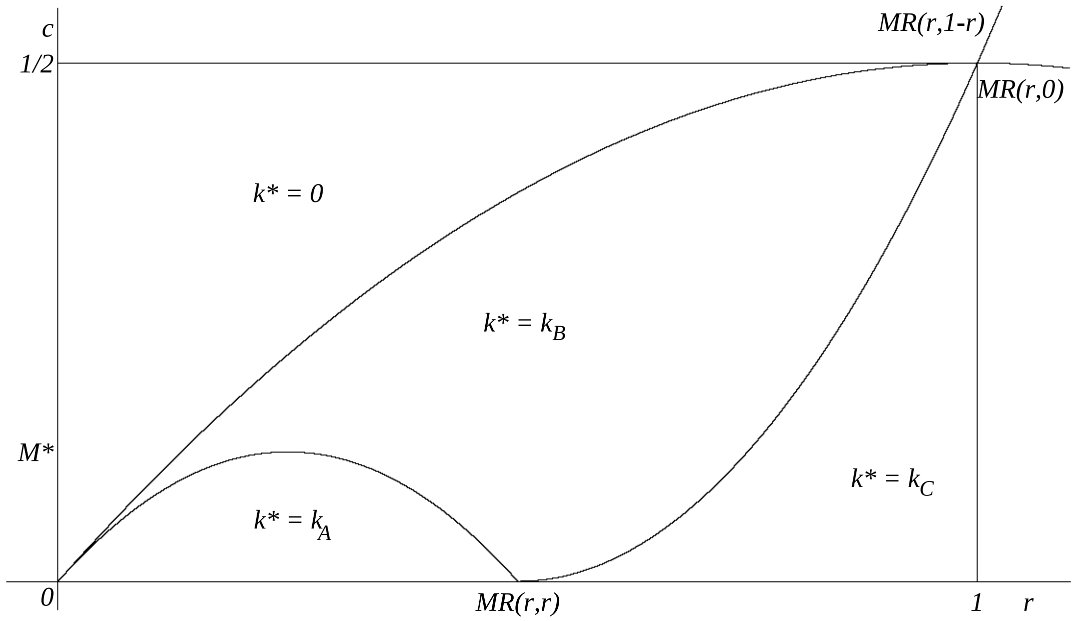

Figure 1.7. Equilibrium Capacity.

Figure 1.7 provides a description of the function $k^*$ for value of $c \in (0, 1/2)$. For $c \leq 1/9$ the equilibrium capacity $k^*(r)$ reaches its maximum at the price cap $r_A^* = \sqrt{c} \in [r_-(c), r_+(c)]$. For $c > 1/9$, the equilibrium capacity $k^*(r)$ reaches its maximum at $r_B^* = \left(1 + 2\sqrt{10c - 1}\right)/5 \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Interestingly, for $c \in (1/9, 1/8)$ the equilibrium capacity $k^*(r)$ is increasing in the interval $(r_-(c), r_+(c))$, and reaches its maximum at $r^*(c) \in (r_+(c), \bar{r}(c))$.

We calculate the expected surplus. If $r < \underline{r}(c)$, then the expected surplus is $\bar{S}(r) = 0$. If $r \in [r_-(c), r_+(c)]$, which requires $c < 1/8$, then the expected surplus is

$$\bar{S}(r) = \frac{r^3\left(1 + 4r^3\right) + 3r^2\left(c\left(c - 2r\left(1 - r\right)\right) - r^3\right) - c^3}{6r^3}.$$

If $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then the expected surplus is

$$\bar{S}(r) = \frac{r}{2}\left(4 - 9r\right) - c(1 + 2r) + \left(c + 2r - \frac{1}{2}\right)\sqrt{2c - r\left(2 - 5r\right)}.$$

And if $r \in [\bar{r}(c), 1]$ then

$$\bar{S}(r) = \frac{1 - 6c}{8} + \frac{\sqrt{2c^3}}{2}.$$

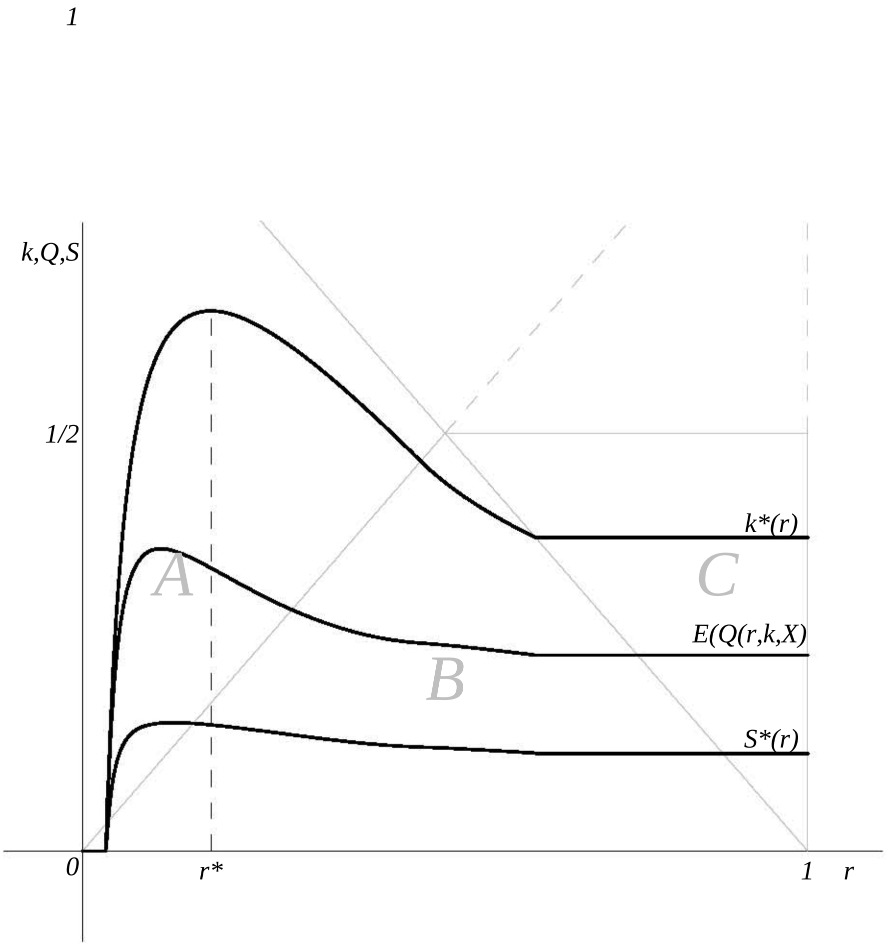


Figure 1.8. Capacity, Expected Output and Surplus for $c = 1/32$.

Figure 1.8 displays the equilibrium capacity and surplus as functions of the price cap when the unit cost of capacity is $c = 1/32$. The price cap that maximizes capacity is $r_A^* = \sqrt{2}/8$ whereas, consistently with Proposition 1.6, the expected surplus is maximized at $r = 1/8 < r_A^*$.

Figure 1.9 shows the graphs of the capacity, the expected output and the expected surplus for $c = 3/25$. For this unit cost of capacity we have $[r_-(c), r_+(c)] = [2/10, 3/10]$. (Note that $c = 3/25 < 1/8$.) The price cap that maximizes capacity, expected output and expected surplus is $r_B^* = (2\sqrt{5}+5)/25 \in (\underline{r}(c), \bar{r}(c))$, i.e., the maximum capacity is reached at a price cap-capacity pair in region $B$, and consistently with Proposition 1.6, the expected surplus is maximal at this price cap.

Suppose that a regulator chooses the level of capacity, assuming that for each demand realization the entire capacity is served to the consumers that value the good the most, in order to maximize surplus. Using the results obtained in Section 1.5 we calculate the expected surplus as a function of the capacity as

$$S^*(k) = \frac{k^2(k-3)}{6} + \frac{k(1-2c)}{2},$$

which is maximized at $k^W = 1 - \sqrt{2c}$.

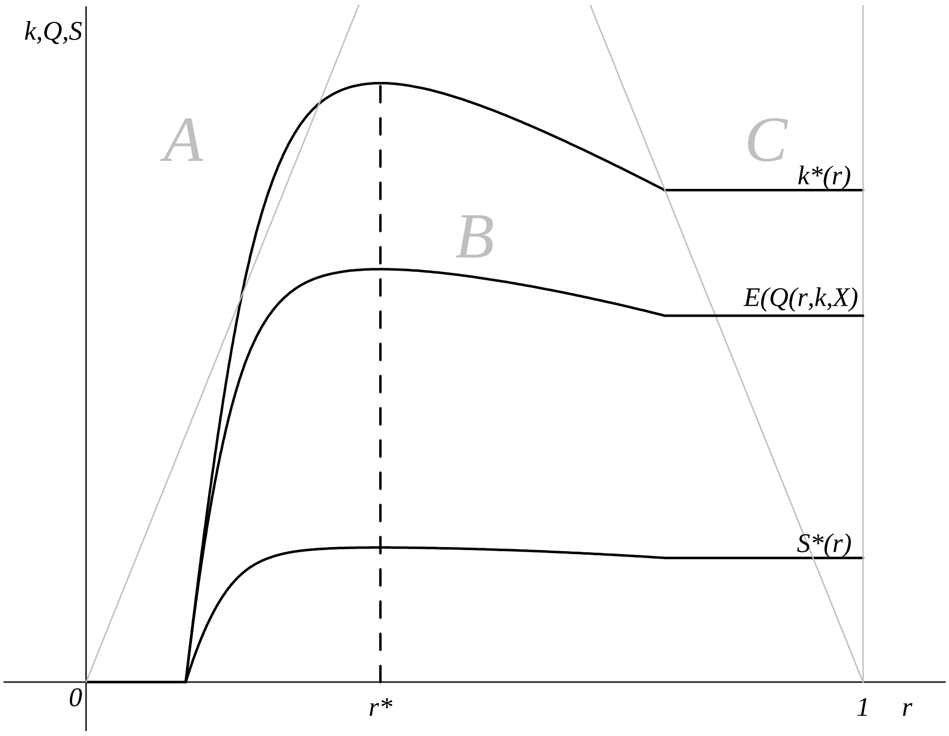


Figure 1.9. Capacity, Expected Output and Surplus for $c = 3/25$.

With capacity withholding, for $c = 1/32$ the optimal capacity is $k^*(r^*) = (0.86)\, k^W$ and the expected surplus is $\bar{S}(k^*(r^*)) = (0.93) S^*(k^W)$. For $c = 3/25$ these numbers are considerably lower, $k^*(r^*) \simeq (0.61) k^W$ and $\bar{S}\,(k^*(r^*)) = (0.81) S^*(k^W)$. These numbers suggest that with capacity withholding price caps are more effective when unit cost of capacity is small than when it is large.

## 1.7. Conclusions

In the absence of capacity precommitment, whether the demand is deterministic or stochastic, price cap regulation provides an effective instrument to mitigate market power and enhance surplus: If firms produce the good with constant returns to scale, for example, decreasing the price cap (while maintaining it above marginal cost) leads to an increase of (expected) output and surplus, and to a decrease of the market price. Moreover, a price cap equal to marginal cost is able to eliminate inefficiencies.

With capacity precommitment and capacity withholding price cap regulation has to deal with a trade off involving the incentives for capacity investment and capacity withholding: decreasing the price cap alleviates capacity withholding but may discourage capacity investment. As a consequence, an optimal price cap may not maximize capacity investment: when the cost of capacity is low, maximizing the expected surplus calls for a low price cap that alleviates capacity withholding, even at the cost of reducing capacity investment. Moreover, under standard regularity assumptions on the demand, the comparative static properties of price caps above the price cap that maximizes capacity are analogous to those obtained in the case of a deterministic demand. Thus, price cap regulation provides useful instrument to mitigate market power and enhance market efficiency, although it cannot restore efficiency.

It is noteworthy that even if capacity withholding is not an issue, i.e., even if the regulator may enforce full capacity utilization, price cap regulation does not provide appropriate incentives for capacity investment either. In fact, both capacity investment and surplus may be smaller with full capacity utilization than with capacity withholding. See the example discuss in Appendix C.

## Appendix 1.A: Proofs

**Proof of Proposition 1.3.** Assume that the hazard rate of $X$, $h(\cdot) = f(\cdot)/[1 - F(\cdot)]$, is increasing. We calculate the equilibrium capacity $k^*(r)$. Let us consider first price caps $r \in [0, 1/2]$. Then $\bar{\Pi}(r, \cdot)$ takes values in regions $A$ and $B$.

If the capacity that maximizes $\bar{\Pi}(r, \cdot)$ is such that $(r, k) \in A$, then solving the equation (1) for $\overline{MR}$ given by (6) yields

$$k_A(r) = F^{-1}(1 - \frac{c}{r}) - r.$$

Hence

$$k_A(r) + r = F^{-1}(1 - \frac{c}{r}) < 1,$$

and therefore $k_A(r) < 1 - r$. If $(r, k_A(r)) \in A$, then $r \leq k_A(r)$. This inequality is equivalent to

$$c \leq r\,(1 - F(2r)) = \overline{MR}(r, r).$$

Write $M(r) := \overline{MR}(r, r)$. Differentiating $M$ yields

$$\frac{dM(r)}{dr} = (1 - F(2r)) - 2rf(2r) = (1 - F(2r))\,(1 - 2rh(2r))\,,$$

which is positive for values of $r$ close to zero and negative for values of $r$ close to $1/2$. Since $h$ is increasing, then the function $M(r)$ is strictly concave and reaches its maximum value $M^*$ on $(0, 1/2)$. If $c < M^*$, then the equation $\overline{MR}(r, r) = c$ has two solutions on $(0, 1/2)$, which we denote by $r_-(c)$ and $r_+(c)$ with $r_-(c) < r_+(c)$. In this case, for $r \in [r_-(c), r_+(c)]$, we have $(r, k_A^*(r)) \in A$. If $r \notin [r_-(c), r_+(c)]$, i.e., $c > \overline{MR}(r, r)$, then $\bar{\Pi}(r, \cdot)$ decreases with $k$ in region $A$, and reaches its maximum in region $B$.

Assume that the capacity that maximizes $\bar{\Pi}(r, \cdot)$ is such that $(r, k) \in B$. Denote by $k_B(r)$ the solution to equation (1) for $\overline{MR}$ given by (4). Hence $k_B(r)$ satisfies

$$0 < k_B(r) < r.$$

(Recall that we are identifying the monopolist capacity for $r < 1/2$, and therefore $k_B(r) < r$ implies $k_B(r) < 1 - r$.) The inequality $k_B(r) < r$ is equivalent to

$$c > \overline{MR}(r, r).$$

If $c \leq \overline{MR}(r,r)$, i.e., $r \in [r_-(c), r_+(c)]$, then $\bar{\Pi}(r,\cdot)$ increases with $k$ in region $B$, and reaches its maximum in region $A$. The inequality $k_B(r) > 0$ is equivalent to

$$c < \int_0^r xf(x)dx + r\,(1 - F(r)) = \overline{MR}(r,0),$$

i.e., the expected marginal revenue when output is zero $\overline{MR}(r,0)$ must be greater than the unit cost of capacity $c$. If this inequality does not hold, then $\bar{\Pi}(r,\cdot)$ decreases with $k$ in region $B$ and reaches its maximum at $k^* = 0$. Since $d\overline{MR}(r,0)/dr = 1 - F(r) > 0$ on $(0,1)$, then the function $\overline{MR}(\cdot,0)$ has an inverse, which we denote by $\underline{r}$. Then the condition $c < \overline{MR}(r,0)$ may be written as $r > \underline{r}(c)$. Since

$$\overline{MR}(r,0) < \int_0^r xf(x)dx + r\,(1 - F(r)) = r,$$

then

$$c = \overline{MR}(\underline{r}(c),0) < \underline{r}(c).$$

Therefore the equilibrium capacity is $k^* = 0$ for a range of price caps above the cost of capacity, $r \in (c, \underline{r}(c)]$. Also, since

$$\overline{MR}(r,0) > r\,(1 - F(r)) > r\,(1 - F(2r)) = \overline{MR}(r,r),$$

then $r < \underline{r}(c)$ (i.e., $c > \overline{MR}(r,0)$) implies $r < r_-(c)$.

Let us now consider price caps $r \in (1/2, 1]$. Then $\bar{\Pi}(r,\cdot)$ takes values in regions $B$ and $C$.

Assume that the capacity that maximizes $\bar{\Pi}(r,\cdot)$ is such that $(r,k) \in B$. If $r \leq \underline{r}(c)$, then $\bar{\Pi}(r,\cdot)$ decreases with $k$ and reaches its maximum at $k = 0$. If $r > \underline{r}(c)$, then $\bar{\Pi}(r,\cdot)$ reaches its maximum in region $B$ if the solution to condition (1), $k_B(r)$, satisfies

$$k_B(r) < 1 - r.$$

This condition is equivalent to

$$c > \int_{2(1-r)}^1 xf(x)dx - 2\,(1-r)\,[1 - F(2\,(1-r))] = \overline{MR}(r, 1-r).$$

Note that

$$\frac{d\overline{MR}(r,1-r)}{dr} = 2(1 - F(2\,(1-r))) > 0.$$

Hence the function $\overline{MR}(r, 1-r)$ has an inverse on $(1/2, 1)$, which we denote by $\bar{r}(c)$, and therefore we may write the above inequality as $r < \bar{r}(c)$. If $r \geq \bar{r}(c)$, then $\bar{\Pi}(r, \cdot)$ increases with $k$ in region $B$ and reaches its maximum in region $C$. Note that for $r = 1$ we have $\overline{MR}(r, 1-r) = \overline{MR}(1, 0) = E(X)$. Hence, since $c < E(X)$ by assumption, we have $\bar{r}(c) < 1$.

Finally, assume that the capacity that maximizes $\bar{\Pi}(r, \cdot)$ is such that $(r, k) \in C$. Denote by $k_C$ the solution to the condition (1) for $\overline{MR}$ given by equation (5). Clearly $k_C$ is independent of the price cap $r$. Also, since $\overline{MR}(r, 1/2) = 0$, then $k_C < 1/2$ for all $c \in (0, E(X))$. Since the expected marginal revenue decreases with $k$, then $k_C > 1 - r$ implies $c < \overline{MR}(r, 1-r)$. Moreover, since $r > 1/2$ and $\overline{MR}$ is decreasing, then $\overline{MR}(r, 1-r) < \overline{MR}(r, r)$. Hence $k_C$ solves the monopolist problem if $r \geq \bar{r}(c)$. Otherwise, i.e., if $r < \bar{r}(c)$, then $\bar{\Pi}(r, \cdot)$ decreases with $k$ in region $C$ and reaches its maximum in region $B$.

As shown above $c < \underline{r}(c)$. If $c < M^*$, then we have $\underline{r}(c) < r_-(c) < r_+(c) < 1/2$. Since $1/2 < \bar{r}(c) < 1$, then

$$c < \underline{r}(c) < r_-(c) < r_+(c) < 1/2 < \bar{r}(c) < 1.$$

If $c \geq M^*$, then $c \geq \overline{MR}(r, r)$ for all $r \in [0, 1/2]$, and the equilibrium capacity lies in region $B$ for all $r \in [0, 1/2]$.

**Lemma 1.1.** *Let $g$ be a real valued function on $\mathbb{R}$, continuously differentiable on some interval $(a, b)$, and satisfying $g'(a) > 0 > g'(b)$, and $g''(y) < 0$ for all $y \in (a, b)$ such that $g'(y) = 0$. Then $g$ has a unique global maximizer on $[a, b]$, $y^* \in (a, b)$, and $g'$ is positive on $(a, y^*)$ and negative on $(y^*, b)$.*

**Proof.** Let $y^* = \sup\{y \in (a, b) \mid g'(y) > 0\}$ and $y^{**} = \inf\{y \in (a, b) \mid g'(y) < 0\}$. Since $g'$ is continuous on $(a, b)$, then $g'(y^*) = g'(y^{**}) = 0$, and therefore $a < y^{**} \leq y^* < b$. We show that $y^* = y^{**}$, which establishes the lemma. Suppose by way of contradiction that $y^{**} < y^*$. Since both $g''(y^*)$ and $g''(y^{**})$ are negative, then for $\varepsilon \in (0, y^* - y^{**})$ sufficiently small

$$g'(y^{**} + \varepsilon) < 0 < g'(y^* - \varepsilon).$$

Hence there is $\bar{y} \in (y^{**} - \varepsilon, y^* + \varepsilon)$ satisfying $g'(\bar{y}) = 0$, and such $g'$ is negative (positive) for $y$ below (above) and near $\bar{y}$. Hence $g''(\bar{y}) > 0$, which is a contradiction.

**Appendix 1.B: Theorem 6 in Earle et al.**

Earle et al. (2007)'s Theorem 6 seemingly establishes that our propositions 1.3 to 1.6 fail for an open and dense subset of probability distributions of the demand parameter $X$. Considering that Earle et al. (2007) seem to have in mind a large set of probability distributions (their proof involves a discontinuous *c.d.f.*), this result is hardly surprising, and is not inconsistent with propositions 1.3 to 1.6. (A generic continuous *p.d.f.* on $[0, 1]$ is nowhere differentiable by Banach-Mazurkiewicz Theorem. Thus, the set continuously differentiable *p.d.f.*'s with an increasing hazard rate is a meagre subset of this set.)

Nonetheless, their claim that the proof of their Theorem 4, which establishes this result in the model of full capacity utilization, also applies to the model with capacity withholding that we study here is incorrect. In this section we show in the example discussed in Section 1.6 perturbing the distribution of the demand parameter $X$ as in the proof of Earle et al. (2007)'s Theorem 4 does not produce the desired results. Of course, this does not prevent the existence of *p.d.f.*'s on $[0, 1]$ for which the conclusions of propositions 1.3 to 1.6 do not hold.

Earle et al. (2007)'s proof of Theorem 4 shows that given a *c.d.f.* $F$ and a *binding* price cap $\widehat{r}$ (i.e., $\widehat{r}$ satisfies $\Pr(X - \widehat{r} > k^*(\widehat{r})) > 0$, which in our setting amounts to $\widehat{r} \in (\underline{r}(c), \bar{r}(c))$), and such that $dk^*(\widehat{r})/dr < 0$, then by perturbing $F$ in a certain way one can obtain another c.d.f. $\tilde{F}$ arbitrarily close to $F$ and such that equilibrium capacity when the demand parameter is distributed according to $\tilde{F}$, $\tilde{k}^*$ satisfies $d\tilde{k}^*(\widehat{r})/dr > 0$. We show that the perturbation used in the proof of their Theorem 4 does not produce this result when the monopolist can withhold capacity.

Assume that $X$ is uniformly distributed, and that the unit cost of capacity is $c = 1/32$. Consider the price cap $\widehat{r} = 2/5 \in [r_-(1/32), r_+(1/32)] = \left[\frac{1}{4} - \frac{1}{8}\sqrt{3}, \frac{1}{4} + \frac{1}{8}\sqrt{3}\right]$. As shown in

Section 1.6 we have $k^*(r) = 1 - \frac{c}{r} - r$. Hence

$$\frac{dk^*(\widehat{r})}{dr} = \frac{c}{\widehat{r}^2} - 1 = -\frac{103}{128},$$

i.e., capacity decreases with the price cap near $\widehat{r}$. (In the language of Earle et al. (2007), the comparative static properties near $\widehat{r}$ are standard.)

Using the results of table 1.1A, we see that for demand realizations such that $X - \widehat{r} < k^*(\widehat{r})$, i.e., $X \in (\tilde{x}, 1]$ where $\tilde{x} = \frac{59}{64}$, the monopolist withholds capacity. Let us study the comparative static properties for a new perturbed distribution of $X$, denoted by $\tilde{F}$ which assigns probability uniformly on $[0, 1]$ except on the interval $[\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$, on which the probability is shifted to the end points, thus creating two atoms at $\tilde{x} - \varepsilon$ and $\tilde{x} + \varepsilon$. The probabilities assigned to these atoms are $2\lambda\varepsilon$ and $2(1 - \lambda)\varepsilon$, where $\varepsilon$ and $\lambda$ are such that the optimal capacity when the price cap $\widehat{r} = 2/5$ remains $k^*(\widehat{r}) = 167/320$; that is, $\varepsilon$ and $\lambda$ are chosen in such a way that

$$\frac{\partial}{\partial k}\left(\int_{\tilde{x}-\varepsilon}^{\tilde{x}} (x - \widehat{r})\,\widehat{r}dF(x) + \int_{\tilde{x}}^{\tilde{x}+\varepsilon} \widehat{r}k^*(\widehat{r})dF(x)\right) = \varepsilon\widehat{r}$$

equals

$$\frac{\partial}{\partial k}\left(\int_{\tilde{x}-\varepsilon}^{\tilde{x}} (x - \widehat{r})\,\widehat{r}d\tilde{F}(x) + \int_{\tilde{x}}^{\tilde{x}+\varepsilon} \widehat{r}k^*(\widehat{r})d\tilde{F}(x)\right) = 2\,(1 - \lambda)\,\varepsilon\widehat{r}.$$

Solving this equation yields $\lambda = 1/2$, independently of $\varepsilon$. Therefore let $\lambda = 1/2$.

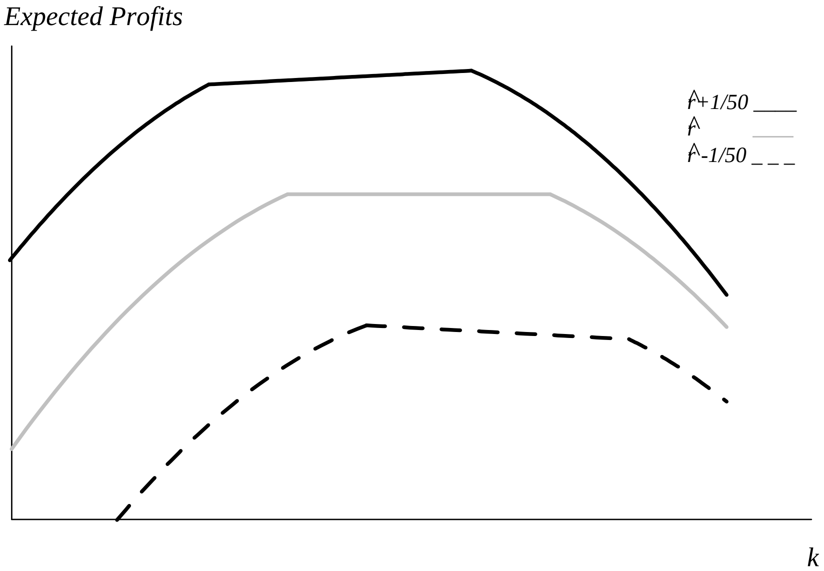


Figure 1.10. Profits near $\hat{r} = \frac{2}{5}$.

