A discusión

INFORMATION ACQUISITION IN AUCTIONS: SEALED BIDS VS. OPEN BIDS*

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ABSTRACT

This paper studies the incentives of a bidder to acquire information in an auction when her information acquisition decision is observed by the other bidders before they bid. Our results show that the sealed bid (second price) auction induces more information acquisition about a common component of the value than the open (English) auction, but less about the private component of the value. Moreover, under our assumptions more information about the private value and less information about the common value improves efficiency and revenue in some sense. Consequently, our results suggest new arguments in favor of the open auction.

JEL classification: D41, D44, D82.

Keywords: auctions, open information acquisition, asymmetric information.

1 Introduction

Most auction theory models assume that bidders have some private information. However, relatively little is known about the origin of this private information and in particular, the incentives of bidders to acquire it. This is not only an important theoretical question but also of practical concern in auction design. The reason is that more or less information acquisition affects the efficiency and the auctioneer's expected revenue in the auction.

In this paper, we shall study a bidder's incentives to acquire information about the value that she may get from the object for sale. In particular, we shall focus on the case in which a bidder's information acquisition decision is observed by the other bidders, i.e. *open information acquisition*. This model allows a richer theoretical problem than the model in which the bidder's information acquisition decision is not observed by the other bidders, i.e. *covert information acquisition*. The reason is that in the open model, information acquisition is a strategic variable in the sense that it may affect the other bidders' behavior, whereas this is not the case in the covert model.

Moreover, there are relevant real-life auctions in which the open information acquisition model is the most appropriate. For instance, bidders that want to acquire information in oil tract auctions use exploratory drills that are easily visible. There are also other cases in which the auctioneer can control whether the bidders' information acquisition decision is observable. Consider again the example of oil tract auctions. It is quite common that bidders that want to run exploratory drills must communicate it to the auctioneer who could decide whether to reveal it.

Even if we are only interested in problems in which the information acquisition decision is unobserved by the other bidders, it is always reasonable to wonder whether bidders have incentives to make public their information acquisition decisions. To answer that question, we need first to understand what happens when the information acquisition decision is observed.

In our paper, we shall compare the incentives to acquire information in two standard auction formats: a (second price) sealed bid auction, and an open (English) auction.

Both formats are similar in the sense that, in both cases, the winner pays the highest losing bid. However, they differ in one important aspect. The sealed bid auction is a static game in which the only information revealed occurs when the auction is over, whereas the open auction is a dynamic game in which there is a lot of information revelation along the game, namely all the losing bids. Thus, the comparison between these two auction formats is interesting not only by itself, but also because it isolates the effect of the information revealed in a dynamic auction on the incentives to acquire information. As a consequence, it sheds new light in the comparison between static and dynamic auctions.

The information revealed along the open auction implies two differences with the sealed bid auction. In the open auction bidders can, first, track the bid behavior of any bidder, for instance, the one who acquires information, and, second, infer in equilibrium the types of the bidders as they leave the auction. This is not the case in the sealed bid auction. Our analysis shows that the first difference is the key to understand why the incentives to acquire information openly in our two auction formats differ.[1]

Our results show that in a model in which the bidder's uncertainty about her value has private components and components common to all bidders, the incentives to acquire more information about the common components in the sealed bid auction are greater than in the open auction. However, the ranking is the opposite, at least if there are sufficiently many bidders, when the information acquisition is about the private component of the bidder's value.

Our results also show that once the open information acquisition decision of bidders is endogenized, in the symmetric equilibrium of the game there is more information

[1]The second difference may play a role in explaining the differences in bidders' incentives to acquire information but not under our assumptions. Intuitively, a bidder has less incentives to acquire information about her value if she expects to learn the other bidders' private information along the auction, at least if there is some kind of substitutability among the bidders' private signals. However, we rule out this effect with an assumption, independency of the bidders' private information. Our reason, as we explain in Section 2, is that the effect we comment in this footnote exists with open or covert information acquisition, and our aim is the study of the effects that are exclusive of open information acquisition.

acquisition in the open auction about the private value and less about the common value than in the sealed bid auction. Moreover, we derive from these results new arguments that suggest that the open auction may give greater expected revenue and be more efficient than the sealed bid auction.

The issue of bidders' open information acquisition in auctions has received very little attention. There are some partial results as a side-product in the work of Engelbrecht-Wiggans, Milgrom, and Weber (1983), and Hernando-Veciana (2004). Larson (2004) and Hernando-Veciana and Tröge (2005) provide an analysis of the value of private information in auctions when information acquisition is observable. These two papers, however, only study the open auction and information acquisition about the common value, whereas we also study the sealed bid auction and the value of additional information about the private value.

One possible explanation for this lack of attention are the technical difficulties inherent to the analysis of what has been called *asymmetric auctions*. These are auctions in which bidders differ from an ex ante point of view, or in other words, where the identity of bidders matters. Most of the work in auction theory requires the study of sophisticated mathematical models that can only be solved explicitly appealing to anonymity assumptions. In fact, it is complex to provide conditions that assure existence, see for instance Athey (2000), or uniqueness of the equilibrium, see Parreiras (2004) and Larson (2004), once asymmetries among bidders are allowed.

But, when bidders' information acquisition decisions are observed before the beginning of the auction game, we must necessarily consider asymmetric auctions. Even if we study a symmetric equilibrium in which bidders acquire the same level of information in equilibrium, we must solve the auction game for deviations of this symmetric equilibrium. In these deviations, a bidder takes a different information acquisition decision than the other bidders. Since the other bidders observe the choice of the bidder who deviates before the auction stage, we can no longer analyze the auction as a symmetric game. In fact, a side-product of our analysis is to provide an equilibrium analysis of these asymmetric auctions.

Thus, our paper is related from a technical point of view to a growing literature on asymmetric auctions in which one bidder is better informed than the others. Models particulary closed are those by Hernando-Veciana (2004), Larson (2004) and Hernando-Veciana and Tröge (2005) that have been already referred, and to an independent and simultaneous paper by Boone and Goeree (2005). This last paper differs in that it concentrates on the optimal auction design and does not consider the question of information acquisition.

There exist some other papers that have studied the problem of covert information acquisition. For instance, two early examples are Matthews (1984) and Lee (1985), and more recently, Persico (2000), Bergemann and Välimäki (2002), Hagedorn (2004) and Hernando-Veciana (2005). The difference with our approach is that in this case information acquisition is not a strategic variable in the sense that it cannot affect the bid behavior of the other bidders since it is not observable. The contribution of our paper is to address this strategic effect.

Compte and Jehiel (2002) also compare information acquisition in the sealed bid auction and the open auction. However, their paper differs from ours in that they study a pure private value model. Under this assumption it is irrelevant whether the information acquisition choices are observed since they do not affect the other bidders' bid behavior, at least in the auction formats that Compte and Jehiel (2002) study.

The rest of the paper is organized as follows. The next section provides the assumptions of the model: basically we study a two-stage game, in a first stage bidders decide how much information to acquire and in the second stage they participate in an auction game. We study the second stage in Section 3, and the first stage in Section 4. In Section 5, we study the implications of our results for the efficiency of the auction and the auctioneer's expected revenue. Finally, Section 6 concludes. We also include an Appendix with the most technical proofs.

2 The Model

We study a model in which one unit of an indivisible object is put up for sale to a set $I \equiv \{1, 2, ..., n\}$, where[2] $n \geq 3$, of risk neutral bidders whose values on the object for sale have private and common components. In particular, we assume that a generic bidder $i \in I$ puts a monetary value of $T_i + \sum_{j \in I} Q_j$ in the consumption of the good. Note that T_i (for taste) is a private value component as it only affects i's preferences whereas Q_i (for quality) is a common value component that affects all bidders' preferences.

Note that we are assuming additive separability of the utility function. This assumption simplifies our problem as it allows us a straightforward application of the techniques developed by Myerson (1981). Moreover, it also simplifies the comparison of the allocation implemented in each of the auction games we study. We could get similar results assuming additive separability only of the private and the common value,[3] i.e. between T_i and $(Q_1, ..., Q_n)$. We also conjecture that a marginal version of our results must also hold true when we relax this assumption. The reason is that any smooth function can be approximately linearized locally.

We shall assume that Bidder $i \in I$ observes privately a noisy signal informative of the common value component Q_i and the bidder's private value T_i. This signal is one element of a family $\{X_i^{\eta_i}\}_{\eta_i \in \mathcal{N}}$ in which the index η_i will be referred as Bidder i's *information precision*. We also introduce some ex ante symmetry assumptions and assume independency between the bidders' private information. In particular, we assume that the random vectors $(T_i, Q_i, (X_i^{\eta_i})_{\eta_i \in \mathcal{N}})$ are independent and identically distributed across bidders.

From a technical point of view, our assumption of independency of the bidders'

[2] We assume that $n \geq 3$ since our open and sealed bid auctions are strategically equivalent for $n = 2$.

[3] We assume additive separability among the common value components to guarantee that the bidders' expected utility is linear not only on the bidder's private component T_i but also on the bidder's common value component Q_i. We are interested in keeping this symmetry between the private and the common component to be able to derive the same notion of a more informative signal, see below, for the private and the common value component.

private information simplifies our problem since we can apply Myerson's (1981) techniques. However, there is a more important reason for this assumption. The bidder's information acquisition has two types of effects: *strategic* and *non-strategic*; the former one because a bidder's information acquisition, if observable, may affect the bids of the other bidders, and the latter one because a more informed bidder can take better decisions. Clearly, the non-strategic effects appear independently of whether the information acquisition is open or covert, whereas the strategic effects are exclusive of the open case.

The non-strategic effects and their differences across auction formats have already been studied in models of covert information acquisition. For instance, Hernando-Veciana (2005) shows that an adaptation of the arguments given by Persico (2000) implies a sealed bid auction gives larger incentives to acquire information covertly than the open auction when signals are affiliated. The contribution of our paper is the study of the strategic effects, and thus we would want to abstract from the non-strategic effects. This is what the assumption of independency of the bidders' private information does for us.

We shall impose some structure on our family of bidders' signals for two reasons. First, we want to associate higher realizations of the signals to higher willingness to pay, and second, we want higher values of η_i to denote more informative signals. We shall derive our assumptions from the results by Athey and Levin (2001).

Under our assumptions of additive separability and independency of the bidders' types, the utility of Bidder i in any auction game, and for some fixed strategies of the other bidders, is linear in both T_i and Q_i. Moreover, Bidder j's utility is also linear in Q_i for the other bidders' strategies fixed. An application of Lemma 1 in Athey and Levin (2001) shows that a sufficient condition to assure that a higher signal induces higher bidding for bidders with value functions that are linear in either T_i or Q_i or both is that $E[T_i|X_i^{\eta_i} = x]$ and $E[Q_i|X_i^{\eta_i} = x]$ are both increasing. In this sense we can say that high realizations of $X_i^{\eta_i}$ correspond to *good news* and low realizations to *bad news*.

To simplify notation in what follows, we normalize the marginal distribution of each private signal $X_i^{\eta_i}$ to be uniform in the interval $[0,1]$. To see why this normalization is without loss of generality, suppose that our original signal $X_i^{\eta_i}$ had a distribution function G that was not uniform. We could define a new signal $\hat{X}_i^{\eta_i} \equiv G(X_i^{\eta_i})$ which has a uniform distribution function on $[0,1]$. Moreover, $\hat{X}_i^{\eta_i}$ is equally informative since it is a monotone transformation[4] of $X_i^{\eta_i}$.

We use a concept of more informative signals that identifies a more informative signal with a more valuable[5] signal in the tradition of Blackwell (1951). However, instead of requiring that a more informative signal must be more valuable for any decision problem, as it was Blackwell's original approach, we restrict to the type of decision problems that bidders face in our model. As we have already argued, these are problems linear in the state. Note that another difference with Blackwell's approach is that we take as given the prior on the state.

