



TESIS DOCTORAL

Evolutionary Games in Networked Populations: Models and Experiments

Autor:

Alberto Antonioni

Directores:

Prof. Angel Sánchez

Prof. Marco Tomassini

DEPARTAMENTO/INSTITUTO

Departamento de Matemáticas

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Autor: *Alberto Antonioni*

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Prof. Marco Tomassini

Firma del Tribunal Calificador:

Firma

Presidente: Dr. D. José Antonio Cuesta Ruiz

Vocal: Dr. D. Daniele Vilone

Secretario: Dr. D. Maxi San Miguel

Calificación:

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Abstract

“All truths are easy to understand once they are discovered; the point is to discover them.”

Galileo Galilei, Italian physicist and mathematician.

Cooperation and coordination is a desirable behavior that is fundamental for the harmonious development of society. People need to rely on cooperation with other individuals in many aspects of everyday life, such as teamwork and economic exchange in anonymous markets. However, cooperation may easily fall prey to exploitation by selfish individuals who only care about short-term gain. For cooperation to evolve, specific conditions and mechanisms are required, such as kinship, direct and indirect reciprocity through repeated interactions, or external interventions such as punishment.

In this dissertation we investigate the effect of the network structure of the population on the evolution of cooperation and coordination. We consider several kinds of static and dynamical network topologies, such as Barabási–Albert, social network models and spatial networks. We perform numerical simulations and laboratory experiments using the Prisoner’s Dilemma and coordination games in order to contrast human behavior with theoretical results.

We show by numerical simulations that even a moderate amount of random noise on the Barabási–Albert scale-free network links causes a significant loss of cooperation, to the point that cooperation almost vanishes altogether in the Prisoner’s Dilemma when the noise rate is high enough. Moreover, when we consider fixed social-like networks we find that current models of social networks may allow cooperation to emerge and to be robust at least as much as in scale-free networks. In the framework of spatial networks, we investigate whether cooperation can evolve and be stable when agents move randomly or performing Lévy flights in a continuous space. We also consider discrete space adopting purposeful mobility and binary birth-death process to discover emergent cooperative patterns. The fundamental result is that cooperation may be enhanced when this migration is opportunistic or even when agents follow very simple heuristics.

In the experimental laboratory, we investigate the issue of social coordination between individuals located on networks of contacts. In contrast to simulations, we find that human players dynamics do not converge to the efficient outcome more often in a social-like network than in a random network. In another experiment, we study the behavior of people who play a pure coordination game in a spatial environment in which they can move around and when changing convention is costly. We find that each convention forms homogeneous clusters and is adopted by approximately half of the individuals. When we provide them with global information, i.e., the number of subjects currently adopting one of the conventions, global consensus is reached in most, but not all, cases. Our results allow us to extract the heuristics used by the participants and to build a numerical simulation model that agrees very well with the experiments. Our findings

have important implications for policymakers intending to promote specific, desired behaviors in a mobile population.

Furthermore, we carry out an experiment with human subjects playing the Prisoner's Dilemma game in a diluted grid where people are able to move around. In contrast to previous results on purposeful rewiring in relational networks, we find no noticeable effect of mobility in space on the level of cooperation. Clusters of cooperators form momentarily but in a few rounds they dissolve as cooperators at the boundaries stop tolerating being cheated upon. Our results highlight the difficulties that mobile agents have to establish a cooperative environment in a spatial setting without a device such as reputation or the possibility of retaliation. i.e. punishment.

Finally, we test experimentally the evolution of cooperation in social networks taking into account a setting where we allow people to make or break links at their will. In this work we give particular attention to whether information on an individual's actions is freely available to potential partners or not. Studying the role of information is relevant as information on other people's actions is often not available for free: a recruiting firm may need to call a job candidate's references, a bank may need to find out about the credit history of a new client, etc. We find that people cooperate almost fully when information on their actions is freely available to their potential partners. Cooperation is less likely, however, if people have to pay about half of what they gain from cooperating with a cooperator. Cooperation declines even further if people have to pay a cost that is almost equivalent to the gain from cooperating with a cooperator. Thus, costly information on potential neighbors' actions can undermine the incentive to cooperate in dynamical networks.

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“Knowledge is in the end based on acknowledgement.”

Ludwig Wittgenstein, Austrian philosopher.

All the knowledge I have acquired during these four years has been an appreciated gift from people to whom I am profoundly grateful. The path to knowledge is challenging but going through it with others is rewarding and efficient. My grandfather always says *La vita ti insegna tutto* and life is a constant process of learning from the ones you are surrounded by.

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Grazie,
Alberto

Lausanne, March 2015

Introduction and Outline

*“Science is organized knowledge.
Wisdom is organized life.”*

Immanuel Kant, German philosopher.

Motivation

Cooperative behavior is at the basis of the harmonious development of animal and human societies. In this context, the Prisoner’s Dilemma game is a widely used paradigm for the investigation of how cooperation might evolve in a population of self-regarding agents. In fact, game-theoretical results predict defection as a Nash equilibrium or as a stable state of the population dynamics. In spite of this, non-negligible amounts of cooperative behavior can be observed daily in the animal kingdom, in the human society, and also in the laboratory, where controlled experiments can be carried out. During the last three decades it has been shown that the evolution of cooperation among agents that pursue their own interest is possible if certain conditions are satisfied. Many mechanisms have been suggested to explain these behaviors, such as direct and indirect reciprocity, kin reciprocity, group reciprocity, and network reciprocity, among others. Network reciprocity considers the structure of the interacting population as a device to promote cooperation and is one of the simplest factors that can change the generalized defection outcome with respect to the well-mixed population case. The population structure of the interacting agents can be generically represented by a relational graph in which two directly linked vertices stand for two interacting agents. This locality of contacts means that only pairs or groups of individuals that are direct neighbors play the game among themselves. This network reciprocity has received much attention in the last decade because it requires minimal rationality and behavioral assumptions to obtain, at least theoretically, the emergence of cooperative behavior.

This dissertation mainly deals with numerical simulation models and human laboratory experiments which investigate the effect of the population structure, usually represented as a network of contacts, on the evolution of cooperation in social and spatial environments. In particular, we are interested in discovering the underlying mechanisms that can allow cooperation to emerge and to spread in a population of selfish agents. Experiments have an important role in this framework. In fact, they allow us to understand better human behavior and to compare theoretical results with empirical observations in order to obtain more reliable models. Numerical simulations are also very useful for the scope of this work since they allow us to explore larger size systems and long-term scenarios which cannot be tested in the experimental laboratory. Particular attention is also given to the issue of coordination in a networked society. In this framework, we want to understand how conventions evolve and spread in a social context.

Plan

The thesis is organized as a collection of papers, whose layout has been made uniform but whose content has not been further edited after publication. In order to avoid repetitions and gain in cohesion, not all the articles that have been published during the doctoral program have been included. The dissertation is organized as follows:

Part I: State of the Art

Chapter 1 provides an informal presentation of the main concepts of Game Theory. Particular focus is given to the explanation of the three most important classes of two-player two-strategy symmetric games: the Prisoner's Dilemma, the Stag Hunt and the Hawk-Dove game. Evolutionary Game Theory is also introduced showing some examples of evolutionary processes, such as replicator dynamics. In Chapter 2 we present complex network theory describing the main features of network construction models used in the literature and in this dissertation. Finally, Chapter 3 provides an introduction of Experimental Game Theory focusing on laboratory experiments in which human subjects play the Prisoner's Dilemma or coordination games on static and dynamical networks.

Part II: Numerical Simulation Models

In this part we introduce the numerical simulation results of this dissertation. Chapter 4 presents a study on the influence of random network fluctuations on the behavior of evolutionary games on Barabási-Albert networks. The paper in Chapter 5 shows how cooperation may evolve and be stable in models of social-like networks. Chapter 6 presents a systematic analysis of population games in two-dimensional Euclidean space and in the presence of agent random mobility. In Chapter 7 we study the effect of unconstrained and contingent Lévy flights on the evolution of cooperation in games played on diluted two-dimensional grids. Chapter 8 focuses on evolutionary games in a diluted grid environment in which agents strategically interact locally but can opportunistically move to other positions within a given migration radius. In Chapter 9 we introduce a binary birth-death dynamics process in evolutionary games played in diluted grids.

Part III: Laboratory Experiments

This part presents the results on games played by human subjects in the experimental laboratory. The work in Chapter 10 focuses on the behavior of people when they play a Stag Hunt game on a cliquish network and on a regular random network. Chapter 11 presents a study on the behavior of people who play a pure coordination game in a spatial environment in which they can move around and when changing convention is costly. In Chapter 12 we analyze the role of short-range mobility on the evolution of cooperation in a laboratory experiment in which people are embedded in a virtual diluted grid. In Chapter 13 we study whether and how the ability to make or break links in social networks fosters cooperation, paying particular attention to whether information on an individual's actions is freely available to potential partners.

Part IV: Concluding Remarks

In this last part, Chapter 14 summarizes the main contributions, briefly discusses open questions and suggests some possible ways forward. A comprehensive list of publications carried out during this doctoral research, including those that are not part of this manuscript, is presented.

Timeline

The exposition of the matter mainly follows a chronological path, which in turn, was led by research questions and uncovered issues. At first, there has been the will to understand how network fluctuations influence the evolution of cooperation in dynamical scale-free networks. This has been also studied in social-like network models which represent a better approximation of actual social networks. At the same time, we investigated the influence of the network structure in coordination games in a laboratory experiment. Then, our attention has been attracted to spatial networks and we studied evolutionary games in spatial environments. This work led to one of the first explorations in this topic of spatial games. In particular, we have investigated the effect of random and contingent mobility using several heuristics, such as random walks, Lévy flights, opportunistic migration and birth-death processes, for the population of agents. Then, we have been interested in testing theoretical models with human behavior and we have performed two experiments considering for the first time in the literature a diluted grid in the laboratory. Finally, we considered again a dynamical relational network to test how purposeful link rewiring can help cooperation to evolve in a social context.

Contents

Jury	iii
Abstract	v
Acknowledgements	vii
Introduction and Outline	ix
Part I State of the Art	
1 The Theory of Games	3
1.1 Prisoner's Dilemma	6
1.2 Coordination games	7
1.3 Conflictual games	8
1.4 Evolutionary Game Theory	9
1.4.1 Replicator dynamics	10
1.4.2 Other selection dynamics	11
References	12
2 Complex Networks	13
2.1 Network topologies	14
2.1.1 Random graphs	14
2.1.2 Barabási–Albert model	15
2.1.3 Social networks	16
2.1.4 Spatial networks	17
2.2 Games on networks	18
References	19
3 Experimental Game Theory	21
3.1 Coordination on static networks	23
3.2 Cooperation on static networks	24
3.3 Cooperation on dynamical networks	25
References	25

Part II Numerical Simulation Models

4	Network Fluctuations Hinder Cooperation in Evolutionary Games	29
4.1	Introduction	29
4.1.1	Games on networks	31
4.2	Results	32
4.2.1	Sequence of random Barabási–Albert networks	32
4.2.2	Fluctuations from network edge swap	34
4.2.3	Fluctuations from edge rewiring	36
4.2.4	Network and strategy noise	38
4.3	Discussion	39
4.4	Methods	40
	References	42
5	Cooperation on Social Networks and Its Robustness	45
5.1	Introduction	45
5.2	Games Studied	46
5.3	Evolutionary Games on Networks	47
5.3.1	Population structure	48
5.3.2	Payoff calculation and strategy revision rules	48
5.3.3	Strategy update timing	49
5.3.4	Simulation parameters	49
5.4	Network Construction and Properties	50
5.5	Results	52
5.5.1	Replicator dynamics	53
5.5.2	Imitation of the best	55
5.5.3	Fermi rule	57
5.5.4	Network links fluctuations	58
5.6	Conclusions	59
	References	60
6	Random Diffusion and Cooperation in Continuous Two-Dimensional Space	63
6.1	Introduction and Previous Work	63
6.2	Model Description	65
6.2.1	The Spatial Environment	65
6.2.2	Games Studied	66
6.2.3	Agent and Population Dynamics	67
6.3	Constant Velocity Results and Discussion	68
6.4	Viscosity and Non-Constant Velocity	72
6.5	Conclusions	75
	References	76

7	Lévy Flights and Cooperation Among Mobile Individuals	79
7.1	Introduction and previous work	79
7.2	Model description	81
7.2.1	Games studied	81
7.2.2	Agent and population dynamics	81
7.2.3	Simulation settings	82
7.3	Results	82
7.3.1	The effect of density	85
7.3.2	The effect of noise	86
7.4	Discussion	88
	References	89
8	Opportunistic Migration in Spatial Evolutionary Games	93
8.1	Introduction	93
8.2	Methods	95
8.2.1	The Games Studied	95
8.2.2	Population Structure	96
8.2.3	Payoff Calculation and Strategy Update Rules	96
8.2.4	Population Dynamics and Opportunistic Migration	97
8.2.5	Simulation Parameters	97
8.3	Results	98
8.3.1	Imitation of the Best and Opportunistic Migration	98
8.3.2	Opportunistic Migration and Noisy Imitation	100
8.4	Discussion and Conclusions	102
	References	103
9	Binary Birth-Death Dynamics and the Expansion of Cooperation by means of Self-Organized Growth	105
9.1	Introduction	105
9.2	Model	106
9.3	Results	107
9.4	Discussion	111
	References	112

Part III Laboratory Experiments

10	Coordination on Networks: Does Topology Matter?	117
10.1	Introduction	117
10.1.1	Coordination games	118
10.1.2	Previous experimental results	120
10.2	Materials and Methods	121
10.2.1	Ethics statement	121
10.2.2	Network design	121
10.2.3	Specific coordination game	122
10.2.4	Implementation	124

10.3 Results	125
10.3.1 Aggregate behavior	125
10.3.2 Estimating individual behavior	125
10.4 Discussion	130
10.4.1 Influence of the network structure	130
10.4.2 Comparison with related experimental results	132
10.5 Conclusions	133
10.6 Supplementary Material	133
10.6.1 Instructions	133
References	136
11 Global Information and Mobility Support Coordination Among Humans	139
11.1 Introduction	139
11.2 Experimental Design	141
11.3 Results	142
11.4 Numerical Simulation Model	146
11.5 Discussion	149
11.6 Methods	150
11.7 Supplementary Information	151
11.7.1 Instructions form	151
11.7.2 Final states	156
11.7.3 Complementary results	158
11.7.4 Numerical simulation model	159
11.7.5 Simulation of the experimental setting	160
References	162
12 Short-Range Mobility and the Evolution of Cooperation: An Experimental Study	165
12.1 Introduction	165
12.2 Experimental Design	167
12.3 Results	168
12.4 Discussion	173
12.5 Methods	174
12.6 Supplementary Information	175
12.6.1 Instructions form	176
12.6.2 Complementary results	182
12.6.3 Typical snapshots	184
References	185
13 Know Thy Neighbor: Costly Information Can Hurt Cooperation in Dynamic Networks	189
13.1 Introduction	189
13.2 Experimental Setup	190
13.3 Results	192
13.4 Discussion	197
13.5 Methods	198

Contents	xvii
----------	------

13.6 Supporting Information	199
13.6.1 Supplementary Results	199
13.6.2 Final Network Topologies	201
13.6.3 Multivariate Regression Analyses	202
13.6.4 Linear Regressions	202
13.6.5 Rational behavior analysis	203
13.6.6 Instructions	205
References	212

Part IV Concluding Remarks

14 Conclusions	217
14.1 Evolutionary games on static and dynamical networks	217
14.2 Cooperation in spatial environments	220
14.3 Coordination in spatial environments	223
References	225

List of Publications	227
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Part I
State of the Art

Chapter 1

The Theory of Games

“Human cooperation must be regarded as a purely modern economic phenomenon – characterizing the last century – which has made such remarkable progress at the beginning of this one, to be present in every aspect of economic life.”

*Sandro Pertini, 7th President of the Italian Republic.
Tesi di Laurea “La Cooperazione”, 1924.*

In this chapter we present a brief introduction of the study of strategic decision making, i.e. the Theory of Games. We then introduce the three most representative two-strategy two-player symmetric games, the Prisoner’s Dilemma, the Stag Hunt, and the Hawk-Dove game, explaining their behavior and showing their Nash Equilibria. Finally, we move into the field of Evolutionary Game Theory where we offer a quick overview on the main evolutionary dynamics and their application in this dissertation.

The Theory of Games, also Game Theory, is a distinct and interdisciplinary approach to the study of human and animal behavior in interactive situations. It deals with social situations of conflict in which two or more individuals with opposed or at best mixed motives, must make decisions based on their best prediction of what the others will do [5]. Specifically, it is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory has important applications in almost all scientific fields but it is mainly used in economics, political science, and psychology, as well as logic, computer science, and biology. Game Theory applies to a wide range of behavioral relations, including both humans and non-humans, e.g. computers, animals.

The first formalization of Game Theory can be found in 1944 in the book by von Neumann and Morgenstern, *Theory of Games and Economic Behavior* [14]. At that time, this book was greeted with great enthusiasm and it was thought that a complete theory of strategic behavior had sprung into existence from nowhere. Moreover, the second edition of this book provided an axiomatic theory of expected utility, which allowed mathematical statisticians and economists to treat decision-making under uncertainty. However, it soon became apparent that von Neumann and Morgenstern had provided only the beginnings of an extremely vast theory. In fact, in the subsequent decades, much effort was devoted to developing cooperative and non-cooperative game theory, and three pioneering game theorists, John Nash, John Harsanyi, and Reinhard Selten, have been awarded in 1994 the Nobel Prize for Economics for their pioneering analysis of equilibria in the theory of non-cooperative games. The concept of Nash Equilibrium, developed in the early 1950s [6], had been revolutionary for this area, but it was not until the early 1970s that it was fully realized what a powerful tool Nash had provided in formulating the equilibrium concept that bears his name.

In Game Theory, the Nash Equilibrium (NE) is a solution concept of a non-cooperative game involving two or more players, in which no player has anything to gain by changing only their own strategy. If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitutes a NE. Informally, imagine there is a game between two people. If both play a strategy that happens to be a NE, then neither person can deviate from this strategy and increase his payoff [7].

We here consider the general payoff matrix between two strategies, A and B:

	A	B
A	(a, a)	(b, c)
B	(c, b)	(d, d)

Table 1.1 Generic form of a symmetric game with two players, the row player and the column player, and two possible strategies, A and B. Letters a, b, c and d are numerical values which represent players' utility or benefit. The first value x in the couple (x, y) is the payoff the row player gets when a certain combination of strategies is obtained. Conversely, the second value y represents the payoff for the column player.

We thus have the following criteria to find a Nash Equilibrium:

- (A, A) is a strict Nash Equilibrium if $a > c$.
- (A, A) is a Nash Equilibrium if $a \geq c$.
- (B, B) is a strict Nash Equilibrium if $d > b$.
- (B, B) is a Nash Equilibrium if $d \geq b$.

For instance, let us consider the following game:

	A	B
A	$(3, 3)$	$(0, 5)$
B	$(5, 0)$	$(1, 1)$

If both players choose A, then one player can improve his payoff by switching to B. If both play B, then neither player can improve his payoff by switching to A. Hence B is a NE. Note that A is dominated by B, that is, each player always gets more choosing B in respect to A, and that playing the NE, B, in this game leads to a lower payoff than playing the dominated strategy, A.

In a game with m strategies and n players, the strategy profile of player i can be represented as:

$$\sigma_i = (p_1, p_2, \dots, p_m)$$

where each p_j represents the probability of adopting strategy j ($\sum_j p_j = 1$). Each σ_i can be then associated to an expected utility $u_i(\sigma_i, \sigma_{-i})$ for player i as a function of other players' strategy profile, $\sigma_{-i} = (\sigma_1, \dots, \sigma_k, \dots, \sigma_n)_{k \neq i}$. Formally, a NE is a strategy profile σ_i^* which satisfies $u_i(\sigma_i^*, \sigma_{-i}) \geq u_i(\sigma_i, \sigma_{-i}), \forall \sigma_i$. This inequality must hold for all players in the population to be in a Nash Equilibrium state. Nash proved that if we allow mixed strategies, then every game with a finite number of players in which each player can choose from finitely many pure strategies has at least one Nash Equilibrium [6].

We now define the general form of the two-player two-strategy symmetric game, presenting the three most relevant ones. These games, the Prisoner's Dilemma (PD), the Stag Hunt (SH) and the Hawk-Dove (HD) game, represent common situations in everyday life and are simple metaphors for different kinds of dilemmas that arise when individual and social interests collide. The normal form of the symmetric two-player two-strategy game is given by the following payoff matrix:

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Table 1.2 Generic payoff matrix for the two-player two-strategy symmetric game. C and D are the possible strategies, and R, T, P , and S are players utility values.

In the matrix of Table 1.2, R stands for the *reward* the two players receive if they both cooperate (C), P is the *punishment* for bilateral defection (D), and T is the *temptation*, i.e. the payoff that a player receives if she defects while the other cooperates. In the latter case, the cooperator gets the *sucker's payoff* S . This payoff labelling comes from the PD game but is usually adopted for other games too. Payoff values may undergo any affine transformation without affecting neither the Nash equilibria, nor the dynamical fixed points; therefore, the parameters' values can be restricted to the "standard" configuration space defined by $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the resulting TS -plane [11], each game's space corresponds to a different quadrant depending on the ordering of the payoffs, as shown in the following Fig. 1.1:

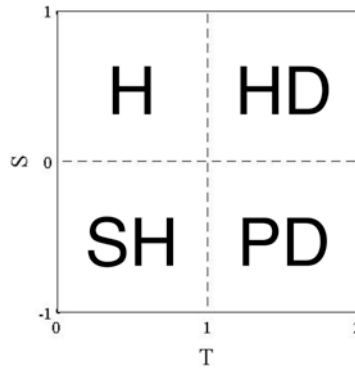


Fig. 1.1 In the TS -plane we can distinguish four different games: Harmony (H), Hawk-Dove (HD), Stag Hunt (SH), and the Prisoner's Dilemma (PD).

The fourth quadrant, where $0 < \{S, T\} < 1$, represents the Harmony game, which is included to complete the square. Here, the strategy C strictly dominates D and the trivial unique NE is (C, C) . Now we turn to a more detailed discussion for each non-trivial game.

1.1 Prisoner's Dilemma

The Prisoner's Dilemma (PD) is a canonical example of a game that shows why two purely *rational* individuals might not cooperate, even if it appears that it is in their best interests to do so. The PD game can be used as a model for many real world situations involving cooperative behavior. Albert W. Tucker formalized the game with prison sentence rewards and gave it the name "prisoner's dilemma" [8], presenting it as follows:

Two bad guys, called Jack and Matthew, jointly committed a crime and they get caught by the police. Each prisoner is in solitary confinement with no means of exchanging messages with the other. The police does not have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, each prisoner is given the opportunity either to betray the other, by testifying that the other committed the crime, or to cooperate with the other by remaining silent. We thus have the following situations:

- If Jack and Matthew each betray the other, each of them serves 2 years in prison;
- If Jack betrays Matthew but Matthew remains silent, Jack will be set free and Matthew will serve 3 years in prison (and vice versa);
- If Jack and Matthew both remain silent, both of them will only serve 1 year in prison.

It is implied that the prisoners will have no opportunity to reward or punish their partner other than the prison sentences they get, and that their decision will not affect their reputation in the future. Because betraying a partner offers a greater reward than cooperating with them, all purely rational self-interested prisoners would betray the other, and so the only possible outcome for two purely rational prisoners is for them to betray each other. The interesting part of this result is that pursuing individual reward logically leads both of the prisoners to betray, when they would get a better reward if they both cooperated. In reality, humans display a systematic bias towards cooperative behavior in this and similar games, much more so than predicted by simple models of "rational" self-interested action. We can thus formalize the above situation obtaining the following payoffs matrix:

	C	D
C	$(-1, -1)$	$(-3, 0)$
D	$(0, -3)$	$(-2, -2)$

It is also possible to transform this game having payoffs T and S in the conventional TS -plane. In fact, adding two units to all payoffs we obtain $R = 1$, $P = 0$, $T = 2$, and $S = -1$. This game is located in bottom-right corner of the TS -plane and can be considered as a rather harsh PD. For the PD game, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique NE.

1.2 Coordination games

Coordinating on a common task or adhering to a shared convention are extremely important behaviors in society. Coordinated behavior is fundamental in work sharing as it permits more efficient outcomes than mis-coordination, as, e.g., in a production line in a factory. In shared conventions and norms, the rationale is that conforming to the behavior of the majority already following the norm confers more welfare than taking a different stance. Examples of the latter abound: driving on a given side of the road depending on the country, speaking the language of the majority, dressing according to generally accepted standards, and so on. How norms and conventions have evolved is a complex process involving many factors [15].

In Game Theory, coordination games are a class of games with multiple pure strategy Nash Equilibria in which players benefit the most when they choose the same actions. Coordination games are a formalization of the idea of a coordination problem, which is widespread in the social sciences, including economics, meaning situations in which all parties can realize mutual gains, but only by making mutually consistent decisions. A common application is the choice of technological standards. Norm-following behavior can be thus described in game-theoretical terms through pure coordination games. In these games the idea is that if the players choose the same action, i.e. they coordinate, then they are at a Nash equilibrium and consequently they have no incentive to unilaterally change their action since this would result in reduced payoff. For example, the following matrix represents a two-player, two-strategy pure coordination game in which one may ideally identify each strategy as a *convention*:

	A	B
A	(a, a)	$(0, 0)$
B	$(0, 0)$	(b, b)

Assuming $a, b > 0$, there are two pure NE in this game: (A, A) and (B, B) . The problem with this approach is that there is no way to choose between the equilibria in the sense that they are both equally valid from the game-theoretical point of view. In practical situations one equilibrium might be better or easier to achieve than the other. For instance, if $a > b$, it is likely that players will tend to coordinate on the payoff-dominant norm A .

Another class of coordination games is the Stag Hunt (SH). Considering Table 1.2, a SH game is defined by the following payoffs ordering: $R > T > P > S$. This situation implies that mutual cooperation (C, C) is the payoff superior outcome and a NE. The second NE, where both players defect (D, D) is less efficient but also less risky. The third mixed-strategy NE in the game is defined by the probability of cooperating:

$$p^* = \frac{P - S}{R + P - S - T}$$

For instance, if we consider the SH game: $R = 1, P = 0, T = 0.5$ and $S = -0.5$, we obtain $p^* = 0.5$. This means that a player who plays against another one who cooperates with frequency p^* must reply with the same probability of cooperating p^* . In this case both players obtain an expected payoff of 0.5, which is lower than the payoff they would get with mutual cooperation.

1.3 Conflictual games

The game of hawk-dove, also known as the chicken game or snowdrift game [12], is an influential model of conflict for two players in game theory. The principle of the game is that while each player prefers not to yield to the other, the worst possible outcome occurs when both players do not yield. The name "chicken" has its origins in a game in which two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken", meaning a coward; this terminology is most prevalent in political science and economics. The name "Hawk-Dove" refers to a situation in which there is a competition for a shared resource and the contestants can choose either conciliation or conflict; this terminology is most commonly used in biology and evolutionary game theory. From a game-theoretic point of view, "chicken" and "hawk-dove" are identical; the different names stem from parallel development of the basic principles in different research areas. The game has also been used to describe the mutual assured destruction of nuclear warfare, especially the sort of brinkmanship involved in the Cuban Missile Crisis [10].

In the HD game, the order of payoffs P and S is reversed in respect to the PD game, yielding $T > R > S > P$, see Table 1.2. Here we give an example:

	C	D
C	(1, 1)	(0.5, 1.5)
D	(1.5, 0.5)	(0, 0)

where $R = 1$, $P = 0$, $T = 2$, and $S = 0.5$.

This game is characterized by the high temptation to defect and the low punishment, thus mutual defection leads to the lowest payoff. When both players cooperate they have a strong incentive to play D , therefore one of the player will adopt defection which is better for that player and worse for the other. On the other hand, if the strategy profile is (D, D) , it is more profitable for both players that one of them chooses cooperation. Finally the solution will be that one player cooperates while the other defects. (C, D) and (D, C) are the NE of the game in pure strategies, and there is a third equilibrium in mixed strategies. This game is in the anti-coordination class and no strategy is strictly dominated. As for the SH game, the mixed NE can be computed by equating the expected payoffs of the strategies C and D for any of the symmetrical players and using the normalization of the probabilities leading to

$$p^* = \frac{P - S}{R + P - S - T}, \quad 1 - p^* = \frac{R - T}{R + P - S - T}$$

this means that when the first player plays C with probability p^* , the second player must reply playing C with probability $1 - p^*$. For instance, we obtain $p^* = 0.5$ in the above example.

1.4 Evolutionary Game Theory

Evolutionary Game Theory (EGT) is applied in numerous fields such as economics, biology and social sciences. Its popularity comes from the fact that the idea of “evolution” is not only appropriate for biological entities, but can also easily be transposed to model the change over time of human behavior, culture, ideas or norms.

The first step towards a combination of evolution with a form of game theory is due to Fischer [1], although he did not state his investigation in game theoretical terms. Some years after him, the engineer Maynard Smith applied the basic notions of Game Theory to biology and he was awarded, among several others, the Crafoord Prize in 1999. In his book, *Evolution and the Theory of Games* [3], Maynard Smith directed game theorists’ attention away from their increasingly elaborate definitions of rationality. After all, insects can hardly be said to think at all, and so rationality cannot be so crucial if game theory somehow manages to predict their behavior under appropriate conditions. Simultaneously the advent of experimental economics brought home the fact that human subjects are no great shakes at thinking either. When they find their way to an equilibrium of a game, they typically do so using trial-and-error methods. EGT is useful in this context by defining a framework of contests, strategies, and analytics into which Darwinian competition can be modeled. At its core there lies John Maynard Smith and George R. Price’s formalization of the way in which such contests can be analyzed as “strategies” and the mathematical criteria that can be used to predict the resulting prevalence of such competing strategies [4]. Two approaches have been successful in evolutionary game theory: evolutionarily stable strategies and replicator dynamics. The first is a static one while the second is based on a purely dynamical systems approach and it will be discussed in more detail in the next section.

Evolutionarily Stable Strategies. In order to define the concept of Evolutionarily Stable Strategy (ESS), suppose that a small group of mutants y appears in a large population of individuals x , all of whom are programmed to play the same strategy σ_x . Suppose also that mutants are all programmed to play some other *mutant* strategy σ_y . A strategy σ_x is said to be *evolutionarily stable* if for every $\sigma_y \neq \sigma_x$ there exists some $\epsilon_y \in (0, 1)$ such that the following inequality holds for all $\epsilon \in (0, \epsilon_y)$:

$$u_x[\epsilon y + (1 - \epsilon)x] > u_y[\epsilon y + (1 - \epsilon)x].$$

An equivalent way of stating the previous result is to say that a strategy σ_x is evolutionarily stable if and only if:

- $u_x(\sigma_x, \sigma_x) \geq u_y(\sigma_y, \sigma_x) \quad \forall y,$
- $u_x(\sigma_x, \sigma_x) = u_y(\sigma_y, \sigma_x) \Rightarrow u_x(\sigma_x, \sigma_y) > u_y(\sigma_y, \sigma_y) \quad \forall y \neq x.$

These two conditions together characterize the evolutionary stability of strategy σ_x [2]. The ESS concept is very close to the Nash Equilibrium but it refers to populations, and it is stricter since there exist games that do not admit of an ESS. For instance, this is the case of the Rock-Scissors-Paper game [2].

Evolutionary game theory differs from classical game theory by focusing more on the dynamics of strategy change as influenced not solely by the quality of the various competing strategies, but by the effect of the frequency with which those various competing strategies are found in the

population. Evolutionary game theory has proven itself to be invaluable in helping to explain many complex and challenging aspects of biology. It has been particularly helpful in establishing the basis of altruistic behaviors within the context of Darwinian process. Despite its origin and original purpose, evolutionary game theory has become of increasing interest to economists, sociologists, anthropologists, and philosophers.

Participants in the evolutionary game aim to become more fit than competitors – to produce as many replicas of oneself as one can, and the payoff is in units of fitness. It is always a multi-player game with a very large population of competitors. Rules describe the contest as in classical Game Theory but for evolutionary games rules include the element of replicator dynamics, in other words the general rules say exactly how the fitter players will spawn more replicas of themselves into the population and how the less fit will be culled out of the player population expressed in a replicator equation. The replicator dynamics in essence models the heredity mechanism, but for simplicity leaves out mutation. Similarly, Evolutionary Game Theory only uses asexual reproduction for the sake of simplicity. Games are run repetitively with no terminating conditions. The results that are studied include the dynamics of changes in the population, the success, or survival, of strategies and any equilibrium states reached in a competing environment. Unlike in classical Game Theory, players do not choose their strategy or have the ability to change it, they are born with that strategy and their offspring will inherit that same identical strategy.

1.4.1 Replicator dynamics

A true evolutionary process is composed of a selection mechanism responsible for favoring the fitter member of a population [7]. The *replicator dynamics*, which if free of any mutation mechanism, provides us with the selection element that leads the population to a stable state. Given an initial distribution of n pure strategies among the agents, evolution in the population is dictated by a system of linear differential equations of the type:

$$\dot{x}_i = [\pi(s_i) - \hat{\pi}]x_i$$

where x_i is the proportion of the population playing pure strategy s_i , $\pi(s_i) = \sum_{j=1}^n x_j \pi(s_i, s_j)$ is the expected payoff of a player programmed to play strategy s_i , and $\hat{\pi} = \sum_{i=1}^n x_i \pi(s_i)$ is the average payoff of the population. Consequently, strategies that do better than average will increase their share in the population, whereas less successful strategies will decrease in time. Note that the *state of a population* at time t can be defined as vector $x(t) = (x_1(t), x_2(t), \dots, x_m(t))$. Hence, a population of individuals programmed to pure strategies can also be interpreted, as far as replicator dynamics is concerned, as a population composed of players all playing the same mixed strategy x which consists of playing each pure strategy s_i with probability $x_i(t)$. Thus we can write $\hat{\pi} = \pi(x, x)$ and

$$\dot{x}_i = [\pi(s_i, x) - \pi(x, x)]x_i$$

which gives the *replicator dynamics* as defined by Taylor and Jonker [13].

The use of replicator dynamics differs in the continuous and discrete cases: in the former, methods from differential equations are used, whereas in the latter the methods tend to be stochastic. Since the replicator equation is non-linear, an exact solution is difficult to obtain, even in simple

versions of the continuous form, so the equation is usually analyzed in terms of stability, trying to find the set of evolutionarily stable states of the population.

Evolutionarily Stable States (ESS) are frequently used to identify solutions to the evolutionary game. A population state $\hat{x}$ is said to be evolutionarily stable if for all $x \neq \hat{x}$ in some neighborhood of $\hat{x}$ we have:

$$\hat{x}^T A x > x^T A x$$

where A is the payoff matrix with N players, $x = (\sigma_1, \sigma_2, \dots, \sigma_N)$ represents the population state and σ_i is the strategy profile of player i . Considering game-theoretical formalization, we can also define an evolutionarily stable strategy as a strategy such that, if all the members of a population adopt it, no mutant strategy can invade following the replicator dynamics equation.

In numerical simulations of evolutionary games on networks it is usually adopted the local *fitness-proportional* update rule to approximate the replicator equation for discrete systems. This update rule is stochastic and gives rise to replicator dynamics in a well mixed population when $N \rightarrow \infty$ [2]. Player i updates her strategy profile σ_i by randomly drawing another player j from her neighborhood, and replacing σ_i by σ_j with probability $p(\sigma_i \rightarrow \sigma_j) = u(\sigma_i) - u(\sigma_j) / K$, where $u(\sigma_i) - u(\sigma_j)$ is the difference of the payoffs earned by player j and player i respectively. This probability is zero when the payoff difference is negative. K ensures proper normalization of the probability and it is defined as a function of game payoffs.

1.4.2 Other selection dynamics

Several update rules are customary in evolutionary game theory. We now describe other game dynamics used in this dissertation that can be adopted to model populations evolutionary process.

Imitation of the best

An imitative strategy update protocol adopted in numerical simulations is to switch to the strategy of the neighbor that has scored the best in the last time step. In contrast with replicator dynamics, this rule is deterministic. This imitation of the best (IB) policy can be described in the following way: the strategy profile $\sigma_i(t)$ of individual i at time step t will be $\sigma_i(t+1) = \sigma_j(t)$, where $j \in \{V_i \cup i\}$ and $u(\sigma_j) = \max_k u(\sigma_k(t))$, $\forall k \in \{V_i \cup i\}$, V_i represents the set of neighbors of player i . That is, individual i adopts the strategy profile of the player with the highest payoff among her neighbors including herself. If there is a tie, the winner individual is chosen uniformly at random.

Fermi rule

A more flexible update rule is the Fermi rule. Here the randomly chosen player i is given the opportunity to imitate the profile of a randomly chosen neighbor j with probability :

$$p(\sigma_i \rightarrow \sigma_j) = \frac{1}{1 + \exp\{-\beta[u(\sigma_j) - u(\sigma_i)]\}}$$

where β is a constant corresponding to the inverse temperature of the system, i.e. high temperature implies that imitation is random to a large extent and depends little on the payoffs. Thus, when

$\beta \rightarrow 0$ the probability of imitating j tends to a constant value of 0.5 and when $\beta \rightarrow \infty$ the rule becomes deterministic: player i imitates player j only if $u(\sigma_j) - u(\sigma_i) > 0$, otherwise it doesn't.

Best-response

The last rule is called best response. This rule stipulates that the agents adopt the strategy profile that gives them the highest payoff given the neighbors' strategies at the previous time step. This rule is different from the other since the players do not imitate their neighbors. The above model rules are common and almost standard in numerical simulation work, which has the advantage that the mathematics is simpler and results can be compared with previous work such as [9, 11]. However, it is far from clear whether these rules are representative of the ways in which human players actually take their strategic decisions, as has been shown by many laboratory experiments. In these experiments it seems that learning and heuristics play an important role. Moreover, players are heterogeneous in their behavior while our stereotyped automata all behave in the same way and never change or adapt.

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Chapter 2

Complex Networks

“A tale is born from an image, and the image extends and creates a network of meanings.”

*Italo Calvino, Italian journalist and writer.
“Our Ancestors”, 1959.*

In this chapter we move into one of the most recent developments of complex systems science: networks. Stimulated by two seminal papers on small-world and scale-free networks published in the late 1990s [5, 24], the science of complex networks, or network science for short, has been rapidly growing and producing novel perspectives, research questions and analytical tools to study various kinds of systems in a number of disciplines, including biology, ecology, sociology, economics, political science, management science, engineering, medicine, and more.

The historical roots of network science can be sought in several disciplines. One is obviously discrete mathematics, especially graph theory where mathematicians study various properties of abstract structures called graphs made of nodes, or vertices, and edges, or links. Another theoretical root is statistical physics, where properties of collective systems made of a large number of entities are studied using analytical means. A more applied root of network science is in social sciences, especially social network analysis [10, 23, 26]. Yet another application-oriented root would be in dynamical systems, especially Boolean networks discussed in theoretical and systems biology [13, 14] and artificial neural networks discussed in computer science [12]. In all of those investigations, the research foci were put on the connections and interactions among components of a system, not just on each individual component.

Network models are different from other more traditional dynamical models in some fundamental aspects. Firstly, the components of the system may not be connected uniformly and regularly, unlike cells in cellular automata that formed regular homogeneous grids. This means that, in a single network, some components may be very well connected while others may not. Such inhomogeneous connectivity makes it more difficult to analyze the system’s properties mathematically; for example, mean-field approximation may not apply to networks so easily. In the meantime, it also gives the model greater power to represent connections among system components closer to reality. We can represent any network topology, i.e., shape of the network, by explicitly specifying in detail which components are connected to which other components, and how. This makes network modeling necessarily data-intensive. No matter whether the network is generated using some mathematical algorithm or reconstructed from real-world data, the created network model will contain a good amount of detailed information about how exactly the components are connected. We need to learn how to build, manage and manipulate these pieces of information in an efficient way.

Secondly, the number of components may dynamically increase or decrease over time in certain dynamical network models. Such growth or decay of the system's topology is a common assumption typically made in generative network models that explain self-organizing processes of particular network topologies. Note, however, that such a dynamic change of the number of components in a system realizes a huge leap from other more conventional dynamical systems models. This is because, if we consider states of the system components, having one more or less component would mean that the system's phase space acquires one more or less dimensions. From a traditional dynamical systems point of view, it sounds almost illegal to change the dimensions of the system's phase space over time, yet things like that do happen in many real-world complex systems. Network models allow us to naturally describe such complicated processes.

2.1 Network topologies

In this section we introduce the main models of networks presented in the related literature and used in this dissertation.

2.1.1 Random graphs

Random graphs may be described simply by a probability distribution, or by a random process which generates them [15]. The theory of random graphs lies at the intersection between graph theory and probability theory. From a mathematical perspective, random graphs are used to answer questions about the properties of typical graphs. Its practical applications are found in all areas in which complex networks need to be modeled – a large number of random graph models are thus known, mirroring the diverse types of complex networks encountered in different areas. However, random graph refers almost exclusively to the Erdős-Rényi random graph model [9].

A random graph is obtained by starting with a set of N isolated vertices and adding successive edges between them at random with probability p . This means that a link between two vertices exists with probability p . Most commonly studied is the one proposed by Edgar Gilbert, denoted $G(N, p)$, in which every possible edge occurs independently with probability $0 < p < 1$. The probability of obtaining any one particular random graph with m edges is thus $p^m(1 - p)^{B-m}$ with the notation $B = \binom{N}{2}$. A closely related model, the Erdős-Rényi model, denoted $G(N, m)$, assigns equal probability to draw one of all graphs with exactly N nodes and m edges [9].

Considering $G(N, p)$ random graphs, we obtain that their average degree tends to $\bar{k} = Np$, for large N . They possess a Poisson degree distribution with mean $\lambda = \bar{k}$. Other known characteristics are the average clustering coefficient ($\bar{c} \sim 1/N$), which tends to zero with $N \rightarrow \infty$, and the average shortest path length, $\ell \sim \log N$ [15]. The average clustering coefficient can be seen as the probability that two random neighbors of a random chosen node in the network are also neighbors between themselves. The average shortest path length in a graph is the mean of all possible shortest paths among all nodes of the connected component of the network.

One can also build random graphs with arbitrary degree distributions just performing many random edge swaps from an existing network with the desired degree distribution. This random process is known as the configuration model.

2.1.2 Barabási–Albert model

The Barabási–Albert (BA) model is an algorithm for generating random scale-free networks using a preferential attachment mechanism. Scale-free networks are widely observed in natural and human-made systems, including the Internet, the world wide web, citation networks, and some social networks. This algorithmic network construction process is named for its inventors Albert-László Barabási and Réka Albert [5].

Many observed networks fall into the class of scale-free networks, meaning that they have power-law, also called scale-free, degree distributions. The degree of a node in a network is the number of connections it has to other nodes and the degree distribution is simply the probability distribution of these degrees over the whole network. The BA model is one of several proposed models that generates scale-free networks. It incorporates two important general concepts: growth and preferential attachment. Both processes exist widely in real networks. Growth means that the number of nodes in the network increases over time. Preferential attachment means that the more connected a node is, the more likely it is to receive new links. Nodes with higher degree have stronger ability to grab links added to the network. Intuitively, preferential attachment can be understood if we think in terms of social networks connecting people. Here a link from A to B means that person A *knows* or *is acquainted with* person B . Heavily linked nodes represent well-known people with lots of relations. When a newcomer enters the community, he or she is more likely to become acquainted with one of those more visible people rather than with a relatively unknown person. Similarly, on the web, new pages link preferentially to hubs, i.e. very well known sites such as Google or Wikipedia, rather than to pages that hardly anyone knows. If someone selects a new page to link to by randomly choosing an existing link, the probability of selecting a particular page would be proportional to its degree. This explains the preferential attachment probability rule. Preferential attachment is an example of a positive feedback cycle where initially random variations are automatically reinforced, thus greatly magnifying differences. This is also sometimes called the Matthew effect, “the rich get richer”.

The steps of the growth of the network of N nodes according to the BA model with parameters m_0 and m is the following. The network begins with an initial fully connected network of m_0 nodes ($m_0 \ll N$). New nodes are added to the network one at a time. Each new node is connected to $m \leq m_0$ existing nodes with a probability proportional to the number of links that the existing nodes already have. Formally, the probability p_i that the new node is connected to node i is:

$$p_i = \frac{k_i}{\sum_j k_j}$$

where k_i is the current degree of node i and the sum is made over all pre-existing nodes j . Heavily linked nodes, also called *hubs*, tend to quickly accumulate even more links, while nodes with only a few links are unlikely to be chosen as the destination for a new link. In fact, the new nodes have a preference to attach themselves to the already heavily linked nodes. This network construction process builds networks which have a degree distribution which follows a power law. In log-log scale the power law function is a straight line and has the form of $P(k) \sim k^{-\lambda}$, where $\lambda = 3$ for the BA model. Moreover, the average path length of the BA model increases approximately logarithmically with the size of the network. The actual form has a double logarithmic correction and goes as $\ell \sim \frac{\ln N}{\ln \ln N}$. This means that the BA model has a systematically shorter average path length than an Erdős–Rényi random graph.

2.1.3 Social networks

Social network analysis examines the structure of relationships between social entities. These entities are often persons, but may also be groups, organizations, nation states, web sites, scholarly publications. Modeling social networks serves at least two purposes. First, it helps us understand how social networks form and evolve. Secondly, in studying network-dependent social processes by simulation, such as diffusion or retrieval of information, successful network models can be used to specify the structure of interaction. A large variety of models have been presented in the physics-oriented complex networks literature in recent years, to explore how local mechanisms of network formation produce global network structure.

According to [21], the models of social networks can be classified into two main categories: those in which the addition of new links is dependent on the local network structure (*network evolution models*, NEMs) [20, 22], and those in which the probability of each link existing depends only on nodal attributes (*nodal attribute models*, NAMs) [7, 25]. NEMs can be further subdivided into growing models, in which nodes and links are added until the network contains the desired number N of nodes, and dynamical models, in which the steps for adding and removing ties on a fixed set of nodes are repeated until the structure of the network no longer statistically changes. All of these models produce undirected networks without multiple links or self-links, and all networks are treated as unweighted, i.e. tie strengths are not taken into account. There exist several models for constructing social-like networks [21]. Among them, we have put much attention on the Toivonen et al.'s social network model [20] (TSN), which was conceived to construct a network with most of the desired features of real-life social networks, i.e., high clustering coefficient, community structure, having an adjustable decay rate of the degree distribution, and a finite cut-off, see Chapter 5. A network is said to have community structure if the nodes of the network can be easily grouped into sets of nodes such that each set of nodes is densely connected internally and has sparser connections between groups. The TSN construction and properties are described in detail in [20]. A realization of the TSN with $N = 500$ nodes is shown in Figure 2.1.

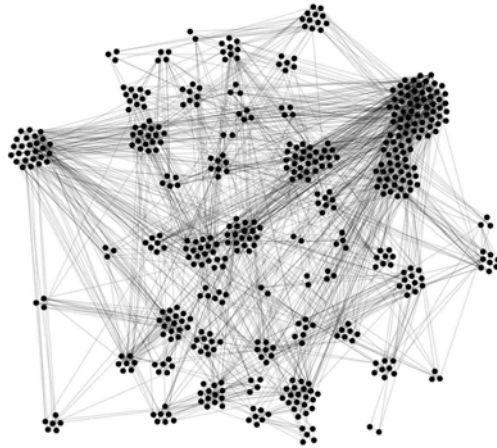


Fig. 2.1 Social networks are usually characterized by high clustering coefficient, positive degree assortativity, positively skewed degree distribution and the presence of community structure.

2.1.4 Spatial networks

Spatial networks were actually long ago the subject of many studies in quantitative geography. Objects of study in geography are locations, activities, flows of individuals and goods, and already in the 1970s the scientists working in quantitative geography focused on networks evolving in time and space [6]. However, spatial networks appear in many different fields, e.g., rivers, roads, wireless networks. Loosely speaking, spatial networks are networks for which the nodes are located in a space equipped with a metric. For most practical applications, the space is the two-dimensional space and the metric is the usual Euclidean distance. This definition implies in general that the probability of finding a link between two nodes will decrease with the distance. However, it does not imply that a spatial network is planar. Indeed, the airline passenger networks is a network connecting through direct flights the airports in the world, but is not a planar network. With this definition of a spatial network links are not necessarily embedded in space: social networks for example connect individuals through friendship relations. In this case, space intervenes in the fact that the connection probability between two individuals usually decreases with the distance between them. For many infrastructure networks, however, planarity is unavoidable. Roads, rail, and other transportation networks are spatial and to a good accuracy planar networks. For many applications, planar spatial networks are the most important, and most studies have focused on these examples.

We here present some of the most important models of spatial networks. According to [6], we may divide these models into five large classes:

- geometric graphs, which are probably the simplest models of spatial networks. They are obtained for a set of vertices located in the plane, and for a set of edges which are constructed according to some geometric condition. An example of this class of models is the Random Geometric Graph (RGG) model [3, 8, 16]. There is a large body of literature on this rather general subject which concerns graphs constructed with an Euclidean proximity condition.
- spatial generalization of the Erdős-Rényi model [9]. These networks are obtained when the probability to connect two nodes depends on the distance between these nodes.
- spatial variants of the Watts-Strogatz model [24], also called spatial small-worlds. In these cases, the starting point is a d -dimensional lattice and random links are added according to a given probability distribution for their length.
- spatial growth models, which can be considered as spatial extensions of the original growth model proposed by Barabási and Albert [5].
- optimal networks obtained by the minimization of a *cost* function. These networks were already considered long ago in different fields (mathematics, transportation science, computer science) and are now back, following the explosion of studies on complex networks.

In this dissertation we have especially made use of the RGG model, see Chapter 6. We also worked on a new model for spatial social networks which takes into account the features of the five classes of spatial networks explained above [4].

2.2 Games on networks

For mathematical convenience, standard Evolutionary Game Theory is based on infinite populations where pairs of individuals are drawn uniformly at random at each step and play the game (well-mixed populations). Correlations are absent by definition and the population has a homogeneous structure. However, everyday observation tells us that in animal and human societies, individuals usually tend to interact more often with some specified subset of partners. Networks are a possible representation of these systems. In fact, social interaction is mediated by networks, in which vertices identify people, firms, etc., and edges identify some kind of relation between the concerned vertices such as friendship, collaboration, economic exchange and so on. Thus, locality of interaction plays an important role. Recently, in the wake of a surge of activity in network research in many fields [15], the dynamical behavior of games on networks that are more likely to represent actual social interactions than regular grids has been investigated, see [19] for comprehensive recent reviews. These studies have shown that there are network structures, such as scale-free and actual social networks that may favor the emergence of cooperation with respect to the fully mixing populations used in the theory [2, 17, 18], see also Chapter 5.

Several analytically rigorous results are available for evolutionary games in well-mixed populations [11], as well as populations with a simple local interaction structure such as rings and grids [19]. These results are very useful; however, while game theory has normative value, its prescriptions are not always reflected in the way people act when confronted with these situations. This has been made manifest by a host of results of experiments with people, see following Chapter. On the other hand, numerical simulations can be extremely useful in order to study evolutionary games. In fact, numerical simulations, with their possibility of modeling many different situations, may shed light on the factors, both endogenous such as strategy update policy and exogenous, such as population structure, that have an influence on the game outcome. In this way, this can be a valuable tool in both theoretical and experimental sides, helping build a bridge between the two. Numerical studies deal with finite, network-structured populations during a limited amount of time, while theoretical results have been established for large well mixed populations in the very long run. Thus, numerical results and theoretical predictions based on different assumptions do not necessarily agree with each other.

The influence of several network topologies on evolutionary games has been studied, including regular lattices, random graphs, scale-free graphs, model and actual social networks [1, 2, 17, 18] using different strategy update rules such as replicator dynamics, imitation of the best, and best response dynamics, see also Chapters 4 and 5. On average, for initially equidistributed strategies, the more efficient strategy is usually enhanced by network reciprocity with respect to what would happen in a well-mixed population. Social networks also favor the Pareto-efficient outcome on average but the steady state population is often dimorphic, i.e. there is a mix of the two strategies. The reason why there can be mixed states in social networks has been attributed to the presence of communities. In fact, social networks can usually be partitioned into recognizable clusters; within these clusters strategies may become dominant as in the pure coordination case just by chance. In other words, as soon as a strategy dominates in a given cluster, it is difficult to eradicate it from outside since other communities, being weakly connected, have little influence.

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Chapter 3

Experimental Game Theory

“The only source of knowledge is experience.”

Albert Einstein.

In this chapter we briefly introduce the vast field of Experimental Game Theory. The use of human subjects in the laboratory has its roots mainly in psychology, social sciences and economics. The data collected in the experiments are fundamental to test the validity of economic theories, estimate the size of the relevant effects and illuminate market mechanisms. Experiments are used to help understand how and why people, markets or other exchange systems function as they do.

The title of Experimental Game Theory refers to experiments whose goal is to learn about general principles of strategic human behavior [7]. Twenty-five years ago it would have been startling to see *experiments* questioning Game Theory or to find an entire issue of an economics journal devoted to game experiments. See, however, the 1995 Games and Economic Behavior special issue on experimental game theory introduced by Ledyard and Palfrey [16]. Twenty-five years ago, the only experiment that had attracted the attention of the entire profession was Smith’s classic study of the competitive double-oral-auction market [26]. His experiment revealed at a time when most economists thought perfect competition required many traders that with perfect information markets with only a few traders on each side can yield competitive outcomes and that market outcomes may actually be more competitive when traders are imperfectly informed. The widespread impact of Smith’s results was probably due largely to their robustness to the details of individual behavior’s details that are the focus of experimental game theory. If this now seems natural, it is a tribute to the increasingly empirical orientation of game theory and the researchers who have made experiments an important tool for the analysis of strategic behavior. Kagel and Roth [13], Crawford [8], Selten [24], and Camerer [6] provide complementary surveys of the experimental literature.

The emergence of Experimental Game Theory can be traced to two factors: the need for empirical information about principles of strategic behavior and the advantages of experiments in providing it. This need was long obscured by the game-theoretic custom of trying to predict behavior entirely by theory, as in normative economics, applying notions of equilibrium and refinements to the structure of the game. The custom may reflect the early conception of game theory as a mathematical investigation of the behavior of idealized *perfectly rational* agents and the view that this would be fully adequate to predict strategic behavior. Schelling gave an early and influential dissenting view [25], which did little to alter the custom among theorists. This custom admits a role for empirical knowledge about players’ preferences, feasible decisions, and information just as in nonstrategic microeconomics, but it precludes any role for empirical input about the principles that

determine how players respond to a given game. Excluding such input is comparatively innocuous in nonstrategic settings, where rationality in the sense of expected-utility maximization often provides a reasonably reliable guide to behavior once preferences, decisions, and information have been identified. But it is far from innocuous in strategic settings, where rationality alone seldom yields definite predictions, reliable or not, and consensus about how to strengthen it is if anything more remote than 25 years ago. As a result, most games of interest in economic applications raise questions about strategic behavior that seem likely to be adequately resolved only by combining theory and empirical knowledge. Economic games usually have multiple rational outcomes and often have multiple equilibria. In principle, the limitations of a purely rationality-based approach could be overcome by more powerful theory, as for example in Harsanyi and Selten [12]; but this program now appears unlikely to succeed without admitting empirical knowledge. The limitations are transformed, but not eliminated, by viewing equilibrium as the outcome of an adaptive learning process, which raises many new issues that theory alone does not satisfactorily resolve.

The need for empirical knowledge about principles of strategic behavior creates a special role for experiments in game theory. The predictions of game theory, particularly noncooperative game theory, which underlies most applications are notoriously sensitive to the details of the structure of the game, and much of this sensitivity is reflected in observed behavior. Such details can seldom be precisely observed or adequately controlled in the field. The laboratory shares some of these problems, but the control and observation that modern experimental techniques allow often give experiments a decisive advantage in identifying the relationship between strategic behavior and the environment. There is nonetheless a history of valuable empirical work using field data from strategic environments, usually with well-specified, observable structures, as for example in auctions or centralized labor markets. Theory and experiment play strongly complementary roles, with theory providing a framework within which to gather and interpret empirical information about behavior, and experiments indicating which parts of the theory are most useful in predicting behavior, and identifying behavioral parameters that theory does not reliably determine.

Despite the importance of these topics, it took quite long until the effects game-theoretic predictions were tested in laboratory experiments. The particular value of the experimental work is that it facilitates more realistic numerical simulations. Thereby it becomes possible to determine payoff values and other model parameters, which are expected to produce interesting effects after an experimentally accessible number of interactions. In fact, experimental games can have qualitatively different outcomes, which are hard to predict without extensive numerical simulations scanning the parameter space. The future of social experimenting lies in the combination of computational and experimental approaches, where numerical simulations optimize the experimental setting and experiments are used to verify, falsify or improve the underlying model assumptions. This concept is schematically represented in Fig. 3.1.

In view of the great theoretical importance of experiments with many interactions and large populations, more large-scale experiments over long time horizons would be desirable. This calls for larger budgets, but also for new concepts. Besides connecting labs in different countries via internet, one may consider to perform experiments in “living labs” on the web itself. One can furthermore hope that lab or web experiments will eventually become standardized measurement instruments to determine indices like the local *level of cooperation* as a function of time, almost like the gross domestic product is measured today. Knowing the *index of cooperativeness* would be good, as it reflects social capital. The same applies to the measurement of social norms, which

are equally important for social order as cooperation, since they determine important factors such as coordination, adaptation, assimilation, integration, or conflict.

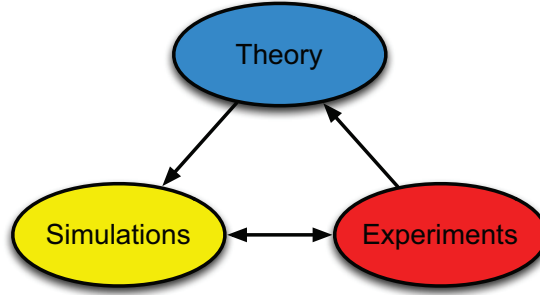


Fig. 3.1 Theoretical models are tested through numerical simulations which can both be compared with laboratory and field experiments; in fact, numerical results inspire new experimental settings and empirical results suggest new models of human behavior; finally, findings on human behavior are tested by numerical simulations to get predictions and they can be integrated in theoretical models.

In the past, there have been a number of studies dealing with randomly mixing populations and groups, we may cite, e.g., Camerer's book [6], where an informative summary is provided. However, given the focus of this dissertation, we concentrate here on situations in which local interaction structures and thus networks play a fundamental role. In the last five years, there has been an increasing number of experiments in which the population structure has been recreated in the laboratory only by local interactions, which can be usually represented by a network structure.

3.1 Coordination on static networks

In this section we describe some laboratory experiments on coordination games presented in the literature. One of the first experiments was conducted by Keser et al. [15], in which they used a ring structure where each player has a neighbor on either side and a well-mixed structure for comparison. Their conclusions are that in the ring the preferred equilibrium is the risk-dominant one, while the payoff-dominant equilibrium was the more frequent result in the globally communicating population. This is in qualitative agreement with the theoretical predictions of Ellison [9] for the ring and of Kandori et al. [14] for the mixing case. In another experiment My et al. [17] performed a comparative experimental study of Stag Hunt games with three different payoff matrices on mixing and structured populations. The population with local structure consisted of a circle of eight people where each player only interacted with her immediate right and left neighbors. They find that the first period modal choice of strategy, which is the payoff dominant one, plays a major role in the final outcome. In the global population case, the steady state generally lies in the same basin of attraction as the initial state. For the ring structure, the convergence to the risk-dominant outcome is more frequent than in the well-mixed case, especially when the payoff matrix values are such that the Pareto-superior basin shrinks. However, still often the system converges to the Pareto-dominant state, which disagrees with the theoretical predictions of Ellison [9]

based on noisy best reply dynamics. By examining the detailed history of play, the experimenters have found that, while in the global population subjects play on average myopic best response, in the ring with local structure a kind of imitation rule fits the data better than best reply. In the study of Berninghaus et al. [5] the authors find that a ring of eight or sixteen players leads to less coordination on the Pareto-efficient strategy in the average than in groups of three completely connected players. In addition, with the same neighborhood size, grids of sixteen individuals are less effective in evolving coordination on the efficient equilibrium. Finally, in the work by Antonioni et al. [1], see also Chapter 10, we considered the modeling and simulation work of Roca et al. [22] to understand the role of social traps [23] in coordination games, although with some unavoidable limitations related to the small size achievable in the laboratory. Roca et al. studied coordination on a couple of actual social networks and identified a different behavior: in one of the networks the Pareto-efficient strategy cannot propagate and the final equilibrium results in a dimorphic population. As will be discussed below we compared human behavior on a cliquish network to behavior on a random network and, in contrast to simulations, we found that human players dynamics do not converge to the efficient outcome more often in the cliquish network than in the random network. So far, no laboratory experiments with dynamical networks have been conducted using coordination games, except in Antonioni et al. [3], see also Chapter 11.

3.2 Cooperation on static networks

One of the mechanisms which may support cooperation has been defined as *network reciprocity* [18]. This network reciprocity does not require any particular psychological propensity or behavior on the part of the agents, but only a heterogeneous distribution of the individuals in the interacting populations. Both theory and quantitative simulations indicate that network reciprocity may explain the implicit cooperation that is at the core of society. To summarize these results, the mere presence of a spatial or relational structure gives rise to evolutionary outcomes in which, thanks to positive assortment, cooperative behavior may evolve and may even lead to fully cooperative states. Recent research tested these predictions by means of targeted experiments with humans in the laboratory, in which the subjects were connected in specific network structures [10, 11, 27]. Surprisingly, these studies found that neither homogeneous nor heterogeneous network structures promote cooperation to a significant extent. Notwithstanding, a very recent work conducted by Rand et al. [21] opened again the debate on static network reciprocity showing some degree of cooperation for low temptation to defect.