When the demand parameter is distributed according to $\tilde{F}$ the expected profit is

$$\begin{aligned}\tilde{\Pi}(r,k) &= \int_0^{2r} \left(\frac{x}{2}\right)^2 dx + \int_{2r}^{k+r} (x-r) r dx + \int_{k+r}^{1} rkdx - ck \\ &= -\frac{r}{2}k^2 + [r\,(1-r) - c]k + \frac{r^3}{6}\end{aligned}$$

if $r + k \in [0, \tilde{x} - \varepsilon)$, it is

$$\begin{aligned}\tilde{\Pi}(r,k) &= \int_0^{2r} \left(\frac{x}{2}\right)^2 dx + \int_{2r}^{\tilde{x}-\varepsilon} (x-r) r dx + \varepsilon\left((\tilde{x}-\varepsilon) - r\right) r \\ &\quad + r\varepsilon k + \int_{\tilde{x}+\varepsilon}^{1} rkdx - ck \\ &= [r(1-\tilde{x}) - c]k + \frac{r}{6}(3\hat{x}^2 - 6\hat{x}r - 3\varepsilon^2 + 4r^2)\end{aligned}$$

if $r + k \in [\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$, and it is

$$\begin{aligned}\tilde{\Pi}(r,k) &= \int_0^{2r} \left(\frac{x}{2}\right)^2 dx + \int_{2r}^{\tilde{x}-\varepsilon} (x-r) r dx + \varepsilon\left((\tilde{x}-\varepsilon) - r\right) r \\ &\quad + r\varepsilon k + \int_{\tilde{x}+\varepsilon}^{k+r} (x-r)\, r dx + \int_{k+r}^{1} rkdx - ck \\ &= -\frac{r}{2}k^2 + \left(r\,(1+\varepsilon-r) - c\right) k + \frac{r^3}{6} - r\varepsilon^2 + r^2\varepsilon - r\hat{x}\varepsilon\end{aligned}$$

if $r + k > \tilde{x} + \varepsilon$.

Figure 1.10 displays the graphs of the expected profit for $r$ near $\widehat{r}$. If $r > \widehat{r}$, then $\tilde{\Pi}(r, \cdot)$ is increasing in capacity. If $r < \widehat{r}$, then $\tilde{\Pi}(r, \cdot)$ is decreasing in capacity. Hence $\tilde{k}^*(r) = \tilde{x} - \varepsilon - r$ if $r > \widehat{r}$, and $\tilde{k}^*(r) = \tilde{x} + \varepsilon - r$ if $r < \widehat{r}$ for $r$ near $\widehat{r}$. That is, the equilibrium capacity is decreasing in the price cap. If $r = \widehat{r}$, then $\tilde{\Pi}(r, \cdot)$ is constant and maximal for $k \in [\tilde{x} - \varepsilon - \widehat{r}, \tilde{x} + \varepsilon - \widehat{r}]$.

Figure 1.11 provides the graphs of $k^*$ and $\tilde{k}^*$ for $\varepsilon = \frac{1}{30}$. Although the mapping $\tilde{k}^*(r)$ becomes a correspondence for $\widehat{r}$, comparative statics for $r$ near $\widehat{r}$ remain standard, i.e., $\partial \tilde{k}^*(r)/\partial r = -1$ near $r = \widehat{r}$. (Except on $\widehat{r}$ itself, where the derivative is not defined since mapping providing the equilibrium capacity becomes a correspondence.) If the monopolist withholds capacity, after this perturbation capacity continues to decrease with the price cap for all price caps in a neighborhood of $\widehat{r}$.

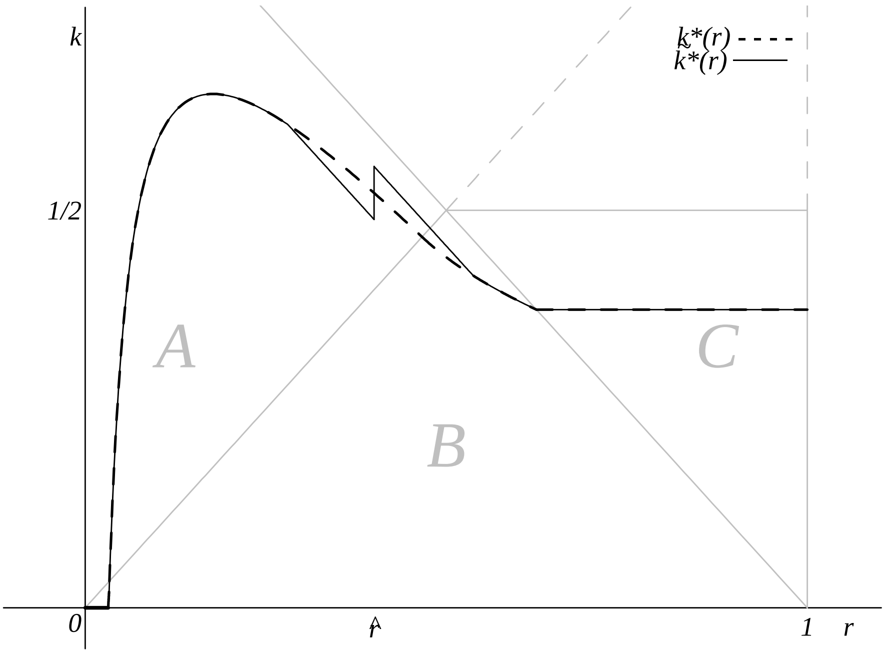


Figure 1.11. Equilibrium Capacity for $F$ and $\tilde{F}$.

Thus, Earle et al. (2007)'s proof, which relies on this perturbation, does not apply to a model where the monopolist may withhold capacity. In fact, this perturbation has an effect on the monopolist profit and the profit maximizing level of capacity akin to that of creating a flat spot on the demand when the demand is known with certainty.

### Appendix 1.C: Full Capacity Utilization

Assume that the monopolist cannot withhold capacity, i.e., must supply its entire capacity for each demand realization. One may interpret this setting as one where the monopolist delivers its output to the market before the demand is realized. This model is studied by Earle et al. (2007) and Grim and Zoettl (2010). We show that the equilibrium and the comparative static properties of price caps in this model are significant different from those of our model where the monopolist may withhold capacity.

#### Monopoly Equilibrium with a Price Cap

Assume that a regulatory agency imposes a price cap $r \in [0, 1]$. Table 1.3A identifies the market equilibrium price for each demand realization if the monopolist installs a capacity $k < 1 - r$ (and supplies it inelastically to the market).

| $X$ | $[0, k)$ | $[k, r + k)$ | $[r + k, 1]$ |
|---|---|---|---|
| $\hat{P}(r, k, x)$ | $0$ | $x - k$ | $r$ |

Table 1.3A: Equilibrium Price for $k \in [0, 1 - r)$.

Table 1.3B identifies the market equilibrium price for each demand realization when the monopolist installs a capacity $k \geq 1 - r$.

| $X$ | $[0, k)$ | $[k, 1]$ |
|---|---|---|
| $\hat{P}(r, k, x)$ | $0$ | $x - k$ |

Table 1.3B: Equilibrium Price for $k \in [1 - r, 1]$.

Note that if $k \geq 1 - r$ the price cap is non-binding.

For $k < 1 - r$ the expected price is

$$E(\hat{P}(r, k, X)) = \int_{k}^{r+k} (x - k) f(x) dx + \int_{r+k}^{1} r f(x) dx.$$

Hence

$$\frac{\partial E(\hat{P}(r, k, X))}{\partial k} = -\int_{k}^{r+k} f(x) dx,$$

and

$$\frac{\partial^2 E(\hat{P}(r, k, X))}{\partial k^2} = f(k) - f(r + k).$$

For $k \geq 1 - r$ the expected price is

$$E(\hat{P}(r, k, X)) = \int_{k}^{1} (x - k) f(x) dx.$$

Hence

$$\frac{\partial E(\hat{P}(r, k, X))}{\partial k} = -\int_{k}^{1} f(x) dx,$$

and

$$\frac{\partial^2 E(\hat{P}(r, k, X))}{\partial k^2} = f(k).$$

The monopolist chooses the level of capacity $k$ in order to maximize its expected profit

$$\hat{\Pi}(r, k) = E\left([\hat{P}(r, k, X) - c]k\right) = [E(\hat{P}(r, k, X)) - c]k,$$

Clearly $\hat{\Pi}$ is continuous on $[0, 1]^2$. In an interior equilibrium $k$ solves

$$\frac{\partial E(\hat{P}(r, k, X))}{\partial k} k + E(\hat{P}(r, k, X)) = c, \tag{13}$$

and satisfies

$$\frac{\partial^2 \hat{\Pi}(r, k)}{\partial k^2} = \frac{\partial^2 E(\hat{P}(r, k, X))}{\partial k^2} k + 2 \frac{\partial E(\hat{P}(r, k, X))}{\partial k} < 0. \tag{14}$$

We have

$$\frac{\partial^2 \hat{\Pi}(r, k)}{\partial k^2} = -k(f(r + k) - f(k)) - 2(F(r + k) - F(k)).$$

for $k < 1 - r$, and

$$\frac{\partial^2 \hat{\Pi}(r, k)}{\partial k^2} = k f(k) - 2 (1 - F(k)).$$

for $k \geq 1 - r$. The sign of these expressions is ambiguous. In fact, it is not difficult to find examples for which the profit function $\hat{\Pi}(r, \cdot)$ is not concave for some values of $r$. (E.g., take $f(x) = 2(1-x)$ and $r = 1/4$.) This property of this model of full capacity utilization stands in contrast with that of our model of capacity withholding, in which the expected profit is a concave function.

In this setting, the surplus realized is independent of $r$. Assuming efficient rationing, the expected surplus is

$$\hat{S}(k) = \int_0^k \frac{x^2}{2} dx + \int_k^1 \frac{1}{2} k (2x - k) \, dx = \frac{1}{6} k \left(k^2 - 3k + 3\right).$$

An Example: The Uniform Distribution

Assume that $X$ is uniformly distributed and $c \in (0, 1/2)$. For $k < 1 - r$ we have

$$E(\hat{P}(r, k, X)) = \frac{1}{2} r (2 - 2k - r),$$

and for $k \geq 1 - r$, we have

$$E(\hat{P}(r, k, X)) = \frac{1}{2} (1 - k)^2.$$

Hence for $k < 1 - r$, we have

$$\frac{\partial^2 \hat{\Pi}(r, k)}{\partial k^2} = -2r.$$

and for $k \geq 1 - r$, we have

$$\frac{\partial^2 \hat{\Pi}(r, k)}{\partial k^2} = -2 + 3k.$$

If the equilibrium capacity is $k < 1 - r$, then equation (13) is

$$-rk + \frac{1}{2} r (2 - 2k - r) = c.$$

Solving this equation we get

$$k_1(r) = \frac{1}{2} \left(1 - \frac{c}{r} - \frac{r}{2}\right).$$

Hence $k_1(r)$ is the solution to the monopolist problem provided $0 < k_1(r) < 1 - r$, i.e.,

$$\underline{r}(c) := 1 - \sqrt{1 - 2c} < r < \frac{1}{3}\sqrt{6c + 1} + \frac{1}{3} := \overline{r}(c).$$

If $r < \underline{r}(c)$, then expected profit decreases with $k$ and the equilibrium capacity is $k^* = 0$. If $r > \overline{r}(r)$, then expected profit increases with $k$ at $k = 1 - r$.

If the equilibrium capacity is $k \geq 1 - r$, then equation (13) is

$$- (1 - k)\, k + \frac{1}{2} (1 - k)^2 = c.$$

Solving this equation we get

$$k_2 = \frac{2 - \sqrt{1 + 6c}}{3}.$$

Note that $k_2 > 0$ for all $c \in (0, 1/2)$. Hence $k_2$ is the solution to the monopolist problem provided $k_2 \geq 1 - r$, i.e., $r \geq \overline{r}(c)$. If $r < \overline{r}(c)$ the expected profit decreases with $k$ at $k = 1 - r$.

The equilibrium capacity is therefore given by

$$\hat{k}^* (r) = \begin{cases} 0 & \text{if } r \leq [0, \underline{r}\,(c)], \\ k_1\,(r) & \text{if } r \in (\underline{r}(c), \overline{r}(c)), \\ k_2 & \text{if } r > [\overline{r}(c), 1]. \end{cases}$$

The maximum capacity is installed for $r^*$ solving

$$\frac{dk_1(r)}{dr} = \frac{1}{2} \left( \frac{c}{r^2} - \frac{1}{2} \right) = 0;$$

i.e., $r^* = \sqrt{2c}$. (Note that $d^2 k_1(r)/dr^2 = -c/r^3 < 0$.) The maximum capacity is

$$k_1(r^*) = \frac{1}{2} - \sqrt{\frac{c}{2}} > k_2.$$

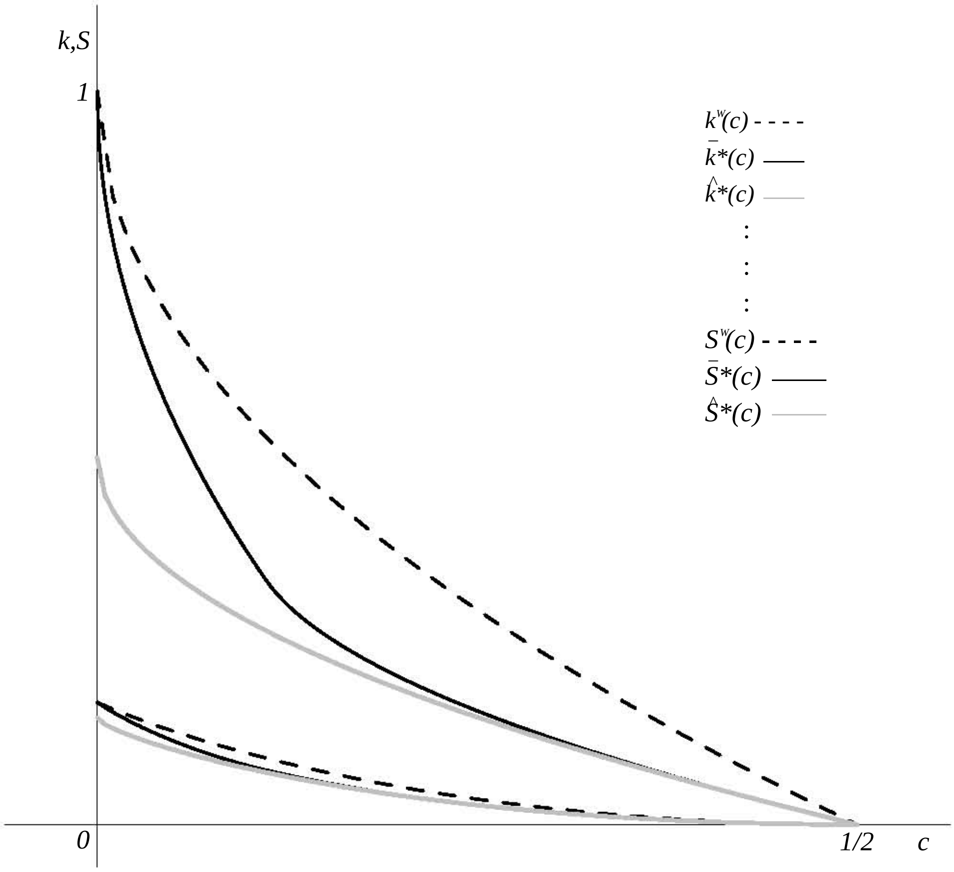


Figure 1.12. Capacity Investment and Surplus with and without Withholding.

As shown in Section 1.6 the optimal capacity is $k^W = 1 - \sqrt{2c} = 2k_1(r^*)$. Hence $r^*$ is indeed the optimal price cap. Moreover, since $\hat{k}^*(r^*) > k_2$, then a binding price increases expected surplus, but is unable to provide incentives for the monopolist to install the optimal level of capacity. A price cap is a poor regulatory instrument also in this framework. In fact, price caps generate a lower expected surplus (and provide worse incentives for capacity investment) under full capacity utilization that when the monopolist can withhold capacity as Figure 1.12 below shows. (Note that $\bar{S}(k^*(r^*(c)))$ is a lower bound to the maximum expected surplus that can be realized with an optimal price cap.)

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## CHAPTER 2. STRATEGIC INCENTIVES FOR KEEPING ONE SET OF BOOKS UNDER THE ARM'S LENGTH PRINCIPLE

### 2.1. Introduction

Policy makers have become increasingly aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions. Transfer pricing policies also have important implications since exports and imports from related parties are a dominant portion of trade flows – see Bernard, Jensen and Schott (2009). To moderate the incentives for firms to use transfer prices to shift profits from high to low tax jurisdictions for reasons unrelated to the economic nature of the transactions, most governments follow taxation policies that are based on the OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the Arm's Length Principle (ALP); i.e., that transfer prices between companies of multinational enterprises for tax purposes be established on a market value basis, thus comparable to transactions between independent (unrelated) parties -see OECD (2010).

Transfer prices serve both the purpose of allocating costs to different subsidiaries and for determining the tax liability of parent firms and subsidiaries. Since using a single transfer price to do this double purpose can distort internal transactions, a growing number of multinational firms use internal transfer prices that differ from those used for tax purposes. This is a legal practice in OECD countries, the only constraint is that being transfer prices for tax purposes must be consistent with the ALP. Given that there is no statutory requirement, incentive and tax transfer prices may differ. Therefore, an immediate question is whether firms separate their internal transfer prices from those used for tax purposes.

Using the terminology of Hyde and Choe (2005) and Dürr and Göx (2011), when firms use the same transfer price for tax reporting and for providing incentives, they keep *one set of books*, while when firms use different transfer prices for each purpose, they keep *two sets of books*.

In the absence of delegation, the choice between keeping one or two sets of books is not a matter. However under delegation, the choice between keeping one or two sets of books is relevant, even if tax rates are equal across jurisdictions.

Theoretical studies regarding the optimal accounting strategy by decentralized firms which comply with tax rules are not conclusive. Specifically, these results depend on considering the presence of competition.

First, abstracting from competition consideration, theoretical literature on this topic has established that keeping two sets of books is optimal whenever tax and incentives objectives are conflicting -see Baldenius, Melumad and Reichelstein (2004).

Second, considering the possibility of competition, Göx (2000) and Dürr and Göx (2011) study the equilibrium accounting and transfer pricing policies in a multinational duopoly with price competition in the final good market. They find that the firms in a duopoly can benefit from strategically using the same transfer price for tax and managerial purposes instead of using separate transfer prices for both objectives. According to their results, firms in industries with a small number of competitors can benefit from using the same transfer price for tax and managerial purposes even if the tax and managerial objectives are conflicting. Therefore if firms keep one set of books, the ALP may reinforce the effect of vertical separation by softening competition – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998).

Empirical evidence on the use of alternative accounting system is also mixed– see Dürr and Göx (2011) for a review of this literature.

In this paper, taking compliance with the tax rules as given (i.e., transfer prices for tax purposes are consistent with the ALP); we study the optimal accounting strategy by decentralized multinational firms which compete in quantities in a context of imperfect competition.[12]

As Göx (2000), Arya and Mittendorf (2008) and Dürr and Göx (2011), we consider accounting policy a commitment device since changing it is associated with high administrative

[12] Quantity competition provides a reduced form model for the analysis of more complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition -see Kreps and Scheinkman (1983) and Moreno and Ubeda (2006).

and consulting costs. Moreover, accounting policies tend to be public (in, for instance, management discussions in annual reports, Securities and Exchange Commission filings and tax authority pricing agreements). For these reasons, choice of a accounting policy may be a publicly observable commitment.

Other means of competitive commitment have been detailed in the literature, including distorting managerial compensation -Fershtman and Judd (1987); Sklivas (1987)-, sinking capacity investments -Dixit (1980); Spence (1977)-, building inventories -Ware (1985)-, limiting information acquisition -Einy et al. (2002); Gal-Or (1988)-, and cost allocation rules -Gal-Or (1993); Hughes and Kao (1998).

In our framework there are two markets, which we refer to as the Latin market (or home market) and the Greek market (or external market). There are two firms engaging in Cournot competition in the Latin market. These firms have subsidiaries, which in turn engage in Cournot competition in the Greek market. As customary, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits. Since competition in the Latin market provides a market price to impose on comparable market transactions, parents use this price to satisfy both cost and tax accounting requirements if keeping one set of books. If parents keep two sets of books, Latin market provides a market price only for tax purposes. Specifically, the analysis is based on a three stage non-cooperative game under complete information. Parents choose their accounting policy and then compete in quantities in the home market and set the prices at which they sell the good to their subsidiaries (either directly or indirectly via their output choices), which in turn compete in quantities in an external market. The decisions of the subsidiaries in the third stage are solely determined by the outcome of the second stage game. We show that parents' accounting policies determine the properties of market outcomes. Before characterizing equilibria of this game, we analyze the properties of each subgame (i.e., when both firms keep one set of books, when both firms keep two sets of books, as well when one firm keeps one set of books and the other keeps two sets of books).

In the subgame where both parents adopt one set of books (i.e., a parent must transfer the good to its subsidiary at the home market price), parent's output decisions must internalize its impact on the transfer price of its subsidiary, and its subsidiary's rival. One set of books thus provides parents with an instrument to soften competition in the external market. Since a parent influences its transfer price via its output decision in the home market, competition may be more aggressive in this market. Total profits under one set of books are above profits at the equilibrium where parents and subsidiaries are integrated. Hence using one set of books may provide a rationale for vertical separation. If tax rates are equal across jurisdictions, maximizing gross or net profits leads to the same result. However, if tax rates are different across jurisdictions, using one set of books also provides tax saving. In particular, when the home market is a tax heaven, the quantity in the home market is cut in order to increase the transfer price and therefore, every additional unit sold in the external market at a higher transfer price reduces the firm's tax liabilities.

In the subgame where both parents adopt two sets of books (i.e., parent firms use internal transfer price that differs from that used for tax purposes), internal transfer prices open up the possibility to gain a Stackelberg advantage in the external market. Parents reduce their internal transfer prices below marginal cost in order to take advantage in the external market, creating a sort of *prisoners' dilemma*. If tax rates are equal across jurisdictions, maximizing gross or net profits lead to a different result: a parent has an incentive to reduce the market price in the home market by increasing its output and at the same time reduces its internal transfer price, thus increasing its subsidiary's rival tax liability without affecting the marginal cost of its own subsidiary. Therefore, if both firms keep two sets of books together with a transfer pricing regulation consistent with the ALP competition intensifies in both markets relative to an equilibrium where parents and subsidiaries are integrated. Thus if tax rates are equal across jurisdictions, neither benefits from competition consideration nor tax liabilities savings exist when parents use two sets of books. Nevertheless if tax rates are different across jurisdictions, two sets of books may reduce tax liabilities.

In the subgame with asymmetric accounting policies (i.e., one parent choosing one set of books and the other parent choosing two sets of books), the parent using two sets of books becomes the dominant producer in the external market, since its internal transfer price is lower than home market price, while the parent using one set of books becomes the dominant producer in the home market because increasing its output in this market alleviates the double marginalization that arises in the external market. Total output (total profits) in both markets are above (below) the standard Cournot level. But profits of the parent using two sets of books exceed this level.

Adding the first stage to the game, whereby parents choose their accounting policy, leads to a variety of equilibrium depending on market sizes and tax rates. Restricting attention to (pure strategy) subgame perfect equilibria, the possible types of the game vary from a *prisoners' dilemma* (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books) to a *game of chicken* (with two pure strategy Nash equilibria, in which one firm uses one set of books and the other uses two sets of books) or a *coordination game* (with two pure strategy Nash equilibria, one in which both parents choose two sets of books, and another one in which choose one set of books). Also, parameter constellations of market sizes and tax rates can be found such the type of the game is a *cooperation game* (with a unique Pareto efficient Nash equilibrium in which both parents choose one set of books).

Parent's strategic behavior implies that keeping one set of books may be sustained as an equilibrium for most of the size differences between markets, when tax rates are high. Moreover, this equilibrium is unique when both markets are similar in size.

Our analysis contributes to the transfer pricing literature by broadening the understanding of the potential incentives for the choice of the accounting policy. A central premise in some related literature is that multinational firms set the same transfer price for tax and incentive purposes (i.e., keeping one set of books) –see Schjelderup and Sorgard (1997), Korn and Lengsfeld (2007), Nielsen et al. (2008) and Lemus and Moreno (2011). In these papers one set of books is taken as given and is not a matter of choice. Here we endogenize that choice and show that one set of books may be sustained as an equilibrium under broad conditions.

Since keeping one set of books provides parents with an instrument to soften competition in the external market, our analysis offers a convincing explanation of how the choice of the accounting policy can serve as a precommitment device. In our setting, taxes commit firms to the adoption of a particular accounting policy (i.e., one or two sets of books).[13]

In addition, our model contributes to the literature on strategic delegation by broadening the understanding of the potential benefits of decentralization, an organizational structure whose motivation is not well understood when firms compete in quantities. Dürr and Göx (2011) analyzed the optimal accounting when firms compete in prices. Their results reinforce the effect of vertical separation in softening competition when firms keep one set of books. Analogous conclusions, when firms compete in prices, were found by Sklivas (1987).

Our analysis does not only broaden the theoretical understanding but it also provides testable empirical predictions depending on the differences in market size and tax rates.

The paper is organized as follows. Section 2.2 introduces the basic setup. Section 2.3 provides an equilibrium analysis under one set of books. Section 2.4 derives results for two sets of books. Section 2.5 studies the equilibrium with asymmetric accounting policies. Section 2.6 characterizes the equilibria of this three stage non-cooperative game. Section 2.7 concludes.

## 2.2. Model and Preliminaries

A good is sold in two markets, which we refer to as the Latin market and the Greek market. The inverse demands in the Latin and Greek markets are $p^d(q) = \max\{0, 1 - bq\}$ and $\rho^d(\chi) = \max\{0, 1 - \beta\chi\}$, respectively, where $b$ and $\beta$ are positive real numbers. Assuming that demands are linear facilitates the analysis and makes it easier to interpret the results.

[13] Arya and Mittendorf (2008) analyze market based transfer pricing as a strategic response in a similar setting. They show that the ALP makes firms more aware of the fact that excessive home market prices depress external production (i.e., the concern is about double marginalization) and may be more aggressive in the home market as a result. However, they do not recognize that ALP increases the prevailing transfer prices and thereby mitigate the prisoner's dilemma in transfer pricing to get an edge in downstream competition. In their model, parents rely on intracompany discounts to manage tensions between the home and the external markets. Intracompany discounts are set prior to the stage of competition in the home market and serve as a precommitment device. Nevertheless, this device is somewhat contrived since parents must credibly bind themselves to these discounts.

We assume that maximum willingness to pay in each market is equal.[14] Differences in the slope of the demands (i.e., of the parameters $b$ and $\beta$) capture the impact of differences in the market size – the demand is greater the smaller the slope. The parameter $s := b/\beta$ is a proxy for the size of Latin market relative to that of the Greek market.[15]

The taxable income in the Latin and Greek markets is determined by this tax $\tau$ and $\tau+\Delta$, respectively. The parameter $\Delta$ is the differential tax rates of the Greek relative to the Latin market. Tax rates are assumed to be less than 1, reflecting the idea that policy makers are unable or unwilling to tax multinational firms with a 100 per cent profit taxation.[16] When $\Delta > 0$ ($\Delta < 0$), the Latin (Greek) market is a tax heaven.

There are two firms producing the good at same constant marginal cost, which is assumed to be zero without loss of generality. Firms engage in Cournot competition in the Latin market, and have subsidiaries which in turn engage in Cournot competition in the Greek market.

We assume throughout that for tax purposes transfer prices must be consistent with the ALP; i.e., that the taxable income of a subsidiary that produces $\chi_i$ is $(\rho - p)\chi_i$, where $\rho$ and $p$ are the market prices in the Greek and Latin markets, respectively. Under this assumption the consolidated profits of firm $i$ as a function of parents' and subsidiaries' outputs is

$$\Pi_i(q_1, q_2, \chi_1, \chi_2) = \tag{2.1}$$
$$= (1-\tau)\, p^d(q_1+q_2)q_i + (1-\tau-\Delta)\,\rho^d\,(\chi_1+\chi_2)\,\chi_i + \Delta p^d(q_1+q_2)\chi_i.$$

We refer to the case where parents use the same transfer prices for internal and tax purposes as keeping *one set of books*. If a parent firm uses an internal transfer price that differs from that used for tax purposes, its subsidiary receives the good at a transfer price

[14] Lemus and Moreno (2011) provide an equilibrium analysis when firms use one set of books, in which willingness to pay in each market are different.

