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Iñarrea, J. Terahertz-Induced Magnetoresistance Oscillations in High-Mobility 2D Electron Systems Under Bichromatic and Multichromatic Excitation. Journal of Elec Materi 46, 3862–3866 (2017).

https://doi.org/10.1007/s11664-017-5285-3

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Noname manuscript No. (will be inserted by the editor)

# Terahertz-induced magnetoresistance oscillations in high mobility 2D electron systems under bichromatic and multichromatic excitation.

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Received: date / Accepted: date

**Abstract** We study the magnetotransport under terahertz radiation in high mobility two-dimensional electron systems. We focus on the irradiation by bichromatic and multichromatic terahertz sources. We study the strong modulation of the Shubnikov-de Haas oscillations at sufficient terahertz radiation power. We obtain that the origin of the modulation is the interference between the average advanced distance by the scattered electrons between irradiated Landau states, and the available initial density of states at a certain magnetic field. In the case of multifrequency illumination we obtain that with the appropriate frequencies the irradiated magnetoresistance can almost be lead to a zero resistance states regime even at moderate radiation power.

Keywords Magnetoresistance · Terahertz · two-dimensional electrons

# 1 Introduction

The effect of microwave-induced magnetoresistance  $(R_{xx})$  oscillations (MIRO)[1, 2] shows up in high mobility two-dimensional electron systems (2DES) when they are illuminated with microwaves (MW) at low temperature  $(T \sim 1K)$  and under low magnetic fields (B) perpendicular to the 2DES. This effect turns into zero resistance states (ZRS) at high enough radiation power (P). If MIRO and ZRS, can be qualified as striking, it is even more remarkable that after more than ten years of important experimental[3–25] and theoretical efforts [26–45], their physical origin still remains controversial and far from reaching a definite consensus among the scientists devoted to this field.

In this paper we report on a theoretical work on magnetotransport in high mobility two-dimensional electron systems under terahertz (TH) radi-

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**Fig. 1** Calculated irradiated magnetoresistance,  $R_{xx}$ , vs the magnetic field *B* for frequencies of 300 GHz and 400 GHz and T=1K. Both panels exhibit total  $R_{xx}$ , with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations, (red curves online).

ation. The interest on TH radiation comes not only from the basic physics standpoint, TH is a very important part of the radiation spectrum from the application point of view too. For instance, sensors for medical applications, imaging for security purposes and large-bandwidth communications. We focus on the interference between the radiation-induced magnetoresistance oscillations (RIRO) and the Shubnikov-de Haas (SdHO) oscillations. The terahertz band offers the possibility of studying this interaction because both kind of oscillations coexist in the same range of B[46,47] while MIRO tend to vanish when the SdHO are more intense and the effects of interaction are more difficult to observe. In this scenario we study the case of bichromatic illumination with terahertz radiation. Thus we are able to predict that the interference effect on the  $R_{xx}$  profile is going to be more intense and complicated. We also consider the situation of three simultaneous sources of terahertz radiation where for the appropriate combination of frequencies we can achieve a  $R_{xx}$ 



Fig. 2 Radiation power dependence of the calculated irradiated  $R_{xx}$  versus B, for increasing radiation power from to 0.1 mW to 6 mW. The subsequent curves are shifted up for clarity.

profile that approaches ZRS for a certain B. To carry out the study, we use the radiation-driven electron orbits model [26,27]. This model was proposed to study the effect of MIRO and ZRS [26,51–53], and according to it when a Hall bar is illuminated, the guiding centers of the irradiated Landau states (LS) perform a classical trajectory consisting in a harmonic motion along the direction of the current. Thus, the electron orbits move in phase and harmonically with each other at the radiation frequency, altering the scattering conditions and giving rise eventually to MIRO and, at higher P, ZRS.