3.3 Cooperation on dynamical networks

Many actual socio-economic networks are dynamic. This fact has not escaped the attention of many researchers, and several models have been proposed for studying cooperation under these conditions [19]. These models differ in their details, but researchers agree that adding these new adjustment margins may lead populations to mainly cooperative and stable states through co-evolution of behavior and connectivity. Empirical tests of dynamic settings include the following works [2, 20]. Rand et al. [20] found that cooperation is supported if participants can rewire connections often enough, and that the evolved networks are more heterogeneous and have more stable links between two cooperators than in less fluid or completely static conditions. Wang et al. [28] investigated the role of link updating frequency on cooperation and found that partner updating significantly increased the cooperation level even at relatively low frequencies. More recently, in the work by Antonioni et al. [2], see also Chapter 13, we studied a setting where individuals can make or break links and need to pay for information on their potential partners' actions. This setting differs from Rand et al. [21] who provided the players with full information on the strategies used by their neighbors in the previous round. Also, Wang et al. [28] provided even richer information, again for free. Players were shown the identities (anonymous labels) and action choices of all players with whom they were connected for up to five previous rounds. While we think that these conditions could be adequate in some situations in which the same people interact repeatedly, we argue that there are many contexts in which there is uncertainty as to potential partners' actions and, by consequence, a decision entails some amount of risk.

Recently, in the forthcoming work [4] we have investigated the role of short-range mobility on the evolution of cooperation, see Chapter 12. In contrast to previous results on purposeful rewiring in relational networks, we have found no noticeable effect of mobility in space on the level of cooperation. Our results highlight the difficulties that mobile agents have to establish a cooperative environment in a spatial setting.

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Part II

Numerical Simulation Models

Chapter 4

Network Fluctuations Hinder Cooperation in Evolutionary Games

Publication: Antonioni and Tomassini (2011),
*Network Fluctuations Hinder Cooperation in
Evolutionary Games*, PLoS ONE 6(10): e25555.
27 October 2011.

Abstract In this paper we study the influence of random network fluctuations on the behavior of evolutionary games on Barabási–Albert networks. This network class has been shown to promote cooperation on social dilemmas such as the Prisoner’s Dilemma and the Snowdrift games when the population network is fixed. Here we introduce exogenous random fluctuations of the network links through several noise models, and we investigate the evolutionary dynamics comparing them with the known static network case. The results we obtain show that even a moderate amount of random noise on the network links causes a significant loss of cooperation, to the point that cooperation vanishes altogether in the Prisoner’s Dilemma when the noise rate is the same as the agents’ strategy revision rate. The results appear to be robust since they are essentially the same whatever the type of the exogenous noise. Besides, it turns out that random network noise is more important than strategy noise in suppressing cooperation. Thus, even in the more favorable situation of accumulated payoff in which links have no cost, the mere presence of random external network fluctuations act as a powerful limitation to the attainment of high levels of cooperation.

4.1 Introduction

In the last decade, thanks to advances in network science, a large number of studies dealing with evolutionary games on networks have appeared. The underlying idea is that the classical infinite, well-mixed populations used in the theory [30] are not a particularly good approximation to the actual shape of the contacts that take place in society, although they do allow rigorous mathematical results to be reached. Indeed, social interactions between agents are mediated by finite networks of contacts, which is why there is a strong interest in the study of this kind of collective systems. For a synthesis of the main results obtained in the last years, we refer the reader to the following reviews which, altogether, give the state of the art in the field of evolutionary games on networks [26, 21, 17].

At the beginning, investigations were targeted at static networks, i.e. networks that do not change during time. A very good summary of this case is provided by Roca et al. [21]. This is an acceptable approximation when network changes are slow with respect to behavioral changes of the agents and it is a useful first step. However, actual social networks are dynamical entities in which agents may leave the network, new agents may join it, and links can be formed and dismissed as well. So, the more general models should be dynamical, and several approaches have

been suggested to deal with these time-dependent aspects of the network structure in evolutionary games; an excellent recent review is contained in [17]. Most models assume a constant population structure, i.e. no agent leaves or join the network, which means that the system is at equilibrium with respect to exchanges of “matter”. This of course removes the need of dealing with the rate of change $\dot{N}$ of N , since $\dot{N} = 0$. The condition also holds when the number of agents entering the system is the same as those leaving it, but this would complicate matters since contacts would change. For this reason it is assumed that there is no flow through the system boundaries. On the other hand, the number of links L may be subject to internal change and, even if L stays constant ($\dot{L} = 0$), it will generally be the case that links are actually being rewired among different pairs of agents. The above is the most often used scenario, although some works have also dealt with growing networks under strategic conditions, e.g. see [19] and the abundant economic literature on strategic network formation as summarized, for example, in [7]. In all cases, only pairwise interactions are considered at first: although n -person interactions are important, it is believed that two-person games are a first useful and necessary step and will be assumed here too.

Now, link rewiring can be either an exogenous random phenomenon, or it can obey some other rule. If it is completely random, then the network drifts toward randomness itself, in the sense that its degree distribution tends to be Poissonian. This is not an interesting case since we already know from the static case [25, 21] that random graphs are not particularly conducive to cooperative interactions between agents. Besides, actual social networks are not random and thus this is not a realistic case either. Thus others, perhaps more socially-inspired relinking patterns, have been postulated. For example, the models presented in [24, 18] try to take into account a kind of strategic “negotiation” between the concerned pair of connected nodes in order to decide whether a given link must be cut or not, while in other cases the decision is unilateral [32, 5] and only affects certain types of links, usually defector-defector ones. The rewiring phase, on the other hand, has often been related to triadic closure i.e., the formation of links among agents that have a neighbor in common have been favored [32, 18]. Starting from a random graph whose clustering coefficient tends to 0 as $N \rightarrow \infty$, this will cause a bias towards an increase of the mean clustering coefficient of the whole network. The reason behind this bias is that triadic closure is a well known observed feature of actual social networks. A recent related theoretical work on the dynamics of link rewiring in games, using explicit rewiring probabilities based on link type and Markov chains analysis has appeared in [31].

In this study we take a different approach and treat network dynamics as an *exogenous* phenomenon that is undoubtedly present to a larger or smaller extent but of which we do not know neither the exact origins nor the actual stochastic behavior. Thus, we shall assume that the network links are simply subject to noise. This point of view is justified by the fact that there is a large variety of social networks and, although some global statistical features such as degree distribution, mean degree, degree correlations, mean clustering coefficient and so on tend to be similar across networks, there is not, as yet, a general theory that explains every single network aspect when it comes to their dynamical behavior. In other words, instead of formulating some mechanisms that are supposed to be responsible for link evolution, we shall take inspiration from empirical data coming from some time-resolved studies of social network evolution to postulate general forms of network noise that are expected to describe, in a statistical sense, how the network links fluctuate. It is not the case that strategic network formation issues are unimportant; we only think that many networks are under the influence of largely unknown exogenous dynamically changing variables and we would like to lump all of them together under the form of unspecified fluctuations. The

following step is to study through numerical simulations the effects of such network fluctuations on the behavior of paradigmatic evolutionary games.

4.1.1 Games on networks

We have studied the four classical two-person, two-strategies games described by the payoff bi-matrix of Table 4.1.

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Table 4.1 Generic payoff bi-matrix for the two-person, two-strategies symmetric games. C and D are the possible strategies, and R, T, P , and S are payoff values as discussed in the text.

In this matrix, R stands for the *reward* the two players receive if they both cooperate (C), P is the *punishment* for bilateral defection (D), and T is the *temptation*, i.e. the payoff that a player receives if she defects while the other cooperates. In the latter case, the cooperator gets the *sucker's payoff* S . The parameters' values are restricted to the standard configuration space defined by $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the resulting TS -plane, each game's space corresponds to a different quadrant depending on the ordering of the payoffs. If the payoff values are ordered such that $T > R > P > S$ then defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only Evolutionarily Stable Strategy (ESS) [30] and we get the *Prisoner's Dilemma* (PD) game. Mutual cooperation would be socially preferable but C is strongly dominated by D .

In the *Snowdrift* (SD) game, the order of P and S is reversed, yielding $T > R > S > P$. Thus, in the SD when both players defect they each get the lowest payoff. (C, D) and (D, C) are NE of the game in pure strategies. There is a third equilibrium in mixed strategies which is the only dynamically stable state, while the two pure NE are not [30]. Players have a strong incentive to play D , which is harmful for both parties if the outcome produced happens to be (D, D) .

With the ordering $R > T > P > S$ we get the *Stag Hunt* (SH) game in which mutual cooperation (C, C) is the best outcome, Pareto-superior, and a NE. The second NE, where both players defect is less efficient but also less risky. The dilemma is represented by the fact that the socially preferable coordinated equilibrium (C, C) might be missed for "fear" that the other player will play D instead. The third mixed-strategy NE in the game is evolutionarily unstable [30].

Finally, the *Harmony* game has $R > S > T > P$ or $R > T > S > P$. C strongly dominates D and the trivial unique NE is (C, C) . This game is non-conflictual by definition and does not cause any dilemma: we include it just to complete the quadrants of the parameter space.

With the above conventions, in the figures that follow, the PD space is the lower right quadrant; the SH is the lower left quadrant, and the SD is in the upper right one. Finally, Harmony is represented by the upper left quadrant.

4.2 Results

Recent research on evolutionary games on static networks has shown that network reciprocity effects may favor cooperation to a fair extent in games, such as the PD, in which it would be doomed if the interacting population were well mixed [25, 14, 21]. In particular, largely degree-inhomogeneous networks topologies such as Barabási–Albert (BA) scale-free networks seem to possess the ingredients that boost cooperation the most. Network reciprocity in this case is facilitated and stabilized by cooperators that get hold of hub nodes, are surrounded mostly by cooperators, and are connected to other cooperator hubs [23]. Social networks also seem to be able to enhance cooperation [10, 11], albeit to a lesser extent than the ideal scale-free case. In social networks too there is degree inhomogeneity expressed by broad-scale degree distribution functions, although usually the tails fall off faster than in scale-free networks. Here other mechanisms play a role besides highly connected nodes: they manifest themselves through clustering and the presence of community boundaries, which are almost absent in BA networks. These features of actual social networks may favor cooperation with respect to well mixed populations. Because they are the best cooperation amplifiers among the studied network models, and thus they represent a kind of upper bound, we focus our numerical simulation study on Barabási–Albert scale-free networks. The construction of BA networks is well known and will be briefly described in the Methods section. The simulations start by randomly distributing cooperators and defectors among the networks' nodes in the same proportion. The simulations then proceed until a steady state is reached and, at this point averages are computed. In a steady state strategy fluctuations are smoothed out both in static and noisy networks. For more details the reader is referred to the Methods section.

4.2.1 Sequence of random Barabási–Albert networks

The first numerical experiment is to compare the behavior of evolutionary games on static BA networks and time-varying networks of the same family. In the latter case, we create a stochastic process $\{G(t), t = 0, 1, \dots\}$ in which each $G(t)$ is an independently generated BA graph with the same size and mean degree while in the static case there is a single graph $G(0)$ which is used all along. Clearly, by construction all the graphs in the sequence $\{G(t)\}$ have equivalent degree distributions. During an epoch t the players, which initially randomly receive a strategy $\sigma \in \{C, D\}$, will synchronously play the given game with their neighbors. In the dynamic case, with a certain frequency $\omega = 1/\Delta t$, the population graph is rebuilt at each Δt time steps. Players are numbered, and their current strategies are conserved when the network changes, but their neighborhood will in general be different. This process is not a likely one socially because it entails too much uncorrelated change, but it is simple and clear from a theoretical point of view. It will thus be used as a benchmark case in the following, as it represents the extreme case in which there is no correlation between successive instances of the network and each new network is an i.i.d. random variable. In all cases we start with the same number of cooperators and defectors randomly distributed over the network nodes. Other initial proportions are also interesting to investigate, as it has been done for the static case by Roca et al. [21]. Here, however, we shall focus on the comparison between the static and the dynamic cases and not so much on the robustness of results with respect to the initial conditions.

The following Figs. 4.1 show the average amount of cooperation at the end of the simulations on dynamically generated BA networks (central and right image) with respect to the static case (leftmost image). The strategy update rule is replicator dynamics (see section 4.4 for details on this revision protocol). The trend is clear: cooperation is negatively affected by the network noise, and the effect is more pronounced the higher the noise from left to right. The two games that are the most affected are the PD and the SH, while the SD game is the least affected. This was expected since, while PD and SH have monomorphic dynamically stable states, in the SD the equilibrium state is a dimorphic population.

Figures 4.2 show the same dynamics but using unconditional imitation of the best instead of replicator dynamics. Here the focal player imitates the strategy of the neighbor having obtained the highest payoff, including himself (see Methods section). Looking at the static case (leftmost image) it is already clear that there is a lower amount of cooperation to start with in the PD

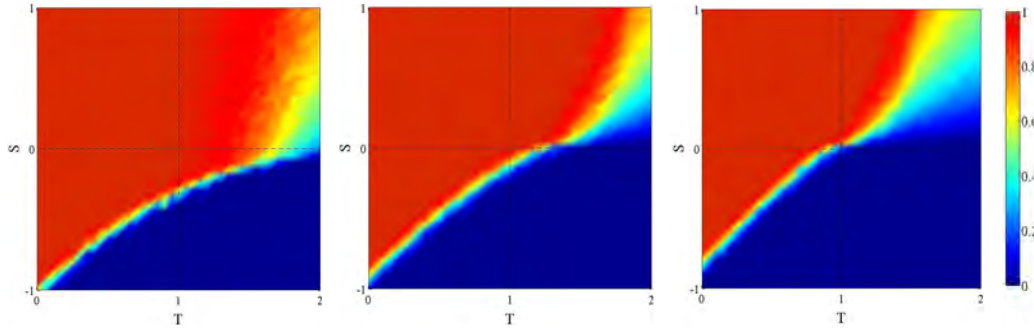


Fig. 4.1 Asymptotic distribution of strategies in the TS plane in static and dynamic BA networks using replicator dynamics as an update rule. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Leftmost image: the static case. Middle image: frequency ω of network generation is 0.1; rightmost image: $\omega = 1$. Values are averages over 100 independent runs.

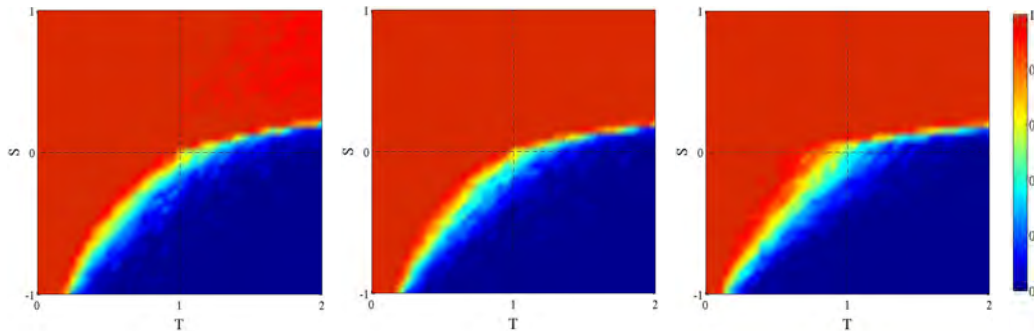


Fig. 4.2 Asymptotic distribution of strategies in the TS plane in static and dynamic BA networks using unconditional imitation of the best neighbor as an update rule. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Leftmost image: the static case. Increasing towards the right: frequency ω of network generation is 0.1 and 1. Values are averages over 100 independent runs.

quadrant, as well as in the SH case with respect to random graphs, while cooperation is high for the SD game (see Roca et al. [21] for a detailed discussion of these effects). Adding network noise has little effect but still the tiny amount of cooperation existing is almost completely lost when the noise level reaches the value one. A tentative qualitative explanation of the relative insensitivity to noise in this case is the following. The way in which a new network is generated in the noisy case (see above) tells us that, on the average, a given player will have more or less the same proportion of cooperators and defectors as neighbors in the new network as in the previous one. Since deterministic unconditional imitation rule depends on the global state of the neighborhood, it seems likely that the network dynamics will not have a large effect in this case.

Table 4.2 summarizes the numerical results by giving the average value of cooperation in the three non-trivial games for static and dynamic networks, and for the two strategy revision rules. From the table, the trend towards loss of cooperation in randomly fluctuating networks becomes very clear.

	PD, ib PD, rd SD, ib SD, rd SH, ib SH, rd					
Static BA networks	0.030	0.131	0.863	0.823	0.597	0.615
Dynamic Network ($\omega = 0.1$)	0.027	0.025	0.890	0.778	0.582	0.490
Dynamic Network ($\omega = 1.0$)	0.021	0.009	0.870	0.572	0.538	0.364

Table 4.2 Asymptotic average cooperation fraction in static and noisy BA networks. ‘ib’ and ‘rd’ stand for ‘imitate the best’ and ‘replicator dynamics’ update rules respectively. PD, SD, and SH design the Prisoners Dilemma, Snowdrift, and Stag Hunt games respectively. The table refers to Figs. 4.1 and 4.2.

4.2.2 Fluctuations from network edge swap

The independent sequence of BA graphs used above constitutes an ideal situation that can be considered as a baseline case, but it is quite far from a realistic link evolution in a given single network. To do a step towards more realism, we shall now assume that the sequence of graphs $\{G(0), G(1), \dots\}$ is generated by successively rewiring an initial given graph as suggested in [12]. We begin with $G(0)$ being a Barabási–Albert network; then, starting at time $t = 1$, each successive graph $G(t)$ in the sequence is generated by swapping two randomly chosen non-adjacent pair of edges in the previous graph $G(t - 1)$. In contrast to the previous case, where the sequence of graphs was an i.i.d. one, this process is a Markov chain since each new graph in the sequence depends on the previous one. The edge swap preserves the degree distribution $P(k)$ of G and obviously the node’s degree and the mean degree as well. The graphs, however, become more and more randomized as time goes by, as they tend to loose the historical degree correlations between hubs that arise in the original BA construction. Note that in this case we assume an asynchronous dynamics since it is, in our opinion, qualitatively more adapted to the new situation. Results are almost the same with either synchronous or asynchronous dynamics as shown in [21]. For the BA networks and replicator dynamics, this is also clear from the leftmost images in Figs. 4.1 and 4.3. Thus, instead of updating all the players’ strategies at once in each time step as before, we randomly choose a player to update (with replacement). This is called an elementary time step.

The period of network rewiring in this case is the number of elementary steps before an edge swap takes place, and the frequency ω is just the reciprocal of this number.

The visual results for this kind of generic network noise under replicator dynamics are shown in Figs. 4.3, while the measured final average values are given in Table 4.3. It is clear that, in agreement with the previous model of noise, even a moderate amount of network links fluctuation gives rise to a serious loss of cooperation in all the non-trivial games. To be more precise, after a good deal of edge swapping, the resulting networks, although they keep the original degree distribution, are close to scale-free random graphs generated according to the configuration model [12]. Cooperation frequencies on random scale-free graphs of the latter type are still better than random Erdős-Rényi graphs results (see [24] and especially [20], where a complete analysis of cooperation in the PD in random scale-free graphs is provided). However, they are significantly lower than those found in BA networks due to the loss of some early hubs interconnections that are present in BA networks because of the temporal growing process [20]. Thus, the reasons for the loss of cooperation are both the topology changes induced by the above loss of interconnected hubs, and especially the noisy neighborhoods induced by the edge swaps.

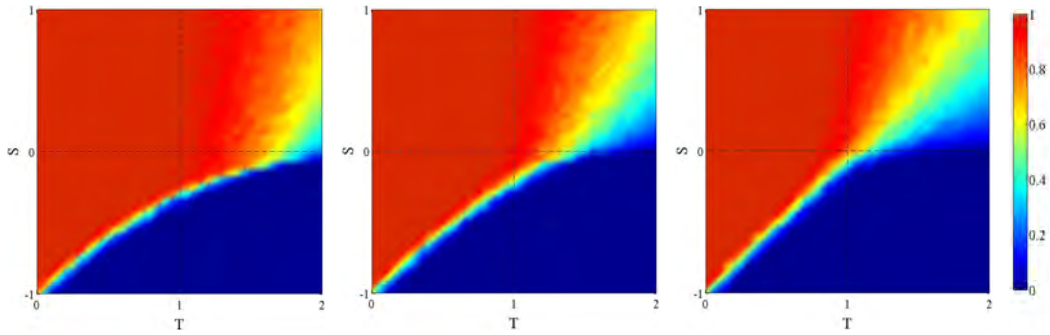


Fig. 4.3 Asymptotic distribution of strategies in the TS plane in static and dynamic BA networks using replicator dynamics as an update rule. Strategy update dynamics is asynchronous and the initial BA graph is rewired as explained in the text. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Leftmost image: the static case. Middle image: frequency ω of network rewiring is 0.1; rightmost image: $\omega = 1$. Values are averages over 100 independent runs.

	PD	SD	SH
Static BA Networks	0.131	0.825	0.617
Dynamic Network ($\omega = 0.1$)	0.055	0.699	0.576
Dynamic Network ($\omega = 1.0$)	0.017	0.591	0.523

Table 4.3 Asymptotic average cooperation fraction in static and dynamic networks with edge swap using replicator dynamics as a strategy update rule. Values refer to Fig. 4.3.

4.2.3 Fluctuations from edge rewiring

Once more, the previous assumed network fluctuation, although it is of theoretical interest, is still far from what happens in real networks. Experimental observations on dynamically changing social networks show that global statistics such as $P(k)$, the mean degree $\langle k \rangle$, the mean clustering coefficient $\langle C \rangle$ all remain similar but not exactly the same, they fluctuate to some extent during network evolution. This is true for growing networks, which are the majority of those that have been observed, but also for time-resolved studies of constant-size ones. These kind of results have been reported, among others, in [3, 8, 28, 9, 15]. Inspired by these considerations, we shall thus examine a third random dynamics that, without making strong assumptions on how players have their links cut and rewired, nevertheless provides fluctuations of the main network quantities similar to what seems to happen in real social networks. This should allow us to check whether the conclusions reached with the two previous models are robust enough starting from a BA network. Rewiring works as follow:

1. a node i is chosen with probability proportional to its degree k_i and one of its neighbors $j \in V(i)$ is selected with uniform probability
2. the corresponding $\{ij\}$ link is suppressed
3. node j creates a new link with a node $l \in V$ anywhere in the graph with probability proportional to l 's degree (preferential attachment)
4. to conserve minimum degree k_{min} , if nodes i or j have degree k_{min} they are not considered for rewiring and two other nodes are selected

This process makes highly connected nodes more likely to loose a link but, on the other hand, it also gives them more probability of being chosen for a new connection. The network statistics do change but they remain relatively close to the starting BA graph. In our simulations, after many rewirings, the network degree distribution function does remain broad-scale in average, but the tail tends to fall off faster than the original power-law. Figure 4.4 shows the degree distribution functions for the original and the rewired networks for two levels of noise averaged over 1000 graph realizations. For the rewired networks, the graphs refer to the final configurations. From the curves, one can see that for low noise ($\omega = 0.1$) the rewired networks have almost the same distribution as the original BA ones. On the other hand, when the noise is high ($\omega = 1$) the networks undergo a more marked change and the resulting degree distributions are closer to an exponential, as seen in the left image of Fig. 4.4 where the scales on the axes are lin-log. Indeed, most empirical degree distributions sampled on actual static social networks do give results that are between these two limit cases, i.e. a power-law and an exponential distribution [2, 4].

The simulations proceed in an asynchronous manner, as explained in the edge swap case above. Figures 4.5 depict the behavior of the four game classes on networks undergoing the above link fluctuations. The leftmost image is given for comparison; it refers to a static graph that has been first rewired so as to randomize the links as explained above, before being used as a fixed population topology. The middle and rightmost images depict the noisy cases with a frequency of rewiring of 0.1 (middle) and 1 (right). It is to be remarked that the static rewired network does not become an Erdős-Rényi random graph, and still allows for a fair amount of cooperation with respect to the pure BA case reported in the leftmost image of Fig. 4.3. Thus, the loss of cooperation observed as the network fluctuates is significant and can lead to full defection for the PD when rewiring and revising strategy have the same time scale (rightmost image). Snowdrift and SH are

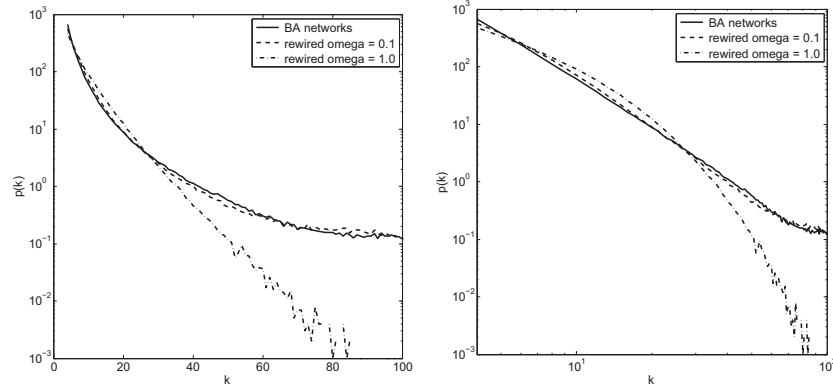


Fig. 4.4 Empirical degree distribution functions for the original BA networks and for the final rewired ones. Left image: lin-log scales; right image: log-log scales. The distributions for the rewired graphs are shown for two levels of network noise. For high levels of noise, distributions tend to the exponential type, otherwise they are closer to the original power-law. Values are averages over 1000 graph realizations for each curve.

less negatively affected. The average final values for the three games are reported in Table 4.4. The conclusion that we can draw from the results obtained with the three network fluctuation models is the following: whatever the source of random link fluctuations, as soon as the amount of noise becomes non-negligible i.e., as soon as network changes are at least ten times slower than strategy revision, the cooperation levels observed on static networks become weaker and they are completely lost when link noise and strategy update occur at the same rate. This conclusion is valid for all the non-trivial games studied, but it is particularly visible in the PD quadrant where defection becomes complete for $\omega = 1$.

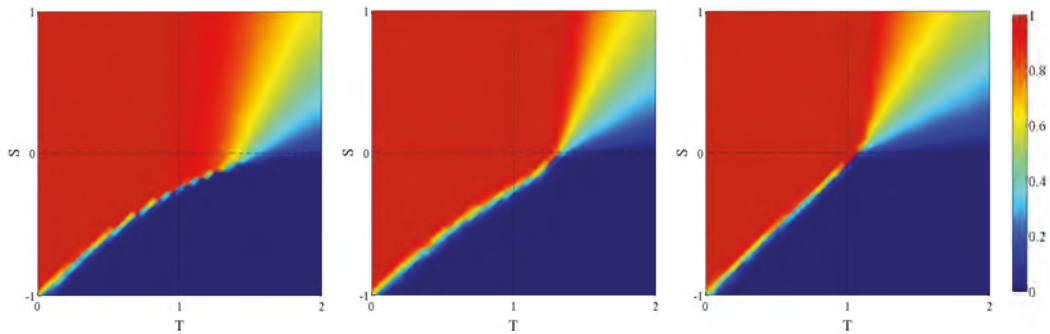


Fig. 4.5 Asymptotic distribution of strategies in the TS plane in rewired networks using replicator dynamics. Strategy update dynamics is asynchronous. Leftmost image: static rewired network (see text). Middle and rightmost images refer to dynamic graphs with frequency ω of network rewiring of 0.1 and 1, respectively. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Values are averages over 100 independent runs.

	PD	SD	SH
Static Networks	0.072	0.696	0.588
Dynamic Network ($\omega = 0.1$)	0.057	0.678	0.595
Dynamic Network ($\omega = 1.0$)	0.009	0.544	0.528

Table 4.4 Asymptotic average cooperation fraction in static and dynamic networks with edge rewiring (see text) using replicator dynamics as a strategy update rule. Values refer to Fig. 4.5.

4.2.4 Network and strategy noise

Until now, we have studied the impact of network fluctuations on typical evolutionary games. Another common source of noise in games arises from strategy errors. These are meant to capture various sources of uncertainty such as deliberate and involuntary decision errors which might play the role of experimentation in the environment, or be related to insufficient familiarity with the game. One easy way to include strategy noise is to use the Fermi function [26] as an update rule (see the Methods section for definitions). The parameter β in the function gives the amount of noise: a low β corresponds to high probability of error and, conversely, high β means that errors will be rare. One may ask how much these errors influence cooperation in networks of contacts, and whether they combine positively or negatively with network noise. As for their influence on static BA networks, the answer has been given in [21], where it is shown that for low noise ($\beta = 10$) the equilibrium behavior is similar to the one seen with replicator dynamics, while values of β close to 0.01 are enough to suppress all residual cooperation in the PD. In this case selection is weak, payoffs and network structure play a less important role. In other words, only comparatively high rates of strategy errors are really detrimental to cooperation. But when network fluctuations are present, cooperation is quickly lost, even for values of β that still allow for a fair amount of cooperation in the static case. Figures 4.6 show this for a static network (leftmost image)

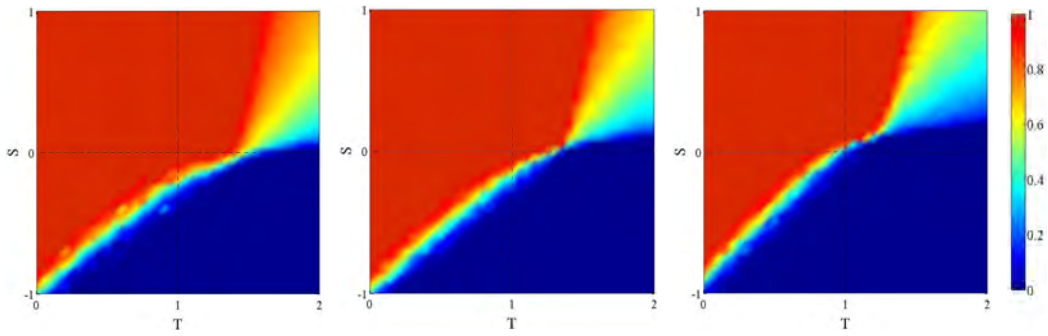


Fig. 4.6 Asymptotic distribution of strategies in the TS plane in static and dynamic BA networks using the Fermi rule (see text). Initial density of cooperators is 0.5 uniformly distributed at random in all cases. In all cases $\beta = 0.1$. Leftmost image: the static case. Middle image: frequency of graph renewal $\omega = 0.1$. Right image: $\omega = 1$. Values are averages over 100 independent runs.

as well as for two levels of network noise (central and right image) for $\beta = 0.1$. Network noise has been created as in our first model, i.e. by generating a sequence of independent BA networks with frequency ω .

Table 4.5 gives the asymptotic average values of cooperation in the three non-trivial games for static and dynamic networks. Although in the static case there is still a certain amount of cooperation in spite of the fact that β is relatively low, adding network noise quickly makes the situation worse. In conclusion, we can say that both kinds of errors tend to hinder cooperation, but network noise is more important than strategy noise in disrupting cooperation on degree-heterogeneous networks.

	PD	SD	SH
Static BA networks	0.066	0.817	0.574
Dynamic Network ($\omega = 0.1$)	0.035	0.739	0.552
Dynamic Network ($\omega = 1.0$)	0.001	0.565	0.416

Table 4.5 Asymptotic average cooperation fraction in static and dynamic BA networks using the Fermi rule with $\beta = 0.1$ (see text). Values refer to Fig. 4.6.

4.3 Discussion

The finding that fixed Barabási–Albert scale-free networks of contacts notably increase cooperation in social dilemmas has been an important one [23, 24] and has raised substantial hope, since scale-free graphs are better representations of actual networks of contacts than the random graphs and regular lattices that have often been used in the past. However, subsequent studies have somehow reduced its scope for various reasons. In the first place, the gains in cooperation can be partially or totally offset if players must pay an extra cost in order to maintain more contacts, as suggested by Masuda [13]. In a similar way if average, instead of accumulated payoff is used, the advantage of degree heterogeneity is lost as the individual’s payoff is divided by its degree in the graph [29, 27]. The latter seems to be an extreme case but it still shows in another way that the cost for an agent to maintain few or many links cannot be the same, although it certainly depends on the social context. Furthermore, if the players’ decision rule is partially conventional, some of the advantage is equally lost. For example, this has been shown to happen when agents have a conformist component to their behavior [16]. Finally, even when none of the above applies, the amount of cooperation gain due to network reciprocity can still be slim or non-existent depending on the strategy update rule and several other factors. This has been shown, among many other things, in the extensive studies of Roca et al. [21, 22] where it appears that using deterministic best response as an update rule instead of an imitative rule such as replicator dynamics causes a serious loss of cooperation in the PD on BA scale-free networks which recover the mean-field case.

In the present work, inspired by the empirical observation that networks are never completely static, we have shown that several forms of random fluctuation of the network links lead to a marked loss of cooperation that affects all the games’ phase space, even for moderate amounts of noise. The result is robust because, irrespective of the precise form of network noise, the same phenomenon manifests itself: asymptotically cooperation tends to disappear in the PD, and it di-

minishes in the other games. Moreover, network fluctuations appear to be more important than strategy noise in provoking a loss of cooperation. All the above refers to BA scale-free graphs and the general conclusion is that these population structures are not robust enough as cooperation amplifiers, as many factors may contribute to impair the ideal results. Network fluctuations, which certainly must occur in real-life, are among the most important factors. As a result, it can be said that, when the amount of noise is non-negligible, the system tends to behave in a mean-field way and thus the well-mixed population description seems to be adequate. This can be seen visually by comparing our figures with $\omega = 1$ with those for complete graphs that appear in [25] and [21]. The main reason for this behavior is the fluctuation of the neighborhood seen by each agent due to global network noise which, to some extent, resembles population mixing.

However, it has to be said that these model networks, although similar in some sense, do not represent well enough actual social networks; for instance, they do not have enough clustering, community structure, and degree correlations, among others. It would be interesting to see what is the effect of noise on games on social networks. Work is in progress in this direction. Finally, the negative conclusion that cooperation in scale-free networks is hindered by exogenous random network dynamics, should be taken with caution. It is valid when strategy evolution and network dynamics are completely uncorrelated as it was the case in the present study. However, it has been shown that when cutting and forming links in a co-evolving network has a strategic dimension to it, then cooperation can thrive and be stable since severing and reforming links is purposeful and based either on game payoff, or on game-related considerations (see, for instance, [32, 24, 18, 17]). From a social point of view, the difference is whether an agent can purposefully manipulate her environment, or is just under the influence of external network forces that she cannot control. In our opinion, both cases, as well as mixed situations may exist in reality. The study presented here belongs to the first stylized situation.

4.4 Methods

Population structure. The population of players is a connected undirected graph $G(V, E)$, where the set of vertices V represents the agents, while the set of edges E represents their symmetric interactions. The population size N is the cardinality of V . The set of neighbors of an agent i is defined as: $V_i = \{j \in V \mid \text{dist}(i, j) = 1\}$, and its cardinality is the degree k_i of vertex $i \in V$. The average degree of the network is called $\langle k \rangle$ and $P(k)$ denotes its degree distribution function, i.e. the probability that an arbitrarily chosen node has degree k . For the network topology we use the classical Barabási–Albert [1] networks. BA networks are grown incrementally starting with a clique of m_0 nodes. At each successive time step a new node is added such that its $m \leq m_0$ edges link it to m nodes already present in the graph. It is assumed that the probability p that a new node will be connected to node i depends on the current degree k_i of the latter. This is called the *preferential attachment* rule. The probability $p(k_i)$ of node i to be chosen is given by $p(k_i) = k_i / \sum_j k_j$, where the sum is over all nodes already in the graph. The model evolves into a stationary network with power-law probability distribution for the vertex degree $P(k) \sim k^{-\gamma}$, with $\gamma \sim 3$. For the simulations, we started with a clique of $m_0 = 9$ nodes and, at each time step, the new incoming node has $m = 4$ links.

Payoff calculation and strategy revision rules. In evolutionary game theory, one must specify how individual's payoffs are computed and how agents decide to revise their present strategy. In the standard theory, there is a very large well-mixed population; however, when the model is applied to a finite population whose members are the vertices of a graph, each agent j can only interact with agents contained in the neighborhood $V(j)$, i.e. only local interactions are permitted.

Let $\sigma_i \in \{\alpha, \beta\}$ be the current strategy of player i and let us call M the payoff matrix of the game. The quantity

$$\Pi_i(t) = \sum_{j \in V_i} \sigma_i(t) M \sigma_j^T(t)$$

is the accumulated payoff collected by agent i at time step t and $\sigma_i(t)$ is a vector giving the strategy profile at time t . Several strategy update rules are commonly used. Here we shall describe three of them that have been used in our simulations.

The first rule is to switch to the strategy of the neighbor that has scored best in the last time step. This *imitation of the best* policy can be described in the following way: the strategy $\sigma_i(t)$ of individual i at time step t will be

$$\sigma_i(t) = \sigma_j(t-1),$$

where

$$j \in \{V_i \cup i\} \text{ s.t. } \Pi_j = \max\{\Pi_k(t-1)\}, \forall k \in \{V_i \cup i\}.$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random, but otherwise the rule is deterministic.

The *local replicator dynamics* rule is stochastic [6]. Player i 's strategy σ_i is updated by drawing another player j from the neighborhood V_i with uniform probability, and replacing σ_i by σ_j with probability:

$$p(\sigma_i \rightarrow \sigma_j) = (\Pi_j - \Pi_i)/K,$$

if $\Pi_j > \Pi_i$, and keeping the same strategy if $\Pi_j \leq \Pi_i$. $K = \max(k_i, k_j)[(\max(1, T) - \min(0, S))]$, with k_i and k_j being the degrees of nodes i and j respectively, ensures proper normalization of the probability $p(\sigma_i \rightarrow \sigma_j)$.

The last strategy revision rule is the *Fermi rule* [26]:

$$p(\sigma_i \rightarrow \sigma_j) = \frac{1}{1 + \exp(-\beta(\Pi_j - \Pi_i))}.$$

This gives the probability that player i switches from strategy σ_i to σ_j , where j is a randomly chosen neighbor of i . $\Pi_j - \Pi_i$ is the difference of payoffs earned by j and i respectively. The parameter β in the function gives the amount of noise: a low β corresponds to high probability of error and, conversely, high β means low error rates. This interpretation comes from physics, where the reciprocal of β is called the temperature. Consequently, payoffs will be more noisy as temperature is raised (β is lowered).

Simulation parameters. The BA networks used in all simulations are of size $N = 2000$ with mean degree $\langle k \rangle = 8$. The TS plane has been sampled with a grid step of 0.05 and each value in the phase space reported in the figures is the average of 100 independent runs, using a fresh graph realization for each run. The initial graph for each run doesn't change in the static case,

while it evolves in the dynamic case, as described in the main text. Note that steady states have always been reached when strategies evolve on a static graph. We first let the system evolve for a transient period of $2000 \times N \simeq 4 \times 10^6$ time steps. After a steady state is reached past the transient, averages are calculated during $200 \times N$ additional time steps. True equilibrium states in the sense of stochastic stability are not guaranteed to be reached by the simulated dynamics. For this reason we prefer to use the terms steady states which are states that have little or no fluctuation over an extended period of time. In the case of fluctuating networks, the system as a whole never reaches a steady state in the sense specified above. This is due to the fact that the link dynamics remains always active. However, the distribution of strategies on the network does converge to a state that shows little fluctuation, i.e. a steady state.

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Chapter 5

Cooperation on Social Networks and Its Robustness

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Abstract In this work we have used computer models of social-like networks to show by extensive numerical simulations that cooperation in evolutionary games can emerge and be stable on this class of networks. The amounts of cooperation reached are at least as much as in scale-free networks but here the population model is more realistic. Cooperation is robust with respect to different strategy update rules, population dynamics, and payoff computation. Only when straight average payoff is used or there is high strategy or network noise does cooperation decrease in all games and disappear in the Prisoner's Dilemma.

5.1 Introduction

Game theory is used in many contexts, particularly in economy, biology, and social sciences to describe situations in which the choices of the interacting agents are interdependent. Evolutionary game theory in particular is well suited to the study of strategic interactions in animal and human populations and has a well defined mathematical structure that allows analytical conclusions to be reached in the context of well-mixed and very large populations (see e.g. [33, 8]). However, starting with the works of Axelrod [3], and Nowak and May [19] population structures with local interactions have been brought to the focus of research since they are supposed to be closer model of real social structures. Indeed, social interactions can be more precisely represented as networks of contacts in which nodes represent agents and links stand for their relationships. Thus, in the wake of the flurry of research on complex networks [18], evolutionary game theory has been recently enhanced with networked population structures. The corresponding literature has already grown to a point that makes it difficult to be exhaustive; however, good recent reviews can be found in [27, 22, 20]. In particular, the problems of cooperation and coordination, leading to social efficiency problems and social dilemmas, have been studied in great detail through some well known paradigmatic games such as the Prisoner's Dilemma, Snowdrift, and the Stag Hunt. Those games, although unrealistic with respect to actual complex human behavior, constitute a simple way for gaining a basic understanding of these important issues. One of the important findings in this line of research is that the heterogeneous structure of many model and actual networks may favor the evolution of socially valuable equilibria, for instance leading to non-negligible amounts of cooperation in a game such as the Prisoner's Dilemma where the theoretical result should be generalized defection [23, 26, 22]. Coordination on the socially preferable Pareto-efficient equi-

librium in the Stag Hunt and an increase of the fraction of cooperators in the Snowdrift game have also been observed on populations interacting according to a complex network structure [26, 22].

Most detailed results on evolutionary games on networks to date have been obtained for network types that are standard in graph theory such as Erdős-Rényi random graphs, scale-free networks, Watts-Strogatz small-world networks, and a few others that are commonly used (see [22] for a good complete review). This is obviously an important first step, since general results have been obtained for these network topologies, especially through numerical simulations and in some cases even analytical ones [27]. However, since the theory is of interest especially when applied in human or animal societies, to move a further step on, the network structures used should be as close as possible to those that can be observed in such contexts. This is the realm of social networks, many of which have been investigated and their main statistics established (see, for instance, [18]). While social networks degree distributions have been found to be broad-scale in general and thus they share this property [1], at least in part, with model scale-free networks they also possess some features that are not found in most common model networks. The three more important for our purposes are: clustering, positive degree assortativity, and community structure [16, 18]. Thus, in the present study we would like to offer a rather systematic study of evolutionary games played on this kind of networks in order to pave the way for the understanding of their behavior in real societies. Actually, there have already been several studies of evolutionary games on particular social networks in the past (see, for example and among others, [9, 12, 13]). Broadly speaking, these investigations all tend to show that socially valuable outcomes such as cooperation and coordination are more likely to evolve than in unstructured populations. However, the particular nature of the few networks used, although it provides some insight, does not allow one to draw more general and statistically valid conclusions. For this reason, here, instead of studying another particular network, we prefer to follow the methodology used in investigations such as [26, 22], where numerical simulation results are validated by using many runs on different networks belonging to the same class. As a model for constructing social networks, among several different possibilities, we choose to use Toivonen et al.'s model [30], which will be discussed in Sect. 5.4. This model has already been investigated in [13], albeit for a different evolution rule and for a reduced game phase space. In this paper, in addition to numerically studying the steady states of evolutionary games on these social network models when using static networks and error-free strategy update rules, we shall also briefly explore the effect of errors on strategy update rules and noise on network structure.

The paper is organized as follows. We first present some background on two-person, two-strategies symmetric games, followed by evolutionary games on networks. This is followed by a description of the model networks used. Next, results for the chosen games are presented and discussed, as well as the effects induced by the introduction of noise. We then offer our conclusions and some ideas for future work.

5.2 Games Studied

We have studied the four classical two-person, two-strategies symmetric games described by the generic payoff bi-matrix of Table 5.1.

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Table 5.1 Generic payoff bi-matrix for the two-person, two-strategies symmetric games. C and D are the possible strategies, and R, T, P , and S are utility values as discussed in the text.

In this matrix, R stands for the *reward* the two players receive if they both cooperate (C), P is the *punishment* for bilateral defection (D), and T is the *temptation*, i.e. the payoff that a player receives if she defects while the other cooperates. In the latter case, the cooperator gets the *sucker's payoff* S . Payoff values may undergo any affine transformation without affecting neither the Nash equilibria, nor the dynamical fixed points; however, the parameters' values are restricted to the "standard" configuration space defined by $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the resulting TS plane, each game's space corresponds to a different quadrant depending on the ordering of the payoffs.

If the payoff values are ordered such that $T > R > P > S$ then defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only Evolutionarily Stable Strategy (ESS) [33]. This famous game is called *Prisoner's Dilemma* (PD). Mutual cooperation would be socially preferable but C is strongly dominated by D .

In the *Snowdrift* (SD) game, the order of P and S is reversed, yielding $T > R > S > P$. Thus, in the SD when both players defect they each get the lowest payoff. (C, D) and (D, C) are NE of the game in pure strategies. There is a third equilibrium in mixed strategies which is the only dynamically stable state, while the two pure NE are not [33]. Players have a strong incentive to play D , which is harmful for both parties if the outcome produced happens to be (D, D) .

With the ordering $R > T > P > S$ we get the *Stag Hunt* (SH) game in which mutual cooperation (C, C) is the best outcome, Pareto-superior, and a NE. The second NE, where both players defect is less efficient but also less risky. The dilemma is represented by the fact that the socially preferable coordinated equilibrium (C, C) might be missed for "fear" that the other player will play D instead. The third mixed-strategy NE in the game is evolutionarily unstable [33].

Finally, the *Harmony* game has $R > S > T > P$ or $R > T > S > P$. C strongly dominates D and the trivial unique NE is (C, C) . This game is non-conflictual by definition and does not cause any dilemma: it is included just to complete the quadrants of the parameter space.

With the above conventions, in the figures that follow, the PD space is the lower right quadrant; the SH is the lower left quadrant, and the SD is in the upper right one. Harmony is represented by the upper left quadrant.

5.3 Evolutionary Games on Networks

In this section we present background material on evolutionary games on finite-size populations of agents represented by networks of contacts to make the paper as self-contained as possible.

5.3.1 Population structure

The population of players is represented by a connected unweighted, undirected graph $G(V, E)$, where the set of vertices V represents the agents, while the set of edges E represents their symmetric interactions. The population size N is $|V|$, the cardinality of V . The set of neighbors of an agent i is defined as: $V_i = \{j \in V \mid \text{dist}(i, j) = 1\}$, and its cardinality $|V_i|$ is the degree k_i of vertex $i \in V$. The average degree of the network is called $\bar{k}$ and $P(k)$ denotes its degree distribution function, i.e. the probability that an arbitrarily chosen node has degree k . The network topologies used are explained in Sect. 5.4.

5.3.2 Payoff calculation and strategy revision rules

In evolutionary game theory, one must specify how individual's payoffs are computed and how agents revise their present strategy. In the standard theory, there is a very large well-mixed population; however, when the model is applied to a finite population whose members are the vertices of a graph, each agent i can only interact with agents contained in the neighborhood V_i , i.e. only local interactions are permitted.

Let $s_i \in \{C, D\}$ be the current strategy of player i and let us call M the payoff matrix of the game. The quantity

$$\Pi_i(t) = \sum_{j \in V_i} \sigma_i(t) M \sigma_j^T(t)$$

is the *accumulated payoff* collected by agent i at time step t and $\sigma_i(t)$ is a vector giving the strategy profile at time t with $C = (1 \ 0)$ and $D = (0 \ 1)$. We also use the *average payoff* $\bar{\Pi}_i(t)$ defined as the average of accumulated payoff collected by a given agent i at time step t :

$$\bar{\Pi}_i(t) = \frac{1}{k_i} \sum_{j \in V_i} \sigma_i(t) M \sigma_j^T(t)$$

Several strategy update rules are commonly used. Here we shall describe three rules belonging to the *imitative* class that have been used in our simulations; the first rule is deterministic, while the following strategy updates are stochastic.

The first rule is to switch to the strategy of the neighbor that has scored best in the last time step. This *imitation of the best* policy can be described in the following way: the strategy $s_i(t)$ of individual i at time step t will be

$$s_i(t) = s_j(t-1),$$

where

$$j \in \{V_i \cup i\} \text{ s.t. } \Pi_j = \max\{\Pi_k(t-1)\}, \forall k \in \{V_i \cup i\}.$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random, but otherwise the rule is deterministic.

The *local replicator dynamics* rule is stochastic [7]. Player i 's strategy s_i is updated by drawing another player j from the neighborhood V_i with uniform probability, and replacing s_i by s_j with probability:

$$p(s_i \rightarrow s_j) = (\Pi_j - \Pi_i)/K,$$

if $\Pi_j > \Pi_i$, and keeping the same strategy if $\Pi_j \leq \Pi_i$. $K = \max(k_i, k_j)[(\max(1, T) - \min(0, S))]$, with k_i and k_j being the degrees of nodes i and j respectively, ensures proper normalization of the probability $p(s_i \rightarrow s_j)$.

The last strategy revision rule is the *Fermi rule* [27]:

$$p(s_i \rightarrow s_j) = \frac{1}{1 + \exp(-\beta(\Pi_j - \Pi_i))}.$$

This gives the probability that player i switches from strategy s_i to s_j , where j is a randomly chosen neighbor of i . $\Pi_j - \Pi_i$ is the difference of payoffs earned by j and i respectively. The parameter β in the function gives the amount of noise: a low β corresponds to high probability of error and, conversely, high β means low error rates. This interpretation comes from physics, where the reciprocal of β is called the temperature. Consequently, payoffs will be more noisy as temperature is raised (β is lowered). In the above expressions we have used the accumulated payoff Π_i . Analogous formulae hold for average payoff $\bar{\Pi}_i$.

5.3.3 Strategy update timing

Usually, agents systems in evolutionary game theory are updated synchronously. However, strictly speaking, simultaneous update is physically unfeasible as it would require a global clock, while real extended systems in biology and society in general have to take into account finite signal propagation speed. Furthermore, simultaneity may cause some artificial effects in the dynamics which are not observed in real systems [10]. However, for evolutionary game theory on networks the common wisdom is that the timing of updates does not influence the system properties in a fundamental manner and results are similar in most cases [22, 6], with asynchronism being sometimes beneficial to the emergence of cooperation [6]. This is called an elementary time step. To compare synchronous and asynchronous dynamics, here we use the customary fully asynchronous update, i.e. updating a randomly chosen agent at a time with replacement. The two dynamics, synchronous and fully asynchronous are the extremes cases. It is also possible to update the agents in a partially synchronous manner where a fraction $f = n/N$ of randomly chosen agents is updated in each time step. When $n = N$ we recover the fully synchronous update, while $n = 1$ gives the extreme case of the fully asynchronous update. Varying f thus allows one to investigate the role of the update policy on the dynamics [21, 6].

5.3.4 Simulation parameters

The networks used in all simulations are of size $N = 4000$ with mean degree $\bar{k} = 8$. The TS plane has been sampled with a grid step of 0.05 and each value in the phase space reported in the figures is the average of 100 independent runs, using a fresh graph realization for each run. The initial graph for each run doesn't change in the static case, while it evolves in the dynamic case, as described in the main text. Note that steady states have always been reached when strategies evolve on a static graph. In the asynchronous dynamics, we first let the system evolve for a transient period

of $4000 \times N \simeq 16 \times 10^6$ elementary time steps. In the synchronous case, the same total number of updates is performed. The averages are calculated at the steady state that is reached after the transient period. True equilibrium states in the sense of stochastic stability are not guaranteed to be reached by the simulated dynamics. For this reason we prefer to use the terms steady states which are configurations that have little or no fluctuation over an extended period of time. In the case of fluctuating networks, the system as a whole never reaches a steady state in the sense specified above. This is due to the fact that the link dynamics remains always active. However, the distribution of strategies on the network does converge to a state that shows little fluctuation, i.e. a steady state.

5.4 Network Construction and Properties

There exist several models for constructing social-like networks [29]. Among them, we have chosen Toivonen et al.'s model [30], called here the Toivonen Social Network (TSN), which was conceived to construct a network with most of the desired features of real-life social networks i.e., assortativity, high clustering coefficient, community structure, having an adjustable decay rate of the degree distribution, and a finite cut-off. The TSN construction and properties are described in detail in [30]. The process we have used to obtain a TSN can be summarized as follows:

1. Start with a small clique formed by m_0 vertices.
2. A new vertex v is added to the network and it is connected to m_r vertices chosen with uniform probability in the existing network. (*random attachment*)
3. Vertex v is connected to m_s vertices chosen with uniform probability within the list of neighbors of its neighbors. (*implicit preferential attachment*)
Every time that vertex v is connected to a new vertex the list of neighbors of its neighbors is updated.
4. Repeat step 2 and 3 until the network has grown to desired size N .

Notice that the process responsible for the appearance of high clustering and community structures is step 3. Moreover, this model is slightly different from the original one, because in the latter the list of neighbors in step 3 is not updated as in the former. This modification does not change the original model properties.

In the simulations, we have used networks of size $N = 4000$ with a clique of $m_0 = 9$ initial nodes as starting network. Every time a new node is added at step 2, its number of initial contacts m_r is distributed with probability: $\mathbb{P}(m_r = 1) = 0.5$ and $\mathbb{P}(m_r = 2) = 0.5$, while the number of its secondary contacts m_s is uniformly distributed between, and including, the integer values 0 and 5. The resulting degree distribution falls below a power law as shown in [30]. The average degree of the network is given by the formula that provides the average degree of a new added vertex:

$$1 \cdot \mathbb{P}(m_r = 1) + 2 \cdot \mathbb{P}(m_r = 2) + \sum_{t=0}^5 t \cdot \mathbb{P}(m_s = t) = 4$$

Thus, the resulting average degree of the network is $\bar{k} = 2 \cdot 4 = 8$.

To compare with another complex network model besides the standard Barabási–Albert (BA) networks [4], we have also used a model proposed by Amaral et al. [1] which also features several

of the characteristics of real complex networks, especially the distribution tail cutoff, and takes into account aging and cost constraints in the pure preferential attachment BA model.

In Fig. 5.1 we depict the degree distribution of a realization of TSN compared with a Barabási–Albert scale-free network, and the Amaral et al. model (in all cases, $N = 4000$ and $\bar{k} = 8$). The degree distribution of both TSN and the Amaral et al. model falls down faster than in a BA scale-free network and they are in fact closer to a stretched exponential or an exponentially truncated power-law. This agrees with many observations on empirically measured social networks, see e.g. [1, 15, 31].

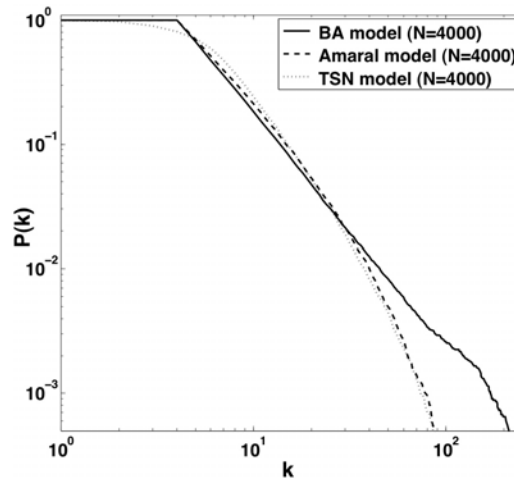


Fig. 5.1 Cumulated degree distribution of a TSN compared with a standard Barabási–Albert network of the same size and mean degree (see text).

Table 5.2 collects a number of statistics on social networks, mainly collaboration networks in various scientific disciplines. These empirical data show that, irrespective of the network size, they are all degree-assortative ($r > 0$) and have a high clustering coefficient. These features are shared by TSNs, and partly by the Amaral model, but not by BA networks. Indeed, it has been shown in the literature that most social networks possess these features (for a review, see [16]).

Finally, another characteristic feature of most social networks is the presence of communities, i.e. roughly speaking, groups of nodes that are more interconnected in the group than between groups [18]. TSNs do reproduce this feature while models such as the BA do not. For the purpose of illustration only, Fig. 5.2 shows a small example ($N = 500$) of a TSN graph in which communities have been detected using an algorithm based multi-level optimization of modularity [5]. Indeed, its modularity score [17] is 0.544 confirms the presence of communities, although to assess its statistical significance would require proper comparison with a suitable null model. Amaral model also gives rise to strongly connected clusters of nodes and its modularity is correspondingly very high, about 0.87.

In conclusion, we shall mainly use the TSN model for generating a family of networks, hoping to be able to reproduce the main features of actual social networks. The Amaral model will also be used in part to compare results of evolutionary games in a couple of more realistic models besides the standard BA one.

	N	$\bar{k}$	C	r	Ref.
Biology	1 520 251	15.53	0.60	0.127	[16]
Mathematics	253 339	3.92	0.34	0.120	[16]
Physics	52 909	9.27	0.56	0.363	[16]
Computer Science	11 994	2.55	0.50	-	[15]
Genetic Programming (GP)	2 809	4.17	0.66	0.130	[31]
TSN model	4 000 $\sim$	8.00	0.36	0.123	-
Amaral model	4 000	8.00	0.22	-0.193	-
BA scale-free networks	4 000	8.00	0.01	-0.042	-

Table 5.2 Some statistical features of model and real networks of coauthorships. N is the network size (number of nodes). Average degree $\bar{k}$; Average clustering coefficient $C = \frac{1}{N} \sum_{j=1, N} C_i$, where C_i is the node's individual clustering coefficient [16]; r is the Pearson's coefficient of neighbors' degree correlation [16].

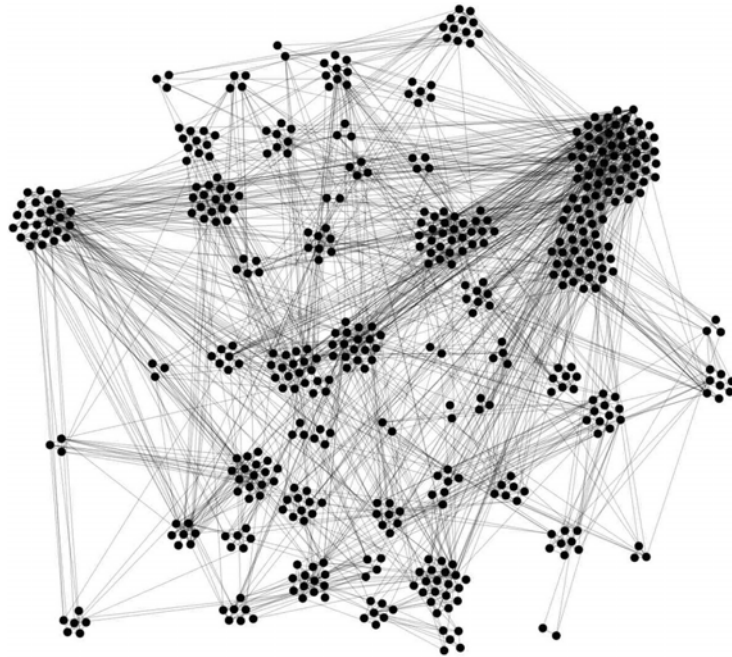


Fig. 5.2 Community structure in a small TSN ($N = 500$).

5.5 Results

In this section we report and discuss our simulation results for the TS -plane using different strategy update rules, network topology, and evolution dynamics.

5.5.1 Replicator dynamics

Figure 5.3 shows average cooperation frequencies for the four games in the case of BA networks (left image), Amaral's networks (middle), and TSNs (right image). The payoff is accumulated payoff (see Sect. 5.3.2). For the BA case, the results are very similar to those obtained in [26, 22] for larger systems with the same mean degree. Both visually and from the average game cooperation values shown next to each quadrant, it is clear that TSNs and the Amaral model networks are as favorable to cooperation as the idealized BA case when the strategy update rule is local replicator dynamics. However, the mechanisms responsible for high cooperation seem to be different. While in BA networks highly connected cooperator hubs play the role of catalyzers for the diffusion of the strategy [23], in social-like networks cooperation may thrive thanks to the higher clustering coefficient and the presence of communities. When a local cluster becomes colonized by cooperators, it tends to be robust against attacks by defectors [22].

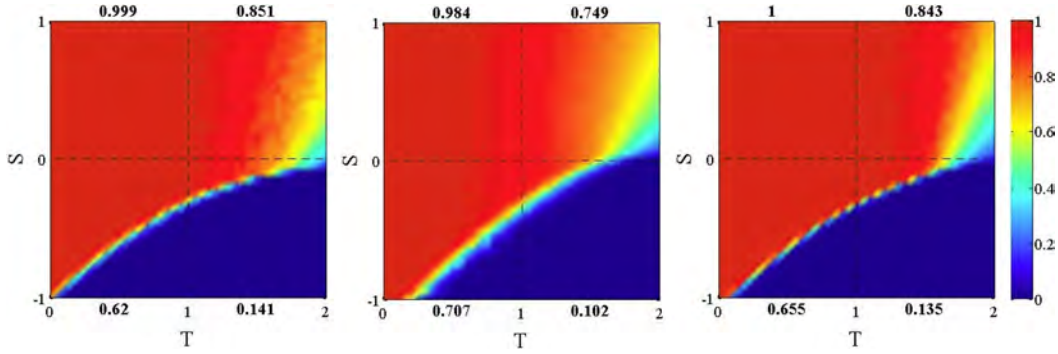


Fig. 5.3 Asymptotic distribution of strategies in the TS plane in BA networks (left), Amaral's model (middle), and TSNs (right) using replicator dynamics and accumulated payoff. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Population evolution is asynchronous. Network size is $N = 4000$ and average degree $\bar{k} = 8$ in both cases. Values are averages over 100 independent runs. The numbers in bold next to each quadrant stand for the average cooperation in the corresponding game.

The results shown are for asynchronous update (see Sect. 5.3.3), results for the synchronous case are very similar and we do not show them. The results on TSNs also confirm those obtained in [13] where a different update rule and parameter space was used, while the BA case was first shown in [26].

In Fig. 5.4 we plot average cooperation levels in all games for the case in which agent's payoffs are averaged over the agent's links (see Sect. 5.3.2). As expected, using average payoffs instead of accumulated ones corresponds to make the network more degree-homogeneous and thus the results resemble those obtained in regular random graphs [22] and are similar for both BA networks and TSNs. Defection becomes almost complete in the PD, values are similar to the well-mixed case in the SD, and in the SH the usual bistable result is recovered.