[15] This assumption about willingness to pay holds if preferences over the good and/or range of income per capita are similar in the Latin market and in the Greek market. As regards market sizes, $\beta > b$ occurs if the number of people demanding the good in the Latin market is larger than in the Greek market.

[16] Dynamic allocative distortions associated with taxations (100 per cent profit taxation removes all incentive to do one thing rather than another) place constraints on profit taxation.

$t_i$ (which is a non market based transfer prices) but the taxable incomes of the parent and subsidiary are determined by $p$. We refer to this case as keeping *two sets of books.*

Parent firms seek to maximize after tax consolidated profits, independently of whether they keep one or two sets of books; since the cost of production is zero, the consolidated profits are just the sum of the after tax revenues of the parent and the subsidiary. A subsidiary maximizes its own profits, which is the difference, after tax, between its revenue and its cost. A subsidiary' unit cost is just its transfer price. We identify a parent and its subsidiary firm with the same subindex $i \in \{1, 2\}$.

We suppose that both parents must make a publicly observable commitment to an accounting policy before competing in the Latin market and determining their transfer prices. After deciding on the choice of the accounting policies the parents compete in quantities in the Latin market and compute the transfer prices according to the accounting policy and communicate them to their subsidiaries.[17] Finally, the subsidiaries compete in quantities in the Greek market. Thus, we consider a three-stage game consisting of the accounting policy choice on stage one, quantities in the Latin market and the well-known transfer pricing on stage two and finally, quantities in the Greek market on stage three.

In the absence of delegation, the choice between keeping one or two sets of books is not a matter. If parents do not delegate but rather compete in quantities also in the Greek market, the equilibrium outcome in both markets is independent of type of accounting.[18] In particular, if tax rates in both markets are identical, the equilibrium outcome is just the Cournot outcome in both markets.

In the Cournot equilibrium of a duopoly where the market demand is $P^d(Q) = \max\{0, 1 - BQ\}$, firms' constant marginal costs are $(c_1, c_2) \in R^2_+$ and the taxable income is determined by this tax $\tau$, the market price $P^C$, the output $Q_i^C$ and profits $\Pi_i^C$ of firm $i$ are

$$(P^C, Q_i^C, \Pi_i^C) = \left( \frac{1 + c_1 + c_2}{3}, \frac{1 - 2c_i + c_{3-i}}{3B}, \frac{(1 - \tau)(1 - 2c_1 + c_2)^2}{9B} \right). \tag{2.2}$$

[17] Since the outcome of the first stage game becomes known before the market stage of the game, subsidiaries can infer the corresponding internal transfer prices from the other firm's accounting policy and perfectly predict the internal transfer price even if it is not observable per se -see Göx (2000).

[18] Hyde and Choe (2005) observe this fact in a monopoly setting.

If the market is monopolized by a single firm whose constant marginal cost is $c \in R_+$, then the market equilibrium price $P^M$, output $Q^M$, and the firm's profits $\Pi^M$ are

$$(P^M, Q^M, \Pi^M) = \left( \frac{1+c}{2}, \frac{1-c}{2B}, \frac{(1-\tau)(1-c)^2}{4B} \right). \tag{2.3}$$

Using these formulae (2.2), we readily calculate the Latin's market Cournot equilibrium price $p^C$, output $q_i^C = q^C$ and profits $\Pi_i^C = \Pi_L^C$ of firm $i$ as

$$(p^C, q^C, \Pi_L^C) = \left( \frac{1}{3}, \frac{1}{3b}, \frac{1-\tau}{9b} \right). \tag{2.4}$$

Using the formulae (2.3), we obtain the monopoly equilibrium price, output, and the monopoly's profits in the Latin market as

$$(p^M, q^M, \Pi_L^M) = \left( \frac{1}{2}, \frac{1}{2b}, \frac{1-\tau}{4b} \right). \tag{2.5}$$

When aggregate output is $q$, the total surplus generated in the market is given by

$$S(q) = \left( 1 - \frac{Bq}{2} \right) q. \tag{2.6}$$

In the Latin market, the surplus at the Cournot equilibrium, $S_L^C$, is therefore

$$S_L^C = \frac{4}{9b}, \tag{2.7}$$

and the surplus at monopoly equilibria, $S_L^M$, is

$$S_L^M = \frac{3}{8b}. \tag{2.8}$$

Replacing $b$ with $\beta$ yields formulas analogous for the Cournot and monopoly equilibria in the Greek market. (These formulas assume that firms' constant marginal cost of production is zero). We use the notation $\chi^C$, $\rho^C$, $\Pi_G^C$, $S_G^C$, and $\chi^M$, $\rho^M$, $\Pi_G^M$, $S_G^M$, for the values of output, price, profits and surplus at the Cournot duopoly equilibrium, and monopoly equilibrium of the market, respectively.

### 2.3. One Set of Books

In this section, we assume that parents use the market price in the Latin market as the transfer price per intrafirm transaction, i.e., parents keep only *one set of books* to satisfy both cost and tax accounting requirements. Of course, this internal pricing scheme is consist with the ALP. We identify the subgame perfect equilibria (SPE henceforth) of the game. In this setup, parents act as "leaders" anticipating the reactions of subsidiary firms.

Assuming that the price in the Latin market is $p \geq 0$, each subsidiary $i \in \{1, 2\}$ chooses its output $\chi_i$ to solve

$$\max_{\chi_i \in \mathbb{R}_+} (1 - \tau - \Delta)\left(\rho^d\left(\chi_1 + \chi_2\right) - p\right)\chi_i.$$

Here $p$ is the constant marginal cost of the subsidiary firms.[19] Using the formulae (2.2), we calculate the equilibrium outputs and price for $p \geq 0$ as

$$\chi_1^* = \chi_2^* = \hat{\chi}(p) = \frac{1-p}{3\beta}.$$

(Note that in the game played by subsidiaries the equilibrium is unique.) The equilibrium outcome depends only on $p$, but do not depend directly on the tax rate in the Greek market $(\tau + \Delta)$.

Therefore, the equilibrium price in the Greek market is

$$\rho^* = \rho^d\left(2\hat{\chi}(p)\right) = \frac{1+2p}{3}.$$

A SPE of the game is profile of actions for parents 1 and 2, $(q_1^*, q_2^*)$, and a pair of functions describing the subsidiaries' strategies $\left(f_1^*\left(q_1^*, q_2^*\right), f_2^*\left(q_1^*, q_2^*\right)\right)$ such that parents maximize consolidated profits and subsidiaries maximize their own profits. Then in a SPE the subsidiaries' strategies are $f_i^*\left(q_1^*, q_2^*\right) = \hat{\chi}_i(p^d(q_1^*, q_2^*))$ for $i \in \{1, 2\}$, and parents, anticipating that subsidiaries' reactions are described by $(\hat{\chi}_1, \hat{\chi}_2)$, choose their actions in order to maximize consolidated profits $(\Pi_i^O)$. Thus, Parent $i$ chooses its output $q_i$ in order to solve

$$\max_{q_i \in \mathbb{R}_+} \Pi_i^O(q_1, q_2),$$

[19] Dürr and Göx (2011) assume that firms can arbitrarily choose a transfer price from an allowable exogenous range of ALP prices, withstanding a possible examination of authorities in the two markets.

where

$$\Pi_i^O(q_1, q_2) = \Pi_i(q_1, q_2, \hat{\chi}_1\left(p^d(q_1+q_2)\right), \hat{\chi}_2\left(p^d(q_1+q_2)\right)),$$

continue to be the same formula as given by (2.1).

The first-order condition for profit maximizing is

$$\begin{aligned}\frac{\partial \Pi_i^O}{\partial q_i} &= (1-\tau)\left(\frac{dp^d}{dq}q_i + p^d\right) + (1-\tau)\frac{dp^d}{dq}\left(\frac{\partial \hat{\rho}}{\partial p}\hat{\chi}_i + \frac{\partial \hat{\chi}_i}{\partial p}\hat{\rho}\right) + \\ &\quad +\Delta\frac{dp^d}{dq}\left(\hat{\chi}_i\left(1-\frac{\partial \hat{\rho}}{\partial p}\right) + \frac{\partial \hat{\chi}_i}{\partial p}(\hat{\rho}-p)\right) = 0.\end{aligned} \tag{2.9}$$

The expression in (2.9) comprises three different terms. In what follows, we refer to first term as Cournot marginal revenue, to second term as competition effect[20] and to the last term as tax effect[21]. Competition effect is a consequence of vertical separation (i.e., delegation).

In the absence of delegation and taxation, the optimal quantity in each market is found by equating Cournot marginal revenue with marginal cost (which in the model is zero). In particular, the equilibrium in both markets is just Cournot output.

The sign of competition effect depends on the price level in the Latin market and the sign of tax effect depends on the sign of $\Delta$:

For $p^d > \frac{3}{4}p^C$, the influence that competition effect has on the marginal profits of parent $i$ is positive from

$$(1-\tau)\frac{dp^d}{dq}\left(\frac{\partial \hat{\rho}}{\partial p}\hat{\chi}_i + \frac{\partial \hat{\chi}_i}{\partial p}\hat{\rho}\right) = (1-\tau)\frac{4s}{9}\left(p^d - \frac{3}{4}p^C\right),$$

in (2.9), so that the optimal quantity in this market, ceteris paribus, is above the Cournot output. Intuitively, this quantity increase is favorable because it reduces the Latin market

[20] Since Latin market price are observable, a parent takes into account that it can influence this price via its output decision in the Latin market. Thus, firms can use Latin market price strategically to affect output decisions for the external market. In this setting, a high Latin market price can be used to reduce the competition in the external market.

[21] If tax rates differ among jurisdiction, firms want to shift profits into the low tax jurisdictions by use of distorted transfer prices.

price and therefore, alleviates the double marginalization problem. Nevertheless, double marginalization problem remains (i.e., $p^d > 0$). By charging transfer prices above marginal cost (zero in this model) both parents can commit their subsidiaries to behave as softer competitors on the final product market. In this setting, a parent takes into account that it can influence its transfer price only via its output decision in the Latin market. Hence, a parent's output decision must internalize its impact on the transfer price of its subsidiary, and its subsidiary's rival. Therefore one set of books provides parents with an instrument to soften competition in the external market.

For $\Delta > 0$ the influence that tax effect has on the marginal profits of parent $i$ is negative from

$$\Delta \frac{dp^d}{dq} \left( \hat{\chi}_i \left( 1 - \frac{\partial \hat{\rho}}{\partial p} \right) + \frac{\partial \hat{\chi}_i}{\partial p} (\hat{\rho} - p) \right) = -\frac{2s}{9} \Delta \left( 1 - p^d \right),$$

in (2.9), so that the optimal output in this market, ceteris paribus, is lower than output in a setting without taxes (or with equal tax rates between markets). Intuitively, this quantity reduction is favorable because it increases the transfer price and every additional unit that is sold in the Greek market at a higher transfer price reduces the subsidiary's tax liabilities.

For $\Delta > 0$ the influence that tax effect has on the marginal profits of subsidiary $i$ is also negative. Intuitively, increasing $p$, given that tax effect in the Latin market is negative, acts as a marginal cost increase for subsidiaries. The opposite holds for $\Delta < 0$.

Solving the system of equations formed by the first-order condition of parents 1 and 2, we obtain their outputs

$$q_1^* = q_2^* = \frac{(1-\tau)(3b + 9\beta)}{b\left((1-\tau)(8b + 27\beta) + 4b\Delta\right)} := q^O. \tag{2.10}$$

The equilibrium price in the Latin market is

$$p^d(2q^O) = \frac{(1-\tau)(2b + 9\beta) + 4b\Delta}{(1-\tau)(8b + 27\beta) + 4b\Delta} := p^O.$$

Substituting the value of $p$ into equations $\hat{\chi}_i(p)$ and $\hat{\rho}(p)$ above, we obtain the subsidiaries' outputs,

$$\chi_1^* = \chi_2^* = \hat{\chi}(p^O) = \frac{(1-\tau)(2b + 6\beta)}{\beta\left((1-\tau)(8b + 27\beta) + 4b\Delta\right)} := \chi^O, \tag{2.11}$$

and the equilibrium price in the Greek market,

$$\rho^d(2\chi^O) = \frac{(1-\tau)(4b+15\beta)+4b\Delta}{(1-\tau)(8b+27\beta)+4b\Delta} := \rho^O.$$

Note that if the taxes differential was zero (i.e., $\Delta = 0$), this outcome would also be optimal in a setting without taxes and maximizing the gross or net profits leads to the same result. For $\Delta > 0$, the output in both markets decreases with $\Delta$. The opposite effect applies to the equilibrium quantity for $\Delta < 0$. Since prices in the Latin market increase with $\Delta$, parents save on tax payments by using one set of books.

In particular, if $\Delta = 0$ and using (2.4) we can rewrite the expression for firms' output in the Latin market (2.10) as

$$q^O = q^C + \frac{1}{3(8b+27\beta)}.$$

Likewise, using the equation (2.5) we can write the expression for firms' output in the Greek market (2.11) as

$$\chi^O = \frac{\chi^M}{2} - \frac{3}{4(8b+27\beta)}.$$

Thus, the output in the Latin market is above the Cournot output and the output in the Greek market is below the Cournot output. Note also that double marginalization imposed by ALP leads to an output in the Greek market that is below the monopoly output.

We have

$$\frac{\partial q^O}{\partial \beta} = -\frac{9}{(8b+27\beta)^2} < 0,$$

and

$$\frac{\partial \chi^O}{\partial b} = \frac{6}{(8b+27\beta)^2} > 0.$$

The output in the Latin (Greek) market decreases (increases) with $\beta$ ($b$). It is worthwhile responding to an increase of the Greek market size (i.e., a smaller $\beta$) with an increase of the output in the Latin market, thus reducing the transfer price (in order to alleviate the double marginalization problem) and avoiding a large reduction of the sales of the subsidiary.

The equilibrium output in the Latin market also satisfies

$$\lim_{\beta \to 0} q^O = q^C + \frac{1}{24b} := q_0^O,$$

and

$$\lim_{\beta \to \infty} q^O = q^C.$$

Thus, as the size of the Greek market becomes large (i.e., $\beta$ becomes small), the output in the Latin market is above the Cournot output. Parents' incentives to increase their output in order to alleviate double marginalization remain as the size of the Greek market becomes arbitrarily large. Of course, as the size of the Greek market becomes arbitrarily small (i.e., $\beta$ approaches infinity), parents tend to ignore the double marginalization problem (as the profits in this market become negligible), and focus on the impact on their output decision on the Latin market, and their output approaches the Cournot output.

The equilibrium output in the Greek market satisfies

$$\lim_{b \to \infty} \chi^O = \frac{\chi^M}{2},$$

and

$$\lim_{b \to 0} \chi^O = \chi^C - \frac{1}{9\beta} = \frac{\chi^M}{2} - \frac{1}{36\beta} := \chi_0^O.$$

Thus, as the size of the Latin market becomes arbitrarily small (i.e., $b$ approaches infinity), the revenues in this market become negligible, and parents' output decisions mainly serve the purpose of committing to high prices in the Greek market.

Interestingly, keeping one set of books (i.e., internal transfer prices are consistent with the ALP) allows parents to attain perfect cooperation (i.e., they are able to sustain the monopoly outcome) when $b$ approaches infinity. In this case, ALP is merely an instrument to avoid competition in the Greek market. When the size of the Latin market becomes arbitrarily large (i.e., $b$ approaches zero), however, revenues mainly come from the Latin market and therefore, parents tend to ignore the impact of double marginalization in the Greek market, producing the Cournot output in the Latin market. Double marginalization leads to an output in the Greek market that is below the monopoly output.

Let us study the total profits and total surplus under one set of books. Total profits can be calculated using (2.1) and (2.4) as

$$\Pi^O = \Pi_L^O + \Pi_G^O = \Pi_L^C + \Pi_G^C + \frac{2(1-\tau)}{9} \frac{4s^2 + 22s + 27}{\beta (8s + 27)^2}, \tag{2.12}$$

and the total surplus can be calculated using (2.6) and (2.7) as

$$S_L^O + S_G^O = S_L^C + S_G^C - \frac{2}{9} \frac{20s^2 + 155s + 297}{\beta \left(8s + 27\right)^2}.$$

We summarize these results in the following proposition.

**Proposition 2.1.** *If both firms use one set of books and* $\Delta = 0$, *then in a SPE:*

*(2.1.1) The output in the Latin market* $q^O$ *is above the Cournot outcome, and increases with the size of the Greek market* $\beta$, *i.e.,*

$$q^O > q^C \text{ and } \frac{\partial q^O}{\partial \beta} < 0,$$

*and the output in the Greek market* $\chi^O$ *is below the Cournot outcome, and decreases with the size of the Latin market* $b$, *i.e.,*

$$\chi^O < \chi^C \text{ and } \frac{\partial \chi^O}{\partial b} > 0.$$

*Further, as* $\beta$ *becomes large* $q^O$ *approaches* $q^C$, *and as* $\beta$ *becomes small* $q^O$ *approaches* $q_0^O$, *where* $q_0^O > q^C$. *And as* $b$ *becomes large* $\chi^O$ *approaches* $\chi^M/2$, *and as* $b$ *becomes small* $\chi^O$ *approaches* $\chi_0^O < \chi^C$, *where* $\chi_0^O < \chi^M/2$.

*(2.1.2) The total profits are above the total profits at the Cournot equilibrium.*

*(2.1.3) The total surplus is below the total surplus at the Cournot equilibrium.*

Keeping one set of books provides parent firms with an instrument to limit aggressive competition in the Greek market, and may allow them to encourage an outcome near the monopoly outcome when the size of the Greek market relative to that of the Latin market is large.[22] Of course, since a parent influences its transfer price only via its output decision in the Latin market, competition in this market is more aggressive and the output is above the Cournot output. Nevertheless, total profits are above at the Cournot profits. Thus, this

[22] Choe and Matsushima (2013) examine the effect of ALP on dynamic competition in imperfectly competitive markets and show that the ALP results in more stable tacit collusion. They consider a vertically related market with two upstream firms which supply to their downstream affiliates and other unrelated buyers in the downstream market. The authors consider the price the upstream firms charge to unrelated buyers as the comparable uncontrolled price for applying the ALP. In our setting, the price in the home market provides a reliable measure of an arm's length result.

accounting policy may provide a rationale for vertical separation. However, total surplus is below the surplus at the Cournot equilibrium, which raises some questions about the use of the ALP as a guideline for regulating transfer prices.

### 2.4. Two Sets of Books

We consider next the case where each parent uses *two sets of books* together with a transfer pricing regulation consistent with the ALP. In this scenario, subsidiary $i$'s taxable income is $(\rho^d(\chi_1+\chi_2)-p)\chi_i$, where $p$ is the price in the Latin market, whereas its gross profits are $\left(\rho^d(\chi_1+\chi_2)-t_i\right)\chi_i$, where $t_i$ is the internal transfer price that parent $i$ uses to allocate costs. Parent $i$'s consolidated net profits as a function of the outputs of parents and subsidiaries continue to be the same formula as given by (2.1), $\Pi_i(q_1,q_2,\chi_1,\chi_2)$. We identify the subgame perfect equilibria (SPE henceforth) of the game as follows.

Assuming that the price in the Latin market is $p \in R_+$ and internal transfer prices are $(t_1,t_2) \in R$, each subsidiary $i \in \{1,2\}$ chooses its output $\chi_i$ to solve

$$\max_{\chi_i \in \mathbb{R}_+} \left(\rho^d(\chi_1+\chi_2)-t_i\right)\chi_i-(\tau+\Delta)(\rho^d(\chi_1+\chi_2)-p)\chi_i.$$

Solving the system of equations formed by the first-order condition of subsidiaries 1 and 2, we calculate their equilibrium outputs as

$$\chi_1^*=\chi_2^*=\tilde{\chi}_1(p,t_1,t_2)=\tilde{\chi}_2(p,t_1,t_2)=\frac{1-\tau-\Delta+(\tau+\Delta)p-2t_i+t_{3-i}}{3\beta(1-\tau-\Delta)}.$$

(Note that in the game played by subsidiaries the equilibrium is unique.) The outcome in the Greek market depends on $p$, $t_i$ and $\tau+\Delta$. Therefore, the outcome depends on tax rate in the Greek market even if tax rates in both markets are identical; i.e., $\Delta=0$.

Assuming that $\tilde{\chi}_1(p,t_1,t_2)+\tilde{\chi}_2(p,t_1,t_1) \le \frac{1}{\beta}$, the market price is

$$\begin{aligned}\tilde{\rho}(p,t_1,t_2) &= \rho^d(\tilde{\chi}_1(p,t_1,t_2)+\tilde{\chi}_2(p,t_1,t_2)) \\ &= \frac{1-\tau-\Delta-2(\tau+\Delta)p+t_1+t_2}{3(1-\tau-\Delta)}.\end{aligned}$$

A SPE of the game is profile of actions for parents 1 and 2, $(q_1^*,q_2^*,t_1^*,t_2^*)$, and a pair of functions describing the subsidiaries' strategies $(f_1^*(q_1^*,q_2^*,t_1^*,t_2^*), f_2^*(q_1^*,q_2^*,t_1^*,t_2^*))$ such that parents maximize consolidated profits and subsidiaries maximize their own profits. Then in a SPE the subsidiaries' strategies are

$$f_i^*\left(q_1^*, q_2^*, t_1^*, t_2^*\right) = \tilde{\chi}_i(p^d(q_1^*, q_2^*), t_1^*, t_2^*) \text{ for } i \in \{1, 2\},$$

and parents, anticipating that subsidiaries' reactions are described by $(\tilde{\chi}_1, \tilde{\chi}_2)$, choose their actions in order to maximize consolidated profits $(\Pi_i^T)$. Thus, Parent $i$ chooses its output $q_i$ and its internal transfer price $t_i$ in order to solve

$$\max_{(t_i, q_i) \in \mathbb{R} \times \mathbb{R}_+} \Pi_i^T(q_1, q_2, t_1, t_2),$$

where

$$\Pi_i^T(q_1, q_2, t_1, t_2) = \Pi_i(q_1, q_2, \tilde{\chi}_1(p^d(q_1 + q_2), t_1, t_2), \tilde{\chi}_2(p^d(q_1 + q_2), t_1, t_2)).$$

Parent $i$'s first-order conditions for profit maximization are

$$\frac{\partial \Pi_i^T}{\partial t_i} = (1 - \tau - \Delta)\left(\frac{\partial \tilde{\rho}}{\partial t_i}\tilde{\chi}_i + \frac{\partial \tilde{\chi}_i}{\partial t_i}\tilde{\rho}\right) + \Delta\left(\frac{\partial \tilde{\chi}_i}{\partial t_i}p\right) = 0, \tag{2.13}$$

and

$$\begin{aligned} \frac{\partial \Pi_i^T}{\partial q_i} &= (1-\tau)\left(p + \frac{dp^d}{dq}q_i\right) + (1-\tau)\frac{dp^d}{dq}\left(\frac{\partial \tilde{\rho}}{\partial p}\tilde{\chi}_i + \frac{\partial \tilde{\chi}_i}{\partial p}\tilde{\rho}\right) + \\ &\quad + \Delta\frac{dp^d}{dq}\left(\tilde{\chi}_i\left(1 - \frac{\partial \tilde{\rho}}{\partial p}\right) + \frac{\partial \tilde{\chi}_i}{\partial p}(\tilde{\rho} - p)\right) = 0. \end{aligned} \tag{2.14}$$

The expression in (2.13) comprises two different terms. Using the same terminology as before, we refer to the first term as the competition effect on the internal transfer price $t_i$[23] and to the second term as tax effect on the internal transfer price $t_i$.

The sign of competition effect on the internal transfer price depends on the output level in the Greek market and the sign of tax effect on the internal transfer price depends on the sign of $\Delta$:

[23] Parent can use the internal transfer prices strategically to impact output decisions for the external market. In this setting, a low transfer price can be used to expand own market share in the external market.

For $\tilde{\chi}_i < \frac{6}{5}\chi^C$ the influence that the competition effect has on the marginal profits of parent $i$ is negative from

$$(1-\tau-\Delta)\left(\frac{\partial\tilde{\rho}}{\partial t_i}\tilde{\chi}_i + \frac{\partial\tilde{\chi}_i}{\partial t_i}\tilde{\rho}\right) = \frac{5}{3}\left(\tilde{\chi}_i - \frac{6}{5}\chi^C\right),$$

in (2.13), so that the internal transfer price $t_i$, ceteris paribus, is lower than the marginal cost (zero in this model).[24] Note $t_i$ is the constant marginal cost of the subsidiary firm. Intuitively, the internal transfer price $t_i$ is lower than the marginal cost in order to render each subsidiary into a low cost competitor that behaves aggressively by increasing its quantity. The transfer price that optimizes managerial incentives $t_i$ (which is a non market transfer pricing) opens up the possibility to gain a Stackelberg advantage in the Greek market. By reducing its internal transfer price below marginal cost, parents attempt to gain a kind of Stackelberg leader status, creating a short of prisoners' dilemma situation. As a consequence of the competition effect, the equilibrium outcome in the Greek market is more efficient than the Cournot outcome. Therefore in the absence of taxation, delegating output decision to subsidiaries encourages parents to compete more aggressively in the Greek market, relative to a setting in which parents exercise direct control of the subsidiary's output.

For $\Delta > 0$ the influence that tax effect has on the marginal profits of parent $i$ is also negative from

$$\Delta\left(\frac{\partial\tilde{\chi}_i}{\partial t_i}p\right) = -\Delta\frac{2p}{3\beta\left(1-\tau-\Delta\right)},$$

in (2.13), so that the internal transfer price $t_i$, ceteris paribus, is lower if Latin market offers a tax advantage over the Greek market. Intuitively, this cost reduction is favorable because it offsets the increase in its subsidiary's taxable income that occurs by the competition effect. The opposite holds for $\Delta < 0$.

The expression in (2.14) comprises three different terms. Again using the above terminology, we refer to first term as Cournot marginal revenue, to the second term as the competition effect on the output $q_i$ and to the third term as the tax effect on the output $q_i$.

[24] If internal transfer price was equal to the marginal cost, the outcome in the Greek market would be Cournot outcome.

The signs of competition effect and tax effect on the output $q_i$ depend on the output level in the Greek market and on $\Delta$, respectively:

For $\tilde{\chi}_i > \frac{3\chi^C}{4}$ the influence that the competition effect has on the marginal profits of parent $i$ is positive from

$$(1-\tau)\frac{dp^d}{dq}\left(\frac{\partial\tilde{\rho}}{\partial p}\tilde{\chi}_i + \frac{\partial\tilde{\chi}_i}{\partial p}\tilde{\rho}\right) = (1-\tau)\frac{4b(\tau+\Delta)}{3(1-\tau-\Delta)}\left(\tilde{\chi}_i - \frac{3\chi^C}{4}\right),$$

in (2.14), so that the output in the Latin market, ceteris paribus, is above the output at the Cournot equilibrium. Intuitively, this quantity increase is favorable because it rises the tax liability of its subsidiary's rival without affecting the marginal cost of its own subsidiary. Each parent can offset exactly its own tax liability increase, reducing its internal transfer price.

For $\Delta > 0$ the influence that the tax effect has on the marginal profits of parent $i$ is negative from

$$\begin{aligned} &\Delta\frac{dp^d}{dq}\left(\tilde{\chi}_i\left(1-\frac{\partial\tilde{\rho}}{\partial p}\right) + \frac{\partial\tilde{\chi}_i}{\partial p}\left(1-2\beta\tilde{\chi}_i - p\right)\right) \\ = \quad &-\Delta\frac{s(1-p)(\tau+\Delta) + 3b(1-\tau-\Delta)\tilde{\chi}_i}{3(1-\tau-\Delta)}, \end{aligned}$$

in (2.14), so that the output in this market, ceteris paribus, is lower if the Latin market is a tax heaven. Intuitively, this quantity reduction is favorable because it increases the transfer price and therefore reduces the firm's tax liabilities. The opposite holds for $\Delta < 0$.