Denote by $F_{V_i}^{\eta}(.|X_i^{\eta})$ the posterior of V_i, $V_i \in \{T_i, Q_i\}$, conditional on signal X_i^{η}, and S_{V_i} the support of V_i. Then:

Definition: We say that signal X_i^{η} is *more informative* of V_i, for $V_i \in \{T_i, Q_i\}$, than signal $X_i^{\eta'}$ if and only if,

$$E\left[\max_a \int u(v_i, a)\, dF_{V_i}^{\eta}(v_i|X_i^{\eta})\right] \geq E\left[\max_a \int u(v_i, a)\, dF_{V_i}^{\eta'}(v_i|X_i^{\eta'})\right],$$

for any function $u : S_{V_i} \times A \to \mathbb{R}_+$ linear in the first argument, continuous in the second one, and A compact.

However, we shall use in our analysis the following result that gives a more tractable characterization of more informative signals.

Lemma 1. *Signal X_i^{η} is more informative of V_i, for $V_i \in t\{T_i, Q_i\}$, than signal $X_i^{\eta'}$ if and only if, $E[V_i|X_i^{\eta} \leq x] \leq E[V_i|X_i^{\eta'} \leq x]$ for any $x \in [0,1]$.*

[4]The argument we have given only works for G continuous and strictly increasing in the support. See Athey and Levin (2001) and Lehmann (1988) for the general case.

[5]We could use instead an ad hoc concept of more informative signals. However, note that a definition with no economic content would always have two drawbacks. First, we can discuss why one definition and not another one. And second, we can always suspect that our results are a consequence of some of the artificial restrictions imposed by our definition.

Proof. The proof is a direct application of Theorem 1 of Athey and Levin (2001). ■

Note that the former characterization also has an intuitive meaning. Recall that low realizations of the signal mean bad news in the sense of a lower conditional expected value. Thus, what this characterization says is that a more informative signal makes bad news to become worse news. It is also straightforward to show that the above characterization is equivalent to a characterization in which all the inequalities are reversed. Intuitively, what this means is that our characterization also implies that a more informative signal makes good news better news.

We show in Appendix D that according to our definition, a signal X_i^η is more informative of V_i, $V_i \in \{T_i, Q_i\}$ than another signal $X_i^{\eta'}$ if and only if the distribution of the conditional expected value $E[V_i|X_i^\eta]$ is dominated in the sense of second order stochastic dominance by the distribution of the conditional expected value $E[V_i|X_i^{\eta'}]$. In this sense, we can say that acquiring more information increases the spread of the distribution of the conditional expected value.

We also introduce some regularity assumptions. We assume that the conditional expected values $E[T_i|X_i^\eta = x]$ and $E[Q_i|X_i^\eta = x]$ have continuous derivatives in x. We also assume that the function $\mu(x, \eta) \equiv E[Q_i|X_i^\eta = x] - E[Q_i|X_i^\eta \leq x]$ is increasing in x, for any[6] η. Finally, we assume that there exists a bound $\nu > 0$ such that,

$$\frac{1}{\nu} < \frac{\partial E[T_i|X_i^\eta = x]}{\partial x}, \frac{\partial E[Q_i|X_i^\eta = x]}{\partial x} < \nu, \text{ for any } x \in [0, 1]$$

Next example illustrates our assumptions:

> **Example A:** Suppose that Q_i and T_i, $\forall i \in I$, follow an independent uniform distribution with support $[0, 1]$, and X_i^η is a lottery that is equal to Q_i with probability $\eta_T(\eta)$, it is equal to T_i with probability $\eta_Q(\eta)$, and with

[6] A sufficient condition for this regularity assumption is that the cumulative distribution function of the conditional expected value $E[Q_i|X_i^\eta]$, is log-concave, an assumption satisfied by many distribution functions, see Bagnoli and Bergstrom (1989), for instance any distribution function of the form $F(q) = q^r$, $r \geq 1$, and any truncated exponential, normal, logistic, extreme-value, chi-square, chi, and Laplace distributions. This is also equivalent to the assumption that the inverse of the function $g(x) = E[Q_i|X_i^\eta = x]$ is log-concave.

the remaining probability, it is equal to an independent random variable with uniform distribution, where η_T and η_Q are weakly increasing functions with domain in $(\frac{1}{\nu}, \nu)$. To see that higher values of η means more informative signals, note that,

$$E[T_i|X^\eta = x] = \eta_T(\eta)x + (1 - \eta_T(\eta))\frac{1}{2},$$

and,

$$E[T_i|X^\eta \leq x] = \eta_T(\eta)\frac{x}{2} + (1 - \eta_T(\eta))\frac{1}{2}.$$

The functions that correspond to Q_i are identical but using η_Q instead of η_T.

We shall focus on two extreme models of information acquisition. In the first one, the *common value information acquisition model*, we assume that a higher information precision η corresponds to a more informative signal of the common value component, but it does not change the conditional expected private value $E[T_i|X_i^\eta = x]$ for any $x \in [0, 1]$. In the other model, the *private value information acquisition model*, we make the symmetric assumption, a higher η corresponds to a more informative signal with respect to the private value component, but it does not change $E[Q_i|X_i^\eta = x]$ for any $x \in [0, 1]$.

It is important to remark that in both models, the bidders' values have private and common value components and bidders have private information about both. The difference is the meaning of additional information. In the first model, to acquire more information means that the signal becomes more informative of the common value component but it remains equally informative about the private value component. The opposite happens in the second model.

Our two models of information acquisition can make sense in some real-life auctions. However, the main point is to offer two theoretical benchmarks that allow clear-cut comparative statics. As we shall see, the crucial assumption is that in one model a bidder's information acquisition changes how informative is her bid (assuming now and in what follows strictly increasing bid functions)[7] either with respect to the common or

[7] We focus on the informativeness of the action rather than in the informativeness of the signal for

the private value components, but not with respect to both components simultaneously. Other models have not made this distinction. In fact, our model seems to lie somewhere in between two opposite models.

On the one hand, models as Hernando-Veciana and Tröge (2005) in which bidder's information acquisition makes the bidder's bid more informative with respect to the common value but at the same time less informative with respect to the private value. This is a natural consequence of two-dimensional bidders' private information, where one dimension is informative of the common value and the other one of the private value.[8] The implicit assumption is that the good has several unrelated characteristics, some of them affect all the bidders' values in exactly the same fashion, and some others only affect to the value of one single bidder.

On the other hand, other models, for instance Bergemann and Välimäki (2002), have assumed the opposite. This is, an increase in the informativeness of the bidder's signal implies an increase in the informativeness of her bid with respect to both the common and the private value. The implicit assumption is that the bidder learns about some characteristics of the good for sale which are valuable to all bidders but more to her than to the other bidders.

We introduce an additional assumption in the common value information acquisition model. We assume that the function $\mu(x, \eta)$ is increasing in η. The reason for this assumption is that it makes our results stronger at a small cost, see after Lemma 3. Note that the definition of more informative signals only implies that $\mu(x, \eta)$ is

two reasons: the first one is to allow a homogeneous comparison of models with one or two dimensional signals, the second is that the strategic effects of information acquisition depend on how informative the bidder's actions are rather than in how informative the bidder's signals are.

[8]For example, consider a two-dimensional signal in which one dimension is perfectly informative of the private value component and the other dimension is completely uninformative, and another signal in which the first dimension is perfectly informative of the private value and the second dimension is perfectly informative of the common value. Certainly, the bids induced by the second signal are more informative of the common value than the first one, but it is easy to see that they are also less informative of the private value. Note, however, that there are examples in which the opposite happens: an increase in the informativeness of the private value dimension of a two-dimensional signal makes the induced bids more informative not only of the private value but also of the common value.

increasing in η for x equal to 0 or to 1. To understand the intuitive meaning of this additional restriction note that $\mu(x,\eta) = E[Q_i|X_i^\eta \geq x, X_i^\eta \leq x] - E[Q_i|X_i^\eta \leq x]$. Thus, we can interpret this last assumption as that a more informative signal makes good news in the sense of $\{X_i^\eta \geq x\}$ become better news when we also condition on $\{X_i^\eta \leq x\}$.

We also introduce an additional assumption in the private value information acquisition model, we assume that $E[T_i|X_i^\eta = 1]$ is strictly increasing. Note that our definition of more informative signals only implies that it is weakly increasing. This assumption is not essential for our analysis but makes the proof of the second item of Proposition 3 simpler.

To model open information acquisition decision we have in mind a two-stage model. In the first stage bidders choose independently and simultaneously the precision of their information at some cost. These decisions are made public at the end of the first stage. In a second stage, bidders bid in an auction game. We shall assume a quite standard structure for the auction stage: each bidder first observes privately the realization of the signal she chose in the first stage and then they all participate in an auction which we assume to be either an open (ascending) auction[9] (O) or a sealed bid (second price) auction[10] (S) with neither a reserve price nor an entry fee.

The simultaneity of the bidders' information acquisition decision may seem in con-

[9] We assume that the auction procedure is as follows, at every moment of time there are two types of bidders: active bidders and inactive bidders. Bidders are active until they manifest that they want to become inactive. Once a bidder has decided to become inactive her decision is irreversible. The identity of the active bidders is publicly observable along the auction. The price is also publicly observable and increases continuously from zero. At each moment in time bidders can decide to become inactive. The price stops increasing whenever there is no more than one active bidder. In this case, the remaining active bidder gets the unit for sale. If there is no active bidder when the price stops, the good is randomly allocated (with equal probability) among the bidders that quit at the last price. The price paid by all the winners is the price at which the auction stopped.

[10] In this auction set-up, all bidders submit simultaneously one bid each. The bidder who has submitted the highest bid gets the good at the price of the second highest bid. If two bidders submit the highest bid, the price equals this bid and the good is allocated randomly among all the bidders that submitted the highest bid, whereby all such bidders have the same probability of being selected.

flict with the assumption that the information acquisition decisions are observable before the auction game. One justification is that bidders information acquisition decision must be taken before the visible part of the process of information acquisition starts. For instance, an oil company may need permissions, hire the equipment and contract consulting services before starting a drill. A more basic reason for this assumption is that we want to focus on the incentives to acquire information abstracting from other strategic issues in the information acquisition game like first mover advantages, predatory strategies and endogenous timing. They seem relevant issues but more appropriate for extensions.

In the next sections, we analyze our model. We start with the auction games and finish with the information acquisition stage.

3 Analysis of the Auction Games

In this section we study the second stage of our game, the auction game. We start by providing an intuitive understanding of the bidders' incentives to change the bid which we use to analyze the strategic effects induced by open information acquisition. Later, we solve for the equilibrium of the auction games and show that the insights that we have learnt explain our equilibrium results.

In both auction formats, the sealed bid auction and the open auction, a bidder's bid only determines whether the bidder wins or loses the auction, but not the price that she pays when she wins, which is equal to the highest bid of the other bidders. In particular, the bidder wins if her bid is greater than the highest bid of the other bidders and loses otherwise.

As a consequence, an increase in the bid only affects the bidder because she can pass from losing to winning when the highest bid of the other bidders is between her old bid and her new bid. Certainly, if the increase in the bid is marginal, the only change happens when the bidder was tying with the highest bid of the other bidders. Thus, a bidder that bids p has incentives to increase (or similarly, decrease) her bid a marginal amount if her expected value of the good conditional on the event that the highest bid of the other bidders is equal to p is greater (respectively, less) than her bid p.