We conclude with [24, 14, 32, 28] that the use of a mean-field for the payoff scheme is detrimental to cooperation. Whether or not this significant factor in social networks is difficult to assess but, obviously, beyond a certain limit, maintaining links becomes expensive in practice and fur-

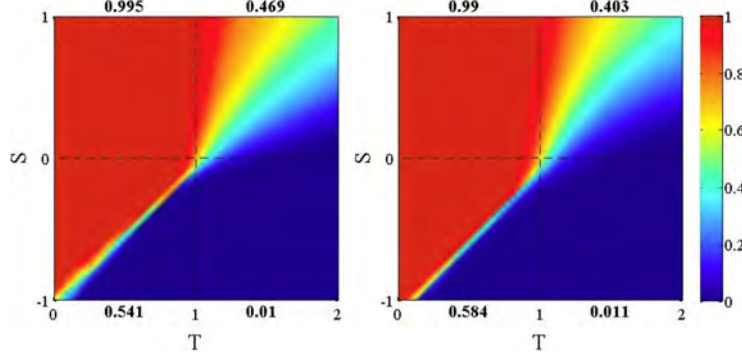


Fig. 5.4 Asymptotic distribution of strategies in the TS plane in BA networks (left) and TSNs (right) using replicator dynamics and average payoff. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Network size is $N = 4000$ and average degree $\bar{k} = 8$ in both cases. Asynchronous population dynamics. Values are averages over 100 independent runs.

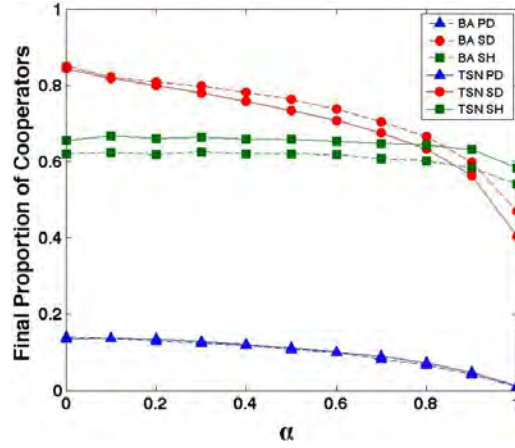


Fig. 5.5 Final average proportion of cooperation in the PD, SD, and SH games as a function of α (see text) for BA and TSN networks. The update is asynchronous and the revision rule is replicator dynamics.

thermore their frequency of usage should decrease. Probably, real situations are somewhere in between these two extreme cases and thus it is interesting to see what happens when average and accumulated payoff computations both happen to some extent.

Finally, following the idea of Szolnoki et al. [28], we compute payoffs using a weight parameter $\alpha \in [0, 1]$ to represent the proportion of average payoff. Thus, the actual payoff to agent i is given by the formula: $\alpha \bar{\Pi}_i + (1 - \alpha) \Pi_i$. In Fig. 5.5 we plot the steady-state average cooperation frequency for several α values. Clearly, in all games cooperation decreases with increasing α but the losses are limited in absolute value, except for the SD. Nevertheless, compared with average cooperation at $\alpha = 0$, which is 0.135 for TSN and 0.141 for BA, the PD loses all residual cooperation.

5.5.2 Imitation of the best

In this section we compare BA networks with TSNs, and the Amaral model using synchronous and asynchronous population dynamics and the imitation of the best strategy update rule (see Sect. 5.3.2). The average cooperation results with accumulated payoff are illustrated in Fig. 5.6. Note that asynchronous dynamics (bottom figures) is more favorable for cooperation in this case for all topologies, as already remarked by Grilo and Correia [6] for the BA case. With this deterministic asynchronous dynamics TSNs are particularly conducive to cooperation, more than 0.5 in the average for the PD, 0.98 in the SD phase space, and 0.95 in the SH quadrant; cooperation is a bit lower for the PD for the Amaral's case but still good in the whole game phase space. The qualitative reason for the diffusion and stability of cooperation seems to be related to the higher clustering coefficients of TSNs with respect to BA networks (see Table 5.2). In these networks, if a tightly linked cluster happens to get a large majority of cooperators, it can spread cooperation more easily to next neighbors. This is also true in the synchronous case (top figures) but to a lesser extent for the PD, and almost to the same extent for the SD and the SH, .

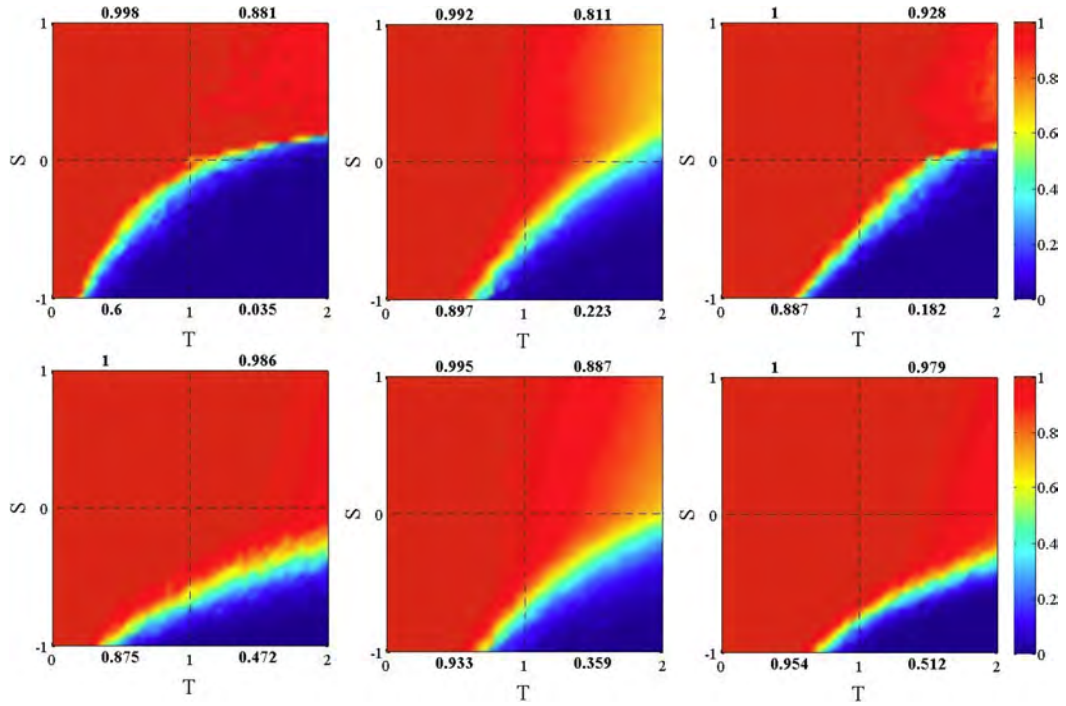


Fig. 5.6 Asymptotic distribution of strategies in the TS plane in BA networks (left column), Amaral's networks (middle), and TSNs (right column) using imitation of the best and accumulated payoff. Top row: synchronous dynamics. Bottom row: asynchronous dynamics. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Network size is $N = 4000$ and average degree $\bar{k} = 8$ in both cases. Values are averages over 100 independent runs.

The results with average payoff for the BA model and TSNs again with synchronous and asynchronous population dynamics are shown in Fig. 5.7. The top row shows images for synchronous update in BA networks (left image) and TSNs (right image). When using fully averaged payoffs, the results become similar to those obtained in a regular random graph with the same average degree, as in [22], with a small advantage in cooperation for the TSNs. In the asynchronous case (bottom row) results are similar with somewhat more cooperation in both cases and again an advantage for the TSNs. The results for BA networks have already been obtained by Grilo and Correia and are consistent with ours [6].

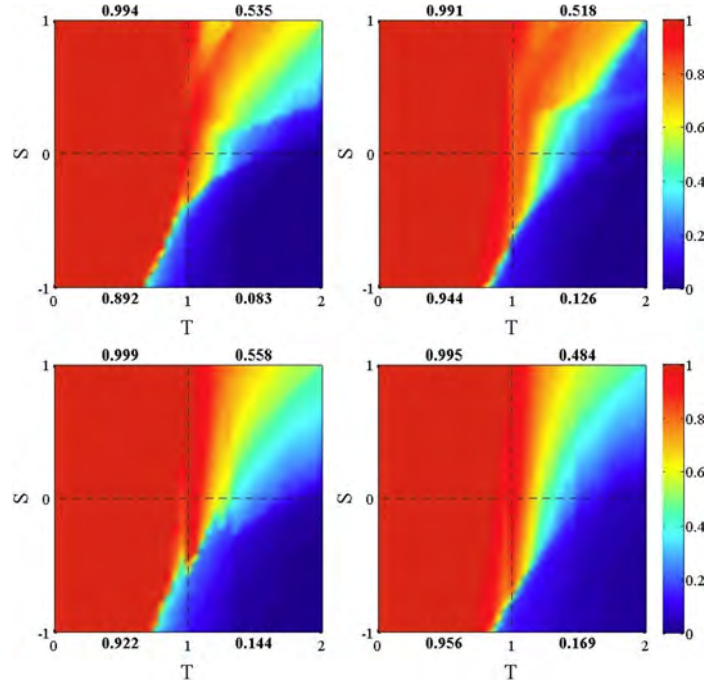


Fig. 5.7 Asymptotic distribution of strategies in the TS plane in BA networks (left column) and TSNs (right column) using imitation of the best and average payoff. Top row: synchronous dynamics. Bottom row: asynchronous dynamics. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Network size is $N = 4000$ and average degree $\bar{k} = 8$ in both cases. Values are averages over 100 independent runs.

Finally, we present in Fig. 5.8 the average amount of cooperation for the asynchronous dynamics as a function of the parameter α as explained in the previous section. The most important remark is that cooperation in the TSN model is almost always higher in the whole range of α for all games with respect to BA networks. Concerning the average/accumulated payoff tradeoff, one can see that there is a large decrease of cooperation in the SD and in the PD games going from pure accumulated ($\alpha = 0$) to pure average ($\alpha = 1$). The SH is less affected.

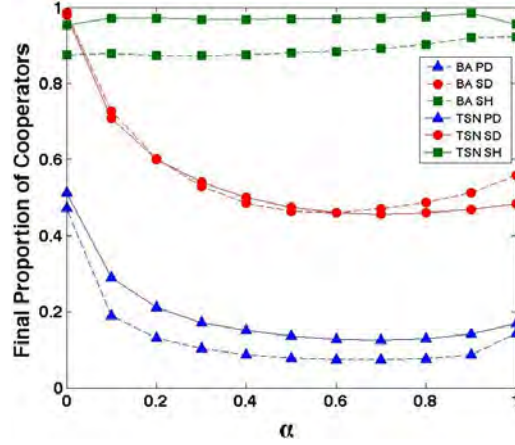


Fig. 5.8 Final average proportion of cooperation in the PD, SD, and SH games as a function of α (see text) for BA and TSN networks. Population update is asynchronous. The strategy revision rule is imitation of the best.

5.5.3 Fermi rule

The Fermi strategy update rule allows for the introduction of random noise in the choice of strategy (see Sect. 5.3.2). In Fig. 5.9 we show average cooperation results for different β values, from left to right $\beta = 10, 1, 0.1, 0.01$. The population dynamics is synchronous and thus the results can be compared with the analogous ones obtained by Roca et al. [22] in the case of BA networks. The results are quite similar, with slightly less cooperation in TSNs, and one can see that average cooperation levels tend to decrease with decreasing β since this corresponds to increasing randomness in the choice of strategy.

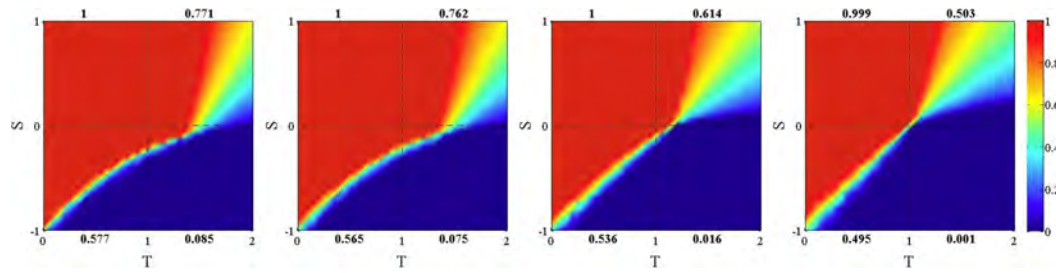


Fig. 5.9 Asymptotic distribution of strategies in the TS plane in TSNs using the Fermi rule with accumulated payoff and synchronous dynamics. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Network size is $N = 4000$ and average degree $\bar{k} = 8$. From left to right: $\beta = 10, 1, 0.1, 0.01$. Values are averages over 100 independent runs.

The asynchronous case is very similar and is not shown to save space. Clearly, those heterogeneous networks make cooperation relatively robust when there is some noise in the choice of

strategies, but when the decision becomes almost random topological considerations do not play an important role any longer.

5.5.4 Network links fluctuations

In a recent study, we have shown by numerical simulations that a certain amount of link rearrangement during evolution may hamper cooperation from emerging in scale-free networks [2]. It is important to note that the dynamics we are referring to is not strategy-related as in other work (see, for instance, [34, 25, 21, 20]); rather, noise is exogenous and random. Under these conditions, we found in [2] that the maintenance of neighborhoods and clusters becomes increasingly difficult with growing noise levels, and this has a negative effect on cooperation. In this section, we shall investigate the same issue for TSNs.

In actual social networks, both the number of links and of nodes tend to increase in time with a speed that is characteristic of a given network, and link formation is not random (see e.g. [11, 31] for two empirical analyses of growing social networks). However, little is known about these complex dynamics and we are not aware of any general theoretical model.

To test for the robustness of cooperation under very simple hypotheses on network fluctuations, and following [2], we have designed a link dynamics well adapted to TSNs through link cutting and rewiring that works as follows:

- a vertex v is chosen uniformly at random in V and its degree k is stored
- **cutting edges to neighbors:** node v loses all links to its neighbors, except to those that would become isolated by doing so and whose number is k_1
- **relinking node v :** vertex v creates $(k - k_1)$ new links:
 1. node v connects itself uniformly at random with another node $u \notin V_v$
 2. vertex v is connected to 4 vertices chosen with uniform probability within the list of neighbors of its neighbors. Every time that vertex v is connected to a new vertex the list of neighbors of its neighbors is updated
 3. Repeat step 1 and 2 until the number of new links equals $(k - k_1)$

This kind of rewiring could be interpreted as a sort of *migration* of node v . In the cutting step all of v 's previous links are suppressed and the same number of links is created in the relinking step in another region of the network. Vertex v keeps its original degree and thus the mean degree stays constant. If the “moving” node v had neighbors of degree one, those will follow it in the new position to avoid isolated nodes. The degree distribution function changes only slightly, the clustering coefficient remains high, above 0.30, and the network is always degree-assortative, all of which is consistent with empirical observations on real social networks [11, 31]. Note, however, that our dynamics represents only one among many reasonable possibilities. Reasons of space prevent us from studying other models here.

The period of network rewiring is the number of strategy update steps before a node rewiring takes place, and the frequency ω is just the reciprocal of this number. Figures 5.10 show average cooperation levels of all games as a function of the rewiring frequency ω . The update rule is replicator dynamics with accumulated payoff and asynchronous population evolution. From left to right: (static case) $\omega = 0$, $\omega = 10^{-3}$, $\omega = 10^{-2}$, $\omega = 2 \times 10^{-2}$. Values of ω larger than

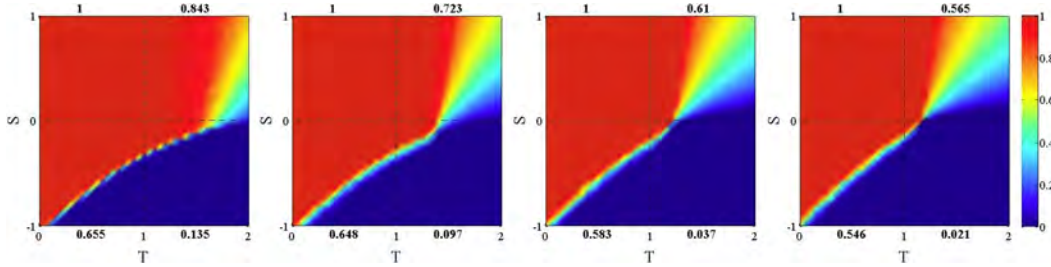


Fig. 5.10 Asymptotic distribution of strategies in the TS plane in TSNs using replicator dynamics with accumulated payoff and asynchronous dynamics. Initial density of cooperators is 0.5 uniformly distributed at random in all cases. Network size is $N = 4000$ and average degree $\bar{k} = 8$. From left to right: $\omega = 0, 10^{-3}, 10^{-2}, 2 \times 10^{-2}$. Values are averages over 100 independent runs.

the latter would cause too fast a rewiring. In actual social networks network dynamics is slow or medium-paced, depending on the type of interaction. The above link dynamics is only intended to qualitatively model those complex phenomena. It is clear that, as in the case of BA networks studied in [2] cooperation is progressively lost with increasing frequency of rewiring ω . This seems to be a general phenomenon in all networks: the loss of cooperation is caused by increasingly fast destruction of an individual's environment in the network. This noise prevents cooperators from forming stable clusters.

5.6 Conclusions

In this work we have presented a systematic numerical study of standard two-person evolutionary games on two classes of social network models. The motivation behind this choice is to make a further step towards more realism in the interacting agents population structure. The networks have been built according to Toivonen et al. model (TSN) [30], one of several social network models used in the literature and, in part, according to the model proposed by Amaral et al. [1].

Previous investigations have shown that broad-scale network models such as Barabási–Albert (BA) networks are rather favorable to the emergence of cooperation, with most strategy update rules and using accumulated payoff [23, 26, 22]. Here we have shown that the same is true in general for TSNs and the Amaral model, almost to the same extent as in BA networks. In addition, synchronous and asynchronous population update dynamics have been compared and the positive results remain true and even better for the asynchronous case when using imitation of the best update. We have also presented results for payoff schemes other than accumulated. In particular, we have studied average payoff and various proportions between the two extreme cases. The general observation is that pure average payoff gives the worst results in terms of cooperation, as already noted in [24, 32, 28]. When going from average to accumulate payoff cooperation tends to increase.

Finally, a couple of sources of noise on the evolutionary process have been investigated in order to get an idea about the robustness of cooperation on TSNs. To introduce strategy errors we have used the Fermi update rule. Cooperation on TSNs is relatively robust against this kind of noise, in

a manner comparable to scale-free graphs [22]. Of course, when the error rate becomes high, the behavior resembles to random and cooperation tends to decrease for all nontrivial games.

With a view to the fact that actual social networks are never really static, we have designed one among many possible mechanisms to simulate link fluctuations. When this kind of network noise is present cooperation tends to decrease and to disappear altogether when the network dynamics is fast enough. Similar effects have been observed in scale-free networks [2].

In conclusion, TSNs and also Amaral's networks appear to be as favorable as scale-free graphs for the emergence of cooperation in evolutionary games. But, with respect to the latter, the additional advantages are that TSNs and Amaral networks are much closer to actual social networks in terms of topological structure and statistical features. Cooperation can thus emerge and be stable in this kind of networks and probably also on related models. This is hopefully good news for co-operation among agents in social networks provided that the relationships are sufficiently stable. However, too much strategy noise or network instability may cause cooperation to fade away as in any other network structure.

Acknowledgments

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Chapter 6

Random Diffusion and Cooperation in Continuous Two-Dimensional Space

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7 March 2014.

Abstract This work presents a systematic study of population games of the Prisoner's Dilemma, Hawk-Dove, and Stag Hunt types in two-dimensional Euclidean space under two-person, one-shot game-theoretic interactions, and in the presence of agent random mobility. The goal is to investigate whether cooperation can evolve and be stable when agents can move randomly in continuous space. When the agents all have the same constant velocity cooperation may evolve if the agents update their strategies imitating the most successful neighbor. If a fitness difference proportional is used instead, cooperation does not improve with respect to the static random geometric graph case. When viscosity effects set-in and agent velocity becomes a quickly decreasing function of the number of neighbors they have, one observes the formation of monomorphic stable clusters of cooperators or defectors in the Prisoner's Dilemma. However, cooperation does not spread in the population as in the constant velocity case.

6.1 Introduction and Previous Work

Cooperative behavior is socially beneficial but difficult to obtain among selfish individuals. In this context, the Prisoner's Dilemma game is a widely used paradigm for the investigation of how cooperation might evolve in a population of self-regarding agents. In fact, game-theoretical results predict defection as a Nash equilibrium or as a stable state of the population dynamics [1, 2]. In spite of this, non-negligible amounts of cooperative behavior can be observed daily in the animal kingdom, in the human society, and also in the laboratory, where controlled experiments can be carried out. Many mechanisms have been suggested to explain these behaviors, such as direct and indirect reciprocity, kin reciprocity, group reciprocity, and population structure among others (see e.g. [3] and references therein for a summary of this vast amount of work).

Among the various reasons that have been advocated, the structure of the interacting population is one of the simplest factors that can change the generalized defection outcome with respect to the well-mixed population case. The population structure of the interacting agents can be generically represented by a relational graph in which two directly linked vertices stand for two interacting agents. This locality of contacts means that only pairs or groups of individuals that are direct neighbors play the game among themselves. By using theoretical models and simulations, it has been found that some network structures appear to be more conducive to cooperation than others, albeit this result is contingent upon the evolutionary dynamics of the model [4, 5, 6, 7]. However,

an earlier way of considering the effect of population structures makes use of the concept of geographical space. Indeed, physical space may be more adequate than generic relational structures in many cases in which territoriality plays an important role. A simple first approximation of physical space is given by a regular discrete lattice in two dimensions. Building on previous work by Axelrod [8], Nowak et al. [9] and Nowak et al. [10] were able to show by extensive simulations that, even when the game is one-shot, i.e. pairs of players interact anonymously, cooperation can evolve and can persist for a non-negligible region of the game phase space thanks to positive assortment among cooperators. Of course, anonymity of neighbors is difficult to maintain in a real unchanging social network environment, but this is the context that has been adopted in previous modeling work. A summary of this and other early work is provided in [11]. Actually, the gains in the PD are relatively limited and depend on the players' strategy update rule used [6]. Meanwhile, the improvements are large in the related game called Stag Hunt (SH) [6, 12] when played on a grid. Evolutionary games on arbitrary static spatially embedded networks have been recently studied in [13].

All the above refers to static environments. However, it is easy to see that fixed environments are the exception rather than the rule. Evolutionary games on dynamic networks have been investigated in recent years, see e.g. [14, 15, 16, 17] and the review article [18]. Although the models differ in their assumptions and the details of the dynamics, there is a consensus emerging on the fact that purposeful, or strategic link update is a further factor allowing cooperating individuals to escape exploiting defectors by cutting links to them and forming new links with fellow cooperators, which facilitates clustering and positive assortment of cooperators, ultimately leading to sustained global cooperation. In a way analogous to the dynamic network case, in the case of spatially embedded agents it is easy to think of mobile rather than fixed individuals. Many examples can be found in biological and ecological sciences, in human populations, and in engineered systems such as ad hoc networks of mobile communicating devices or mobile robot teams. Mobility may have positive or negative effects on cooperation, depending on several factors. An early brief investigation of random grids and spatial drift is to be found in Nowak et al. [10]. Another study was carried out by Enquist and Leimar [19] whose main conclusion of [19] was that mobility may seriously restrict the evolution of cooperation. In the last decade there have been several new studies of the influence of mobility on the behavior of various games in spatial environments covering essentially two strands of research: one in which the movement of agents is seen as a random walk, and a second one in which movement may contain random elements but it is purposeful, or strategy-driven. Examples of the latter kind of work are to be found in [20, 21, 22, 23, 24, 25]. In spite of the difference among the proposed models, the general message of this work is that purposeful contingent movement may lead to highly cooperating stable or quasi-stable population states depending on the individuals' density and the degree of mobility.

As said above, the other line of investigation is centered on random diffusion of the mobile agents through space, either in continuous space [26] or, more commonly, on diluted grids [27, 28]. Random diffusion, with its tendency to mix-up the population has been thought to hinder cooperation by weakening the possibility of cooperator cluster formation. In spite of this, the work of [27, 28] shows that cooperation can be maintained with respect to the static case and even enhanced for some parameters' ranges. In the continuous space case of [26] cooperation can be maintained only for low velocities and low temptation to defect. Within this framework, there has also been work on n -person Prisoner's Dilemma and public goods games, either in the one-shot case [29], as well as in the iterated, short memory case [30]. The effect of diffusion in a spatial

ecological public goods game has been studied by Wakano et al. [31] using a partial differential equation formalism.

The present investigation belongs to the random diffusion category and deals with memoryless agents performing random movements and interacting in pairs with other agents in continuous space. Indeed, we believe that while grids are interesting because of their simplicity, a continuous space approach is more natural and less restricted. Our approach follows Meloni et al. [26] but it largely extends and completes it in various ways. Indeed, Meloni et al. studied the weak Prisoner's Dilemma, which is the segment at the frontier between the genuine Prisoner's Dilemma game space and the Hawk-Dove game [1]. Here we explore the full conventional Prisoner's Dilemma space and also the regions belonging to the Stag Hunt and Hawk-Dove games. Furthermore, we use a second strategy update rule besides their fitness-proportional one. Finally, while the velocity of the agents was held constant and the same for all individuals in the population in [26], we explore the effects of having players diffusing with different velocities. Some relationships with the results found in the grid-based diffusion systems proposed in [27, 28] will also be discussed.

6.2 Model Description

In this section we describe our model and the numerical simulations parameters. We also describe what is new with respect to the previous work.

6.2.1 The Spatial Environment

The environment in which the set of agents N interact and move is a square in the Euclidean plane of side $L = 1$ thus having unit area. For the purposes of the dynamics the square is wrapped around into a torus. Agents are initially distributed uniformly at random over the space. Every agent j has an interaction neighborhood which has the same extension for all agents and is given by a circle of radius r around the given agent. All the agents that fall into this circle at a given time t are considered to be neighbors $\mathcal{N}(j, t)$ of the agent, i.e. $\mathcal{N}(j, t) = \{\forall k \in N \mid \text{dist}(j, k) < r\}$, where $\text{dist}(j, k)$ is the Euclidean distance between agents (points) j and k . Agents are simply material points, they do not have an area. Since the spatial region area has unit value, the density ρ of the agents is $\rho = |N|$.

Given the above setting, at any point in time the current implicit network of contacts between the agents turns out to be a Random Geometric Graph (RGG) [32] as illustrated in Fig. 6.1. The average degree $\bar{k}$ of a RGG is $\bar{k} = \pi \rho r^2$. Thus it is possible to consider $\bar{k}$ as a parameter of RGGs, instead of the radius r . Therefore, in order to construct an RGG with an average degree that tends to $\bar{k}$, it is sufficient to use the radius $r = \sqrt{\bar{k}/(\pi \rho)}$. This class of networks has an high average clustering coefficient [32] and positive degree-degree correlations [33].

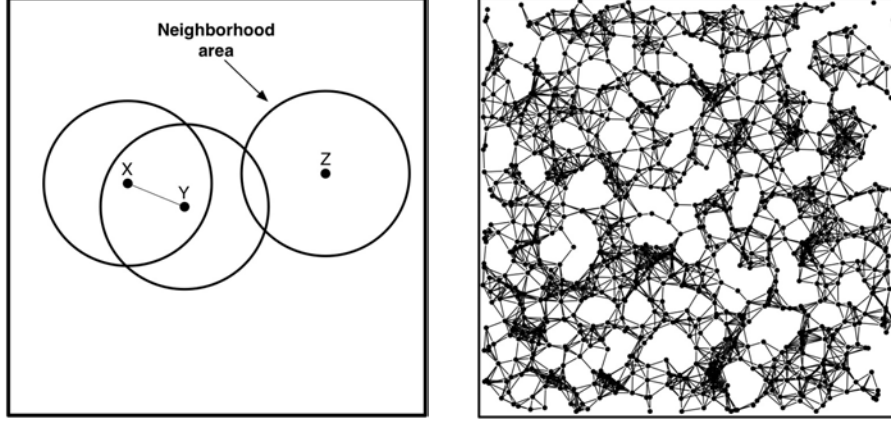


Fig. 6.1 Neighborhood area (left) and an example of a RGG with $|N| = 1000$ and $\bar{k} = 10$ (right).

6.2.2 Games Studied

Agents in our system, when they interact in pairs, play one of three common two-person, two-strategy, symmetric game classes, namely the Prisoner's Dilemma (PD), the Hawk-Dove Game (HD), and the Stag Hunt (SH). These three games are simple metaphors for different kinds of dilemmas that arise when individual and social interests collide. The games have the generic payoff matrix M (eq. 6.1) which refers to the payoffs of the row player. The payoff matrix for the column player is simply the transpose M^T since the games are symmetric.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \quad (6.1)$$

The set of strategies is $\Lambda = \{C, D\}$, where C stands for “cooperation” and D means “defection”. In the payoff matrix R stands for the *reward* the two players receive if they both cooperate, P is the *punishment* if they both defect, and T is the *temptation*, i.e. the payoff that a player receives if he defects while the other cooperates getting the *sucker's payoff* S .

For the PD, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only fixed point of the replicator dynamics [1].

In the HD game, the order of P and S is reversed, yielding $T > R > S > P$. Players have a strong incentive to play D , which is harmful for both parties if the outcome produced happens to be (D, D) . The only dynamically stable equilibrium is in mixed strategies [1, 2].

In the SH game, the ordering is $R > T > P > S$, which implies that mutual cooperation (C, C) is the payoff superior outcome and a NE. The second NE, where both players defect is less efficient but also less risky. The third mixed-strategy NE in the game is evolutionarily unstable [1].

Finally, in the Harmony game, which is included to complete the square, $R > S > T > P$ or $R > T > S > P$. In this case C dominates D and the trivial unique NE is (C, C) .

In order to study the usual standard parameter space [5, 6], we restrict the payoff values in the following way: $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the resulting TS -plane each game class corresponds to a different quadrant depending on the above ordering of the payoffs: PD is the lower right quadrant, HD corresponds to the upper left region, and SH belongs to the lower left quadrant.

6.2.3 Agent and Population Dynamics

The population dynamic is simulated by the following stochastic process at each time step t :

1. An agent is chosen uniformly at random among the population
2. The agent plays the game with each of his neighbors in turn and accumulates his payoff
3. The chosen agent undergoes a strategy revision phase based on his current payoff and the payoff of his neighbors
4. The agent moves randomly to another position in space

Step 1 means that the population evolution is asynchronous in time, which amounts to a Monte Carlo process simulation. At the end of step 2 the player computes his total payoff accumulated through the two-person games played with his neighbors $\mathcal{N}_i$. Let $\sigma_i(t)$ be a vector giving the strategy profile at time t with $C = (1, 0)$ and $D = (0, 1)$ and let M be the payoff matrix of the game (eq. 6.1). The quantity

$$\Pi_i(t) = \sum_{j \in \mathcal{N}_i} \sigma_i(t) M \sigma_j^\top(t) \quad (6.2)$$

is the cumulated payoff collected by player i at time step t .

In step 3 the chosen agent adopts a strategy in $\Lambda = \{C, D\}$ according to a microscopic revision rule. Several update rules are customary in evolutionary game theory [4]. Two imitative protocols have been used in our simulations: *fitness difference proportional* and *imitation of the best*. The fitness difference proportional rule (FDP) is linear and stochastic [34, 35]. Player i 's strategy σ_i is updated by randomly drawing another player j from the neighborhood $\mathcal{N}_i$, and replacing σ_i by σ_j with probability:

$$p(\sigma_i \rightarrow \sigma_j) = \begin{cases} (\Pi_j - \Pi_i)/K & \text{if } \Pi_j > \Pi_i \\ 0 & \text{if } \Pi_j \leq \Pi_i \end{cases} \quad (6.3)$$

where $\Pi_j - \Pi_i$ is the difference of the payoffs earned by j and i respectively.

$$K = \max(k_i, k_j)[\max(1, T) - \min(0, S)]$$

ensures proper normalization of the probability $p(\sigma_i \rightarrow \sigma_j)$.

Another imitative strategy update protocol is to deterministically switch to the strategy of the neighbor that has scored best in the last time step [9]. This imitation of the best (IB) policy can be described in the following way: the strategy $\sigma_i(t)$ of individual i at time step t will be

$$\sigma_i(t) = \sigma_j(t-1), \quad (6.4)$$

where

$$j \in \{\mathcal{N}_i \cup i \mid \Pi_j = \max\{\Pi_k(t-1)\}, \forall k \in \{\mathcal{N}_i \cup i\}\}. \quad (6.5)$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random.

Finally, in Step 4 the individual moves to another position in the plane in the following way. First, the magnitude of the displacement v is constant and the same for all agents for a given simulation run. Next, a direction is chosen by randomly drawing an angle in the range $[0, 2\pi]$.

The iteration of steps 1, 2, 3, and 4 gives rise to a sequence of RGGs $\{G(t)\}_t$, and a sequence of population strategy profiles $\{\sigma_1(t), \dots, \sigma_{|N|}(t)\}_t$, with $t \in \mathbb{N}$. Given our assumptions, both sequences are Markov chains.

In another set of simulations the velocity v is not constant for all individuals but will instead depend on a given individual's neighborhood, see Sect. 6.4.

6.3 Constant Velocity Results and Discussion

The parameters used in the simulations were the following. The number of players was $|N| = 1000$; the mean degrees of the RGGs constituting the populations were either $\bar{k} = 4$ or $\bar{k} = 8$. The maximum number of simulation steps was 2000. Here one step means the update of all the $|N|$ players in the average, since updating is asynchronous. Simulations end either when a monomorphic population has been obtained, or when the maximum number of steps has been reached. Cooperation frequencies in the population are computed when the run stops. As for the game space, the TS -plane has been sampled in steps of 0.05 in the T and S directions thus forming a 41×41 grid. As stated above, at the beginning of each run agents are randomly distributed over the space with an agent being a cooperator with probability 0.5. Cooperation frequencies for each game space grid point are computed as the average value of 100 runs. The last parameter is the individual's velocity v . As stated, in each given simulation run, all the agents are assigned the same constant velocity v . We have studied the cases $v = 0$ (the static case), $v = 0.01$, and $v = 0.001$.

Figures 6.2 and 6.3 show the results for the entire games' phase space when players update their strategy using the FDP rule (see Sect. 6.2.3) with $\bar{k} = 4$ and $\bar{k} = 8$ respectively. In the figures, velocity decreases from left to right and the $v = 0$ case refers to a static RGG. Since Meloni et al. [26] used the same update rule, we first compare their results with ours. The game space sampled in [26] corresponds to the segment at the frontier between the PD and HD game spaces with $R = 1$, $P = 0$, $S = 0$, and $1 \leq T \leq 2$ (weak PD game). When our velocities are suitably normalized in order to be comparable with the corresponding values in [26], our results do qualitatively agree with theirs for the weak PD, i.e. cooperation can only evolve for low T values, small velocity, and small mean degrees, or radii used in the simulations shown in the figures. However, when one looks at the entire PD game space, one can see that the gains in cooperation are indeed very small or non-existent when the agents can move. Only in the case of $v = 0.001$ cooperative outcomes are found in the small area in the upper left corner of the PD quadrant with S slightly less than 0, and T between 1.0 and about 1.1. It is likely that the separation between defection and cooperation in this small region would become crisper if we could perform extremely long runs and use larger population, both of which contribute to dramatically increase the computing time. However, these considerations do not change the global result, i.e. defection prevails almost completely in the PD.

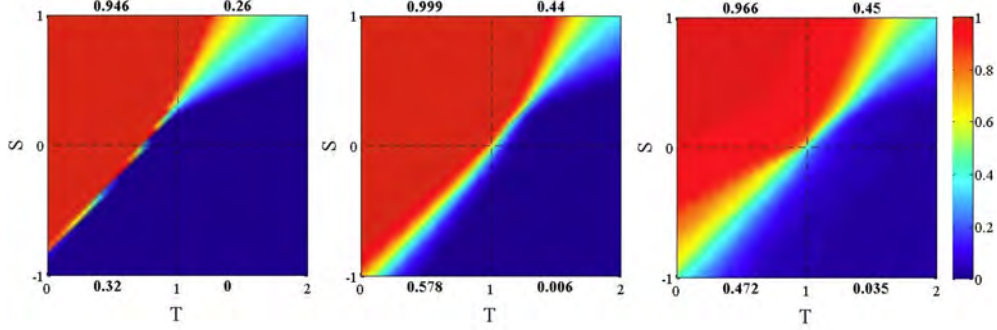


Fig. 6.2 Average cooperation values using FDP as an update rule. The mean degree is $\bar{k} = 4$. From left to right: $v = 0.01$, $v = 0.001$, and $v = 0$. The bold-face numbers next to each quadrant are average cooperation values for the corresponding game. The results are averages over 100 runs.

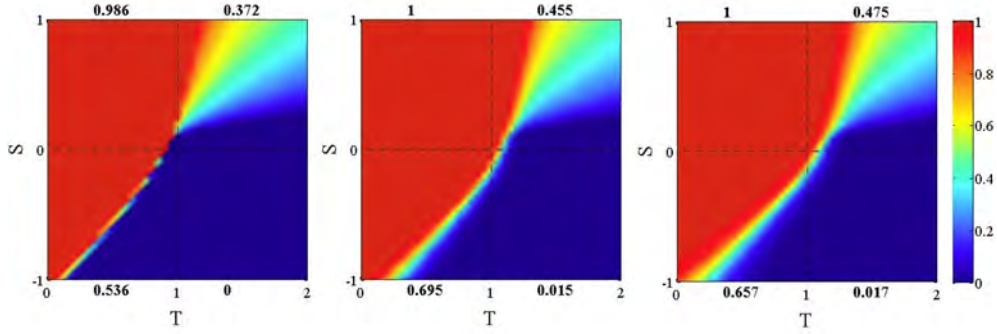


Fig. 6.3 Average cooperation values using FDP as an update rule. The mean degree is $\bar{k} = 8$. From left to right: $v = 0.01$, $v = 0.001$, and $v = 0$. The bold-face numbers next to each quadrant are average cooperation values for the corresponding game. The results are averages over 100 runs.

For the other games, the HD behavior is practically not influenced by the agents' movement except for the case $\bar{k} = 4$ and $v = 0.01$ where there is more defection than what one observes in the static case, i.e. $v = 0$. This is due to the particular status of the HD in which the only stable state is the mixed one. When small cooperator or defectors clusters form, they are subsequently invaded by the other strategy and their composition remain close to the proportions of the mixed NE. The situation is different and more interesting in the SH case which features bistability. Here cooperation benefits from the agents' diffusion with low velocity $v = 0.001$ because slow movement favors the spreading of the efficient strategy. Instead, when $v = 0.01$ the populations are less cooperative than in the static case.

While Meloni et al. [26] did not use the IB rule, Vainstein et al. [27] and Sicardi et al. [28] performed an extensive study of random migration in diluted grids employing this rule. They investigated the influence of the density of players and of their mobility on cooperation when the agents have the possibility of migrating to an empty neighboring site in the grid, if any, with a certain probability. It must be said that our continuous space system is not directly comparable to

the setting of [27, 28]. Indeed, in their case the mobility parameter, a quantity that is related to the players' displacements, and thus to their implicit velocity, depends on the density of players in the grid, while in our continuous collisionless model velocity is a well-defined independent quantity. Keeping in mind the differences, it is nevertheless interesting to compare the results of [27, 28] with ours. In Figs. 6.4 and 6.5 we show average cooperation levels in the whole T - S space when the update rule is imitation of the best. In [27, 28] equilibrium cooperation values for only some points in the PD, HD, and SH spaces were reported, albeit using a much larger population size. In [27, 28] each player has at most four neighbors which means that the mean degree is less than four, since there are empty sites when their density $\rho < 1$; we should thus compare their results with those of Fig. 6.4. Sicardi et al. found that for intermediate densities ρ and for the games sampled in the T - S plane, cooperation states are reached in the three games PD, HD, and SH. This result is qualitatively confirmed and generalized by looking at the two leftmost images of Fig. 6.4. Indeed, when agents can move the SH quadrant becomes fully cooperative, the HD results are improved with respect to the static case, and a larger cooperative region appears in the PD space too. Results are better in terms of cooperation for lower velocities. Fig. 6.5 confirms all the trends when the mean connectivity of the agents doubles; indeed, more connections seem to further favor cooperation because cooperator cluster formation becomes easier.

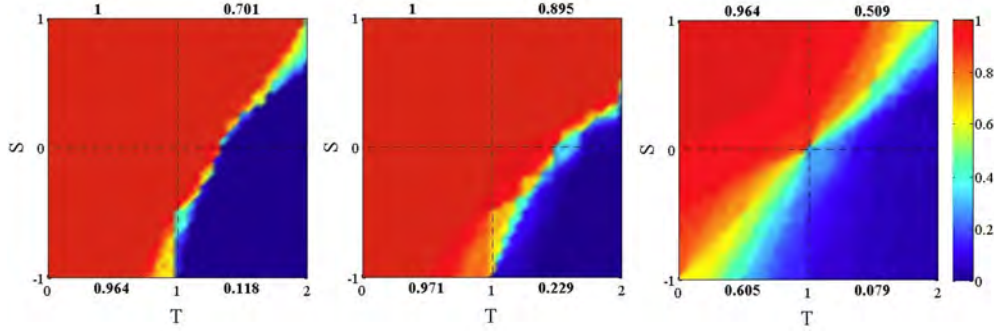


Fig. 6.4 Average cooperation values using imitation of the best as an update rule. The mean degree is $\bar{k} = 4$. From left to right: $v = 0.01$, $v = 0.001$, and $v = 0$. The bold-face numbers next to each quadrant are average cooperation values for the corresponding game. The results are averages over 100 runs.

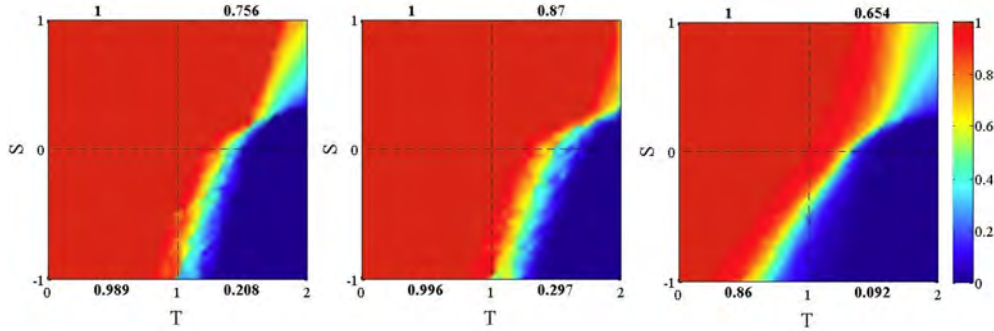


Fig. 6.5 Average cooperation values using imitation of the best as an update rule. The mean degree is $\bar{k} = 8$. From left to right: $v = 0.01$, $v = 0.001$, and $v = 0$. The bold-face numbers next to each quadrant are average cooperation values for the corresponding game. The results are averages over 100 runs.

For the sake of illustration, Fig. 6.6 shows spatial snapshots of a particular but typical run at successive time steps. The particular game shown here, which converges to full cooperation, has $R = 1$, $P = 0$, $S = 0$, and $T = 1.3$, $v = 0.001$, $\bar{k} = 4$, and thus it belongs to the weak PD along the segment between the PD and HD games (see Fig. 6.4 middle image). The sequence of figures represent evolving RGGs. Starting from a 50 – 50 initial configuration, there is first a fall in cooperation followed by a steady increase caused by cooperation spreading around small cooperative subgraphs. Although the last image still contains a few defectors, those are likely to disappear if the simulation would last longer. This mechanism is common to the SH game too, which converges to cooperation everywhere in the parameters space, and also in the cooperative HD and PD regions. The graph and strategy evolution found with FDP instead of IB when the final state is full cooperation are similar but take longer times.

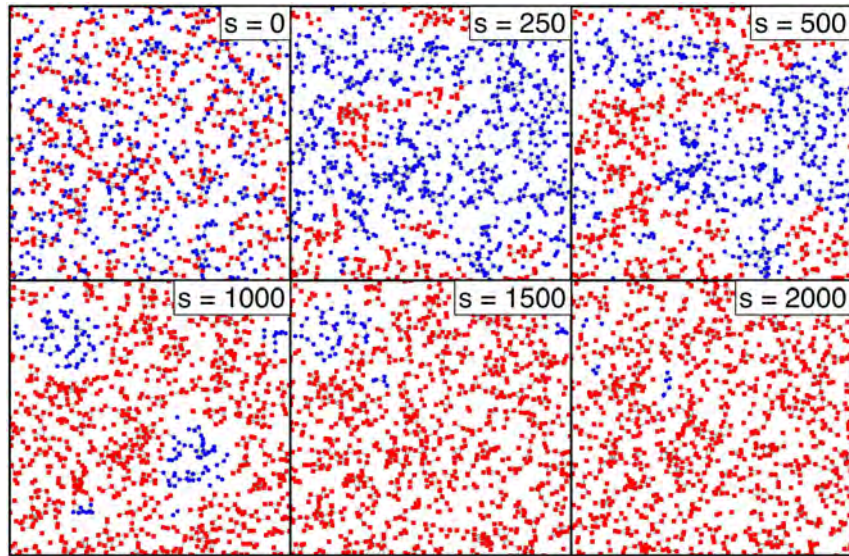


Fig. 6.6 Time evolution of a particular run with $R = 1$, $P = 0$, $S = 0$, $T = 1.3$, $v = 0.001$, and $\bar{k} = 4$. Red squares represent cooperators; blue circles stand for defectors; The initial proportion of cooperators is 0.5. s gives the time step at which the corresponding snapshot has been taken.

In conclusion, it appears that cooperation tends to be enhanced for a sizable part of the parameters space when agents update their strategies using the IB rule, and are able to move around. In addition, since our results and the partial ones of [27, 28] are coherent between themselves starting from two rather different models, the phenomenon can be considered robust. Some caveats are in order however. In the first place, the IB strategy update rule requires knowing the payoffs of the neighbors, which is likely to be cognitively unfeasible in many biological situations of interest. Perhaps it is conceivable, at least in a stylized manner, in human societies. As we saw above, the results are not as good with FDP update, which is also based on payoff differences but has a stochastic component that could be thought to roughly represent some uncertainty in the decision process.

6.4 Viscosity and Non-Constant Velocity

In the basic model agents move with a constant pre-defined velocity v which is the same for all of them. However, this does not seem very realistic. If one looks at a crowd moving in space, for instance, one sees that the magnitude of the displacements of an agent strongly depends on the density of people around him. We have tried to model this effect in a very simple but general way, although it might not be suitable for any particular given real situation. Now the velocity becomes a function of the instantaneous degree k of the agent such that the more neighbors she has, the more her movement is hindered. For simplicity, we have chosen a negative exponential function form. Thus an agent of degree k will have an instantaneous velocity $v(k)$ given by:

$$v(k) = v_0 e^{-\beta k} \quad (6.6)$$

Here v_0 is the agent velocity when there are no neighbors, and β is a scaling factor to be chosen empirically which influences the speed of the decay and thus the mobility. For $\beta = 0$ we recover the constant velocity model, while for $\beta \rightarrow \infty$ viscosity is maximal and we recover the static case for $k \neq 0$. After some numerical experimentation, we have chosen to study the effect of two different values of β : 1 and 0.1.

The following discussion is for an initial contact radius that corresponds to $\bar{k} = 4$, the strategy update rule is IB, and $v_0 = 0.01$. However, as time goes by, owing to the non-uniform random diffusion caused by viscosity effects, the average degree is not maintained and will tend to increase. Let us first examine the case $\beta = 0.1$. In this case the displacements are less hindered and thus viscosity is comparatively low. Before presenting average results for the whole $T - S$ plane which are globally interesting but could hide some important dynamical phenomena, we will show a particular, suitably chosen game. Figure 6.7 shows the time evolution for a game in the weak PD segment with $R = 1$, $P = 0$, $S = 0$, and $T = 1.3$. The phenomena observed are very different from those that appeared in the same game with constant velocity shown in Fig. 6.6. With constant velocity, this game region led to full cooperation. Instead, with velocity damping, although average cooperation is still of the order of the initial 50 – 50 proportion, the spatial distribution of cooperators and defectors is no longer random: there is cluster formation of each

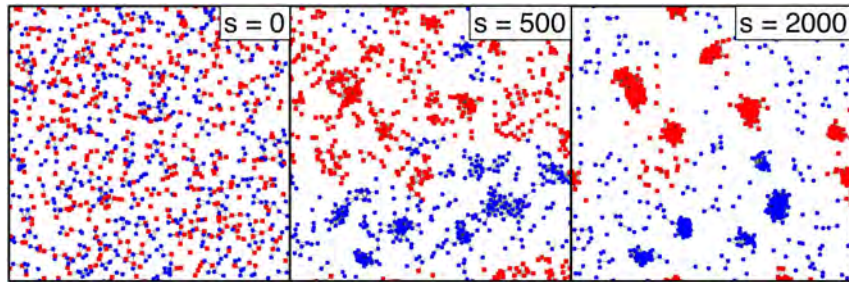


Fig. 6.7 Time evolution of a particular run with $R = 1$, $P = 0$, $S = 0$, $T = 1.3$, initial $\bar{k} = 4$, $\beta = 0.1$ and $v_0 = 0.01$. Red squares represent cooperators; blue circles stand for defectors; The initial proportion of cooperators is 0.5. s gives the time step at which the corresponding snapshot has been taken. The final cooperation frequency at $s = 2000$ is 0.57.

strategy which is essentially caused by progressive velocity reduction due to viscosity. Once the dense clusters are formed, agents cannot move away easily just because viscosity is too high.

It is thus clear that cooperation cannot spread past the clusters as in the case with constant velocity, and the clusters are just connected components which are disconnected from the other parts. In fact, in the limit of very long simulation times, the clusters become more and more dense because of the lack of a minimal repulsion distance in the model.

Now we describe the case of $\beta = 1$. Taking the same game as before, we obtained the dynamics depicted in the snapshots of Fig. 6.8. With a larger β value one would expect that viscosity plays a more important role in hindering the agents' movements. This is indeed the case. At the very beginning agents do not diffuse much already but are still able to move enough to join other agents and form more elongated spread-out clusters in which nodes have few connections, as opposed to the dense clusters observed with $\beta = 0.1$. Indeed, for the higher β value a small number of neighbors is already sufficient to severely hinder the agents' motion. Now, because viscosity is high, these configurations do not change their shapes much as time goes by. After an initial loss, non-negligible levels of cooperation are recovered at steady state and, again, strategies cluster together but with a definite difference in cluster shape with respect to the previous case.

Figure 6.9 illustrates the behavior of the average velocity and the mean degree in the system as a function of time for a single typical run with the same $v_0 = 0.01$. We remark that this behavior is general and does not depend on the particular game parameters or agent strategy. For the sake of studying long-term system behavior, we have performed 20000 steps to draw these figures. Clearly, as velocity and degree are related in Eq. 6.6, one would expect that the lower β the higher the mean speed in the system, and the same should be true for the mean degree. In fact, Fig. 6.9 left image shows that velocity indeed decreases exponentially for $\beta = 0.1$. The mean velocity can never go to zero as the lowest possible velocity for such a finite system is $v_0 e^{-\beta(N-1)}$. Actually, towards the end of the simulation, the mean velocity is the average of individuals that have not yet joined a densely connected cluster (see Fig. 6.7 rightmost image) and move around in space more freely, and those within a cluster which are almost at rest. The network counterpart of this velocity damping is the large increase of mean degree $\bar{k}$ (Fig. 6.9, right image) which is due to the fact that individuals are densely packed into clusters. In the case $\beta = 1$ mobility is more restricted from the start but, since the clusters that form are less dense, the velocity decreases more slowly.

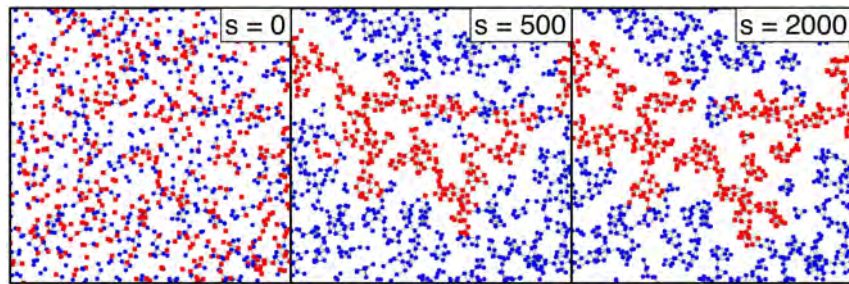


Fig. 6.8 Time evolution of a particular run with $R = 1$, $P = 0$, $S = 0$, $T = 1.3$, initial $\bar{k} = 4$, $\beta = 1$ and $v_0 = 0.01$. Red squares represent cooperators; blue circles stand for defectors; The initial proportion of cooperators is 0.5. s gives the time step at which the corresponding snapshot has been taken. The final cooperation frequency at $s = 2000$ is 0.38.

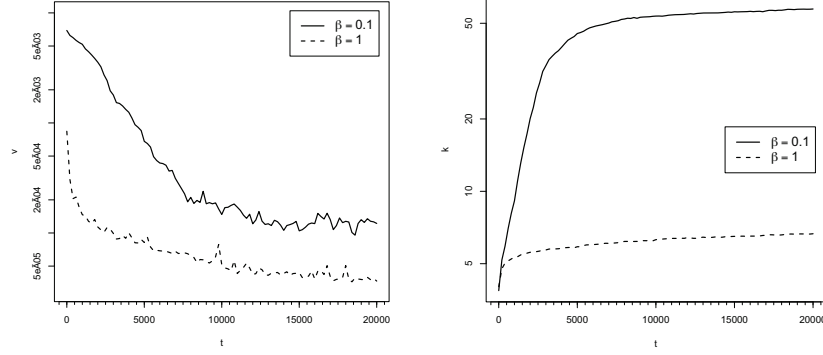


Fig. 6.9 Average system velocity for $\beta = 0.1$ and $\beta = 1$ (left image) and evolution of the mean degree $\bar{k}$ for the same values of β (right image); $v_0 = 0.01$.

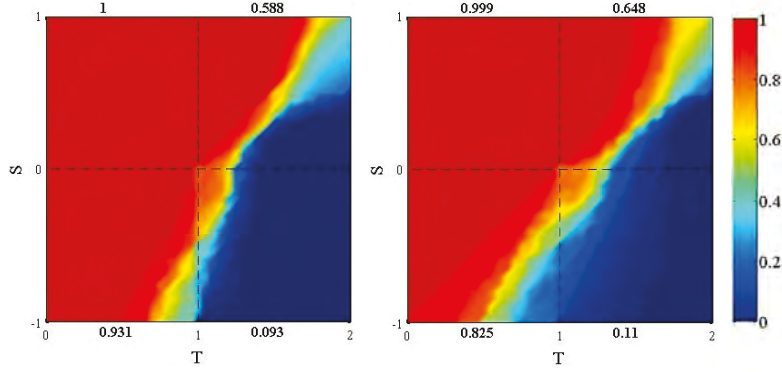


Fig. 6.10 Average cooperation values using imitation of the best as an update rule in the viscous model. The initial mean degree is $\bar{k} = 4$ but it evolves during the run. Left: $\beta = 0.1$; Right: $\beta = 1$. The results are averages over 100 runs.

To conclude this section, we show the global cooperation results in Fig. 6.10. Although these average results do not provide information on the fine system dynamics, as we said above, we still observe that, even in the more realistic simulation of a viscous system, there are non-negligible gains of cooperation compared to the RGG static case (rightmost image of Fig. 6.4). On the other hand, it doesn't make much sense to compare the present results with those at constant velocity (left and middle images of Fig. 6.4) since there are variable scale factors between the two cases; nevertheless, the global cooperation pattern is similar to the one found with constant velocity $v = 0.01$.

6.5 Conclusions

We have presented a systematic study of some population games by extensive numerical simulations in two-dimensional Euclidean space under two-person, one-shot game-theoretic interactions, and in the presence of agent random mobility. The goal was to investigate whether cooperation can evolve and be stable when agents can move randomly in space. Individuals in the population only interact with other agents that are contained in a circle of radius r around the given agent, where r is much smaller than the spatial dimension occupied by the whole population. For the game-theoretic interactions we have used two common microscopic payoff-based strategy update rules: fitness difference proportional and imitation of the best neighbor. We have studied a large game parameter space which comprises the Prisoner's Dilemma, the Stag Hunt, and the Hawk-Dove class of games. We have investigated two models which differ only in their mobility aspects.

In the first model, the velocity is the same for all agents in the population and it remains constant throughout the dynamics. Our main results with this model are the following. Under fitness difference proportional update, the effect of mobility on cooperation is very small and there is little difference with the case in which the agents sit at the nodes of a random geometric graph and don't move. These results extend previous ones obtained by Meloni et al. [26] which were limited to the weak Prisoner's Dilemma region, the segment with $S = 0$ and $1 \leq T \leq 2$ in the game's phase space.

However, when the imitation of the best neighbor rule is used instead, random mobility promotes cooperation in all the games' parameter space. Indeed, in the steady state we observe full cooperation in the Stag Hunt games while cooperation is also boosted in the Hawk-Dove. The Prisoner's Dilemma class of games is the most critical one but random mobility seems to significantly improve the situation opening up to cooperation a large region of the PD space. The results are qualitatively the same with two different radii r that lead to network mean degrees 4 and 8 respectively. The main mechanism leading to the evolution of cooperation is the random initial formation of small clusters of cooperators followed by other cooperators joining the initial clusters thanks to their mobility, together with defectors slowly becoming cooperators because of the latter higher payoff.

In the second model agents do not move with constant scalar velocity; rather, velocity is assumed to be a negative exponential function of the agent's connectivity degree. This introduces a damping factor which can be seen as a kind of viscosity due to the accumulation of individuals around a given agent, leading to a more hindered random movement. The numerical simulation study of the average cooperation levels in the TS -plane in this case leads to results that are qualitatively similar to those obtained in the constant velocity case, although the gains in cooperation with respect to a static population represented by a RGG are slightly less. However, average values do not reveal the particular dynamics that are at work. To study this aspect, we have simulated a particular Prisoner's Dilemma game with two different velocity damping factors β , one giving rise to low viscosity and the second to a higher viscosity. With low viscosity, starting with a uniform distribution of the agents in the plane, the system evolves toward the formation of dense monomorphic clusters of cooperators or defectors. In these clusters agents are almost at rest in the steady state and only individuals that have not joined a cluster still move. Under these conditions, contrary to the case with constant velocity, cooperation cannot spread past the cluster boundaries because of the lack of individual dispersion. With high viscosity the agents' movements are more

hindered from the beginning but they are still able to join clusters of their kind. The situation is similar to the previous case, i.e. clusters of C's and D's do form and remain stable, with the important difference that now they are much less dense and, consequently, the mean degree of the population is smaller. Again, viscosity and progressive velocity loss do not allow cooperation to spread to the whole population. One can thus conclude that random agent movements in physical space that take into account the natural fact that crowding effects have an effect on the agents' mobility may still lead to cooperative outcomes in many cases. However, the dynamics lead to cluster formation and condensation which hinders further spreading of cooperators especially in the harder Prisoner's Dilemma case. In future work we would like to address the detailed study of cluster dynamics and the effect of strategy noise on the system evolution.

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Chapter 7

Lévy Flights and Cooperation Among Mobile Individuals

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Abstract Mobility is a general feature of human and animal populations living in physical space. In particular, it has been observed that often these movements are of the Lévy flight type. In this article we study the effect of such power-law distributed displacements on the evolution of cooperation in simple games played on diluted two-dimensional grids. Using numerical simulations, our results show that unconditional Lévy flights do not favor the evolution of cooperation. However, when Lévy jumps are performed as a reaction to a growing number of defectors in an individual's neighborhood, all games studied here, including the harder Prisoner's Dilemma, remarkably benefit from this simple heuristics and cooperation can evolve in the population.

7.1 Introduction and previous work

Cooperative behavior is at the basis of the harmonious development of animal and human societies. In this context, the Prisoner's Dilemma game (PD) is a widely used paradigm for the investigation of how cooperation might evolve in a population of self-regarding agents [1]. During the last three decades it has been shown that the evolution of cooperation among agents that pursue their own interest is possible if certain conditions are satisfied. The huge amount of work that has led to these conclusions has been usefully summarized in several reviews, e.g. [2, 3, 4]. Here we just emphasize that assortment mechanisms are the key, as first shown by W.D. Hamilton [5] and, more recently in, e.g., Simon et al. [6]. Among the various ways in which positive assortment can be generated in a population the structure of the latter, represented by a graph in which two directly linked vertices stand for two interacting agents, has received attention because it requires minimal rationality and behavioral assumptions. By using theoretical models and numerical simulations, it has been found that some network structures appear to be more favorable for cooperation than others, although this result depends upon the evolutionary dynamics of the model [7, 8, 9, 10, 11, 12, 13]. An early way of considering the effect of population structure makes use of the concept of geographical space which is a natural one when territoriality plays an important role. A simple approximation of physical space is given by a regular discrete lattice in two dimensions, as in the pioneering work of Kimura and Weiss [14] on population genetics. These kind of structures were introduced in evolutionary games by Axelrod [1] and by Nowak et al. [15, 16] who showed, in agreement with previous work by Hamilton [5] that, even when the game is one-shot, i.e. pairs of players interact anonymously, cooperation can evolve thanks to positive assortment among cooperators. A

summary of this and other early work on grids is provided in [17]. More recent extensive numerical simulation work showed that the gains in the PD are limited and depend on the players' strategy update rule used [9]. Evolutionary games on arbitrary spatially embedded networks have been recently studied in [18].