Solving the system of equations formed by the first-order conditions of parents 1 and 2 we obtain their outputs and the internal transfer prices. In the Latin market, parents' outputs are

$$q_1^* = q_2^* = q^C + \frac{1}{\beta}\frac{d(\tau,\Delta)}{\theta(\tau,\Delta,s)} := q^T,$$

where

$$d(\tau,\Delta) = \tau(1-\tau) - \frac{\Delta}{3}\left(2(1-\tau-\Delta) + (1+\tau)\right),$$

and

$$\theta(\tau,\Delta,s) = 15(1-\tau)^2 - \Delta\left(15(1-\tau) - 2s(\tau-\Delta)\right).$$

Assuming that $2q^T \leq \frac{1}{b}$, the market price is

$$p^d(2q^T) = p^C - 2s\frac{d(\tau, \Delta)}{\theta(\tau, \Delta, s)} := p^T.$$

The equilibrium internal transfer prices are $t_1^* = t_2^* := t^T$, where

$$\begin{aligned} t^T &= -\frac{1}{5} - \frac{10\tau(1-\tau)(s\tau - 4(1-\tau))}{5\theta(\tau, \Delta, s)} \\ -\Delta &\frac{2(s(\Delta - \tau(6-5\Delta)) + 5(1-\tau)(4\tau - (1-\tau-\Delta)))}{5\theta(\tau, \Delta, s)}. \end{aligned}$$

Substituting the values $p^T$ and $t^T$ into equations above we obtain the subsidiaries' outputs

$$\chi_1^* = \chi_2^* = \tilde{\chi}_1(p^*, t_1^*, t_2^*) = \tilde{\chi}_2(p^*, t_1^*, t_2^*) = \frac{6}{5}\chi^C - \frac{\Delta}{\beta}\frac{\delta(\tau, \Delta, s)}{\theta(\tau, \Delta, s)} := \chi^T,$$

and market price in the Greek market,

$$\rho^d(2\chi^T) = \frac{3}{5}\rho^C + 2\Delta\frac{\delta(\tau, \Delta, s)}{\theta(\tau, \Delta, s)} := \rho^T,$$

where $\delta(\tau, \Delta, s) = \frac{4}{5}s(\tau - \Delta) - 2(1-\tau)$.

For $\Delta > 0$, the output in the Latin market decreases with $\Delta$ if $\tau < \frac{1}{2}$ (see Appendix 2.A for a proof of this assertion). Thus the tax effect on the output $q_i$ prevails over the competition effect. Tax incentives make a high price desirable and therefore, parents increase the market price in the home market by reducing their outputs. Since the Latin market price increases with $\Delta$, parents save on tax payments by using two sets of books.

Increased tax rates on the Greek market may have a pro-competitive effect in this market by encouraging lower internal transfer price. Thus the reduction in the internal transfer price may prevail over the increase of tax liabilities as a result of increased tax rates and prices in the Latin market. Whether or not output in the Greek market decreases with $\Delta$ depends on the size difference between markets and on the value of $\Delta$ (see Appendix 2.B for a proof of this assertion).

In particular assuming that $\Delta = 0$ and using again (2.4), we can rewrite the expression for firms' output in the Latin market as

$$q^T = \begin{cases} \frac{3}{2}q^C - \frac{1}{6b}\frac{(\tau) - s}{(\tau)} = q^C + \frac{1}{2\ (\tau)}\chi^C & \text{if } s < \quad (\tau) \\ \frac{3}{2}q^C & \text{if } s \geq \quad (\tau) \end{cases},$$

the output in the Greek market as

$$\chi^T = \frac{6}{5}\chi^C,$$

and the internal transfer prices as

$$t^T = -\frac{1}{5} - \frac{\tau}{3\ \ (\tau)}s + \frac{8}{15}\tau,$$

where $\ \ (\tau) = \frac{5}{2}\frac{(1-\tau)}{\tau}$ (the gray curve in Figure 2.2 is the graph of $\ \ $).

Thus, the outputs in the Latin market and the Greek market are above the output at the Cournot equilibrium. On the one hand, parents reduce their internal transfer prices below marginal cost in order to take advantage in the external market, creating a short of prisoners' dilemma. On the another hand, parents increase their output (i.e., reducing the market price in the home market) in order to increase their subsidiary's rival tax liability without affecting the marginal cost of their own subsidiaries.

We have

$$\frac{\partial q^T}{\partial \beta} = -\frac{1}{6\beta^2\ \ (\tau)} < 0,$$

and

$$\frac{\partial \chi^T}{\partial b} = 0,$$

Thus, the output in the Latin market decreases with $\beta$. Parents respond to an increase of the size of the Greek market (i.e., as $\beta$ becomes small) with an increase of the output in the Latin market, thus reducing the Latin market price, in order to raise the tax liability of its rival's subsidiary without affecting the marginal cost of its own subsidiary. The output in the Greek market is independent of the size $b$.

We have

$$\frac{\partial q^T}{\partial \tau} = \frac{1}{15\beta\,(1-\tau)^2} > 0,$$

and

$$\frac{\partial \chi^T}{\partial \tau} = 0.$$

The output in the Latin market increases with $\tau$. The higher tax rates are, the larger output in the Latin market is. Parents respond to an increase of tax with an increase of the output in the Latin market, thus reducing the Latin market price, in order to raise the tax liability of its rival's subsidiary without affecting the marginal cost of its own subsidiary. The output in the Greek market is independent of $\tau$.

Let us study the total profits and total surplus under two sets of books.

Firms' profits in the Latin and Greek markets can be calculated using (2.1) and (2.4) as

$$\Pi_L^T = \begin{cases} \Pi_L^C - \frac{\tau}{45\beta}\frac{s+ \ (\tau)}{(\tau)} & \text{if } s < \ (\tau) \\ 0 & \text{if } s \geq \ (\tau) \end{cases},$$

and $\Pi_G^T = \frac{18}{25}\Pi_G^C$, respectively. Therefore, total profits are

$$\Pi^T = \Pi_L^T + \Pi_G^T = \Pi_L^C - \frac{\tau}{45\beta}\frac{s + \ \ (\tau)}{(\tau)} + \frac{18}{25}\Pi_G^C \text{ if } s < \ \ (\tau)\,, \tag{2.15}$$

and

$$\Pi^T = \Pi_L^T + \Pi_G^T = \frac{18}{25}\Pi_G^C \text{ if } s \geq \ \ (\tau)\,. \tag{2.16}$$

The surplus in the Latin and Greek markets can be calculated using (2.6) and (2.7) as

$$S_L^T = \begin{cases} \frac{9}{8}S_L^C - \frac{1}{18b}\left(\frac{(\tau)-s}{(\tau)}\right)^2 = S_L^C + \frac{1}{18\beta}\frac{2 \ (\tau)-s}{(\tau)^2} & \text{if } s < \ (\tau) \\ \frac{9}{8}S_L^C & \text{if } s \geq \ (\tau) \end{cases},$$

and $S_G^T = \frac{27}{25}S_G^C$, respectively. Therefore, total surplus is

$$S^T = S_L^T + S_G^T = \frac{9}{8}S_L^C + \frac{27}{25}S_G^C - \frac{1}{18b}\left(\frac{(\tau) - s}{(\tau)}\right)^2 \text{ if } s < \ \ (\tau)\,, \tag{2.17}$$

and

$$S^T = S_L^T + S_G^T = \frac{9}{8}S_L^C + \frac{27}{25}S_G^C \text{ if } s \geq \ \ (\tau)\,. \tag{2.18}$$

We summarize these results in the following proposition.

**Proposition 2.2.** *If both firms use two sets of books and* $\Delta = 0$, *then in a SPE:*

*(2.2.1) The output in the Greek market is*

$$\chi^T = \frac{6}{5}\chi^C,$$

*and the output in the Latin market is*

$$q^T = \begin{cases} \frac{3}{2}q^C - \frac{1}{6b}\frac{(\tau)-s}{(\tau)} = q^C + \frac{1}{2 \ (\tau)}\chi^C & \text{if } s < \ (\tau) \\ \frac{3}{2}q^C & \text{if } s \geq \ (\tau) \end{cases}.$$

*Moreover, $q^T$ increases with $\beta$ and converges to the efficient outcome as $\beta$ becomes large.*
*(2.2.2) The total profits are below total profits at the Cournot equilibrium.*
*(2.2.3) The total surplus is above the total surplus at the Cournot equilibrium.*

In summary, keeping two sets of books adhering to the ALP generates a subtle link between markets that may intensify competition in both markets. On the one hand, each parent attempts to make the subsidiary a lower cost competitor, in order to gain a competitive advantage in the external market, by reducing its internal transfer price. On the another hand, each parent attempts to increase the tax liability of its subsidiary's rival, in order to gain a competitive advantage in the external market, by reducing the Latin market price (i.e. increasing its production). Therefore, using two sets of books opens the possibility to gain a competitive advantage in the external market by reducing own costs and increasing rival's one.

In the absence of the ALP, parents have also an incentive to employ below cost transfer prices in order to compel their subsidiaries to be more aggressive in the external market. However incentives in the home market are unchanged and the equilibrium outcome is just the Cournot outcome -see Lemus and Moreno (2011). Therefore, if both firms keep *two sets of books* together with a transfer pricing regulation consistent with the *ALP* competition intensifies in the external market relative to the equilibrium where both firms using transfer prices for tax purposes not linked to the external market price.

## 2.5. Asymmetric Accounting Policies

In this section we consider the case in which parent firms use asymmetric accounting policies. We assume that parent 1 uses the market price in the Latin market as the transfer price per intrafirm transaction, i.e., it keeps only *one set of books* to satisfy both cost and tax accounting requirements, while parent 2 uses *two sets of books*. Subsidiaries observe the price in the Latin market and the internal transfer policy before competing in quantities. We identify the subgame perfect equilibria (SPE henceforth) of the game. In this set up, parents act as "leaders" anticipating the reactions of the subsidiary firms. We assume throughout this section that $\Delta = 0$ (i.e., equal tax rates between markets).

Assuming that the price in the Latin market is $p \in R_+$, subsidiary 1 chooses its output $\chi_1$ to solve

$$\max_{\chi_1 \in \mathbb{R}_+} (1-\tau)\left(\rho^d\left(\chi_1+\chi_2\right)-p\right)\chi_1.$$

Subsidiary 2, knowing the internal transfer price used by its parent $t_2 \in R$ chooses its output $\chi_2$ to solve

$$\max_{\chi_2 \in \mathbb{R}_+} \left(\rho^d\left(\chi_1+\chi_2\right)-t_2\right)\chi_2 - \tau(\rho^d\left(\chi_1+\chi_2\right)-p)\chi_2.$$

Thus, the reaction functions of subsidiaries 1 and 2 are

$$R_1^\chi\left(\chi_2, p\right) = \max\left(\frac{1-p}{2\beta} - \frac{1}{2}\chi_2, 0\right),$$

and

$$R_2^\chi\left(\chi_1, p, t_2\right) = \max\left(\frac{1-p}{2\beta} + \frac{p-t_2}{2\beta\left(1-\tau\right)} - \frac{1}{2}\chi_1, 0\right),$$

respectively.

An equilibrium of the Greek market is a profile of the subsidiaries' outputs $(\overline{\chi}_1(p,t_2), \overline{\chi}_2(p,t_2))$ satisfying the system of equations

$$\begin{aligned} \chi_1 &= R_1^\chi\left(\chi_2, p\right), \\ \chi_2 &= R_2^\chi\left(\chi_1, p, t_2\right). \end{aligned}$$

Solving this system we get

$$\overline{\chi}_1(p,t_2) = \begin{cases} 0 & \text{if } t_2 < p - \left(1-\tau\right)\left(1-p\right), \\ \frac{(1-\tau)(1-p)-(p-t_2)}{3\beta(1-\tau)} & \text{if } p - \left(1-\tau\right)\left(1-p\right) < t_2 < p + \frac{(1-\tau)(1-p)}{2}, \\ \frac{1-p}{2\beta} & \text{if } t_2 > p + \frac{(1-\tau)(1-p)}{2}, \end{cases} \tag{2.19}$$

and

$$\overline{\chi}_2(p,t_2) = \begin{cases} \frac{1-p}{2\beta} + \frac{p-t_2}{2\beta(1-\tau)} & \text{if } t_2 < p - \left(1-\tau\right)\left(1-p\right), \\ \frac{(1-\tau)(1-p)+2(p-t_2)}{3\beta(1-\tau)} & \text{if } p - \left(1-\tau\right)\left(1-p\right) < t_2 < p + \frac{(1-\tau)(1-p)}{2}, \\ 0 & \text{if } t_2 > p + \frac{(1-\tau)(1-p)}{2}. \end{cases} \tag{2.20}$$

Note that in the game played by subsidiaries the equilibrium is unique.

A SPE of the game is profile of actions for parents 1 and 2, $(q_1^*, q_2^*, t_2^*)$, and a pair of functions describing the subsidiaries' strategies $(f_1^*(q_1, q_2, t_2), f_2^*(q_1, q_2, t_2))$ such that parents maximize consolidated profits and subsidiaries maximize their own profits. As discussed above, the subsidiaries' game has a unique equilibrium. Then in a SPE the subsidiaries' strategies are $f_i^*(q_1, q_2, t_2) = \overline{\chi}_i(p^d(q_1, q_2), t_2)$ for $i \in \{1, 2\}$, and parents, anticipating that subsidiaries' reactions are described by $(\overline{\chi}_1, \overline{\chi}_2)$, choose their actions in order to maximize consolidated profits $\left(\bar{\Pi}_i\right)$. Thus, Parent 1 chooses $q_1$ to solve

$$\max_{q_1 \in \mathbb{R}_+} \bar{\Pi}_1(q_1, q_2, t_2),$$

where

$$\bar{\Pi}_1(q_1, q_2, t_2) = \Pi_1(q_1, q_2, \overline{\chi}_1\left(p^d(q_1 + q_2), t_2\right), \overline{\chi}_2\left(p^d(q_1 + q_2)\right), t_2).$$

Denote by $R_1^q(t_2, q_2)$ the reaction function of Parent 1, i.e., the solution to Parent 1's profit maximization problem.

Likewise, Parent 2 chooses its output $q_2$ and its internal transfer price $t_2$ in order to solve

$$\max_{(t_2, q_2) \in \mathbb{R} \times \mathbb{R}_+} \bar{\Pi}_2(q_1, q_2, t_2),$$

where

$$\bar{\Pi}_2(q_1, q_2, t_2) = \Pi_2(q_1, q_2, \overline{\chi}_1\left(p^d(q_1 + q_2), t_2\right), \overline{\chi}_2\left(p^d(q_1 + q_2)\right), t_2).$$

Denote by $(R_2^q(q_1), R_2^t(q_1))$ the reaction functions of Parent 2, i.e., the solution to Parent 2's profit maximization problem.

Hence in a SPE of the game the profile of parents' actions, $(q_1^*, q_2^*, t_2^*)$, satisfy the system

$$\begin{aligned} q_1^* &= R_1^q(t_2^*, q_2^*), \\ q_2^* &= R_2^q(q_1^*), \\ t_2^* &= R_2^t(q_1^*). \end{aligned}$$

In an interior SPE, i.e., such that the outputs of parent and subsidiaries are positive, the subsidiaries' outputs are

$$\chi_1^* = \overline{\chi}_1\left(p^d(q_1^* + q_2^*), t_2^*\right) = \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) - (p^d(q_1^* + q_2^*) - t_2^*)}{3\beta(1-\tau)} > 0,$$

and

$$\chi_2^* = \overline{\chi}_2\left(p^d(q_1^*+q_2^*), t_2^*\right) = \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)+2(p^d(q_1^*+q_2^*)-t_2^*)}{3\beta\,(1-\tau)} > 0.$$

Using these formulae we can solve the system of equations formed by parents 1 and 2 reaction functions to obtain

$$\begin{aligned}
q_1^* &= \frac{(1-2\tau)\,s^2+2\,(5-4\tau)\,s+12\,(1-\tau)}{2b\,((1-2\tau)\,s+18\,(1-\tau))},\\
q_2^* &= -\frac{(1-2\tau)\,s^2+2\,(5-4\tau)\,s-12\,(1-\tau)}{2b\,((1-2\tau)\,s+18\,(1-\tau))},\\
t_2^* &= -\frac{(1-2\tau)\,((1-3\tau)\,s+12\,(1-\tau))}{2\,((1-2\tau)\,s+18\,(1-\tau))}.
\end{aligned}$$

We calculate the equilibrium price in the Latin market,

$$p^d(q_1^*+q_2^*) = \frac{(1-2\tau)\,s+6\,(1-\tau)}{(1-2\tau)\,s+18\,(1-\tau)}.$$

Substituting the values $t_2^*$ and $p^d(q_1^*+q_2^*)$ into the equations for $\chi_1^*$ and $\chi_2^*$ above we obtain the subsidiaries' outputs,

$$\begin{aligned}
\chi_1^* &= -\frac{1}{2\beta}\frac{(1-2\tau)\,s}{(1-2\tau)\,s+18\,(1-\tau)},\\
\chi_2^* &= \frac{1}{\beta}\frac{(1-2\tau)\,s+12\,(1-\tau)}{(1-2\tau)\,s+18\,(1-\tau)}.
\end{aligned}$$

For tax rates $\tau \in [0, 1/2)$, the equation above yields $\chi_1^* < 0$, and therefore an interior SPE does not exist.

**Proposition 2.3.** *Assume that parent firms use asymmetric accounting policies and $\Delta = 0$. If $\tau \in [0, 1/2)$, then an interior SPE does not exist.*

Since in almost all countries tax rates are below one half, we turn to studying the (corner) SPE that arise for $\tau \in [0, 1/2)$.[25] Let us be given a SPE. Note that a SPE is identified by $(q_1^*, q_2^*, t_2^*)$, since subsidiaries' outputs are given by

$$(\chi_1^*, \chi_2^*) = \left(\overline{\chi}_1\left(p^d(q_1^*+q_2^*), t_2^*\right), \overline{\chi}_2\left(p^d(q_1^*+q_2^*), t_2^*\right)\right)$$

[25] Auerbach et al (2008) present evidence on trends in corporation tax revenues and the industrial composition of revenues for the G7 countries (France, United Kingdom, Germany, Italy, Japan, United States and Canada) over the period 1979 to 2006. They show that statutory corporation tax rates have been falling across the G7 economies and provide some evidence of convergence to main rates between 30% to 40%.

in the equations (2.19) and (2.20). We establish some properties of SPE.

**Claim 2.1.** *If* $\chi_2^* > 0$, *then* $t_2^* < p^*$ *and* $t_2^* \in (-\frac{1}{2}, \frac{1}{4})$.

**Proof.** Assume that $\chi_2^* > 0$. If $\chi_1^* > 0$, then the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t\left(q_1^*, q_2^*\right) = -\frac{1 - \tau + (1 - 5\tau)\, p^d(q_1^* + q_2^*)}{4}.$$

Since $p^* = p^d(q_1^* + q_2^*) \geq 0$, then

$$\begin{aligned} t_2^* - p^* &= -\frac{1 - \tau + (1 - 5\tau)\, p^*}{4} - p^* \\ &= -\frac{1 - \tau}{4}\left(1 + 5p^*\right) < 0. \end{aligned}$$

Moreover, since $t_2^*$ increases with $\tau$ and $p^* \in (0, 1)$, then $t_2^* \in (-\frac{1}{2}, \frac{1}{4})$.

If $\chi_1^* = 0$, then the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t\left(q_1^*, q_2^*\right) = -\tau p^d(q_1^* + q_2^*).$$

Since $p^* = p^d(q_1^* + q_2^*) \geq 0$, then

$$t_2^* - p^* = -\left(1 - \tau\right) p^* < 0.$$

Moreover, since $p^* \in (0, 1)$ and $\tau \in [0, 1/2)$, then $t_2^* \in (-\frac{1}{2}, 0)$. □

**Claim 2.2.** *If* $q_2^* = 0$, *then* $q_1^* > 0$.

**Proof.** Assume $q_2^* = 0$. If $t_2^* < p^d(q_1^* + q_2^*) - (1 - \tau)\left(1 - p^d(q_1^* + q_2^*)\right)$, then $\chi_1^* = 0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q\left(t_2^*, 0\right) = q^M > 0.$$

If $p^d(q_1^* + q_2^*) - (1 - \tau)\left(1 - p^d(q_1^* + q_2^*)\right) < t_2^* < p^d(q_1^* + q_2^*) + \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right)}{2}$, then $\chi_1^*, \chi_2^* > 0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q\left(t_2^*, 0\right) = \frac{1}{2b}\frac{9\left(1 - \tau\right)^2 + \left(5\left(1 - 2\tau\right) + 3\tau^2\right) s + \left(1 + \tau\right) s t_2^*}{n(\tau, s)},$$

where $n(\tau, s) := 9\left(1 - \tau\right)^2 + \left(1 - 2\tau\right)\left(2 - \tau\right) s$. Note that $n(\tau, s) > 0$ on $[0, 1/2)$.

Since $\tau < 1/2$ by assumption, and $t_2^* > -\dfrac{1}{2}$ by Claim 2.1, we have

$$\begin{aligned} q_1^* &> \frac{1}{2b}\frac{9(1-\tau)^2 + \left(5(1-2\tau)+3\tau^2\right)s + (1+\tau)s(-\frac{1}{2})}{n(\tau,s)} \\ &= \frac{1}{2b}\frac{9(1-\tau)^2 + \frac{3s}{2}(1-2\tau)(3-\tau)}{n(\tau,s)} > 0. \end{aligned}$$

Finally, if $t_2^* > p^d(q_1^*+q_2^*) + \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_2^* = 0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q(t_2^*,0) = \frac{1}{b}\frac{s+2}{s+4} > 0. \square$$

**Claim 2.3.** $q_1^* > 0$.

**Proof.** Assume by way of contradiction that $q_1^* = 0$. If $t_2^* < p^d(q_1^*+q_2^*) - (1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)$, then $\chi_1^* = 0$, and therefore

$$\chi_2^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta} + \frac{p^d(q_1^*+q_2^*)-t_2^*}{2\beta(1-\tau)},$$

by equation (2.20). Since $q_2^* > 0$ by Claim 2.2, then the first-order conditions for Parent 2's profit maximization are

$$\begin{aligned} q_2^* &= \frac{1}{b}\frac{2(1-\tau)^2+\tau^2 s}{\tau^2 s + 4(1-\tau)^2} - \frac{1}{\beta}\frac{\tau}{\tau^2 s + 4(1-\tau)^2}t_2^*, \\ t_2^* &= \tau(1-bq_2^*). \end{aligned}$$

Solving this system of equations we get $(q_2^*, t_2^*, \chi_2^*) = (q^M, \frac{\tau}{2}, \chi^M)$. However,

$$q_1^* = R_1^q\left(\frac{\tau}{2}, q^M\right) = \frac{q^M}{2} > 0,$$

contradicting that $q_1^* = 0$.

If $p^d(q_1^*+q_2^*) - (1-\tau)\left(1-p^d(q_1^*+q_2^*)\right) < t_2^* < p^d(q_1^*+q_2^*) + \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_1^*, \chi_2^* > 0$, and therefore

$$\begin{aligned} \chi_1^* &= \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right) - \left(p^d(q_1^*+q_2^*)-t_2^*\right)}{3\beta(1-\tau)}, \\ \chi_2^* &= \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right) + 2\left(p^d(q_1^*+q_2^*)-t_2^*\right)}{3\beta(1-\tau)}. \end{aligned}$$

by equations (2.19) and (2.20). Since $q_2^* > 0$ by Claim 2.2, then the first-order conditions for Parent 2's profit maximization are

$$
\begin{aligned}
q_2^* &= \frac{1}{2b}\frac{s\left(5\left(1-2\tau\right)+3\tau^2-3\left(3-5\tau\right)\right)+9\left(1-\tau\right)^2}{g\left(\tau,s\right)}+\frac{1}{2\beta}\frac{1-5\tau}{g\left(\tau,s\right)}t_2^*,\\
t_2^* &= -\frac{1-3\tau}{2}+\frac{b}{4}\left(1-5\tau\right)q_2^*,
\end{aligned}
$$

where $g\left(\tau,s\right) := 9\left(1-\tau\right)^2-\left(1-2\tau\right)\left(1+\tau\right)s$.

Solving this system of equations we get

$$
\left(q_2^*, t_2^*, \chi_1^*, \chi_2^*\right) = \left(\frac{2}{b}\frac{2-s}{8-s}, \frac{\tau\left(7+s\right)-3}{8-s}, -\frac{1+s}{\beta\left(8-s\right)}, \frac{6}{\beta\left(8-s\right)}\right).
$$

Hence either $\chi_1^* < 0$ or $\chi_2^* < 0$, and therefore such a profile cannot be an SPE.

If $t_2^* > p^d(q_1^*+q_2^*)+\frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_2^* = 0$, and therefore

$$
\chi_1^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta},
$$

by equation (2.19). Since $q_2^* > 0$ by Claim 2.2, then the first-order condition for Parent 2's profit maximization yields

$$
q_2^* = q^M.
$$

However,

$$
q_1^* = R_1^q\left(q^M\right) = \frac{1}{2b}\frac{s+2}{s+4} > 0,
$$

contradicting that $q_1^* = 0$. □

**Claim 2.4.** $\chi_1^* > 0$.

**Proof.** Assume by way of contradiction that $\chi_1^* = 0$. Then

$$
\chi_2^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta}+\frac{p^d(q_1^*+q_2^*)-t_2^*}{2\beta\left(1-\tau\right)} > 0,
$$

by equation (2.20). Since $q_1^* > 0$ by Claim 2.3, the first-order condition for Parent 1's profit maximization yields

$$
q_1^* = \frac{1-bq_2^*}{2b},
$$

and the first-order conditions for Parent 2's profit maximization yield the system

$$\begin{aligned} q_2^* &= \max\left(0, \frac{1}{b}\frac{2\left(1-\tau\right)^2+\tau^2 s}{\tau^2 s+4\left(1-\tau\right)^2} - \frac{1}{\beta}\frac{\tau}{\tau^2 s+4\left(1-\tau\right)^2}t_2^* - \frac{2\left(1-\tau\right)^2+\tau^2 s}{\tau^2 s+4\left(1-\tau\right)^2}q_1^*\right), \\ t_2^* &= \tau\left(1-b\left(q_1^*+q_2^*\right)\right). \end{aligned}$$

Solving this system of equations we get $(q_1^*, q_2^*, t_2^*) = \left(q^C, q^C, \frac{\tau}{3}\right)$. Substituting these values into equation (2.19) yields

$$\chi_1^* = \bar{\chi}_1\left(p^d(2q^C), \frac{\tau}{3}\right) = \frac{1}{12\beta} > 0,$$

contradicting that $\chi_1^* = 0$. □

**Claim 2.5**. $\chi_2^* > 0$.

**Proof**. Assume by way of contradiction that $\chi_2^* = 0$. Then

$$\chi_1^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta} > 0,$$

by equation (2.19). Since $q_1^* > 0$ by Claim 2.3, the first-order condition for profit maximization of parents 1 and 2 yield the system

$$\begin{aligned} q_1^* &= \frac{1}{b}\frac{s+2}{s+4} - \frac{s+2}{s+4}q_2^*, \\ q_2^* &= \max\left(0, \frac{1}{2b}\left(1-bq_1^*\right)\right). \end{aligned}$$

Solving this system of equations we get

$$(q_1^*, q_2^*) = \left(\frac{s+2}{b\left(s+6\right)}, \frac{2}{b\left(s+6\right)}\right).$$

In a SPE, the level of output $q_2^* = 2/b\left(s+6\right) > 0$ must maximize Parent 2's profit taking as given $q_1^* = \frac{s+2}{b(s+6)}$ and the subsidiaries' reactions $(\bar{\chi}_1, \bar{\chi}_2)$. Then $q_2^*$ solves the system given by the first-order conditions for Parent 2's profit maximization

$$\begin{aligned} q_2^* &= \frac{1}{2b}\frac{2s\left(13\tau-5\left(2-\tau^2\right)\right)+36\left(1-\tau\right)^2-s^2\left(1-\tau\right)\left(2-\tau\right)}{\left(s+6\right)g\left(\tau,s\right)} + \frac{1}{2\beta}\frac{1-5\tau}{g\left(\tau,s\right)}t_2^*, \\ t_2^* &= -\frac{1}{4}\frac{2\left(5-13\tau\right)+s\left(1-\tau\right)}{s+6} + \frac{b}{4}\left(1-5\tau\right)q_2^*. \end{aligned}$$

The solution to this system is

$$\hat{q}_2 = \frac{s\,(s+10)-16}{b\,(s-8)\,(s+6)}.$$

For $s > 0$, $\hat{q}_2 \neq 2/b\,(s+6)$, which leads to a contradiction. Hence $\chi_2^* > 0$. □

With these results in hand, we can now identify the parameter values of $\tau$ and $s = b/\beta$ for which a pure strategy SPE exists, and identify the equilibrium outputs and profits. Define $l(\tau) := 3\,(1-\tau)\,/\,(2-\tau)$, and $h(\tau) := 12\,(1-\tau)\,/\,(1+\tau)$. The functions $l$ and $h$ are both decreasing, and $l(\tau) < h(\tau)$ on $[0,1]$ – in Figure 2.1 the thin (resp. thick) curve is the graph of $l$ (resp. $h$). Also write $r(\tau, s) := (5-7\tau)\,s + 24\,(1-\tau)$. Note that $r(\tau, s) > 0$ on $[0, 1/2)$.