# 2 Theoretical Model

In the theory of the radiation driven electron orbit model, the corresponding time-dependent Schrödinger equation can be exactly solved and the solution for the total wave function [26,51–53] reads:  $\Psi_n(x,t) \propto \phi_n(x - X_0 - x_{cl}(t),t)$ , where  $\phi_n$  is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function representing the LS is the same as the one of the standard quantum harmonic oscillator where the guiding center of the LS,  $X_0$  without radiation, is displaced by  $x_{cl}(t)$ .  $x_{cl}(t)$ is the classical solution of a negatively charged, forced and damped, harmonic oscillator[54,55],

$$x_{cl}(t) = \frac{-eE_o}{m^*\sqrt{(w_c^2 - w^2)^2 + \gamma^4}}\cos(wt - \beta) = -A\cos(wt - \beta)$$
(1)

where  $E_0$  is the amplitude of the radiation electric field and  $\beta$  is a phase constant.  $\beta$  is the phase difference between the radiation-driven guiding center and the driving radiation.  $\gamma$  is a phenomenologically-introduced damping factor for the interaction of electrons with the lattice ions giving rise to the emission of acoustic phonons. For high-mobility 2DES[56],  $\beta \rightarrow \frac{\pi}{2}$ , then, the time-dependent guiding center is,  $X(t) = X_0 + x_{cl}(t) = X_0 - A \sin wt$ . This physically implies that the orbit guiding centers oscillate harmonically at the radiation frequency, w.

To calculate the longitudinal conductivity  $\sigma_{xx}$  in the 2DES we use the Boltzmann transport theory. With this theory and within the relaxation time approximation,  $\sigma_{xx}$  is given by the following equation [57,58]:

$$\sigma_{xx} = 2e^2 \int_0^\infty dE \rho_i(E) (\Delta X)^2 W_I\left(-\frac{df(E)}{dE}\right)$$
(2)

being E the energy and  $\rho_i(E)$  the density of initial LS.  $W_I$  is the remote charged impurity scattering rate, given, according to the Fermi's Golden Rule, by  $W_I = \frac{2\pi}{\hbar} |\langle \Psi_f | V_s | \Psi_i \rangle |^2 \delta(E_f - E_i)$ , where  $E_i$  and  $E_f$  are the energies of the initial and final LS.  $\Psi_i$  and  $\Psi_f$  are the wave functions corresponding to the initial and final LS respectively.  $V_s$  is the scattering potential for charged impurities[57],  $\Delta X$  is the average distance advanced by the electron between orbits in every scattering jump in the x direction and is given by[56],  $\Delta X = \Delta X^0 - A \sin(2\pi \frac{w}{w_c})$ .  $\Delta X^0$  is the advanced distance without radiation.

After some algebra we get to an expression for  $\sigma_{xx}$ [58–60]:

$$\sigma_{xx} = \frac{2e^2m^*}{\pi\hbar^2} (\Delta X)^2 W_I \left[ 1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi F}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right) \right]$$
(3)

where  $X_s = \frac{2\pi^2 k_B T}{\hbar w_c}$ ,  $\Gamma$  is the Landau level width and  $E_F$  the Fermi energy. To find the expression for  $R_{xx}$  we use the well-known tensorial relation  $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$ , where  $\sigma_{xy} \simeq \frac{n_e e}{B}$ ,  $n_e$  being the electron density, and  $\sigma_{xx} \ll \sigma_{xy}$ . Finally, the complete expression of  $R_{xx}$  reads:

$$R_{xx} = \frac{2e^2m^*}{\pi\hbar^2} \left(\frac{B}{n_e e}\right)^2 W_I \left[\Delta X^0 - A\sin\left(2\pi\frac{w}{w_c}\right)\right]^2 \left[1 + \frac{2X_s}{\sinh(X_s)}e^{-\frac{\pi F}{\hbar w_c}}\cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right] \tag{4}$$

With this expression we want to stand out the terms that are going to be responsible of the interference between radiation-induced resistance oscillations (RIRO), first bracket, and the SdHO, second bracket.