The previously mentioned work on regular grids and spatial graphs refers to a static situation in which agents are fixed at the vertices of the network in which positive assortment may result as a consequence of fluctuations in the initial strategy distribution. This is an excellent first approximation but mobile, rather than fixed, individuals seem to be closer to biological reality. Besides animal populations, many examples can be found in mobility patterns of human populations, e.g. [19], in engineered systems such as ad hoc networks of mobile communicating devices (see e.g. [20, 21]), and mobile robot teams [22, 23] among others. Mobility may have positive or negative effects on cooperation, depending on several factors. An early brief investigation of random spatial drift on diluted grids is to be found in Nowak et al. [16]. Another early study was carried out by Enquist and Leimar [24] whose main conclusion was that mobility may seriously restrict the evolution of cooperation. In the last decade there have been several new studies of the influence of mobility on the behavior of various games in spatial environments, either in the case in which agents essentially perform random walks or when they move according to heuristic strategy-driven considerations. Random diffusion of agents playing games has been studied in continuous space [25, 26] as well as two-dimensional diluted grids [27, 28]. In a more refined and complex island model that takes into account kinship and offspring diffusion using a Moran process, the role of empty sites in a patch has been thoroughly investigated in Alizon and Taylor [29] who showed that altruistic behavior can be promoted in these conditions. Random diffusion in diluted grids has been thought to hinder cooperation by weakening the possibility of cooperator cluster formation. However the works [27, 28, 26] show that cooperation can be maintained with respect to the static case and sometimes even enhanced for low mobility, depending on the strategy update rule used by the agents. When agents are allowed to move according to some simple heuristic rule instead of fully randomly, it has been shown that the population may evolve a certain degree of self-organization [30, 31, 32, 33, 34]. The general message that transpires from this work is that purposeful contingent movement may lead to highly cooperating stable or quasi-stable population states depending on the individuals' density and the degree of mobility.

One form of random movement are "Lévy flights" [35]. Lévy flights resemble the standard Brownian motion process [36] but, instead of drawing the displacements from a Gaussian distribution, they come from a power-law probability distribution of the jump length x given by $P(x) = Cx^{-\alpha}$, where C is a normalization constant such that $C \int_1^\infty x^{-\alpha} dx = 1$ and $1 < \alpha \leq 3$. The implication is that while most displacements are short, large amplitude movements have non-negligible probability, contrary to the Gaussian case, and the global behavior is dominated by those rare events.

Lévy flights are important in animal movements in real landscapes. Observation of animal search patterns in several different species have revealed that optimal foraging patterns when resources are distributed at random and there is no information on their location are of the Lévy flight type [37, 38, 39, 40, 41]. But Lévy flights have also been found to play an important role in human behaviour as well, for instance in large-scale movement of individuals [19] and even in memory retrieval of words [42]. Thus, Lévy flights seem to be an ubiquitous feature in Nature and so there is scope for investigating their effect on evolutionary games. The only use of Lévy distributions in evolutionary games we are aware of has been published by M. Perc [43, 44], who

studied the influence of Lévy distributed stochastic payoffs compared to Gaussian noise in the Prisoner's Dilemma. However, the present study of the influence of Lévy-distributed jumps on games played by mobile agents has not yet been performed to our knowledge.

In the following we show that the introduction of Lévy flights in the agents' movements together with a very simple heuristic to decide whether to migrate to another place or not can notably help cooperation to evolve in spatial evolutionary games. We first introduce our game-theoretical and mobility models. This is followed by a discussion of the results of extensive numerical simulations, and we finally present our conclusions.

7.2 Model description

7.2.1 Games studied

Agents interact in pairs playing one of three common two-person, two-strategy, symmetric games, namely the Prisoner's Dilemma (PD), the Hawk-Dove Game (HD), and the Stag Hunt (SH). These three games are simple metaphors for different kinds of dilemmas that arise when individual and social interests are not aligned. The set of strategies is $\{C, D\}$, where C stands for "cooperation" and D means "defection". The two players receive the *reward* R if they both cooperate; they get the *punishment* P if they both defect; the payoff T that a player receives if he defects while the other cooperates is the *temptation*. In this case the latter receives the *sucker's payoff* S .

For the PD, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only stable fixed point of the replicator dynamics [45, 46].

In the HD game, the order is $T > R > S > P$. Players have a strong incentive to play D , which is harmful for both parties if the outcome produced happens to be (D, D) . The only dynamically stable equilibrium is in mixed strategies [46].

In the SH game, the ordering is $R > T > P > S$, which implies that mutual cooperation (C, C) is the payoff superior outcome and a NE. The second NE, where both players defect is less efficient but also less risky. The third mixed-strategy NE in the game is evolutionarily unstable [46].

In order to study the usual standard parameter space [8, 9], we restrict the payoff values in the following way: $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the resulting TS -plane each game class corresponds to a different quadrant depending on the above ordering of the payoffs.

7.2.2 Agent and population dynamics

The population dynamic is simulated by the following stochastic process:

1. An individual is chosen uniformly at random among the population
2. The individual plays the game with each of his neighbors in turn and accumulates its payoff
3. The individual updates its strategy by imitating the neighbor with the highest payoff including itself
4. If unsatisfied, the individual performs a Lévy flight to another grid cell if the latter is empty, otherwise it doesn't move
5. The above process is repeated N times for each time step t , where N is the population size

Step 1 means that the population evolution is asynchronous in time. At the end of step 2 the focal player computes his total payoff accumulated through the two-person games played with his neighbors.

In step 3 the chosen agent adopts a strategy in $\{C, D\}$ according to a microscopic revision rule. Among several possible update rules [7], we use the imitative update consisting in deterministically switching to the strategy of the neighbor that has scored best in the last time step [15, 16]. If there is a tie, the winner individual is chosen uniformly at random. Since this rule is fully deterministic, in order to simulate strategy update errors that are likely to occur for various reasons in a more realistic setting, we also introduce a mutation rate $\beta \ll 1$ such that an agent switches to the wrong strategy with probability β .

Finally, in Step 4 the individual performs a Lévy flight to another position in the grid if unsatisfied. We implemented two models: a baseline one in which agents are always unsatisfied and therefore always try to move, and a second one in which agents are satisfied when at least half of their neighbors are cooperators. This heuristic is sufficiently simple to be credible in many real human and animal societies, and similar ones have been used in [31, 32]. The magnitude $d \geq 1$ of the displacement is drawn from a power-law distribution with a given exponent α . Next, a direction is chosen by drawing uniformly at random an angle in the range $[0, 2\pi]$. After adjusting the resulting position and taking cyclic boundary conditions into account such that it falls into a grid cell, the agent moves there if the cell is empty, otherwise it stays at its original position.

7.2.3 Simulation settings

The population is structured as a diluted square lattice of side $L = 100$ with toroidal borders. Each cell of the grid can be occupied by one player or be empty. The *density* is $\rho = N/L^2$, where $N \leq L^2$ is the number of players. Players interact with the closest $k = 8$ neighbors resulting in the customary Moore neighborhood. The TS -plane has been sampled with a grid step of 0.05 and each value in the phase space reported in the figures is the average of 100 independent runs. The evolution proceeds by first initializing the population by creating a player in each cell of the underlying lattice with probability $\rho = 2/3$. Then the players' strategies are initialized uniformly at random such that each strategy has a fraction of approximately 1/2. Agents may move to a free position by performing Lévy flights with $\alpha = 2.0$. This value has been found to be optimal for animal random searches [38]. However, the exact α value does not seem to be important here, as we checked that $\alpha = 1.5$ and $\alpha = 2.5$ give very similar results with slightly less cooperation for $\alpha = 1.5$. To compute global statistics, we let the system evolve during 10000 time steps and cooperation values are sampled at the end of this process.

7.3 Results

Figure 7.1 shows the average cooperation for the whole TS -plane. The left image refers to the case in which agents are always unsatisfied and thus they always migrate using a Lévy flight. It is apparent that the PD quadrant (bottom right) is all defect, while in the HD quadrant (top right) there is coexistence of cooperators and defectors as prescribed by evolutionary game theory since migrations tend to randomize the population structure. The only game in which mobility on the grid enhances cooperation with respect to the well mixed case is the SH depicted in the bottom left

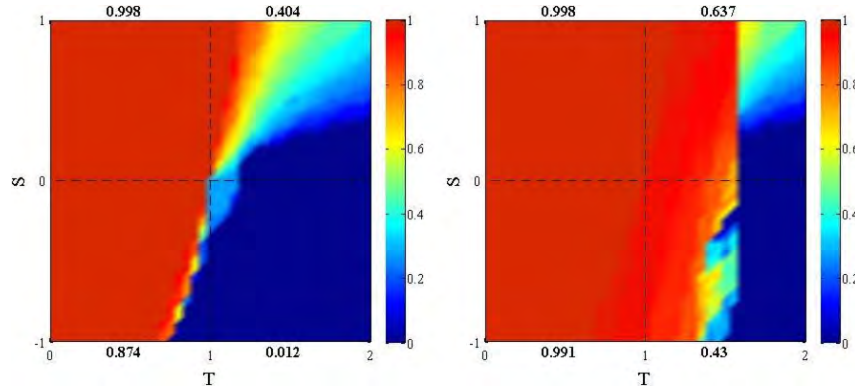


Fig. 7.1 Average cooperation levels over 100 runs per grid point in the TS -plane for $\rho = 2/3$. Agents migrate by performing Lévy flights with exponent $\alpha = 2.0$. In the left image migration is unconditional while in the right image agents only migrate when there is a majority of defectors in their neighborhood. The upper left quadrant corresponds to the trivial *Harmony* game in which cooperation is always stable. The numbers next to each quadrant are the average cooperation levels in the respective region.

quadrant. We note in passing that this is also the case in full grids [9]. These results do not differ qualitatively from those obtained in diluted grids with random walk mobility for particular games in the game space [27, 28] and in continuous two-dimensional space [26]. Indeed, in the PD space mobility through Lévy flights tends to erode cooperators clusters that are slowly forming due to the fact that, from time to time, defectors make long jumps that bring them inside forming cooperator clusters. Those defectors earn there a larger payoff than cooperators and thus their number grows at the expense of cooperators. This effect is also present in the SH but here, since cooperators between themselves have the largest payoff the effect is much smaller. The HD case is different since in this game the only stable equilibrium is the mixed one. By consequence, although full cooperation is globally reached with adaptive migration in the left part of the quadrant, the grid patterns that form contain both cooperators and defectors in the other cases.

The right image of Fig. 7.1 shows the results obtained when migration by Lévy flights is contingent on the satisfaction of players, i.e. both defectors and cooperators migrate only when there are more defectors than cooperators in their neighborhood. The results when players employ this very simple reactive heuristic is that there is full cooperation in the SH and cooperation is strongly enhanced in the HD and the PD games. This result is in agreement with those obtained for the weak PD only (the segment at the frontier between the PD and HD spaces) with similar heuristics coupled with standard random walks [31]. Much more elaborate heuristics that require the agents to do an exhaustive or random exploration of their environment within a given radius such as “success driven” [30] and “opportunistic” migration [34], give comparable gains. However, these kinds of migration require superior rational capabilities on the part of the agents. One surprising feature of the right image of Fig. 7.1 that requires explanation is the vertical line at $T = 1.6$ crisply separating full or almost full cooperation from full or almost full defection. The phenomenon is due to the combination of the deterministic threshold rule update function and a particular temptation T value. For $T = 1.6$, defectors surrounded by five cooperators get a payoff of 8, which is exactly the same as the payoff of cooperators fully surrounded by other cooperators in a cluster. In these

conditions, the update rule prescribes that imitation is random with probability $1/2$. Thus, a cooperator at the border of a cooperator cluster may imitate the defector. This gives rise to a cascade of defection that will enter the cluster and eventually will break it apart. On the other hand, for T just a bit less than 1.6, say $T = 1.59$, the phenomenon is much less likely to happen.

The examination of particular, but typical, runs of the simulations is revealing, see Fig. 7.2. The left picture shows the evolution of cooperation in a population playing a SH game with $S = -0.9$ and $T = 0.9$ which is in the defection area in this game's space in a diluted grid with unconstrained Lévy flights (see Fig. 7.1). Indeed, the population quickly sets in a full defection state (dotted curve). In contrast, when adaptive Lévy jumps are performed, after a short transient in which many players shift to defection, the population quickly converges toward an almost fully cooperative state. The right image of Fig. 7.2 shows the same time-dependent behavior for a PD with $S = -0.4$ and $T = 1.4$ which should lead to full defection. Here too, full defection is in fact quickly reached with unconstrained Lévy flights (dotted curve). However, when migration is heuristic, escaping from defector ridden places allows cooperation to recover and to finally dominate in the population. It should be noted that the relaxation time here is much longer than in the SH case: this game is much harder for cooperation to evolve as it is at the extreme border of the cooperating region in the PD quadrant (Fig. 7.1).

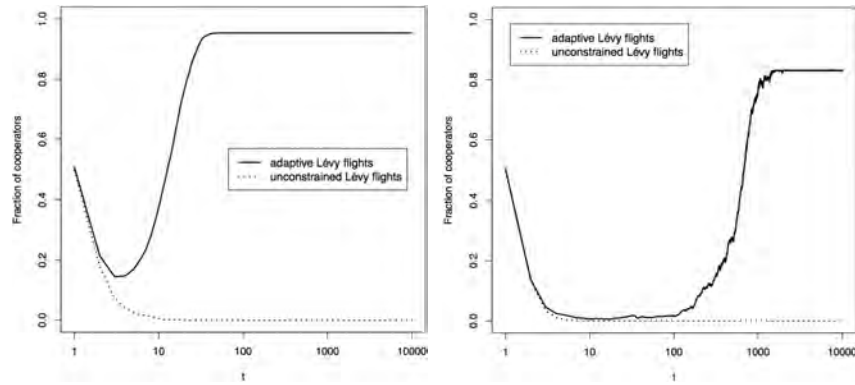


Fig. 7.2 Fraction of cooperation as a function of time. Left: SH game with $S = -0.9$ and $T = 0.9$. The dotted curve corresponds to unconstrained migration while the thick curve is the heuristic migration case. Right: PD game with $S = -0.4$ and $T = 1.4$.

Defectors too leave neighborhoods in which they are in the majority but they do not cluster together. Figure 7.3 is a visual illustration of the global phenomenon for the above PD game. A similar image for the previously used SH game is given in Fig. 7.4. Although the game is in the defection basin in the left image of Fig. 7.1, thanks to adaptive Lévy diffusion, the evolution ends up in almost full cooperation.

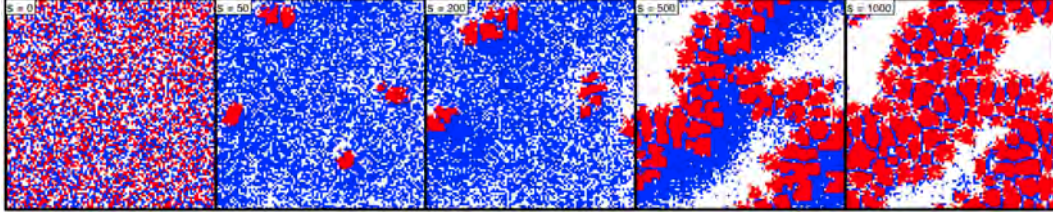


Fig. 7.3 Snapshots for a particular PD game with $S = -0.4$ and $T = 1.4$ when agents perform adaptive Lévy flights. Cooperators are represented in red and defectors in blue. The snapshots are taken at time step $s = 0, 50, 200, 500, 1000$. At the beginning cooperators are at risk but later on they are successful in expanding some of the initial clusters and cooperation finally prevails. Defectors cannot form clusters between themselves but they are interspersed among cooperator clusters.

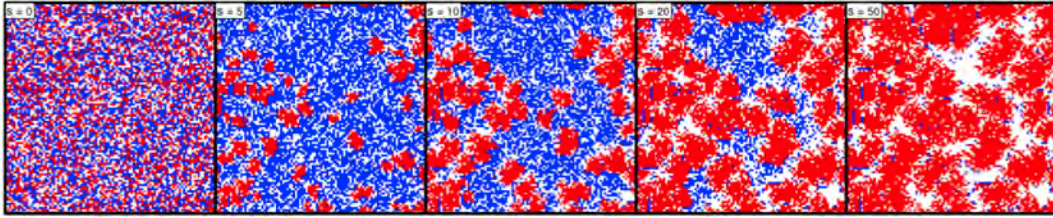


Fig. 7.4 Snapshots for a particular SH game with $S = -0.9$ and $T = 0.9$ when agents perform adaptive Lévy flights. Cooperators are represented in red and defectors in blue. The snapshots are taken at time step $s = 0, 5, 10, 20, 50$. With respect to the PD in this game convergence is much faster. Also, cooperator clusters are more stable because defectors jumping at their borders or in the interior are likely to become cooperators instead of eroding the cluster.

7.3.1 The effect of density

We have also studied densities $\rho = 1/3$ and $1/2$ and average cooperation results are shown in Fig. 7.5. For unconstrained mobility the results are similar to the $2/3$ case (see the left column of Fig. 7.5), except that there is less cooperation in the SH quadrant because the sparseness of agents makes clustering more difficult to achieve, especially in the $1/3$ case (left image). In fact, due to the abundant free space, when some cooperators happen to cluster together, weak disturbances by defectors approaching the cluster may easily break it apart. This dynamical phenomenon due to random mobility at low densities has already been observed (e.g. in [34]). In this case, Ohtsuki's et al. criterion for cooperation [10] in static networks, which would predict more cooperation for lower average degrees, cannot be expected to fully apply because the underlying graph is constantly changing and the population evolution is not described by a Moran process.

When migration is contingent the above phenomena are mitigated by the increased trend of cooperators to escape defectors thus making clusters more stable, when they form. The right column of Fig. 7.5 shows the average cooperation results in this case for $\rho = 1/3$ and $\rho = 1/2$. The SH region remains fully cooperative, while the PD quadrant shows a small cooperation loss with respect to the case $\rho = 2/3$. Densities higher than $2/3$ give results that smoothly tend to those on complete grids and full results can be seen in [9].

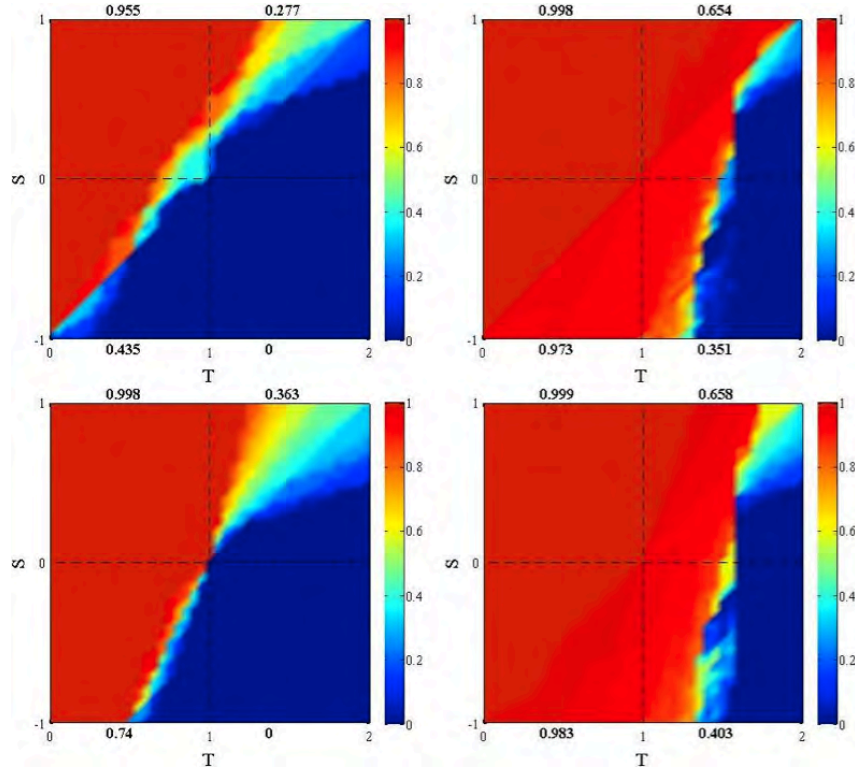


Fig. 7.5 Average cooperation levels over 100 runs per grid point in the TS -plane for different densities. Agents migrate by performing Lévy flights. In the left column images migration is unconditional while in the right column images agents only migrate when there is a majority of defectors in their neighborhood. Player density $\rho = 1/3$ for the left images and $\rho = 1/2$ for the right images.

7.3.2 The effect of noise

The results discussed above do not take errors and “trembles” into account. However, in a real situation it is likely that, from time to time, agents will not take the “right” decision because of slips of attention, insufficient information, and other reasons. This random noise is represented here following the standard approach of [30, 31] as a small error probability β in the strategy update phase (see Sect. 7.2.2). Figure 7.6 reports the results obtained in the whole game parameter space with $\beta = 0.02$ (left image) and $\beta = 0.03$ (right image) when agents migrate by performing contingent Lévy flights. Comparing the results with those depicted in Fig. 7.1 (right image), it is apparent that noise has an adverse effect on cooperation due to the fact that, at all times, some cooperator in a forming cooperator cluster may turn into a defector with a small but non-negligible probability. From there defectors may spread and destroy the cluster if the temptation T is large enough and S is sufficiently negative. The effect is proportional to the amount of noise since it increases for $\beta = 0.03$ (Fig. 7.6, right image). However, in spite of these errors, cooperation is still higher in the PD and HD games than when noiseless agents only perform unrestrained Lévy

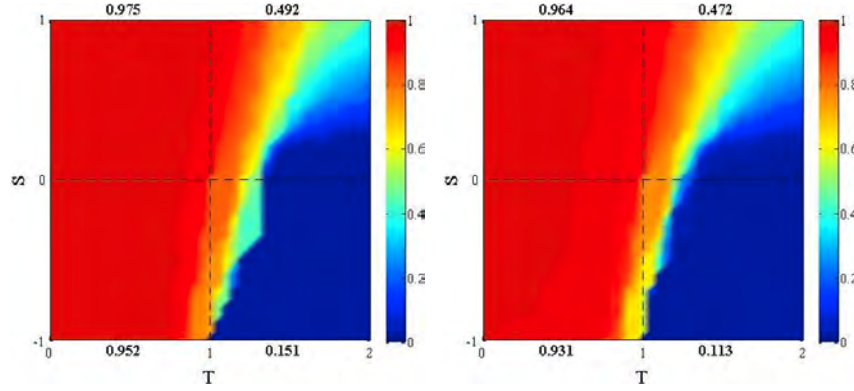


Fig. 7.6 Average cooperation levels over 100 runs per grid point in the TS -plane with strategy noise. Agents migrate by performing adaptive Lévy flights. The error rate β is 0.02 in the left picture and it is 0.03 in the right picture.

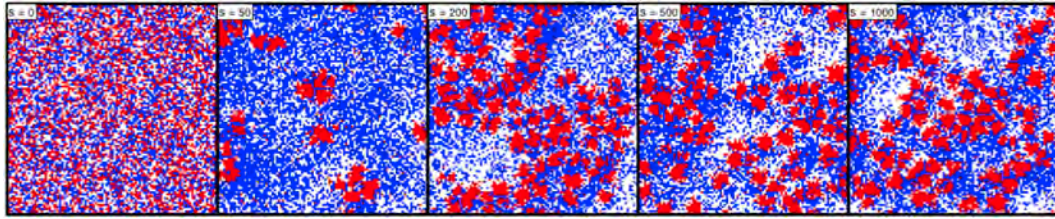


Fig. 7.7 Snapshots for a particular PD game with $S = -0.3$ and $T = 1.3$ when agents perform adaptive Lévy flights and the error rate β is 0.02. Cooperators are represented in red and defectors in blue. The snapshots are taken at time step $s = 0, 50, 200, 500, 1000$. The cooperation level becomes stable, with some fluctuations, from time step 200 on. However, due to mutations, defectors continuously break cooperator clusters, which start to form elsewhere. In this sense, the population never settle into a true equilibrium.

flights as in the left image of Fig. 7.1, while it remains almost unaffected in the SH. A visual example is shown in Fig. 7.7 for a particular PD game.

It is also of interest to investigate noisy migrations. The decision to migrate is given by a count of the defectors around either a focal defector or a cooperator. We assume that, with some low rate ϵ , players may make errors in this count and, for simplicity, we attribute the same error proneness to both cooperators and defectors. In other words, players may decide to migrate when they shouldn't and not to migrate when they should with probability ϵ . Figure 7.8 shows that these kinds of blunders affect final average cooperation levels but only slightly, at least for the two rates studied, i.e. $\epsilon = 0.02, 0.05$ and $\rho = 2/3$. When $\epsilon \rightarrow 1$ one recovers the random migration case shown in the left image of Fig. 7.1. Comparing Figs. 7.6 and 7.8 it appears that the strategy mutation noise has a stronger effect on cooperation than the migration noise.

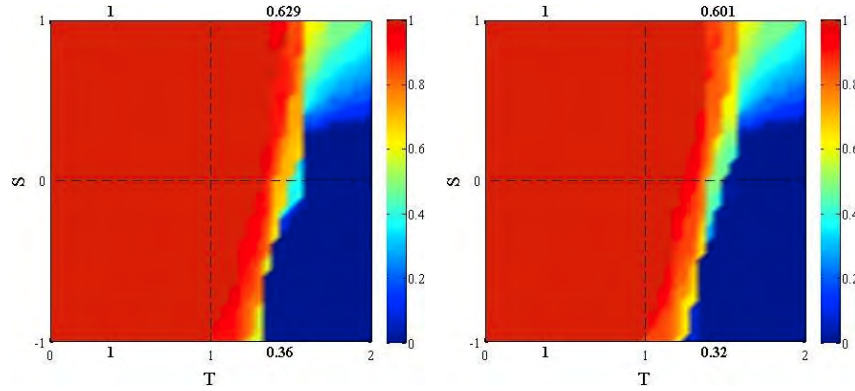


Fig. 7.8 Average cooperation levels over 100 runs per grid point in the TS -plane with migration errors. Agents migrate by performing adaptive Lévy flights but, with probability $\epsilon = 0.02$ (left) and $\epsilon = 0.05$ (right) they make mistakes (see text).

7.4 Discussion

Mobility is a general feature of human and animal populations living in physical space. Random mobility is customarily modeled by Gaussian or constant step random walks. However, it has been shown in several field studies that random walks of the Lévy flight type are an ubiquitous form of movement in animal and human populations [37, 38, 39, 40, 41, 19]. These movements have the remarkable characteristics of being generally short but, from time to time, they can be very long, giving a special behavior to the agents' mobility patterns. In this study we started from this observation and we applied Lévy flights to the problem of cooperation in populations that are laid out on a two-dimensional discrete grid space with a given amount of free cells. Our results show that unrestricted mobility by Lévy flights alone do not allow cooperation to evolve when agents play a PD game with their neighbors, which is the same conclusion reached in models where agent diffusion is through standard random walks [27, 28, 25, 26]. However, when an agent's movement is conditional on some property of the agent's neighborhood, the results are very different. We used a very simple and natural heuristic rule according to which both cooperators and defectors try to migrate by trying to do a Lévy jump to a vacant place when they are surrounded by a majority of defectors. In this case, all the SH games in the games' space lead to full cooperation, and the HD and PD games are favorably influenced too. The difference can be explained qualitatively by numerically observing the dynamics induced in the two cases. When migration is unrestricted there are two self-reinforcing negative effects working against cooperation. On the one hand, cooperator clusters that may possibly develop, are not stable enough because cooperators keep moving, preventing clusters to grow and leaving empty spaces at the border or inside the cluster. On the other hand, defectors may randomly jump into partially formed cooperator clusters and from there break the cooperator groups apart thanks to their superior payoff. When migration is contingent, the first effect above is no longer operating since cooperators are satisfied when they start to be surrounded by other cooperators: they don't jump away, and the mean cooperator cluster size increases as a result. Defectors may still enter a cooperator cluster but they find it more difficult to propagate into it. Overall, this explains why cooperation may evolve under contingent migration.

The above conclusions were obtained for a density of players of $2/3$. We therefore investigated how this parameter influences these processes by numerically simulating systems with lower density ($1/2$ and $1/3$). The result is that lower densities negatively influence the evolution of cooperation in the unrestricted migration case, while the mechanisms at play in contingent migration are not much affected, with only little loss of cooperation. Thus, we conclude that cooperation is robust under the migration mechanism produced by adaptive Lévy flights.

Since in Nature noise and errors can and do occur, we experimented with two kinds of low-probability disturbances: random strategy mutations and migration errors. When the system is noisy and some strategy update errors creep in, the results remain very good in terms of cooperation although the latter has more trouble to evolve in the PD region. On the other hand, migration errors are less important and do not influence results to the same extent as strategy update errors. We should point out that each kind of noise was investigated independently of the other in order to understand their separate effects. It would be useful to analyze their cumulated impact on cooperation.

Summarizing, although similar improvements in cooperation have been found previously by using other opportunistic dynamical models, they were either restricted to a small region of the games' space [31], or they required a rather high degree of rationality on the part of the playing agents [30, 34]. We believe that the main result of this work is that we showed that cooperation can evolve and be stable using very simple behavioral devices that only require primitive adaptive capabilities that seem to be common in Nature.

There are still several open issues and questions that could be investigated. The present study was limited to the imitate the best strategy update rule. This choice has been motivated by the desire to compare our results with a number of previous related works that used the same update rule [16, 27, 28, 26, 30, 31]. It would certainly be interesting and enlightening to use other well-known rules, especially the biologically relevant ones called birth-death and death-birth [47]. This however would make the article too long and we choose to leave it for follow-up work. Also, Lévy flights might be coupled with a more intelligent exploration of the neighborhood at destination such that migration is more payoff-driven, similar to what has been suggested in [30, 34]. This approach could be interesting for populations endowed with higher rational capabilities.

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Chapter 8

Opportunistic Migration in Spatial Evolutionary Games

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Abstract We study evolutionary games in a spatial diluted grid environment in which agents strategically interact locally but can also opportunistically move to other positions within a given migration radius. Using the imitation of the best rule for strategy revision, it is shown that cooperation may evolve and be stable in the Prisoner's Dilemma game space for several migration distances but only for small game interaction radius while the Stag Hunt class of games become fully cooperative. We also show that only a few trials are needed for cooperation to evolve, i.e. searching costs are not an issue. When the stochastic Fermi strategy update protocol is used cooperation cannot evolve in the Prisoner's Dilemma if the selection intensity is high in spite of opportunistic migration. However, when imitation becomes more random, fully or partially cooperative states are reached in all games for all migration distances tested and for short to intermediate interaction radii.

8.1 Introduction

Spatially embedded systems are very important in biological and social sciences since most interactions among living beings or artificial actors take place in physical two- or three-dimensional space [1]. Along these lines, game-theoretical interactions among spatially embedded agents distributed according to a fixed structure in the plane have been studied in detail, starting from the pioneering works of Axelrod [2] and Nowak and May [3]. The related literature is very large; see, for instance, the review article by Nowak and Sigmund [4] and references therein for a synthesis. Most of this work was based on populations of agents arranged according to planar regular grids for mathematical simplicity and ease of numerical simulation. Recently, some extensions to more general spatial networks have been discussed in [5]. The study of strategic behavior on fixed spatial structures is necessary in order to understand the basic mechanisms that may lead to socially efficient global outcomes such as cooperation and coordination. However, in the majority of real situations both in biology and in human societies, actors have the possibility to move around in space. Many examples can be found in biological and ecological sciences, in human populations, and in engineered systems such as ad hoc networks of mobile communicating devices or robot teams. Mobility may have positive or negative effects on cooperation, depending on several factors. An early investigation was carried out by Enquist and Leimar [6] who concluded that mobility may seriously restrict the evolution of cooperation. In the last decade there have been

several new studies of the influence of mobility on the behavior of various games in spatial environments representing essentially two strands of research: one in which the movement of agents is seen as a random walk, and a second one in which movement may contain random elements but it is purposeful, or strategy-driven.

Random diffusion of mobile agents through space, either in continuous space or, more commonly, on diluted grids has been investigated in [7, 8, 9]. In the present study we focus on situations where, instead of randomly diffusing, agents possess some basic cognitive abilities and they actively seek to improve their situation by moving in space represented as a discrete grid in which part of the available sites are empty and can thus be the target of the displacement. This approach has been followed, for example, in [10, 11, 12, 13, 14, 15, 16, 17]. The mechanisms invoked range from success-driven migration [10], adaptive migration [12], reputation-based migration [15], risk-based migration [14], flocking behavior [16], and cooperators walking away from defectors [17]. In spite of the difference among the proposed models, the general qualitative message of this work is that purposeful contingent movement may lead to highly cooperating stable or quasi-stable population states if some conditions are satisfied. Another related line of research has dealt with the case in which the grid is diluted but there is no migration. Recent work has shown that in this case cooperation is optimally promoted when the population density is close to the percolation threshold of the lattice [18, 19]. This interesting result could be somehow seen as a base case in the study of cooperation in diluted lattices with migration.

Our approach is based on numerical simulation and is inspired by the work of Helbing and Yu [11, 10] which they call “success-driven migration” and which has been shown to be able to produce highly cooperative states. In this model, locally interacting agents playing either defection or cooperation in a two-person Prisoner’s Dilemma are initially randomly distributed on a grid such that there are empty grid points. Agents update their strategies according to their own payoff and the payoff earned by their first neighbors but they can also “explore” an extended square neighborhood by testing all the empty positions up to a given distance. If the player finds that it would be more profitable to move to one of these positions then she does it, choosing the best one among those tested, otherwise she stays at her current place. Helbing and Yu find that robust cooperation states may be reached by this mechanism, even in the presence of random noise in the form of random strategy mutations and random agent relocation. Our study builds upon this work in several ways. In the first place, whilst Helbing and Yu had a single game neighborhood and migration neighborhood, we systematically investigate these two parameters showing that only some combination do foster cooperation using success-driven migration. Secondly, cost issues are not taken into account in [10]. However, it is clear that moving around to test the ground is a costly activity. In a biological setting, this could mean using up energy coming from metabolic activity, and this energy could be in short supply. In a human society setting, it is the search time that could be limited in a way or another. Additionally to physical energy, cognitive abilities could also limit the search. We present results for a whole game phase space including the Hawk-Dove class of games, and the Stag Hunt coordination class. Helbing’s and Yu’s agents based their strategy change on the imitation of the most successful neighbor in terms of accumulated payoff. We kept this rule but also added the Fermi strategy-updating rule, a choice that allows us to introduce a parametrized amount of imitation noise. With the imitation of the best policy we find that cooperation prevails in the Stag Hunt and may evolve in the Prisoner’s Dilemma for small interaction radius. With the Fermi rule fully cooperative states are reached for the standard neighborhoods independently of the migration distances when the rate of random strategy imitation is high enough.

8.2 Methods

8.2.1 The Games Studied

We investigate three classical two-person, two-strategy, symmetric games classes, namely the Prisoner's Dilemma (PD), the Hawk-Dove Game (HD), and the Stag Hunt (SH). These three games are simple metaphors for different kinds of dilemmas that arise when individual and social interests collide. The Harmony game (H) is included for completeness but it doesn't originate any conflict. The main features of these games are well known; more detailed accounts can be found elsewhere e.g. [20, 21, 22]. The games have the generic payoff matrix M (equation 8.1) which refers to the payoffs of the row player. The payoff matrix for the column player is simply the transpose M^T since the game is symmetric.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \quad (8.1)$$

The set of strategies is $\Lambda = \{C, D\}$, where C stands for “cooperation” and D means “defection”. In the payoff matrix R stands for the *reward* the two players receive if they both cooperate, P is the *punishment* if they both defect, and T is the *temptation*, i.e. the payoff that a player receives if he defects while the other cooperates getting the *sucker's payoff* S . For the PD, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE). In the HD game the payoff ordering is $T > R > S > P$. Thus, when both players defect they each get the lowest payoff. (C, D) and (D, C) are NE of the game in pure strategies. There is a third equilibrium in mixed strategies which is the only dynamically stable equilibrium [22, 20]. In the SH game, the ordering is $R > T > P > S$,

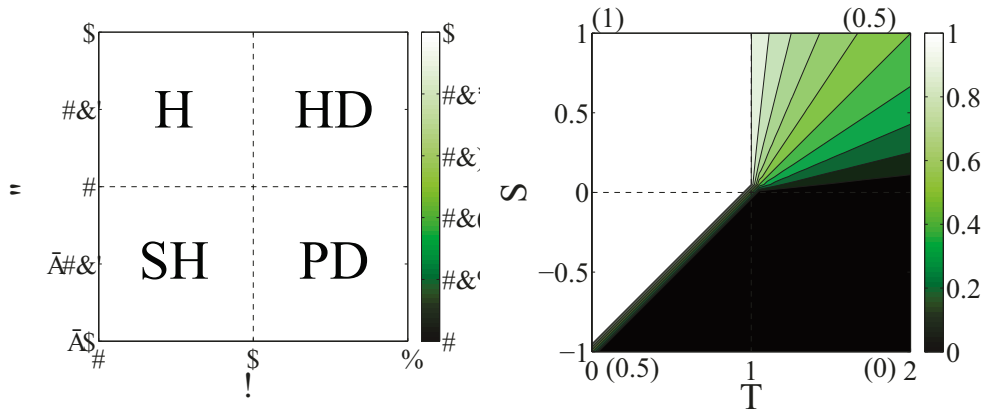


Fig. 8.1 Left image: The games phase space (H= Harmony, HD = Hawk-Dove, PD = Prisoner's Dilemma, and SH = Stag Hunt) as a function of S, T ($R = 1, P = 0$). Right image: replicator dynamics stable states [22, 20] with 50% cooperators and defectors initially in a well mixed population for comparison purposes. Lighter tones stand for more cooperation. Values in parentheses next to each quadrant indicate average cooperation in the corresponding game space.

which means that mutual cooperation (C, C) is the best outcome and a NE. The second NE, where both players defect is less efficient but also less risky. The third NE is in mixed strategies but it is evolutionarily unstable [22, 20]. Finally, in the H game $R > S > T > P$ or $R > T > S > P$. In this case C strongly dominates D and the trivial unique NE is (C, C) . The game is non-conflictual by definition; it is mentioned to complete the quadrants of the parameter space.

There is an infinite number of games of each type since any positive affine transformation of the payoff matrix leaves the NE set invariant [22]. Here we study the customary standard parameter space [23, 24], by fixing the payoff values in the following way: $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. Therefore, in the TS plane each game class corresponds to a different quadrant depending on the above ordering of the payoffs as depicted in Fig. 8.1, left image. The right image depicts the well mixed replicator dynamics stable states for future comparison.

8.2.2 Population Structure

The Euclidean two-dimensional space is modeled by a discrete square lattice of side L with toroidal borders. Each vertex of the lattice can be occupied by one player or be empty. The *density* is $\rho = N/L^2$, where $N \leq L^2$ is the number of players. Players can interact with k neighbors which lie at an Euclidean distance smaller or equal than a given constant R_g . Players can also migrate to empty grid points at a distance smaller than R_m . We use three neighborhood sizes with radius 1.5, 3, and 5; they contain, respectively, 8, 28, and 80 neighbors around the central player.

8.2.3 Payoff Calculation and Strategy Update Rules

Each agent i interacts locally with a set of neighbors V_i lying closer than R_g . Let $\sigma_i(t)$ be a vector giving the strategy profile at time t with $C = (1, 0)$ and $D = (0, 1)$ and let M be the payoff matrix of the game (equation 8.1). The quantity

$$\Pi_i(t) = \sum_{j \in V_i} \sigma_i(t) M \sigma_j^\top(t) \quad (8.2)$$

is the cumulated payoff collected by player i at time step t .

We use two imitative strategy update protocols. The first is the Fermi rule in which the focal player i is given the opportunity to imitate a randomly chosen neighbor j with probability:

$$p(\sigma_i \rightarrow \sigma_j) = \frac{1}{1 + \exp(-\beta(\Pi_j - \Pi_i))} \quad (8.3)$$

where $\Pi_j - \Pi_i$ is the difference of the payoffs earned by j and i respectively and β is a constant corresponding to the inverse temperature of the system. When $\beta \rightarrow 0$ the probability of imitating j tends to a constant value 0.5 and when $\beta \rightarrow \infty$ the rule becomes deterministic: i imitates j if $(\Pi_j - \Pi_i) > 0$, otherwise it doesn't. In between these two extreme cases the probability of imitating neighbor j is an increasing function of $\Pi_j - \Pi_i$.

The second imitative strategy update protocol is to switch to the strategy of the neighbor that has scored best in the last time step. In contrast with the previous one, this rule is deterministic.

This *imitation of the best* (IB) policy can be described in the following way: the strategy $\sigma_i(t)$ of individual i at time step t will be

$$\sigma_i(t) = \sigma_j(t-1), \quad (8.4)$$

where

$$j \in \{V_i \cup i\} \text{ s.t. } \Pi_j = \max\{\Pi_k(t-1)\}, \forall k \in \{V_i \cup i\}. \quad (8.5)$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random.

8.2.4 Population Dynamics and Opportunistic Migration

We use an asynchronous scheme for strategy update and migration, i.e. players are updated one by one by choosing a random player in each step with uniform probability and with replacement. Then the player migrates with probability $1/2$, otherwise it updates its strategy. If the pseudo-random number drawn dictates that i should migrate, then it considers N_{test} randomly chosen positions in the disc of radius R_m around itself. The quantity N_{test} could be seen as a kind of “energy” available to a player for moving around and doing its search. N_{test} being fixed for a given run, it follows that an agent will be able to make a more complete exploration of its local environment the smaller the R_m . For each trial position the player computes the payoff that it would obtain in that place with its current strategy. The positions already occupied are just discarded from the possible choices. Then player i stays at its current position if it obtains there the highest payoff, or migrates to the most profitable position among those explored during the test phase. If several positions, including its current one, share the highest payoff then it chooses one at random. We call this migration *opportunistic* or *fitness-based*. The protocol described in Helbing and Yu [10] is slightly different: the chosen player chooses the strategy of the best neighbor including itself with probability $1-r$, and with probability r , with $r \ll 1-r$, its strategy is randomly reset. Before this imitation step i deterministically chooses the highest payoff free position in a square neighborhood of size $(2M+1) \times (2M+1)$ cells surrounding the current player and including itself, where M can take the values 0, 1, 2, 5. If several positions provide the same payoff, the one that is closer is selected.

8.2.5 Simulation Parameters

The TS plane has been sampled with a grid step of 0.1 and each value in the phase space reported in the figures is the average of 50 independent runs. The evolution proceeds by first initializing the population by distributing $N = 1000$ players with uniform probability among the available cells. Then the players’ strategies are initialized uniformly at random such that each strategy has a fraction of approximately $1/2$. To avoid transient states, we let the system evolve for a period of $\tau = 1000$ time steps and, in each time step, N players are chosen for update. At this point almost always the system reaches a steady state in which the frequency of cooperators is stable except for small statistical fluctuations. We then let the system evolve for 50 further steps and take the average cooperation value in this interval. We repeat the whole process 50 times for each grid point and, finally, we report the average cooperation values over those 50 repetitions.

8.3 Results

8.3.1 Imitation of the Best and Opportunistic Migration

In this section we study cooperation with the IB rule and fitness-based opportunistic migration, and we explore the influence of different radii R_m and R_g and other parameters such as the density ρ and the number of trials N_{test} . The left image of Fig. 8.2 displays the TS plane with the IB rule, a density $\rho = 0.5$, and $N_{test} = 20$. For small $R_g = 1.5$ full cooperation is achieved in the SH quadrant for all R_m . The average levels of cooperation in the PD games are 0.33, 0.31, 0.30 for $R_m = 1.5, 3, 5$ and $R_g = 1.5$ respectively. It is remarkable that cooperation emerges in contrast to the well mixed population case (Fig. 8.1, right image), and also that better results are obtained with respect to a fully populated grid in which agents cannot move [24]. The HD doesn't benefit in the same way and the cooperation levels are almost the same in the average. Cooperation remains nearly constant as a function of R_m for a given R_g value but increasing R_g has a negative effect. For higher game radius, $R_g \in \{3, 5\}$ cooperation is progressively lost in the PD games while there is little variation in the HD quadrant among the different cases due to the dimorphic structure of these populations. In the SH quadrant there is a large improvement compared to the well mixed case but the gain tends to decrease with increasing R_g . In the PD with high R_g , cooperators cannot increase their payoff by clustering, since the neighborhood of defectors covers adjacent small clusters of cooperators, the payoff of defectors becomes higher and they can invade cooperators clusters. Figure 8.3 illustrates in an idealized manner what happens to a small cooperators cluster when the game radius R_g increases using a full grid for simplicity.

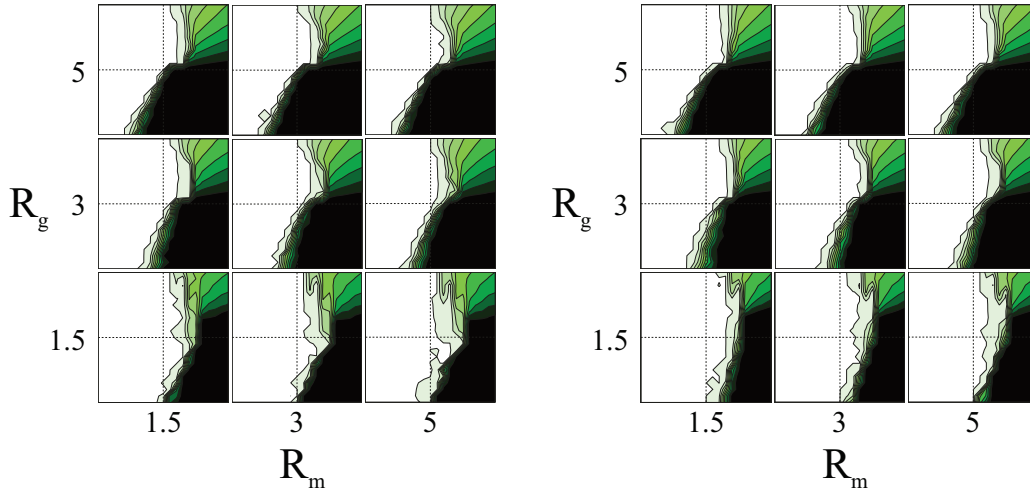


Fig. 8.2 Average cooperation levels with opportunistic migration and IB rule as a function of R_g and R_m . Left: $N_{test} = 20$; Right: $N_{test} = 1$. The size of the population is 1000 players and the density ρ is 0.5. In all cases the initial fraction of cooperators is 0.5 randomly distributed among the occupied grid points.

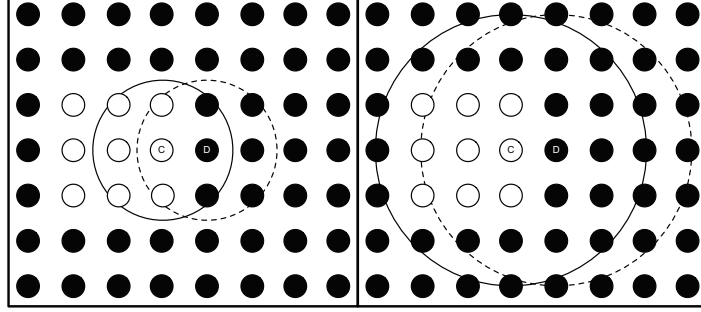


Fig. 8.3 Illustration of the effect of the playing radius R_g on the payoff of individuals. Left image: $R_g = 1.5$, right image: $R_g = 3$. The drawings refer to a locally full grid and are intended for illustrative purposes only (see text).

For $R_g = 1.5$ (Fig. 8.3, left image) the cooperator cluster is stable as long as $8R > 3T$ since the central cooperator gets a payoff of $8R$, while the best payoff among the defectors is obtained by the individual marked D (and by the symmetrically placed defectors) and is equal to $3T$ since $P = 0$. Under this condition all the cooperators will thus imitate the central one. On the other hand, the defector will turn into a C as long as $5R + 3S > 3T$, thus provoking cooperator cluster expansion for parameter values in this range. On the contrary, for $R_g = 3$ (Fig. 8.3, right image) the central cooperator gets $8R + 20S$ whilst the central defector at the border has a payoff of $7T$. Thus the cooperator imitates the defector if $7T > 8R + 20S$, i.e. $7T > 8 + 20S$ since $R = 1$. This qualitative argument helps to explain the observed cooperation losses for increasing R_g . This inequality is satisfied almost everywhere in the PD quadrant except in a very small area in its upper left corner.

Helbing and Yu [10] found very encouraging cooperation results in their analysis but they only had a small game radius corresponding to the Von Neumann neighborhood which is constituted, in a full lattice, by the central individual and the four neighbors at distance one situated north, east, south, and west. We also find similar results for our smallest neighborhood having $R_g = 1.5$, which corresponds to the eight-points Moore neighborhood but, as R_g gets larger, we have just seen that a sizable portion of the cooperation gains are lost. We think that this is an important point since there are certainly situations in which those more extended neighborhoods are the natural choice in a spatially extended population.

The number of trials N_{test} could also be a critical parameter in the model. The right image of Fig. 8.2 refers to the same case as the left one, i.e. the IB update rule with opportunistic migration and $\rho = 0.5$, except for the number of trials which is one instead of 20. We observe that practically the same cooperation levels are reached at steady state in both cases for $R_m = 5$ and $R_m = 3$, while there is a small increase of the average cooperation in the PD games for $R_g = 1.5$ which goes from 0.33, 0.31, and 0.30 for $N_{test} = 20$ to 0.41, 0.36, and 0.33 for $N_{test} = 1$, for $R_m = 1.5, 3, 5$ respectively. On the whole, it is apparent that N_{test} does not seem to have a strong influence. However, one might ask whether the times to convergence are shorter when more tests are used, a fact that could compensate for the extra work spent in searching. But Figs. 8.4 show that convergence times are not very different and decrease very quickly with the number of essays

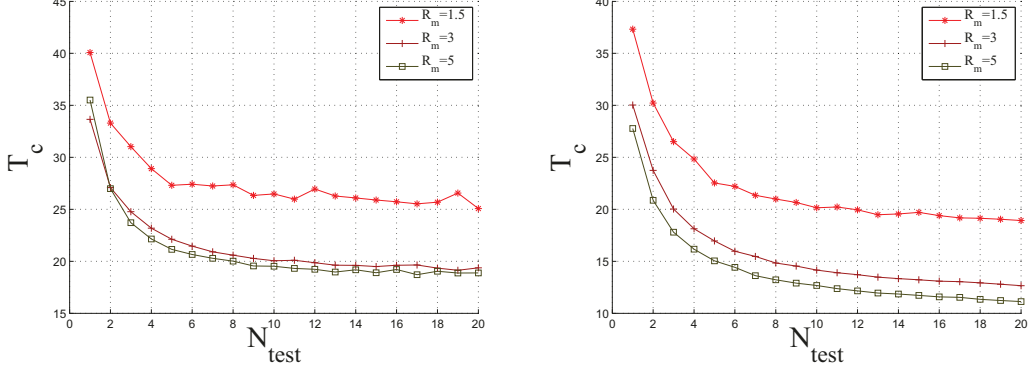


Fig. 8.4 Average convergence time T_c with IB rule as a function of N_{test} for $R_g = 1.5$ and $R_m = 1.5, 3, 5$. Left image: $S = -0.5, T = 0.5$. Right image: $S = -0.1, T = 1.1$. The time to convergence is defined as the number of simulation steps needed for the number of cooperators N_c or defectors N_d to be smaller than $0.1N$. Times of convergence are averaged over 500 independent runs.

N_{test} . This is shown for two particular games, one in the middle of the SH quadrant (left image), and the other near the upper left corner of the PD space (right image). Thus, a shorter time does not compensate for the wasted trials. Since moving around to find a better place is a costly activity in any real situation, this result is encouraging because it says that searching more intensively doesn't change the time to convergence for more than four tests. Thus, quite high levels of cooperation can be achieved by opportunistic migration at low search cost, a conclusion that interestingly extends the results presented in [10].

In diluted grids, density is another parameter that influences the evolution of cooperation [8, 9], also in the presence of intelligent migration [10, 12]. Too high densities should be detrimental because clusters of cooperators are surrounded by a dense population of defectors, while low densities allow cooperator clusters to have less defectors in their neighborhood once they are formed. We have performed numerical simulations for two other values of the density besides 0.5, $\rho = 0.2$ and $\rho = 0.8$. We do not show the figures to save space but the main remark is that there is a monotone decrease of cooperation going from low to higher densities in the low S region that influences mainly the PD and, to a smaller extent, the SH games.

8.3.2 Opportunistic Migration and Noisy Imitation

In this section we use the more flexible strategy update protocol called the Fermi rule which was described in Sect. 8.2.3 and in which the probability to imitate a random neighbor's strategy depends on the parameter β . We have seen that using the IB rule with adaptive migration leads to full cooperation in the SH quadrant and improves cooperation in a part of the PD quadrant (Fig. 8.2). This result does not hold with the Fermi rule with $\beta \geq 1$, and we are back to full defection in the PD and almost 50% cooperation as in the well mixed case in the SH; this behavior can be appreciated in the leftmost image of Fig. 8.5.

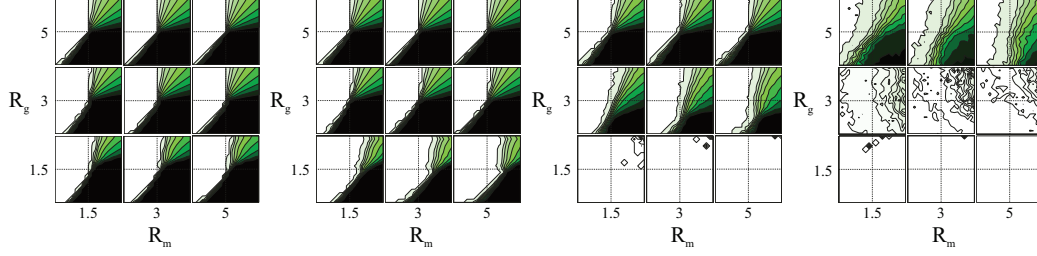


Fig. 8.5 Average cooperation levels with opportunistic migration and the Fermi rule as a function of R_g and R_m . From left to right $\beta = 1.0, 0.1, 0.01, 0.001$. The density ρ is 0.5 and $N_{test} = 20$. The size of the population is 1000 players. In all cases the initial fraction of cooperators is 0.5 randomly distributed among the population.

An interesting new phenomenon appears when β becomes small, of the order of 10^{-2} . In this case, the levels of cooperation increase in all games for R_g values up to 3 and cooperation raises to almost 100% in all game phase space for $R_g = 1.5$, for all migration radii, see the third image of Fig. 8.5. The positive trend continues with decreasing β (see rightmost image) and cooperation prevails almost everywhere. As we said above, the Fermi rule with $\beta = 0.01$ or less implies that the decision to imitate a random neighbor becomes almost random itself. Thus, the spectacular gains in cooperation must depend in some way from opportunistic migration for the most part. Figure 8.6 illustrates the dynamical behavior of a particular case in the PD space. Here $T = 1.5$, $S = -0.5$, $R = 1$, $P = 0$; that is, the game is in the middle of the PD quadrant. The other parameters are: $\beta = 0.01$, $R_g = 1.5$, and $R_m = 3$. This particular game would lead to full defection in almost all cases but here we can see that it leads to full cooperation instead.

This is a surprising phenomenon that needs an explanation. At the beginning, due to opportunistic migration, cooperators will be likely to form small clusters between themselves more than defectors, as the latter tend to follow cooperators instead of clustering between themselves since the (D, D) payoff is equal to 0. The low β value will make strategy change close to random and

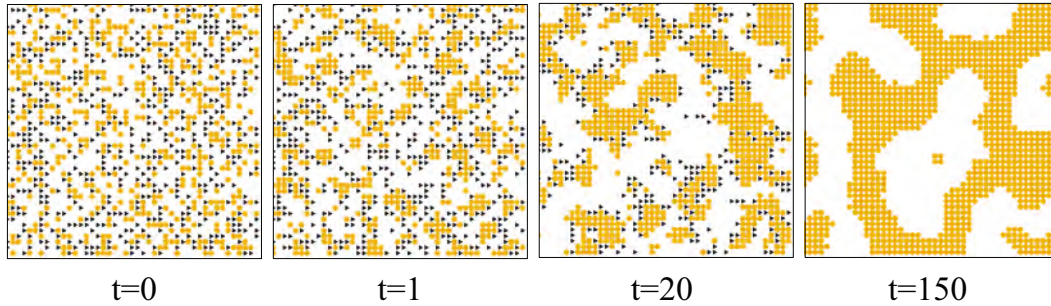


Fig. 8.6 Time evolution for the case of a PD with $T = 1.5$, $S = -0.5$, $R = 1$, $P = 0$. Here $\beta = 0.01$, $R_g = 1.5$, $R_m = 3$. The density $\rho = 0.5$ and $N_{test} = 20$. There are 1000 players and the strategies are initially attributed uniformly at random in a 50 – 50 proportion.

thus strategy update will have a neutral effect. Indeed, as soon as cooperator clusters form due to migration, defectors that enter a cooperator cluster thanks to random imitation cannot invade them. The situation there is akin to a full grid and the number of defectors inside the cluster will fluctuate. Meanwhile, defectors at the border of a cooperator cluster will steadily turn into cooperators thus extending the cluster. This is due to the fact that lone defectors at the border will tend to imitate cooperators since defectors are less connected, and strategy imitation is almost random. Finally, the defectors inside the clusters will reach the border and turn into cooperators as well. The phenomenon is robust with respect to the migration radius R_m , as can be seen in the lower part of the third and fourth images of Fig. 8.5. Cooperation prevails even when P becomes positive which increases the payoff for defectors to aggregate. We have simulated the whole phase space for $P = 0.2$ and $P = -0.2$. The results are similar to those with $P = 0$ except that cooperation decreases slightly with increasing P . On the same images it can be seen that the game radius R_g has a large influence and cooperation tends to be lost for radii larger than 1.5. The reasons for this are very similar to those advocated in Sect. 8.3.1 where Fig. 8.3 schematically illustrates the fact that increasing R_g makes the situation more similar to a well mixed population. In these conditions, the payoff-driven strategy imitation process becomes more important and may counter the benefits of opportunistic migration. However, since we believe that system possessing locality are important in practice, the findings of this section seem very encouraging for mobile agents that are better at finding more profitable positions and moving to them rather than at strategic reasoning.

8.4 Discussion and Conclusions

In this work we have explored some possibilities that arise when agents playing simple two-person, two-strategy evolutionary games may also move around in a certain region seeking better positions for themselves. The games examined are the standard ones, like the Prisoner's Dilemma, the Hawk-Dove, or the Stag Hunt. In this context, the ability to move around in space is extremely common in animal as well as human societies and therefore its effect on global population behavior is an interesting research question. As already pointed out by other researchers [10, 11, 12, 15, 16, 17], adding a form of contingent mobility may result in better capabilities for the population to reach socially valuable results. Among the existing models, we have started from a slightly modified form of the interesting Helbing's and Yu's model [10] and have explored some further avenues that were left untouched in the latter work. In the model agents live and move in a discrete two-dimensional grid space in which part of the cells are unoccupied. Using a strategy update rule that leads an agent to imitate her most successful neighbor as in [10], and having the possibility to explore a certain number of free positions around oneself to find a better one, the gains in cooperative behavior are appreciable in the Prisoner's Dilemma, in qualitative agreement with [10]. In the Hawk-Dove games the gains in cooperation are small but, in addition, we find that cooperation is fully promoted in the class of Stag Hunt games which were not considered in [10]. In Helbing and Yu the exploration of the available cells in search of a better one was fixed and deterministic. The question of the amount of effort needed to improve the agent's situation was left therefore open, although this is clearly an important point, given that in the real world more exploration usually entails an increasing cost be it in terms of energy, time, or money. By using a similar search strategy but to random positions within a given radius, and by varying

the number of searches available to the agent, we have seen that the convergence times to reach a given average level of cooperation do not degrade significantly by using fewer trials. This is a reassuring finding, given the above remarks related to the search cost.

Helbing and Yu explored migration effects under a number of sizes of the square neighborhood around a given agent. However, they only had a single neighborhood for the game interactions, the standard five-cells Von Neumann neighborhood. We have explored this aspect more deeply and presented results for several combinations of game radius R_g and migration radius R_m . In fact, it turns out that increasing the interaction radius has an adverse effect on cooperation to the point that, at $R_g = 5$, cooperation levels are similar to those of a well mixed population, in spite of fitness-based migration. Thus, positive results are only obtained when agents interact locally in a relatively small neighborhood which, fortunately, seems to be a quite common condition in actual spatial systems.

Most importantly, we have explored another important commonly used strategy update rule, the Fermi rule. This rule is also imitative but allows to control the intensity of selection by varying a single parameter β . When β is high, i.e. larger than one, almost all the cooperation gains observed with the imitation of the best rule are lost and we are back to a scenario of defection in the Prisoner's Dilemma space and the Stag Hunt games are also influenced negatively. Migration does not help in this case. However, when β is low, of the order of 0.01, a very interesting phenomenon emerges: cooperation prevails everywhere in the game space for small game radius and for all migration radii, including in the PD space, which is notoriously the most problematic class of games. With $\beta = 0.01$ or lower the strategy update is close to random; however, fitness-based migration is active and thus we see that migration, and not strategy update, is the main force driving the population towards cooperation and we have hypothesized a qualitative mechanism that could explain this striking result. Cooperation is robust with respect to the migration radius R_m but increasing R_g affects the results negatively for $R_g \geq 3$. The effect is mitigated the more random the strategy update, i.e. by further decreasing β .

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Chapter 9

Binary Birth-Death Dynamics and the Expansion of Cooperation by means of Self-Organized Growth

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Abstract Natural selection favors the more successful individuals. This is the elementary premise that pervades common models of evolution. Under extreme conditions, however, the process may no longer be probabilistic. Those that meet certain conditions survive and may reproduce while others perish. By introducing the corresponding binary birth-death dynamics to spatial evolutionary games, we observe solutions that are fundamentally different from those reported previously based on imitation dynamics. Social dilemmas transform to collective enterprises, where the availability of free expansion ranges and limited exploitation possibilities dictates self-organized growth. Strategies that dominate are those that are collectively most apt in meeting the survival threshold, rather than those who succeed in exploiting others for unfair benefits. Revisiting Darwinian principles with the focus on survival rather than imitation thus reveals the most counterintuitive ways of reconciling cooperation with competition.