The event that the highest bid of the other bidders is equal to p is the intersection of two other events: the event that the highest bid of the other bidders is greater than p, and the event that the highest bid of the other bidders is less than p. The first event is good news about the common value, and hence it induces greater incentives to increase the bid. This event has been called the *loser's curse* as a bidder who ignores it will bid too low and may regret losing. The second event is bad news about the common value, and hence it induces lower incentives to increase the bid. This event has been called the *winner's curse* as a bidder who ignores it will bid too high and may regret winning.[11]

For the sake of simplicity, we shall only consider auction games in which one single bidder acquires different information than the other bidders. This analysis is sufficient to study equilibria of the first stage in which all bidders acquire the same level of information because we only need to consider deviations of one single bidder. We thus call this bidder the *deviating bidder* and denote her index by $d \in I$ and her information precision by η_d. We refer to the other bidders as *non-deviating bidders* and to their information precision by η.

The strategic effect associated to the information acquisition decision of the deviating bidder comes from the change in the bid behavior of the non-deviating bidders. We, thus, focus on the incentives to change the bid of the non-deviating bidders, first, in the open auction and, second, in the sealed bid auction.

The open auction is a dynamic game with several information sets. However, a non-deviating bidder generally ties with the highest bid of the other bidders only in information sets in which the bidder that makes that bid is the only other bidder who remains in the auction. Moreover, since we are interested in the incentives to change the bid of a non-deviating bidder only because its effect on the expected utility of the deviating bidder we focus on the case in which the deviating bidder is the only other

[11] The description of the winner's curse and loser's curse in terms of statistical events was first used by Pesendorfer and Swinkels (1997). They also introduced the notion of loser's curse in auctions.

remaining bidder.

In the information sets we mention above, the highest bid of the other bidders is certainly the deviating bidder's bid and thus the loser's curse means that her bid is greater than p. Moreover, the types of all the bidders who have already left the auction can be inferred in equilibrium from the price at which they have quit, and hence they are public information. Consequently, the winner's curse only means that the bid of the deviating bidder is less than p. We can illustrate these two effects with a formalization of the incentives of a non-deviating bidder, say Bidder i, with type x_i to change marginally her bid around p:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, \underbrace{b_d^O(X_d^{\eta_d}) \geq p}_{\text{loser's curse}}, \underbrace{b_d^O(X_d^{\eta_d}) \leq p}_{\text{winner's curse}}, \mathcal{X} \right] - p, \tag{1}$$

where $\mathcal{X}$ is the information about the other bidders inferred along the equilibrium path and b_d^O denotes the bid function used by the deviating bidder in the above information sets.

The incentives of a non-deviating bidder in a sealed bid auction are similar to the open auction but with two differences. The first one is that a given non-deviating bidder has uncertainty about the identity of the bidder that submits the highest bid of the other bidders: it may be the deviating bidder or another non-deviating bidder. This difference affects the loser's curse which in this auction set-up means that with some probability, say[12] ρ, the bid of the deviating bidder is greater than p and with the complementary probability it is the the bid of another non-deviating bidder.

The second difference is that there is no information revelation along the auction, and thus, the bidder does not have any additional information about the other bidders' types. This difference makes the winner's curse stronger in the sense that it gives bad news about the types of more other bidders.

[12]This probability must be computed conditional on the event that it is relevant for the bidder, this is that the highest bid of the other bidders is equal to p. Moreover, we shall see later that this probability depends on p and on the shape of the bidders' bid functions. We have abstracted from these complications in the current description to make our arguments clearer. Although we take them into account later in the equilibrium analysis.

We shall see that the difference in the loser's curse between our two auction formats explains our ranking of auctions in terms of incentives to acquire information. However, and because of our assumptions of additive separability and independency of the bidders' types, the difference in the winner's curse does not have any implication on the comparison of our auction formats.

Again, to illustrate the loser's curse and the winner's curse, we formalize the incentives of a non-deviating bidder with type x_i to change her bid marginally around a price p:

$$(1-\rho)\, E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, \underbrace{b_{nd}^{S}(X_j^{\eta}) \geq p}_{\text{loser's curse}}, \underbrace{b_d^{S}(X_d^{\eta_d}) \leq p, \{b_{nd}^{S}(X_l^{\eta}) \leq p\}_{l \neq d,i}}_{\text{winner's curse}} \right] +$$

$$\rho\, E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, \underbrace{b_d^{S}(X_d^{\eta_d}) \geq p}_{\text{loser's curse}}, \underbrace{b_d^{S}(X_d^{\eta_d}) \leq p, \{b_{nd}^{S}(X_l^{\eta}) \leq p\}_{l \neq d,i}}_{\text{winner's curse}} \right] - p, \quad (2)$$

where b_d^S denotes the bid function of the deviating bidder and b_{nd}^S the bid function used by all the other non-deviating bidders.

Consider first the common value information model. If the deviating bidder acquires a more informative signal, it increases both the loser's and the winner's curse of the non-deviating bidders, and hence affects the non-deviating bidders' incentives to change their bids. The final effect on the incentives to increase the bid is unclear. It may be positive if the effect on the loser's curse dominates and negative if it is the effect on the winner's curse which dominates.[13]

Nevertheless, while the increase of the winner's curse affects both auction formats with the same magnitude, the increase of the loser's curse is stronger in the open auction than in the sealed bid auction. The reason, as we explain above, is that the private information of the deviating bidder affects the loser's curse only with probability ρ. As a consequence, we expect the bids of the non-deviating bidders to increase less (or

[13]It may be shown that the effect is positive, i.e. induces higher bidding, for high prices and negative, i.e. induces lower bidding, for low prices, at least in the open auction. We do not explore this argument here because it is not essential for our results. Hernando-Veciana and Tröge (2005) have provided a more detailed analysis.

decrease more) in the sealed bid auction than in the open auction. In principle, this effect should give greater incentives to acquire information in the sealed bid auction than in the open auction. This is shown in Proposition 3.

Consider next the private value information model. In this case, there is no direct effect because the private information of the deviating bidder only affects the incentives of the non-deviating bidders through the common value, and by assumption the informativeness of the signal of the deviating bidder with respect to the common value is kept constant. However, a more informative signal induces a spread (in the sense of second order stochastic dominance, see Appendix D) of the deviating bidder's conditional expected private value $E[T_d|X_d^{\eta_d}]$ and, consequently, of her bids. It approximately means that high bids become higher and low bids become lower. We shall argue that this effect has the opposite consequences on the non-deviating bidders' bids, this is, it makes them less spread.

The fact that the deviating bidder makes higher her high bids affects the non-deviating bidders' incentives to submit high bids. The reason is that higher bids by the deviating bidder means that the bad news of the winner's curse becomes worse and the good news of the loser's curse not so good. Figure 1 illustrates these two effects. We can see that for a fixed price p sufficiently high, the steeper the bid function is, the lower the signals that the loser's curse indicates, and hence the less good the good news of the loser's curse are. Similarly, for a fixed price p sufficiently high, the steeper the bid function is, the lower the signals the winner's curse indicates, and hence, the worse the bad news of the winner's curse are.

The combination of both effects reduces the incentives of the non-deviating bidders to increase the bid and hence we would expect that in equilibrium the non-deviating bidders lower their high bids.[14] Symmetric reasons explain that low bids of the deviating bidder lower induces the non-deviating bidders to make their low bids higher.

The reduction of the spread of the bids of the non-deviating bidders does not have

[14]This change in the bid of the non-deviating bidders may induce a similar effect but of opposite direction in the deviating bidder's bids. Moreover, this new change of the deviating bidder's bids should reinforce the change in the non-deviating bids creating a feedback loop. This effect was already pointed out by Bulow, Huang, and Klemperer (1999) for the open auction.

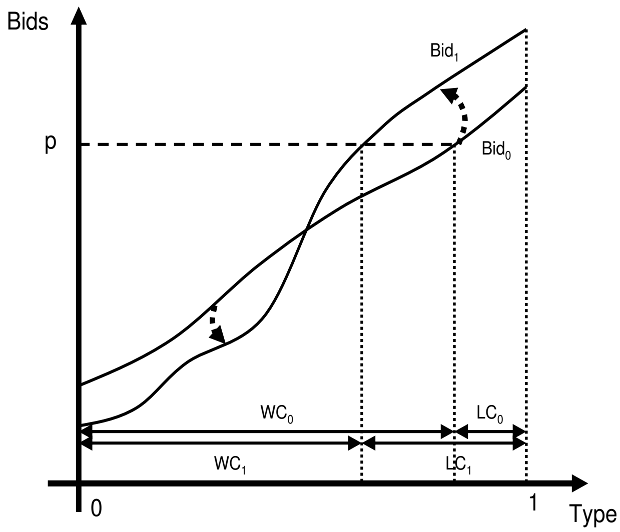

Figure 1: Change in the winner's curse ($WC_0 \rightarrow WC_1$) and loser's curse ($LC_0 \rightarrow LC_1$) for a high bid p as a consequence of a change in the bid function ($Bid_0 \rightarrow Bid_1$).

obvious consequences on the expected utility of the deviating bidder but in one case, when the number of bidders is sufficiently large. In this case, the highest bid of the non-deviating bidders is with high probability a high bid, and hence, the reduction of the high bids of non-deviating bidders is more important than the increase of their low bids. Consequently, this strategic effect should give greater incentives to acquire information. More importantly, this effect should be stronger, and thus the incentives to acquire information, in the open auction than in the sealed bid auction because of the difference in the loser's curse we pointed out above. This result is proved in Proposition 3.

We now move to the equilibrium analysis of our two auction games. This analysis will show that the effects studied in this section translate into some equilibrium results that support the ranking of auctions with respect to the incentives to acquire information that we have suggested.

Our equilibrium analysis will be based on the study of the allocation implemented in the equilibrium of each of the auction formats. In the case of an equilibrium in which all the non-deviating bidders use the same strictly increasing bid strategy (and the good is always sold), an allocation can be characterized by a function, the *allocation function*, that maps types of the deviating bidder into types of the non-deviating bidders. The good goes to the non-deviating bidder with highest type if its image is higher than the deviating bidder's type, and otherwise, it goes to the deviating bidder.

There are two reasons to focus on allocation functions. First, it may be shown that in our model, the comparison between the bids across auction formats is equivalent, in some sense, to the comparison of the allocations functions. Second, and more important, the incentives of the deviating bidder to acquire information depend on how her expected utility varies with changes in the her information precision, and, as we shall see in Lemma 5, the expected utility of a deviating bidder is characterized by the allocation function.

An allocation function maps types of the deviating bidder into types of the non-deviating bidders, if any, that make the same bid. We, thus, deduce the allocation functions that correspond to each auction format from some equilibrium conditions that relate these types. In particular, we use the condition that a bidder does not have incentives neither to increase not to decrease her bid marginally in equilibrium.

3.1 Allocation Function of the Open Auction

Denote by x_i and x_d the types of a non-deviating bidder, say Bidder i, and the deviating bidder, respectively, that bid in a given equilibrium the same price p in the information sets in which they are the only bidders who remain in the auction. Denote also by $\mathcal{X}$ the information that these two bidders can infer along the equilibrium path about the types of the other bidders. For the reasons explained at the beginning of the section, our equilibrium condition for the non-deviating bidder is:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \middle| X_i^{\eta} = x_i, X_d^{\eta_d} = x_d, \mathcal{X}\right] - p = 0, \tag{3}$$

and similar arguments let us deduce that our equilibrium condition for the deviating bidder is:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \middle| X_d^\eta = x_d, X_i^\eta = x_i, \mathcal{X}\right] - p = 0. \tag{4}$$

Subtracting both equations, and after some simplifications, we get the following equation that relates x_d and x_i:

$$E[T_d|X_d^{\eta_d} = x_d] - E[T_i|X_i^\eta = x_i] = 0. \tag{5}$$

Under our assumptions, the left hand side of Equation (5) is continuous in x_i and x_d, strictly increasing in x_d and strictly decreasing in x_i. Hence, it defines implicitly a strictly increasing function ϕ_O that maps types of the deviating bidder x_d into types of the non-deviating bidder x_i that solve the above equation and whose graph splits the set $[0,1]^2$ into two subsets.