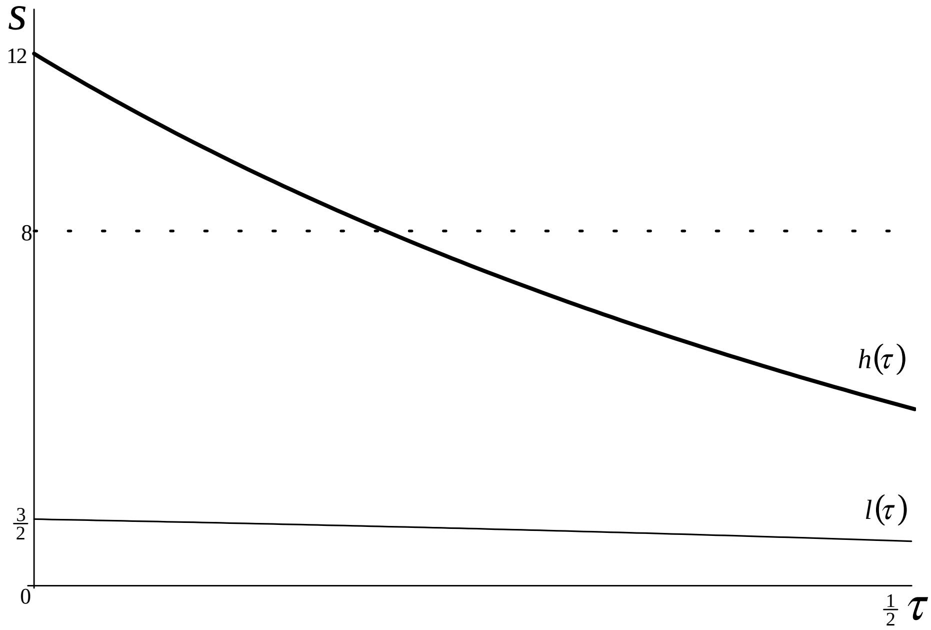


Figure 2.1. Functions $l$ and $h$.

**Proposition 2.4**: *Assume that $\tau < 1/2$ and $\Delta = 0$. If Parent 1 uses one set of books and Parent 2 uses two sets of books, then a unique (pure strategy) SPE exist whenever $l(\tau) < s < 8$, whereas no (pure strategy) SPE exists otherwise. Moreover:*

*(2.4.1) If $l(\tau) < s < \min\{h(\tau), 8\}$, then the outputs of parents and subsidiaries in the unique SPE are*

$$(q_1^*, q_2^*) = \left(2q^C + \frac{4(2-\tau)}{3b}\frac{s - l(\tau)}{r(\tau,s)}, 0\right),$$

*and*

$$(\chi_1^*, \chi_2^*) = \left(\frac{3}{4}\left(\chi^C - \frac{(1+\tau)}{\beta}\frac{h(\tau)-s}{r(\tau,s)}\right), \frac{3}{2}\left(\chi^C + \frac{(1+\tau)}{3\beta}\frac{h(\tau)-s}{r(\tau,s)}\right)\right),$$

*and parents' profits are*

$$\begin{aligned}
\Pi_1^* &= \frac{9}{16}\Pi_G^C - \frac{\left(1-\tau^2\right)\left((7-17\tau)s^2 - 12(1-\tau)(s+16)\right)}{16b}\frac{h(\tau)-s}{r(\tau,s)^2}, \\
\Pi_2^* &= \frac{9}{8}\Pi_G^C + \frac{3\left(1-\tau^2\right)\left((3-5\tau)s + 20(1-\tau)\right)}{8\beta}\frac{h(\tau)-s}{r(\tau,s)^2}.
\end{aligned}$$

*(2.4.2) If $h(\tau) \leq s < 8$, then the outputs of parents and subsidiaries are*

$$(q_1^*, q_2^*, \chi_1^*, \chi_2^*) = \left(3q^C, 0, \frac{3}{4}\chi^C, \frac{3}{2}\chi^C\right),$$

*and the parents' profits are*

$$(\Pi_1^*, \Pi_2^*) = \left(\frac{9}{16}\Pi_G^C, \frac{9}{8}\Pi_G^C\right).$$

**Proof.** Since $q_1^*$,$\chi_1^*$,$\chi_2^* > 0$, by claims 2.3, 2.4 and 2.5, and since by Proposition 2.3 there is no SPE such that these inequalities and $q_2^* > 0$ hold, then in a (pure strategy) SPE, when it exists, we have $q_2^* = 0$. Since $\chi_1^*$,$\chi_2^* > 0$ by claims 2.4 and 2.5, then

$$\begin{aligned}
\chi_1^* &= \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) - (p^d(q_1^* + q_2^*) - t_2^*)}{3\beta(1-\tau)}, \\
\chi_2^* &= \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) + 2(p^d(q_1^* + q_2^*) - t_2^*)}{3\beta(1-\tau)},
\end{aligned}$$

by equations (2.19) and (2.20). Since $q_1^* > 0$ by Claim 2.2 and $q_2^* = 0$, the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q(t_2^*, 0) = \frac{1}{2b}\frac{s\left(5(1-2\tau) + 3\tau^2\right) + 9(1-\tau)^2}{n(\tau,s)} + \frac{1}{2\beta}\frac{1+\tau}{n(\tau,s)}t_2^*,$$

and the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t(q_1^*, 0) = -\frac{1-3\tau}{2} + \frac{b}{4}(1-5\tau)q_1^*.$$

Solving this system of equations we get

$$q_1^* = \frac{1}{b} - \frac{(1+\tau)}{b}\frac{h(\tau)-s}{r(\tau,s)},$$

$$t_2^* = -\frac{1-\tau}{4} - \frac{1+\tau}{4}\frac{h(\tau)-s}{r(\tau,s)}(1-5\tau),$$

$$\chi_1^* = \frac{(2-\tau)}{\beta}\frac{s-l(\tau)}{r(\tau,s)} \text{ and}$$

$$\chi_2^* = \frac{2}{\beta}\frac{(1-2\tau)\,s+9\,(1-\tau)}{r(\tau,s)}.$$

(These values for $q_1^*$, $\chi_1^*$ and $\chi_2^*$ can be readily rewritten using the formulae given in (4.1) of Proposition 2.4.) Thus, $\chi_1^* \leq 0$ whenever $s \leq l(\tau)$. Since in equilibrium $\chi_1^* > 0$ by Claim 2.5, then a SPE does not exist whenever $s \leq l(\tau)$. Assume that $l(\tau) < s$. The equilibrium prices in the Latin is

$$p^* = p^d(q_1^*) = \frac{h(\tau)-s}{(1+\tau)\,r(\tau,s)}.$$

Thus, in order for $p^* > 0$ we must have $s < h(\tau)$. Assume that $h(\tau) > s$. The equilibrium price in the Greek markets is

$$\rho^* = \rho^d(\chi_1^* + \chi_2^*) = \frac{(1-2\tau)\,s+9\,(1-\tau)}{r(\tau,s)} > 0.$$

In order to verify that the profile identified is SPE we need to show that the level of output $q_2^* = 0$ maximizes Parent 2's profits taking $q_1^*$ as given. The system given by the first-order conditions for Parent 2's profit maximization is

$$q_2^* = \frac{1}{2b}\frac{3\,(1-\tau)\left(36\,(1-\tau)^2 - s\,(27-\tau\,(32+11\tau)) + s^2\tau\left(\frac{29}{3}-\tau\right)\right) - 8s^2}{g(\tau,s)\,r(\tau,s)} + \frac{1}{2\beta}\frac{1-5\tau}{g(\tau,s)}t_2^*,$$

$$t_2^* = -\frac{s\,(1-\tau\,(2-3\tau)) + 3\,(3-7\tau)\,(1-\tau)}{r(\tau,s)} + \frac{b}{4}(1-5\tau)\,q_2^*.$$

Solving this system we get

$$\bar{q}_2 = \frac{1}{b}\frac{4s\,(s+10) - 8s\tau\,(s+4) - 48\,(1-\tau)}{(s-8)\,r(\tau,s)}.$$

In order for $\bar{q}_2 \leq 0$ we must have

$$(\tau) := \frac{\sqrt{37-4\tau\,(19-10\tau)} - 5 + 4\tau}{1-2\tau} \leq s \leq 8.$$

Since $s > l(\tau) > \quad (\tau)$ on $[0, 1/2)$, for $\bar{q}_2 \leq 0$ we must have $s < 8$. In summary, the profile of parents and subsidiaries' outputs as well as the transfer price of parent 2 identified above forms a SPE when $l(\tau) < s < h(\tau)$ and $s < 8$, i.e., $l(\tau) < s < \min\{h(\tau), 8\}$. Thus, when this is the case there is a unique SPE and it is given by the formulae given in (4.1) of Proposition 2.4.

Now suppose that $s \geq h(\tau)$. Then in equilibrium $p^* = 0$, and therefore $q_1^* \geq \frac{1}{b}$. Then the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t\left(\frac{1}{b}, 0\right) = -\frac{1-\tau}{4},$$

and therefore,

$$(\chi_1^*, \chi_2^*) = (\frac{3}{4}\chi^C, \frac{3}{2}\chi^C),$$

by equations (2.19) and (2.20). The equilibrium price in the Greek markets is

$$\rho^* = \rho^d(\chi_1^* + \chi_2^*) = \frac{3}{4}\rho^C.$$

In order for $q_2^* = 0$ to maximize the profits of Parent 2 taking as given $q_1^* = \frac{1}{b}$, the solution to the system defined the first-order conditions,

$$\begin{aligned} q_2^* &= -\frac{1-\tau}{2\beta}\frac{2-\tau}{g(\tau, s)} + \frac{1}{2\beta}\frac{1-5\tau}{g(\tau, s)}t_2^*, \\ t_2^* &= -\frac{1-\tau}{4} + \frac{b}{4}(1-5\tau)\, q_2^*. \end{aligned}$$

Solving this system of equations we get

$$\tilde{q}_2 = \frac{1}{\beta}\frac{1}{s-8}.$$

For $\tilde{q}_2 \leq 0$ we must also have $s < 8$. Hence the profile of outputs and transfer price define above forms a SPE when $h(\tau) < s < 8$.

Finally, if $s \leq l(\tau)$, then $\chi_1^* \leq 0$, and since in equilibrium $\chi_1^* > 0$ by Claim 2.5, then a SPE does not exist. And if $s \geq 8$, then whether $s < h(\tau)$, or $s \geq h(\tau)$ neither of the two candidate equilibria identified are SPE, and therefore a pure strategy SPE does not exist either.

The parents' equilibrium profits for the cases $l(\tau) < s < \min\{h(\tau), 8\}$ and $h(\tau) < s < 8$ are readily obtained simply by substituting parents' and subsidiaries' outputs into the formulae of the consolidated profits. □

In an equilibrium in which parents use asymmetric accounting policies, the parent that uses one set of books, say Parent 1, has an incentive to increase its output in order to alleviate double marginalization (i.e., to decrease the cost of its subsidiary), whereas the parent that uses two sets of books, Parent 2, decreases its output all the way to zero in order to increase the cost of its subsidiary's rival. Thus Parent 1 becomes the dominant producer in the home market. Since $t_2^* < p^*$ by claims 2.1 and 2.5, Subsidiary 2 becomes the dominant producer in the external market. The equilibrium profits of Parent 2 uses two sets of books dominate equilibrium profits of Parent 1 uses one set of books (i.e., $\Pi_2^* > \Pi_1^*$, see Appendix 2.C).

Assume that $l(\tau) < s < \min\{h(\tau), 8\}$. Then the total output in the Latin market satisfies

$$q_1^* + q_2^* = q_1^* = 2q^C + \frac{4\,(2-\tau)}{3b}\frac{s-l(\tau)}{r(\tau,s)} > 2q^C,$$

and the total output in the Greek market satisfies

$$\chi_1^* + \chi_2^* = 2\chi^C + \frac{2-\tau}{3\beta}\frac{s-l(\tau)}{r(\tau,s)} > 2\chi^C.$$

Hence the surplus in both markets is above the surplus at the Cournot equilibrium, i.e., $S_L^* > S_L^C$ and $S_G^* > S_G^C$. Since $S_L^C > S_L^O$ and $S_G^C > S_G^O$ by Proposition 2.1, the surplus in both markets is above under one set of books. In the Appendix 2.D we show that $S_L^* + S_G^* < S^T$ and therefore, the total surplus is below under two sets of books.

We have

$$\frac{\partial q_1^*}{\partial \beta} = -\frac{12\,(1-\tau)}{\beta^2 r(\tau,s)^2}\,(7-5\tau) < 0,$$

and

$$\frac{\partial\,(\chi_1^* + \chi_2^*)}{\partial b} = \frac{3\,(1-\tau)}{\beta^2 r(\tau,s)^2}\,(7-5\tau) > 0.$$

Thus, the output in the Latin (Greek) market decreases (increases) with $\beta$ ($b$). Parent 1 responds to an increase of the size of the Greek market (i.e., a smaller value of $\beta$) with an increase of the output in the Latin market, thus reducing the Latin market price (in order to alleviate the double marginalization problem) and avoiding a large reduction of the sales of its subsidiary. The market share of subsidiary 2 increases with the size of the Latin since its output decreases with $b$. Subsidiary 1 is more (less) aggressive competitor in the Greek market as the profits in the Latin market become negligible (large).

Also we have

$$\frac{\partial q_1^*}{\partial \tau} = 12\frac{s+2}{\beta r(\tau,s)^2} > 0,$$

and

$$\frac{\partial\left(\chi_1^* + \chi_2^*\right)}{\partial \tau} = 3s\frac{s+2}{\beta r(\tau,s)^2} > 0.$$

The output in the Latin market of parent 1 increases with $\tau$. The higher tax rates are, the larger output in the Latin market of parent 1 is. This occurs because a larger the Latin market output (to compensate for $q_2^* = 0$) tends to reduce the difference between the tax bill paid at the Latin and the Greek markets. The output of subsidiary 1 (2) increases (decreases) with $\tau$. Parent 1 responds to an increase of tax with an increase of the output in the Latin market, thus reducing the Latin market price. A decrease in the Latin market price encourages the subsidiary 1 to behave more aggressively by expanding its output in the Greek market and thus causes subsidiary 2 to become less aggressive by reducing its outcome.

If $h(\tau) < s < 8$, then

$$q_1^* + q_2^* = q_1^* = 3q^C > 2q^C,$$

and

$$\chi_1^* + \chi_2^* = \frac{9}{4}\chi^C > 2\chi^C.$$

Hence the surplus in both markets is above the surplus at the Cournot equilibrium, i.e., $S_L^* > S_L^C$ and $S_G^* > S_G^C$. Since $S_L^C > S_L^O$ and $S_G^C > S_G^O$ by Proposition 2.1, the surplus in both markets is above under one set of books. In the Appendix 2.D we show that $S_L^* + S_G^* < S^T$ and therefore, the total surplus is below under two sets of books.

Of course, our results would be symmetric if Parent 1 uses two sets of books and Parent 2 uses one set of books. Henceforth we use the superscripts $\bar{O}T$ and $O\bar{T}$ to refer to the outputs and profits of the firm using one and two sets of books, respectively, in a situation where parents use asymmetric accounting policies; i.e., $q^{\bar{O}T} = q_1^*$, $\chi^{\bar{O}T} = \chi_1^*$ and $\Pi^{O\bar{T}} = \Pi_1^*$, whereas $q^{O\bar{T}} = q_2^*$, $\chi^{O\bar{T}} = \chi_2^*$ and $\Pi^{O\bar{T}} = \Pi_2^*$, where the star values are those given in Proposition 2.4 above.

### 2.6. Endogenizing the Choice of Accounting Policies

We now turn to study parents' choice of accounting policies. We assume that parents can commit to keeping either *one set of books* or *two sets of books.* This assumption is reasonable if, for example, the costs associated with changing the accounting policy are sufficiently high. Göx (2000) notes that a new accounting policy usually requires substantial investments in developing or acquiring software and in training employees and/or hiring consultants. By choosing to keep one set of books, a parent commits to using the Latin market price as the transfer price per intrafirm transaction, regardless of its competitor actions. Likewise, by choosing to keep two sets of books, a parent commits to using an internal transfer price to allocate costs, whatever action of its competitor.

In section 2.3, we identified the parents' profits when both parents choose one set of books, $\Pi^O$ and in section 2.4, we identified the parents' profits when both parents choose two sets of books, $\Pi^T$. Likewise, in section 2.5 we identified the profits in a (pure strategy) SPE when parents choose asymmetric accounting policies, $\Pi^{\overline{O}T}$ and $\Pi^{O\overline{T}}$, where the superscripts $\bar{O}T$ and $O\bar{T}$ refer to the parent using one and two sets of books, respectively. Thus, at the stage of choosing their accounting policies parents, assuming that following their decisions a (pure strategy) SPE follows, parents' payoffs are described by the following matrix:

| | $O_2$ | $T_2$ |
|---|---|---|
| $O_1$ | $\Pi^O, \Pi^O$ | $\Pi^{\bar{O}T}, \Pi^{O\bar{T}}$ |
| $T_1$ | $\Pi^{O\bar{T}}, \Pi^{\bar{O}T}$ | $\Pi^T, \Pi^T$ |

Table 2.1: Parents' Choice of Accounting Policies.

We study the equilibria of this game. Recall from Proposition 2.2 the function defining $\Pi^T$ differ in the space $(\tau, s)$ depending on the sign of the inequality $s \gtreqless \quad (\tau)$. Likewise, from

Proposition 2.4 the functions defining $\Pi^{\bar{O}T}$ and $\Pi^{O\bar{T}}$ differ in the space $(\tau, s)$ depending on the sign of the inequality $s \gtreqless h(\tau)$. In Appendix 2.E we study the sign of $\Pi^{T} - \Pi^{\bar{O}T}$, which is the profit gain or loss to a parent that *deviates* to choosing *one set of books* from a situation where both parents choose *two sets of books.* In Appendix 2.F we study the sign of $\Pi^{O} - \Pi^{O\bar{T}}$, which is the profit gain or loss to a parent that *deviates* to choosing *two sets of books* from a situation where both parents choose *one set of books.*

On the parameter space $(\tau, s)$ parents' profits configure the game described in Table 2.1 as a *prisoners' dilemma* (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books), a *game of chicken* (with one parent choosing one set of books and the other parent choosing two sets of books), a *coordination game* (in which both parents choose two sets of books or both parents choose one set of books), or even to a *cooperation game* with a unique Pareto efficient Nash equilibrium (in which both parents choose one set of books).

In Appendixes 2.3 and 2.4 we show that $\Pi^{T} > \Pi^{\bar{O}T}$ and $\Pi^{O} > \Pi^{O\bar{T}}$ whenever $\min\{\ \ (\tau), h(\tau)\} < s < 8$. In this region, characterized by relatively high tax rates ($\tau > \frac{1}{5}$) and a large value of the size of the Greek market relative to that of the Latin ($s > \frac{5}{2}$), the game in Table 2.1 is a Coordination Game *(CO)* that has two pure strategy Nash equilibria, one in which both parents choose two sets of books, and another one in which choose one set of books.

When $s < \min\{\ \ (\tau), h(\tau)\}$ identifying the signs of $\Pi^{T} - \Pi^{\bar{O}T}$ and $\Pi^{O} - \Pi^{O\bar{T}}$ is cumbersome. We show that if $l(\tau) < s < 1.385$, then $\Pi^{T} < \Pi^{\bar{O}T}$. If $1.385 < s < \min\{\ \ (\tau), h(\tau)\}$ and the tax rates are not too high, then $\Pi^{T} > \Pi^{\bar{O}T}$ (see Figure 2.3 in Appendix 2.E). We also show that if $1.23 < s < 2.26$, then $\Pi^{O} < \Pi^{O\bar{T}}$. If $l(\tau) < s < 1.23$ or $2.26 < s < \min\{\ \ (\tau), h(\tau)\}$, there is a critical value $\widetilde{\tau}$ such that $\Pi^{O} \lesseqgtr \Pi^{O\bar{T}}$ whenever $\tau \lesseqgtr \widetilde{\tau}$ (see Figure 2.4 in Appendix 2.F). These results allow to identify the possible types of the game that Table 2.1 may give rise depending of the values of $s$ and $\tau$:

(i) If $l(\tau) < s < 1.23$, then $\Pi^{T} < \Pi^{\bar{O}T}$, and Table 2.1 describes either a Game of Chicken *(CH)* (when $\Pi^{O} < \Pi^{O\overline{T}}$) or a Cooperation Game *(CP)* (when $\Pi^{O} > \Pi^{O\overline{T}}$) depending on whether the value of $\tau$ is high or very high, respectively. In a *CH* game there are two pure strategy Nash equilibria, in these equilibria one firm uses one set of books and the other uses two sets of books. In a *CP* game it is a dominant strategy for both firms to use one set of books.

(ii) If $1.23 < s < 1.385$, then $\Pi^{O} < \Pi^{O\bar{T}}$ and $\Pi^{T} < \Pi^{\bar{O}T}$, and hence Table 2.1 describes a *CH* game.

(iii) If $1.385 < s < 2.26$, then $\Pi^{O} < \Pi^{O\bar{T}}$, and the game in Table 2.1 is either a Prisoners' Dilemma game *(PD)* (when $\Pi^{T} > \Pi^{\bar{O}T}$) or a *CH* game (when $\Pi^{T} < \Pi^{\bar{O}T}$), depending on whether $\tau$ is low or high, respectively. In a *PD* game keeping two sets of books is the unique equilibrium (and is in dominant strategies).

(iv) If $2.26 < s < \min\{\ \ (\tau), h(\tau)\}$, then there are parameter constellations such that $\Pi^{T} \lesseqgtr \Pi^{\bar{O}T}$ and/or $\Pi^{O} \lesseqgtr \Pi^{O\bar{T}}$. In this case, all four types of games (*PD*, *CO*, *CH* and *CP*) may emerge as the tax rate $\tau$ increases from low, to intermediate, to high values.

In Figure 2.2 below, the gray curve is the graph of the function  , the thin curve is the graph of the function $l$ and thick curve is the graph of the function $h$. The figure indicates the regions of parameters for which the game of Table 2.1 is a member of the different classes in the taxonomy described above. A *PD* game arises for low tax rates if $s > 1.385$. For high tax rates (i.e., for value of $\tau$ near $\frac{1}{2}$) a *CP* game arises when the Latin market is not too small relative to the Greek market (i.e., when $s$ is near 1), and a *CO* arises when the Latin market is significantly smaller than the Greek market. For intermediate tax rates *CH* game arises when the Latin market is not too small relative to the Greek market, and a *CO* arises when the Latin market is significantly smaller than the Greek market.

We summarize these results in Proposition 2.5 below.

**Proposition 2.5:** *Assume that $\tau < 1/2$ and $\Delta = 0$. Depending on the values of $\tau$ and $s$ the game facing parents when they choose their accounting policies may be a Coordination Game, a Cooperation Game, a Game of Chicken or a Prisoners' Dilemma Game. In particular for most of the size difference between markets, when the tax rates are high, there is an equilibrium in which parents keep one set of books (this equilibrium is unique when the both markets are similar in size) and when tax rates are low, keeping two sets of books is*

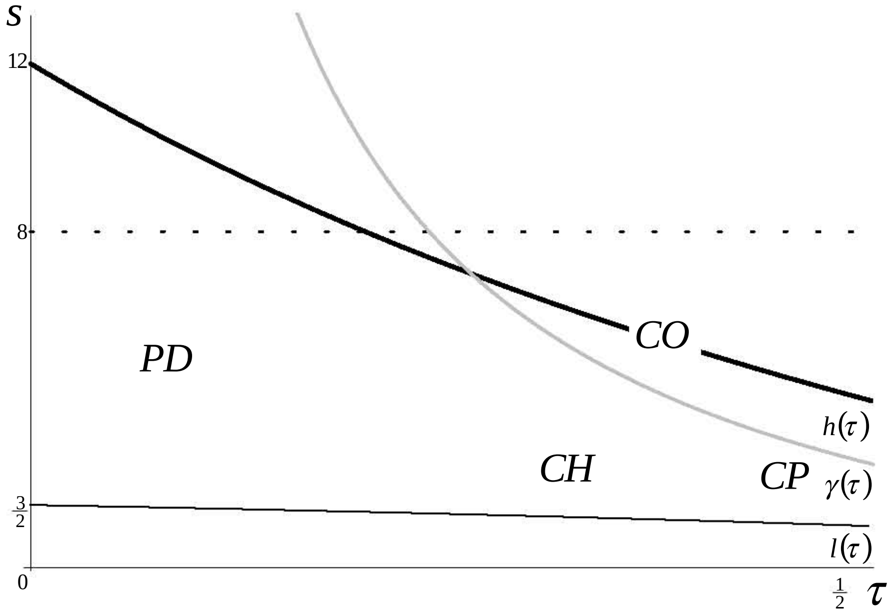


Figure 2.2. Nature of the Game in Table 2.1.

*the unique equilibrium. Asymmetric accounting policies, where one parent keeps one set of books and the other keeps two sets of books, may be sustained in equilibrium when the size of the Latin market is not too small relative to that of the Greek market.*

Propositions 5 provides a rationale for the mixed empirical evidence on the use of alternative accounting system. Also it identifies the parameter constellations for which there are strategic incentives for maintaining one set of books, i.e., for using the same transfer prices for tax reporting and for managerial purposes. Since keeping one set of books provides parents with an instrument to soften competition in the Greek market, our analysis provides a convincing explanation of how the choice of the accounting policy can serve as a precommitment device. In our setting, the regulatory constraint (i.e., transfer prices for tax purposes must be consistent with the ALP) introduces possibilities for tacit coordination, and provides a rationale for why parents delegate quantity decisions to subsidiaries.