9.1 Introduction

In *The Origin of Species*, Darwin laid out a beautiful theory according to which generations of organisms change gradually over time to give rise to the astonishing diversity of life we witness today. Only organisms that survive long enough to reproduce are able to pass on their genetic material, and in time those characteristics that allow survival and reproduction become more common [1]. Individual fitness is key to success, as those who fail to reproduce are destined to disappear through natural selection. But if only the fittest survive, why is there so much cooperation in nature? Eusocial insects like ants and bees are famous for their large-scale cooperative behavior, even giving up their own reproductive potential to support that of the queen [2]. Cooperative breeding in birds prompts allomaternal behavior where helpers take care for the offspring of others [3]. Microorganisms cooperate with each other by sharing resources and joining together to form biofilms [4]. Humans have recently been dubbed supercooperators [5] for our unparalleled other-regarding abilities and cooperative drive. But why should an organism carry out an altruistic act that is costly to perform, but benefits another? Altruistic cooperation is the most important challenge to Darwin's theory, and it is fundamental for the understanding of the main evolutionary transitions that led from single-cell organisms to complex animal and human societies [6].

Hamilton's kin selection theory has been applied prolifically to solve the puzzle [7]. It rests on the fact that by helping a close relative to reproduce still allows indirect passing of the genes to the

next generation. But since cooperation is common not only between relatives, kin selection can only be part of the solution. Other mechanisms that may explain cooperation have recently been reviewed in [8], including various forms of reciprocity and group selection. Network reciprocity in particular [9], has recently attracted considerable attention in the physics community, as it became clear that methods of nonequilibrium statistical physics [10, 11] can inform relevantly on the outcome of evolutionary games on structured populations [12, 13, 14]. While the basic idea behind network reciprocity is simple — cooperators do better if they are surrounded by other cooperators — the manifestation of this fact and the phase transitions leading to it depend sensitively on the structure of the interaction network and the type of interactions [12], as well as on the number and type of competing strategies [15, 16]. Physics-inspired studies have led to significant advances in our understanding of the evolution of cooperation, for example by revealing the importance of time scales in evolutionary dynamics [17], the positive impact of heterogeneity of interaction networks [18], the dynamical organization of cooperation [19] in conjunction with population growth [20, 21], as well as the emergence of hierarchy among competing individuals [22].

While the infusion of physics is relatively recent development, evolutionary game theory [23, 24, 25] is long established as the theory of choice for studying the evolution of cooperation among selfish individuals. Competing strategies vie for survival and reproduction through the maximization of their utilities, which are traditionally assumed to be payoffs that are determined by the definition of the contested game. The most common assumption underlying the evolution on structured populations has been that the more successful strategies are imitated and thus spread based on their success in accruing the highest payoffs [12, 13, 14]. As such, imitation has been considered as the main driving force of evolution, reflecting the individual struggles for success and the pressure of natural selection. In harsh environments, however, imitation may be prohibitively slow, and the extreme conditions may render a strategy viable or not in a binary way. Either the conditions for survival by an individual are met and may lead to offspring, or they are not and the individual perishes. Here we explore the consequences of such a binary birth-death evolutionary dynamics on structured populations, which effectively shifts the focus of Darwinian selection from the imitation of the fittest to the survival of the viable.

9.2 Model

We study pairwise evolutionary games on a square lattice of size L^2 , where mutual cooperation yields the reward R , mutual defection leads to punishment P , and the mixed choice gives the cooperator the sucker's payoff S and the defector the temptation T . Within this setup we have the prisoner's dilemma (PD) game if $T > R > P > S$, the snowdrift game (SG) if $T > R > S > P$, and the stag-hunt (SH) game if $R > T > P > S$, thus covering all three major social dilemma types. Without loss of generality we set $R = 1$ and $P = 0$, thus leaving the remaining two payoffs to occupy $-1 \leq S \leq 1$ and $0 \leq T \leq 2$, as illustrated in Fig. 9.1. Initially, either the whole or part of the lattice is populated by cooperators (C) and defectors (D), who are distributed randomly amidst empty sites (E), all in equal proportion. We conduct Monte Carlo simulations by randomly selecting a player x from the population, who acquires its payoff P_x by playing the game with its four nearest neighbors. We refer to the latter as the R_1 interaction range, but we also consider n -level next-nearest interactions, which we thus refer to as an R_n interaction range. If $P_x \geq Q$

player x is allowed to place an offspring on a randomly chosen empty site within its interaction range (in the absence of an empty site player x survives but does not place an offspring). If $P_x < Q$ player x dies out, leaving behind an empty site. Here Q is the survival threshold and is considered a key free parameter. Each full Monte Carlo step gives a chance to every player to either reproduce (survive in the absence of space) or die once on average.

9.3 Results

Three qualitatively different scenarios exist depending on Q . If $Q > P_x$ even for the most individually favorable strategy configurations (for example a defector being surrounded solely by cooperators in the PD game), the entire population dies out. If Q is very low, both strategies are viable and essential spread randomly as determined by the toss of a coin. For intermediate values of Q , however, fascinating evolutionary outcomes emerge. Figure 9.1 summarizes the possibilities on the $T-S$ plane for two different values of Q . For $Q = 2$ (left) cooperators dominate across the PD and SH quadrant since the threshold is high enough to support only cooperative behavior. Defectors are able to prevail only in small regions of the HG quadrant and in the SD quadrant, where they benefit from the game parametrization that awards cumulatively highest payoffs to mixed $C + D$ pairs. If the threshold is lowered to $Q = 1.5$, defectors are able to survive in the presence of cooperators even if $S < 0$. Counterintuitively, increasing the value of S can support defectors before the transition to the SD quadrant. This is akin to the phenomenon observed in games of cyclic dominance, where the direct support of prey will often strengthen predators [26, 27]. In our case, if the threshold is not too high, cooperators (prey) can survive even if they are exploited by defectors (predators). Increasing S supports cooperators, but if they are surrounded by defectors the latter are supported even more since they can continue with their exploitation. Thus, relatively low Q can sustain a mixed state even if the overall productivity of the population is marginal. It is important to note that these results are in stark contrast with the outcome obtained if the evolution-

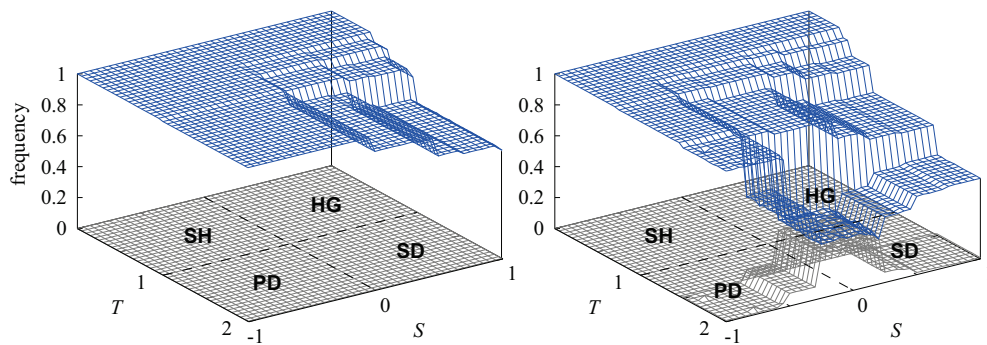


Fig. 9.1 Stationary frequency of cooperators (blue wire) and empty sites (gray wire) on the $T-S$ parameter plane for $Q = 2$ (left) and $Q = 1.5$ (right). The square lattice with nearest-neighbor interaction range (R_1) was used, and initially cooperators, defectors and empty sites were distributed uniformly at random. Dashed lines delineate the four different types of social games.

ary process is governed by strategy imitation or other proportional rules where the more successful individuals produce offspring with higher probability (see e.g. [13]).

To illustrate the dependence on Q explicitly, we show in fig. 9.2 the stationary frequency of cooperators ρ_C for the prisoner's dilemma and the snowdrift game at R_2 . The wider interaction range as used in fig. 9.1 allows us to observe a broader range of the Q dependence, because the maximally attainable payoff of the players is accordingly larger. Regardless of this, however, the main features of the outlined threshold dependence are robust and obviously independent of the applied interaction range. It can be observed that there exists an optimal intermediate interval of Q values, for which the level of cooperation in the population is maximal. Interestingly, and in agreement with our previous observations related to the results presented in fig. 9.1, harsh external conditions that are constituted by the prisoner's dilemma confer a stronger evolutionary advantage to the cooperators.

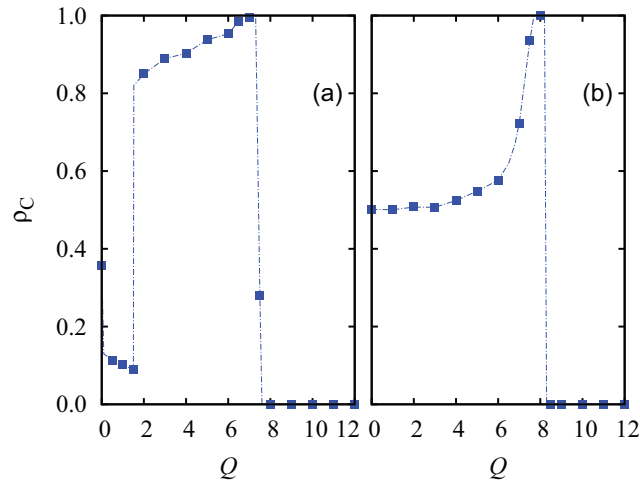


Fig. 9.2 Stationary frequency of cooperators ρ_C in dependence on the threshold value Q for two representative $T - S$ pairs. Panel (a) shows the result for the prisoner's dilemma game obtained by using $T = 1.5$ and $S = 0.5$, while panel (b) shows the result for the snowdrift game obtained by using $T = 1.5$ and $S = 0.5$. Regardless of game parametrization, there exists an intermediate interval of Q values within which cooperation thrives best. We have used the R_2 interaction range on a square lattice with linear size $L = 3200$ and random initial conditions. The displayed results are averages over 100 independent runs.

Snapshots presented in Fig. 9.3 reveal fundamental differences in the way cooperators and defectors spread if compared to imitation-based dynamics. In the latter case, cooperators form compact clusters to protect themselves against the exploitation by defectors (see e.g. [9]). Binary birth-death dynamics also introduces clusters of cooperators (c,d), but their role is not to protect cooperators, but rather to provide sufficiently high payoffs to those defectors who manage to cling onto one of the interfaces. Those defectors that fail to do so die out. This fact is illustrated clearly in the bottom row, where an initial state with ample expansion range (e) allows cooperators to free themselves from defectors (f) and expand unboundedly in a self-organized way (g,h). Defectors remain frozen within their initial radius of existence and cannot spread further. The random initial

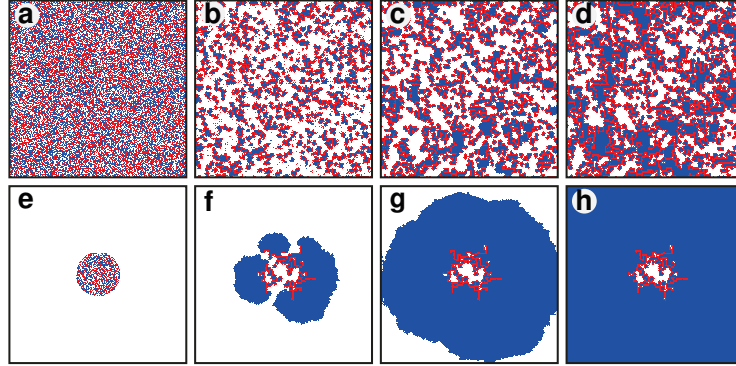


Fig. 9.3 Evolution of cooperators (blue) and defectors (red) from an initially fully (a-d) and partially (e-h) populated square lattice with R_1 . Although the distribution of strategies and empty sites (white) is the same within the populated area, the final outcome is significantly different. Defectors need the presence of cooperators to survive (b,d), and they stifle proliferation of cooperative behavior in the absence of free expansion ranges (d). If the latter exist (e), cooperators spread successfully (f,g), and their compact domains are not penetrable by defectors (h). Parameter values are $T = 1.5$, $S = -0.2$, and $Q = 1.5$.

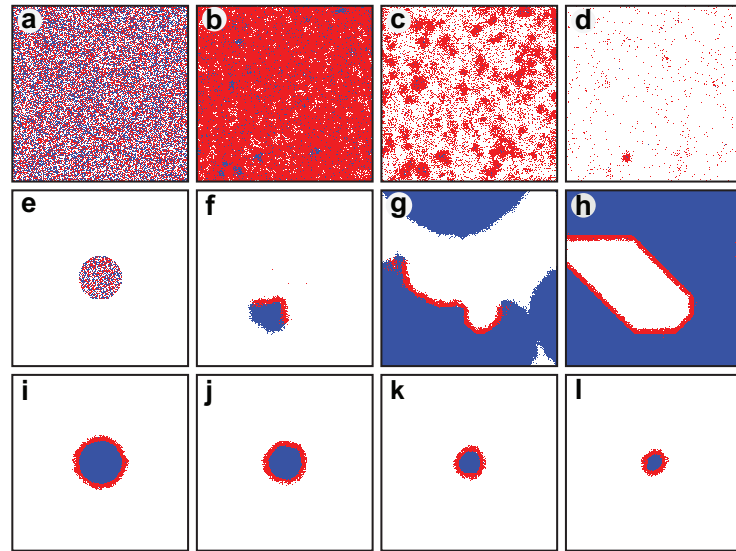


Fig. 9.4 Increasing the interaction range from R_1 (see Fig. 9.3) to R_4 (four times the elementary nearest-neighbor range) increases the vulnerability of cooperators. In the absence of free expansion ranges (a) defective behavior thrives (b), but as soon as cooperators vanish defectors alone are unable to reach the survival threshold Q and die out (c,d). If free expansion ranges do exist (e), chances are that some cooperators can evade the exploitation and spread in a given direction (f). However, defectors that succeed in maintaining contact to cooperators form a persistent front, the width of which cannot exceed the interaction range (g,h). The evolutionary stability is determined by the curvature of the invasion front, which leads to symmetry breaking in favor of defection if it is positive (i-l) (see Fig. 9.5). Parameter values are $T = 2$, $S = -1$, and $Q = 29$ (the higher threshold is possible due to the larger interaction range).

state depicted in (a), which was also used for producing Fig. 9.1, is thus not necessarily the most compatible with birth-death dynamics, as it may fail to reveal the entire potential of cooperative behavior. If a smaller portion of the lattice is used like in (e), then the final outcome will in fact always be a cooperator-dominant state, regardless of the values of Q (assuming of course Q is within reach of a cooperative domain), T and S . Here a destructive strategy like defection cannot expand towards the empty space, but can only exploit other cooperators locally to ensure its own mere existence.

Further supporting these conclusions are snapshots presented in Fig. 9.4, where the larger R_4 interaction range allows defectors to exploit cooperators more efficiently. In the absence of free expansion ranges (a) defectors initially thrive (b), but subsequently fall victim to the absence of cooperators and die out (c,d). This is a vivid demonstration of an actual tragedy of the commons [28]. Initial conditions in (e) yield a very different evolutionary outcome. Although the majority of the population within the circle dies out, some cooperators at the edge nevertheless succeed in forming a homogeneous domain and reaching Q (f). They spread towards the empty space, but defectors form a persistent front that never vanishes (g,h). It turns out that the curvature of this front is crucial for cooperators to survive. If the curvature is positive (i), symmetry breaking favors defection and the whole populations is again destined to die out — as soon as cooperators vanish, defectors are soon to follow, as illustrated in (c,d).

Details of the symmetry breaking are explained in Fig. 9.5. Accordingly, from the viewpoint of cooperators the interface is stable if it is either straight or has negative curvature. The defector's point of view is slightly different because $T > R$ breaks the symmetry, and a defector can thus reach Q even if the curvature is positive. As panels (g,h) of Fig. 9.4 demonstrate, straight and negative defective fronts indeed fail to upset cooperators, while a belt of defectors can easily suffocate a cooperative domain (i vs. l). We note that the curvature-dependent propagation of the

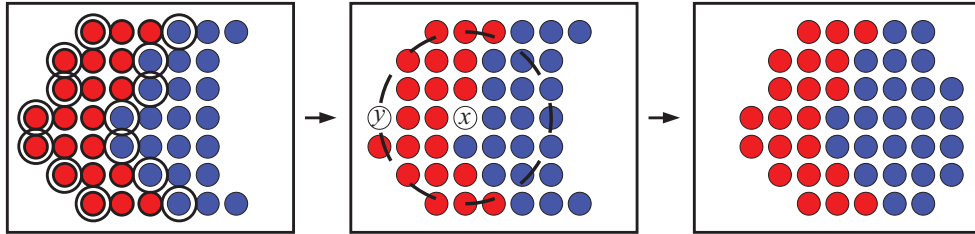


Fig. 9.5 Schematic explanation of symmetry breaking of domain wall movement that preserves the constant width of invasion fronts of defectors depicted in Fig. 9.4. Cooperators (blue) that are unable to reach Q and defectors (red) that are unable to reach Q if a cooperator dies out are encircled (left). When cooperator x dies, the empty site can be populated by an offspring from any cooperator that is within the interaction range (dashed circle in the middle panel). But since the interface has a positive curvature from the point of view of cooperators, there are even more defectors to place an offspring on x . Due to the symmetry breaking it is more likely that the site x will be populated by a defector. As soon as this happens, however, a defector at the edge of the interface will die out (site y). And in this way, all the encircled cooperators in the left panel will be replaced by a defector, while all the encircled defectors will die out. At the same time, cooperators will spread further towards the empty space (right). Thus, the domains move and are stable, but the width of the defective front remains constant.

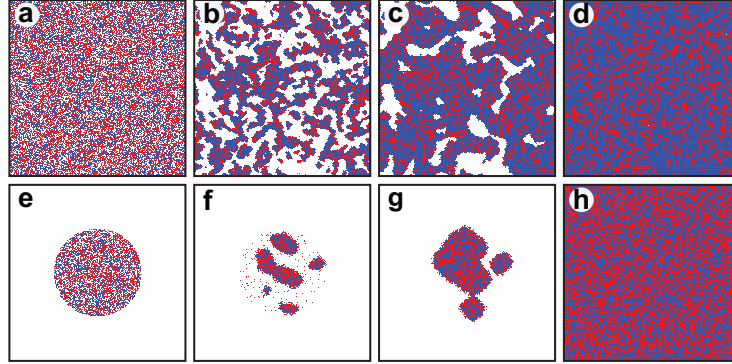


Fig. 9.6 If the prisoner’s dilemma is replaced by the snowdrift game, cooperators (blue) and defectors (red) have the potential to spread independently. But since the mixed $C + D$ pairing yields the highest cumulative payoff above the $T + S = 2$ line, a sufficiently high threshold value Q can evoke the optimal solution. As panels (a-d) illustrate, the role separation emerges spontaneously from a random initial state, and although the birth-death rule acts locally, the stationary state is globally optimal. This emergence is clearer still if the R_1 interaction range (a-d) is replaced by the R_4 interaction range (e-g), and if initially the population is bounded to a small fraction of the whole lattice (e). While a random mixture of strategies fails to preclude extinction (f), spontaneously emerging compact $C + D$ clusters grow unbounded. This is a self-organizing growth. Defectors are unable to survive alone — they need cooperators to utilize on the high value of T — but the relatively high value of S sustains cooperators as well. Parameter values are $T = 2$, $S = 1$, and $Q = 3$ (a-d) 29 (e-h).

$C - D$ interface is more robust at larger interaction range (R_3 or R_4) because the width of the defective front at R_1 or R_2 is very narrow and may thus break apart easily.

Having revealed the mechanisms that may sustain the dominance of cooperators even in the most adverse social dilemma ($T = 2$, $S = -1$), it remains of interest to study the evolutionary dynamics in the SD quadrant. Snapshots presented in Fig. 9.6 demonstrate self-organized growth towards the globally optimal mixed $C + D$ state. While a random mixture of cooperators and defectors is unable to survive (b), especially not at R_4 (f), once the role-separation sets in, the mixed clusters with proper distribution of strategies expand until the whole population is occupied (c,d and g,h). Although the applied binary birth-death dynamics affects exclusively individual players, a sufficiently high threshold value Q spontaneously selects the global optimum through self-organization.

9.4 Discussion

Summarizing, we have shown that replacing imitation with a binary birth-death rule in spatial evolutionary games creates a new class of solutions of social dilemmas. If free expansion ranges are paired with limited exploitation possibilities, cooperative behavior dominates the prisoner’s dilemma and the stag-hunt game by means of unbounded self-organized expansion that sets in as soon as cooperators find a niche to expand. If defectors are given the opportunity to exploit cooperators more effectively through the application of larger interaction ranges, cooperative be-

havior may still thrive, although it relies on a special type of symmetry breaking that determines the direction of invasion based on the curvature of the interface that separates the two competing strategies. Counterintuitively, higher levels of cooperation are observed in the prisoner's dilemma than in the snowdrift quadrant, since in the latter case self-organized growth favors mixed $C + D$ domains with proper role separation. Regardless of the governing social dilemma, and despite of the fact that the birth-death dynamics acts locally, the stationary state is always globally optimal in that the payoff of the entire population is maximal. We note that all the presented results are robust to variations of the interaction network, and can be observed also in off-lattice simulations [29], thus indicating a high degree of universality.

Our results also highlight the importance of initial conditions, in particular the fact that a random initial state without obvious chances of expansion may fail to reveal all the benefits of cooperative behavior. This may be relevant in experimental setups. Recent experiments with microbial metapopulations indeed support the conclusion that range expansion may promote cooperation [30], and recent theoretical work highlights the importance of the eco-evolutionary feedback even when evolution is governed by frequency-dependent selection [31]. Although special initial conditions with ample empty space may appear somewhat artificial in the realm of mathematical modeling, they do mimic rather accurately the conditions in a Petri dish [32], and together with birth-death dynamics they appear to hold the key for understanding the success of cooperation from an entirely different perspective.

Acknowledgments

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Part III
Laboratory Experiments

Chapter 10

Coordination on Networks: Does Topology Matter?

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Abstract Effective coordination is key to many situations that affect the well-being of two or more humans. Social coordination can be studied in coordination games between individuals located on networks of contacts. We study the behavior of humans in the laboratory when they play the Stag Hunt game – a game that has a risky but socially efficient equilibrium and an inefficient but safe equilibrium. We contrast behavior on a cliquish network to behavior on a random network. The cliquish network is highly clustered and resembles more closely to actual social networks than the random network. In contrast to simulations, we find that human players dynamics do not converge to the efficient outcome more often in the cliquish network than in the random network. Subjects do not use pure myopic best-reply as an individual update rule. Numerical simulations agree with laboratory results once we implement the actual individual updating rule that human subjects use in our laboratory experiments.

10.1 Introduction

Game theory [1] is a useful tool in the study of economic, social, and biological situations for describing interactions between agents having possibly different and often conflicting objectives. Paradigmatic games such as the Prisoner's Dilemma have been used in order to represent the tension that appears in society when individual objectives are in conflict with socially desirable outcomes. Indeed, most of the vast research literature has focused on conflicting situations in order to uncover the mechanisms that could lead to cooperation instead of socially harmful outcomes. However, there are important situations in social and economic contexts that do not require players to use aggressive strategies but simply to coordinate their actions on a common goal, since in many cases the best course of action is to conform to the standard behavior. For example, if one is used to drive on the right side of the road and travels to a country where the norm is reversed, it pays off to follow the local norm. Games that express this extremely common kind of interactions are called *coordination games*.

One important consideration is the interaction structure of the playing agents. It is frequently assumed, especially in theoretical work but also in many laboratory experiments, that agents interact globally with any other agent in the population. However, everyday observation tells us that in animal and human societies, individuals usually tend to interact more often with some specified subset of partners; for instance, teenagers tend to adopt the fashions of their close friends group;

closely connected groups usually follow the same religion, and so on. In short, social interaction is mediated by networks, in which vertices identify people, firms etc., and edges identify some kind of relation between the concerned vertices such as friendship, collaboration, economic exchange and so on. Thus, locality of interaction plays an important role. The dynamical behavior of games on networks has been investigated both theoretically and by numerical simulation methods (see [2, 3, 4, 5, 6] for comprehensive recent reviews).

Several analytically rigorous results are available for coordination games in well-mixed populations [7, 8], as well as populations with a simple local interaction structure such as rings and grids [9, 10]. These results are very useful and will be summarized later on; however, while game theory has normative value, its prescriptions are not always reflected in the way people act when confronted with these situations. This has been made manifest by a host of results of experiments with people [11]. Coordination games are no exception and also confront the theory with many puzzles. For coordination games on small-worlds and regular networks the laboratory experiments carried out in [12] and in [13, 14, 15] are particularly relevant.

In this paper we describe and discuss a laboratory experiment on coordination games using particular local network structures that are characteristic of real social interactions and thus go beyond the simple and well-known rings and grids usually employed in such experiments. By doing so we wish to understand how well the theoretical or simulated actions of automata align with choices by humans. If theory and simulations line up well with actual choices in the laboratory this re-inforces the use of these relatively cheap tools to understanding behavior. In contrast, if the laboratory reveals results that are not to be expected from theory or simulations, there is a need to refine theory and simulation methods.

The paper is organized as follows. In the next section we present a brief introduction to the subject of coordination games and we summarize the main known theoretical results in order to provide the right context for the experimental part. The following sections deal with the main theme of the present study where, after a discussion of previous related work, we present the setup and the results of our laboratory experiment related to the relevant network structures. Finally, we present a detailed discussion of the results in the context of related work and give our conclusions.

10.1.1 Coordination games

As in most previous work, we shall restrict ourselves to two-person, two-strategies, symmetric coordination games. General two-person, two-strategies coordination games have the normal form of Table 10.1.

	α	β
α	a, a	c, d
β	d, c	b, b

Table 10.1 A general two-person, two-strategies symmetric game.

Here we shall assume that $a > b$ and $(a - d) \leq (b - c)$; then (β, β) is the risk-dominant equilibrium, while (α, α) is the Pareto-dominant one. This simply means that players get a higher payoff by coordinating on (α, α) but they risk less by using strategy β instead. There is also a third equilibrium in mixed strategies but it is evolutionarily unstable. The Pareto-efficient equilibrium (α, α) is socially preferable but mis-coordination may happen easily leading to inefficient outcomes. This type of game is the so-called Stag-Hunt game [16]; it has been extensively studied analytically using stochastic processes [7, 9] and by numerical simulation on several model network types [16, 17, 6].

In well-mixed populations, agents may use *myopic best-reply* [8] to revise their strategy. This is a deterministic, bounded-rationality adaptive learning rule in which, in each time step, an agent has the opportunity of revising her current strategy with probability p . She does so by considering the previous actions of the rest of the population and switches to the action that would maximize her payoff if the other players stick to their previous choices. In other words, $\hat{\sigma}_i$ is a myopic best-reply for player i if $\Pi_i(\hat{\sigma}_i(t), \sigma_{-i}(t-1)) > \Pi_i(\sigma_i(t), \sigma_{-i}(t-1)), \forall \sigma_i$, where $\sigma_{-i}(t-1)$ is the strategy profile of all players other than i at time $t-1$. In case of a tie, agent i keeps its current strategy.

For best reply both monomorphic populations of all α and all β are asymptotically stable states [18]. However, if some noise is introduced in best response dynamics to simulate strategy update errors of various kinds then the stochastically stable state in the long run will be the risk-dominant strategy β since the risk-dominant strategy has the largest basin [8, 7].

When the population has a network structure the strategy-revision rule described above is slightly modified in such a way that it works for pairs of agents that are neighbors [9, 8, 6]. For populations structured as rings, the risk-dominant strategy β should take over the population in the long run [9, 8] if the agents play according to myopic best-reply. If, instead, agents imitate the strategy of their most successful neighbor and the neighborhood size is large enough, then the payoff-dominant strategy becomes the unique long-run equilibrium [4]. In two-dimensional grids, both equilibria can be reached depending on the evolution rules considered and, most remarkably, dimorphic states, i.e. population states in which α and β players coexist in a stable manner, become possible [19, 10, 16, 4]. An important feature of these local models is that the convergence is faster than in global interaction models [4].

No general theoretical results on coordination games are available for arbitrary networks. However, the simulation results show that the presence of a local interaction structure provided by a network tends to increase the region of the game's parameter space in which the Pareto-dominant outcome prevails [16, 6]. Moreover, dimorphic populations may be stable in complex networks thanks to the existence of recognizable communities of tightly linked agents [20].

The conclusion of this brief summary on theoretical results is that either the all- α or all- β convention can be reached as a stable state in well-mixed populations depending on details such as agent matching, noise, and strategy revision rule. On rings, the stable state in the long-run is most probably the risk-efficient equilibrium all- β , although all- α can also arise if agents imitate the best neighbor and neighborhoods are large. On grids and complex networks in general both monomorphic and dimorphic populations can be stable, thus both strategies can coexist.

10.1.2 Previous experimental results

We have seen that theory alone is not discriminating enough to solve the equilibrium selection problem by analytical means and thus empirical approaches are very valuable. Indeed, coordination games have been the object of a number of experimental works in the last two decades. Among the most well-known studies dealing with randomly mixing populations and groups, we may cite e.g. [21, 22, 23, 24, 25] and chapter seven of Camerer's book [11], where an informative summary is provided.

Given the focus of our work, we concentrate here on situations in which local interaction structures and thus networks play a fundamental role. To our knowledge, there have been few experiments in which the population structure that has been recreated in the laboratory only allows for local interactions. Possibly among others, the works of My et al. [13], of Keser et al. [14], of Berninghaus et al. [15], and of Cassar [12] are relevant in this context.

Keser et al. used a ring structure where each player has a neighbor on either side and a well-mixed structure for comparison. Their conclusions are that in the ring the preferred equilibrium is the risk-dominant one, while the payoff-dominant equilibrium was the more frequent result in the globally communicating population. This is in qualitative agreement with the theoretical predictions of Ellison [9] for the ring and of Kandori et al. [7] for the mixing case.

My et al. performed a comparative experimental study of Stag Hunt games with three different payoff matrices on mixing and structured populations. The population with local structure was composed by a circle of eight people where each player only interacted with her immediate right and left neighbors. They find that the first period modal choice of strategy, which is the payoff dominant one, plays a major role in the final outcome. In the global population case, the steady state generally lies in the same basin of attraction as the initial state. For the ring structure, the convergence to the risk-dominant outcome is more frequent than in the well-mixed case, especially when the payoff matrix values are such that the Pareto-superior basin shrinks. However, still often times the system converges to the Pareto-dominant state, which disagrees with the theoretical predictions of Ellison [9] based on noisy best reply dynamics. By examining the detailed history of play, the experimenters have found that, while in the global population subjects on average play myopic best response, in the ring with local structure a kind of imitation rule fits the data better than best reply.

In the study of Berninghaus et al. the authors find that a ring of eight or sixteen players leads to less coordination on the Pareto-efficient strategy α in the average than in groups of three completely connected players. In addition, with the same neighborhood size, grids of sixteen individuals are less effective in evolving coordination on the efficient equilibrium.

Our study is close to three other studies. The first study is the modeling and simulation work of Roca et al. [26], with some unavoidable limitations related to the small size achievable in the laboratory. Roca et al. studied cooperation and coordination on a couple of actual social networks and identified a different behavior: in one of the networks the Pareto-efficient strategy α cannot propagate and the final equilibrium results in a dimorphic population. They attributed the phenomenon, which is deemed to be quite general, to the existence of *topological traps*, which are local network features characterized by local bridges [27] and scarcity of redundant paths. These structures make it difficult for any flow to easily propagate past the trap.

The second is the study of Cassar [12], which is the closest one from the standpoint of the present paper as it investigates network structures that are more realistic than the ring and the two-dimensional lattice, although the ring is also used in the experiments for comparison. Basically, the main finding of Cassar was that small-world networks were apparently the more conducive to coordination on the Pareto-efficient outcome, and she attributed this effect to the higher clustering of these networks with respect to random structures. We shall discuss her settings and results in more detail later.

The third article describes a recent experimental investigation on coordination games on various kinds of small-size networks [28]. The authors focus on equilibrium selection in these networks by the experimental subjects as compared with theoretical predictions. They found little support for the prediction that network effects have an influence on the emergence of a given equilibrium, as most groups coordinated on the efficient equilibrium irrespective of the network shape. However, they did find a difference between inexperienced subjects and those that have already played the game. After the first run, most groups coordinated on the efficient equilibrium very quickly. We also observed a strong initial bias towards playing strategy α at the beginning of a run, a tendency that becomes stronger when subjects have gained some better understanding of the game (see section Results). Although the study is interesting, it is not really comparable to ours since the network size is very small (six) and this of course makes network effects more difficult to assess. Moreover, in most runs the participants had full information on the strategies and positions of all the other players, while in our case knowledge was effectively restricted to the first neighbors.

10.2 Materials and Methods

10.2.1 Ethics statement

The use of human subjects in economics laboratory experiments has been approved by the ethics committee of the University of Lausanne. The participants were fully informed of the nature of the experiment and signed an informed consent to participate. Their anonymity has been guaranteed at all stages of the experiment.

10.2.2 Network design

We designed two basic network topologies containing 20 nodes each: a random network and a cliquish network. The random network, shown in Fig. 10.1a, is a regular random graph of degree five and is used as a baseline case in which no topological traps are present. The average clustering coefficient of this graph is 0.15. Two different matches of this topology, denoted by R_1 and R_2 respectively, were tested with random relabeling of nodes between them. The idea behind a constant degree and relabeling of nodes is to avoid confusing the effect of the topology itself with that of the particular location of the node in it. This structure was used as a baseline against which the following is to be compared with respect to the game behavior.

The cliquish network is reproduced in Fig. 10.1b. Here each agent has the same number of neighbors (five) but she is more strongly connected to four of them, those belonging to the five-

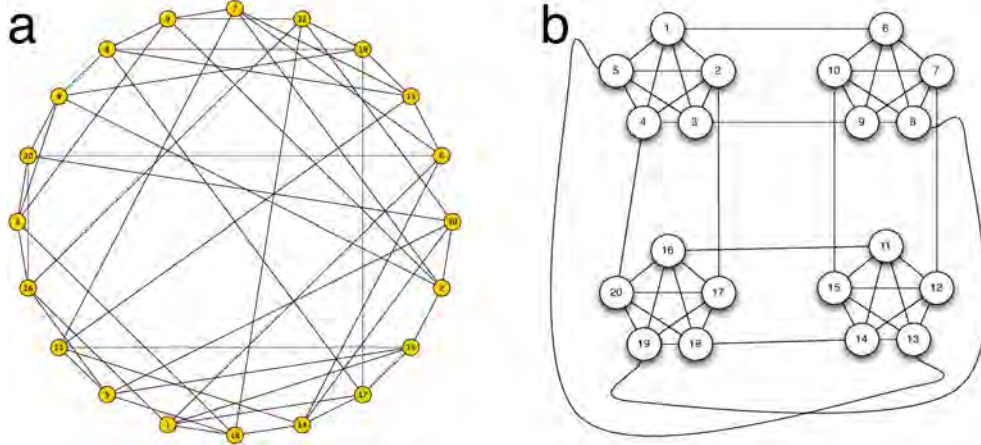


Fig. 10.1 Network topologies used in the experiments. a: regular random graph; b: cliquish network;.

clique structure. This network was designed to reproduce the kind of topological traps of Roca et al. [26] introduced above, as the low number of links between cliques could, in principle, play the role of traps. This graph has an average clustering coefficient of 0.6, much higher than the random graph one (see above). Again, two matches of this topology were tested, that we denote as C_1 and C_2 .

In our opinion, the non-random network used in the experiments is much closer to actual social networks in its local structure than any previously used topology in the laboratory as rings and grids. Indeed, we found that comparable structures were independently used by Suri and Watts in a Web-based experimental study of public goods games [29].

10.2.3 Specific coordination game

With reference to Table 10.1, we have chosen a particular game in the coordination game space by fixing $a = 1$, $b = 0$, and $c = -1$. Then, using numerical simulations, we have varied the last parameter d in the interval $[0, 1]$ in such a way as to choose the value of d that approximately maximizes the difference in equilibrium fractions of α strategists and β strategists between the random network and the cliquish one. Figure 10.2 shows the average results of 100 simulation runs on each of the networks, considering myopic best-response as update rule and an initial fraction of 0.7 α -strategists. This last value is rather typical, being close to the initial frequencies of α that have been observed in many laboratory experiments. Absolute values of the differences in steady state frequencies of α between the random graph and the cliquish network are reported on the y-axis as a function of d and are systematically higher for d about $0.35 \leq d \leq 0.75$. We fixed $d = 0.5$, a value that lies in the interior of the interval that discriminates well between the cliquish and the random network. From these values, in order to obtain positive integer payoff values, we have performed an affine transformation of the matrix which leaves the NE invariant and leads to the matrix used in the experiment shown in Table 10.2.

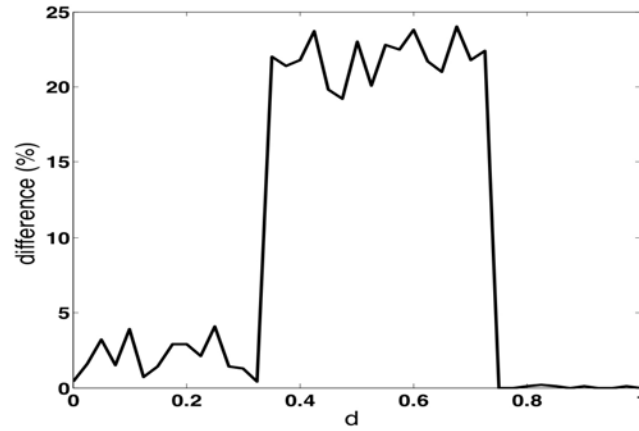


Fig. 10.2 x-axis: payoff value d ; y-axis: absolute value of the difference between fractions of α strategy at steady state in random and cliquish networks. The values are averages over 100 simulation runs for each network structure (see text).

	α	β
α	5, 5	1, 4
β	4, 1	3, 3

Table 10.2 Specific coordination game used in the experiment.

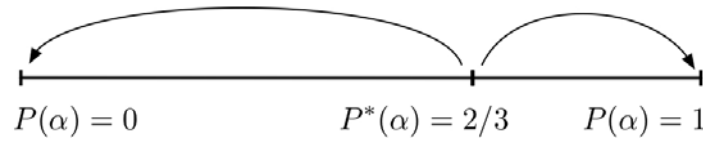


Fig. 10.3 Basins of attraction for α strategy (right basin) and for β (left basin) when using the numerical payoffs of the experimental coordination game. $P^*(\alpha) = 2/3$ is the unstable dimorphic equilibrium.

For the particular coordination game represented by this matrix, the mixed equilibrium is found to be $(\alpha = 2/3, \beta = 1/3)$ and the corresponding basins are sketched in Fig. 10.3. It is worth noting that the game used here is formally equivalent to the one employed in Cassar's experiments [12].

We performed many numerical simulations on the graph structures shown in Fig. 10.1. We only present a summary of the results to save space; detailed data are available on request. Using pure best response as a strategy update rule on the cliquish network (Fig. 10.1b) and starting with 50% α -strategists the dynamics converges on all- β 90% of the time, after 1000 repetitions; otherwise there is one clique of α players that remain stable and the rest of the population plays β . With an initial fraction of α of about 80%, we never observe convergence on a β monomorphic population. Instead, in 96% of the runs the game dynamics converged on dimorphic populations with cliques conquered either by α or β strategists and only 4% went to the all- α equilibrium. This last result shows that, in the absence of noise and errors of some kind, weak links do indeed cause freezing of the strategies in some parts of the network. In a series of runs we perturbed deterministic

best response by adding a 0.1 probability of making errors. In these conditions we always found convergence on the all- β fixed point with both initial proportions. This is understandable as, with noise added, dimorphic configurations that were stable with pure best response, are destabilized and ultimately broken.

The baseline case of the regular random network of Fig. 10.1a is indeed rather different. With pure best response and an initial proportion of 50% α , the dynamics always converges to all β . With 80% α initially, convergence is on all α in 92% of the cases, with some runs converging to dimorphic populations. As soon as noise is added to pure best response, all the runs converged to the β fixed point for both initial conditions.

10.2.4 Implementation

We conducted a total of four experimental sessions that counted 20 participants each using the z-Tree environment [30]. Participants were recruited from a subject pool that includes students from several faculties. In each session, subjects played the coordination game in four different network topologies, and each topology lasted for 30 periods. In other words, the location of nodes in a network remained unchanged during 30 periods. Table 10.3 summarizes the order in which the different network topologies were implemented in each session.

Session	Date	Subjects	Network 1	Network 2	Network 3	Network 4
1	03.10.2011	20	C_1	C_2	R_1	R_2
2	07.10.2011	20	R_1	R_2	C_1	C_2
3	07.10.2011	20	C_1	R_1	C_2	R_2
4	14.10.2011	20	R_1	C_1	R_2	C_2

Table 10.3 Summary of experimental sessions

Each period counted two stages. In the first stage, players had to select one of the two strategies, that we called “square” and “circle” instead of α and β in order to avoid suggesting an implicit ranking. Subjects were allowed to take as much time as they wanted to reach a decision. The average response time was 2 seconds so most subjects were very fast in selecting their strategy. In the second stage, subjects observed on the screen their own choice, the number of neighbors that selected each strategy and their own gain of the period. In particular, they were never informed about their neighbors’ payoffs, nor about their individual strategy choices. This implies that payoff-based imitation rules are ruled out, since subjects cannot identify the most successful strategy.

Students read a detailed description of the experiment before the started playing the game. After reading the instructions, subjects had to respond to a set of control questions that insured common understanding of the game and the computation of payoffs. A translation of the instructions distributed to subjects is provided as supplementary material to this paper. After one round of 30 periods, subjects were informed that they would play the same game for another 30 periods, but that their neighbors (and their neighbors’ neighbors) would be different than the ones met in the previous round. They were not informed about the particular network topology of the society, but they were aware that they (and everyone else) would always play the game with five neighbors. Each session lasted for about 80 minutes and subjects earned, on average, 36.6 swiss francs, or about 30 EUR (37 USD).

10.3 Results

10.3.1 Aggregate behavior

Fig. 10.4 reports the proportion of players choosing the efficient strategy α aggregated over all sessions and periods. Strategy α is the preferred choice at any time period in both topologies. Players on random networks coordinate a bit more often on the payoff-dominant strategy α than players on the cliquish network (Fig. 10.4a). Yet, a standard t-test that accounts for clustering across individuals does not reject the hypothesis that the proportion of α -strategists is the same in both topologies.

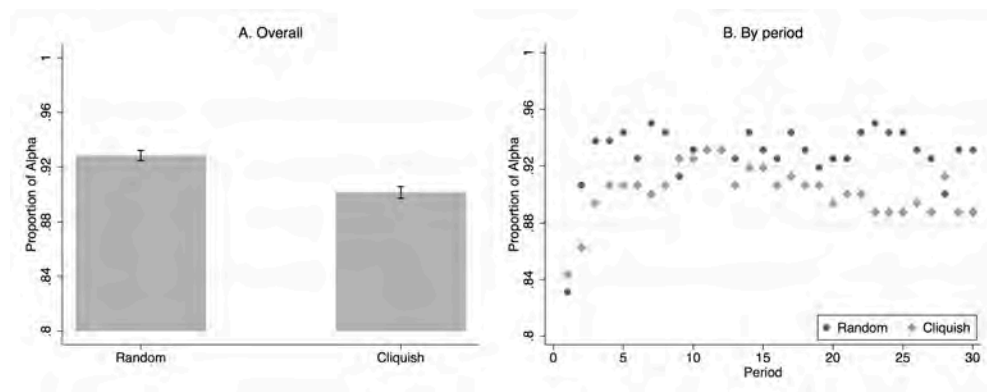


Fig. 10.4 Proportion of α -choices by network topology. Proportions are aggregated over all sessions and periods in A, and over session in B.

When disaggregating by period (Fig. 10.4b), we see that subjects seem to understand payoff-dominance from the very first period, where no more than 20% of the population minimizes risk by choosing strategy β . From this high initial rate of α -choices, convergence to almost full coordination on the payoff-dominant strategy is quite rapid.

10.3.2 Estimating individual behavior

Our interest lies in understanding how subjects make their choices in response to their neighbor's choices. The key graph compares the choice of strategy with the information received about the strategy choice of neighbors in the previous period. Fig. 10.5 plots the proportion of individual α -choices against the proportion of α -choices in the neighborhood in the previous period.

First of all, it is evident that most subjects are sensitive to the information regarding their neighbors' choices, since their own decisions are correlated to it. Moreover, the effect of this information is monotone: the higher the proportion of α -choices in my neighborhood, the higher the probability that I also choose α in the next period. Second, the effect seems to be non-linear or, more precisely, S-shaped. This would suggest that the biggest change in individual behavior occurs at

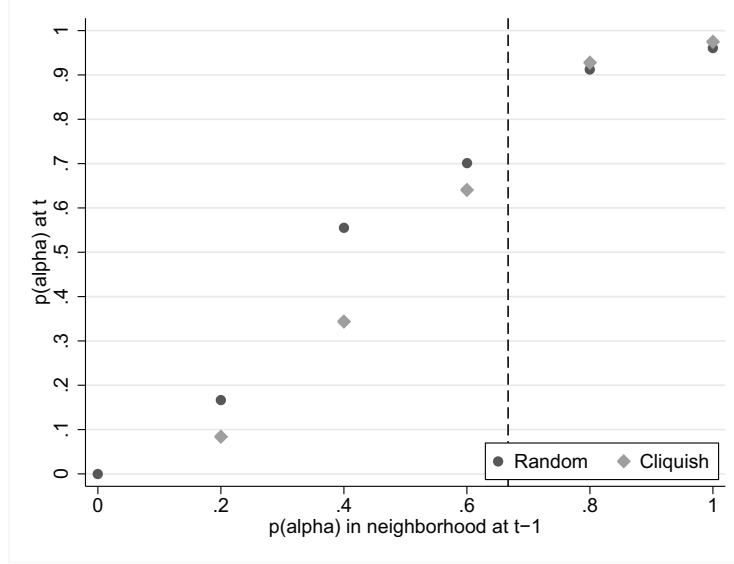


Fig. 10.5 Proportion of α -choices as a function of the fraction of neighbors that chose α in the previous period. Dashed line at two thirds: at the left of the dashed line a myopic best replier chooses α with probability zero; at the right of the dashed line a myopic best replier chooses α with probability one.

intermediate values of the neighborhood's distribution of α -choices. We now discuss how to use this evidence to infer whether individuals play myopic best-reply or not.

Remember that a player's strategy σ in any given period is a myopic best-reply if, assuming that the distribution of her neighbors' strategies remains unchanged, it gives her a higher payoff than any other strategy.

Let $\bar{p}_{i,t-1}(\alpha)$ denote the fraction of neighbors of individual i that chose $\sigma = \alpha$ in the previous period. That is

$$\bar{p}_{i,t-1}(\alpha) = |N_i|^{-1} \sum_{j \in N_i} \alpha_{j,t-1} \quad \text{with} \quad \alpha_{j,t} = \begin{cases} 1 & \text{if } \sigma_{j,t} = \alpha \\ 0 & \text{if } \sigma_{j,t} = \beta \end{cases}$$

where N_i is the neighborhood of i and $|N_i|$ its cardinality (i.e. the degree of node i). Given the payoffs used in this experiment, the (myopic) expected gains to subject i of choosing each strategy in period t are

$$\begin{aligned} E[II_{i,t} | \sigma_{i,t} = \alpha] &= 5 \times \bar{p}_{i,t-1}(\alpha) + 1 \times [1 - \bar{p}_{i,t-1}(\alpha)] \\ E[II_{i,t} | \sigma_{i,t} = \beta] &= 4 \times \bar{p}_{i,t-1}(\alpha) + 3 \times [1 - \bar{p}_{i,t-1}(\alpha)] \end{aligned}$$

Thus, choosing the payoff-dominant strategy α is a best response if

$$\begin{aligned} E[II_{i,t} | \sigma_{i,t} = \alpha] - E[II_{i,t} | \sigma_{i,t} = \beta] &> 0 \\ \Leftrightarrow \Delta_{i,t} \equiv 3\bar{p}_{i,t-1}(\alpha) - 2 &> 0 \end{aligned} \tag{10.1}$$

Borrowing the terminology used in Cassar [12], we will refer to the amount $\Delta_{i,t}$ as the “payoff advantage” of choosing strategy α . Myopic best-reply means that an individual switches to playing

α as soon as more than 2 out of 3 of her neighbors play α , i.e. as soon as the payoff advantage becomes positive.

If a subject uses myopic best-reply as update rule, we should observe $\sigma_{i,t} = \alpha$ whenever (10.1) holds, i.e. the payoff advantage of choosing α is positive, and $\sigma_{i,t} = \beta$ otherwise. Hence, we can estimate a model of the form

$$Prob(\sigma_{i,t} = \alpha | \bar{p}_{i,t-1}(\alpha)) = G(\gamma_0 + \gamma_1 \Delta_{i,t} + \gamma_2 C_{i,t} + \gamma_3 \Delta_{i,t} C_{i,t} + x'_{i,t} \phi) \quad (10.2)$$

where $C_{i,t} = \mathbf{1}(\Delta_{i,t} > 0)$, and $x_{i,t}$ includes other relevant control variables. $G(\cdot)$ is a cumulative distribution function.

The parameter γ_1 measures the effect of the payoff advantage on the probability of choosing α , when this advantage is negative. If the subject follows myopic best-reply this parameter is zero because a negative payoff advantage, regardless of its size, should translate into a zero probability of choosing strategy α . The parameter γ_2 measures the discrete jump in the probability of playing α once more than two thirds of neighbors play α . A myopic best-replier should go from choosing α with zero probability to choosing it with certainty as soon as the payoff advantage becomes positive. This means that γ_2 is 1 for a pure myopic best-replier. Finally, $\gamma_1 + \gamma_3$ measures the effect of the payoff advantage on the probability of choosing α , when this advantage is positive. Again, γ_1 and γ_3 are zero for a player who plays myopic best-reply.

The specification (10.2) amounts to fitting the S-shaped relationship observed in the data of Fig. 10.5. To estimate the model, we consider $G(\cdot)$ to be the uniform cdf and we include as control variables the lagged individual choice (to control for lock-in due to either inertia or unobserved heterogeneity), dummies for 5-period intervals and for the order in which the particular topology was played in the session. We have also explored estimating model (10.1) adopting a logistic specification for $G(\cdot)$, but did not pursue this approach since players ended up playing the α strategy with probability 1 in one repetition of the experiment. The logistic specification rules out such cases whereas the uniform specification allows for it. Table 10.4 reports the sensitivity of the probability of playing α to the excess payoff ($\Delta_{i,t}$), the change in that probability as the fraction playing α exceeds 2 out of 3 ($C_{i,t}$), and the change in the sensitivity as the fraction of players playing α exceeds two thirds ($\Delta_{i,t} C_{i,t}$). Inference is based on standard errors that account for clustering within individuals.

The parameter γ_1 is estimated to be positive and significantly different from zero in both network topologies. This means that subjects are more likely to play α the less negative the payoff advantage of playing α is. Second, there is some evidence of an increase in the probability of choosing α when the payoff advantage becomes positive, but the magnitude of the shift is small (γ_2 close to zero). Subjects are no longer sensitive to the payoff advantage as soon as more than two out of three of their neighbors have switched to playing α . These results reject the hypothesis that all subjects in the laboratory adopted pure myopic best reply as update rule.

Regarding the other variables, we see evidence of lock-in in that past individual choices are correlated to current ones. This could either be due to inertia or, as stressed by Berninghaus et al. [15], to unobserved heterogeneity. Players are significantly less likely to play α in rounds 6 to 20 (compared to rounds 1 to 5) in the random network. This effect is not present in the cliquish network. Moreover, the order in which the topologies were played do not matter.

Results suggest that players are somewhat more sensitive to their neighbor's choices in cliquish networks than in random ones. Moreover, clustering could also be present across individuals within

	Random	Cliquish
$\Delta_{i,t}$	0.202* (0.085)	0.271*** (0.046)
$C_{i,t}$	0.065 (0.047)	0.088* (0.033)
$\Delta_{i,t}C_{i,t}$	-0.193* (0.084)	-0.247*** (0.046)
$\alpha_{i,t-1}$	0.572*** (0.097)	0.500*** (0.099)
periods6to10	-0.023** (0.007)	-0.008 (0.009)
periods11to15	-0.022** (0.008)	-0.009 (0.007)
periods16to20	-0.023** (0.008)	-0.010 (0.008)
periods21to25	-0.014 (0.007)	-0.007 (0.006)
periods26to30	-0.019 (0.010)	-0.008 (0.008)
order2	0.029 (0.018)	0.006 (0.009)
order3	0.018 (0.011)	0.003 (0.008)
order4	0.022 (0.014)	0.002 (0.009)
Session dummies	Yes	Yes
N	4640	4640

Notes: standard errors clustered by individual in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Table presents marginal effects from a linear probability model. $\Delta_{i,t}$ is the payoff advantage of choosing α given the distribution of neighbors' choices in previous period; $C_{i,t} = \mathbf{1}(\Delta_{i,t} > 0)$; $\alpha_{i,t} = \mathbf{1}(\sigma_{i,t} = \alpha)$; *periods#* are dummies for 5-period intervals; *Order#*=1 if network played in the #th order in a session; *cliquish*=1 if cliquish topology, =0 if random. Session dummies were included but turned out to be not significant.

Table 10.4 Estimation of α -choice

the same network. Table 10.5 reports an empirical specification that allows testing whether the updating rule differs between the cliquish and the random network. Column “All repetitions” reports results that use all repetitions in the experiment. Column “First repetition” reports results based on the first network topology that subjects played. All estimates report the standard errors clustered by individual in parentheses. The “first repetition” estimates report the standard error in brackets that allow for clustering at the individual level and for correlation between individual i 's choice in period t with the decisions of her neighbors in $t-1$.¹

¹ We do not report these standard errors in column 1 since we have been able to figure out how to calculate these standard errors only for the first repetition. The standard solution to account for clustering within sessions would be to

	All repetitions	First repetition
Delta	0.213 (0.086)* {0.011}***	0.232 (0.087)** {0.006}*** [0.112]*
C	0.074 (0.048) {0.016}*	0.069 (0.056) {0.036} [0.062]
Delta*C	-0.197 (0.084)*	-0.220 (0.090)* [0.105]*
Delta*cliquish	0.044 (0.090)	0.014 (0.120) [0.118]
C*cliquish	0.003 (0.057)	-0.011 (0.077) [0.081]
Delta*C*cliquish	-0.034 (0.092)	0.048 (0.128) [0.112]
cliquish	-0.005 (0.057)	-0.015 (0.072) [0.058]
l.alpha	0.540 (0.092)***	0.482 (0.064)*** [0.025]***
Constant	0.364 (0.087)***	0.406 (0.073)*** [0.023]***
N	9280	2320
r ²	0.525	0.378
Order dummies	yes	no
Session dummies	yes	no
Period-interval dummies	yes	yes

Standard errors clustered by individual in parentheses. Restricted standard errors in square brackets (assuming independent sessions, no correlation between unconnected subjects in a session, no contemporaneous correlation between connected subjects, no correlation at lags > 1 between connected subjects). * $p < .05$, ** $p < .01$, and *** $p < .001$ (t-distribution with $G - 1$ degrees of freedom, with G the number of clusters. The restricted standard errors consider the same distribution as the individual clustering.)

Table 10.5 Estimation of α -choice (interacted specification)

Table 10.5 shows two main results. First, the parameters in the update rule do not differ by topology. The terms $\Delta_{i,t} \text{cliquish}$, $C_{i,t} * \text{cliquish}$, and $\Delta_{i,t} * C_{i,t} * \text{cliquish}$ measure the difference in update rule parameters between the cliquish and the random network. None of these three parameters is significantly different from zero, regardless of the standard errors we use to perform the test. Second, column two indicates that our main result that subjects do not use myopic best reply is valid, again regardless of the type of standard error we use.

Figure 10.6 plots the predicted probabilities of choosing α against the actual choices as a function of the previous period proportion of α -choices in the neighborhood. It appears that model (10.2) does a very good job in fitting the observed data, in particular for cliquish networks.

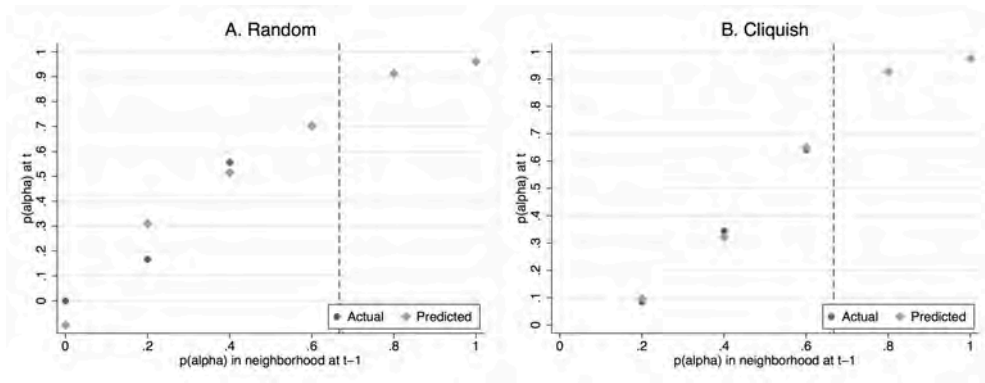


Fig. 10.6 Actual and predicted proportion of α -choices as a function of the fraction of neighbors that chose α in the previous period.

10.4 Discussion

10.4.1 Influence of the network structure

The analysis of the previous section has shown that human subjects do not use myopic best reply as an update rule. The analysis also shows that interaction structure has no influence on the aggregate fraction of individuals playing the efficient strategy α . We also do not find that network structure is important for how subjects adapt to their neighbor's choices.

It is worthwhile to try to relate our observations with the theoretical predictions of Roca et al. [26]. In the experimental results we didn't see clear signs of the fact that *topological traps* re-created in the laboratory by having cliques of players weakly connected to each other (see Fig. 10.1) do actually cause freezing of inefficient β coordination zones or, equivalently, prevent α to spread further past the trap. There can be several reasons for this. First of all, while the strategy revision rule for the artificial agents was noiseless and always the same for all players, human

assume arbitrary clustering within networks. While this assumption is realistic we do not have a sufficient number of sessions to apply this solution.

subjects make errors and may try to experiment to gain more knowledge about the neighbors' behavior. In particular, even a stable situation may become unstable if agents do not apply strict best response at the next time step. In fact, the update rule in Roca et al. [26] was payoff-based imitation dynamics, which cannot arise in any form here since the players are not informed about the payoffs of their neighbors. In addition, in the experiments strategy α is always predominant in the first time step. Whatever the psychological or strategic reasons for that, it makes it more difficult for strategy β to gain a stronghold in a clique. Taken together, these two reasons make stable dimorphic states more difficult to attain than in theory or simulations based on simple and invariant protocol revision rules.

Moreover, while we were able to study small human networks of twenty people because of financial and equipment limitation, reasons that are common in this kind of laboratory work, simulations can be done on much larger systems in which the number of possible dimorphic stable states is higher. Finally, because of the size limitations of our networks, the degree correlation between agents at distance two and three, to which Roca et al. attribute an important role, are not meaningful. In spite of this, dimorphic situations in which some cliques were playing β have been observed in the experiment. We identified such a state when at least one clique counted at least four α -strategists during the last five periods of a round and, at the same time, at least one clique counted at least four β -strategists during the last five periods of a round. Indeed, two out of the eight cliquish networks implemented ended up in a dimorphic stable state. In four of the cliquish networks all cliques became α -stable, while in the remaining two cliquish networks some cliques became α -stable while the others remained unstable. In terms of cliques, we find that 75 % of the cliques end up in a state where all players choose the α strategy in the final round, 12.5 % of the cliques have 4 out of 5 players choosing α , and the remaining cliques 12.5 % of the cliques were uniformly distributed between no-one choosing α and 3 out of 5 choosing α .

Interestingly, our results differ strongly from our own simulations that were based on the exact *same* network structures as in the laboratory and best response. The simulations predicted that from a 85% initial proportion of α -strategies, (i) most random networks end up in an all- β equilibrium while some of them end up in dimorphic states, and (ii) most cliquish networks end up in dimorphic states while a few of them end up in an all- α equilibrium. Figure 10.7 reports the difference in the proportion of players choosing α in the random network compared to the cliquish network assuming that the initial proportion of players choosing α equals 85 % (this is the average fraction we observe in the laboratory over the four treatments). The simulations that assume myopic best reply as an update rule predict that the random network will have about 31 % fewer players choosing α than the cliquish network from round 11 onwards. This prediction contrasts sharply with the main result from the laboratory experiment that the proportion of players choosing α does not differ by topology. Interestingly, we are able to reproduce this laboratory result once we run simulations that implement the empirical updating rule that players use in the laboratory experiment (we implement the updating rule reported in Figure 10.5, averaged over topologies). This result highlights the usefulness of laboratory experiments to update the common behaviors implemented in the automata used in numerical simulations. See also the work of Grujić et al. [31] that does another step in this direction.