The function ϕ_O defined above characterizes an allocation, however, it is convenient to make sure that the allocation function is defined in the domain $[0,1]$. We call the *extension* of ϕ_O in the domain $[0,1]$ to a function that we denote by $\hat{\phi}_O(x_d)$ and which is equal to 0 if x_d is to the left of the domain of ϕ_O, to $\phi_O(x_d)$ if x_d is in the domain of ϕ_O, and to one, if x_d is to the right of the domain of ϕ_O. Similarly, we denote by ϕ_O^{-1} the inverse of ϕ_O and by $\hat{\phi}^{-1}$ the extension of ϕ_O^{-1} to $[0,1]$. Figure 2 illustrates the extension of a function ϕ in $[0,1]$.

The following proposition shows that there exists an equilibrium that implements the allocation function $\hat{\phi}_O$. The reader may find a characterization of the equilibrium in its proof.

Proposition 1. *There exists an equilibrium of the open auction that implements the allocation function $\hat{\phi}_O$.*

Proof in the Appendix.

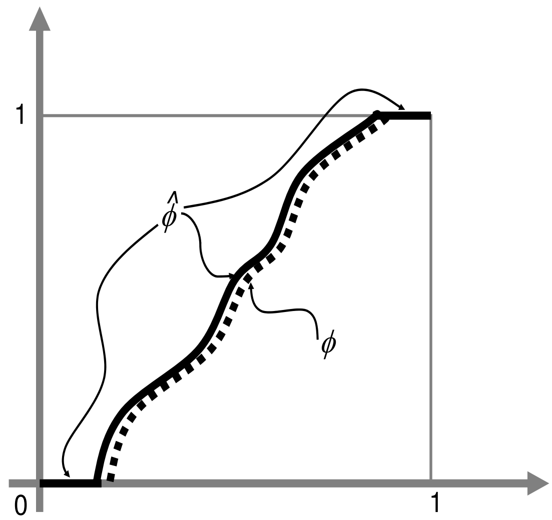

Figure 2: Extension of a function ϕ in $[0,1]$.

3.2 Allocation Function of the Sealed Bid Auction

In this subsection we analyze the sealed bid auction. Denote by x_i and x_d the types of a non-deviating bidder, say Bidder i, and the deviating bidder, respectively, who bid in a given equilibrium the same price p. Again for the reasons explained at the beginning of the section, our equilibrium condition for the non-deviating bidder is:

$$(1-\rho)\,E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = X_j^{\eta} = x_i, X_d^{\eta_d} \le x_d, \{X_l^{\eta} \le x_i\}_{l \neq d,i,j}\right] + \rho\,E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, X_d^{\eta_d} = x_d, \{X_l^{\eta} \le x_i\}_{l \neq d,i}\right] - p = 0, \quad (6)$$

where, ρ is the probability that i ties with the deviating bidder given that Bidder i ties with the maximum bid of the other bidders at price p, this is,

$$\rho \equiv \frac{\frac{x_i^{n-2}}{b_d^{S\prime}(x_d)}}{\frac{x_i^{n-2}}{b_d^{S\prime}(x_d)} + (n-2)\frac{x_i^{n-3}x_d}{b_{nd}^{S\prime}(x_i)}},$$

where b_{nd}^S and b_d^S denote the equilibrium bid functions of the non-deviating bidder and the deviating bidder, respectively.

Similarly, our equilibrium condition for the deviating bidder with type x_d is in this case:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = x_d, X_i^{\eta} = x_i, \{X_j^{\eta} \leq x_i\}_{j \neq d,i}\right] - p = 0. \tag{7}$$

Suppose now that ϕ_S is the allocation function implemented in the sealed bid auction. This means that $b_d^S(x) = b_{nd}^S(\phi_S(x))$, which implies by the implicit function theorem that $\phi_S'(x) = \frac{b_d^{S\prime}(x)}{b_{nd}^{S}{}'(\phi_S(x))}$. If we use this fact, and combine Equation (6) and Equation (7) eliminating p, we get after some algebra the following equation:

$$\begin{aligned}\phi_S'(x)\left(E[T_d|X_d^{\eta_d} = x] - E[T_i|X_i^{\eta} = \phi_S(x)] + \mu(x,\eta_d) - \mu(\phi_S(x),\eta)\right) = \\ \left[\frac{\phi_S(x)}{(n-2)x}\right]\left(E[T_i|X_i^{\eta} = \phi_S(x)] - E[T_d|X_d^{\eta_d} = x]\right).\end{aligned} \tag{8}$$

The right hand side of Equation (8) corresponds to the case in which the highest bid of the other bidders is the deviating bidder's, i.e. $\rho = 1$, as it was the case in the open auction. In fact, this right hand side equals to zero if $\phi_S(x) = \phi_O(x)$ in the domain of ϕ_O. This holds true in spite of the differences on the information on which the bidders condition in each auction because of our assumption of additive separability and independency of the bidders' types.

The left hand side of Equation (8) corresponds to the case in which the highest bid of the other bidders is the bid of another non-deviating bidder, i.e. $\rho = 0$. Making it equal to zero we get the following equation in x_i and x_d:

$$E[T_d|X_d^{\eta_d} = x_d] - E[T_i|X_i^{\eta} = x_i] + \mu(x_d,\eta_d) - \mu(x_i,\eta) = 0. \tag{9}$$

Under our assumptions, the left hand side of Equation (9) is continuous in x_d and x_i, strictly increasing in x_d and strictly decreasing in x_i. As a consequence, Equation (9) defines a implicit function ϕ_* that maps types of the deviating bidder x_d into types of the non-deviating bidders x_i. We define in a similar way to $\hat{\phi}_O$, the extension of ϕ_* and ϕ_*^{-1} on the domain $[0,1]$ and we denote them by $\hat{\phi}_*$ and $\hat{\phi}_*^{-1}$ respectively.

The next lemma shows that there exists a solution to the differential equation (8) that lies between $\hat{\phi}_O$ and $\hat{\phi}_*$. Note that one difference with ϕ_O is that this solution depends on n because Equation (8) does so. For this reason we denote by $\phi_{S,n}$ the solution, with a slight abuse of notation. We also define the extensions of $\phi_{S,n}$ and $\phi_{S,n}^{-1}$ on $[0,1]$ in a similar way as we defined $\hat{\phi}_O$ and $\hat{\phi}_O^{-1}$, and denote them by $\hat{\phi}_{S,n}$ and $\hat{\phi}_{S,n}^{-1}$ respectively.

Lemma 2. *For a fixed value of n, there exists a continuous strictly increasing function $\phi_{S,n}$ whose graph splits $[0,1]^2$ into two sets, and whose extensions satisfy that $\hat{\phi}_{S,n}(1) \geq \hat{\phi}_*(1)$ and $\hat{\phi}_{S,n}^{-1}(1) \geq \hat{\phi}_O^{-1}(1)$. Moreover, the graph of $\phi_{S,n}$ lies between the graph of ϕ_O and the graph of ϕ_*; and the graph of $\phi_{S,n+1}$ lies between the graph of $\phi_{S,n}$ and the graph of ϕ_*.*

Proof in the Appendix.

The conditions on the extensions of $\hat{\phi}_{S,n}$ and $\hat{\phi}_{S,n}^{-1}$ in Lemma 2 are necessary to ensure that the allocation function $\hat{\phi}_{S,n}$ can be implemented in an equilibrium of the sealed bid auction.

Note that Lemma 2 also states that the increase in the number of bidders make the allocation function $\hat{\phi}_{S,n}$ approach, in some sense, the auxiliary function $\hat{\phi}_*$. Intuitively, the greater is n, the less probable is that the highest bid of the other bidders is the bid of the deviating bidder, and thus, the closer the equilibrium allocation function should be to the auxiliary function $\hat{\phi}_*$. This result ensures that as n grows in Proposition 3 there is no counter effect to the differences in allocations between auction formats shown in Lemma 4.

The next proposition shows that there exists an equilibrium that implements the allocation function $\hat{\phi}_{S,n}$. Again, the reader may found a characterization of the equilibrium in the proof.

Proposition 2. *There exists an equilibrium of the sealed bid auction that implements the allocation function $\hat{\phi}_{S,n}$.*

Proof in the Appendix.

3.3 Comparison of the Equilibrium Allocation Functions

We shall show next how the equilibrium allocation functions $\hat{\phi}_O$ and $\hat{\phi}_{S,n}$ have the properties that correspond to the strategic effects that we analyze at the beginning of the section.

Our first result shows that the increase in the information precision in the common value information model allows the deviating bidder to win more often in the sealed bid auction than in the open auction. The opposite happens when the deviating bidder decreases her information precision.

Lemma 3. *In the common value information model and for any* $x \in [0,1]$*:*

- *If* $\eta_d > \eta$*, then* $\hat{\phi}_{S,n}(x) \geq \hat{\phi}_O(x) = x$*.*
- *If* $\eta_d < \eta$*, then* $\hat{\phi}_{S,n}(x) \leq \hat{\phi}_O(x) = x$*.*

Proof. The definition of $\hat{\phi}_O$ in Equation (5) implies that $\hat{\phi}_O$ is invariant to changes in η_d in the common value information model. Moreover, our symmetry assumption implies that $\hat{\phi}_O(x) = x$ for all $x \in (0,1)$. For the sealed bid auction we analyze the case $\eta_d > \eta$, the other case is symmetric. By our additional assumption in the common value information model $\mu(x,\eta_d) > \mu(x,\eta)$, which means that,

$$E[T_d|X_d^{\eta_d} = x] - E[T_i|X_i^{\eta} = x] + \mu(x,\eta_d) - \mu(x,\eta) > 0.$$

Since $E[T_i|X_i^{\eta} = x] + \mu(x,\eta)$ is increasing in x, to satisfy Equation (9), $\phi_*(x)$ must be greater than x. Since $\hat{\phi}_O(x) = x$, the application of Lemma 2 concludes the proof. ∎

A version of Lemma 3 also holds when we do not assume that $\mu(x,\eta)$ is increasing in η. In that case, however, the result is only true for x close to one. We could still derive the first item of Proposition 3, but in this case only for n sufficiently large.

Our second result looks at the private value information model. We show that the increase in the information precision of the deviating bidder makes her win more often in the open auction than in the sealed bid auction, at least for high bids, i.e. realizations of the private signal close to one. The opposite happens when the deviating bidder decreases her information precision.

Lemma 4. *In the private value information model and for any x sufficiently close to one:*

- *If $\eta_d > \eta$, then $\hat{\phi}_O^{-1}(x) < \hat{\phi}_{S,n}^{-1}(x) < \hat{\phi}_*^{-1}(x) < x$.*
- *If $\eta_d < \eta$, then $\hat{\phi}_O(x) < \hat{\phi}_{S,n}(x) \leq \hat{\phi}_*(x) < x$.*

Proof. We only study the case $\eta_d > \eta$, the other one is similar. By our assumptions in the private value information acquisition model $E[T_d|X_d^{\eta_d} = 1]$ strictly increases with η_d, which means that $E[T_d|X_d^{\eta_d} = 1] - E[T_i|X_i^{\eta} = 1] > 0$, and thus that $\phi_O^{-1}(1) < 1$, and,

$$E[T_d|X_d^{\eta_d} = \phi_O^{-1}(1)] - E[T_i|X_i^{\eta} = 1] + \mu(\phi_O^{-1}(1), \eta_d) - \mu(1, \eta) < 0.$$

Moreover,

$$E[T_d|X_d^{\eta_d} = 1] - E[T_i|X_i^{\eta} = 1] + \mu(1, \eta_d) - \mu(1, \eta) > 0,$$

since $\mu(1, \eta)$ is constant with respect to η in the private value information model. We can thus conclude that $\phi_O^{-1}(1) < \phi_*^{-1}(1) < 1$. An application of Lemma 2 implies that $\phi_O^{-1}(1) < \phi_{S,n}^{-1}(1) = \phi_*^{-1}(1) < 1$, which implies the lemma by continuity and by Lemma 2. ■

4 Analysis of the Game of Information Acquisition

In this section, we study the first stage game, the *game of information acquisition.* In this game bidders choose simultaneously and independently an information precision η each at some monetary cost that we do not model explicitly yet. We assume that the bidders' continuation payoffs are those that correspond to the equilibrium in Propositions 1 and 2 in the former section.