## 2.7. Conclusions

The OECD's recommendation that transfer prices between parent firms and their subsidiaries be consistent with the ALP for tax purposes does not restrict internal pricing policies. Since transfer prices serve both to allocate costs to subsidiaries and to determine tax liability in the jurisdictions where firms operate, the incentive and tax transfer prices would be different. Thus, in practice, taxes commit firms to the adoption of a particular accounting policy. When firms use the same transfer price (and hence, a transfer price consistent with the ALP) for tax reporting and for providing incentives, they keep one set of books, and when firms use different transfer prices for each purpose, they keep two sets of books.

In a context of imperfectly competitive markets where firms are vertical separated, we find that accounting policies determine the properties of market outcomes: if parents keep one set of books, competition in the external (home) market softens (intensifies) relative to an equilibrium where parents and subsidiaries are integrated. In contrast, if firms keep two sets of books or keep asymmetric accounting policies, competition intensifies in both markets.

In this paper we show that the choice between one or two sets of books may serve as a precommitment device. When parents choose their accounting policies there exists a wide variety of game forms for alternative parameter depending on the differences in the market size and tax rates. The possible types of the game varies from a prisoners' dilemma (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books) to a game of chicken (with one parent choosing one set of books and the other parent choosing two sets of books ) or a coordination game (in which both parents choose two sets of books or both parents choose one set of books). There also exist parameter constellations of market sizes and tax rates such that the type of game is a cooperation game (with a unique Pareto efficient Nash equilibrium in which both parents choose one set of books).

Our results provide a possible explanation for the mixed empirical evidence on the use of alternative accounting systems. Particularly, the choice of a Pareto superior strategy (i.e., one set of books) can be supported as an equilibrium action under broad conditions. Specifically, for most of the size difference between markets, when tax rates are high, there is an equilibrium in which parents keep one set of books. Interestingly, the prospect to tacit coordination may contribute to a better understanding of why firms decentralize. Therefore, vertical separation of parent and subsidiary firms, whose motivation is not well understood in the absence of frictions when quantities are strategic substitutes, may be justified if firms keep one set of books.

**Appendix 2.A.**

*If* $\tau < \frac{1}{2}$*, then* $\frac{\partial q^T}{\partial \Delta} < 0$ *for all* $s$*.*

**Proof.** We have

$$\frac{\partial q^T}{\partial \Delta} = -\frac{(1-\tau)}{\beta \left(15 (1-\tau) (\Delta - (1-\tau)) + 2s\Delta (\Delta - \tau)\right)^2} \Psi,$$

where

$$\Psi = 5 (3 - 7\tau) + 10 \left(\Delta (\Delta - 2 (1-\tau)) + 2\tau^2\right) + 2s (\Delta - \tau)^2 .$$

Write $\overline{\mu} (\Delta, s)$ and $\overline{\overline{\mu}} (\Delta, s)$ for the value $\tau$ that solves $\Psi = 0$ given $\Delta$ and $s$. We omit the expressions of $\overline{\mu} (\Delta, s)$ and $\overline{\overline{\mu}} (\Delta, s)$ because of its length. Then we have $\Psi > 0$, whenever $\overline{\mu} (\Delta, s) < \tau < \overline{\overline{\mu}} (\Delta, s)$ and $\Psi < 0$, otherwise. Since $\tau = \frac{1}{2}$ is the minimum value of $\overline{\mu} (\Delta, s)$ which is yield when $\Delta = \frac{1}{2}$ for all $s$, then $\tau < \frac{1}{2}$ implies $\tau < \overline{\mu} (\Delta, s)$, and therefore $\Psi > 0$. Thus if $\tau < \frac{1}{2}$, then $\Psi > 0$ and therefore $\frac{\partial q^T}{\partial \Delta} < 0$ for all $s$. $\square$

**Appendix 2.B.**

*If* $\Delta$ *is sufficiently low, there is a critical value* $\overline{s}$ *such that* $\frac{\partial \chi^T}{\partial \Delta} \lesseqgtr 0$ *whenever* $s \gtreqless \overline{s}$*.*

**Proof.** We have

$$\frac{\partial \chi^T}{\partial \Delta} = \frac{2 (1-\tau)}{\beta \left(2s\Delta (\Delta - \tau) - 15 (1-\tau) (1 - (\tau + \Delta))\right)^2} \overline{\Psi},$$

where

$$\overline{\Psi} = 15 (1-\tau)^2 - 2s \left(2\Delta^2 - 3 (1-\tau) (2\Delta - \tau)\right) .$$

Write $\mu (\Delta, s)$ for the value of $\tau$ that solve $\overline{\Psi} = 0$ given $\Delta$ and $s$. We omit the expression of $\mu (\Delta, s)$ because of its length. Then we have $\overline{\Psi} \gtreqless 0$, and therefore $\frac{\partial \chi^T}{\partial \Delta} \gtreqless 0$, whenever $\tau \lesseqgtr \mu (\Delta, s)$. In the limit, as $s$ approaches zero, $\mu = 1$ for all $\Delta$ and as $s$ approaches infinity, $\mu = 0$ if $\Delta = 0$. Note that $\tau \in (0, 1)$ and $\mu$ decreases with $s$ for all $\Delta$. Since $\lim_{s \to 0} \mu (\Delta, s) = 1$ and $\lim_{s \to \infty} \mu (0, s) = 0$, then $\mu$ decreases with $s$ for all $\Delta$ implies there is a critical value $\overline{s}$ such that $\tau \lesseqgtr \mu (0)$ whenever $s \gtreqless \overline{s}$, and therefore $\overline{\Psi} \lesseqgtr 0$. Thus we have $\overline{\Psi} \lesseqgtr 0$, and therefore $\frac{\partial \chi^T}{\partial \Delta} \lesseqgtr 0$, whenever $s \gtreqless \overline{s}$. $\square$

**Appendix 2.C.**

*If* $l(\tau) < s < 8$, *then* $\Pi_2^* - \Pi_1^* > 0$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$ and $l(\tau) < s < 8$. If $s < h(\tau)$, we calculate the difference of profits between parent 2 and parent 1 at the equilibrium described in (4.1) of Proposition 2.4 as

$$\Pi_2^* - \Pi_1^* = \frac{3\,(1-\tau)}{br(\tau,s)^2}\left(s\,(1-\tau)\,(43-35\tau+3s\,(3-7\tau)) - s^3\tau\,(1-2\tau) - 48\,(1-\tau)^2\right).$$

We omit the expression of $^{-}(\tau)$ and $^{=}(\tau)$ for the values of $s$ that solves $\Pi_2^* - \Pi_1^* = 0$ for $\tau < \frac{1}{2}$ because of its length. We have $\Pi_2^* - \Pi_1^* > 0$ whenever $^{-}(\tau) < s < {}^{=}(\tau)$. Since $^{-}(\tau) < l(\tau) < s < \min\{h(\tau), 8\} < {}^{=}(\tau)$, then $\Pi_2^* - \Pi_1^* > 0$. If $s > h(\tau)$, we calculate the difference of profits between parent 2 and parent 1 at the equilibrium described in (4.2) of Proposition 2.4 as $\Pi_2^* - \Pi_1^* = \frac{9}{16}\Pi_G^C > 0$. Therefore if $l(\tau) < s < 8$, then whether $s < h(\tau)$, or $s \geq h(\tau)$, $\Pi_2^* - \Pi_1^* > 0$. $\square$

**Appendix 2.D.**

Let us study the total surplus in a situation where parents use asymmetric accounting policies in term of the total surplus when both firms use two sets of books:

*If* $h(\tau) \leq s < 8$ *and* $(\tau) < s$, *then* $S_L^* + S_G^* < S^T$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$, $h(\tau) \leq s < 8$ and $(\tau) < s$. Using equations (2.6) and (2.18) we calculate the total surplus at the equilibrium described in (4.2) of Proposition 2.4 as

$$S_L^* + S_G^* = S^T - \frac{9}{800\beta},$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

*If* $(\tau) > s > h(\tau)$, *then* $S_L^* + S_G^* < S^T$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$ and $(\tau) > s > h(\tau)$. Using equations (2.6) and (2.17) we calculate the total surplus at the equilibrium described in (4.2) of Proposition 2.4 as

$$S_L^* + S_G^* = S^T + \frac{S}{7200b\,(1-\tau)^2},$$

where

$$S = 64s^2\tau^2 + 400\,(1-\tau)^2 - s\,(239\tau + 81)\,(1-\tau)\,.$$

We omit the expression of $\omega\,(\tau)$ for the value of $s$ that solves $S = 0$ given $\tau$ because of its length. Then we have $S \gtreqless 0$, and therefore $S^{OT} \gtreqless S^T$, whenever $s \gtreqless \omega\,(\tau)$. Since

$$\omega\,(\tau) - \quad (\tau) < 0,$$

for all $\tau$, then $\omega\,(\tau) - \quad (\tau) < 0$ implies

$$s < \omega\,(\tau)\,,$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

*If* $\quad(\tau) < s < h(\tau)$ *and* $s < 8$, *then* $S_L^* + S_G^* < S^T$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $\quad(\tau) < s < h(\tau)$ and $s < 8$. Using equations (2.6) and (2.18) we calculate the total surplus at the equilibrium described in (4.1) of Proposition 2.4 as

$$S_L^* + S_G^* = S^T - \frac{\overline{S}}{50br\,(\tau, s)^2},$$

where

$$\overline{S} = s\left(3\,(283 - 683\tau)\,(1-\tau) - 3s^2\tau\,(10 - 17\tau) + s\,(235 + \tau\,(589\tau - 724))\right) + 3600\,(1-\tau)^2\,.$$

We omit the expression of $\varpi\,(\tau)$ for the value of $s$ that solves $\overline{S} = 0$ given $\tau$ because of its length. Then we have $\overline{S} \gtreqless 0$, and therefore $S^{OT} \gtreqless S^T$, whenever $s \gtreqless \varpi\,(\tau)$. Since

$$\varpi\,(\tau) - h\,(\tau) < 0,$$

for all $\tau$, then $\varpi\,(\tau) - h\,(\tau) < 0$ implies

$$s < \varpi\,(\tau)\,,$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

*If* $s < h(\tau)$, $s < \ (\tau)$ *and* $l(\tau) < s < 8$, *then* $S_L^* + S_G^* < S^T$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $s < h(\tau)$, $s < \ (\tau)$ and $l(\tau) < s < 8$. Using equations (2.6) and (2.17) we calculate the total surplus at the equilibrium described in (4.1) of Proposition 2.4 as

$$S_L^* + S_G^* = S^T + \frac{\widetilde{S}}{450b\,(1-\tau)^2\, r\,(\tau, s)^2},$$

where

$$\begin{aligned}\widetilde{S} \;=\; & 4s^4\tau^2\,(5-7\tau)^2 - 18\,000\,(1-\tau)^4 - s^3\tau\,(1-\tau)\,(230-\tau\,(1631-1865\tau)) \\ & -2s^2\,(745-\tau\,(2474\tau-17))\,(1-\tau)^2 - 3s\,(493\tau+547)\,(1-\tau)^3\,.\end{aligned}$$

Since $\widetilde{S}$ is negative for all $s$ if $\tau = \frac{1}{2}$, then $\widetilde{S}$ increases with $\tau$ (recall $\frac{\partial q^T}{\partial \tau} > 0$ and $\frac{\partial \chi^T}{\partial \tau} = 0$) implies $\widetilde{S}$ is negative in the space $(\tau, s)$, and therefore

$$S_L^* + S_G^* < S^T.\square$$

**Appendix 2.E.**

Let us study total profits of the firm using one set of books in a situation where parents use asymmetric accounting policies in term of total profits if both parents use two sets of books:

*If* $h(\tau) \leq s < 8$ *and* $\ (\tau) < s$, *then* $\Pi^{\overline{O}T} < \Pi^T$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$, $h(\tau) \leq s < 8$ and $\ (\tau) < s$. Using (2.16), we calculate firm's total profits under one set of books when its competitor keeps two sets of books described in (4.2) of Proposition 2.4 as

$$\Pi^{\overline{O}T} = \Pi^T - \frac{7\,(1-\tau)}{400\beta},$$

and therefore

$$\Pi^{\overline{O}T} < \Pi^T.\square$$

*If* $\quad(\tau) > s > h(\tau)$, *then* $\Pi^{\overline{O}T} < \Pi^{T}$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$ and $\quad(\tau) > s > h(\tau)$. Using (2.15), these profits can be calculated as

$$\Pi^{\overline{O}T} = \Pi^{T} + \frac{\Pi}{3600b\,(1-\tau)},$$

where $\Pi = 32s^2\tau^2 - s\,(63 - 143\tau)\,(1-\tau) - 400\,(1-\tau)^2$. We omit the expression of $\phi\,(\tau)$ for the value of $s$ that solves $\Pi = 0$ given $\tau$ because of its length. Then we have $\Pi \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \gtreqless \Pi^{T}$, whenever $s \gtreqless \phi\,(\tau)$. Since

$$h(\tau) - \phi\,(\tau) < 0 \text{ and } \quad(\tau) - \phi\,(\tau) < 0,$$

for all $\tau$, then $h(\tau) - \phi\,(\tau) < 0$ and $\quad(\tau) - \phi\,(\tau) < 0$ implies

$$s < \phi\,(\tau)\,,$$

and therefore

$$\Pi^{\overline{O}T} < \Pi^{T}.\square$$

*If* $\quad(\tau) < s < h(\tau)$ *and* $s < 8$, *then* $\Pi^{\overline{O}T} < \Pi^{T}$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $\quad(\tau) < s < h(\tau)$ and $s < 8$. Using (2.16), we calculate firm's total profits under one set of books when its competitor keeps two sets of books described in (4.1) of Proposition 2.4 as

$$\Pi^{\overline{O}T} = \Pi^{T} - \frac{3\,(1-\tau)}{25br(\tau,s)^2}\overline{\Pi},$$

where

$$\overline{\Pi} = (1-\tau)\left(s\,(109 + 91\tau) + s^2\,(85 - 149\tau)\right) - s^3\tau\,(5 - 16\tau) - 1200\,(1-\tau)^2\,.$$

Write $\overline{\phi}_1\,(\tau)$ for the value of $s$ that solves $\overline{\Pi} = 0$ if $\tau < \frac{5}{16}$ and $\overline{\phi}_2\,(\tau)$ for the value of $s$ that solves $\overline{\Pi} = 0$ if $\tau \notin \left(0, \frac{5}{16}\right)$ (i.e., there are two real roots for the value of $s$ that solves $\overline{\Pi} = 0$: one in the interval $\tau \in \left(0, \frac{5}{16}\right)$ and the other $\tau \notin \left(0, \frac{5}{16}\right)$). We omit the expressions of $\overline{\phi}_1\,(\tau)$ and $\overline{\phi}_2\,(\tau)$ for the value of $s$ that solves $\overline{\Pi} = 0$ given $\tau$ because of its length.

We have $\overline{\Pi} \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^{T}$, whenever $s \gtreqless \overline{\phi}_1\,(\tau)$. Since

$$(\tau) - \overline{\phi}_1\,(\tau) > 0,$$

for $\tau < \frac{5}{16}$, then $\quad(\tau) - \phi_1(\tau) > 0$ implies

$$s > \overline{\phi}_1(\tau),$$

and therefore $\Pi^{\overline{O}T} < \Pi^T$. We have $\overline{\Pi} \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^T$, whenever $s \gtreqless \phi_2(\tau)$. Since

$$(\tau) - \overline{\phi}_2(\tau) > 0,$$

for $\tau > \frac{5}{16}$, then $\quad(\tau) - \overline{\phi}_2(\tau) > 0$ implies

$$s > \overline{\phi}_2(\tau),$$

and therefore $\Pi^{\overline{O}T} < \Pi^T$. Thus whether $\tau \in \left(0, \frac{5}{16}\right)$, or $\tau \notin \left(0, \frac{5}{16}\right)$,

$$\Pi_2^* - \Pi_1^* > 0.\square$$

*If* $s < h(\tau)$, $s < \quad(\tau)$ *and* $l(\tau) < s < 8$, *then* $\Pi^{\overline{O}T} < \Pi^T$ *whenever* $s < 1.385$, *whereas* $\Pi^{\overline{O}T} \lesseqgtr \Pi^T$ *whenever* $\tau \lesseqgtr \widehat{\tau}$ *and* $s > 1.385$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $s < h(\tau)$, $s < \quad(\tau)$ and $l(\tau) < s < 8$. Using (2.15), these profits can be calculated as

$$\Pi^{\overline{O}T} = \Pi^T + \frac{\widehat{\Pi}}{225b(1-\tau)\, r(\tau, s)^2},$$

where

$$\begin{aligned} \widehat{\Pi} = &-(1-\tau)^2 \left(3s(2981 - 2941\tau)(1-\tau) + 4s^2\left(730 - 2317\tau + 1444\tau^2\right)\right) \\ &-s^3\tau(1-\tau)\left(437\tau - 5\tau^2 - 260\right) + 18\,000(1-\tau)^4 + 2s^4\tau^2(5-7\tau)^2. \end{aligned}$$

There is no closed form solutions for the value of $s$ that solves $\widehat{\Pi}(\tau, s) = 0$. Figure 2.3 below are the graphs of the function $\widehat{\Pi}$ for different values of $\tau$. As graphically displayed by the Figure 2.3 if $\tau = \frac{1}{2}$, the values of $s$ must lie between 1 and $\frac{5}{2}$ and $\widehat{\Pi}$ is positive for all $s$ and if $\tau = 0$, the values of $s$ must lie between $\frac{3}{2}$ and 8 and $\widehat{\Pi}$ is negative for all $s$. Also $\widehat{\Pi}$ increases with $\tau$ if $s > 1.385$ and decreases with $\tau$, otherwise. Therefore for $s > 1.385$, since $\widehat{\Pi} < 0$ if $\tau = 0$ and $\widehat{\Pi} > 0$ if $\tau = \frac{1}{2}$, then $\widehat{\Pi}$ increases with $\tau$ implies there is a critical value $\widehat{\tau}$ such that $\widehat{\Pi} \lesseqgtr 0$ whenever $\tau \lesseqgtr \widehat{\tau}$. Then we have $\widehat{\Pi} \lesseqgtr 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^T$, whenever $\tau \lesseqgtr \widehat{\tau}$. For $s < 1.385$, since $\widehat{\Pi} > 0$ for $\tau = \frac{1}{2}$, then $\widehat{\Pi}$ decreases with $\tau$ implies $\widehat{\Pi} > 0$ for all $\tau$ and therefore $\Pi^{\overline{O}T} > \Pi^T$.$\square$

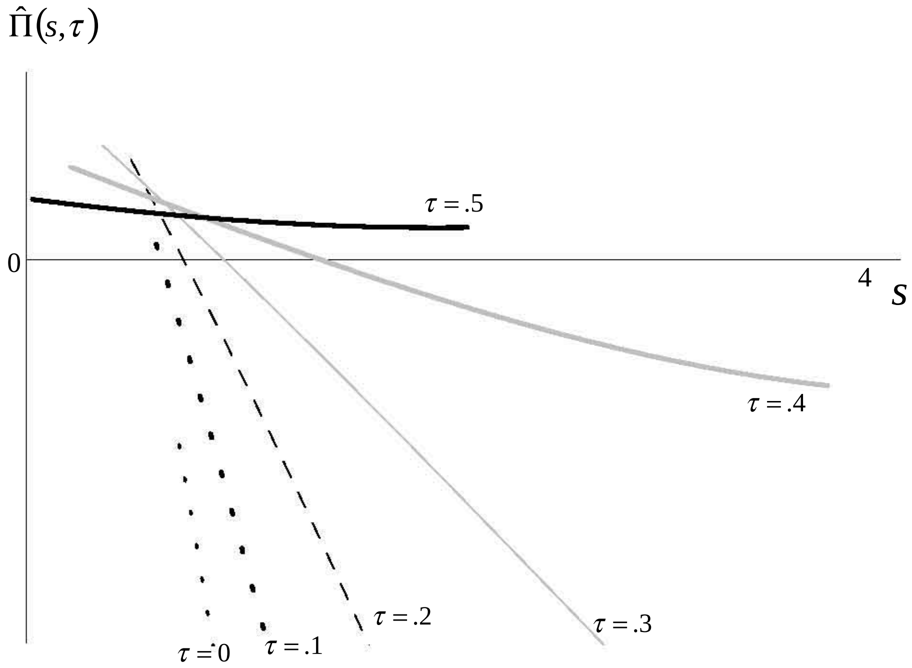


Figure 2.3. Graphs of the function $\hat{\Pi}(s)$ for different values of $\tau$.

**Appendix 2.F.**

Let us study the profits of the firm using two sets of books in a situation where parents use asymmetric accounting policies in term of total profits if both parents use one set of books:

*If* $h(\tau) \leq s < 8$*, then* $\Pi^{\overline{O}T} < \Pi^{T}$.

**Proof.** Assume $\tau < 1/2$, $\Delta = 0$ and $h(\tau) \leq s < 8$. Using (2.12), we calculate firm's total profits under two sets of books when its competitor keeps one set of books described in (4.2) of Proposition 2.4 as

$$\Pi^{O\overline{T}} = \Pi^{O} - \frac{3(1-\tau)}{8b(8s+27)^2}\left(216 + 117s + 16s^2\right),$$

and therefore

$$\Pi^{O\overline{T}} < \Pi^{O}.\square$$

*If* $l(\tau) < s < h(\tau)$ *and* $s < 8$*, then* $\Pi^{O\overline{T}} > \Pi^{O}$ *whenever* $s \in (1.23, 2.26)$*, whereas*

$\Pi^{O\overline{T}} \lesseqgtr \Pi^O$ *whenever* $\tau \gtreqless \widetilde{\tau}$ *and* $s \notin (1.23, 2.26)$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $l(\tau) < s < h(\tau)$ and $s < 8$. Using (2.12), we calculate firm's total profits under two sets of books when its competitor keeps one set of books described in (4.1) of Proposition 2.4 as

$$\Pi^{O\overline{T}} = \Pi^O + \frac{3(1-\tau)}{b(8s+27)^2 r(\tau,s)^2}\widetilde{\Pi},$$

where

$$\begin{aligned}\widetilde{\Pi} &= 3s^3(555 - \tau(2222 - 1769\tau)) - \left(8s^5(1+\tau)(3-5\tau) + 4s^4(21 + \tau(130 - 203\tau))\right) \\ &\quad - \Big(15\,552(1-\tau)^2 - 9\left(s^2(1009 - \tau(2450 - 1429\tau)) + 18s(1-\tau)(43 - 27\tau)\right)\Big).\end{aligned}$$

There is no closed form solutions for the value of $s$ that solves $\widetilde{\Pi} = 0$. Figure 2.4 below are the graphs of the function $\widetilde{\Pi}$ for different values of $\tau$. As graphically displayed by the Figure 2.4 if $\tau = \frac{1}{2}$ the values of $s$ must lie between 1 and 4 and $\widetilde{\Pi}$ is positive for all $s \in (1.23, 2.26)$ and if $\tau = 0$, the values of $s$ must lie between $\frac{3}{2}$ and 8 and $\widetilde{\Pi}$ is positive for all $s$. Also $\widetilde{\Pi}$ decreases with $\tau$ for all $s$. For $s \in (1.23, 2.26)$, since $\widetilde{\Pi} > 0$ for $\tau = \frac{1}{2}$, then $\widetilde{\Pi}$ decreases with $\tau$ implies $\widetilde{\Pi} > 0$ for all $\tau$ and therefore $\Pi^{O\overline{T}} > \Pi^O$. For a given $s \notin (1.23, 2.26)$, since $\widetilde{\Pi} < 0$ for $\tau = \frac{1}{2}$ and $\widetilde{\Pi} > 0$ for $\tau = 0$, then $\widetilde{\Pi}$ decreases with $\tau$ implies there is a critical value $\widetilde{\tau}$ such that $\widetilde{\Pi} \lesseqgtr 0$ whenever $\tau \gtreqless \widetilde{\tau}$. Then we have $\widetilde{\Pi} \lesseqgtr 0$, and therefore $\Pi^{O\overline{T}} \lesseqgtr \Pi^O$, whenever $\tau \gtreqless \widetilde{\tau}$.□

*If* $(\tau) < s < h(\tau)$, *then* $\Pi^{O\overline{T}} < \Pi^O$.

**Proof.** Assume that $\tau < 1/2$, $\Delta = 0$, $(\tau) < s < h(\tau)$. Since the sign of $\Pi^{\overline{O}T} - \Pi^T$ is positive, we discuss the sign of $\Pi^O - \Pi^{O\overline{T}}$ in order to characterize the SPE in this region of parameters. For $s = (\tau)$, the equation above yields $\widetilde{\Pi} < 0$ for all $\tau$. Since $\widetilde{\Pi} < 0$ if $s = (\tau)$, then $\widetilde{\Pi}$ decreases with $\tau$ implies $\widetilde{\Pi} < 0$ for all $s > (\tau)$ and therefore $\Pi^{O\overline{T}} < \Pi^O$.□

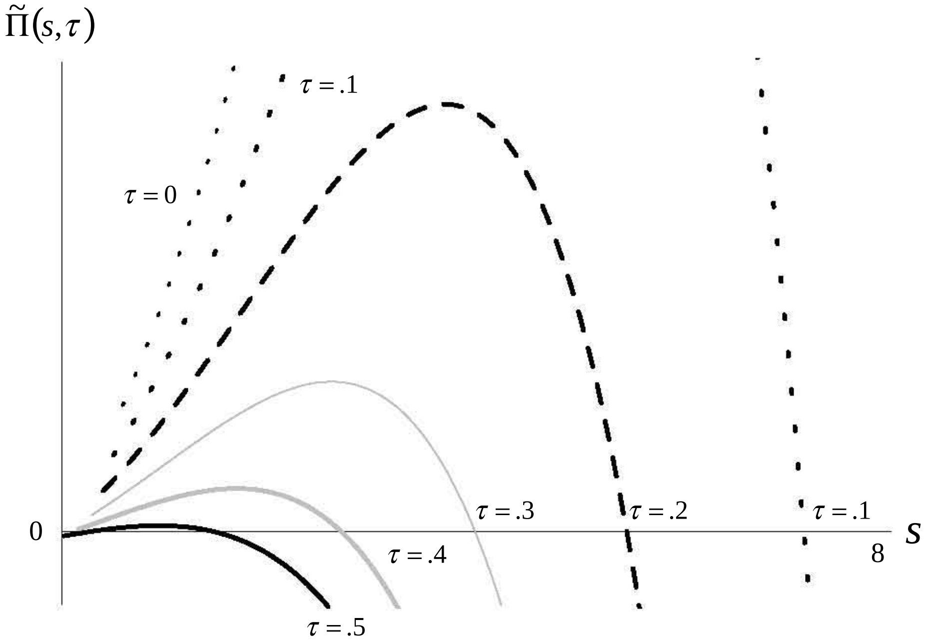


Figure 2.4. Graphs of the function $\widetilde{\Pi}\ (s)$ for different values of $\tau$.