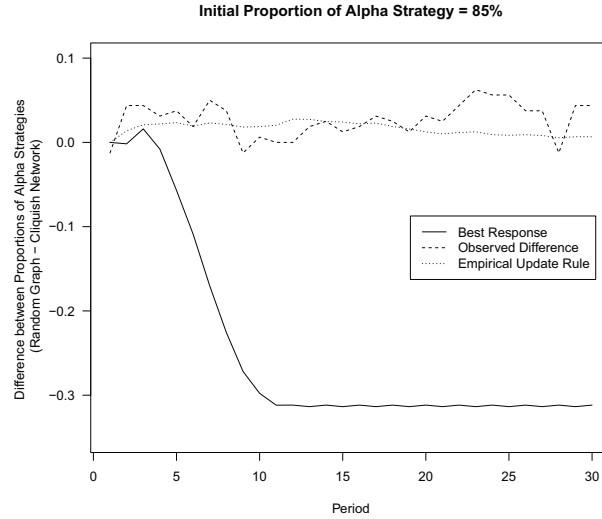


Fig. 10.7 Difference in proportion of players choosing α between random network and cliquish network. The figure reports simulations that use myopic best reply, the actual observed difference in the laboratory, and simulations that use the observed updating rule in Figure 10.5, averaged over topologies

10.4.2 Comparison with related experimental results

Here we compare our results with the conclusions of a recent experimental study by Cassar [12]. Cassar's work is the more related to ours since it is the only one we could find in the literature that explicitly tests for the influence of complex network structure on coordination games, with the exception of [28] which however deals with very small networks. She conducted a laboratory study in which eighteen subjects were virtually disposed as networks of three types: ring, small-world of the Watts-Strogatz class [32], and a random network, all with four neighbors per agent on average. The small-world networks in particular have a high clustering coefficient and a short path length. Aside from that, neither the ring nor the random or small-world networks featured cliques and weak links as in our settings (Fig.10.1b). Each run consisted of eighty periods on average and three realizations of each network class were used. Ten runs were monitored in total for the three network types. The information available to the subjects was similar to ours including the number of randomly assigned neighbors and the fact that they were to stay the same during a given run. Cassar studied both the Prisoner's Dilemma as well as the Stag Hunt games. Here we only comment about the Stag Hunt case. The payoff matrix for the coordination game in Cassar is equivalent to ours through an affine transformation. Most of Cassar's runs started with high initial rates of α strategies and ended in the all α state, with the small-world networks being apparently the more conducive to coordination on the Pareto-efficient outcome. The differences, however, are small and their statistical significance is doubtful (see also the discussion of Cassar's Prisoner Dilemma results in [29], where some doubt is cast on their statistical interpretation).

Our results also show a consistent preference for α initially as well as later in the runs. However, we do not find that network structure plays a role. In the Prisoner's Dilemma case, similar con-

clusions have been reached in a recent very large-scale experimental study on grids and scale-free networks by Gracia-Lázaro et al. [33] in which the authors conclude that the level of cooperation reached in both structures is the same. Likewise, Suri and Watts conclude that the network topology had no statistically significant effect on the average contribution in a public goods game [29].

Cassar also analyzed the individual player decision making and came to the conclusion that best reply and inertia are significant in explaining behavior. While we also find evidence of inertia, we reject that subjects in our experiment use best-reply as update rule.

10.5 Conclusions

We study the role of network topology for coordination decisions in a Stag Hunt game. Numerical simulations of the setting suggest that populations of 20 players will end up in a dimorphic state more often in the cliquish network than in the random network. Also, players choose the efficient strategy α more often in the random network. While we find that human subjects in a laboratory setting do converge more often to dimorphic states in cliquish networks than in random networks, there is no difference in terms of the proportion coordinating on the efficient outcome between the two topologies. Moreover, subjects do not use best-reply as update rule. Numerical simulations agree with laboratory results once we implement the actual updating rule that human subjects in our laboratory experiments use.

This evidence suggests that numerical simulations can be a useful tool to understanding coordination in small scale societies. However, they should incorporate more empirical knowledge on their strategy update functions, which are currently too simplistic. These methods can then be updated and improved, hopefully not only for small scale but also for large scale societies – settings where laboratory studies are hard and very costly to implement.

10.6 Supplementary Material

10.6.1 Instructions

(The following instructions were originally written in French.)

Explanation for this part of the experiment

Welcome to this experiment! You'll have to make decisions that will affect your income as well as the income of other participants. Although we express all income in terms of points, these points will be exchanged at the end of the experiment using the following exchange rate:

100 points = CHF 1.-

From this moment, it is **strictly forbidden to talk with other participants**. If you have any questions, please contact the assistants. If you do not follow this rule, we will have to exclude you from the experiment. In this study, each participant interacts with five neighbors (other participants in the room). Your neighbors are always the same for this part of the experiment. However, the other neighbors of your neighbors can be other participants with whom you have no interaction.

What is it about?

There will be 30 rounds. In each round you must choose a “strategy” between two options:

SQUARE and CIRCLE

Your neighbors will have the same choice to make as you. Your gain in each round will be based on the combination of your strategy with those of your five neighbors. We now explain the gains made by each combination of your strategy with the strategy of one of your neighbors:

- You choose SQUARE, your neighbor chooses SQUARE: you earn 5 points.
- You choose SQUARE, your neighbor chooses CIRCLE: you earn 1 point.
- You choose CIRCLE, your neighbor chooses SQUARE: you earn 4 points.
- You choose CIRCLE, your neighbor chooses CIRCLE: you earn 3 points.

Your final gain in each round is calculated as the **sum** of points earned in your interaction with each of your five neighbors. The following examples illustrate the computation of your gain in a round:

Example 1: Your strategy is CIRCLE, the strategy of all of your neighbors is also CIRCLE.

Your final gain: $3 + 3 + 3 + 3 + 3 = 15$ points.

Example 2: Your strategy is CIRCLE, the strategy of all of your neighbors is SQUARE.

Your final gain: $4 + 4 + 4 + 4 + 4 = 20$ points.

Example 3: Your strategy is SQUARE, the strategy of all of your neighbors is also SQUARE.

Your final gain: $5 + 5 + 5 + 5 + 5 = 25$ points.

Example 4: Your strategy is SQUARE, the strategy of all of your neighbors is CIRCLE.

Your final gain: $1 + 1 + 1 + 1 + 1 = 5$ points.

Example 5: Your strategy is SQUARE, the strategy of three of your neighbors is also SQUARE, while the strategy of your other two neighbors is CIRCLE.

Your final gain: $5 + 5 + 5 + 1 + 1 = 17$ points.

Example 6: Your strategy is CIRCLE, the strategy of four of your neighbors is SQUARE, while the strategy of your fifth neighbor is also CIRCLE.

Your final gain: $4 + 4 + 4 + 4 + 3 = 19$ points.

What will you do?

In each round, you must decide your strategy. You can do this using the following screen (Fig. 10.8):

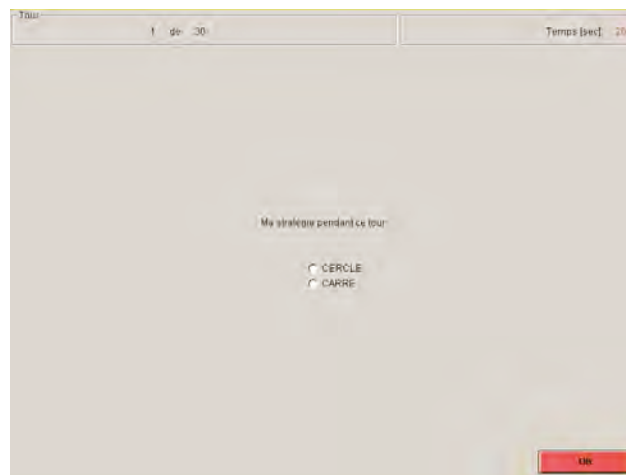


Fig. 10.8 (Text in Figure) My strategy during this round: CIRCLE, SQUARE.

Once you have made your choice, click the “OK” button at the bottom of the screen. At the end of each round, your gain will be shown as well as the information about the strategies adopted by your neighbors. This will be shown as in the following screen (Fig. 10.9):



Fig. 10.9 (Text in Figure) My strategy during this round: SQUARE. My gain in this round: 17. The number of neighbors that have chosen SQUARE: 3. The number of neighbors that have chosen CIRCLE: 2.

Did you understand?

Before starting the rounds that will account for your earnings in this experiment, we want to be sure that you and all other participants have understood the decisions to make. Please answer the four questions that appear on your screen. When you finish with a question, click on the “OK” button at the bottom of the screen.

(Then, all the participants must answer correctly to 4 trial open questions before the session begins. In each trial question they have to answer which is their final gain in one of the following situations: your strategy is (CIRCLE/SQUARE), (0/1/2/3/4/5) of your neighbors have adopted the strategy CIRCLE and (5/4/3/2/1/0) of your neighbors have adopted the strategy SQUARE. What is your final gain?)

Instructions (2nd, 3rd and 4th topology)

(The following instructions were originally written in French.)

Explanation for this part of the experiment

This part of the experiment is almost identical to the previous one. The only difference is that your five neighbors, and the neighbors of your neighbors, have changed. This means that you will now interact with five other participants in the room that are different from the ones you interacted with in the previous part. Everything else is exactly the same as in the previous part. You can thus see the instructions distributed at the beginning of this experiment.

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Chapter 11

Global Information and Mobility Support Coordination Among Humans

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Coordination Among Humans, Scientific Reports 4:*
6458.
24 September 2014.

Abstract Coordination among different options is key for a functioning and efficient society. However, often coordination failures arise, resulting in serious problems both at the individual and the societal level. An additional factor intervening in the coordination process is individual mobility, which takes place at all scales in our world, and whose effect on coordination is not well known. In this experimental work we study the behavior of people who play a pure coordination game in a spatial environment in which they can move around and when changing convention is costly. We find that each convention forms homogeneous clusters and is adopted by approximately half of the individuals. When we provide them with global information, i.e., the number of subjects currently adopting one of the convention, global consensus is reached in most, but not all, cases. Our results allow us to extract the heuristics used by the participants and to build a numerical simulations model that agrees very well with the experiments. Our findings have important implications for policymakers intending to promote specific, desired behaviors in a mobile population.

11.1 Introduction

Coordinating on a common task or adhering to a shared convention are extremely important behaviors in society. Coordinated behavior is fundamental in work sharing as it permits more efficient outcomes than miscoordination, as, e.g., in a production line in a factory. In shared conventions and norms, the rationale is that conforming to the behavior of the majority already following the norm confers more welfare than taking a different stance. Examples of the latter abound: driving on a given side of the road depending on the country, speaking the language of the majority, dressing according to generally accepted standards, and so on. How norms and conventions have evolved is a complex process involving many factors [23]. It is commonly believed that norms and conventions become established, and can change over time through a process of evolution and learning among the members of a population [21, 9]. But it may well be that a given norm is just imposed through authority and enforcement against non-conforming behavior. This was the case in Sweden, for example, where the whole country shifted from driving on the left side of the road to the right side on 3 September, 1967. Of course, to be successful, the move had been prepared much in advance.

Norm-following behavior can be described in game-theoretical terms through *pure coordination games*. In these games the idea is that if the players choose the same action, i.e. they coordinate, then they are at Nash equilibrium and consequently they have no incentive to unilaterally change their action since this would result in reduced payoff. For example, the following matrix 11.1 represents a two-player, two-strategy pure coordination game in which one may ideally identify each strategy as a “convention”.

$$\begin{array}{cc} & \begin{array}{cc} \alpha & \beta \end{array} \\ \begin{array}{c} \alpha \\ \beta \end{array} & \begin{pmatrix} a, a & 0, 0 \\ 0, 0 & b, b \end{pmatrix} \end{array} \quad (11.1)$$

There are two pure Nash equilibria in this game: (α, α) and (β, β) . The problem with this approach is that there is no way to choose between the equilibria in the sense that they are both equally valid from the game-theoretical point of view. In practical situations one equilibrium might be better or easier to achieve than the other. For instance, if a is larger than b in the above game, it is likely that players will tend to coordinate on the payoff-dominant norm α . But when payoffs are the same and there is no apparent reason to select a particular convention, the multiple equilibria problem and the ensuing indeterminacy led Schelling to the notion of a “focal point” or “saliency” [17] which is a piece of shared information such that people with similar background will be more likely to choose the coordinated solution that looks more “salient” to both of them. This and other similar concepts have been developed to try to restrict the possible solutions in these games.

In the present work, however, we take another point of view. We do accept that there may be multiple competing norms and that people have to weight the advantages and the drawbacks of sticking with a norm or switching to another one. This problem is crucially important in modern societies where, thanks to easy mobility, people of different backgrounds, nationality, and culture may meet in large cities or even in whole countries. Those conventions may represent languages, religious beliefs, dressing attitudes, preferred food, and many others. Some of the questions that one can ask in this context are the following: if at least two different conventions are present and actors can move around, which one, if any, will prevail in a population in the long run? Can different conventions coexist in a sufficiently stable manner? And what could be the main factors that influence the outcome? Since theory alone cannot provide all the answers to these questions, in order to shed some light on this socially important matter, researchers have tried to observe people behavior when confronted with that kind of choice. There have been a fair amount of experiments on pure coordination games in the past; most of them, however, have dealt with how people coordinate on equivalent conventions in the absence or presence of communication and extra-game knowledge [14, 13, 2, 3]. Here psychological and cultural considerations are fundamental in the selection of a “prominent” convention. Many more experiments have been performed on the “stag hunt” class of coordination games [19] in which equilibria have different properties, see e.g. [6, 12]. Experimental work on the stag hunt has also recently been done on networked populations, e.g. [20, 1]. A complete review of the experimental work in the field of coordination games up to 2002 can be found in Camerer’s book [4]. As previously said, our goal is related but different and so we designed a new type of laboratory experiment which is explained below.

The article is organized as follows. In the next section we describe the design of the experiment. Subsequently, we present and discuss the main experimental results. The following section presents a numerical simulation model based on the previous experimental observations and its

application to larger populations and longer time horizons. The paper concludes with a discussion of the key findings.

11.2 Experimental Design

The main distinguishing features of our experimental setting are that we have a spatial dimension and participants can move around. Twenty participants occupy the cells of a virtual two-dimensional 8×8 grid that wraps around itself by assuming cyclic boundary conditions. An experimental treatment consists of 30 rounds. Each participant can only see a local neighborhood composed by the eight cells around himself. At each round, people play with their current neighbors according to the following pure coordination game matrix:

$$\begin{array}{c} \alpha \qquad \beta \\ \alpha \left(\begin{array}{cc} 2, 2 & -1, -1 \\ -1, -1 & 2, 2 \end{array} \right) \\ \beta \end{array} \quad (11.2)$$

This differs from the pure coordination game of matrix 11.1: the Nash equilibria are always the diagonal pairs of strategies but miscoordination has been assigned a negative payoff. This stands for the inconvenience that may arise in many situations when interacting with the opposite convention. After having given the necessary information to the subjects (see SI sect. 1 for details), the experiment unfolds as follows. At the beginning, participants receive an initial endowment of 100 points and they are assigned to grid cells randomly such that no two players go into the same cell. Moreover, conventions α and β , being of equal value for partners of the same type, are also distributed at random among players such that there are ten participants for each convention. Before taking synchronous decisions, subjects see the following information on their screen: their current convention, their amount of cumulated points, and the position and convention of their current neighbors.

This is schematically shown in Fig. 11.1. For each player, a round of the treatment consists of deciding whether to keep the current convention or to switch to the alternative one, which can be done at a cost of 10 points. We decided to set a non-zero cost since changing conventions

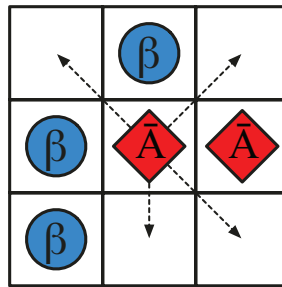


Fig. 11.1 Schematic view of the neighborhood seen by participants. In this case, the central player could choose to switch to action β by paying a given cost, and/or to migrate to one of the empty cells indicated by the arrows. To avoid collisions, when more than one player decides to move to a neighboring empty cell, a random player is chosen to move during that round.

always requires an effort; for instance, when traveling abroad, if one wants to interact with people often a new language has to be learned. Next, each player decides whether or not to freely move to a neighboring empty cell. To avoid multiple cell occupancy, if more than one player wants to move to the same cell only one is chosen at random. The setting just described refers to the “local information” treatment, simply called local treatment from now on. In another setting, called “global information” treatment, the conditions are exactly the same except that participants also receive the information about the number of α and β conventions present in the population at each round, without seeing their positions in space. Henceforth, this is named as the global treatment.

The local treatment is clearly related to the early cellular automata spatial segregation model of Schelling [18]. In the simplest form of this famous model, two kinds of agents live close to each other in a two-dimensional grid, but each prefers that a minimum fraction of his neighbors be of his own type. If this threshold, which is shared by all the agents, is not reached then the given agent will try to migrate to a free neighboring spot. Numerical simulations of the model show that when there is a sufficient number of “unsatisfied” agents the dynamics leads from mixing to segregated clusters of the two kinds of agents. While the diluted grid spatial setting is identical with the one used in our experiments, our agents are real, not automata. Indeed, Schelling’s automata all obey the same deterministic or probabilistic threshold rule, which is not the case, in principle, for human subjects. In addition, Schelling’s automata cannot change their action; in other words, behaviors are hard-wired. Thus, while Schelling’s model has had inspirational value for the present work, our experiments include payoff-based human decisions besides mobility.

On the other hand, with the global treatment setting we aim to represent the fact that a world-wide information, such as the percentage of English speakers in the country, is considered common knowledge among the population and it can thus be found without much effort. Indeed, thanks to information media and the Internet in particular, acquiring information about the behavior of people who live outside our local environment is becoming increasingly cheaper. Here, we are interested in investigating how this outside-world information may influence group dynamics and whether it can be helpful in reaching higher coordination levels. This setting is therefore realistic for a variety of social phenomena in which a global information is available.

11.3 Results

We begin the analysis of the data obtained through the experiments by describing first the results of the local treatment. In this setting, in all cases (9 out of 9 times) the system stabilized in a clustered form in which there were tight α and β clusters. These two groups were also spatially separated because of the negative payoff that occurs for interactions between opposite types. In other words, the population segregates according to the two conventions. This can be seen in Fig. 11.2 where we report the average size of the dominant convention over all the treatments (black symbols). Moreover, an example of a local treatment population observed at the end of a single run is shown in Fig. 11.3 (a). All other final population states can be found in the SI sect. 2.

The analogous results for the global treatment are also given in Fig. 11.2 (red symbols). It is immediately apparent that, on average, global information significantly influences the outcome causing the prevalence of one convention over the other in most cases. However, mean values hide the fact that in seven out of twelve runs all the players aggregated into a single convention

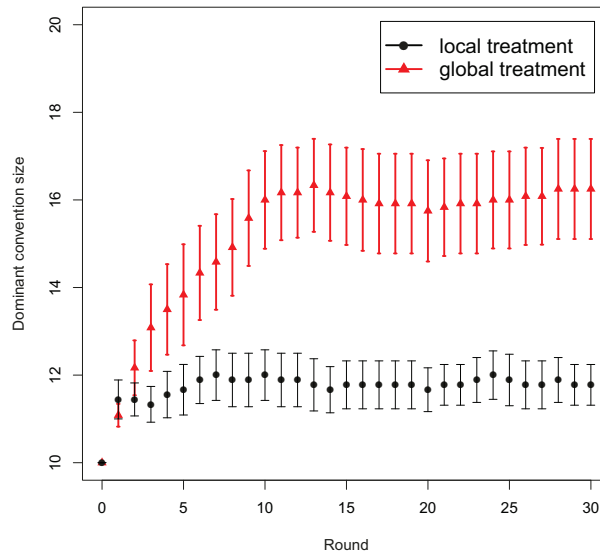


Fig. 11.2 Dominant convention size for local and global treatments. This measure represents the global coordination level achieved by participants. Both values are averages and are depicted with standard error bars. At the beginning, conventions are equally distributed among participants, but, as time goes by, they become unbalanced reaching higher coordination levels in the global treatment case. The difference between final values has statistical significance ($P < 0.01$).

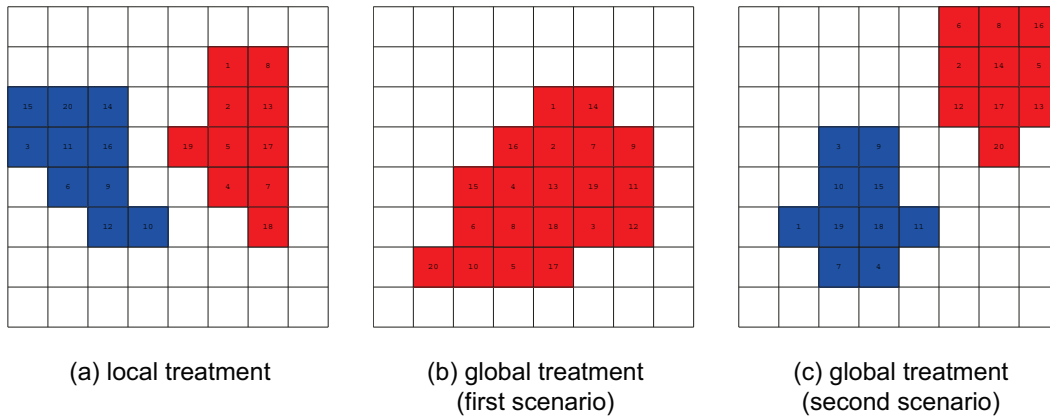


Fig. 11.3 Characteristic arrangement of participants conventions at the end of local and global treatments (see SI sect. 2 to find all other final configurations). The viewpoint of the toroidal space has been chosen so as to see the clusters in the middle of the grid. (a) In the local treatment participants always form (9 out of 9 times) two groups of similar size that adopt different conventions; clusters repel each other because of the negative payoff for the interaction between different conventions. (b) In the first scenario of the global treatment participants fully, or almost fully, converge (7 out of 12 times) to the same convention and they form a unique group maximizing their number of connections. (c) In the second scenario of the global treatment (5 out of 12 times), conventions balance remains equilibrate during the whole run and participants behavior looks analogous to the one seen in the local treatment case.

cluster, while in the other five cases we observed a behavior similar to the local treatment in which no convention really prevails. Figs. 11.3 (b) and (c) show two end of game snapshots illustrating these two cases.

We now turn to discussing the evolution of the number of neighbors adopting the same convention in local and global treatments. At the beginning of a run the expected value of this quantity is about one, since the initial disposition of the players in the grid is random and because of the equally distributed number of conventions among participants. However, as time goes by, players following the same convention tend to stick together and the value steadily increases for both treatments until it reaches a value around four. This value is almost equal to the number of connections participants have at the end of the treatment. This means that participants neighborhoods are mostly composed by players adopting the same convention. However, in the global treatment, participants are able to reach a slightly larger number of neighbors of the same convention because of the higher global coordination level achieved (see SI Fig. S5 for further details). The difference between final values of local and global treatments is statistically significant (aggregating per treatment, $P < 0.1$; considering single individuals per treatment, $P < 0.05$).

The following Fig. 11.4 depicts one interesting aspect of the dynamics; it reports the fraction of participants that changed convention at each round averaged over all the sessions for local (black symbols) and global (red symbols) treatments. People change action at low rate at the beginning but later on they tend to stick to their current convention. This is true for both local and global treatments and there is no statistically significant difference between the two. Figure 11.4 also

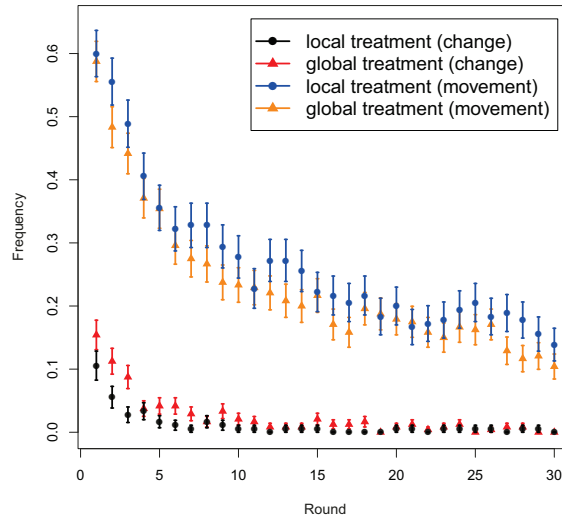


Fig. 11.4 Average frequency of movement and convention changing decisions per round. Values are depicted with standard error bars. Black and red values represent convention changing frequency for local and global treatments, respectively. Only a small percentage of people changed convention during the whole experiment and almost all of them did it during the initial rounds. Blue and orange values represent subjects mobility for local and global treatments, respectively. At the beginning mobility is high but it tends to decrease with time for both treatments. Mobility never attains zero because of the persistent presence of unsatisfied players who move around the border of their cluster. Differences are not statistically significant ($P > 0.35$).

reports the fraction of participants moving to another position for local (in blue) and global (in orange) treatments. Here we see that the fraction of participants moving to another position starts at about 0.6 and goes down with time, but never becomes zero. In other words, even at the end of the run some participants are still moving around. Overall, subjects prefer to move in order to meet other people adopting the same convention rather than changing convention since the latter is costly while moving to another cell is cost-free.

A key point in the experiment was to get an understanding of the reasons that motivate the participants' convention change and displacement decisions. In order to do so, we studied the frequency of these decisions as a function of the number of neighbors of the same or the opposite type they see in their neighborhood. These quantities, a priori, seemed to play an important role. Figure 11.5 (a) shows the average frequency of convention changes and Fig. 11.5 (b) is the average frequency of walking to a neighboring empty cell. From Fig. 11.5 (a) it appears that when participants see some players of the same type they are reluctant to switch to the other convention and this trend becomes stronger as the number of neighbors of the same type increases. Conversely, when they see at least two neighbors of the opposite type they have a tendency to pay the cost and switch to the other convention. The propensity to switch conventions in this case is higher in the global information treatment thanks to the reinforcement effect of the global signal.

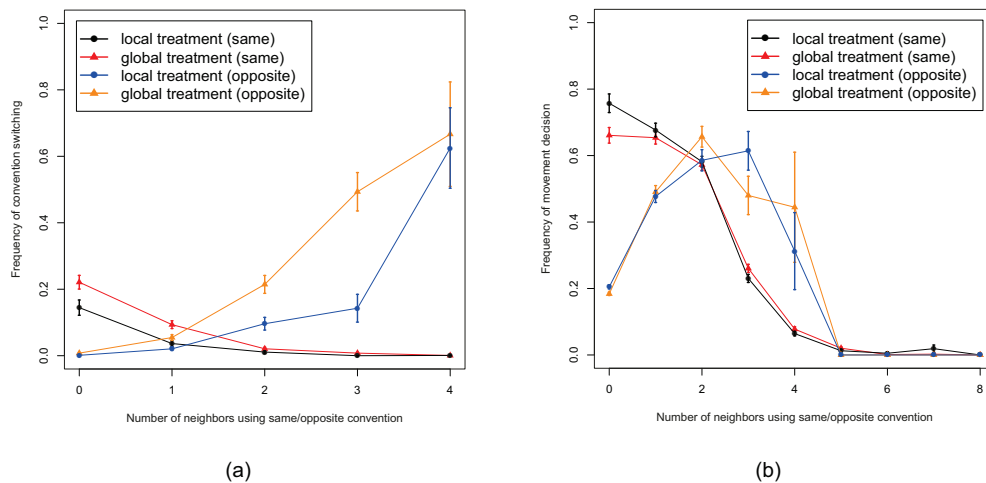


Fig. 11.5 Average frequency of convention change (a) and of displacement (b) as a function of the number of neighbors adopting the same or the opposite convention. Values are depicted with standard error bars. There are no relevant observations for values greater than four. Black and red values represent local and global treatments, respectively, as a function of the number of the same type of neighbors. Blue and orange values stand for local and global treatments, respectively, as a function of the number of opposite type neighbors. (a) People are reluctant to change convention when they have at least one neighbor of the same type. It is apparent that the larger the number of opposite type individuals in the neighborhood, the higher the frequency of convention change. (b) Movement decision frequencies tend to quickly decrease with the number of neighbors of the same type, i.e. players are satisfied with at least four neighbors of their type. On the other hand, opposite convention curves are not monotonic and have a maximum between two and three opposite type neighbors.

As for the frequency of movement (Fig. 11.5 (b)), when there are four or more neighbors of the same type, i.e. about 50%, participants are satisfied and have little incentive to move, a situation reminiscent of Schelling's cellular automaton model. However, when neighbors of the same type are not enough, people tend to move with positive probability. This unsatisfied behavior also justifies mobility in the last rounds (see Fig. 11.4). The mobility behavior as a function of the number of neighbors of the opposite type is different and more interesting. We see that mobility is maximal around a value of two or three different neighbors. With more neighbors adopting the opposite convention it appears from Fig. 11.5 (a) that participants are more likely to change convention rather than to move. Indeed, it seems rational to pay a price in order to benefit from a future gain in the expectation that the new configuration will remain stable at least for a couple of time steps. These results can be seen in a complementary way by plotting the empirical frequencies of convention change and of displacement as a function of the difference between the number of neighbors of the same type minus those of the opposite type (see SI Fig. S7).

From these results we conclude that unsatisfied participants prefer to move rather than to change convention, given that the latter entails an effort in real life represented as a cost in our experiment. When only local information is available, the society of players tends to split into two clusters, one for each convention. On the other hand, when global information on the fraction of strategies in the whole population is provided, the most likely result is the convergence towards a cluster of the same type, with α and β being equally likely by definition. However, even with global information, we sometimes observed the formation of two different type clusters having almost the same size. A study of the fine dynamics of the evolution shows that the main drive for the dynamics can be attributed to the combination of the numbers of subjects of the same and of the opposite type in the neighborhood of a given player.

11.4 Numerical Simulation Model

Starting from the pioneering work of Schelling [17], several theoretical models of convention evolution in spatial populations have been published. Among others, Mukherjee et al. [15] proposed a simulation model for the problem in which artificial agents use learning in a complete grid to choose a convention but without mobility. Dall'Asta et al. published a theoretical paper of the original Schelling model based on statistical physics techniques [7]. Collard and Mesmoudi [5] have proposed a model based on Schelling in which there is both heterogeneity of agents' preferences and learning. Finally, Zhang proposes a theoretical model of residential segregation evolution on a grid using the theory of stochastic games [24].

These theoretical and simulation models are interesting but would not be able to forecast or reproduce any particular set of data coming from a laboratory experiment such as ours because of their lack of diversity and heterogeneity of behavior. Instead, here we would like to follow another more empirical approach based on our experimental observations. The goal is not a general abstract model of how people behave in spatial convention evolution but rather, more modestly, to be able to numerically simulate larger societies of "average agents" that behave similarly to our experimental subjects. This seems to us a useful step because for technical and financial reasons larger populations for longer time horizons cannot be studied experimentally.

The important feature of the model is to state precisely how subjects decide to change convention and how they decide to move. From the observation of the participants' behavior, we make the hypothesis that the key factor for changing conventions and for moving is the number and type of individuals in the neighborhood around a given player. Thus, let us define a variable

$$\Delta = \#\{\text{same type neighbors}\} - \#\{\text{opposite type neighbors}\}$$

for the neighborhood of a given focal individual. We then define for the local information model the probability function $p_{c,L}(\Delta)$ of changing convention and the probability function $p_{m,L}(\Delta)$ of moving to a neighboring empty cell. Here c and m stand for convention change and movement, respectively, and L refers to the local model. Functions $p_{c,G}(\Delta, i)$ and $p_{m,G}(\Delta, i)$ are used for the global information model (G). The global information value $0 \leq i \leq 1$ is defined as the percentage of players in the population adopting the opposite convention. The form of $p_{c,L}(\Delta)$ used here is similar in spirit to the one proposed by Romero et al. [16] in the study of hashtag diffusion in Twitter.

From the experimental data points (see SI Fig. S8), we found $p_{c,L} = 0$, for $\Delta \geq 0$, and $p_{c,L} = 1$, for $\Delta \leq -5$. Then, the simplest assumption which qualitatively agrees with the data is to use the straight line $p_{c,L} = -\Delta/5$ in the range $-5 < \Delta < 0$. To estimate $p_{m,L}$ and to keep things as simple as possible, we fitted experimental data points in the range $-5 \leq \Delta \leq 5$ with two symmetric lines with a maximum of 0.75 for $\Delta = 0$. The corresponding equations are: $p_{m,L} = 0.75 + 0.15\Delta$, for $-5 \leq \Delta \leq 0$, and $p_{m,L} = 0.75 - 0.15\Delta$, for $0 < \Delta \leq 5$. $p_{m,L} = 0$ for all other values.

From the experimental data we observed that the empirical frequency of moving has the same shape for both the local and global information treatments (see SI Fig. S7). Therefore, we assumed $p_{m,L} = p_{m,G}$. On the other hand, experimental results clearly show that action changes are influenced by the availability of global information. We model this situation introducing eq. 11.3:

$$p_{c,G}(\Delta, i) = \begin{cases} 0 & \text{if } \lambda < 0 \\ \lambda & \text{if } 0 \leq \lambda \leq 1 \\ 1 & \text{otherwise,} \end{cases} \quad (11.3)$$

where $\lambda = (2i - 1) + 2(1 - i)p_{c,L}$.

We can see that when global information $i = 0.5$, i.e. when there is an equal number of the two conventions in the grid, i does not provide any positive or negative signal to the individuals, and thus equation 11.3 reduces to the local information model, $p_{c,G} = p_{c,L}$. Conversely, when $i \rightarrow 1$, it follows that $p_{c,G} \rightarrow 1$ since most, or all the other players follow the opposite convention. Finally, for $i \rightarrow 0$, the probability $p_{c,G}$ to change convention is positive only if $p_{c,L} > 0.5$ that is, only if the opposite convention is strongly dominant in its neighborhood (see SI Fig. S9 for further details).

Having defined how agents take their decisions, the population evolution is simulated by two stochastic processes at each time step t for each agent in the population: the update of agent's current action according to probability p_c , and the movement to a neighboring free position according to probability p_m . Since the population evolution is synchronous in time, each step occurs simultaneously as in the laboratory experiment. When an agent decides to move to a neighboring free position she does so in an opportunistic way, choosing the most profitable position among those available. In taking this decision the agent only sees the subset of cells that belong to her

original neighborhood; she has no information on the rest of her new neighborhood. When two or more agents try to move to the same empty cell, the collision is resolved randomly. If there are no empty cells around a given agent, the agent doesn't move.

To validate the model, we provide results for the experimental laboratory setting values in the SI sect. 5. But, as said above, one of the advantages of simulations is that they can be applied to larger systems and for longer time horizons, both of which are very useful to overcome the technical and financial limitations of experiments with human participants, once a suitable model is available. Besides, other values of the system parameters can be quickly tested, as we do below with the system density. The results are reported in Fig. 11.6. Figure 11.6 (a) shows the behavior of the predominant convention size for several grid sides L from $L = 4$ up to $L = 40$; the agent density is $1/3$. The curves depict this quantity at three different time steps in the evolution: $t = 30$, as in the experiment, $t = 300$, and $t = 3000$. Figures 11.6 (b) and (c) show the same results but for density $1/2$ and $2/3$ respectively. The important phenomenon to note here is that, in the local model and for all tried densities, convergence to a single convention only occurs for small grid sizes. When the populations evolve in larger grids clusters of individuals following different conventions form but do not merge and they remain segregated. This does not happen when global information is made available to the agents, in which case the tendency is toward convergence of the population to a single convention in all cases given enough time. In a different setting related results have recently been obtained by Gleeson et al. [10]. These authors study the likelihood of adopting applications in Facebook as a function of the number of applications installed in a given time window. They find that recent levels of application's installation activity are much more important than cumulative adoption levels, which means that local data in the temporal sense are more important than global ones, differently from what we found in our spatially extended system.

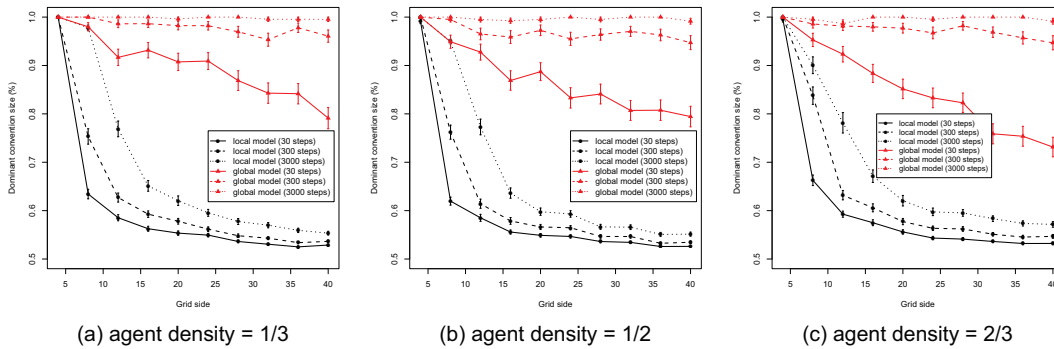


Fig. 11.6 Dominant convention size for local and global models at different density, grid side, and time horizon. The values are averages over 100 runs in each case and standard error bars are shown. Black curves correspond to the local model while red curves are for the global model. The population density, from left to right, is $1/3$ (a), $1/2$ (b), and $2/3$ (c). Numerical results for the experimental laboratory setting are reported in Fig. 11.2 (a) where agent density is $1/3$ and for a grid side of eight cells. Values at 30 time steps for local and global models qualitatively agree with those observed in the laboratory experiment (see final values of Fig. 11.2).

11.5 Discussion

Our main finding is that the type of information available to the population has a dramatic effect on the possibility of reaching general consensus. In the local treatment we have observed that the population segregates into two spatially isolated groups, one for each convention. We stress that we have never observed convergence to adopting a unique convention in this setup. On the contrary, when global information is provided, people can reach higher levels of global coordination, albeit we observe two different scenarios. In the first one, equilibrium between the two conventions remains stable during the whole experimental session and people cannot recognize a dominant convention in the population. This leads to a final population structure that is very similar to that observed in the local treatment. However, when equilibrium between the two conventions is not stable and an appreciable majority begins to form, the knowledge of the convention dominating the population allows people to reach a consensus and to aggregate in a unique monomorphic cluster.

Beyond our findings in terms of consensus, we have been able to extract the behavioral pattern of the average individual. Thus, we have observed that, given that changing conventions is costly, subjects accordingly try to move when they are in a neighborhood with few occupied cells and those occupied adhere more to the opposite convention, and only change convention when they are in the presence of several neighbors, almost of all them agreeing. Based on these simple and quite natural rules, we have proposed a model that allows us to study the dependence of our results on variables such as 'world' size, population density, and longer duration of interactions. The model, which is in good qualitative agreement with our experimental results, shows that in very small environments with only local information available, consensus can be reached as with global information, but after much longer interaction times. In larger systems, however, when global information is provided to individuals they can always converge to a unique convention in the population (given enough time), but local information alone leads to smaller coordination levels and spatial segregation. These results are qualitatively the same for a range of population densities, in so far as they are not so small as to prevent interaction, or so large that mobility is seriously hindered.

This experimental work has important implications in a number of real life contexts. Thus, our results suggest that when changing a convention is costly and interacting with the other convention detrimental, like in situations where a sizable group of people who should work together use different standards, or in collaborations where a common language is needed, population mobility is not enough to induce a common choice. Indeed, mobility would only lead to segregation by conventions, and when the segregation process is completed interaction ceases. On the basis of our results, one could think that providing information on how the adoption of conventions is going could help, but as we have seen something else is required, namely that this information conveys the feeling that an overwhelming majority is forming and that it is for the best to conform to their convention. While the adoption rates are still close, the process could still end up in segregation. Therefore, providing such global information as a way of policy making to encourage consensus is not enough, and should be complemented with incentives for part of the population to change their choice, so a majority begins to form. Even then, we have also seen that there are individuals who never change even when they are the last ones left of their choice. Whether this was due to a misunderstanding of the instruction or to unreasonable stubbornness is debatable, but in any event it is a clear indication that one cannot expect in general to achieve perfect consensus. In fact, in a

large, mobile population such subjects may end up grouping and becoming a prejudicial influence. On the other hand, the fact that people are reluctant to change conventions is in good agreement with the daily life observation of the coexistence of choices (such as different phone companies, or operating systems, or even languages). When individuals are locally satisfied because they have many neighbors with the same preference, they do not have incentives to switch to the other option or to move. Of course, in the long run the rational preference should be to go along with the majority, but as our simulations show, this may be a very long term limit (not dissimilar to the ultra long runs discussed by Young [22]), and therefore in practice those convention may coexist forever. Thus, it becomes quite clear that the process of convergence to consensus may certainly be complicated and, while our experiment opens the door to understanding how mobile populations evolve or not towards global agreement, further research is needed to fully unveil all the effects relevant for such an ubiquitous problem.

11.6 Methods

The use of human subjects in this experiment has been approved by the Ethics Committee of the University of Lausanne. The participants were fully informed of the nature of the experiment and signed an informed consent to participate. Their anonymity has been guaranteed at all stages of the experiment.

We conducted a total of seven experimental sessions in October 2013 in a specially equipped laboratory using the z-Tree environment [8]. Each session involved 20 participants and a total of 140 subjects took part in the experiment. Each session consisted of three repetitions of the same treatment played by the same group but with new random initial conditions. Participants were recruited using ORSEE [11] from a subject pool that includes students from several faculties of the University of Lausanne and of the Swiss Federal Institute of Technology (EPFL).

Students read a detailed description of the experiment before playing the game. After reading the instructions, subjects had to respond to a set of control questions that insured common understanding of the game and the computation of payoffs. An English translation of the instructions distributed to subjects is provided (see SI sect. 1). Each session lasted about 90 minutes. Participants earned a certain number of points during the experiment and their final score in points was converted at an exchange rate of 1.- CHF = 30 points. The average gain per student was 33.5 CHF (about 38 USD). All statistical results have been obtained performing *t*-test analysis and assuming an independent observation for each new treatment. Observations are also assumed to be unpaired samples and with unequal variance.

Acknowledgments. We gratefully acknowledge financial support by the Swiss National Science Foundation (under grant n. 200020-143224) and by the Rectors' Conference of the Swiss Universities (under grant n. 26058983). This work has been supported in part by Ministerio de Ciencia e Innovación (Spain) through grant PRODIEVO.

11.7 Supplementary Information

The supplementary information (SI) provides details on the exact instructions form that participants received (section 11.7.1), an overview of the final states for each treatment and group (section 11.7.2), complementary results of experimental data not shown in the main text (section 11.7.3), complementary information of the numerical simulation model (section 11.7.4), and simulation results of the experimental setting model (section 11.7.5).

11.7.1 Instructions form

Each participant read the following set of instructions in detail before the experiment started. Here we present the instructions form for the global treatment case. The only difference between the two treatments is that in the local treatment case participants never know the current number of conventions present in the population at each round.

(The following instructions were originally written in French.)

Explanation for the first part of the experiment

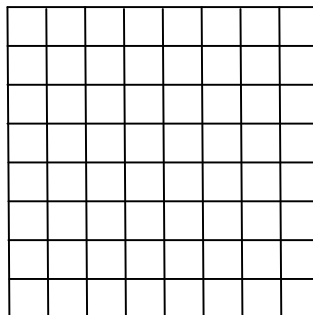
Welcome to this experiment!

You will have to make decisions that will affect your income as well as the income of other participants. Although we express all income in terms of points, these points will be exchanged at the end of the experiment using the following exchange rate:

30 pts. = 1.- CHF

From now on, it is **strictly forbidden to talk with other participants**. If you have any questions, please contact the assistants. If you do not follow this rule, we will have to exclude you from the experiment.

In this study, each one of the 20 participants is placed on a cell of a chessboard, i.e. a 8×8 grid (see below).



Each cell cannot be occupied by more than one participant. Consequently, $\frac{64 \text{ (total number of cells)}}{20 \text{ (participants)}} = 44$ cells will always be empty.

During each round of this experiment, you interact with participants placed in a cell of your neighborhood. Your neighborhood is defined as the 8 cells of the chessboard that are around your current position. You can find a representation of your neighborhood here below: your position is in the middle (X) and 8 cells (x) are around you.

x	x	x
x	X	x
x	x	x

Border conditions: if your cell is one at the border of the grid or one of the four corners, your neighborhood always contains 8 cells since the grid is virtually wrapped around itself as in the figures here below.

			x	x	x		
			x	x	x		
			x	X	x		

x						x	X
x						x	x
x						x	x

However, during the whole experiment, you will only see your neighborhood and you will not be able to see what happens in the rest of the chessboard.

What is it about?

There will be 30 rounds. In the first round, a profile between options **A** and **B** will be randomly assigned to you. At the beginning, in the whole chessboard there will be 10 participants with the **A** profile and 10 participants with the **B** profile but, as you will see below, this may change during the experiment.

Your gain for each round depends on the combination of your profile and those of your neighbors. More precisely, you get **2 points** for each of your neighbors with the same profile as yours. On the other hand, you will loose **1 point** for each of your neighbors with a profile different than yours.

The following examples illustrate your payoff computation at the end of a round. Please note that your profile is shown in the central cell with a capital letter, while profiles of your neighbors are represented in lowercase letters. In the following examples your profile is **A**.

Example 1: Your profile is **A** and you are in the following situation:

a	a	a
a	A	a
a	a	a

Your gain for this round is: $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$ points.
Note that this is the best situation you can achieve.

Example 2: Your profile is **A** and you are in the following situation:

b	b	b
b	A	b
b	b	b

Your gain for this round is: $-1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 = -8$ points.
Note that this is the worst situation you can find.

Example 3: Your profile is **A** and you are in the following situation:

	a	b
a	A	b
a		

Your gain for this round is: $2 + 2 + 2 - 1 - 1 = 4$ points.

Example 4: Your profile is **A** and you are in the following situation:

	A	

Your gain for this round is: 0 points.

What will you do ?

At the beginning of this experiment you will receive an initial endowment of **100 points**. These points can be used as follows. At each round, you take two decisions:

Profile switching

You may decide to change your profile switching to the other profile paying a cost of 10 points. If you don't have a sufficient number of point you won't be able to change your profile.

Movement

You may decide to move yourself to an empty cell in your neighborhood, if any. If you try to move to a cell which is already occupied by another participant, your displacement won't be accepted. Moreover, if more than one participant want to move into the same cell, only one participant will be chosen while the others won't move for that round.

You will take these decisions using the following screen (Fig. 11.7):

The screenshot shows a web-based interface for an experiment. At the top, it indicates 'Période 1 de 1' and 'Temps restant (sec): 10'. The main area is divided into three sections:

- VOTRE VOISINAGE:** A 3x3 grid representing a neighborhood. The center cell contains 'B'. The top-right cell contains 'a', and the bottom-right cell contains 'b'.
- Left Panel:** Displays 'Votre profil actuel: B'. Below it, 'Participants ayant le profil A: 10' and 'Participants ayant le profil B: 10'. At the bottom, 'Votre gain cumulé: 100'.
- Right Panel:** Contains two decision sections. The first asks 'Souhaitez-vous changer votre profil?' with radio buttons for 'oui' and 'non'. The second asks 'Où souhaitez-vous aller?' with a 3x3 grid of directional buttons: NO, N, NE, O, C, E, SO, S, SE.

Fig. 11.7 Your neighborhood is shown at the top of the screen, while your current profile and your cumulated payoff are illustrated on the left of the screen. The number of participant having an **A** (or **B**) profile is shown and updated at each round. On the right side you can choose to change your profile and if you want to move during this round. Note that if you want to keep your position for the current round you may click on the "C" button.

At the end of each round, you will see your final profit for that round and you will be informed about your new neighborhood. This information will be shown you as follows (Fig. 11.8):

The screenshot shows the end of a round in the experiment. At the top, it indicates 'Période 1 de 1' and 'Temps restant (sec): 7'. The main content area displays a 3x3 grid titled 'VOTRE NOUVEAU VOISINAGE'. The grid contains the letter 'B' in the center and the letter 'b' in the middle-right cell. Below the grid, a message states 'Vous avez gardé votre position'. A summary box at the bottom provides the following information:

Votre profil actuel:	B
Vous vous êtes déplacé:	non
Votre gain dans cette période:	2
Participants ayant le profil A:	9
Participants ayant le profil B:	11
Votre gain cumulé:	102

A 'Continuer' button is located at the bottom right of the screen.

Fig. 11.8 Your new neighborhood is shown at the top of the screen. Your current profile, a message for your eventual displacement, your profit for the current round, the total number of A and B in the whole chessboard, and your cumulated payoff are shown at the bottom of the screen.

Did you correctly understand the instructions?

Before starting the experiment, we would like to be sure that you and the other participants have correctly understood the decisions that you are going to make. To this end, please answer the questions that will appear on the screen.

(Then, all the participants must answer correctly to 7 trial questions before the first treatment begins. After have played for 30 rounds, participants receive another instructions form (here below) before playing for another treatment. Each group of 20 participants play for a total of 3 treatments.)

Explanation for the second/third part of the experiment

This part of the experiment is almost the same of the previous one. At the beginning, your profile and your position on the chessboard will be randomly initialized as before. The only difference is that at the beginning your position and your profile may have changed. You will receive another initial endowment of **100 points**.

11.7.2 Final states

This section presents all 21 final states we obtained. We have run 9 treatments for the local information setting, while 12 treatments have been performed for the global information setting. Each group of 20 participants played the same setting, local or global, for three times.

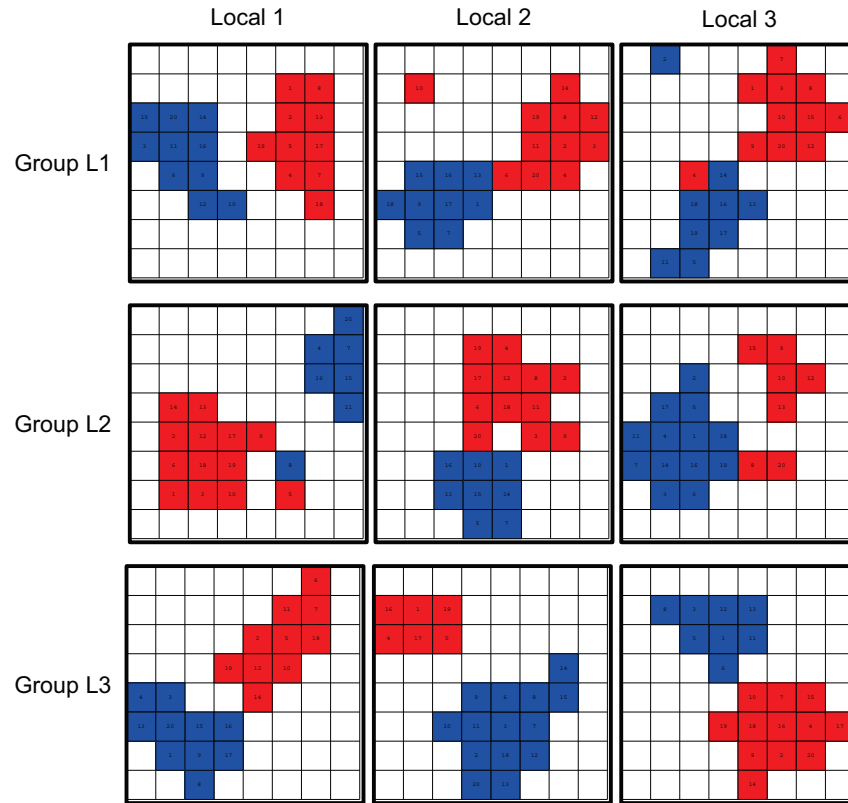


Fig. 11.9 Final states for the local information treatments. Red stands for the A profile and blue for the B profile. Full convergence to one monomorphic cluster was never reached. The maximal difference (6A–14B) has been obtained by Group L3 during the second treatment (Local 2).

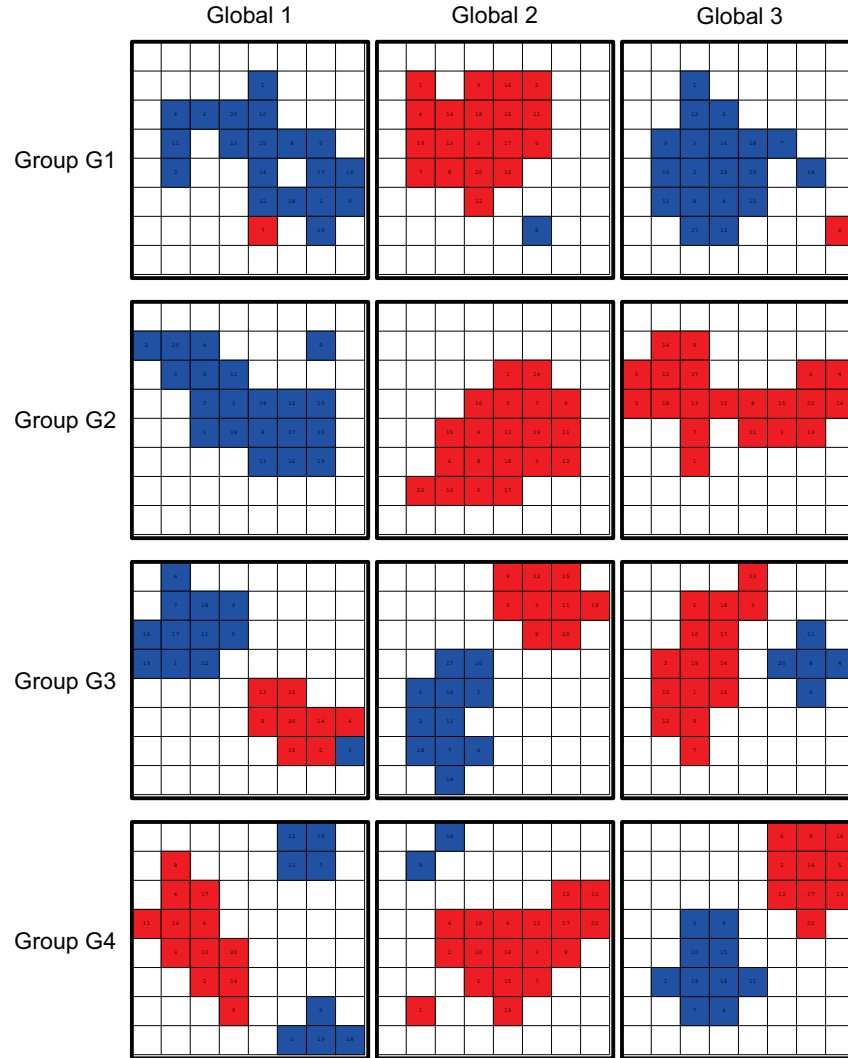


Fig. 11.10 Final states for the local information treatments. Red stands for the **A** profile and blue for the **B** profile. Full (or almost full) convergence to one monomorphic cluster was reached for all treatments of Group G1 and Group G2. Group3 obtained a maximal difference of 15A–5B during the third treatment and never played the same convention. Group 4 obtained almost full convergence at the end of the second treatment (Global 2), while they maintained a stable equilibrium of conventions during other treatments (12A–8B, 10A–10B).

11.7.3 Complementary results

Here we show the complementary results of the experimental study.

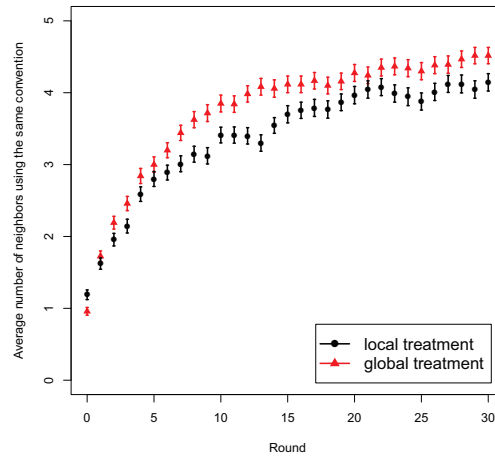


Fig. 11.11 Average number of neighbors adhering to the same convention per round for local and global treatment. Both values are depicted with standard error bars. Participants tend to aggregate and to form clusters of players adopting the same convention. In the global treatment case this phenomenon is enhanced because of the higher global coordination level achieved. The difference between final values of local and global treatments has statistical significance (aggregating per treatment, $P < 0.1$; considering single individuals per treatment, $P < 0.05$).

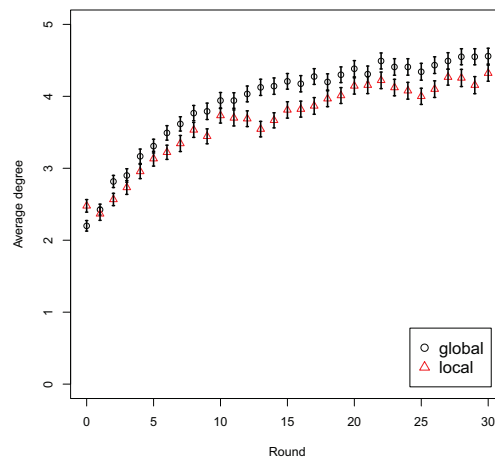


Fig. 11.12 Average number of neighbors per round for local and global treatment. Both values are depicted with standard error bars. Participants tend to aggregate and to form clusters of players. In the global treatment case this phenomenon is enhanced because of the higher global coordination level achieved.

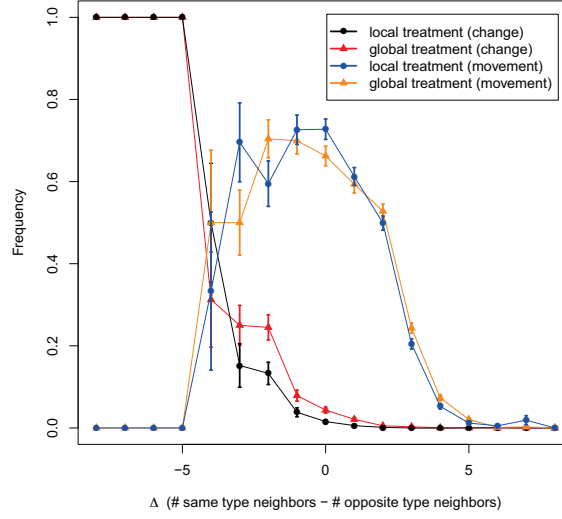


Fig. 11.13 Average frequency of convention change and of displacement as a function of Δ (the number of same type neighbors minus the number of opposite type neighbors). Values are depicted with standard error bars.

11.7.4 Numerical simulation model

Here we show the complementary information for the numerical simulation model.

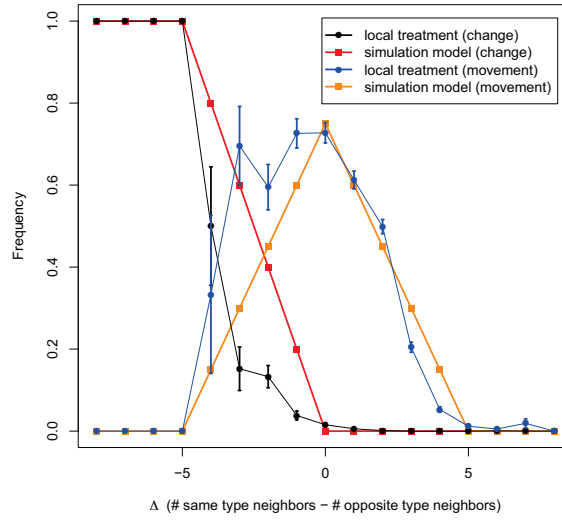


Fig. 11.14 Here there are depicted two probability functions $p_{c,L}$ (red line) and $p_{m,L}$ (orange line) compared to experimental results in local treatment (black line for frequency of convention change and blue line for frequency of movement decision). Variable Δ is defined as the difference between the number of same type neighbors and the number of opposite type neighbors in the neighborhood of a given player. Moreover, we assumed that $p_{m,L} = p_{m,G}$.

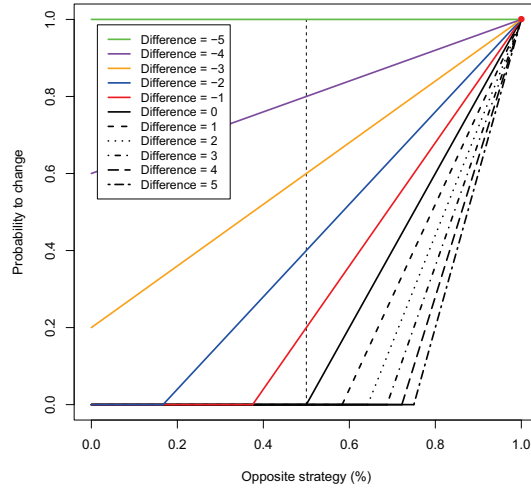


Fig. 11.15 Plot of probability function $p_{c,G}$ as a function of Δ (here, Difference) and the global information value i (here, opposite strategy (%)). We can note that for high value of i all probabilities of changing convention converge to one because it is present a dominant majority of opposite type players in the population. On the other hand, when this information value is low, the local environment, represented as the quantity Δ , influences more the convention switching decision.

11.7.5 Simulation of the experimental setting

Here we show the results of applying the above model to a grid of the same size and density as that of the actual experiment. It is clear that the results are expected to be somewhat self-referential since to set the model parameters we used the experimental data. Nevertheless, in the simulation the agents are homogeneous in the sense that they move and change convention with the same rules. Figure 11.16 shows the evolution of the mean number of neighbors in the experiment compared to the numerical simulation results for local (a) and global (b) models. Although the correspondence is not quantitative, the trends are certainly correct in both cases.

Figure 11.17 refers to the evolution of the dominant convention size in the local (a) and global (b) cases. Here too, the results of the experiments are reproduced by the artificial agents, at least in a qualitative way. In conclusion, it seems to us that this simple model adequately replicates the qualitative behavior observed in the real experiments and can thus be considered a good starting point for larger system simulations.

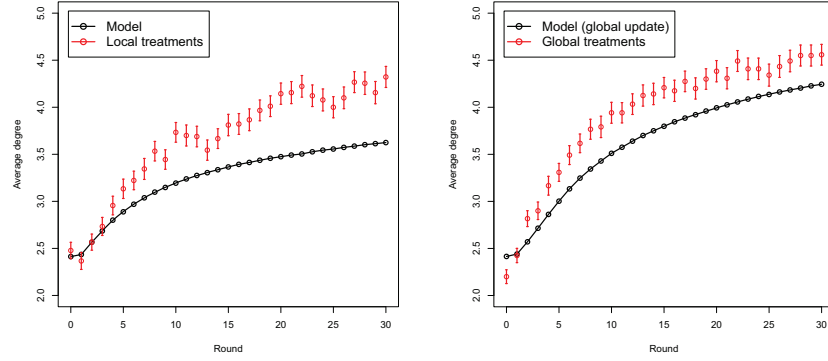


Fig. 11.16 Average degree for local and global treatments compared to numerical simulation model results.