To compute the continuation payoffs, note that under our assumption of independent types, and by the arguments of the analysis of Myerson (1981), the allocation function and the expected utility of the minimum types characterizes the expected utility of the bidders. Moreover, the additive separability of the bidders' utility function makes specially simple the expression of the bidder's expected utility. Next lemma shows these claims.

Lemma 5. *Suppose that there exists an equilibrium for a given auction mechanism in which the allocation function ϕ is implemented. Then, the ex-ante expected utility of the deviating bidder in this equilibrium of the auction mechanism is equal to:*

$$\int_0^1 (1-x)\phi(x)^{n-1} \frac{\partial E[T_d + Q_d | X_d^{\eta_d} = x]}{\partial x} dx$$

plus the expected utility that the deviating bidder gets when she has type 0.

Proof. Straightforward adaptation of the arguments by Myerson (1981). ■

We denote by $U^a(\eta_d, \eta)$ the expected utility of a deviating bidder in auction format $a \in \{S, O\}$ when her information precision is η_d and all the other bidders' information precision is equal to η. We also define $\frac{\Delta U^a}{\Delta \eta}(\eta_d, \eta) \equiv \frac{U^a(\eta_d,\eta) - U^a(\eta,\eta)}{\eta_d - \eta}$ and call it the *incentives to acquire information.* Note that $\frac{\Delta U^a}{\Delta \eta}(\eta_d, \eta)$ is the expected gains, or losses, that a deviating bidder gets when she acquires more, respectively less, information than the others and divided by $\eta_d - \eta$.

There is some kind of revenue equivalence between both auction formats when all bidders have the same level of information precision, in the sense that $U^O(\eta, \eta) = U^S(\eta, \eta)$. The reason is that in a symmetric equilibrium, the allocation function is the identity in both auction formats and the minimum type always loses and thus gets zero expected utility. This implies that the comparison of the incentives to acquire information is equivalent to the comparison of $U^O(\eta_d, \eta)$ and $U^S(\eta_d, \eta)$ which depends basically on the allocation functions $\hat{\phi}_O$ and $\hat{\phi}_{S,n}$. The next proposition, which is central in our analysis, makes use of this feature:

Proposition 3.

- *In the common value information model, the sealed bid auction gives greater incentives to acquire information than the open auction, in the sense that for any $\eta_d \neq \eta$, $\frac{\Delta U^O}{\Delta \eta}(\eta_d, \eta) \geq \frac{\Delta U^S}{\Delta \eta}(\eta_d, \eta)$.*
- *In the private value information model, the open ascending auction gives greater incentives to acquire information than the sealed bid auction, in the sense that for any $\eta_d \neq \eta$, $\frac{\Delta U^O}{\Delta \eta}(\eta_d, \eta) \geq \frac{\Delta U^S}{\Delta \eta}(\eta_d, \eta)$ if n is large enough.*

Proof. From the arguments above we can conclude that the first item only requires to prove that $U^O(\eta_d, \eta) - U^S(\eta_d, \eta)$ is positive if $\eta_d > \eta$ and negative otherwise. This is a more or less straightforward consequence of Lemmas 3 and 5. The only difficulty that arises in the proof is with respect to the expected utility of a deviating bidder with type zero. This type gets zero expected utility in the open auction because she loses with probability one as $\hat{\phi}_O(0) = 0$. Moreover, this type gets non-negative expected utility in the sealed bid auction if $\eta_d > \eta$ because then $\hat{\phi}_{S,n}(0) \geq 0$; and zero expected utility if $\eta_d < \eta$ because then $\hat{\phi}_{S,n}(0) = 0$.

The second item is slightly more complicated. We start with the case $\eta_d > \eta$ which is simpler. We can easily derive from Lemmas 4 and 5 that:

$$\lim_{n\to\infty} U^O(\eta_d, \eta) = \int_{\hat{\phi}_O^{-1}(1)}^{1} (1-x) \frac{\partial E[T_d + Q_d | X_d^{\eta_d} = x]}{\partial x} dx. \tag{10}$$

Note that we ignore the expected utility of the deviating bidder with type zero. The reason is that it tends to zero as n grows to infinity since $\hat{\phi}_O^{-1}(1) > 0$ and thus, the probability that this type wins goes to zero.

A similar argument holds for the sealed bid auction. The only difference is that we also need to use the monotonicity of $\hat{\phi}_{S,n}(x)$ with respect to n in Lemma 2 and the monotone convergence theorem. The corresponding limit is:

$$\lim_{n\to\infty} U^S(\eta_d, \eta) = \int_{\hat{\phi}_{S,\infty}^{-1}(1)}^{1} (1-x) \frac{\partial E[T_d + Q_d | X_d^{\eta_d} = x]}{\partial x} dx, \tag{11}$$

where $\hat{\phi}_{S,\infty}^{-1}(1)$ is the limit of $\hat{\phi}_{S,n}^{-1}(1)$.

That the limit in Equation (10) is strictly less than the limit in Equation (11) follows from an application of Lemma 4. We can thus conclude that there must exists a bound on the number of bidders such that for any n above this bound, it holds true that $U^O(\eta_d, \eta) > U^S(\eta_d, \eta)$ for $\eta_d > \eta$.

The case $\eta_d < \eta$ has a similar proof although in this case, we have to divide both expected utilities by $\hat{\phi}_O(1)^n$ before we compute their limits and compare them. Otherwise both limits are equal to zero, and hence, do not provide any information to the comparison for a finite n. ■

An additional question of interest is whether the strategic effects in our model are such that they make the returns of additional private information negative. This was for instance the concern of the work of Larson (2004), and Hernando-Veciana and Tröge (2005). To give an answer to this question, it refine our concept of more informative signals. In particular, we assume in the remaining of the paper that: $\frac{\partial E[Q_i|X_i^\eta=x]}{\partial x}$ is strictly increasing with respect to η in the common value information acquisition model;[15] and that $\frac{\partial E[T_i|X_i^\eta=x]}{\partial x}$ is strictly increasing with respect to η in the private value information acquisition model. This refinement of the concept of more informative signals has been used before by Hagedorn (2004), and intuitively means that the expected posterior is more sensitive to the signal.

Proposition 4.

- *In the common value information model, the incentives to acquire information are strictly positive in both auction formats, in the sense that* $\frac{\Delta U^a}{\Delta\eta}(\eta_d,\eta) > 0$ *if* $\eta_d \neq \eta$ *and for* $a \in \{O, S\}$.
- *In the private value information model, the incentives to acquire information are strictly positive in both auction formats, in the sense that* $\frac{\Delta U^a}{\Delta\eta}(\eta_d,\eta) > 0$ *if* $\eta_d \neq \eta$ *and for* $a \in \{O, S\}$*, if the number of bidders is sufficiently large.*

Proof. The first item is a direct consequence of Lemmas 3 and 5 and our assumption that $\frac{\partial E[Q_i|X_i^\eta=x]}{\partial x}$ is strictly increasing with respect to η. The second item can be proved with a similar argument using Lemmas 4 and 5 and our assumption that $\frac{\partial E[T_i|X_i^\eta=x]}{\partial x}$ is strictly increasing with respect to η. The only difference is that we have to take limits with respect to n and argue that the sign in the limit still holds for n sufficiently large. ■

The first result contrasts with the results of Hernando-Veciana and Tröge (2005) which imply that additional information about the common value may decrease the

[15]To see that this new assumption is a refinement of our concept of more informative signals note that $\frac{\partial E[V_i|X_i^\eta=x]}{\partial x}$ strictly increasing with respect to η for all $x \in [0,1]$ implies that $E[V_i|X_i^\eta \leq x]$ decreases with respect to η for all $x \in [0,1]$.

expected utility of the bidder. The reason for this discrepancy is that in the model of Hernando-Veciana and Tröge (2005), when one bidder acquires more information about the common value, her bid becomes less informative of the private value, and in our model we have explicitly ruled out this effect. See our discussion in page 12.

Finally, we show that the results derived in this section may be used to prove that the auction with greater incentives to acquire information induce more information acquisition. To prove so, we introduce some additional assumptions. The reason is to assure that there exists a unique symmetric equilibrium in the game of information acquisition. We follow a similar approach to Persico (2000).

We assume that $\mathcal{N}$ is an interval $[0,1]$ and that the cost of acquiring information precision is the same for all bidders and equal to $C(\eta) = \frac{\alpha}{2}\eta^2$, for $\alpha \in (0,\infty)$. We also require the function $U^a(\eta_d, \eta)$ to satisfy certain technical assumptions. In particular, we assume it to have continuous differentials with respect to both η_d and η. Finally, we assume that the limit properties with respect to n in the second bullet of Propositions 3 and 4 hold not only pointwise as they are stated but uniformly in the set $\{(\eta, \eta_d) \in \mathcal{N}^2 : \eta \neq \eta_d\}$.

Proposition 5.

- *In the common value information model and if α is sufficiently large, there exist a unique symmetric Nash equilibrium in the game of information acquisition in both the sealed bid and the open auction, and in this equilibrium there is more information acquisition in the sealed bid auction than in the open auction.*

- *In the private value information model and if α and n are sufficiently large, there exist a unique symmetric Nash equilibrium in the game of information acquisition in both the sealed bid and the open auction, and in this equilibrium there is more information acquisition in the open auction than in the sealed bid auction.*

Proof. We start with the first item. A necessary condition for η to be a symmetric Nash equilibrium is that $\frac{\partial U^a}{\partial \eta_d}(\eta, \eta) = C'(\eta)$, $a \in \{O, S\}$. This equation has a unique solution for α sufficiently large follows since $C'(\eta) = \alpha\,\eta$, $\frac{\partial U^a}{\partial \eta_d}(\eta, \eta) \geq 0$ by Proposition 4 and

U^a has continuous differentials. This implies uniqueness of the equilibrium. We denote by η_* the solution to the former equation and only candidate for an equilibrium point. To prove existence, it is sufficient to show that $\frac{\Delta U^a}{\Delta \eta}(\eta_d, \eta_*) - \frac{C(\eta_d)-C(\eta_*)}{\eta_d - \eta_*} < 0$ if and only if $\eta_d < \eta_*$. This is the case for α sufficiently large since $\frac{C(\eta_d)-C(\eta_*)}{\eta_d-\eta_*} = \frac{\alpha}{2}(\eta_d + \eta_*)$. Once we have proved existence and uniqueness of the equilibrium, the result in the first item of Proposition 3 imply that there is more information acquisition in the sealed bid auction than in the open auction. The proof of the second item is similar but we need to start choosing an n large enough so that the results in Proposition 3 and Proposition 4 apply. ■

5 Revenue and Efficiency

We can derive from the results in Proposition 5 some conclusions with respect to the revenue and efficiency comparison of our two auction formats. To do so, we shall assume in this section that the equilibria that correspond to Proposition 5 are played in each of our two auctions.