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# CHAPTER 3. THE NON-NEUTRALITY OF THE ARM'S LENGTH PRINCIPLE WITH IMPERFECT COMPETITION

## 3.1. Introduction

International tax authorities have become increasingly aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions. Transfer pricing policies have important implications since exports and imports from related parties are a dominant portion of trade flows – see Bernard, Jensen and Schott (2009). In order to discourage tax shifting activities by multinational firms, most countries follow taxation policies that are based on the OECD's Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the *Arm's Length Principle (ALP)*; i.e., that transfer prices between companies of multinational enterprises for tax purposes be established on a market value basis, thus comparable to transactions between independent (unrelated) parties – see OECD (2010). Tax authorities from all OECD member nations rely on the *ALP* to protect their revenue base by preventing incomes shifting from one country to another for reasons unrelated to the economic nature of the transactions. We study the consequences of adopting the *ALP* when markets are imperfectly competitive.[26]

Hirshleifer (1956) shows that the application of the *ALP* is inconsequential under perfect competition. The simplest version of Hirshleifer's (1956) model assumes a decentralized firm consisting of a *headquarters* and two divisions, the *upstream* and *downstream* divisions. The upstream division produces an intermediate good and supplies it to the downstream division. The downstream division processes this intermediate good and sells it in the final

[26] Under the *ALP* firms are free to charge their subsidiaries either the same or different prices to those used for tax purposes, i.e., firms may keep either one set of books or two sets of books. Lemus (2011) provides an analysis of firms' strategic incentives for choosing either alternative, and shows that under broad conditions keeping one set of book is an equilibrium. Here we assume that adopting the *ALP* leads parent firms to keep one set of books, thus transferring the good to their subsidiaries at market prices.

good market. Each division maximizes it own profits ignoring the impact of its decisions on the profits of the other division or the firm as a whole. The problem of headquarters consists of finding a transfer pricing policy that coordinates the decisions of the two divisions so that consolidated profits are maximized. The efficient level of internal trade can be implemented by setting transfer prices at the opportunity cost of the intermediate good. If there is a competitive market for the intermediate good, the opportunity cost of the intermediate good is equal to the market price. If no market exists, the optimal transfer price equals the marginal cost of the intermediate good. Thus, setting the transfer price equal to the market price is consistent with the Arm's Length Principle, and leads to an efficient allocation of resources. Hirshleifer's result depends crucially on the assumption that the intermediate good market is perfectly competitive. As we shall see, under imperfect competition the $ALP$ significantly distorts the resource allocation (as well as firms' tax liabilities).

In this paper, abstracting from issues arising due to differences on tax rates in each jurisdiction, we examine the consequences of adopting transfer pricing policies adhering to the $ALP$ under imperfect competition and vertical separation. (If firms are vertically integrated, then transfer pricing policies are irrelevant.) In our setting parents compete in quantities in a home market and set the prices at which they sell the good to their subsidiaries (either directly or indirectly via their output choices), which in turn compete in quantities in an external market. As customary, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits.

Contrary to the conventional wisdom that views regulatory constraints as impediments to effective management, our results suggest that regulatory restrictions leading parent firms to set transfer prices at market value may serve as a precommitment device, thus playing a strategic role beneficial to firms: the Arm's Length Principle serves to credibly convey to external parties that the related party price is above marginal cost, ensuring commitment and observability.

In the absence of the $ALP$, it has been established that vertical separation intensifies or alleviates competition depending on the nature of the oligopolistic competition: When firms compete in prices, vertical separation softens competition, whereas when firms compete in quantities vertical separation induces firms to compete more aggressively – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998). When the adoption of the *ALP* leads to market based transfer pricing, our results provide a rationale for vertical separation also when firms compete in quantities. Göx (2000) and Dürr and Göx (2011) show that when firms compete in prices, the *ALP* reinforces the effect of vertical separation on softening competition. Contrary to Göx's (2000) claim that this result does not "... carry over to the case of quantity competition because quantities are strategic substitutes...," our results show the *ALP* softens competition even in this case. Moreover, quantity competition provides a reduced form model for the analysis of more complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition – see Kreps and Scheinkman (1983) and Moreno and Ubeda (2006).

In our framework there are two markets, which we refer to as the Latin market and the Greek market. There are two firms engaging in Cournot competition in the Latin market. These firms have subsidiaries, which in turn engage in Cournot competition in the Greek market. We begin by considering two alternative transfer pricing schemes for intrafirm transactions. Since competition in the Latin market provides a market price to impose on comparable market transactions, we study *market based transfer pricing* (MB) as the equivalent to the *ALP* as the OECD recommends.[27] Alternatively, we consider transfer pricing not linked to the Latin market, i.e., *non-market based transfer pricing* (NMB). We show that MB transfer pricing typically leads to a lower total surplus, and may lead to larger profits, than NMB transfer pricing.

Under NMB transfer pricing a parent's decisions of how much to produce in the Latin market and what transfer price to charge to its subsidiary are independent. In equilibrium, parents set transfer prices below marginal cost in an attempt to gain a Stackelberg advantage

[27] Choe and Matsushima (2013) examine the effect of ALP on dynamic competition in imperfectly competitive markets and show that the ALP results in more stable tacit collusion. They consider a vertically related market with two upstream firms which supply to their downstream affiliates and other unrelated buyers in the downstream market. The authors consider the price upstream firms charge to unrelated buyers as the comparable uncontrolled price for applying the ALP. In our setting, the price in the home market provides a reliable measure of an arm's length result.

in the Greek market; i.e., both parents act in a Stackelberg fashion. The equilibrium output in the Greek market is greater than the Cournot output, and consolidated profit is below the sum of profits at the Cournot equilibria of both markets. These results reproduce those of Vickers (1985) in our framework.

Under MB transfer pricing a parent must transfer the good to its subsidiary at the Latin market price. Hence, a parent's output decision must internalize its impact on the transfer price of its subsidiary and its subsidiary's rival. *MB transfer pricing* thus provides parents with an instrument to soften competition in the Greek market.[28] Since a parent influences its transfer price via its output decision in the Latin market, competition may be more aggressive in this market. Thus, total profits under MB transfer pricing may be above that under NMB transfer pricing. Hence the Arm's Length Principle provides a rationale for vertical separation.[29] However, total surplus under MB transfer pricing is typically below that under NMB transfer pricing, which raises some questions about the use of the *ALP* as a guideline for regulating transfer prices.

We also consider the consequences of applying the *ALP* less rigorously by studying a variation of the model of MB transfer pricing where parents may introduce *discounts*. Un-

[28] Arya and Mittendorf (2008) analyze market based transfer pricing as a strategic response in a similar setting. They show that ALP makes firms more aware of the fact that excessive home market prices depress external production (i.e., the concern is about double marginalization). Thus, as a result firms may become more aggressive in the home market. However, the authors do not acknowledge that ALP increases prevailing transfer prices and thereby mitigates the prisoner's dilemma to downstream competition. In their model, parents rely on intracompany discounts to manage tensions between the home and the external markets. Intracompany discounts are set prior to the stage of competition in the home market and serve as a precommitment device. Nevertheless, this device is somewhat artificial since parents must credibly bind themselves to these discounts. In our setting, it is regulatory restriction (i.e., ALP) that serves to credibly convey to external parties that the home market price is above marginal cost.

[29] Arya and Mittendorf (2007) provide an alternative rationale for vertical separation in a model in which subsidiaries use two inputs, one that is produced internally and another one that is purchased from an external supplier. They observe that delegating quantity decisions to a subsidiary results in a lower price from the external supplier, overcompensating the negative effect on profits of transfer prices for the internal input above marginal cost.

der this scheme of *market based transfer pricing with discounts* (MBD) each parent can compensate the effect of a high price in the Latin market on its subsidiary's cost by applying a discount. Discounts open up the possibility to gain a Stackelberg advantage in the Greek market, bringing back the kind of prisoners' dilemma that firms face under NMB transfer pricing. However, whereas under MBD transfer pricing the equilibrium output in the Greek market is the same as under NMB transfer pricing, the equilibrium output in the Latin market is less competitive under MBD transfer pricing than under NMB transfer pricing: a parent has an incentive to increase the price in the Latin market by reducing its output and at the same time increase the discount to its subsidiary, thus increasing its subsidiary's rival transfer price without affecting the transfer price of its own subsidiary. These incentives lead to a smaller output and a smaller total surplus in the Latin market than under NMB.

In summary, a transfer pricing policy consistent with the Arm's Length Principle is likely to induce a surplus loss relative to the NMB transfer pricing. Thus, contrary to common wisdom based on competitive models, under imperfect competition the adoption of the $ALP$ is non neutral, but has an significant impact on market outcomes as it softens competition either in the external market (when it is applied rigorously) or in the home market (when its application is more lax).

The paper is organized as follows. Section 3.2 introduces the basic setup. Section 3.3 derives results for NMB transfer pricing. Section 3.4 provides an equilibrium analysis of MB transfer pricing, and compares the properties of equilibrium under the two transfer pricing schemes. Section 3.5 studies the impact of introducing discounts into the MB transfer pricing scheme. Section 3.6 concludes.

### 3.2. Model and Preliminaries

A good is sold in two markets, which we refer to as the Latin market and the Greek market. The inverse demands in the Latin and Greek markets are $p^d(q) = \max\{0, a - bq\}$ and $\rho^d(\chi) = \max\{0, \alpha - \beta\chi\}$, respectively, where $a, b, \alpha$, and $\beta$ are positive real numbers.

Assuming that demands are linear facilitates the analysis and makes it easier to interpret the results. Comparing the constant terms in each demand (i.e., the parameters $a$ and $\alpha$) allows us to consider the impact of differences in the maximum willingness to pay in each market. The parameter $u := a/\alpha$ is a proxy for the maximum willingness to pay in the Latin market relative to that of the Greek market. Differences in the slope of the demands (i.e., of the parameters $b$ and $\beta$) capture the impact of differences in the market size – the demand is greater the smaller the slope. The parameter $s := \beta/b$ is a proxy for the size of the Greek market relative to that of the Latin market.

There are two firms producing the good at the same constant marginal cost, which is assumed to be zero without loss of generality. Firms engage in Cournot competition in the Latin market, and have subsidiaries which in turn engage in Cournot competition in the Greek market. Each subsidiary receives the good from its parent firm at a transfer price. Parent firms seek to maximize consolidated profits; since the cost of production is zero, consolidated profits are just the sum of revenues of the parent and the subsidiary. A subsidiary maximizes its own profits, which is the difference between its revenue and its cost. A subsidiary' unit cost is just its transfer price. We identify the parent and subsidiary firms with the same subindex $i \in \{1, 2\}$.

Clearly, if parents do not delegate but rather compete in quantities in the external market as well, then in equilibrium firms produces their Cournot output in each market, and transfer pricing policies are irrelevant.

In the Cournot equilibrium of a duopoly where the market demand is $P^d(Q) = \max\{0, A - BQ\}$ and firms' constant marginal costs are $(c_1, c_2) \in \mathbb{R}^2_+$, the market price $P^C$, the output $Q_i^C$ and profits $\Pi_i^C$ of firm $i$ are

$$(P^C, Q_i^C, \Pi_i^C) = \left( \frac{A + c_1 + c_2}{3}, \frac{A - 2c_i + c_{3-i}}{3B}, \frac{(A - 2c_1 + c_2)^2}{9B} \right). \tag{3.1}$$

If the market is monopolized by a single firm whose constant marginal cost is $c \in \mathbb{R}_+$, then the market equilibrium price $P^M$, output $Q^M$, and the firm's profits $\Pi^M$ are

$$(P^M, Q^M, \Pi^M) = \left( \frac{A + c}{2}, \frac{A - c}{2B}, \frac{(A - c)^2}{4B} \right). \tag{3.2}$$

Using these formulae (3.1), we readily calculate the Cournot equilibrium in the Latin market as

$$(p^C, q^C, \Pi_L^C) = \left(\frac{a}{3}, \frac{a}{3b}, \frac{a^2}{9b}\right). \tag{3.3}$$

Using the formulae (3.2), we obtain the monopoly equilibrium in the Latin market as

$$(p^M, q^M, \Pi_L^M) = \left(\frac{a}{2}, \frac{a}{2b}, \frac{a^2}{4b}\right). \tag{3.4}$$

Note that $q^M = \frac{3}{4}(2q^C)$; i.e., in a monopoly the equilibrium output is 75% of the output in a Cournot duopoly.

When aggregate output is $q$, the total surplus generated in the market is given by

$$S(q) = \left(A - \frac{Bq}{2}\right) q. \tag{3.5}$$

In the Latin market, the surplus at the Cournot equilibrium, $S_L^C$, is

$$S_L^C = \frac{4a^2}{9b}, \tag{3.6}$$

and the surplus at monopoly equilibrium, $S_L^M$, is

$$S_L^M = \frac{3a^2}{8b}. \tag{3.7}$$

Replacing $a$ with $\alpha$ and $b$ with $\beta$ yields formulae analogous for the Cournot and monopoly equilibria in the Greek market. (These formulae assume that firms' constant marginal cost of production is zero). We use the notation $\chi^C$, $\rho^C$, $\Pi_G^C$, $S_G^C$, and $\chi^M, \rho^M, \Pi_G^M, S_G^M$, for the values of output, price, profits and surplus at the Cournot duopoly equilibrium, and monopoly equilibrium of the market, respectively.

### 3.3. Non-Market Based Transfer Pricing

Assume that the parent firms simultaneously decide the transfer prices they charge to their subsidiaries, knowing that these firms will engage in Cournot competition in the Greek market; i.e., each parent firm $i \in \{1, 2\}$ sets its transfer price $t_i \in \mathbb{R}$ so as to maximize consolidated profits. (Of course, a parent firm may provide the good to a subsidiary at a subsidized cost, which implies, since the unit cost is zero, that transfer prices may be negative.) The equilibrium under this scheme of non-market based (NMB) transfer pricing is determined as follows.

For $(t_1, t_2)$, the equilibrium in the Greek market is that of a Cournot duopoly where firms' constant marginal costs are $(t_1, t_2)$; i.e., the output of firm $i \in \{1, 2\}$ is

$$\chi_i^* = \bar{\chi}_i\left(t_1, t_2\right) = \frac{\alpha - 2t_i + t_{3-i}}{3\beta}.$$

Thus, parent $i$ solves the problem

$$\max_{(q_i, t_i) \in \mathbb{R}_+ \times \mathbb{R}} p^d(q_1 + q_2)q_i + \rho^d(\bar{\chi}_1(t_1, t_2) + \bar{\chi}_2(t_1, t_2))\bar{\chi}_i(t_1, t_2).$$

Since parent $i$'s choice of transfer prices $t_i$ does not affect its revenue in the Latin market, nor its output decisions in the Latin market $q_i$ affect its revenue in the Greek market. Hence, these two decisions can be treated independently; i.e., $q_i$ $(t_i)$ is chosen to maximize revenue in the Latin (Greek) market. Thus, the equilibrium outcome in the Latin market is just the Cournot equilibrium outcome.

We calculate the equilibrium outcome in the Greek market. Parent $i$ chooses its transfer price $t_i$ so as to maximize its subsidiary's revenue in the Greek market,

$$\rho^d\left(\bar{\chi}_1\left(t_1, t_2\right) + \bar{\chi}_2\left(t_1, t_2\right)\right)\bar{\chi}_i\left(t_1, t_2\right).$$

Hence, parent $i$'s reaction to the transfer price set up by its competitor, $t_{3-i}$, is

$$r_i(t_{3-i}) = -\frac{t_{3-i} + \alpha}{4}.$$

Therefore, the equilibrium transfer prices are

$$t_1^* = t_2^* = -\frac{\alpha}{5}.$$

Substituting these values into the equation for $\bar{\chi}_i\left(t_1, t_2\right)$ and using (3.1) we get the subsidiaries' outputs

$$\bar{\chi}_1\left(t_1^*, t_2^*\right) = \bar{\chi}_2\left(t_1^*, t_2^*\right) = \frac{2\alpha}{5\beta} = \frac{6}{5}\chi^C := \chi^{NMB}.$$

Hence the equilibrium price in the Greek market is

$$\rho^d\left(2\chi^{NMB}\right) = \frac{\alpha}{5} = \frac{3}{5}\rho^C := \rho^{NMB}.$$

Total profits are

$$\begin{aligned} \Pi_L^{NMB} + \Pi_G^{NMB} &= p^C q^C + \rho^{NMB}\chi^{NMB} \\ &= \Pi_L^C + \frac{18}{25}\Pi_G^C. \end{aligned} \tag{3.8}$$

And total surplus is

$$\begin{aligned} S_L^{NMB} + S_G^{NMB} &= S_L^C + \left(\alpha - \frac{\beta}{2}\left(2\chi^{NMB}\right)\right) 2\chi^{NMB} \\ &= S_L^C + \frac{27}{25}S_G^C. \end{aligned} \tag{3.9}$$

We summarize these results in the following proposition.

**Proposition 3.1.** *Under non-market based transfer pricing:*

*(3.1.1) The equilibrium output in the Latin market is the Cournot output, i.e.,*

$$q^{NMB} = q^C.$$

*(3.1.2) The equilibrium output in the Greek is above the Cournot output, i.e.,*

$$\chi^{NMB} = \frac{6}{5}\chi^C.$$

*(3.1.3) Firms' profits are*

$$(\Pi_L^{NMB}, \Pi_G^{NMB}) = (\Pi_L^C, \frac{18}{25}\Pi_G^C).$$

*Hence, total profits are below their profits at the Cournot equilibria of these markets.*

*(3.1.4) The surpluses in the Latin and Greek markets are*

$$(S_L^{NMB}, S_G^{NMB}) = (S_L^C, \frac{27}{25}S_G^C).$$

*Thus, the total surplus is above the surplus at the Cournot equilibria of these markets.*

The strategic considerations behind this result are clear: delegating output decision to subsidiaries induces parents to compete more aggressively in the Greek market, relative to a setting in which parents exercise direct control of the subsidiary's output. By reducing its transfer price below marginal cost, parents attempt to gain a kind of Stackelberg leader status, creating a sort of prisoners' dilemma situation. As a consequence, the equilibrium outcome in the Greek market is more efficient than the Cournot outcome. Analogous results are found by Vickers (1985), Judd and Fershtman (1987), Sklivas (1987), and Alles and Datar (1998).

### 3.4. Market Based Transfer Pricing

In this section, we assume, consistently with the Arm's Length Principle, that subsidiaries buy the good from parents at the price at which the good trades in the Latin market, which is known to the firms competing in the Greek market at the time of making output decisions. In this setup, parents act as "leaders" anticipating the reactions of subsidiary firms. The equilibrium under this scheme of market based (MB) transfer pricing is determined as follows.[30]

Assuming that the price in the Latin market is $p \geq 0$, each subsidiary $i \in \{1, 2\}$ chooses its output $\chi_i$ to solve the problem

$$\max_{\chi_i \in \mathbb{R}_+} (\rho^d(\chi_1 + \chi_2) - p)\chi_i.$$

Here $p$ is the constant marginal cost of the subsidiary firms. Using the formulae (3.1), we calculate equilibrium outputs for $p \geq 0$ as

$$\chi_1^* = \chi_2^* = \hat{\chi}(p) = \frac{\alpha - p}{3\beta}.$$

Parents, anticipating the outputs in the Greek market, choose their output $q_i$ in order to

[30] Dürr and Göx (2011) assume that firms can arbitrarily choose a transfer price from an allowable exogenous range of *ALP prices*, withstanding a possible examination of authorities in the two markets. In the next section we consider a lax application of the *ALP* where effective transfer prices are determined endogenously.

solve

$$\max_{q_i \in \mathbb{R}_+} p^d(q_1+q_2)q_i + \rho^d(\hat{\chi}_1(p^d(q_1+q_2)) + \hat{\chi}_2(p^d(q_1+q_2)))\hat{\chi}_i(p^d(q_1+q_2)).$$

Solving the system of equations formed by the first-order condition for profit maximization of parents 1 and 2 we obtain their outputs,

$$q_1^* = q_2^* = \frac{(4b+9\beta)a - b\alpha}{b\,(8b+27\beta)} := q^{MB}. \tag{3.10}$$

The equilibrium price in the Latin market is

$$p^d(2q^{MB}) = \frac{9a\beta + 2b\alpha}{8b+27\beta} := p^{MB}.$$

Substituting the value of $p^{MB}$ into equation $\hat{\chi}(p)$ we obtain the equilibrium subsidiaries' outputs,

$$\chi_1^* = \chi_2^* = \hat{\chi}(p^{MB}) = \frac{(2b+9\beta)\,\alpha - 3\beta a}{\beta(8b+27\beta)} := \chi^{MB}. \tag{3.11}$$

The equilibrium price in the Greek market is

$$\rho^d(2\chi^{MB}) = \frac{6a\beta + 4b\alpha + 9\alpha\beta}{8b+27\beta} := \rho^{MB}.$$

For the equilibrium to be interior we must have

$$(4b+9\beta)\,a - 4b\alpha > 0,$$

i.e.,

$$u > \frac{1}{4+9s} := l(s),$$

and

$$(9\beta+2b)\,\alpha - 3\beta a > 0,$$

i.e.,

$$u < 3 + \frac{2}{3s} := g(s).$$

Thus, equilibrium is interior whenever

$$l(s) < u < g(s), \tag{3.12}$$

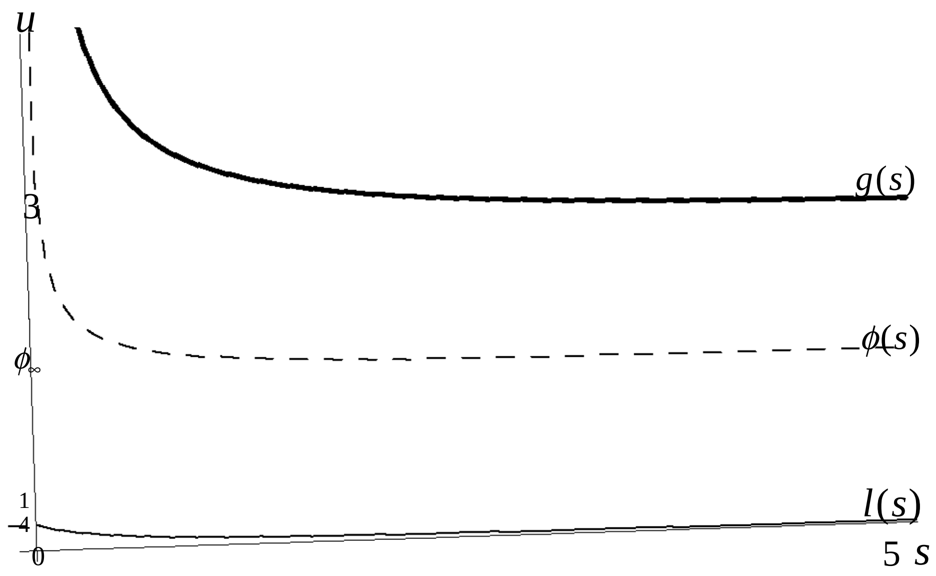


Figure 3.1. Total profits under MB and NMB transfer pricing.

holds. The thin and thick curves in Figure 3.1 below displays the graphs of the functions $l$ and $g$, respectively. For parameter constellations $(s, u)$ lying between these curves the equilibrium is interior.

If $u \geq g(s)$, then firms' equilibrium outputs are $q^{MB} = q^C$ and $\chi^{MB} = 0$; that is, for parameter constellations lying above the thick curve of Figure 3.1 double marginalization leads to a complete shut down of the Greek market. And if $u \leq l(s)$, then firms' equilibrium outputs are $q^{MB} = 0$ and $\chi^{MB} = (\alpha - a)/3\beta$; that is, for parameter constellations below the thin line of Figure 3.1, it pays to shut down the Latin market in order to soften competition in the Greek market among subsidiaries as much as possible.

Assuming that (3.12) holds, so that both markets are active, and using again (3.3), we can rewrite the expression for firms' output in the Latin market (3.10) as

$$q^{MB} = q^C + \frac{4\alpha}{3\left(8b + 27\beta\right)}\left(u - \frac{3}{4}\right).$$

Likewise, using equations (3.3) and (3.4) we can write the expression for firms' output in the Greek market (3.11) as

$$\begin{aligned}\chi^{MB} &= \chi^{C} - \frac{9a\beta + 2b\alpha}{3\beta\,(8b + 27\beta)} \\ &= \frac{\chi^{M}}{2} - \frac{3\alpha}{(8b + 27\beta)}\left(u - \frac{3}{4}\right).\end{aligned}$$

Thus, under MB transfer pricing whether the output in the Latin market is above or below the Cournot output (which is also their output under NMB pricing by Proposition 3.1) depends on the sign of $u - 3/4$. This term is positive whenever the maximum willingness to pay in the Latin market relative to that in the Greek market is sufficiently large (at least 75%), and it is negative otherwise. However, the output in the Greek market is always below the Cournot output (and therefore, it is below the output under NMB transfer pricing by Proposition 3.1). Note also that double marginalization imposed by MB transfer pricing leads to an output in the Greek market that is below the monopoly output when $u > 3/4$.

We have

$$\frac{\partial q^{MB}}{\partial \beta} = -\frac{36\alpha}{(8b + 27\beta)^2}\left(u - \frac{3}{4}\right),$$

and

$$\frac{\partial \chi^{MB}}{\partial b} = \frac{24\alpha}{(8b + 27\beta)^2}\left(u - \frac{3}{4}\right).$$

Hence, the signs of these derivatives are also determined by the sign of $u - 3/4$. If $u > 3/4$, then the output in the Latin (Greek) market decreases (increases) with $\beta$ ($b$). It is easy to see why: only if the willingness to pay in the Latin market is sufficiently large relative to that of the Greek market (i.e., $u > 3/4$), it is worthwhile responding to an increase of the Greek market size (i.e., a smaller $\beta$) with an increase of the output in the Latin market, thus reducing the transfer price and avoiding a large reduction of the sales of the subsidiary.

The equilibrium output in the Latin market satisfies

$$\lim_{\beta \to 0} q^{MB} = q^{C} + \frac{\alpha}{6b}\left(u - \frac{3}{4}\right) := q_0^{MB},$$

and

$$\lim_{\beta \to \infty} q^{MB} = q^{C}.$$

Thus, as the size of the Greek market becomes large (i.e., $\beta$ becomes small), the output in the Latin market is above or below the Cournot output depending on the sign of $u - 3/4$. If $u > 3/4$, then parents' incentives to increase their output in order to alleviate double marginalization remains as the size of the Greek market becomes arbitrarily large. When $u < 3/4$, however, parents reduce their output in the Latin market as a way to commit to high prices in the Greek market. Of course, as the size of the Greek market becomes arbitrarily small (i.e., $\beta$ approaches infinity), parents tend to ignore the double marginalization problem (as the profits in this market become negligible), and focus on the impact on their output decision in the Latin market, and their output approaches the Cournot output, independently of the sign of $u - 3/4$.

The equilibrium output in the Greek market satisfies

$$\lim_{b \to \infty} \chi^{MB} = \frac{\chi^M}{2},$$

and

$$\lim_{b \to 0} \chi^{MB} = \chi^C - \frac{a}{9\beta} = \frac{\chi^M}{2} - \frac{\alpha}{9\beta}\left(u - \frac{3}{4}\right) := \chi_0^{MB}.$$

Thus, as the size of the Latin market becomes arbitrarily small (i.e., $b$ approaches infinity), the revenues in this market become negligible, and parents' output decisions mainly serve the purpose of committing to high prices in the Greek market.

Interestingly, MB transfer pricing allows parents to attain perfect cooperation (i.e., they are able to sustain the monopoly outcome) when $b$ approaches infinity. In this case, MB transfer pricing is merely an instrument to avoid competition in the Greek market. When the size of the Latin market becomes arbitrarily large (i.e., $b$ approaches zero), however, revenues mainly come from the Latin market and therefore, parents tend to ignore the impact of double marginalization in the Greek market, producing the Cournot output in the Latin market. Double marginalization leads to an output below the Cournot output, and has its worst effects whenever $u > 3/4$, in which case output falls even below the monopoly output.

We summarize these results in Proposition 3.2.