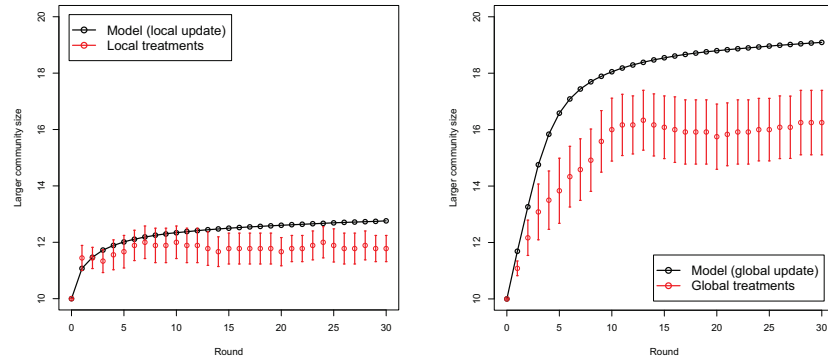


Fig. 11.17 Dominant convention size for local and global treatments compared to numerical simulation model results.

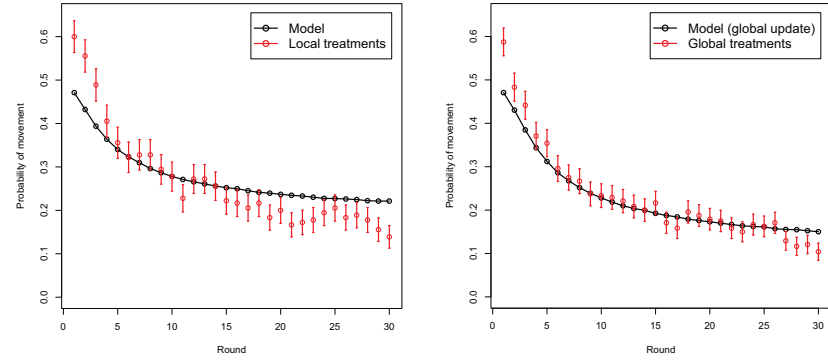


Fig. 11.18 Frequency of movement decision for local and global treatments compared to numerical simulation model results.

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Chapter 12

Short-Range Mobility and the Evolution of Cooperation: An Experimental Study

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Short-Range Mobility and the Evolution of
Cooperation: An Experimental Study, submitted.

Abstract A pressing issue in biology and social sciences is to explain how cooperation emerges in a population of self-interested individuals. Theoretical models suggest that one such explanation may involve the possibility of changing one's neighborhood by removing and creating connections to others, but this hypothesis has problems when random motion is considered and lacks experimental support. To address this, we have carried out experiments on diluted grids with human subjects playing a Prisoner's Dilemma. In contrast to previous results on purposeful rewiring in relational networks, we have found no noticeable effect of mobility in space on the level of cooperation. Clusters of cooperators form momentarily but in a few rounds they dissolve as cooperators at the boundaries stop tolerating being cheated upon. Our results highlight the difficulties that mobile agents have to establish a cooperative environment in a spatial setting.

12.1 Introduction

Cooperation is a desirable behavior that is fundamental for the harmonious development of society. However, cooperation may easily fall prey to exploitation by selfish individuals who only care about short-term gain. For cooperation to evolve, specific conditions and mechanisms are required, such as kinship, direct and indirect reciprocity through repeated interactions, or external interventions such as punishment. Reputation in the case of repeated interactions [32] and assortment mechanisms that favor cooperator-cooperator interactions [14] are the key, as first shown by W.D. Hamilton [23] in the case of genetic relatedness. In principle, positive assortment among cooperators might also result when agents interact through network relationships. The reviews [10, 29] aptly summarize the vast amount of work that has been accomplished in the last two decades on the study of such cooperation-enhancement mechanism. Theory and numerical simulations suggest that network reciprocity can explain the evolution of cooperation in a population of self-regarding agents under certain circumstances (see, e.g., [40, 44, 37, 33]). But what can be said about real people? Recent research tested these predictions by means of targeted experiments with humans in the laboratory, in which the subjects were connected in specific network structures, including large-scale ones [46, 20, 18, 43]. Surprisingly, these studies found that neither homogeneous nor heterogeneous network structures promote cooperation to a significant extent [46, 20, 22, 21, 17, 18].

However, when people are allowed to change their neighborhood by deleting unsatisfying relationships and trying to form better ones, theoretical and numerical models agree in concluding that cooperation may evolve (see, e.g., [42, 50, 39, 34] among others). Remarkably, and in contrast to the static case, empirical laboratory tests of dynamic settings performed in the last few years did confirm experimentally that fluid networks allow cooperation to evolve [35, 48]. In other words, unless there is uncertainty about the behavior of neighbors that is costly to resolve [2], dismissing relationships towards exploiting defectors as a form of direct punishment usually allows cooperation to prevail, even when rewiring an undesired link requires paying a cost [6]. This is certainly an encouraging result and the laboratory settings used, except for their reduced size, do match to a significant extent in spirit present-day internet-mediated relational social networks in which links are essentially independent of location in physical space.

Nevertheless, there also exist many situations in which geographical space does play a fundamental role in biological and ecological systems, as well as human societies. When agents find themselves in geographical space they usually can move around. Many examples can be found in mobility patterns of human populations, e.g. [8], in engineered systems such as ad hoc networks of mobile communicating devices (see e.g. [28, 7]), and mobile robot teams [16, 15] among others. In the case of agents interacting in physical space through game-theoretical principles, mobility might turn out to have an important effect for cooperation but, surprisingly, there has been comparatively little research in evolutionary game theory on this subject. Early on fixed regular lattices were used to represent spatial interactions in populations in a simplified manner in evolutionary games by Axelrod [5] and by Nowak and May [30]. They showed that, even when the game is one-shot, i.e. pairs of players interact anonymously, cooperation can evolve thanks to positive assortment among cooperators. A summary of this and other early work on grids is provided in [31]. However, more recent extensive numerical simulation work showed that the gains in the paradigmatic Prisoner's Dilemma game depend on the players' strategy update rule used [37]. Mobility of individuals can readily be introduced in these models through the use of a *diluted* grid, i.e. a lattice only partially filled by players such that agents can change location by moving to an empty cell in the grid. Mobility may have positive or negative effects on cooperation, depending on several factors. An early study was carried out by Enquist and Leimar [11] whose conclusion was that mobility may seriously restrict the evolution of cooperation as a result of randomization and invasion by defectors. In the last decade there have been several new studies of the influence of mobility on the behavior of various games in spatial environments, either in the case in which agents essentially perform random walks, or when they move according to heuristic strategy-driven considerations. Random diffusion of agents playing games has been studied in two-dimensional diluted grids [47, 41], as well as in continuous space [27, 4]. Random diffusion has been thought to hinder cooperation by randomizing the population and reducing the possibility of cooperator cluster formation. However, the works [47, 41, 4] show that cooperation can be maintained with respect to the static case and sometimes even enhanced for low mobility, depending on the strategy update rule used by the agents. When agents are allowed to move according to some simple heuristic rule instead of randomly, it has been shown that the population may evolve a certain degree of self-organization [24, 26, 1, 38, 9, 25, 45]. Clearly, intermediate situations also exist whereby the movement is partly random and partly contingent on some criterion.

A general conclusion about all the above works is that it is far from clear whether cooperation may be helped or hindered by random mobility as the result depends on many factors. Only purposeful contingent movements seem to be able to lead to highly cooperating popula-

tion states [24, 26, 1, 38, 9, 25, 45]. In view of the practical importance of individuals' mobility in geographical space it is clear that, like the case of static and dynamic networks, some light could be shed on the issue by experimental work with human subjects. Indeed, to our knowledge, no laboratory experiment of this kind has been published to date. In the rest of the paper we discuss our experimental setting with its limitations and constraints, the results we obtain, and some possible interpretations of the observed players' behavior.

12.2 Experimental Design

Our experimental setup is based on the customary Prisoner's Dilemma (PD) game [36, 5]. In this two-person game players must decide whether to cooperate or to defect. If both cooperate, each gets a payoff R . If one defects and the other cooperates, the defector gets T and the cooperator receives the payoff S . If both defect, each gets P . Since $T > R > P > S$, defection is a dominating strategy and a rational payoff-maximizing player will choose to defect, although mutual cooperation yields a higher collective payoff, whence the dilemma. Evolutionary reasoning leads to the same result, as defectors will reproduce at a higher rate due to their superior payoff [49]. This simple game is a good metaphor for the tension that exists between socially desirable outcomes and self-interested individual actions. In our experiment, subjects played a series of two-person Prisoner's Dilemma games with their immediate neighbors in the network. We took $T = 8$, $R = 6$, $P = 2$, and $S = 0$.

Our experimental setting is new in that we introduce a virtual spatial dimension represented by a square grid that wraps around itself into a torus, as in the experiments reported in [46, 20], but we also allow for player movements, as explained below, like in the coordination experiment reported in [3].

After having given the necessary information to the subjects (see SI sect. 1 for details), the experiment unfolds as follows. At the beginning, each of the twenty participants occupies a randomly assigned cell in an 8×8 grid giving a partially filled grid with a density of players $\rho = 20/64 = 0.3125$, i.e. about $1/3$. Participants must then choose a cooperate or defect strategy. These are called simply A and B in the experiment so as not to create a psychological bias (the A and B meaning is switched for different groups).

Before deciding on the next actions to perform, subjects see the following information on their screen: their current strategy, their cumulated gain in conventional points (which is translated into actual monetary payment at the end of the experiment), and the position and strategy of their current neighbors in the 8-cell neighborhood as schematically shown in Fig. 12.1a. As in most previous works, each player uses the same strategy against all her neighbors. Using different strategies against different neighbors (see e.g. [12]) might be more adequate if the neighbors were distinguishable, but not in our setting because players do not have labels to tell them apart and, owing to mobility, they can leave their positions and be replaced by other anonymous players.

For each player, a round of the treatment consists first in deciding whether to keep the current strategy or to switch to the other one. Next, each player decides whether or not to move to a neighboring empty cell, if any. To avoid multiple cell occupancy, if more than one player wants to move to the same cell only one is chosen at random. An illustration of a possible move decision by the central C player is given in Fig. 12.1b. It is clear that, for human players, the decision of

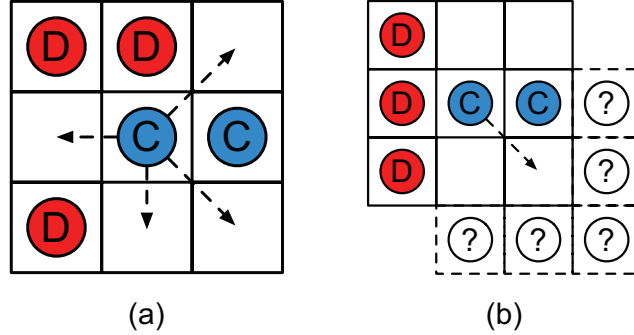


Fig. 12.1 (a) An illustration of the neighborhood seen by participants (they don't see the arrows). In this case, the central player could choose to switch to defection, and/or to migrate to one of the empty cells indicated by the arrows. (b) When it comes to decide whether to move or not, if the central player moves to the cell indicated by the arrow, it will keep the other cooperator as a neighbor while the other positions may contain either players with an unknown strategy, or are empty.

the cell to move to might be made according to some heuristic rather than fully randomly. Finding those heuristics and comparing them with theoretical models would be one relevant contribution of our experimental approach. Furthermore, the decision will be done under some uncertainty represented by the “?” symbols in the figure, which stand for a possible new neighbor with an unknown strategy or an empty cell. At the end of this process, participants accumulate their payoffs obtained by playing the PD in pairs against all their non-empty neighbors. These decisions are taken synchronously by all participants. After all players have completed these steps, another round begins.

Our spatial simulated setting is a necessary but, in our opinion, acceptable compromise between the obvious technical limitations of the laboratory and the much more general numerical simulation models that have been proposed in the literature [47, 41, 24, 1, 38, 9, 25, 45]. Participants in the laboratory setting can only perform jumps to neighboring empty cells like in [47, 41, 26]. Given the number of available cells (64), long jumps like those of [25, 45] are out of the question, given the available laboratory equipment, for both technical and financial reasons because too many participants would be required.

In the following sections we first present our experimental results and then we discuss the global statistical behavior of the participants. Starting from these results we then try to uncover the rules of the participants' decision making.

12.3 Results

We now turn to the discussion of the main experimental results. The first and most important observation is the global amount of cooperative acts in the population averaged over all treatments. This is shown in Fig. 12.2a where it is seen that cooperation could never increase past the initial fraction of about 0.4 and stabilized itself around 0.2. Although it is well-known that, contrary to theoretical predictions, cooperation almost never goes to zero in experimental work on the PD, it

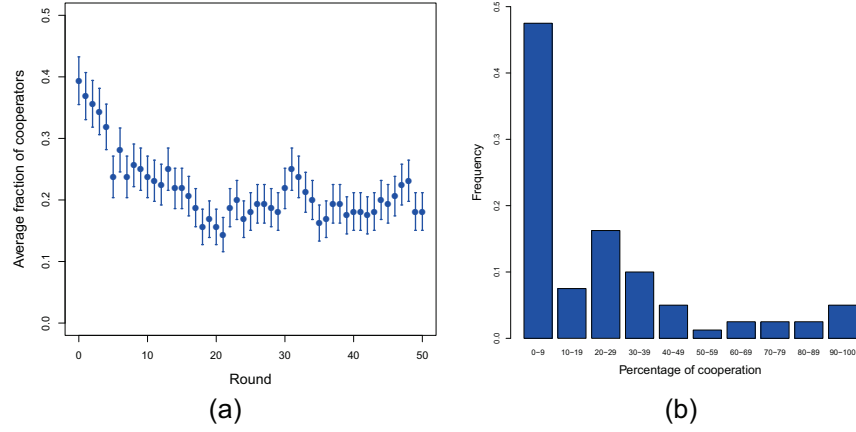


Fig. 12.2 (a) Average fraction of cooperators as a function of time for all treatments. (b) Overall participants' frequency of cooperation in deciles cumulated for all treatments. Most participants belong to the 0-10 decile and thus cooperate very little. In the low-middle range (10-50) we find participants who behave as “moody conditional cooperators” (see text).

is clear that migration does not help cooperation to evolve. Indeed, after a transient period of a few rounds, typical cooperation levels are similar to those found by Traulsen et al. [46] in their experiment on a fixed full grid in spite of the widely different settings. Related experiments have been performed by Grujić et al. [20] and Gracia-Lázaro et al. [18] in which much larger full grids were used and, as in [46], participants can be reassigned randomly to a different position but no autonomous mobility in the sense of our setting is provided. Here again the fraction of cooperation tends to stabilize around 0.2 after a transient period.

Another view of the global cooperation results is given in Fig. 12.2b where we report the fraction of participants as a function of the percentage of cooperative actions cumulated over all participants and all sessions. For instance, the highest bar to the left represents the fraction of participants that cooperated between 0% and 10% of the times. Although it is clear that defection prevailed among the participants, it is interesting to note the presence of a region comprised between about 10% and 50% where people cooperated a fair amount of times. Another remarkable thing is that there exists a small fraction of players that cooperate more than 50% and even some participants that cooperate nearly always, as shown by the last bar to the right.

We have looked in more detail at the behavior of the group cooperating between 10% and 50% of the time in order to compare it with the findings of earlier experiments on the PD [20, 22, 21, 17, 18]. As Fig. 12.3 shows, individuals in this group show a behavior consistent with the moody conditional cooperation found in those previous experiments: a cooperative decision is followed by another one with relatively high probability, the higher the number of cooperative partners in the previous round. On the contrary, a defection is most likely to be followed by another one. While we here show the dependence in terms of the difference between cooperators and defectors in the neighborhood, plotting the same two quantities as a function of the number of cooperative neighbors yields qualitatively the same results (see SI, Fig. S3). Therefore, we conclude that, even if subjects can move around in our spatial setting, the decision to cooperate or to defect is very

much determined by a reaction to the observed behavior in the neighborhood and the players' own mood, as in the PD experiments on fixed lattices.

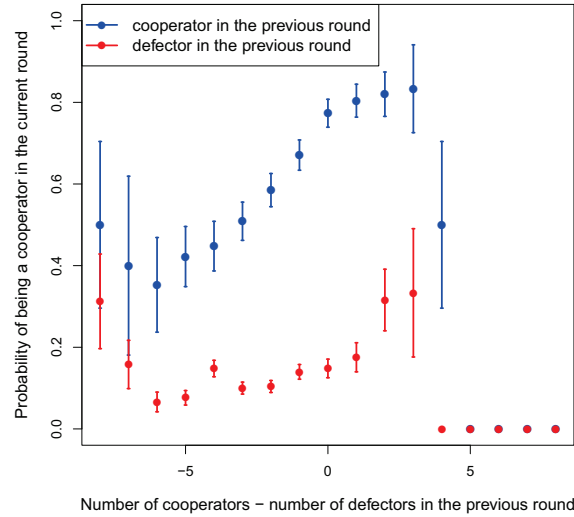


Fig. 12.3 Probability of cooperating in the current round having cooperated (blue curve) or defected (red curve) in the previous one, as a function of the difference between the number of cooperators and defectors in the neighborhood in the previous round. The data are those of the participants cooperating between 10% and 50% of the time (see Fig. 12.2b).

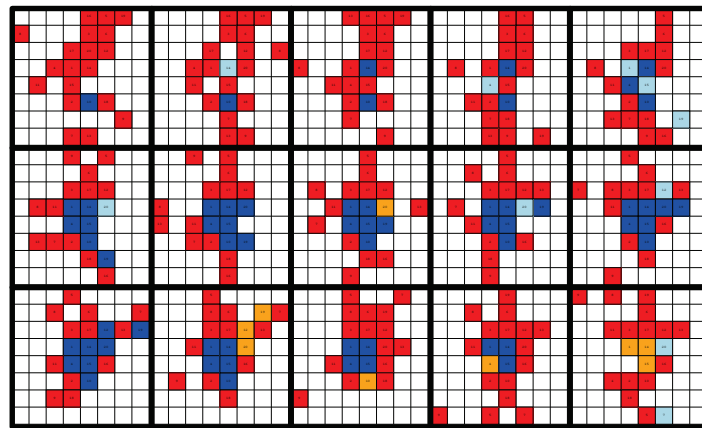


Fig. 12.4 Snapshots for one of the treatments going from round 25 to round 39. One can see the beginning of the formation and the subsequent dissolution of a cooperator cluster. Cooperators are in blue and defectors are painted red. A light blue cell stands for a cooperator that was a defector in the previous round and an orange cell indicates a defector that was a cooperator in the previous round.

The evolutionary process is a complex one but we hope to offer a gist of it with the time evolution shown in Fig. 12.4. This figure represents a particular instance in our experiment but it is absolutely typical of all treatments. The time frames in the figure represent rounds 25 to 39 of the first treatment. As can be seen from the sequence of snapshots, sometimes a few subjects that happen to be close to each other cooperate simultaneously and initiate a cluster of cooperators. However, contrary to the pioneering suggestion in [30], these clusters do not spread: they are not stable and, after a few rounds they vanish as cooperators become tired of being exploited by defectors at the boundary. This phenomenon, which we insist we have observed in most sessions (see SI, sect. 3 for more snapshots), shows the stability of the asymptotic state of the population or, in other words, that once a defective behavior pattern sets in, it is practically impossible to revert it to a cooperative one.

Having discussed the emergence and decay of cooperation in general, taking into account previous experiments and intuitions, we now turn to the unique feature of our setting: the possibility of changing position that is offered to the participants and its possible effects on cooperation. In all our sessions, we have observed that players are rather mobile during the first rounds but, as time goes by, mobility decreases slightly and they tend to settle at some position in space, although movement never ceases until the end of the experiment (see SI, Fig. S4). In fact, a cooperator, unless she finds herself in a very favorable cooperation environment, tends to escape from incoming defectors. Conversely, a defector moves because she is always seeking a cooperation environment in order to accumulate more payoff. We also note that mobility and the incentive to aggregate in order to get a larger payoff cause the mean degree to increase from the initial value of 2.41 to about 4 (see SI, Fig. S5).

Besides the global figures, it is perhaps of more interest to plot the average mobility behavior of cooperators, respectively defectors, as a function of the composition of the local neighborhood they experience, since this is the only information on which they can base their decisions (let us recall here that we do not show information on the earnings of their partners; in this respect, it is important to keep in mind that it has been recently shown that players do not take their partner's payoff into account when making their choices [21]). Figure 12.5 shows the corresponding plots for the number of cooperators (a), respectively defectors (b), in the neighborhood in the previous round.

In Fig. 12.5a, we observe that the mobility of both cooperators and defectors is lower on average than in Fig. 12.5b and decreases faster with the number of cooperating neighbors. This is in agreement with our interpretation that both cooperators and defectors aim at a cooperator environment as far as possible. Cooperators move more than defectors because the latter are more satisfied with a given number of cooperator neighbors. The right panel is somewhat more difficult to understand, but it shows that the mobility behavior of both cooperators and defectors is similar when the number of defecting neighbors is between two and four; however, the mobility of cooperators increases when this number is more than four. In this case cooperators feel exploited and try to evade defectors although the average number of free cells around them tends to decrease. In the other limit, i.e., when there are only a few defectors around, cooperators are more patient in tolerating exploitation while defectors move more frequently, searching for cooperators to exploit. This happens because with a mean degree tending to about four, there are about one or two other cooperators around the focal one and so the latter is relatively satisfied and shows less tendency to move. A caveat is in order here regarding mobility when many neighboring sites are occupied, irrespective of the actions of the individuals occupying them: in those situations, players have less

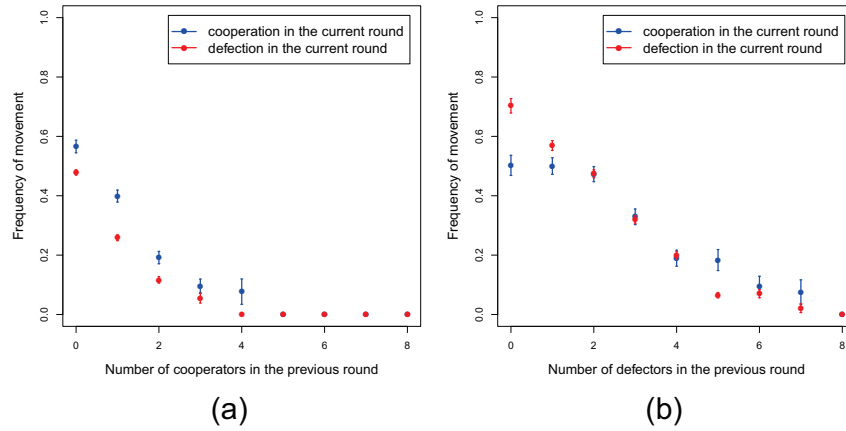


Fig. 12.5 Average mobility of players in their neighborhood. (a) Frequency of movement of cooperators and defectors as a function of the number of cooperators in the previous round. (b) Frequency of movement of cooperators and defectors as a function of the number of defectors in the previous round.

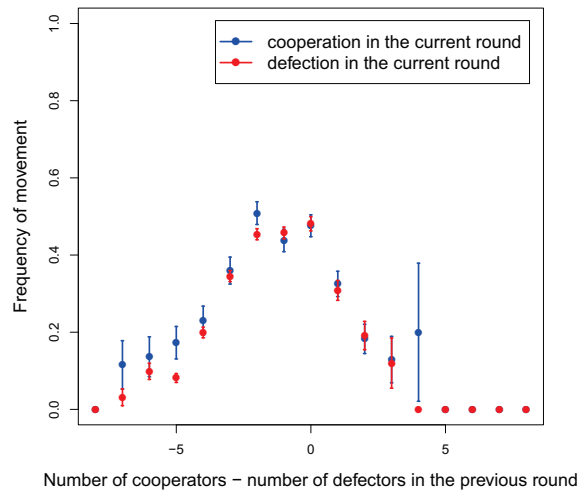


Fig. 12.6 Frequency of movement of cooperators and defectors as a function of the difference between the number of cooperators and defectors in their neighborhood in the previous round.

options to move (in fact, if they were completely surrounded, which is an uncommon phenomenon, they would not be able to move at all). This is an extra factor influencing their behavior and can also be responsible for part of the decay of the mobility in both panels.

A complementary view of local mobility is provided by Fig. 12.6, in which the average mobility of both cooperators and defectors is reported as a function of the difference between the number of cooperators and defectors in the neighborhood of the focal player. It appears that mobility is maximal when this difference is around 0, meaning one cooperator less than the number of defecting neighbors. This is understandable because in this situation neither a cooperator nor a

defector is satisfied. For instance, if a cooperator has a defector and another cooperator in his neighborhood, he will tend to move closer to the cooperator. However, as discussed above, when there are many cooperators and defectors in the neighborhood, most of the time the movement cannot take place, either because of collisions or because of lack of free cells. At the extremes of the curve, where the difference is large in absolute value, either the neighborhood is too crowded to allow movements, or the cases are rare and have a high standard deviation, or they have not been observed. An interesting observation arising from Fig. 12.6 is that cooperators and defectors exhibit the same mobility, within the error bars, in the whole range of differences, which means that even if they are likely to move for different motives, their behavior could be described by the same, simple heuristics, opening the door to simulating larger systems and studying other parameter values.

12.4 Discussion

In this paper, we have presented the results of an experiment intended to shed light on the hitherto unclarified issue of the relevance of mobility in a geographical context to cooperation. In particular, important differences between random and purposeful motion in their ability to support cooperative interactions had been reported from a theoretical viewpoint, but experimental counterparts to those results were lacking. In the context of this previous literature, our most relevant result is that mobility does not promote cooperation: in fact, as in most experiments involving a Prisoner's Dilemma, we have found that the fraction of cooperators decays from an initial value close to half the population to residual ones of approximately a 20%, a value that is almost universally found in the laboratory. In fact, a vast majority of players can be classified as defectors or as moody conditionally cooperators, i.e., as players whose probability of cooperation depend not only on the actions of their partners but on their own previous actions, a type first identified in [20, 21, 18]. This indicates that the possibility to move around in space does not change very much the way players choose their actions.

Remarkably, our experiments also contribute to the understanding of the possible assortment of cooperators in order to support cooperation. The numerical simulations reported in [30] suggested that cooperators may survive by forming clusters in which they mainly interact with other cooperators. In our experiments, we have indeed observed that such clusters appear with non-negligible frequency; however, their lifetime is quite limited because the possibility to move allows cooperating agents at the boundary of the cluster to separate from it to sever their interactions with defectors, or to choose defection themselves. This is in contrast to what has been observed on dynamic relational networks [35, 48], where allowing players to cut and make links at will does lead to clustering of cooperators and to an increase of the cooperation level. The reason for this difference can be traced back to the fact that, if links evolve indirectly by motion of the players in geographical space, they cannot be cut one by one, and when moving away from defectors play will many times cut also their links to cooperators. Therefore, we conclude that for clusters to be an important factor in the promotion of cooperation, individuals must have complete control on their choices of partners, a condition that has never been put forward before.

Regarding mobility, we have found that players move considerably at the beginning of the experiment, but the average fraction of individuals deciding to move decreases and by the end

of the experiment only some 10%-20% of players are moving. We believe that this behavior is connected to the observation in the previous paragraph: players realize that the decision to move has very frequently pros and cons as it affects their connections in an indiscriminate manner, and at some point they conclude that they are not going to find a safe haven against defectors. On the other hand, it is worth noticing that in our experiment there is no punishment for interacting with a defector, and therefore all the residual motions observed in late stages must arise from spite, i.e., from subjects preventing others to benefit from them even if they are not harmed by those partners' actions. This is in agreement with our finding that cooperators tend to move somewhat more often than defectors, implying that while the latter just move trying to find others to exploit, cooperators have the additional motivation to punish defecting partners. In addition, we have also observed that mobility of all players is maximum when there is more or less the same number of cooperators or defectors in the neighborhood. Of course, to interpret these results one needs to bear in mind that when a player has many partners her mobility is also reduced by the lack of available cells to move to. With this caveat, it appears that when the number of cooperators and defectors is approximately the same around a given subject, she will try to move to increase her interaction with cooperators irrespective of her own action, as can be expected. On the contrary, when there are many neighbors of the same type, mobility becomes less relevant and perhaps impossible, this being the reason why we observe a maximum.

In conclusion, we stress that the interaction between behaviour and mobility does not seem to increase the level of cooperation in a human population set on a geographical framework. The main reason for this phenomenon turns out to be the fact that setting and breaking links cannot be done independently for every player as the mechanism for rewiring is motion in space. Interestingly, these results pose important questions about the emergence of cooperation in neighboring human groups, which could be most relevant in interactions in a socio-ecological context among hunter-gatherer groups, either in our recent evolutionary past or in presently existing populations. Furthermore, the type of mechanism we have unveiled with our experiment is most certainly not a very sophisticated way to make decisions, and therefore similar conclusions might apply to spatially structured populations of many other animals or bacteria. In this respect, our findings may provide a new perspective to interpret observational data on cooperative behavior in social animals, pointing to other behavioral traits (e.g., in terms of deciding to move or to change action) that coevolve with the geographical distribution of the population in its ecological environment.

12.5 Methods

We conducted a total of four experimental sessions in February-March 2014 in a specially equipped laboratory using the z-Tree environment [13]. The use of human subjects in this experiment was approved by the Ethics Committee of the University of Lausanne. The participants were fully informed of the nature of the experiment and signed an informed consent to participate. Their anonymity was guaranteed at all stages of the experiment. Each session involved 20 participants and a total of 80 subjects took part in the experiment. Each session consisted of two repetitions of the same treatment played by the same group but with new random initial conditions. An experimental treatment consists of 50 rounds but, to avoid end-of-treatment effects, this number is unknown to participants who are told that there will be between 30 and 60 rounds. Par-

ticipants were recruited using ORSEE [19] from a subject pool that includes students from several faculties of the University of Lausanne and of the Swiss Federal Institute of Technology (EPFL).

Students read a detailed description of the experiment before playing the game. After reading the instructions, subjects had to respond to a set of control questions that insured common understanding of the game and the computation of payoffs. An English translation of the instructions distributed to subjects is provided (see SI sect. 1). Each session lasted about 60 minutes. Participants earned a certain number of points during the experiment and their final score in points was converted at an exchange rate of 1.- CHF = 50 points. The average gain per student was 22.5 CHF (about 19 EUR).

Acknowledgments.

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12.6 Supplementary Information

The supplementary information (SI) provides details on the exact instructions form that participants received (section 12.6.1), complementary results of experimental data not shown in the main text (section 12.6.2), and an overview of some typical configurations observed during the experiment (section 12.6.3).

12.6.1 Instructions form

Each participant read the following set of instructions in detail before the experiment started. Here we present the instructions form for the global treatment case. The only difference between the two treatments is that in the local treatment case participants never know the current number of conventions present in the population at each round.

(The following instructions were originally written in French.)

Explanation for the first part of the experiment

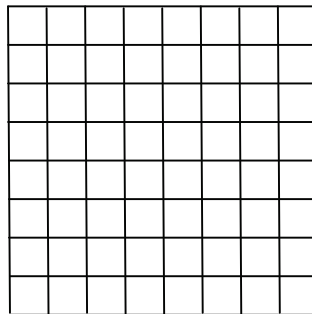
Welcome to this experiment!

You will have to make decisions that will affect your income as well as the income of other participants. Although we express all income in terms of points, these points will be exchanged at the end of the experiment using the following exchange rate:

50 pts. = 1.- CHF

From now on, it is **strictly forbidden to talk with other participants**. If you have any questions, please contact the assistants. If you do not follow this rule, we will have to exclude you from the experiment.

In this study, each one of the 20 participants is placed on a cell of a chessboard, i.e. a 8×8 grid (see below).



Each cell cannot be occupied by more than one participant. Consequently, 64 (total number of cells) – 20 (participants) = 44 cells will always be empty.

During each round of this experiment, you interact with participants placed in a cell of your neighborhood. Your neighborhood is defined as the 8 cells of the chessboard that are around your current position. You can find a representation of your neighborhood here below: your position is in the middle (X) and 8 cells (x) are around you.

x	X	x
x	X	x
x	x	x

Border conditions: if your cell is one at the border of the grid or one of the four corners, your neighborhood always contains 8 cells since the grid is virtually wrapped around itself as in the figures here below.

			x	x	x		
			x	x	x		
			x	X	x		

x						x	X
x						x	x
x						x	x

However, during the whole experiment, you will only see your neighborhood and you will not be able to see what happens in the rest of the chessboard.

What is it about?

There will be a number of rounds between 30 and 60. In the first round, you must choose a profile between options **A** and **B** but, as you will see below, this may change during the experiment.

Your gain for each round depends on the combination of your profile and those of your neighbors. Here below we explain the gain you and one of your neighbors get for each possible combination of your profiles:

- Your profile is **A**, the profile of your neighbor is **A**: you gain **6** points and your neighbor also gets **6** points.
- Your profile is **A**, the profile of your neighbor is **B**: you gain **0** points and your neighbor gets **8** points.
- Your profile is **B**, the profile of your neighbor is **A**: you gain **8** points and your neighbor gets **0** points.
- Your profile is **B**, the profile of your neighbor is **B**: you gain **2** points and your neighbor also gets **2** points.

The following examples illustrate your gain in a round. Your profile is shown in the central cell represented by a capital letter, while your neighbors' profiles are represented with lower case letters.

Example 1: Your profile is **A** and you are in the following situation:

	a	a
	A	
		a

Your gain for this round is: $6 + 6 + 6 = 18$ points.

Example 2: Your profile is **A** and you are in the following situation:

	b	
b	A	
b		

Your gain for this round is: $0 + 0 + 0 = 0$ points.

Example 3: Your profile is **B** and you are in the following situation:

		a
a	B	
a		

Your gain for this round is: $8 + 8 + 8 = 24$ points.

Example 4: Your profile is **B** and you are in the following situation:

	B	b
b		b

Your gain for this round is: $2 + 2 + 2 = 6$ points.

Example 5: Your profile is **A** and you are in the following situation:

	a	b
a	A	b
a		

Your gain for this round is: $6 + 6 + 6 + 0 + 0 = 18$ points.

Example 6: Your profile is **B** and you are in the following situation:

a		b
b	B	b
		a

Your gain for this round is: $8 + 8 + 2 + 2 + 2 = 22$ points.

Example 7: Your profile is **A** and you are in the following situation:

	A	

Your gain for this round is: 0 point.

Example 8: Your profile is **B** and you are in the following situation:

	B	

Your gain for this round is: 0 point.

What will you do ?

At each round, you take two decisions:

Profile switching

You may decide to change your profile switching to the other profile. This profile will be yours until you decide to change again.

Movement

You may decide to move yourself to an empty cell in your neighborhood, if any. If you try to move to a cell which is already occupied by another participant, your displacement won't be accepted. Moreover, if more than one participant want to move into the same cell, only one participant will be chosen while the others won't move for that round.

You will take these decisions using the following screen (Fig. 12.7):

The screenshot shows a web-based interface for an experiment. At the top right, a timer indicates 'Temps restant (séc) : 10'. The main area is divided into three sections:

- VOTRE VOISINAGE**: A 3x3 grid representing the neighborhood. The center cell contains 'A'. The top-left cell contains 'b', and the middle-right cell contains 'a'. All other cells are empty.
- Left Panel**: Displays 'Votre profil actuel: A' and 'Votre gain cumulé: 0'.
- Right Panel**: Contains two decision sections. The first asks 'Souhaitez-vous changer votre profil?' with radio buttons for 'oui' and 'non'. The second asks 'Où souhaitez-vous aller?' with a 3x3 grid of buttons labeled 'NO', 'N', 'NE', 'O', 'C', 'E', 'SO', 'S', 'SE'.

Fig. 12.7 Your neighborhood is shown at the top of the screen, while your current profile and your cumulated payoff are illustrated on the left of the screen. On the right side you can choose to change your profile and if you want to move during this round. Note that if you want to keep your position for the current round you may click on the "C" button.

At the end of each round, you will see your final profit for that round and you will be informed about your new neighborhood. This information will be shown you as follows (Fig. 12.8):

The screenshot shows a game interface with a 3x3 grid titled "VOTRE NOUVEAU VOISINAGE". The grid contains the following values:

b		
	A	a

Below the grid, the text "Vous avez gardé votre position" is displayed. At the bottom, a box contains the following information:

Votre profil actuel:	A
Vous vous êtes déplacé:	non
Votre gain dans cette période:	6
Votre gain cumulé:	6

A "Continuer" button is located at the bottom right of the interface. In the top right corner of the window, a timer shows "Temps restant (sec): 10".

Fig. 12.8 Your new neighborhood is shown at the top of the screen. Your current profile, a message for your eventual displacement, your profit for the current round and your cumulated payoff are shown at the bottom of the screen.

Did you correctly understand the instructions?

Before starting the experiment, we would like to be sure that you and the other participants have correctly understood the decisions that you are going to make. To this end, please answer the questions that will appear on the screen.

(Then, all the participants must answer correctly to 7 trial questions before the first treatment begins. After have played for 50 rounds, participants receive another instructions form (here below) before playing for another treatment. Each group of 20 participants play for a total of 2 treatments.)

Explanation for the second part of the experiment

This part of the experiment is almost the same of the previous one. At the beginning, you must decide your starting profile and your position on the chessboard will be randomly initialized as before.

12.6.2 Complementary results

Here we show the complementary results of the experimental study.

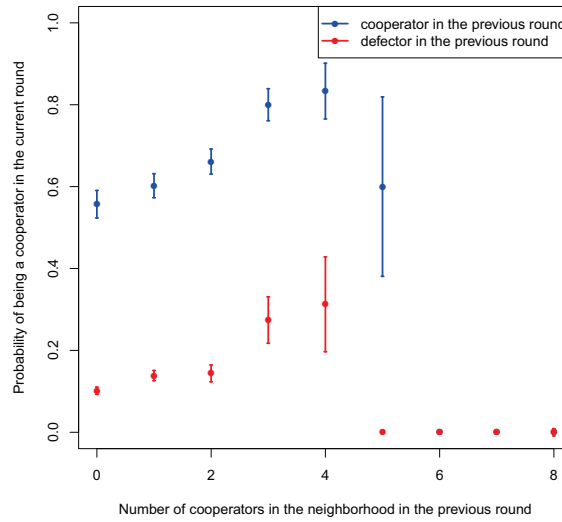


Fig. 12.9 Probability of cooperating in the current round having cooperated (blue curve) or defected (red curve) in the previous one, as a function of the number of cooperators in the neighborhood. The data are those of the participants cooperating between 10% and 50% of the time.

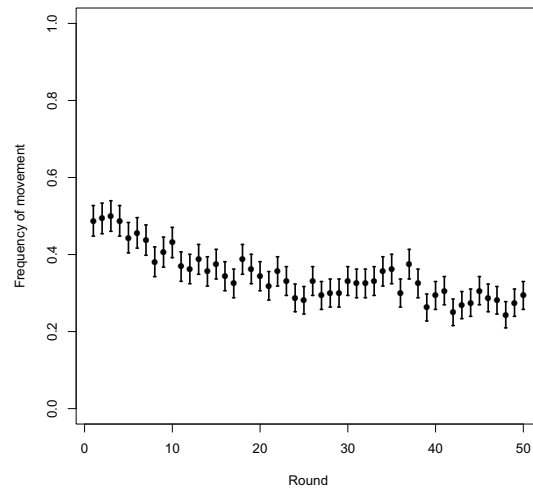


Fig. 12.10 Average mobility of participants per round during the experiment cumulated over all treatments.

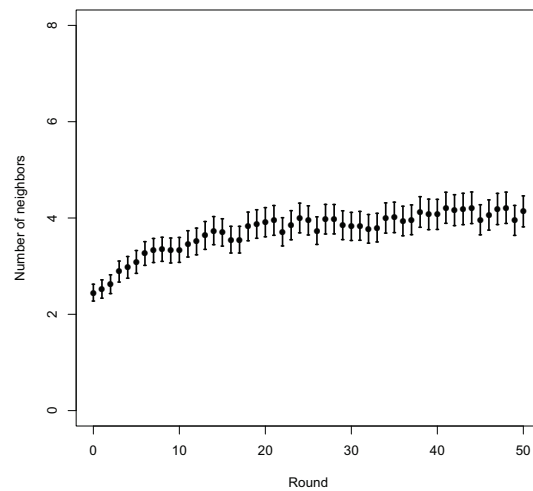


Fig. 12.11 Evolution of the average number of neighbors (mean degree) cumulated over all treatments.

12.6.3 Typical snapshots

This section presents two complete treatments of 50 rounds we observed. We have run 8 treatments. Each group of 20 participants played the same setting for two times.

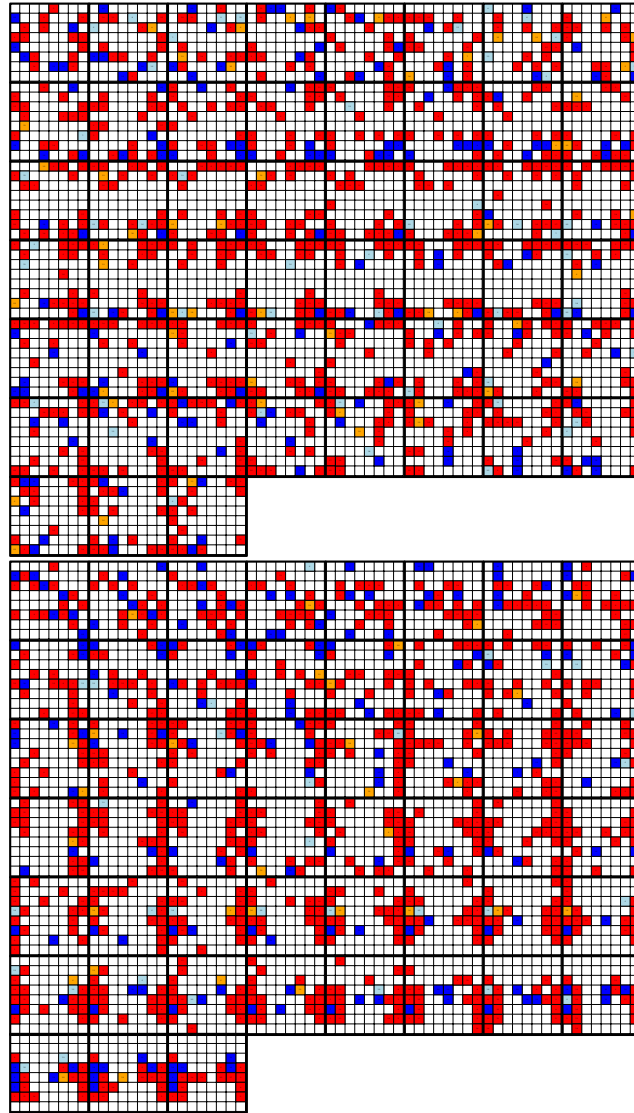


Fig. 12.12 Time evolution of the strategies and positions in the grid during a session of the experiment. Top: group 1, first treatment. Bottom: group 1, second treatment. The snapshots go from round 0 to round 50. Cooperators are in blue and defectors are painted red. A light blue cell stands for a cooperator that was a defector in the previous round and an orange cell indicates a defector that was a cooperator in the previous round.

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Chapter 13

Know Thy Neighbor: Costly Information Can Hurt Cooperation in Dynamic Networks

Publication: Antonioni, Cacault, Lalive, Tomassini (2014), *Know Thy Neighbor: Costly Information Can Hurt Cooperation in Dynamic Networks*, *PLoS ONE* 9(10): e110788. 30 October 2014.

Abstract People need to rely on cooperation with other individuals in many aspects of everyday life, such as teamwork and economic exchange in anonymous markets. We study whether and how the ability to make or break links in social networks fosters cooperate, paying particular attention to whether information on an individual's actions is freely available to potential partners. Studying the role of information is relevant as information on other people's actions is often not available for free: a recruiting firm may need to call a job candidate's references, a bank may need to find out about the credit history of a new client, etc. We find that people cooperate almost fully when information on their actions is freely available to their potential partners. Cooperation is less likely, however, if people have to pay about half of what they gain from cooperating with a cooperator. Cooperation declines even further if people have to pay a cost that is almost equivalent to the gain from cooperating with a cooperator. Thus, costly information on potential neighbors' actions can undermine the incentive to cooperate in fluid networks.

13.1 Introduction

Cooperation is a widespread behavior and a necessary condition for the advancement of social institutions and society as a whole. However, cooperation may easily fall prey to exploitation by selfish individuals who only care about short-term gain [1]. For cooperation to evolve, specific conditions and mechanisms are required, such as kinship, direct and indirect reciprocity through repeated interactions, or a particular structure in the interaction networks that are the fabric of society (see, e.g., [2] for a concise and insightful summary of a vast amount of work, or [3] for a presentation in layman's terms). This network reciprocity does not require any particular psychological propensity or behavior on the part of the agents, but only a heterogeneous distribution of the individuals in the interacting populations. Both theory and quantitative simulations indicate that network reciprocity explains the implicit cooperation that is at the core of society (see, e.g., [4, 5, 6]). To summarize these results, the mere presence of a spatial or relational structure gives rise to evolutionary outcomes in which, thanks to positive assortment, cooperative behavior may evolve and may even lead to fully cooperative states. Recent research tested these predictions by means of targeted experiments with humans in the laboratory, in which the subjects were connected in specific network structures [7, 8, 9, 10]. Surprisingly, these studies found that

neither homogeneous nor heterogeneous network structures promote cooperation to a significant extent [8, 11, 9].

The above analyses relate to networks that do not change with time. However, many actual socio-economic networks are dynamic. This fact has not escaped the attention of researchers, and several models have been proposed for studying cooperation under these conditions (see, e.g., [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] among others, and the recent review in [22]). There are also model studies of the coevolving ultimatum game [23].

These models differ in their details, but researchers agree that adding these new adjustment margins may lead populations to mainly cooperative and stable states through co-evolution of behavior and connectivity. Empirical tests of dynamic settings include [24, 25, 26, 27]. Rand et al. [24] found that cooperation is supported if participants can rewire connections often enough, and that the evolved networks are more heterogeneous and have more stable links between two cooperators than in less fluid or completely static conditions. Wang et al. [26] investigated the role of link updating frequency on cooperation and found that partner updating significantly increased the cooperation level even at relatively low frequencies.

We study a setting where individuals can make or break links and need to pay for information on their potential partners' actions. This setting differs from Rand et al. [24] who provided the players with full information on the strategies used by their neighbors in the previous round. Also, Wang et al. [26] provided even richer information, again for free. Players were shown the identities (anonymous labels) and action choices of all players with whom they were connected for up to five previous rounds. While we think that these conditions could be adequate in some situations in which the same people interact repeatedly, we argue that there are many contexts in which there is uncertainty as to potential partners' actions and, by consequence, a decision entails some amount of risk. This is the case in today's widespread large online systems, where changing and multiple identities are easy to create and use and confer a certain amount of anonymity to the participants. We believe that this is an important situation to investigate, and we have tried to introduce this factor in our experiments by imposing a cost on a player to "discover" the current strategy of a potential partner.

13.2 Experimental Setup

Our experimental setup is based on the Prisoner's Dilemma game [28, 29, 30]. In this two-person game, players must decide whether to cooperate or to defect. If both cooperate, each gets a payoff R . If one defects and the other cooperates, the defector gets T and the cooperator receives the payoff S . If both defect, each gets P . Since $T > R > P > S$, defection is a dominating strategy and a rational payoff-maximizing player will choose to defect, although mutual cooperation yields a higher collective payoff, whence the dilemma. Evolutionary reasoning leads to the same result, as defectors will reproduce at a higher rate due to their superior payoff [31]. This simple game perfectly displays the tension between socially desirable outcomes and self-interested individual actions. In our experiment, subjects played a Prisoner's Dilemma game with their immediate neighbors in the network, with $T = 20$, $R = 10$, $P = 0$, and $S = -10$. These payoff values are the same as those used in [24], except for an unimportant scale factor. The initial set of connections between the participants was chosen to be a regular random graph of degree 4. Partic-

Participants played 15 periods of the game described below, although this exact number was unknown to them; they were only told that they would play for at least 10 periods. Each period consisted of the following five stages:

1. Action choice
2. Link proposals
3. Information acquisition choice
4. Link acceptance decision
5. Feedback on payoffs

In the first stage, players had to select one of two actions, “square” or “circle,” where “square” implied “cooperation” and “circle” implied “defection.” We chose to label actions in a neutral fashion to rule out framing effects. The association between the label (“circle” or “square”) and the actions (cooperation or defection) was randomized across sessions.

In the second stage, subjects received information on their own action and the number of current neighbors that selected each of the two actions. Subjects then chose one and only one of the following actions: do nothing at this stage, break a link with one of their current neighbors who chose “square,” break a link with one of their current neighbors who chose “circle,” or ask to be matched with a randomly chosen individual who is not yet their neighbor.

In the third stage, subjects saw how many individuals wanted to link with them (those who asked to be linked and were randomly assigned to the subject and his or her new partner if he or she asked for one). Subjects decided whether to pay a cost of c per connection to be informed about the current action (“square” or “circle”) of each potential partner.

In the fourth stage, subjects saw the information they paid for and decided whether or not to accept each pending connection. Link deletion was unilateral. Link creation required mutual consent from both partners.

After these decision stages, subjects were informed of their current payoff as well as their accumulated payoff. They were neither informed about their neighbors’ payoffs nor about their neighbors’ individual strategy choices; they only knew the number of people playing each of the two strategies among the neighbors. Participants never knew the full network topology.

The crucial experimental variation concerned the cost of obtaining information on the partner’s decision in stage three. We implemented a baseline condition with cost $c = 0$. Results from that baseline allow us to compare our design with the existing designs. We expected to obtain similar levels of cooperation, since we implemented a fluid dynamic network design with free information. Crucially, we also implemented two settings in which participants needed to pay before they got access to information on their potential partners’ actions. Participants paid a cost of $c = 4$ in the low-cost condition and a cost of $c = 8$ in the high-cost condition. Note that these costs are small in the sense that the cost of getting the information is smaller than the benefit of playing the game with a cooperator, i.e., $c < R$. Paying the cost is also rational for a cooperator who fears being paired with a defector, since the cooperator pays c to avoid a payoff of $S = -10$ (see Supporting Information file for details on rational behavior analysis).

We repeated this experiment twice for each group of 20 participants. We re-initialized the network to be a new regular random graph of degree 4, and participants played the same game in the same treatment condition for another 15 periods.

13.3 Results

We begin by examining the amount of cooperation that prevailed during the last five rounds of the experiment. Figure 13.1 depicts the average level of cooperation in the last five periods as a function of the information-gathering cost. Recall that participants knew that the experiment would last at least 10 periods. During periods 11 through 15, participants expected that the experiment could end in any period. This feature could have triggered “last round” effects, where participants typically return to the Nash Equilibrium of the stage game. The final five rounds are therefore a strong test for cooperation.

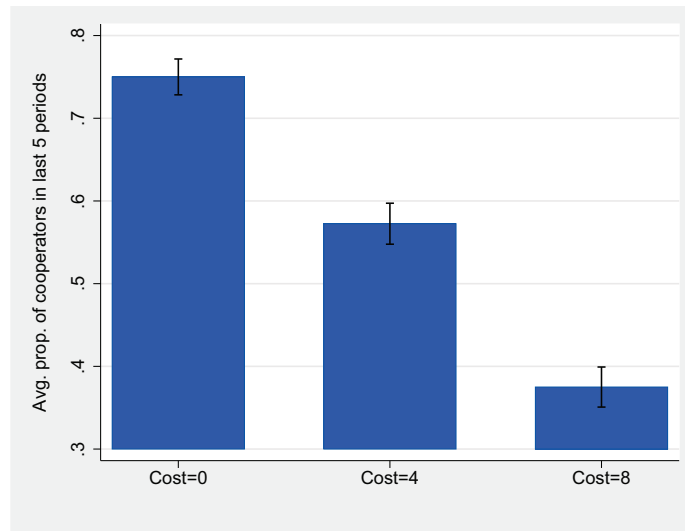


Fig. 13.1 Fraction of individuals cooperating during the last 5 periods by cost of obtaining information on new partner. Cooperation is the dominant action if information on the new partner’s action is available for free. Cooperation plummets as costs of obtaining information on new partner’s actions are introduced. The capped spikes plot the standard errors of the mean.

Results in Fig. 13.1 indicate that cooperation attains a high level when information is freely available (Cost=0). About three out of four decisions are cooperation decisions (0.75). Cooperation declines substantially once information on the potential partner’s action is introduced. Merely two out of five players cooperate in the high-cost treatment (the fraction cooperating is 0.38, significantly lower than in the no-cost treatment, $P < .05$). Cooperation levels are also lower if participants need to pay an intermediate cost in order to access information on their future partners’ actions (the fraction cooperating is 0.57, significantly lower than in the no-cost treatment, $P < .05$). Thus, high costs for information on potential partners’ actions hurts cooperation.

We now turn to the evolution of cooperation over time. Figure 13.2 reports the fraction of cooperators per period for the three different cost treatments. The shaded area is the 95% confidence interval for the baseline treatment ($c=0$). Cooperation increases steadily when information is free. About 80% of all participants decide to cooperate in the final period. Cooperation also increases in the low-cost treatment with $c = 4$, but merely 55% of all subjects cooperate in the final round.

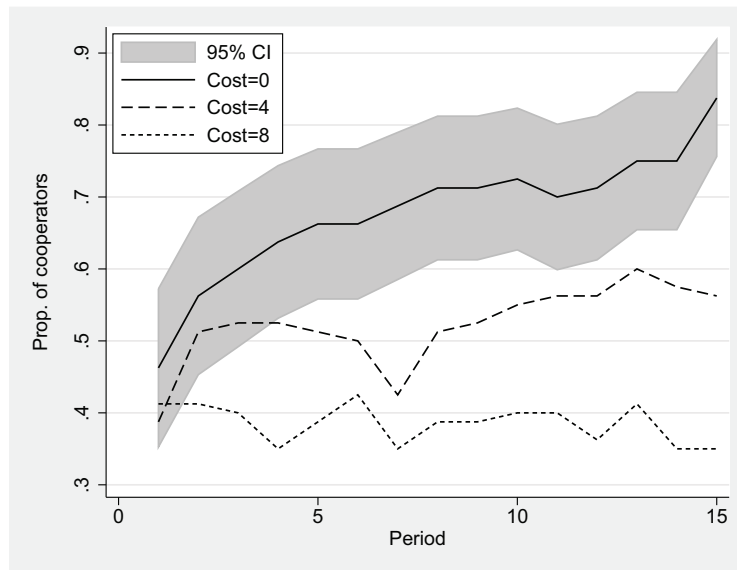


Fig. 13.2 Fraction of cooperators by period and treatment (cost of information). Cooperation starts at just below 50% in all treatments. Cooperation rapidly increases over periods in the treatment with free information on potential partners' actions. Cooperation builds up less rapidly in the low-cost (Cost=4) treatment and remains almost at the initial level for the high cost (Cost=8) treatment. The grey area plots the 95% CI for the Cost=0 treatment.

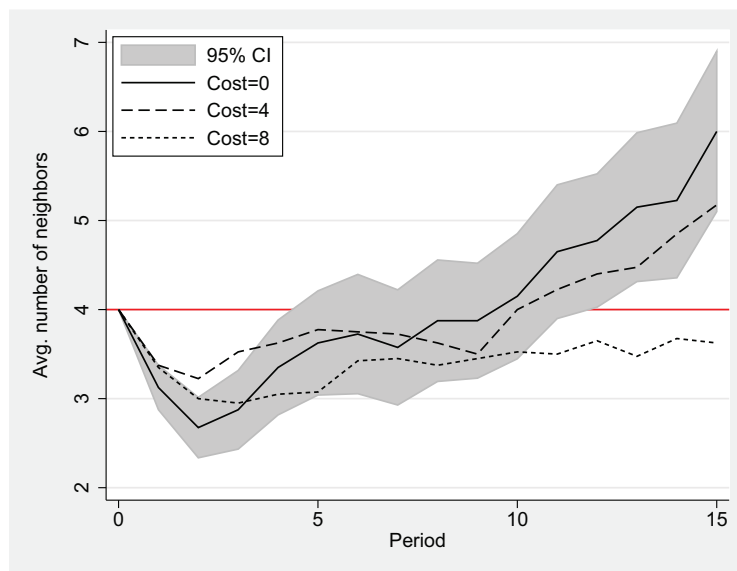


Fig. 13.3 Average number of neighbors by period and treatment (cost of information). The grey area plots the 95% CI for the cost=0 treatment.

Cooperation ceases to build up over time in the high-cost treatment with $c = 8$. Merely 35% of all individuals cooperate in the final period. Thus, the cost of obtaining information on a future partner's action dramatically reduces the capacity of fluid networks to sustain cooperation.

In the following we discuss the effects of costly information on the average number of neighbors of each participant, i.e., the participant's degree. Figure 13.3 shows the average number of neighbors by period for the different cost treatments. Initially, all individuals have four neighbors in all three conditions. Interestingly, subjects initially sever a number of existing links and average degree decreases from the initial level of four neighbors to a level of just under three neighbors between period 0 and period 3 (see Supporting Information file for a detailed analysis of link proposal decisions). Average degree then increases rapidly and reaches a level of about 6 links in the free-information condition. Average degree increases to a very similar extent in the low-cost condition, with participants having about five neighbors in the final round. Participants who had to pay the high cost also accepted new links but at a much lower rate. Average degree remains below the initial level of four neighbors in the high cost condition. This analysis indicates that the social networks become sparser as the cost of obtaining information on neighbors' actions increases. Related results that tend to confirm this trends by numerical simulations have been published by J. Tanimoto [32, 33]. The total accumulated payoff earned by all members of the population is clearly related to the number of neighbors and the fraction of cooperators. This quantity is shown in Fig. 13.4 where it is clearly seen that a higher mean degree in a population mainly composed by cooperators is highly favorable for social wealth.

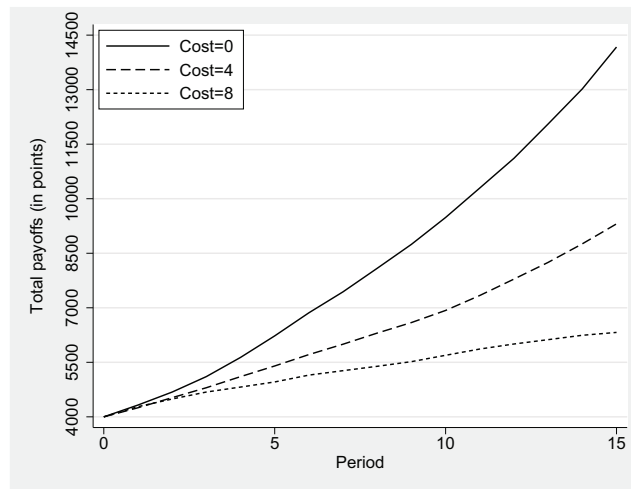


Fig. 13.4 Total wealth in the population by period as a function of the cost required to discover information on partners. Participants start with an initial endowment of 200 points each.

We now turn to how the cost of obtaining information on neighbors affects the resulting social networks. Figure 13.5 displays the topology of networks that formed in the final period. Fig. 13.5 presents the final topology we obtained in four of the total 12 runs of the experiment. These four topologies are representative of the total we obtained (see Figs. 4, 5, and 6 in the Supporting Information file for all final topologies). Network (a) displays the final network state in cases in which

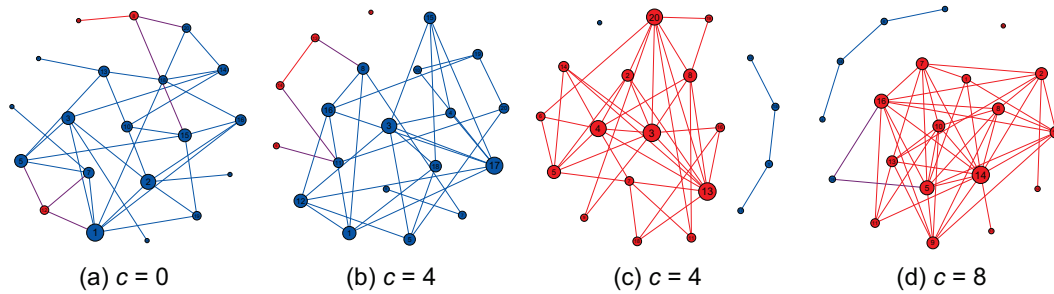


Fig. 13.5 This figure displays the final state of four networks. Blue stands for cooperation and red for defection. The size of each node represents the number of neighbors of a node. The figure shows that the cost of obtaining information dramatically alters the final state of the network. The free information treatment results in a majority of cooperators playing each other in a densely connected social network (a). The high-cost treatment results in a majority of defectors, again fairly densely connected (d). The low-cost treatment produces a case that is similar to the low-cost treatment (compare (b) to (a)) or a state that is similar to the high cost treatment (compare (c) to (d)). Cooperation in dynamic social networks exhibits path dependence in the low-cost treatment.

information on neighbors' actions is freely available. This network consists of densely connected cooperators with only a few poorly connected defectors scattered around. Networks (b) and (c) resulted from the low-cost ($c = 4$) treatment. They are representative of the two tendencies we observed for this cost value. Players either tended toward a clear majority of cooperators, as in (b) (very similar to $c = 0$ in case (a)), or they tended towards a state with many defectors and only a few poorly connected cooperators (see image (c)). Network (d) resulted from the treatment with $c = 8$ and is typical of this condition: defectors prevailed in all cases. Interestingly, it appears that the cost $c = 4$ is a threshold cost such that for cost values less than 4 the population tends to self-organize in a mainly cooperative structure, while for higher costs people seem to be more conservative and tend toward defection. Information-gathering costs are important for the emergence of cooperation in dynamic anonymous networks.