We start analyzing efficiency. We shall distinguish two concepts of efficiency. We talk of the *ex post efficiency* as the expected value of the winning bidder.[16] This concept captures the allocative efficiency of the auction. We also talk of *ex ante efficiency* as the expected value of the winning bidder net of the information acquisition costs in which all the bidders incur, in a symmetric equilibrium $n\,C(\eta)$.

We first provide results for *standard-symmetric auctions*. These are auctions in which all bidders have the same information precision, the bidder with highest type wins, and only the winning bidder makes a payment to the auctioneer. Note that our two auction formats are standard-symmetric when all bidders have the same information precision, as it happens in the equilibrium path in Proposition 5.

Lemma 6. *In a standard-symmetric auction:*

[16] A social planner may be interested in ex post efficiency to avoid costly renegotiation after the auction. Moreover, note that allocating a monopoly license to the minimum cost bidder maximizes the consumer surplus.

- *The ex post efficiency of the auction is increasing in η in the private value information acquisition model and remains constant in the common value information acquisition model.*

- *The ex ante efficiency of the auction is decreasing in η in the common value information acquisition model.*

Proof. The ex post efficiency is equal to $E[\max\{E[T_i|X_i^\eta]\}_{i\in\mathcal{I}}] + n\,E[Q_i]$. That this expression is constant in the common value information acquisition model is obvious. This also implies the second bullet, i.e. that the ax ante efficiency of the auction is decreasing in η in the common value information acquisition model. The proof that ex post efficiency increases with η is a direct consequence of the convexity of the maximum function and the fact that a more informative signal in the private value information acquisition model spreads the distribution of $E[T_i|X_I^\eta]$ in the sense of second order stochastic dominance, see Lemma 8 in Appendix D. ■

Corollary 1.

- *The open auction induces greater ex post efficiency than the sealed bid auction in the private value information acquisition model, at least when the number of bidders is sufficiently large.*

- *The open auction induces greater ex ante efficiency than the sealed bid auction in the common value information acquisition model.*

Finally, the next lemma studies the auctioneer's expected revenue.

Lemma 7. *In a standard-symmetric auction:*

- *The expected revenue of the auctioneer is decreasing in η in the common value information acquisition model.*

- *The expected revenue of the auctioneer is increasing in η in the private value information acquisition model if the number of bidders is sufficiently large.*

Proof. The expected revenue of the auctioneer is equal to the surplus generated in the auction minus the bidders' expected utility, both gross of information acquisition costs. The former is equal to the expected value of the bidder who wins, i.e.:

$$\int_0^1 E[T_i|X_i^\eta = x]\, dx^n + n\, E[Q_i], \tag{12}$$

and the latter can be computed from Lemma 5 taking expectations with respect to the bidder's type and after some algebra to be equal to:

$$\int_0^1 (1-x)\frac{\partial E[T_i + Q_i|X_i^\eta = x]}{\partial x}\, dx^n. \tag{13}$$

The first result follows from the fact that Equation (12) is constant and Equation (13) is increasing with respect to η in the common value information acquisition model. To prove the second result, we can combine Equations (12) and (13) to show after some algebra that the auctioneer's expected revenue is equal to:

$$\int_0^1 \left(E[T_i|X_i^\eta = x] - (1-x)\frac{\partial E[T_i|X_i^\eta = x]}{\partial x} \right) dx^n + \\ n\, E[Q_i] - \int_0^1 (1-x)\frac{\partial E[Q_i|X_i^\eta = x]}{\partial x}\, dx^n.$$

The second integral is constant with respect to η in the private value information acquisition model, whereas the integrand of the first integral is increasing in η for values of x close to one. Consequently, for values of n sufficiently large, the above expression is increasing in η. ■

Corollary 2. *The open auction gives greater expected revenue to the auctioneer than the sealed bid auction: in the common value information acquisition model, for any number of bidders; and in the private value information acquisition model, for a number of bidders sufficiently large.*

6 Conclusions

In this paper, we have studied the strategic effects associated to open information acquisition. This strategic effects originate from a bidder's information acquisition decision affecting the other bidders' bid behavior. In particular, we have shown that these strategic effects are such that a bidder has greater incentives to acquire information about the common value in a sealed bid auction than in an open auction. However, we have also shown that if the information acquisition is about the private value, the incentives are greater in an open auction than in a sealed bid auction, at least when the number of bidders is sufficiently large.

We have shown that there is more information acquisition about the common value and less about the private value in the sealed bid auction than in the open auction. We have also shown that these results may imply that the open auction is more efficient and generates more expected revenue than the sealed bid auction once the bidders' information acquisition decisions are endogenized.

APPENDIX

This Appendix has four parts. In Appendix A we prove Proposition 1. Appendix B shows that there exists a solution to the differential equation of Lemma 2 and in Appendix C we prove Proposition 2. Finally, in Appendix D we provide an auxiliary result that shows the equivalence between a more informative signal and a more spread mean posterior.

Appendix A: Proof of Proposition 1

We start defining some strategies making use of the allocation function $\hat{\phi}_O$. We proceed sequentially, first, information sets in which nobody has left the auction yet:

- Bid function of the deviating bidder:

$$b_d^0[x|\emptyset] \equiv E\left[T_d + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = x, X_i^{\eta} = \hat{\phi}_O(x), \left\{X_j^{\eta} = \hat{\phi}_O(x)\right\}_{j \neq d,i}\right].$$

- Bid function of a non-deviating bidder[17] $i \neq d$:

$$b_i^0[x|\emptyset] \equiv E\left[T_i + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = \hat{\phi}_O^{-1}(x), X_i^{\eta} = x, \left\{X_j^{\eta} = x\right\}_{j \neq d,i}\right].$$

Next we define the bid function in information sets in which k bidders have left the auction and where p_l is the price at which the l-th bidder in declaring inactive has quit, and j_l is her identity. First, when the non-deviating bidder is not among the k bidders who have left the auction. To shorten notation, we do not include the range of sub-index l which is always from 1 to k.

- Bid function of the deviating bidder:

[17] We index each bid function by the identity of the bidder for convenience in the notation. However, note that all the non-deviating bidders use the same bid function.

$$b_d^k\left[t|\{(p_l, j_l)\}\right] \equiv$$
$$E\left[T_d + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = x, \left\{X_j^{\eta} = \hat{\phi}_O(x)\right\}_{\substack{j\notin\{j_l\}\\ j\neq d}}, \left\{b_{j_l}^{l-1}\left[X_{j_l}^{\eta}|\{(p_q, j_q)\}_{q=1}^{l-1}\right] = p_l\right\}\right],$$

where we adopt the convention that $\{(p_q, j_q)\}_{q=1}^{0} = \emptyset$.

- Bid function of a non-deviating bidders, $i \neq d$:

$$b_i^k\left[t|\{(p_l, j_l)\}\right] \equiv$$
$$E\left[T_i + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = \hat{\phi}_O^{-1}(x), \left\{X_j^{\eta} = x\right\}_{\substack{j\notin\{j_l\}\\ j\neq d}}, \left\{b_{j_l}^{l-1}\left[X_{j_l}^{\eta}|\{(p_q, j_q)\}_{q=1}^{l-1}\right] = p_l\right\}\right].$$

And when the deviating bidder is among the bidders who have left the auction, $i \neq d$:

$$b_j^k\left[t|\{(p_l, j_l)\}\right] \equiv$$
$$E\left[T_i + \sum_{j=1}^{n} Q_j \middle| \left\{X_j^{\eta} = x\right\}_{j\notin\{j_l\}}, \left\{b_{j_l}^{l-1}\left[X_{j_l}^{\eta_{j_l}}|\{(p_q, j_q)\}_{q=1}^{l-1}\right] = p_l\right\}\right],$$

where $\eta_{j_l} = \eta$ for any $j_l \neq d$.

Note that the proposed strategies implement $\hat{\phi}_O$. First, all non-deviating bidders use the same strictly increasing bid function, and thus the highest type of the non-deviating bidders outbids all the other non-deviating bidders. Second, the deviating bidder with a generic type x_d outbids the non-deviating bidder with maximum type, say x_i if and only if $x_i < \hat{\phi}_O(x_d)$. This is because the deviating bidder also uses a strictly increasing bid function and the deviating bidder with type x_d submits the same bid as a non-deviating bidder with type $\hat{\phi}_O(x_d)$ if $\hat{\phi}_O(x_d) \in (0, 1)$.

To see why these strategies form an equilibrium, we show that the deviating bidder does not have incentives to deviate. The proof is similar for non-deviating bidders.

Note first that the expected valuation of the deviating bidder conditional on a realization of the vector of bidders' types $(x_1, x_2, ..., x_n) \in (0,1)^n$ is equal to:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = x_d, \left\{X_j^{\eta} = x_j\right\}_{j \neq d}\right].$$

On the other hand, if all the non-deviating bidders follow the proposed strategy, the price that the deviating bidder pays if she wins is equal to the bid of the bidder with highest type among the non-deviating bidders, say Bidder i. This bid is equal to:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \middle| X_d^{\eta_d} = \hat{\phi}_O^{-1}(x_i), \left\{X_j^{\eta} = x_j\right\}_{j \neq d}\right].$$

After some algebra, we may show that the difference between these two values is equal to:

$$E[T_d|X_d^{\eta_d} = x_d] - E[T_i|X_i^{\eta} = x_i] + E[Q_d|X_d^{\eta_d} = x_d] - E[Q_d|X_d^{\eta_d} = \hat{\phi}_O^{-1}(x_i)]. \quad (14)$$

The deviating bidder cannot improve with a deviation because she wins with our proposed strategy if and only if $x_d \geq \hat{\phi}_O^{-1}(x_i)$, and these are all the cases in which the former expression is non-negative. To see why, note that the expression is increasing in x_d and by definition of $\hat{\phi}_O$, the expression evaluated at $x_d = \hat{\phi}_O^{-1}(x_i)$ is equal to zero if $\hat{\phi}_O^{-1}(x_i) \in (0,1)$, weakly negative if $\hat{\phi}_O^{-1}(x_i) = 1$, and weakly positive if $\hat{\phi}_O^{-1}(x_i) = 0$.■

Appendix B: Proof of Lemma 2

We look for a strictly increasing and continuous solution to Equation (8) whose graph splits the set $[0,1]^2$ into two subsets. This means that the required solution must start at a point either in the left or down boundaries of the set $[0,1]^2$ and end at a point in either the right or top boundary of $[0,1]^2$.

To show that there exists a solution to Equation (8) with the required conditions, we distinguish three disjoint subsets of $(0,1)^2$. The first subset that we denote by S^M includes points (x, ϕ) such that $\phi = \phi_O(x) = \phi_*(x)$. The second subset, and that we denote by S^L, includes points (x, ϕ) such that $\phi \in (\hat{\phi}_O(x), \hat{\phi}_*(x))$. The last subset, which will be denoted by S^R, includes points (x, ϕ) such that $\phi \in (\hat{\phi}_*(x), \hat{\phi}_O(x))$. We decompose the set S^L in a collection of disjoint sets $\{S_l^L\}$, where $\{S_l^L\}$ is the collection of open connected sets with minimum number of elements that covers the set S^L. Similarly, we decompose the set S^R in a collection of disjoint sets $\{S_l^R\}$, where $\{S_l^R\}$ is the collection of open connected sets with minimum number of elements that covers the set S^R. Figure 3 illustrates the set S^M and the collections $\{S_l^L\}$ and $\{S_l^R\}$.