**Proposition 3.2.** *Under market based transfer pricing:*

*(3.2.1) If* $1/(4+9s) < u < 3+2/3s$*, then the equilibrium is interior. In equilibrium: The output in the Latin market* $q^{MB}$ *is above or below the Cournot output, and decreases or increases with the size of the Greek market* $\beta$ *depending on whether* $u$ *is above or below* $3/4$*, i.e.,*

$$q^{MB} \gtreqless q^{C} = q^{NMB} \text{ and } \frac{\partial q^{MB}}{\partial \beta} \lesseqgtr 0 \text{ if and only if } u \gtreqless \frac{3}{4}.$$

*The output in the Greek market* $\chi^{MB}$ *is below the Cournot outcome, i.e.,*

$$\chi^{MB} < \chi^{C} < \chi^{NMB},$$

*and is below or above the monopoly output and increases or decreases with the size of the Latin market* $b$ *depending on whether* $u$ *is above or below* $3/4$*, i.e.,*

$$\chi^{MB} \lesseqgtr \frac{\chi^{M}}{2} \text{ and } \frac{\partial \chi^{MB}}{\partial b} \gtreqless 0 \text{ if and only if } u \gtreqless \frac{3}{4}.$$

*Further, as* $\beta$ *becomes large* $q^{MB}$ *approaches* $q^{C}$*, and as* $\beta$ *becomes small* $q^{MB}$ *approaches* $q_0^{MB}$*, where* $q_0^{MB} \gtreqless q^{C}$ *whenever* $u \gtreqless 3/4$*. And as* $b$ *becomes large* $\chi^{MB}$ *approaches* $\frac{\chi^{M}}{2}$*, and as* $b$ *becomes small* $\chi^{MB}$ *approaches* $\chi_0^{MB} < \chi^{C}$*, where* $\chi_0^{MB} \gtreqless \frac{\chi^{M}}{2}$ *whenever* $u \lesseqgtr 3/4$*.*

*(3.2.2) If* $u \leq 1/(4+9s)$*, then equilibrium outputs are* $q^{MB} = 0$ *and* $\chi^{MB} = (\alpha - a)/3\beta$*. And if* $u \geq 3+2/3s$*, then equilibrium outputs are* $q^{MB} = q^{C}$ *and* $\chi^{MB} = 0$*.*

Let us analyze the profits under MB transfer pricing. In an interior equilibrium firms' total profits can be calculated using (3.8) as

$$\Pi_L^{MB} + \Pi_G^{MB} = \Pi_L^{NMB} + \Pi_G^{NMB} + \frac{b^2\alpha^2\bar{\Pi}}{64b^2\beta + 432b\beta^2 + 729\beta^3},$$

where

$$\bar{\Pi} = -\left(30s^2 + \frac{64}{9}s\right)u^2 + \left(8s + 36s^2\right)u + \frac{567}{25}s^2 + \frac{436}{25}s + \frac{72}{25}.$$

Write

$$\phi(s) = \frac{810s^2 + 180s + \sqrt{2}\,(24+81s)\sqrt{155s^2+36s}}{10s\,(135s+32)},$$

for the value of $u$ that solves $\bar{\Pi} = 0$ given $s$. Then we have $\bar{\Pi} \gtreqless 0$, and therefore $\Pi_L^{MB} + \Pi_G^{MB} \gtreqless \Pi_L^{NMB} + \Pi_G^{NMB}$, whenever $u \lesseqgtr \phi(s)$.

The dashed curve in Figure 3.1 above displays the function $\phi$. (Recall that the thin and thick curves represent the functions $l$ and $g$, respectively.) For the equilibrium to be interior, the values of $s$ and $u$ must lie between these two curves. Note $\phi$ is decreasing in $s$ and

$$\lim_{s\to\infty}\phi(s)=\frac{3}{5}\left(1+\frac{\sqrt{310}}{10}\right):=\phi_\infty\simeq 1.6564.$$

Thus, when equilibrium is interior and $u$ is below $\phi_\infty$ total profits under MB transfer pricing are greater than under NMB transfer pricing even if the size of the Greek market is small relative to that of the Latin market (i.e., $s$ is large).

We also examine total profits at corner equilibria. When $u \geq g(s)$, then firms' equilibrium outputs are $q^{MB}=q^{C}=q^{NMB}$ and $\chi^{MB}=0<\chi^{NMB}$. Hence total profits are

$$\Pi_L^{MB}+\Pi_G^{MB}=\Pi_L^{NMB}+0<\Pi_L^{NMB}+\Pi_G^{NMB}.$$

When $u \leq l(s)$, then firms' equilibrium outputs are $q^{MB}=0<q^{NMB}$ and $\chi^{MB}=\frac{(\alpha-a)}{3\beta}<\chi^{C}<\chi^{NMB}$. Hence total profits are

$$\Pi_G^{MB}=\Pi_L^{NMB}+\Pi_G^{NMB}+\frac{\alpha^2\hat{\Pi}}{225\beta},$$

where

$$\hat{\Pi}=7-25u\left(2u+su-1\right).$$

Hence, we have $\hat{\Pi}\gtreqless 0$, and therefore $\Pi_L^{MB}+\Pi_G^{MB}\gtreqless\Pi_L^{NMB}+\Pi_G^{NMB}$, whenever

$$u\lesseqgtr\frac{5+\sqrt{28s+81}}{20+10s}:=\hat{\phi}(s).$$

Since

$$l(s)-\hat{\phi}(s)<0,$$

for all $s$, then $u\leq l(s)$ implies

$$u<\hat{\phi}(s),$$

and therefore

$$\Pi_L^{MB}+\Pi_G^{MB}>\Pi_L^{NMB}+\Pi_G^{NMB}.$$

Thus, in corner equilibria that arise when the willingness to pay in the Latin market relative to that of the Greek market $u$ is small (i.e., when $u \leq l(s) < 1/4$) firms' total profits under MB transfer pricing are greater than under NMB, whereas in the corner equilibria that arise when $u$ is large (i.e., when $u \geq g(s) > 3$), firms' total profits under MB transfer pricing are smaller than under NMB transfer pricing.

In summary, for parameter constellations $(s, u)$ that lie below (above) the graph of $\phi$ (the dashed curve in Figure 3.1) firms' profits under MB transfer pricing are above (below) their profits under NMB transfer pricing. Proposition 3.3 summarizes our results.

**Proposition 3.3.** *Total profits under market based transfer pricing are above or below total profits under non-market based transfer pricing depending on whether $u$ is above or below $\phi(s)$; i.e.,*

$$\Pi_L^{MB} + \Pi_G^{MB} \gtreqless \Pi_L^{NMB} + \Pi_G^{NMB} \text{ if and only if } u \lesseqgtr \phi(s).$$

*In particular, if $u < \phi_\infty \simeq 1.6564$, then total profits under market based transfer pricing are above total profits under non-market based transfer pricing.*

Let us study the total surplus under MB transfer pricing. In an interior equilibrium we calculate the surplus in the Latin market under MB transfer pricing using equation (3.5) as

$$S_L^{MB} = S_L^{NMB} + \frac{8\alpha\left(27a\beta + b\left(4a + 3\alpha\right)\right)}{9\left(8b + 27\beta\right)^2}\left(u - \frac{3}{4}\right).$$

Therefore $S_L^{MB} \gtreqless S_L^{NMB}$ whenever $u \gtreqless 3/4$. Using again equation (3.5), we calculate the surplus in the Greek market under MB transfer pricing as

$$S_G^{MB} = S_G^{NMB} - \frac{6}{25}\frac{\left(5a\beta + \alpha\left(2b + 3\beta\right)\right)\left(15a\beta + 2\alpha\left(7b + 18\beta\right)\right)}{\beta\left(8b + 27\beta\right)^2}.$$

Hence $S_G^{MB} < S_G^{NMB}$.

Thus, in an interior equilibrium the comparison of total surplus under MB and NMB transfer pricing is as follows: if $u \leq 3/4$, then the surplus under MB transfer pricing is below the surplus under NMB transfer pricing in both markets, and so is total surplus, i.e., $S_L^{MB} + S_G^{MB} < S_L^{NMB} + S_G^{NMB}$. If $u > 3/4$, then we have $S_L^{MB} > S_L^{NMB}$, but $S_G^{MB} < S_G^{NMB}$.

Thus, the comparison of total surplus under MB and NMB transfer pricing is ambiguous. We have

$$S_L^{MB} + S_G^{MB} = S_L^{NMB} + S_G^{NMB} + \frac{2b^2\alpha^2\bar{S}}{225\beta\,(8b + 27\beta)^2},$$

where

$$\bar{S} = 25s\,(27s + 16)\,u^2 - 2700s\,(3s + 1)\,u - 2916s^2 - 3303s - 756.$$

Write

$$(s) = \frac{4050s^2 + 15\,(27s + 8)\sqrt{7s\,(16s + 3)} + 1350s}{400s + 675s^2}.$$

for the solution to the equation $\bar{S} = 0$ given $s$. Hence $\bar{S} \gtreqless 0$, and therefore $S_L^{MB} + S_G^{MB} \gtreqless S_L^{NMB} + S_G^{NMB}$, whenever $u \gtreqless \ \ (s)$.

The dashed curve in Figure 3.2 displays the function  . (Here again the thin and thick curves in Figure 3.2 represents the functions $l$ and $g$, respectively. Recall that the equilibrium is interior under MB transfer pricing for parameter constellations $(s, u)$ lying between these two curves.)

The minimum value of  is $\underline{\ \ } = \frac{27}{20}\sqrt{7} + 6 \simeq 9.5718$. Thus, for $u < \underline{\ \ }$ the total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing. Only for parameter constellations $(s, u)$ satisfying $\ \ (s) < u < g(s)$ we have

$$S_L^{MB} + S_G^{MB} > S_L^{NMB} + S_G^{NMB}.$$

As Figure 3.2 illustrates, these parameter constellations involve a large willingness to pay in the Latin market relative to that of the Greek market $u$ (larger than $249/25 \simeq 9.96$), and a small size of the Greek market relative to that of the Latin market $s$ (smaller than $25/261 \simeq .095$), and form a small subset of the parameter space.

Let us examine the total surplus at corner equilibria. If $u \geq g(s)$, then firms' equilibrium outputs are $q^{MB} = q^C = q^{NMB}$ and $\chi^{MB} = 0 < \chi^{NMB}$, and the total surplus satisfies

$$S_L^{MB} + S_G^{MB} = S_L^{NMB} + 0 < S_L^{NMB} + S_G^{NMB}.$$

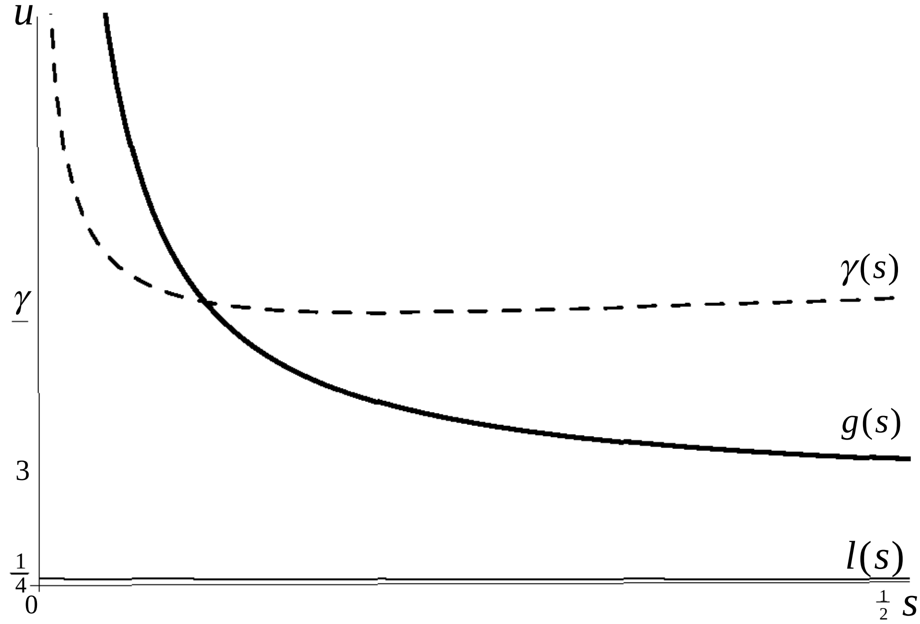


Figure 3.2. Total welfare under MB and NMB transfer pricing.

If $u \leq l(s)$, then firms' equilibrium outputs are $q^{MB} = 0 < q^{NMB}$ and $\chi^{MB} = \frac{(\alpha - a)}{3\beta} < \chi^{C} < \chi^{NMB}$. Hence $S_L^{MB} = 0$ and $S_G^{MB} < S_G^{NMB}$. Therefore

$$S_L^{MB} + S_G^{MB} < S_L^{NMB} + S_G^{NMB}.$$

Thus, in every corner equilibrium the total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing.

The total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing except for the small set of parameter constellations $(s, u)$ in the area below the graph of $g$ and above the graph of , i.e., for $(s, u)$ satisfying $(s) < u < g(s)$. As Figure 3.2 illustrates, for these parameter constellations the increment in surplus due to the increment in output in the Latin market under MB transfer pricing relative to that under NMB transfer pricing, $q^{MB} > q^{C} = q^{NMB}$, more than compensates the reduction in surplus due to the reduction of the output in the Greek market, $\chi^{MB} < \chi^{C} < \chi^{NMB}$. Proposition 3.4 states these results.

**Proposition 3.4.** *The total surplus under market based transfer pricing is typically smaller than under non-market based transfer pricing. Specifically, only if* $(s, u)$ *satisfies*

$$(s) < u < g(s)$$

*is the total surplus under market based transfer pricing larger than under non-market based transfer pricing. This condition requires that the maximum willingness to pay in the Latin market relative to that of the Greek market* $u$ *be large (larger than 9.95) and the size of the Latin market relative to that of the Greek market* $s$ *be small (smaller than 0.095).*

MB transfer pricing provides parent firms with an instrument to limit aggressive competition in the Greek market, and may allow them to induce an outcome close to the monopoly outcome when the size of the Greek market relative to that of the Latin market is large. Of course, since a parent influences its transfer price only via its output decision in the Latin market, competition in this market may be more aggressive than under NMB transfer pricing, provided the maximum willingness to pay in this market is not too small compared to that of the Greek market. For some parameter constellations, total profits under MB transfer pricing are above that under NMB transfer pricing. Thus, under quantity competition the Arm's Length Principle provides a rationale for vertical separation. However, total surplus under MB transfer pricing is typically below that under NMB transfer pricing, which raises some questions about the use of the *ALP* as a guideline for regulating transfer prices.

### 3.5. Market Based Transfer Pricing with Discounts

In order to discuss the consequences of a lax application of the *ALP*, we consider an alternative setting where transfer prices are market based, but parents apply *discounts* to their subsidiaries. Such practices are common. Baldenius, Melumad, and Reichelstein (2004) argue that this is a frequent practice, which is justified due to cost differences between internal and external transactions. Bernard, Jensen and Schott (2006) examine U.S. international export transaction between 1993 and 2000, and find that prices of U.S. exports are substantially larger than transfer prices for subsidiaries. In addition, they find that the wedge between the market prices and related-party prices is negatively correlated with destination-country corporate tax rates, and positively correlated with both destination-country import tariffs and other characteristics indicating greater market power. Baldenius and Reichelstein (2005) also cite examples of firms adjusting prevailing market prices for internal transfers. Of course, failure to comply with the Arm's Length Principle may result in penalties, which firms may have to optimally trade off. We abstract away from penalties, and focus our analysis on the strategic consequences of a lax application of the *ALP*.

In our setting, each parent firm chooses simultaneously its output in the Latin market as well as the discount that will apply to its subsidiary. Then each subsidiary, knowing the price in the Latin market, its own discount and that of its rival, competes in quantities in the Greek market.[31]

The equilibrium under this scheme of market based transfer pricing with discounts (MBD) is determined as follows. Assuming that the price in the Latin market is $p \in \mathbb{R}_+$ and discounts are $(\delta_1, \delta_2) \in \mathbb{R}_+^2$, each subsidiary $i \in \{1, 2\}$ chooses its output $\chi_i$ to solve the problem

$$\max_{\chi_i \in \mathbb{R}_+} (\rho^d (\chi_1 + \chi_2) - (p - \delta_i))\chi_i,$$

Here the term $p - \delta_i$ is the constant marginal cost of subsidiary $i$. Using the equation (3.1), we calculate the equilibrium outputs in the Greek market as a function of the price in the Latin market and the parents' discounts, which are given by

$$\chi_i^* = \tilde{\chi}_i(p, \delta_1, \delta_2) = \frac{\alpha - p + 2\delta_i - \delta_{3-i}}{3\beta}.$$

Parent firm $i$, anticipating the outputs and market price in the Greek market, chooses its

[31] Arya and Mittendorf (2008) analyze transfer pricing policy as a strategic response to external competition in a similar setting. In their model, however, discounts are set prior to the stage of competition in the Latin market.

outputs $q_i$ and its discount $\delta_i$ in order to solve the problem

$$\max_{(q_i,\delta_2)\in\mathbb{R}^2_+} p^d(q_1+q_2)q_i+\rho^d(\tilde{\chi}_1\left(p^d(q_1+q_2),\delta_1,\delta_2\right)+\tilde{\chi}_2\left(p^d(q_1+q_2),\delta_1,\delta_2\right))\tilde{\chi}_i\left(p^d(q_1+q_2),\delta_1,\delta_2\right)$$

Solving the system of equations formed by the first-order conditions for profit maximization of parents 1 and 2 we obtain their outputs and discounts in an *interior* equilibrium. In the Latin market, parents' outputs are

$$q_1^* = q_2^* = \frac{a}{3b} - \frac{\alpha}{15\beta} := q^{MBD},$$

and the market price is

$$p^d(2q^{NMD}) = \frac{a}{3} + \frac{2}{15}\frac{b\alpha}{\beta} := p^{MBD}, \tag{3.13}$$

Equilibrium discounts are

$$\delta_1^* = \delta_2^* = \frac{5a\beta + 2b\alpha + 3\alpha\beta}{15\beta} := \delta^*. \tag{3.14}$$

and thus, transfer prices are given by

$$p^{MBD} - \delta^* = -\frac{\alpha}{5}.$$

Note that transfer prices are negative, i.e., transfer prices are below marginal cost. Substituting these values into the equation above, we obtain the subsidiaries' outputs

$$\tilde{\chi}_i\left(p^{MBD},\delta^*,\delta^*\right) = \frac{2}{5}\frac{\alpha}{\beta} := \chi^{MBD}.$$

The market price in the Greek market is

$$\rho^d(2\chi^{MBD}) = \frac{\alpha}{5} := \rho^{MBD}.$$

For the equilibrium to be interior we must have

$$\frac{a}{\alpha} > \frac{b}{5\beta},$$

i.e.,

$$u > h(s) := \frac{1}{5s}. \tag{3.15}$$

If $u \leq h(s)$, then in equilibrium $q^{MBD} = 0$ and $\chi^{MBD} = \frac{2}{5}\frac{\alpha}{\beta}$. The solid curve in Figure 3.3 below represents the function $h$ and the area above the graph of $h$ corresponds to the parameter constellations $(s,u)$ for which the equilibrium is interior.

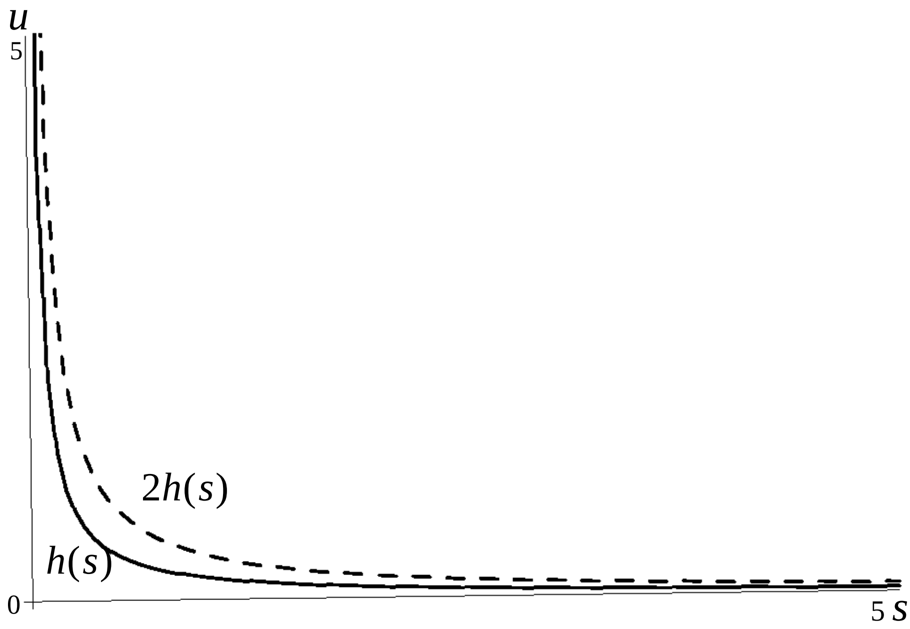


Figure 3.3. Total profits under MBD and NMB transfer pricing.

Using again (3.3) and (3.10), we can rewrite the expression for firms' output in the Latin market as

$$q^{MBD} = q^C - \frac{1}{5}\chi^C,$$

and the output in the Greek market as

$$\chi^{MBD} = \frac{6}{5}\chi^C.$$

Since $q^{NMB} = q^C$ and $\chi^{NMB} = \frac{6}{5}\chi^C$ by Proposition 3.1, then $q^{MBD} < q^{NMB}$ and $\chi^{MBD} = \chi^{NMB}$; that is, under MBD transfer pricing the output in the Latin (Greek) market is below (equal to) the output under NMB transfer pricing.

It is also interesting to compare the output under MBD and MB transfer pricing. We have

$$\begin{aligned} q^{MB} - q^{MBD} &= \frac{4\alpha}{3\,(8b+27\beta)}\left(u - \frac{3}{4}\right) + \frac{1}{5}\chi^{C} \\ &= \frac{4}{15}\frac{\alpha}{\beta\,(8b+27\beta)}\,(2b+3\beta+5u\beta) \\ &> 0, \end{aligned}$$

i.e., $q^{MB} > q^{MBD}$. Also, propositions 1 and 2 and the results above imply $\chi^{MBD} > \chi^{MB}$. Hence the equilibrium outcome in the Latin (Greek) market is less (more) competitive under MBD than under MB transfer pricing; i.e., a lax application of the *ALP* makes competition softer (more aggressive) in the parents' (subsidiaries') market.

Discounts open up the possibility to gain a Stackelberg advantage in the Greek market, and bring back a prisoner's dilemma analogous to that firms face under NMB transfer pricing. Under MBD transfer pricing, however, parents' output decisions in the two markets are not independent: a parent by reducing its output in the Latin market and simultaneously increasing its discount, rises the marginal cost of its subsidiary's rival without affecting the marginal cost of its own subsidiary. Therefore, linking the cost of its subsidiary's rivals to the price in the Latin market makes competition more aggressive in the Greek market and less aggressive in the Latin market. In fact, when condition (3.15) does not hold, parents choose to completely shut down the Latin market. Note that a parent's incentive to reduce its output in order to increase the transfer price of its subsidiary's rival increases with both the maximum willingness to pay and the size of the Greek market relative to those of the Latin market. These results are stated in Proposition 3.5.

**Proposition 3.5.** *Under market based transfer pricing with discounts, the output in the Greek market is*

$$\chi^{MBD} = \frac{6}{5}\chi^{C} = \chi^{NMB} > \chi^{MB}.$$

*Moreover, if* $u > 1/5s$, *then the output in the Latin market is*

$$q^{MBD} = q^{C} - \frac{1}{5}\chi^{C} < q^{NMB},$$

*satisfies* $q^{MBD} < q^{MB}$, *and approaches* $q^{C}$ *as* $\beta$ *becomes large and/or* $\alpha$ *becomes small, and if* $u \leq 1/5s$, *then* $q^{MBD} = 0$.

Let us study profits under MBD transfer pricing. If $u > h(s)$, then the equilibrium is interior and we can calculate firms' profits in the Latin market under MBD transfer pricing using (3.8) as

$$\begin{aligned} \Pi_L^{MBD} &= \Pi_L^{NMB} + \frac{\alpha^2}{45\beta}\left(u - \frac{2}{5s}\right) \\ &= \Pi_L^{NMB} + \frac{\alpha^2}{45\beta}\left(u - 2h(s)\right). \end{aligned}$$

Since, $\Pi_G^{MBD} = \Pi_G^{NMB}$, we have $\Pi_L^{MBD} + \Pi_G^{MBD} \lesseqqgtr \Pi_L^{NMB} + \Pi_G^{NMB}$ if and only if $u \lesseqqgtr 2h(s)$.

If $u \le h(s)$, then in equilibrium $q^{MBD} = 0 < q^{NMBD}$ and $\chi^{MBD} = \frac{2}{5}\frac{\alpha}{\beta} = \chi^{NMB}$. Hence

$$\Pi_L^{MBD} + \Pi_G^{MBD} = 0 + \Pi_G^{NMB} < \Pi_L^{NMB} + \Pi_G^{NMB}.$$

Therefore $\Pi_L^{MBD} + \Pi_G^{MBD} < \Pi_L^{NMB} + \Pi_G^{NMB}$ if and only if $u < 2h(s)$. The dashed curve in Figure 3.3 displays the function $2h$. Parameter constellations $(s, u)$ that lie above (below) this curve correspond to those for which total profits under MBD transfer pricing are greater than (less than or equal to) total profits under NMB transfer pricing. This result is established in Proposition 3.6.

**Proposition 3.6.** *Under market based transfer pricing with discounts, total profits are above (below) total profits under non-market based transfer pricing whenever $u$ is above (below) $2h(s)$.*

Finally, we study the total surplus under MBD transfer pricing. If the equilibrium is interior, i.e., if $u > h(s)$, then the surplus in the Latin market is

$$S_L^{MBD} = S_L^{NMB} - \frac{2}{45}\frac{\alpha^2}{\beta}\left(u + \frac{1}{5s}\right).$$

Hence, $S_L^{MBD} < S_L^{NMB}$. Since $S_G^{MBD} = S_G^{NMB}$, we have

$$S_L^{MBD} + S_G^{MBD} < S_L^{NMB} + S_G^{NMB}.$$

In a corner equilibrium, i.e., when $u \le h(s)$, we have $q^{MBD} = 0 < q^{NMB}$ and $\chi^{MBD} = (6/5)\chi^C = \chi^{NMB}$, and therefore

$$S_L^{MBD} + S_G^{MBD} = 0 + S_G^{NMB} < S_L^{NMB} + S_G^{NMB}.$$

Hence total surplus under MBD transfer pricing is unambiguously below total surplus under NMB transfer pricing. This result is stated in Proposition 3.7.

**Proposition 3.7.** *Under market based transfer pricing with discounts, total surplus is unambiguously below total surplus under non-market based transfer pricing.*

In summary, market based transfer pricing with discounts generates a subtle link between markets that softens competition in the home market as each parent attempts to increase the transfer price of its subsidiary's rivals in order to gain a competitive advantage in the external market.

### 3.6. Conclusions

While a regulatory policy requiring that transfer prices be consistent with the Arm's Length Principle does not affect market outcomes under perfect competition, in imperfectly competitive markets with vertically separated firms it modifies the strategic nature of firms' interactions and ultimately has an impact on market outcomes. Specifically, the application of the $ALP$ serves as a commitment device that softens competition. When the $ALP$ is applied rigorously, the result is a softer competition in the subsidiaries' (external) market that is not compensated for by a more aggressive competition in the parents' (home) market. A more lax application of the $ALP$ softens competition in the home market. Interestingly, vertical separation, an organizational structure whose motivation is not well understood in the absence of frictions, may be justified under transfer pricing policies based on the $ALP$.

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