We now turn to a more detailed discussion of how players interacted with their neighbors. Fig. 13.6 shows the frequency of links that involved two cooperators (CC), two defectors (DD), or one cooperator and one defector (CD/DC). The population ends up mostly cooperating, and the majority of links are between cooperators in the baseline treatment (see case (a)). Links initially involve two cooperators only in 35% of all cases, but their frequency steadily increases and displaces both links among defectors and the mixed links. Introducing a low cost of obtaining information on neighbors' actions dramatically changes the dynamics of link types. Both links involving only cooperators and links involving only defectors can be present in the final state, depending on the type of final network (see case (b)). The picture fundamentally changes with high costs. Links involving two defectors now displace mixed links. Cooperator links remain fairly stable but at a low level. Interestingly, CD links tend to disappear, meaning that subjects refuse to be exploited and punish defectors by severing links to them. Note also that links between two defectors are stable in our setting because participants neither gain nor lose in that interaction.

Now we discuss how the cost of obtaining information on neighbors' actions affected the demand for that information. Figure 13.7 presents information on how many subjects wanted to know the strategy of a potential new partner; we call this action "scouting." The figure plots the propor-

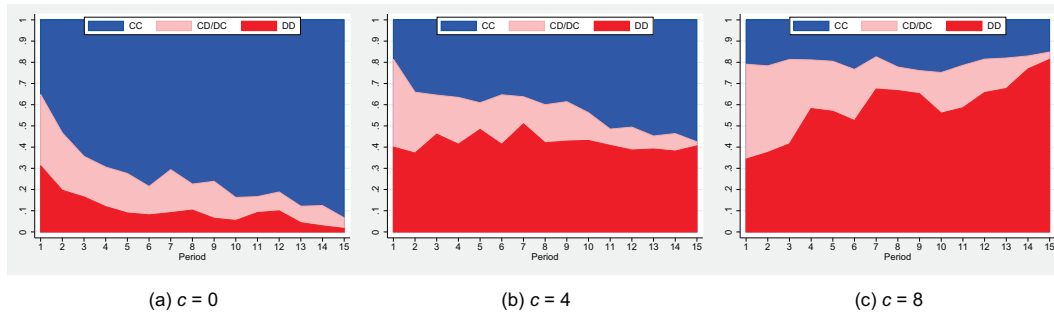


Fig. 13.6 Evolution of link type by cost of obtaining information. Links between two cooperators are shaded blue, links between two defectors are shaded solid red, and mixed links are shaded light red. Links between two cooperators displace other link types almost completely in the free information treatment (a). Links among two defectors displace the other link types almost completely in the high-cost treatment (c). The low-cost treatment has either cooperator or defector links, depending on the final state of the network (b).

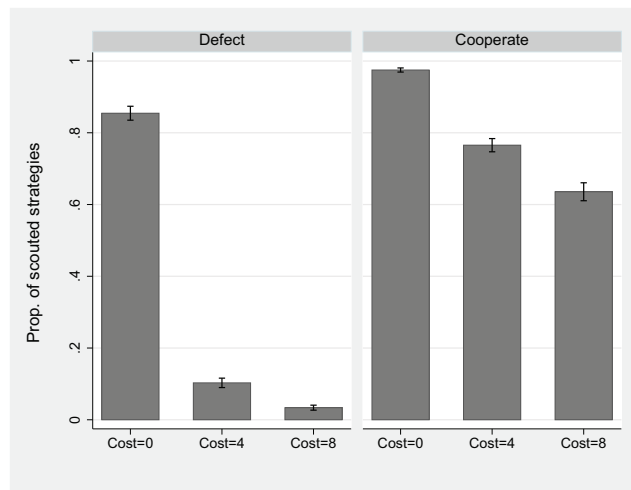


Fig. 13.7 Fraction of scouted links among potential links by individual action and treatment (cost of information). “Scouting” means that the player asked to see the action of the potential neighbor. Both defectors and cooperators scout when information is freely available. Defectors almost cease to scout once information on the neighbor’s action becomes costly. Cooperators continue to scout as information becomes costly, albeit at a lower rate than in the free-information treatment. The capped spikes plot the standard errors of the mean.

tion of scouted strategies as a function of the information cost, for both cooperators and defectors. The majority of participants chose to discover the decision of a potential partner when it was free, regardless of their own decision. However, their behavior was different when scouting entailed a cost. Defectors were reluctant to pay for the information on their neighbors’ action because they did not incur losses when playing another defector (both earned the payoff $P = 0$), and they gained by playing a cooperator (defector gains $T = 20$). In contrast, cooperators incurred losses when playing a defector, so knowing the potential partner’s strategy continued to be valuable to

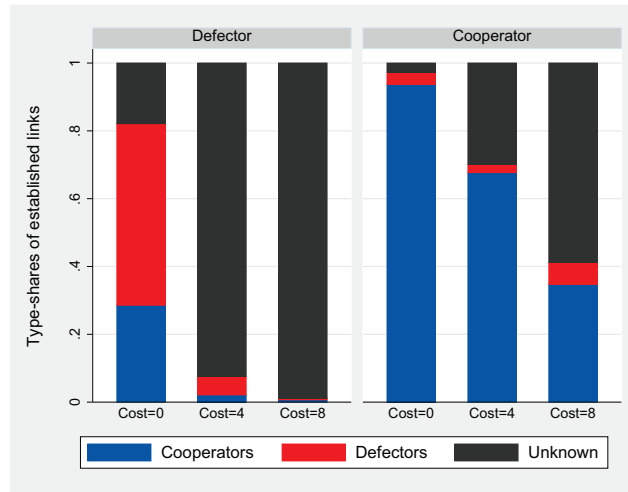


Fig. 13.8 Proportion of established links with defectors, cooperators, and unknown partners according to players' current type and as a function of the cost c .

them. Nonetheless, the fraction of cooperators paying to scout their future neighbors' action decreased substantially, from 95% in the free-information treatment to below 80% in the low-cost treatment, and to about 60% in the high-cost treatment. This behavior in turn reduced the incentive for players to cooperate, since defection could go undetected in the high-cost treatment.

Finally, we now discuss how costly information shapes link acceptance. Figure 13.8 plots the proportion of each type of effectively created link by treatment and according to the participants' current strategies. When information is free, defectors create few links with cooperators because the latter do not give their consent. Most new links of defectors are to other defectors, and the rest are to unknown types. Cooperators, on the other hand, always establish links to cooperators and reject defectors except for some rare cases. When information about others' strategies entails a cost, defectors no longer scout and link only to unknown types, with few exceptions. Cooperators continue to pay for the information, since the expected gain is positive (see Supporting Information file, rational behavior analysis section). However, the expected gain from acquiring the information decreases with its cost. Hence, the proportion of new links to unknown types increases as the cost rises.

13.4 Discussion

We have experimentally investigated the cooperation behavior in networks where participants playing a Prisoner's Dilemma game are allowed to update their connections by making and breaking ties. While recent experiments have focused on contexts in which information on the types of players is either fully available or easy to obtain, such as in groups that interact frequently [24, 26], our conditions resemble to today's online networks in which a certain degree of anonymity is easy to achieve. The players only know how many of their current direct neighbors are cooperators and how many are defectors; information on the rest of the network and on the neighbors' payoff is

not provided. A subject may suppress an unwanted relationship just by cutting the corresponding link unilaterally at no cost, but forming new links to an unknown participant can be done either “blindly” at no cost or by paying a certain cost to know the participant’s current type, i.e., whether he or she is a cooperator or a defector, and the link is effectively created only if both partners agree to accept it. We experimented with two values for the cost of acquiring the information about the potential partner’s type: one that is relatively low with respect to the payoff values used in the experiment, and another which is higher but not so high that a rational player would never choose to pay for it. We compared results in the costly setting with a baseline situation in which the information is available for free.

In the baseline free treatment, the network quickly evolved toward full or almost full cooperation. The resulting networks had a high average degree, since it is beneficial for a cooperator to have many links to other cooperators. At intermediate cost an interesting phenomenon emerges, wherein the population may evolve in two ways: either cooperators or defectors may be in the majority. In the high-cost case we are again in a rather clear-cut situation: introducing the high cost favors defection because participants are more reluctant to pay for linking information. The consequence is that cooperators have trouble finding other cooperators to form stable clusters, and the resulting networks are sparser.

Our results suggest that there is a cost barrier that depends on the actual payoffs, beyond which people have less propensity to pay for information, in spite of the fact that knowing the potential partner type improves decision making. We believe that these conditions are representative of many of today’s network relationships, in which a certain degree of anonymity can be maintained, and thus our results should be relevant in these contexts. As a consequence, cooperation is more difficult to achieve in situations in which costly or uncertain information about a partner’s behavior is the rule. The present work could be extended in a number of directions. For example, in the present setting, breaking a link has no cost and does not require the agreement of the current partner. However, there are situations in which this is not possible, e.g., when there is a contract or when breaking the link would entail adverse social consequences. One could also consider different payoffs for the game, such as having a negative P , and their influence on cooperation and network dynamics.

13.5 Methods

Ethics Statement

This research was approved by the ethics committee on the use of human subjects in research of Lausanne University. All participants signed an informed consent describing the nature of the experiment before they entered into the laboratory.

Procedure

We conducted a total of six experimental sessions in October 2013. Participants were recruited from the pool of undergraduate students from all disciplines of the University of Lausanne and

the Ecole Polytechnique Fédérale of Lausanne using ORSEE [34]. Subject-subject anonymity was granted at all stages, and the experiment was computerized using the z-Tree environment [35]. The use of human subjects in this experiment has been approved by the ethics committee of the University of Lausanne, and participants signed an informed consent describing the nature of the experiment before they entered into the laboratory. Before making decisions, participants read detailed instructions and responded to a set of control questions that insured common understanding of the game and the computation of payoffs. A translation of these instructions from the original French is provided as Supporting Information to this paper. Each session included 20 participants (a total of 120 subjects took part in the experiment) and lasted about one and a half hours. Participants received a show-up fee of 10 CHF (about \$11 USD), and their final score in points was converted at an exchange rate of 1 CHF = 30 points. The average payoff per student was 40.85 CHF (about \$46.2 USD).

All statistical analyses are at the level of the individual using linear regression. Because multiple observations of an individual are not independent, we cluster observations at the individual level. Because individuals play in dynamic networks, we also allow for arbitrary clustering of the error terms between individuals who were neighbors in period $t - 1$. Our results are not sensitive to allowing for clustering at the session level. Differences in cooperation were assessed by means of two dummy variables (Cost 4 takes the value 1 if the individual faced cost $c = 4$, and zero otherwise; Cost 8 takes the value 1 if the individual faced $c = 8$, and zero otherwise; see Supporting Information section).

13.6 Supporting Information

The supplementary information (SI) provides supplementary results on link proposal decisions and final topologies (section 13.6.1), presents the statistical analyzes of the main results on cooperation (section 13.6.3), discusses whether the scouting decision is rational (section 13.6.5), and a translation of the instructions that participants received (section 13.6.6).

13.6.1 Supplementary Results

Link Proposal Decisions

Figures 13.9, 13.10, and 13.11 analyze decisions on link proposals (stage 2). Figure 13.9 reports the fraction of subjects who ask to break a link, with a defector in almost all cases. About half of all the subjects ask to cut a link with a defector in the first period. The fraction breaking rapidly decreases and reaches a level of about 20 % from period 4 to 15. The cost of acquiring information does not affect the decision to break a link substantially.

Figure 13.10 reports the fraction of subjects who ask to be matched with a new neighbor. The proportion wanting to make a link is initially at around 30%, but increases rapidly to reach a level of between 60 and 80 %. The proportion wanting to establish a new link is somewhat higher in the treatment condition with free information on neighbor's actions than in the high cost of information treatment.

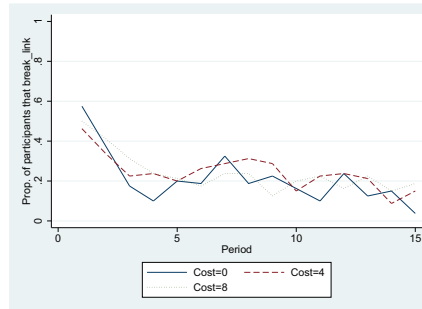


Fig. 13.9 Fraction of players choosing to break a link by period and cost of acquiring information.

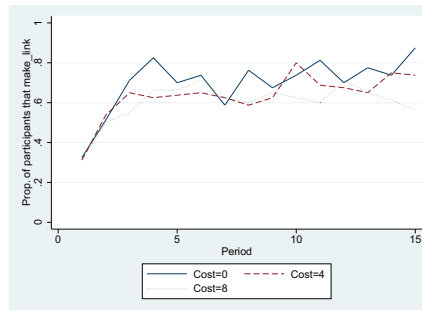


Fig. 13.10 Fraction of players asking to be linked with a new player by period and cost of acquiring information.

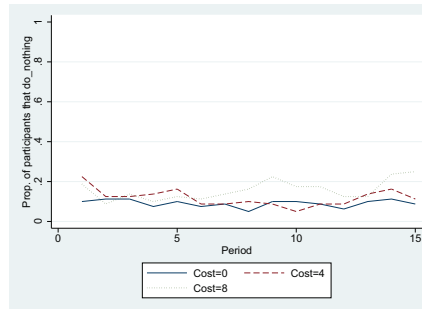


Fig. 13.11 Fraction of players remaining idle by period and cost of acquiring information.

Figure 13.11 reports the fraction of players who decide to do nothing at the link proposal stage. This proportion remains at 10 %, a very low level, throughout all periods of play. The proportion of idle players is somewhat larger in the high cost treatment, compared to the free information treatment.

13.6.2 Final Network Topologies

This subsection presents final network topologies in all 12 sessions.

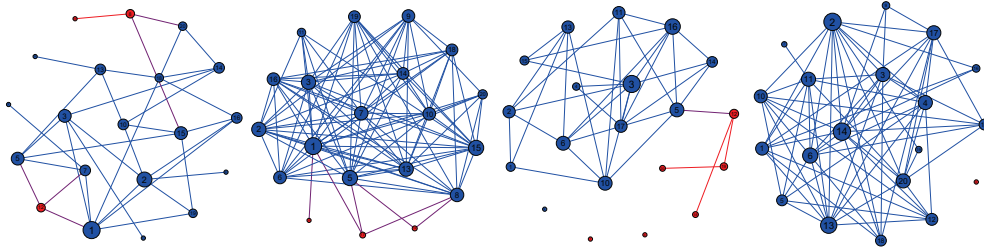


Fig. 13.12 Final network topologies for $c = 0$ treatments. Blue stands for cooperation and red for defection. The size of each node represents the number of neighbors of a node. First and third images represent the sessions performed as first by the subjects, while second and fourth images stand for the second experimental run.

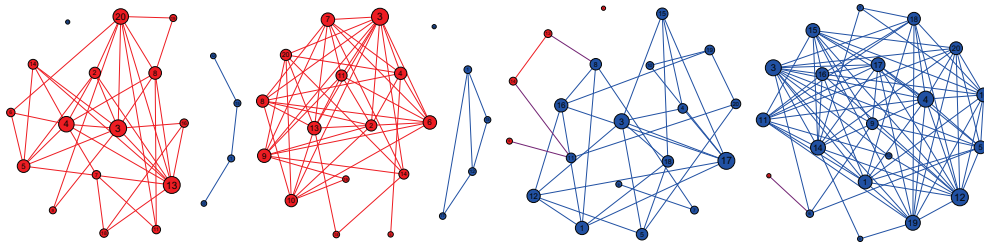


Fig. 13.13 Final network topologies for $c = 4$ treatments.

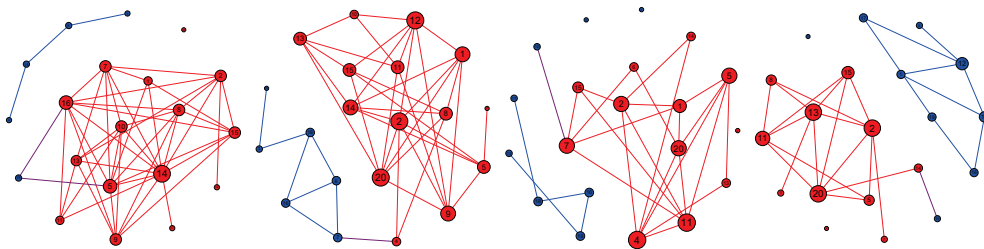


Fig. 13.14 Final network topologies for $c = 8$ treatments.

13.6.3 Multivariate Regression Analyses

13.6.4 Linear Regressions

Table 13.1 presents a statistical analysis of cooperation decisions in the experiment. Recall that all subjects played two repetitions of the experiment. The analysis considers only the decisions of individuals in the first repetition of the experiment to exclude learning effects. The dependent variable in all analyses is a binary variable taking the value 1 if the subject decided to play the cooperative action, and zero otherwise.

It is not trivial to conduct proper inference in networks settings. We have explored three approaches to estimating the standard errors of the parameters measuring the effects of the cost treatment. The first approach reports standard errors in parentheses in Column (1) of Table 13.1 that allow for arbitrary clustering across sessions [37]. This approach provides an approximation to standard errors that is appropriate in settings with a large number of clusters. In our setting, the approach is potentially mis-leading since our data is based on six different sessions whereas the clustering approach requires 30 or more clusters.

Our second approach calculates standard errors that allow for arbitrary correlation of errors of individuals that currently are neighbors or have been neighbors at any point in time. The remaining correlations are set to zero. Standard errors from this approach are reported in square brackets in Column (1) of Table 13.1.

Our third approach to inference applies methods that have been developed in the analysis of spatial data (Column 2 in Table 13.1). The spatial error model allows for correlation between error terms of different individuals through their linked neighbors [38]. The parameter λ measures the strength of the correlation between error terms of different individuals.

Approaches two and three require that individuals be connected to at least one other neighbor. We discard 92 observations of individuals who did not have any neighbor.

Column (1) of Table 13.1 presents an ordinary least squares (OLS) analysis of average cooperation decisions across all periods of the experiment. In the free information treatment, 58.3 % of all participants decided to cooperate (see Constant). The cooperation rate was 12.7 % lower in the intermediate cost treatment (Cost4 indicator). The cooperation was 22.7 % lower in the high cost treatment (Cost8 indicator). Column (2) provides somewhat different estimates of the treatment effects since estimates in these columns are based on a maximum likelihood estimation of the spatial error model. The treatment effects are both significantly different from zero, regardless of the approach that we use to assess statistical significance. Our conclusions are robust to the exact approach to assessing statistical significance.

The dependent variable is binary. Column (3) in Table 13.1 reports estimates of this binary decision variable in a Logit framework. We report the marginal effects of being exposed to an intermediate cost (or a high cost) on cooperation decisions. The treatment effects in the Logit model are virtually identical to effects that are based on OLS analysis (Columns (1) and (2)). Our conclusions are robust to the exact estimation method chosen to measure the treatment effects. Non-linear Logit analysis and linear OLS methods tend to give similar results in situations where the average of the dependent variable is not too close to either zero or one [36].

Table 13.1 OLS, Spatial Error and Logit regressions for cooperation (1st repetition)

	OLS cluster (1)	Spatial Error (2)	Logit (avg. mg. eff.) (3)
Cost4	-0.127 (0.054)* [0.038]**	-0.112 (0.043)***	-0.123 (0.029)***
Cost8	-0.227 (0.078)** [0.038]***	-0.238 (0.042)***	-0.222 (0.028)***
Constant	0.583 (0.004)*** [0.027]***	0.586 (0.030)***	
λ		0.343 (0.024)***	
Observations	1588	1588	1588

Notes: (1) Standard errors clustered by session (G-matrix) in parentheses (square-brackets) (2) Standard errors in parentheses (3) Delta-method Standard errors in parentheses. *** $P < 0.01$ ** $P < 0.05$ * $P < 0.1$.

Cooperate=1 if current strategy is cooperate, =0 if defect.

Cost4(8)=1 if participant in cost=4(8) treatment, =0 otherwise.

92 observations are excluded because they were not connected to any other player in the experiment.

13.6.5 Rational behavior analysis

This section discusses an approach to assessing whether players acted rationally in the model.

Cooperator dynamics

In the rewiring stage cooperators estimate their expected payoffs from the three possible actions: do nothing, cut a link with a defector, and ask for the creation of a new link with a random participant.

We define $p_c = p_c(c)$ as the probability that a cooperator pays the cost c in order to know the potential partner strategy. This probability has been estimated empirically as follows:

$$p_c \sim 1.0 - 0.05c \rightarrow p_c = 1.0, 0.8, 0.6 \text{ for } c = 0, 4, 8, \text{ respectively.}$$

n_c : current number of cooperators in the current neighborhood

n_d : current number of defectors in the current neighborhood

T, R, P, S : payoffs; $T = 20$, $R = 10$, $P = 0$ and $S = -10$ in our experiment.

Here it follows the estimation of the expected payoffs for a cooperator:

- do nothing:

$$\bar{\Pi}_N = n_c R + n_d S$$

- cut a link with a defector (if any):

$$\bar{\Pi}_C = n_c R + (n_d - 1)S$$

- ask for a new link: this expected payoff is estimated assuming that a cooperator pays the cost c with probability p_c , as explained before. Then, it is assumed that a rational cooperator accepts an unknown potential partner with probability r . This parameter of the model can be seen as the proportion of cooperators in the rest of the population. Figure 13.15 shows how it is calculated this expected payoff as a function of p_c and r .

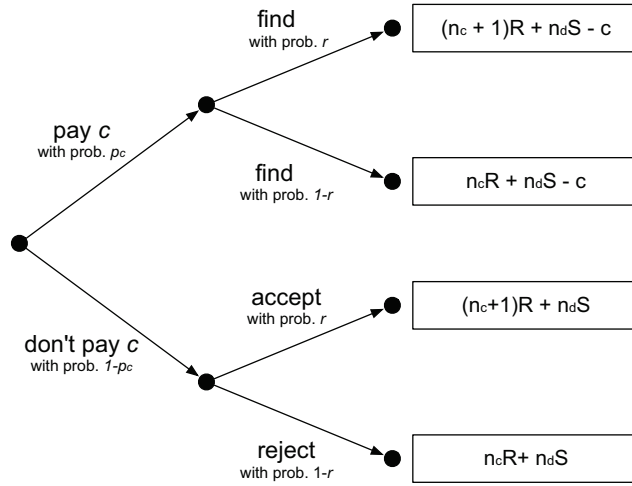


Fig. 13.15 A cooperator pays (or does not) the cost c with probability p_c ($1 - p_c$), while the proposed partner is a cooperator (defector) with probability r ($1 - r$).

According to Fig. 13.15:

$$\begin{aligned} \bar{\Pi}_L = & p_c r ((n_c + 1)R + n_d S - c) + p_c (1 - r) (n_c R + n_d S - c) + \\ & + (1 - p_c) r ((n_c + 1)R + n_d S) + (1 - p_c) (1 - r) (n_c R + n_d S) \end{aligned}$$

A rational cooperator is thus defined as a cooperator that performs the action which has the best expected payoff among $\bar{\Pi}_N$, $\bar{\Pi}_C$ and $\bar{\Pi}_L$.

Defectors dynamics

In the rewiring stage a rational defector always ask for a new link.

Experimental results

Cooperator dynamics. Figure 13.16 shows the proportion of times a cooperator chooses the *rational* action as a function of r .

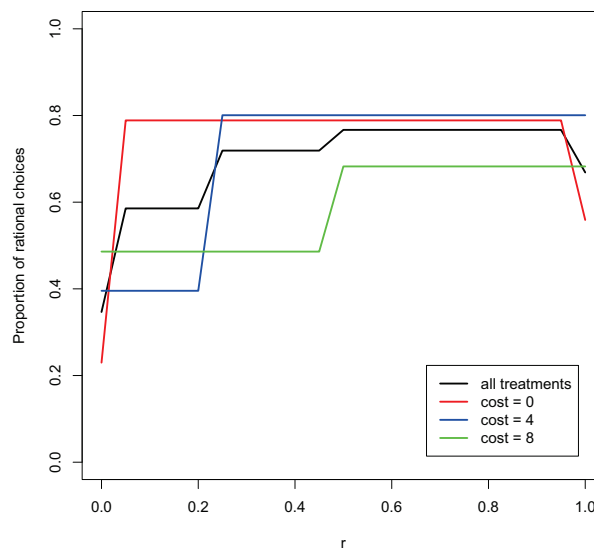


Fig. 13.16

Defector dynamics. A defector chose the *rational* action, i.e. ask for a new link, with frequency rate ~ 0.68 .

13.6.6 Instructions

Each participant read the following set of instructions in detail before the experiment started.

Explanation for this part of the experiment

(The following instructions were originally written in French.)

Welcome to this experiment! You will have to make decisions that will affect your income as well as the income of other participants. Although we express all income in terms of points, these points will be exchanged at the end of the experiment using the following exchange rate:

30 pts. = 1.- CHF

From now on, it is **strictly forbidden to talk with other participants**. If you have any questions, please contact the assistants. If you do not follow this rule, we will have to exclude you from the experiment. In this study, each one of the 20 participants interacts with some “neighbors” (other participants in the room) according to a network structure. At the beginning of the experiment all the participants have exactly four neighbors each but this may change during the experiment as you will see below.

During this experiment you will only see your direct neighborhood, i.e. the people that are directly connected to you in the network. You won’t be able to see what happens in the rest of the network (for instance, you won’t be able to know what the neighbors of your neighbors do).

At the beginning you’ll receive a non-renewable endowment of **200 pts**.

What is it about?

There will be at least 10 rounds in the game, but you will not be informed on the exact number. Each round has five stages:

1. Action choice
2. Link proposals
3. Information acquisition choice
4. Link acceptance decision
5. Feedback on payoffs

Those decisions are now explained in more detail below.

1. Action choice

In this first stage you will have to choose an “action” among these two options:

CARRE or CERCLE

Your neighbors will have to make the same decision. Your action is “unique” that is, you’ll play this same action against all your direct neighbors (you can’t use a different action against different neighbors). Your profit in each round will be calculated as a function of your current action and the strategies chosen by your neighbors.

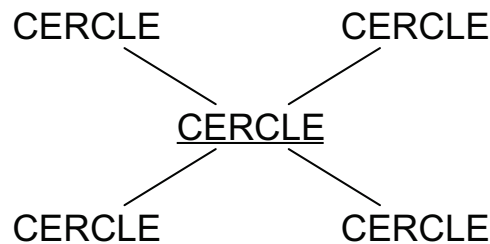
Here are the gains that you’ll get depending on your action and the action of your neighbors:

- you choose CARRE, a neighbor chooses CARRE:
your payoff is 0 points.
- you choose CARRE, a neighbor chooses CERCLE:
your payoff is 20 points.
- you choose CERCLE, a neighbor chooses CARRE:
you lose 10 points (-10 points).
- you choose CERCLE, a neighbor chooses CERCLE:
your payoff is 10 points.

Your accumulated payoff in each round will be calculated as the sum of the payoffs earned with each one of your current neighbors. The actual neighbors that count for the payoff computation are those with which you'll be linked **at the end of the round** since you will have the possibility of modifying your neighborhood in the next stages, as explained below.

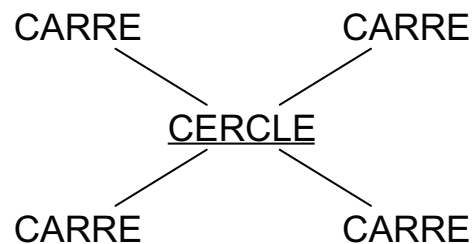
The following examples illustrate your payoff computation at the end of a round. Please note that in the examples you have four neighbors but this number could be different during the experiment. In the drawings you are represented as the central player, whose action is underlined.

Example 1: your action is CERCLE, the action of all your neighbors is also CERCLE.



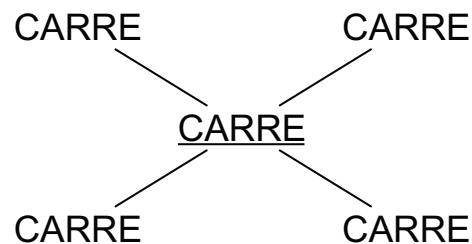
Your payoff is : $10 + 10 + 10 + 10 = 40$ points.

Example 2: your action is CERCLE, the action of all your neighbors is CARRE.



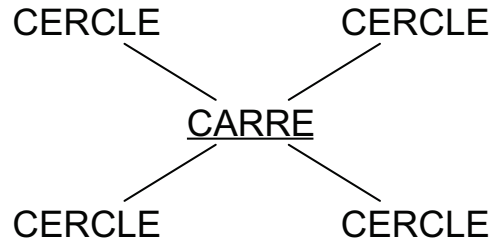
Your payoff is : $-10 - 10 - 10 - 10 = -40$ points.

Example 3: your action is CARRE, the action of all your neighbors is also CARRE.



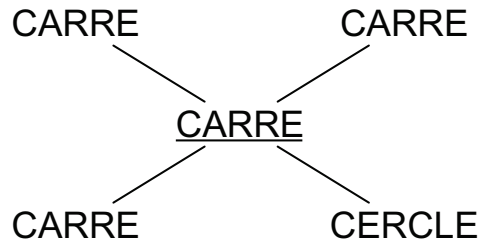
Your payoff is : $0 + 0 + 0 + 0 = 0$ points.

Example 4: your action is CARRE, the action of all your neighbors is CERCLE.



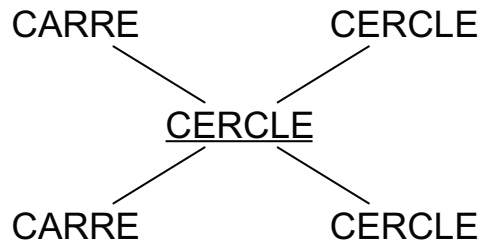
Your payoff is : $20 + 20 + 20 + 20 = 80$ points.

Example 5: your action is CARRE, the action of three of your neighbors is also CARRE while the action of the fourth neighbor is CERCLE.



Your payoff is : $0 + 0 + 0 + 20 = 20$ points.

Example 6: your action is CERCLE, the action of two of your neighbors is CARRE, while the action of the two remaining neighbors is CERCLE.



Your payoff is : $10 + 10 - 10 - 10 = 0$ points.

In this stage you will choose the action to adopt in the present round with the help of the following screenshot:

Vous êtes actuellement en lien avec -- participant(s)

Ma stratégie pendant ce tour:

☐ CARRE

☐ CERCLE

OK

("--" will be replaced by the actual number during the experiment)

2. Link proposals

In this second stage you will be requested to decide whether you would like to change your neighborhood. On the screen you'll see the number of neighbors of each action that you have in this round and you will have to choose one, and only one, of the following actions:

- do nothing in this stage.
- cut a link with a neighbor playing the action CARRE. The link will be deleted automatically.
- cut a link with a neighbor playing the action CERCLE. The link will be deleted automatically.
- create a new link with a randomly chosen participant who is not one of your current neighbors. This link will only be created if both participants accept it in the acceptance stage.

The modifications, if any, will be performed through the following screenshot:

Votre stratégie pendant ce tour: -----

Nombre de partenaires actuels qui ont choisi CARRE: --

Nombre de partenaires actuels qui ont choisi CERCLE: --

Voulez-vous couper un lien existant ou créer un nouveau lien?

☐ Non, merci.

☐ Oui, je veux créer un lien avec un nouveau partenaire

☐ Oui, je veux couper un lien avec un partenaire actuel qui a choisi CERCLE

☐ Oui, je veux couper un lien avec un partenaire actuel qui a choisi CARRE

OK

3. Information acquisition

In this stage you may want to know the action of each one of your potential partners. A potential partner is either the random participant that you have asked for, or one or more participants that are asking to create a link to you. The cost for uncovering the action information is **8 points** per potential partner. For example, if you decide to uncover the action of two among your potential partners, you'll pay $2 \times 8 = 16$ points. If you don't have enough points, you won't be able to acquire the information. In this stage you'll be able to implement the previous decision through the following screenshot:

The screenshot shows a game interface with the following elements:

- At the top, a text prompt: "Vous avez décidé de -----" followed by a dropdown menu showing "-- participant(s) demandent de créer un lien avec vous".
- A horizontal line separates this from the next section.
- Below the line, a text prompt: "Voulez-vous connaître la stratégie du participant avec lequel vous demandez de créer un lien?".
- Below this prompt are two radio buttons: "oui" (selected) and "non".
- Below the radio buttons, another text prompt: "Voulez-vous connaître les stratégies des participants qui demandent de créer un lien avec vous?".
- Below this prompt is a text input field.
- Below the input field, a text prompt: "Entrez le nombre de stratégies que vous voulez connaître (entre 0 et --)".
- At the bottom right, there is a red button labeled "OK".

4. Acceptance of new links

Finally, in this stage you are going to decide which links you would like to accept with your potential partners. If you paid to uncover the action of one or more partners in the previous step, those strategies will appear on the screen before you. On the other hand, if you chose not to pay to see them, the corresponding strategies will remain unknown. Please note that a given link will be actually created only if **both partners** have accepted it.

According to the decisions of all participants a new network will be formed. Your payoff for this round, after cost deduction, will be calculated as a function of your action and the action of your current neighbors in the actualized network.

You will complete this stage through the following screenshot:

Tour

Vous avez choisi de connaître les stratégies de - participant(s), parmi les - qui demandent un lien avec vous (vos partenaires potentiels).

La stratégie de - partenaire(s) potentiel(s) reste INCONNU(E)

Désirez-vous établir des liens - rec vos partenaires potentiels?

Nombre de liens que je désire établir parmi les partenaires potentiels dont la stratégie est CERCLE: ----

Nombre de liens que je désire établir parmi les partenaires potentiels dont la stratégie est CARRE: ----

Nombre de liens que je désire établir parmi les partenaires potentiels dont la stratégie est INCONNU(E): ----

OK

5. Feedback on payoffs

At the end of the current round, a screenshot will appear showing the number of links that have actually been created with you, the number of deleted links to you (note that this could happen even if you didn't ask to cut a link: one or more of your neighbors could have decided to cut the link with you), your current number of neighbors, the respective numbers of the two strategies in your neighborhood, your payoff for this round, and your total accumulated gain.

-- nouveaux liens ont été créés entre vous et d'autres participants

-- liens existants ont été coupés entre vous et d'autres participants

Vous êtes maintenant en lien avec - participant(s)

Votre stratégie pendant ce tour: ----

Nombre de partenaires qui ont choisi CERCLE: --

Nombre de partenaires qui ont choisi CARRE: --

Votre gain de ce tour: ----

Votre gain cumulé: ----

Did you correctly understand the instructions?

Before starting the experiment, we would like to be sure that you and the other participants have correctly understood the decisions that you are going to make. To this end, please answer the questions that will appear on the screen.

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Part IV
Concluding Remarks

Chapter 14

Conclusions

*“Science never solves a problem
without creating ten more.”*

George Bernard Shaw.

In this final chapter, we will try to assess what the contributions of the thesis are, what problems remain open, and what are the possibilities for further research. We divide the discussion in three parts. The first part considers games played on static and dynamic networks, the second part treats cooperation in spatial environments, while the third section focuses on a laboratory experiment dealing with pure coordination games played in diluted grids. In each section we present numerical simulation and experimental results and we list open problems and future research related to the topic.

14.1 Evolutionary games on static and dynamical networks

The finding that fixed Barabási-Albert (BA) scale-free networks of contacts notably increase cooperation in social dilemmas has been an important one [21] and has raised substantial hope, since scale-free graphs are better representations of actual networks of contacts than the random graphs and regular lattices that have often been used in the past. However, subsequent studies have somehow reduced its scope for various reasons. In the first place, the gains in cooperation can be partially or totally offset if players must pay an extra cost in order to maintain more contacts, as suggested by Masuda [14]. In a similar way if average, instead of accumulated payoff is used, the advantage of degree heterogeneity is lost as the individual’s payoff is divided by its degree in the graph [24]. Furthermore, if the players’ decision rule is partially blind imitation, some of the advantage is equally lost. For example, this has been shown to happen when agents have a conformist component to their behavior [17]. Finally, even when none of the above applies, the amount of cooperation gain due to network reciprocity can still be slim or non-existent depending on the strategy update rule and several other factors. This has been shown, among many other things, in the extensive studies of Roca et al. [20] where it appears that using deterministic best response as an update rule instead of an imitative rule such as replicator dynamics causes a serious loss of cooperation in the Prisoner’s Dilemma (PD) on BA scale-free networks which recover the mean-field case. In the work in Chapter 4 [5], inspired by the empirical observation that networks are never completely static, we have shown that several forms of random fluctuation of the network links lead to a marked loss of cooperation that affects all the games’ phase space, even for moderate amounts of noise. The result is robust because, irrespective of the precise form of network noise, the same phenomenon manifests itself: asymptotically cooperation tends to disap-

pear in the PD, and it diminishes in the other games. Moreover, network fluctuations appear to be more important than strategy noise in provoking a loss of cooperation. All the above refers to BA scale-free graphs and the general conclusion is that these population structures are not robust enough as cooperation amplifiers, as many factors may contribute to impair the ideal results. Network fluctuations, which certainly must occur in real-life, are among the most important factors. As a result, it can be said that, when the amount of noise is non-negligible, the system tends to behave in a mean-field way and thus the well-mixed population description seems to be adequate. Finally, the negative conclusion that cooperation in scale-free networks is hindered by exogenous random network dynamics, should be taken with caution. It is valid when strategy evolution and network dynamics are completely uncorrelated as it was the case in the above-mentioned study [5]. However, it has been shown that when cutting and forming links in a co-evolving network has a strategic dimension to it, then cooperation can thrive and be stable since severing and reforming links is purposeful and based either on game payoff, or on game-related considerations (see, for instance, [18]). From a social point of view, the difference is whether an agent can purposefully manipulate her environment, or is just under the influence of external network forces that she cannot control. In our opinion, both cases, as well as mixed situations may exist in reality.

However, it has to be said that these model networks, although similar in some sense, do not represent well enough actual social networks; for instance, they do not have enough clustering, community structure, and degree correlations, among others. In the work in Chapter 5 we have presented a systematic numerical study of standard two-person evolutionary games on two classes of social network models [6]. The motivation behind this choice is to make a further step towards more realism in the interacting agents population structure. The networks have been built according to Toivonen et al. model [25], one of several social network models used in the literature and, in part, according to the model proposed by Amaral et al. [1]. We have shown that the positive result about BA is true in general for this class of social networks models and the Amaral model, almost to the same extent as in BA networks. In addition, synchronous and asynchronous population update dynamics have been compared and the positive results remain true and even better for the asynchronous case when using imitation of the best update. We have also presented results for payoff schemes other than accumulated. In particular, we have studied average payoff and various proportions between the two extreme cases. The general observation is that pure average payoff gives the worst results in terms of cooperation, as already noted in [24]. When going from average to accumulated payoff cooperation tends to increase. With a view to the fact that actual social networks are never really static, in a similar way to what we did in [5], we have designed one among many possible mechanisms to simulate link fluctuations. When this kind of network noise is present cooperation tends to decrease and to disappear altogether when the network dynamics is fast enough. In conclusion, Toivonen's and Amaral's networks appear to be as favorable as scale-free graphs for the emergence of cooperation in evolutionary games. But, with respect to the latter, the additional advantages are that Toivonen and Amaral networks are much closer to actual social networks in terms of topological structure and statistical features.

One of the goals of this dissertation has been to compare the behavior of actual people under controlled conditions with the results indicated by theoretical models. In order to understand the role of a static network topology when humans are considered, we performed a laboratory experiment in which human subjects played a coordination game of the Stag Hunt class on random graphs and social-like networks with the same degree [8], see also Chapter 10. In this framework, numerical simulations of the setting suggest that populations of 20 players will end up in

a dimorphic state more often in a social-like cliquish network than in the random network. Also, players choose the efficient strategy more often in the random network. While we find that human subjects in a laboratory setting do converge more often to dimorphic states in cliquish networks than in random networks, there is no difference in terms of the proportion coordinating on the efficient outcome between the two topologies. Moreover, subjects do not use best-reply as update rule. Numerical simulations agree with laboratory results once we implement the actual updating rule that human subjects in our laboratory experiments use. This evidence suggests that numerical simulations can be a useful tool to understanding coordination in small scale societies. However, they should incorporate more empirical knowledge on their strategy update functions, which are currently too simplistic. These methods can then be updated and improved, hopefully not only for small scale but also for large scale societies, which are settings where laboratory studies are very costly to implement.

Finally, we have experimentally investigated the cooperation behavior in networks where participants playing a Prisoner's Dilemma game are allowed to update their connections by making and breaking ties [9], see also Chapter 13. While recent experiments have focused on contexts in which information on the types of players is either fully available or easy to obtain, such as in groups that interact frequently [19, 28], our conditions resemble to today's online networks in which a certain degree of anonymity is easy to achieve. The players only know how many of their current direct neighbors are cooperators and how many are defectors; information on the rest of the network and on the neighbors' payoff is not provided. A subject may suppress an unwanted relationship just by cutting the corresponding link unilaterally at no cost, but forming new links to an unknown participant can be done either "blindly" at no cost or by paying a certain cost to know the participant's current type, i.e., whether he or she is a cooperator or a defector, and the link is effectively created only if both partners agree to accept it. We experimented with two values for the cost of acquiring the information about the potential partner's type: one that is relatively low with respect to the payoff values used in the experiment, and another which is higher but not so high that a rational player would never choose to pay for it. We compared results in the costly setting with a baseline situation in which the information is available for free. In the baseline free treatment, the network quickly evolved toward full or almost full cooperation. At intermediate cost an interesting phenomenon emerges, wherein the population may evolve in two ways: either cooperators or defectors may be in the majority. In the high-cost case we are again in a rather clear-cut situation: introducing the high cost favors defection because participants are more reluctant to pay for linking information. The consequence is that cooperators have trouble finding other cooperators to form stable clusters, and the resulting networks are sparser. Our results suggest that there is a cost barrier that depends on the actual payoffs, beyond which people have less propensity to pay for information, in spite of the fact that knowing the potential partner type improves decision making. We believe that these conditions are representative of many of today's network relationships, in which a certain degree of anonymity can be maintained, and thus our results should be relevant in these contexts. As a consequence, cooperation is more difficult to achieve in situations in which costly or uncertain information about a partner's behavior is the rule. This work could be extended in a number of directions. For example, in the present setting, breaking a link has no cost and does not require the agreement of the current partner. However, there are situations in which this is not possible, e.g., when there is a contract or when breaking the link would entail adverse social consequences. One could also consider different payoffs for the game, such as having negative payoffs, and their influence on cooperation and network dynamics.

14.2 Cooperation in spatial environments

Mobility is a general feature of human and animal populations living in physical space. We discuss here one of the main conclusions reached in this dissertation on the role of random and contingent mobility in evolutionary games when players are embedded in a discrete or continuous spatial environment.

In the work in Chapter 6 [4], we have presented a systematic study of some population games by extensive numerical simulations in two-dimensional Euclidean space under two-person, one-shot game-theoretic interactions, and in the presence of agent random mobility. The goal was to investigate whether cooperation can evolve and be stable when agents can move randomly in space. Individuals in the population only interact with other agents that are contained in a circle of a given radius around the focal agent. In the first model, the velocity is the same for all agents in the population and it remains constant throughout the dynamics. Under fitness difference proportional update, the effect of mobility on cooperation is very small and there is little difference with the case in which the agents sit at the nodes of a random geometric graph and do not move. These results extend previous ones obtained by Meloni et al. [15] which were limited to the weak Prisoner's Dilemma region. However, when the imitation of the best neighbor rule is used instead, random mobility promotes cooperation in all the games' parameter space. The main mechanism leading to the evolution of cooperation is the random initial formation of small clusters of cooperators followed by other cooperators joining the initial clusters thanks to their mobility, together with defectors slowly becoming cooperators because of the latter higher payoff. In the second model agents do not move with constant scalar velocity; rather, velocity is assumed to be a negative exponential function of the agent's connectivity degree. This introduces a damping factor which can be seen as a kind of viscosity due to the accumulation of individuals around a given agent, leading to a more hindered random movement. The numerical simulation study of the average cooperation levels in this case leads to results that are qualitatively similar to those obtained in the constant velocity case. However, average values do not reveal the particular dynamics that are at work. To study this aspect, we have simulated a particular Prisoner's Dilemma game with two different velocity damping factors, one giving rise to low viscosity and the second to a higher viscosity. With low viscosity, starting with a uniform distribution of the agents in the plane, the system evolves toward the formation of dense monomorphic clusters of cooperators or defectors. In these clusters agents are almost at rest in the steady state and only individuals that have not joined a cluster still move. Under these conditions, contrary to the case with constant velocity, cooperation cannot spread past the cluster boundaries because of the lack of individual dispersion. With high viscosity the agents' movements are more hindered from the beginning but they are still able to join clusters of their kind. The situation is similar to the previous case, i.e. clusters of cooperators and defectors do form and remain stable, with the important difference that now they are much less dense and, consequently, the mean degree of the population is smaller. Again, viscosity and progressive velocity loss do not allow cooperation to spread to the whole population. One can thus conclude that random agent movements in physical space that take into account the natural fact that crowding effects have an effect on the agents' mobility may still lead to cooperative outcomes in many cases. However, the dynamics lead to cluster formation and condensation which hinders further spreading of cooperators especially in the harder Prisoner's Dilemma case. In future work we would like

to address the detailed study of cluster dynamics, the effect of strategy noise on the system evolution, and the effect of having agents of finite size instead of material points. Moreover, it would be of great interest to test these numerical results in the laboratory performing targeted experiments.

We have then explored some possibilities that arise when agents playing simple two-person, two-strategy evolutionary games may also move around in a certain region seeking better positions for themselves [10], see also Chapter 8. As already pointed out by other researchers [13], adding a form of contingent mobility may result in better capabilities for the population to reach socially valuable results. Among the existing models, we have started from a slightly modified form of the interesting Helbing and Yu's model [13] and have explored some further avenues that were left untouched in the latter work. In the model agents live and move in a discrete two-dimensional grid space in which part of the cells is unoccupied. Using a strategy update rule that leads an agent to imitate her most successful neighbor, and having the possibility to explore a certain number of free positions around oneself to find a better one, the gains in cooperative behavior are appreciable in the Prisoner's Dilemma. In the Hawk-Dove games the gains in cooperation are small but, in addition, we find that cooperation is fully promoted in the class of Stag Hunt games. Positive results are only obtained when agents interact locally in a relatively small neighborhood which, fortunately, seems to be a quite common condition in actual spatial systems. In addition, this behavior requires a minimum of heuristic decision making on the past of the agents. The model is thus more adequate for human or animal societies than for bacteria for instance.

Furthermore, it has been shown in several field studies that random walks of the Lévy flight type are an ubiquitous form of movement in animal and human populations [26]. These movements have the remarkable characteristics of being generally short but, from time to time, they can be very long, giving a special behavior to the agents' mobility patterns. In our study we started from this observation and we applied Lévy flights to the problem of cooperation in populations that are laid out on a two-dimensional discrete grid space with a given amount of free cells. Our results show that unrestricted mobility by Lévy flights alone does not allow cooperation to evolve when agents play a Prisoner's Dilemma game with their neighbors, which is the same conclusion reached in models where agents diffuse through standard random walks [4, 15, 22]. However, when an agent's movement is conditional on some property of the agent's neighborhood, the results are very different. We used a very simple and natural heuristic rule according to which both cooperators and defectors try to migrate by trying to do a Lévy jump to a vacant place when they are surrounded by a majority of defectors. In this case, all the Stag Hunt games lead to full cooperation, and the Hawk-Dove and Prisoner's Dilemma games are favorably influenced too. When migration is unrestricted there are two self-reinforcing negative effects working against cooperation. On one hand, cooperator clusters that may possibly develop are not stable enough because cooperators keep moving, preventing clusters to grow and leaving empty spaces at the border or inside the cluster. On the other hand, defectors may randomly jump into partially formed cooperator clusters and from there break the cooperator groups apart thanks to their superior payoff. When migration is contingent, the first effect above is no longer operating since cooperators are satisfied when they start to be surrounded by other cooperators: they do not jump away, and the mean cooperator cluster size increases as a result. Defectors may still enter a cooperator cluster but they find it more difficult to propagate into it. Overall, this explains why cooperation may evolve under contingent migration. There are still several open issues and questions that could be investigated. The present study was limited to the imitate the best strategy update rule. This choice has been motivated by the desire to compare our results with a number of previous related works that used the same

update rule [4, 10, 13, 15, 22]. It would certainly be interesting and enlightening to use other well-known rules, especially the biologically relevant ones. Also, Lévy flights might be coupled with a more intelligent exploration of the neighborhood at destination such that migration is more payoff-driven, similar to what has been suggested in Helbing and Yu [13] and Buesser et al. [10]. This approach could be interesting for populations endowed with higher rational capabilities.

Always in a two-dimensional space but with a rather different evolutionary dynamics we have studied a binary birth-death rule for the evolution of the players population [23], see also Chapter 9. We have shown that replacing imitation with this rule in spatial evolutionary games creates a new class of solutions of social dilemmas. If free expansion ranges are paired with limited exploitation possibilities, cooperative behavior dominates the Prisoner's Dilemma and the Stag Hunt game by means of unbounded self-organized expansion that sets in as soon as cooperators find a niche to expand. If defectors are given the opportunity to exploit cooperators more effectively through the application of larger interaction ranges, cooperative behavior may still thrive, although it relies on a special type of symmetry breaking that determines the direction of invasion based on the curvature of the interface that separates the two competing strategies. We note that all the presented results are robust to variations of the interaction network, and can be observed also in off-lattice simulations [3], thus indicating a high degree of generality.

Following our approach of having experimental counterparts to numerical models whenever possible, we tested the influence of short-range mobility in a laboratory experiment intended to shed light on the hitherto unclarified issue of the relevance of mobility in a geographical context to cooperation [7], see also Chapter 12. Indeed, important differences between random and purposeful motion in their ability to support cooperative interactions had been reported from a theoretical viewpoint, but experimental tests of those results were lacking. In the context of this previous literature, our most relevant result is that mobility does not promote cooperation: in fact, as in most experiments involving a Prisoner's Dilemma, we have found that the fraction of cooperators decays from an initial value close to half the population to residual ones of approximately a 20%, a value that is almost universally found in the laboratory. In fact, a vast majority of players can be classified as defectors or as moody conditionally cooperators, i.e., as players whose probability of cooperation depend not only on the actions of their partners but on their own previous actions, a type first identified in [11, 12, 27]. This indicates that the possibility to move around in space does not change very much the way players choose their actions. Remarkably, our experiments also contribute to the understanding of the possible assortment of cooperators in order to support cooperation. The numerical simulations reported in [16] suggested that cooperators may survive by forming clusters in which they mainly interact with other cooperators. In our experiments, we have indeed observed that such clusters appear with non-negligible frequency; however, their lifetime is quite limited because the possibility to move allows cooperating agents at the boundary of the cluster to separate from it to sever their interactions with defectors, or to choose defection themselves. This is in contrast to what has been observed on dynamic relational networks [19, 28], where allowing players to cut and make links at will does lead to clustering of cooperators and to an increase of the cooperation level. The reason for this difference can be traced back to the fact that, if links evolve indirectly by motion of the players in geographical space, they cannot be cut one by one, and when moving away from defectors play will many times cut also their links to cooperators. Therefore, we conclude that for clusters to be an important factor in the promotion of cooperation, individuals must have complete control on their choices of partners, a condition that has never been put forward before. Regarding mobility, we have found that players move

considerably at the beginning of the experiment, but the average fraction of individuals deciding to move decreases and by the end of the experiment only some 10%-20% of players are moving. We believe that this behavior is connected to the observation in the previous paragraph: players realize that the decision to move has very frequently pros and cons as it affects their connections in an indiscriminate manner, and at some point they conclude that they are not going to find a safe haven against defectors. On the other hand, it is worth noticing that in our experiment there is no punishment for interacting with a defector, and therefore all the residual motions observed in late stages must arise from spite, i.e., from subjects preventing others to benefit from them even if they are not harmed by those partners' actions. This is in agreement with our finding that cooperators tend to move somewhat more often than defectors, implying that while the latter just move trying to find others to exploit, cooperators have the additional motivation to punish defecting partners. In addition, we have also observed that mobility of all players is maximum when there is more or less the same number of cooperators or defectors in the neighborhood. Of course, to interpret these results one needs to bear in mind that when a player has many partners her mobility is also reduced by the lack of available cells to move to. With this caveat, it appears that when the number of cooperators and defectors is approximately the same around a given subject, she will try to move to increase her interaction with cooperators irrespective of her own action, as can be expected. On the contrary, when there are many neighbors of the same type, mobility becomes less relevant and perhaps impossible, this being the reason why we observe a maximum. In conclusion, we stress that the interaction between behavior and mobility does not seem to increase the level of cooperation in a human population set on a geographical framework. The main reason for this phenomenon turns out to be the fact that setting and breaking links cannot be done independently for every player as the mechanism for rewiring is motion in space. Interestingly, these results pose important questions about the emergence of cooperation in neighboring human groups, which could be most relevant in interactions in a socio-ecological context among hunter-gatherer groups, either in our recent evolutionary past or in presently existing populations. Furthermore, the type of mechanism we have unveiled with our experiment is most certainly not a very sophisticated way to make decisions, and therefore similar conclusions might apply to spatially structured populations of many other animals or bacteria. In this respect, our findings may provide a new perspective to interpret observational data on cooperative behavior in social animals, pointing to other behavioral traits (e.g., in terms of deciding to move or to change action) that coevolve with the geographical distribution of the population in its ecological environment.

14.3 Coordination in spatial environments

In this last section and always in connection with the agents' mobility issue, we discuss the main conclusions of our experimental work in Chapter 11 [2], where we consider a virtual spatial environment, i.e. a diluted grid, in which human subjects play a pure coordination game with two distinct possible equivalent choices and can move around. We study the behavior of people when changing convention is costly. Our main finding is that the type of information available to the population has a dramatic effect on the possibility of reaching general consensus. In the local treatment we have observed that the population segregates into two spatially isolated groups, one for each convention. We stress that we have never observed convergence to adopting a unique

convention in this setup. On the contrary, when global information is provided, people can reach higher levels of global coordination, albeit we observe two different scenarios. In the first one, equilibrium between the two conventions remains stable during the whole experimental session and people cannot recognize a dominant convention in the population. This leads to a final population structure that is very similar to that observed in the local treatment. However, when equilibrium between the two conventions is not stable and an appreciable majority begins to form, the knowledge of the convention dominating the population allows people to reach a consensus and to aggregate in a unique monomorphic cluster.

Beyond our findings in terms of consensus, we have been able to extract the behavioral pattern of the average individual. Thus, we have observed that, given that changing conventions is costly, subjects accordingly try to move when they are in a neighborhood with few occupied cells and those occupied adhere more to the opposite convention, and only change convention when they are in the presence of several neighbors, almost of all them agreeing. Based on these simple and quite natural rules, we have proposed a model that allows us to study the dependence of our results on variables such as “world” size, population density, and longer duration of interactions. The model, which is in good qualitative agreement with our experimental results, shows that in very small environments with only local information available, consensus can be reached as with global information, but after much longer interaction times. In larger systems, however, when global information is provided to individuals they can always converge to a unique convention in the population (given enough time), but local information alone leads to smaller coordination levels and spatial segregation. These results are qualitatively the same for a range of population densities, in so far as they are not so small as to prevent interaction, or so large that mobility is seriously hindered.

This experimental work has important implications in a number of real life contexts. Thus, our results suggest that when changing a convention is costly and interacting with the other convention detrimental, like in situations where a sizable group of people who should work together use different standards, or in collaborations where a common language is needed, population mobility is not enough to induce a common choice. Indeed, mobility would only lead to segregation by conventions, and when the segregation process is completed interaction ceases. On the basis of our results, one could think that providing information on how the adoption of conventions is going could help, but as we have seen something else is required, namely that this information conveys the feeling that an overwhelming majority is forming and that it is for the best to conform to their convention. While the adoption rates are still close, the process could still end up in segregation. Therefore, providing such global information as a way of policy making to encourage consensus is not enough, and should be complemented with incentives for part of the population to change their choice, so a majority begins to form. Even then, we have also seen that there are individuals who never change even when they are the last ones left of their choice. Whether this was due to a misunderstanding of the instruction or to unreasonable stubbornness is debatable, but in any event it is a clear indication that one cannot expect in general to achieve perfect consensus. In fact, in a large, mobile population such subjects may end up grouping and becoming a prejudicial influence. On the other hand, the fact that people are reluctant to change conventions is in good agreement with the daily life observation of the coexistence of choices (such as different phone companies, or operating systems, or even languages). When individuals are locally satisfied because they have many neighbors with the same preference, they do not have incentives to switch to the other option or to move. Of course, in the long run the rational preference should be to go along with

the majority, but as our simulations show, this may be a very long term limit (not dissimilar to the ultra long runs discussed by Young [29]), and therefore in practice those conventions may coexist forever. Thus, it becomes quite clear that the process of convergence to consensus may certainly be complicated and, while our experiment opens the door to understanding how mobile populations evolve or not towards global agreement, further research is needed to fully unveil all the effects relevant for such an ubiquitous problem.

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List of Publications

Publications arising from this thesis

In this manuscript

Part II, Chapter 4: Alberto Antonioni and Marco Tomassini (2011), *Network Fluctuations Hinder Cooperation in Evolutionary Games*, PLoS ONE 6(10): e25555.

Part II, Chapter 5: Alberto Antonioni and Marco Tomassini (2012), *Cooperation on Social Networks and Its Robustness*, Advances in Complex Systems 15(1): 1250046.

Part III, Chapter 10: Alberto Antonioni, Maria Paula Cacault, Rafael Lalive, Marco Tomassini (2013), *Coordination on Networks: Does Topology Matter?*, PLoS ONE 8(2): e55033.

Part II, Chapter 8: Pierre Buesser, Marco Tomassini, Alberto Antonioni (2013), *Opportunistic Migration in Spatial Evolutionary Games*, Physical Review E 88(4): 042806.

Part II, Chapter 6: Alberto Antonioni, Marco Tomassini, Pierre Buesser (2014), *Random Diffusion and Cooperation in Continuous Two-Dimensional Space*, Journal of Theoretical Biology 344: pp. 40-48.

Part II, Chapter 9: Attila Szolnoki, Alberto Antonioni, Marco Tomassini, Matjaž Perc (2014), *Binary Birth-Death Dynamics and the Expansion of Cooperation by means of Self-Organized Growth*, EPL (Europhysics Letters) 105(4): 48001.

Part III, Chapter 11: Alberto Antonioni, Angel Sánchez, Marco Tomassini (2014), *Global Information and Mobility Support Coordination Among Humans*, Scientific Reports 4: 6458.

Part III, Chapter 13: Alberto Antonioni, Maria Paula Cacault, Rafael Lalive, Marco Tomassini (2014), *Know Thy Neighbor: Costly Information Can Hurt Cooperation in Dynamic Networks*, PLoS ONE 9(10): e110788.

Part II, Chapter 7: Marco Tomassini and Alberto Antonioni (2015), *Lévy Flights and Cooperation Among Mobile Individuals*, Journal of Theoretical Biology 364: pp. 154-161.

Part III, Chapter 12: Alberto Antonioni, Marco Tomassini, Angel Sánchez (2015), *Short-Range Mobility and the Evolution of Cooperation: An Experimental Study*, submitted.

In related work

- (i) Enea Pestelacci, Marco Tomassini, Alberto Antonioni (2011), *Coordination Games on Small-Worlds: Artificial Agents vs. Experiments*, Advances in Artificial Life, ECAL 2011 Conference Proceedings: pp. 654-661.
- (ii) Alberto Antonioni, Attila Szolnoki, Matjaž Perc, Marco Tomassini. *Threshold-Driven Birth-Death Dynamics in Off-Lattice Populations*, under preparation.
- (iii) Alberto Antonioni, Marco Tomassini, Angel Sánchez. *Reputation vs. Cooperation: A Laboratory Experiment in Dynamical Populations*, under preparation.
- (iv) Brais Alvarez-Pereira, Alberto Antonioni, Claire Lagesse, Francesca Lipari, Luis A. Martínez-Vaquero, *The Chicken-Egg Dilemma: Homophily vs. Contagion*, under preparation.

Publications arising from side projects

Random Geometric Graphs

- (i) Alberto Antonioni and Marco Tomassini (2012), *Degree Correlations in Random Geometric Graphs*, Physical Review E 86(3): 037101.

Spatial Social Networks

- (i) Alberto Antonioni, Mattia Egloff, Marco Tomassini (2013), *An Agent-Based Model for Spatial Social Networks*, ECAL 2013 Conference Proceedings: pp. 226-231.
- (ii) Alberto Antonioni, Seth Bullock, Marco Tomassini (2014), *REDS: An Energy-Constrained Spatial Social Network Model*, ALIFE 2014 Conference Proceedings: pp. 368-375.
- (iii) Alberto Antonioni, Seth Bullock, Christian Darabos, Mario Giacobini, Bryan N. Iotti, Marco Tomassini, *Epidemiological Dynamics on REDS Networks*, under preparation.

In Santa Fe Institute Complex Systems Summer School 2014 Proceedings

- (i) Alberto Antonioni, Luis A. Martínez-Vaquero, Nicholas Mathis, Leto Peel, Massimo Stella (2014), Dynamical Game Theory, aka Drunk Game Theory (DGT).
- (ii) Cecilia Andreazzi, Alberto Antonioni, Alireza Goudarzi, Sanja Selakovic, Massimo Stella (2014), *Disease Spreading on Ecological Multiplex Networks*.
- (iii) Flávia M. Darcie-Marquitti, Degang Wu, Luis A. Martínez-Vaquero, Alberto Antonioni, Massimo Stella, Claudius Graebner, Blaž Krese (2014), *Persistence of Pollination Systems*.
- (iv) Alberto Antonioni, Alex Brummer, Morgan Edwards, Bernardo A. Furtado, James Holden, Michael Kalyuzhny, Claire Lagesse, Diana LaScala-Gruenewald, Ernest Y. Liu, Rohan Mehta (2014), *Can Simple Models Reproduce Complex Transportations Networks: Human Cities and Ant Colonies*.