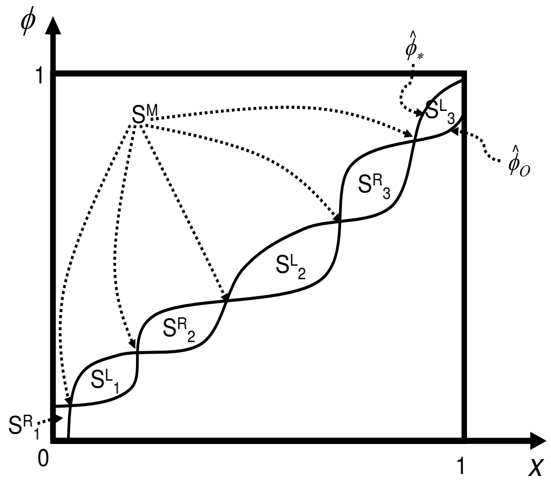

Figure 3: Illustration of the sets S^M and the collections $\{S_l^L\}$ and $\{S_l^R\}$ for some arbitrary functions $\hat{\phi}_O$ and $\hat{\phi}_*$.

Since the functions $\hat{\phi}_O$ and $\hat{\phi}_*$ are continuous, strictly increasing and their graphs split the set $[0,1]^2$, there are continuous and strictly increasing paths that cross each of the sets S^M, $\{S_l^L\}$ and $\{S_l^R\}$ and split the set $[0,1]^2$ into two. We shall show that one of these paths satisfies the conditions of the lemma.

In sets S^M we have no choice but to chose $\phi_{S,n}(x)$ equal to $\phi_O(x)$ and $\phi_*(x)$. To define $\phi_{S,n}$ outside S^M, we introduce an auxiliary function to rewrite Equation (8):

$$\Phi(x,\phi) \equiv \frac{\phi}{x(n-2)} \frac{E[T_i|X_i^{\eta}=\phi]-E[T_d|X_d^{\eta_d}=x]}{E[T_d|X_d^{\eta_d}=x]-E[T_i|X_i^{\eta}=\phi]+\mu(x,\eta_d)-\mu(\phi,\eta)}, \tag{15}$$

Thus, our differential equation can be written as $\phi'_{S,n}(x)=\Phi(x,\phi_{S,n}(x))$. In what follows we shall show how to construct our function $\phi_{S,n}$ in a given set S_l^R as a solution to the former differential. We shall only consider the case in which the boundary of S_l^R does not contain any point in the boundary of $[0,1]^2$. The proof when this is not the case and for the remaining subsets $\{S_l^L\}$ and $\{S_l^R\}$ is very similar. We point out the differences below.

In our proof we show that there exists a path from the lower-left corner to the upper-right corner of S_l^R that solves our differential equation in S_l^R. We follow several steps.

- **Remark 1:** There exists a unique solution to $\phi'_{S,n}(x) = \Phi(x, \phi_{S,n}(x))$ that passes by any given point (x_0, ϕ_0) in S_l^R. Moreover, the solution is strictly increasing.

  This is a direct consequence of the fact that Φ is differentiable, and thus satisfies a Lipschitz condition at (x_0, ϕ_0), and thus we can apply Coddington and Levinson (1984)[Theorem 2.2, pag. 10]. That the solution is strictly increasing follows because the denominator and the numerator of the expression that defines Φ are strictly negative. This is because the numerator is equal to zero at $\phi = \phi_O(x)$ and it is strictly increasing in ϕ, and moreover, the denominator is equal to zero at $\phi = \phi_*(x)$ and it is strictly decreasing in ϕ.

- **Remark 2:** The solutions of our differential equation in the set S_l^R do not cross and they converge to the upper-right corner of S_l^R as we continue them to the right.

  That they do not cross is a consequence of Remark 1. The convergence to the upper-right is because any solution to our differential equation cannot cross neither ϕ_O nor ϕ_* as it is continued to the right. First, it cannot cross ϕ_O because the slope of ϕ_O is bounded away from zero, which can be proved applying the implicit function theorem to Equation (14), and the slope of any solution tends to zero, as it approaches the graph of ϕ_O from below. And second, the solution cannot cross ϕ_* because the slope of ϕ_* has derivative bounded from above, which can be proved applying the implicit function theorem to Equation (9), and the slope of any solution tends to infinity, as it approaches the graph of ϕ_* from above.

- **Remark 3:** There exists a solution to our differential in S_l^R and with boundary condition any point in the boundary of S_l^R but the lower-left and the upper-right corners.

  For points in ϕ_O the claim can be proved as in Remark 1 by showing that there exists a solution that passes by the corresponding point, and noting that since Φ equals zero for these points, the solution can be continued towards the interior of S_l^R. For points in ϕ_* the proof is similar, but in this case it is more convenient to operate with an auxiliary differential equation that corresponds to the inverse of our original differential equation, i.e. $\psi'(\phi) = \frac{1}{\Phi(\psi(\phi),\phi)}$. By the same arguments as with boundary conditions in ϕ_O, there exists a solution to the auxiliary differential equation and it can be continued towards the interior. Moreover, since in the interior of S_l^R the solution must be strictly increasing, it is invertible, and thus, its inverse must be solution to our original differential equation.

- **Remark 4:** There exists a solution to our differential equation in S_l^R that starts at the lower-left corner, say (x_L, ϕ_L) and ends at the top-right corner.

  Take a conditionally decreasing sequence $\{x_\xi\}$ that converges to x_L and define two sequences of solutions to our differential equation in S_l^R. The first sequence is characterized by the sequence of boundary conditions $\{(x_\xi, \phi_O(x_\xi))\}$ and the second sequence by the sequence of boundary conditions $\{(x_\xi, \phi_*(x_\xi))\}$. Note that Remark 3 implies the required solutions exist. Moreover, by Remark 2 none of the solutions of the two sequences can cross. This implies three things. First, the first sequence is a decreasing sequence, and the second sequence is an increasing sequence, and thus both sequences have point-wise limits[18] that we denote by $\overline{\phi}$ and $\underline{\phi}$, respectively. The second implication is that $\underline{\phi}(x) \leq \overline{\phi}(x)$. The third implication is that any solution to our original differential equation that is

[18] A careful reader may realize that the domain of the functions in our two sequences does not remain constant. A simple way of dealing with this problem is to define any element of the first sequence $\phi_\xi(x) \equiv \phi_O(x)$ if $x \in [x_L, x_\xi]$, and similarly, any element of the second sequence $\phi_\xi(x) \equiv \phi_*(x)$ if $x \in [x_L, x_\xi]$.

between $\underline{\phi}$ and $\overline{\phi}$ for some values of x must lie between $\underline{\phi}$ and $\overline{\phi}$ in all the domain. Moreover, since $\underline{\phi}$ and $\overline{\phi}$ go from the lower-left corner to the upper-right corner of S_l^R, the continuation of any solution between $\underline{\phi}$ and $\overline{\phi}$ to the right converges to the upper-right corner of S_l^R and to the left converges to the down-left corner.

There exists a selection of the solutions that correspond to Remark 4 that satisfies that $\phi_{S,n+1}$ lies between $\phi_{S,n}$ and $\phi_{S,n}$. To see why note that an increase in n decreases the function that generates our differential equation Φ, see Equation (15). As a consequence the vector field associated should become flatter at any point in S_l^R. This means that the solutions constructed from left to right when the number of bidders is $n+1$ go below the corresponding solutions when the number of bidders is n, which implies the result.

In other sets S_l^R or in sets S_l^L the construction of the solution is similar. It is only worth remarking two points. First, in sets S_l^R we proceed basically as above extending all the auxiliary solutions to the right. As we have shown in Remark 2, the properties of the vector field generated by Φ assures in this case that the solution does not escape from the set S_l^R. However, in sets S_l^L we proceed differently. We extend the auxiliary solutions to the left as it is what the properties of the vector field generated by Φ require to ensure that the auxiliary solutions do not escape from S_l^L. Second, in a set S_l^L whose boundary intersects the upper and right boundaries of $[0,1]^2$, we shall chose a solution with boundary condition $(1, \hat{\phi}_*(1))$, if $\hat{\phi}_*(1) < 1$; $(\hat{\phi}_O^{-1}(1), 1)$ if $\hat{\phi}_O^{-1}(1) < 1$; and $(1,1)$ otherwise. These boundary conditions are sufficient to ensure that $\hat{\phi}_{S,n}(1) \geq \hat{\phi}_*(1)$ and $\hat{\phi}_{S,n}^{-1}(1) \geq \hat{\phi}_O^{-1}(1)$.

Appendix C: Proof of Proposition 2

We start proposing some strategies and later we show that they form an equilibrium. In our construction we use the function $\hat{\phi}_{S,n}$ of Lemma 2.

- Bid function of the deviating bidder:

$$b_d^S(x) \equiv E\left[T_d + \sum_{j=1}^{n} Q_j \,\middle|\, X_d^{\eta_d} = x, X_i^{\eta} = \hat{\phi}_{S,n}(x), \left\{ X_j^{\eta} \leq \hat{\phi}_{S,n}(x) \right\}_{j \neq d,i} \right].$$

- Bid function of a non-deviating bidder $i \neq d$:

$$b^S_{nd}(x) \equiv$$

$$(1-\hat{\rho}(x))\, E\left[T_i + \sum_{j=1}^{n} Q_j \,\Big|\, X_i^\eta = X_j^\eta = x, X_d^{\eta_d} \leq \hat{\phi}_{S,n}^{-1}(x), \{X_l^\eta \leq x\}_{l\neq d,i,j}\right] +$$

$$\hat{\rho}(x)\, E\left[T_i + \sum_{j=1}^{n} Q_j \,\Big|\, X_i^\eta = x, X_d^{\eta_d} = \hat{\phi}_{S,n}^{-1}(x), \{X_l^\eta \leq x\}_{l\neq d,i}\right]$$

where, $\hat{\rho}(x) \equiv 0$ if $x \geq \hat{\phi}_{S,n}(1)$, $\hat{\rho}(x) \equiv 1$ if $x \leq \hat{\phi}_{S,n}(0)$, and otherwise,

$$\hat{\rho}(x) \equiv \frac{x}{x + (n-2)\ \hat{\phi}_{S,n}^{-1}(x)\ \hat{\phi}'_{S,n}(\hat{\phi}_{S,n}^{-1}(x))}.$$

That the proposed strategies implement $\hat{\phi}_S$ can be proved exactly as in the proof of Proposition 1. It only remains to be shown that the bid function b^S_{nd} is indeed strictly increasing, and in particular for x in the interval $[\hat{\phi}_{S,n}(0), \hat{\phi}_{S,n}(1)]$. Outside this interval the monotonicity is trivial. In the interior of this interval, and by definition of $\hat{\phi}_{S,n}$, the bid is equal to:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^\eta = x, X_d^{\eta_d} = \hat{\phi}_{S,n}^{-1}(x), \left\{X_j^\eta \leq x\right\}_{j\neq d,i}\right],$$

and thus, it is strictly increasing.

In the lower end of the interval, i.e. $x = \hat{\phi}_{S,n}(0)$, we may have problems only if $\hat{\phi}_{S,n}(0) > 0$ and there is a discontinuity. In this case, $\hat{\phi}_{S,n}^{-1}(x) = 0$, and thus the limit of the bid function when we approach x from the left is equal to:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\Big|\, X_i^\eta = x, X_d^{\eta_d} = 0, \{X_l^\eta \leq x\}_{l\neq d,i}\right],$$

which is clearly less than,

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\Big|\, X_i^\eta = X_j^\eta = x, X_d^{\eta_d} = 0, \{X_l^\eta \leq x\}_{l\neq d,i,j}\right].$$

We can conclude that the bid function must be increasing at $x = \hat{\phi}_{S,n}(0)$ because the limit of the bid function when we approach x from the right is a weighted average of the two expected values above.

In the upper end of the interval, i.e. $x = \hat{\phi}_{S,n}(1)$, we may have problems only if $\hat{\phi}_{S,n}(1) < 1$ and there is a discontinuity. In this case, $\hat{\phi}_{S,n}^{-1}(x) = 1$, and thus the limit of the bid function from the right is equal to:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = X_j^{\eta} = x, X_d^{\eta_d} \leq 1, \{X_l^{\eta} \leq x\}_{l \neq d,i,j}\right].$$

By Lemma 2, $\hat{\phi}_{S,n}(1) \geq \hat{\phi}_*(1)$ and hence $x \geq \hat{\phi}_*(1)$ which means that,

$$E[T_i | X_i^{\eta} = x] + \mu(x, \eta) \geq E[T_d | X_d^{\eta_d} = 1] + \mu(1, \eta_d).$$

This implies that the limit from the right is greater than:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x, X_d^{\eta_d} = 1, \left\{X_j^{\eta} \leq x\right\}_{j \neq d,i}\right].$$

By definition of $\hat{\phi}_{S,n}$, this last expected value is equal to the limit of the bid function as we approach x from the left.

Finally, we shall prove that our proposed strategies form an equilibrium by showing that an individual bidder does not have incentives to deviate when all the other bidders follow our proposed strategies. In particular, we show that our proposed strategies ensures the bidder that she wins if and only if it is profitable for her to win.

We start with the deviating bidder. Her expected value of the good conditional on her type x_d, and the realization of the maximum of the other bidders' types, say the type of Bidder i and denote it by x_i, is equal to:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \,\middle|\, X_d^{\eta_d} = x_d, X_i^{\eta} = x_i, \left\{X_j^{\eta} \leq x_i\right\}_{j \neq d,i}\right].$$

Moreover, if the deviating bidder wins the auction, she pays the bid of the highest type of the non-deviating bidders, i.e. the bid of Bidder i.

If $x_i \geq \hat{\phi}_{S,n}(1)$, then the bid of Bidder i is equal to:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = X_j^{\eta} = x_i, X_d^{\eta_d} \leq 1, \left\{X_j^{\eta} \leq x_i\right\}_{j \neq d,i,j}\right].$$

The difference between value and price is equal after some simplifications to:

$$E[T_d|X_d^{\eta_d} = x_d] + E[Q_d|X_d^{\eta_d} = x_d] - E[T_i|X_i^{\eta} = x_i] - E[Q_d|X_d^{\eta_d} \leq 1] - \mu(x_i, \eta),$$

which is less than,

$$E[T_d|X_d^{\eta_d} = 1] - E[T_i|X_i^{\eta} = x_i] + \mu(1, \eta_d) - \mu(x_i, \eta).$$

Since $x_i \geq \hat{\phi}_{S,n}(1)$ and by Lemma 2, $\hat{\phi}_{S,n}(1) \geq \hat{\phi}_*(1)$, then $x_i \geq \hat{\phi}_*(1)$, which means that the above expression must be negative. Our proposed bid function ensures the deviating bidder that she loses in these cases.

If $x_i \in (\hat{\phi}_{S,n}(0), \hat{\phi}_{S,n}(1))$, then by definition of $\hat{\phi}_{S,n}$, the bid of Bidder i and thus the price is equal to:

$$E\left[T_d + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, X_d^{\eta_d} = \hat{\phi}_{S,n}^{-1}(x_i), \left\{ X_j^{\eta} \leq x_i \right\}_{j \neq d,i} \right].$$

The difference between value and price in this case is equal to:

$$E[T_d|X_d^{\eta_d} = x_d] + E[Q_d|X_d^{\eta_d} = x_d] - E[T_d|X_d^{\eta_d} = \hat{\phi}_{S,n}(x_i)] - E[Q_d|X_d^{\eta_d} = \hat{\phi}_{S,n}(x_i)],$$

which is positive if and only if $x_d \geq \hat{\phi}_{S,n}(x_i)$. Consequently, our proposed strategy assures the deviating bidder that she wins if it is profitable to win and loses otherwise.

Finally, if $x_i < \phi_{S,n}(0)$, then the price is equal to:

$$E\left[T_i + \sum_{j=1}^{n} Q_j \,\middle|\, X_i^{\eta} = x_i, X_d^{\eta_d} = 0, \left\{ X_j^{\eta} \leq x_i \right\}_{j \neq d,i} \right].$$

The difference between value and price after some simplifications becomes:

$$E[T_d|X_d^{\eta_d} = x_d] + E[Q_d|X_d^{\eta_d} = x_d] - E[T_i|X_i^{\eta} = x_i] - E[Q_d|X_d^{\eta_d} = 0],$$

which is greater than,

$$E[T_d|X_d^{\eta_d} = 0] - E[T_i|X_i^{\eta} = x_i].$$

This last expression is positive since it may be shown that $\hat{\phi}_*(0)$ must be less than $\hat{\phi}_O(0)$ and hence $x_i \leq \hat{\phi}_O(0)$. The proposed strategy assures the deviating bidder that she wins in these cases.

The proof that the non-deviating bidders do not have incentives to deviate is slightly different. We start denoting by p the price that a non-deviating bidder pays when she wins. Suppose first that $p > b_{nd}(1)$. Then, the price must be fixed by the deviating bidder and her type x_d must be strictly greater than $\phi^{-1}(1)$. We can use arguments as above to show that a non-deviating bidder gets negative expected utility if she wins at a price that equals the bid of such types of the deviating bidder. Next note that it cannot happen that $p < b_{nd}(0)$. The reason is that there is always another non-deviating bidder, and this other non-deviating bidder never bids above $b_{nd}(0)$ with probability one.

Finally, suppose that $p \in [b_{nd}(0), b_{nd}(1)]$. To compute Bidder i's expected value of the good conditional on winning the auction at a price p we first compute the conditional probability that it is the deviator who has bid p. By construction our bid functions satisfy that $\hat{\phi}'_{S,n}(x_j) = \frac{b'_d(\phi_{S,n}(x_j))}{b'_{nd}(x_j)}$ where x_j is the type of the non-deviating bidders that corresponds to a bid p according to our proposed bidding function, i.e. $x_j \equiv b_{nd}^{-1}(p)$. Thus, we can show that the former probability is equal to $\hat{\rho}(x_j)$. This means that the corresponding conditional expected value is equal to:

$$\hat{\rho}(x_j)E\left[T_i + \sum_{l=1}^{n} Q_l \middle| X_d^{\eta_d} = \hat{\phi}_S^{-1}(x_j), X_i^{\eta} = x_i, \left\{X_l^{\eta} \leq x_l\right\}_{l \neq d,i}\right] +$$
$$(1-\hat{\rho}(x_j))E\left[T_i + \sum_{l=1}^{n} Q_l \middle| X_i^{\eta} = x_i, X_j^{\eta} = x_j, X_d^{\eta_d} \leq 1, \left\{X_l^{\eta} \leq x_j\right\}_{l \neq d,i,j}\right].$$

But, by definition of x_j, the price p is equal to the bid of a non-deviating bidder with type x_j which is equal to:

$$\hat{\rho}(x_j)E\left[T_j + \sum_{l=1}^{n} Q_l \middle| X_d^{\eta_d} = \hat{\phi}_S^{-1}(x_j), X_j^{\eta} = x_j, \left\{X_l^{\eta} \leq x_j\right\}_{l \neq d,i}\right] +$$
$$(1-\hat{\rho}(x_j))E\left[T_j + \sum_{l=1}^{n} Q_l \middle| X_i^{\eta} = X_j^{\eta} = x_j, X_d^{\eta_d} \leq \hat{\phi}_S^{-1}(x_j), \left\{X_l^{\eta} \leq x_j\right\}_{l \neq d,i,j}\right].$$

We can easily conclude by subtracting the former expected values that the difference between value and price is positive if and only if $x_i \geq x_j$, and hence that our proposed strategy assures Bidder i that she wins if and only if it is profitable to win. As a result, Bidder i does not have incentives to deviate.■

Appendix D: More Informative Signals and Second Order Stochastic Dominance

Definition: We say that the distribution F_1 dominates in the sense of second order stochastic dominance the distribution F_2 if,

$$\int_{-\infty}^{\psi} \left(F_1(\tilde{\psi}) - F_2(\tilde{\psi})\right) d\tilde{\psi} \leq 0, \forall \psi \in \mathbb{R}.$$

Lemma 8. *A signal X_i^{η} is more informative of V_i, $V_i \in \{T_i, Q_i\}$, than another signal $X_i^{\eta'}$ if and only if the distribution of the conditional expected value $E[V_i|X_i^{\eta}]$ is dominated in the sense of second order stochastic dominance by the distribution of the conditional expected value $E[V_i|X_i^{\eta'}]$.*

Proof. Let $\Psi_\eta(x) \equiv E[V_i|X_i^\eta = x]$ for $x \in [0,1]$, and Ψ_η^{-1} its inverse. First note that the cumulative distribution function of $E[V_i|X_i^\eta]$ is equal to Ψ_η^{-1}. This is because the probability of the event $\{E[V_i|X_i^\eta] \leq \psi\}$ is equal to $\Psi_\eta^{-1}(\psi)$ for any ψ in the support of $E[V_i|X_i^\eta]$. For completeness we also define $\Psi_\eta^{-1}(\psi) \equiv 0$ for ψ below the support of $E[V_i|X_i^\eta]$, and $\Psi_\eta^{-1}(\psi) \equiv 1$ above the support of $E[V_i|X_i^\eta]$. Note also that $E[V_i|X_i^\eta \leq x] = \int_0^x \Psi_\eta(x) \frac{d\tilde{x}}{x}$. Hence, we have to prove that:

$$\int_0^x \left(\Psi_{\eta'}(\tilde{x}) - \Psi_\eta(\tilde{x})\right) d\tilde{x} \geq 0, \forall x \in [0,1] \Leftrightarrow \int_{-\infty}^{\psi} \left(\Psi_{\eta'}^{-1}(\tilde{\psi}) - \Psi_\eta^{-1}(\tilde{\psi})\right) d\tilde{\psi} \leq 0, \forall \psi \in \mathbb{R}.$$

We only prove "$\Leftarrow$". The proof of "$\Rightarrow$" is symmetric. First, note that,

$$\int_0^x \Psi_\eta(\tilde{x})\, d\tilde{x} = \Psi_\eta(x)x - \int_0^x \tilde{x}\, d\Psi_\eta(\tilde{x}) = \Psi_\eta(x)x - \int_{-\infty}^{\Psi_\eta(x)} \Psi_\eta^{-1}(\tilde{\psi})\, d\tilde{\psi}.$$

Consequently,

$$\int_0^x \left(\Psi_{\eta'}(\tilde{x}) - \Psi_\eta(\tilde{x})\right) d\tilde{x} =$$
$$\int_{\Psi_{\eta'}(x)}^{\Psi_\eta(x)} \left(x - \Psi_\eta^{-1}(\tilde{\psi})\right) d\tilde{\psi} + \int_{-\infty}^{\Psi_{\eta'}(x)} \left(\Psi_{\eta'}^{-1}(\tilde{\psi}) - \Psi_\eta^{-1}(\tilde{\psi})\right) d\tilde{\psi}.$$

The second integral is positive by our starting assumption. To compute the sign of the first integral assume first that $\Psi_\eta(x) \geq \Psi_{\eta'}(x)$. Then the integral is positive because $\Psi_\eta^{-1}(\tilde{\psi}) \leq x$ for any $\tilde{\psi} < \Psi_\eta(x)$. Suppose next that $\Psi_\eta(x) < \Psi_{\eta'}(x)$. Then the integral is positive because $\Psi_\eta^{-1}(\tilde{\psi}) \geq x$ for any $\tilde{\psi} > \Psi_\eta(x)$. ■

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