## 3 Results and discussion.

The simulations that we present in this paper are based on a set of parameters given by  $m^* = 0.067$  for the GaAs effective mass,  $n_e = 3 \times 10^{11} cm^{-2}$  for the electron density, T = 1K for the temperature and  $\Gamma = 10^{-5}$  eV for the LL



Fig. 3 a) Calculated irradiated magnetoresistance,  $R_{xx}$ , vs the magnetic field *B* for frequencies of 300 GHz and 400 GHz. Total  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations or  $R_{xx,RIRO}$ , (red curves online). b) Same frequencies as in a), calculated irradiated  $R_{xx}$  (black color online),  $R_{xx}$  without SdHO, ( $R_{xx,RIRO}$ ) (red color on line), and the difference of both,  $\delta R_{xx}$  (blue color online) vs  $w/w_c$ . Total  $R_{xx}$  curve is shifted up for clarity. It is observed the rise of beats, more clearly in the case of  $R_{xx,RIRO}$ .

width. For the latter we have considered a constant value, i.e., independent of B, that is a rather good approximation for low values of B, as in our case.

Figure 1 exhibits irradiated  $R_{xx}$  vs B for 300 GHz in the upper panel and 400 GHz in the lower panel. For both panels we represent the total  $R_{xx}$ (black curves online) and the averaged out  $R_{xx}$  (without SdHO, red curves online), in order to stand out only the effect of RIRO. Then, we can see intense RIRO in the TH regime that clearly fulfill for both frequencies the periodicity in  $B^{-1}$  and the 1/4-cycle phase shift of the oscillations minima, ( $w/w_c =$  5/4, 9/4, 13/4...). Besides, it is interesting to observe with the TH regime, how the radiation induced oscillations overlap with the more rapidly varying with the magnetic field SdHO giving rise to a strong modulation of the latter. This modulation is explained, according to our model, by the interference effect between the harmonic terms showing up in Eq. 4. Thus, this effect is mainly dependent on the radiation frequency and on the Fermi energy or electron density.

In Fig. 2, we present the P dependence of the TH irradiated  $R_{xx}$  versus B for increasing P, from to 0.1 mW to 6 mW. We observe that RIRO's amplitude increases with P. This can be straightforward explained according to our model since the radiation electric field  $E_0$  shows up in the numerator of the amplitude of RIRO, A. The most interesting effect can be observed around B = 0.6 T. In this region we obtain the evolution of SdHO as a function of increasing P. As in experiment[49,50], the SdHO vanish as  $R_{xx}$  tends to zero. In other words, we obtain the suppression of SdHO in the region of radiation-induced zero resistance states. According to our model this is because this region corresponds to a situation where the advanced distance  $\Delta X \to 0$ , making smaller and smaller the obtained  $R_{xx}$ , including resistance background and SdHO.

We consider now the magnetoresistance of the 2DES irradiated by two different sources with different frequencies of the TH band. In the framework of our model is relatively simple to extend the theory to two or more different radiation frequencies[61–63]. This is a consequence of the application of the superposition principle. The latter is, in turn, a consequence of the *linear* characteristic of the equation of the driven and damped classical oscillator. Then if we add two or more radiation sources to the system the displacement of the driven LL guiding center will be equal to the sum of the corresponding individual solutions. We would expect for  $R_{xx}$  a much richer and more complex profile since we would have more independent sources of interference. And the more frequencies are included the more elaborated profile would be obtained. It is straightforward to get to an expression for  $R_{xx}$  in the case of a multifrequency scenario:

$$R_{xx} \propto \left[\Delta X^0 - \sum_{i} \left[A\sin\left(2\pi\frac{w_i}{w_c}\right)\right]\right]^2 \left[1 + \frac{2X_s}{\sinh(X_s)}e^{-\frac{\pi\Gamma}{\hbar w_c}}\cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right]$$
(5)

In Fig. 3 we exhibit calculated magnetoresistance for the irradiation by two simultaneous sources of different frequencies: 300 and 400 GHz. For this specific case  $R_{xx}$  can be expressed as:

$$R_{xx} \propto \left[ \Delta X^0 - \left[ A_1 \sin\left(2\pi \frac{w_1}{w_c}\right) + A_2 \sin\left(2\pi \frac{w_2}{w_c}\right) \right] \right]^2 \\ \times \left[ 1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi F}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right) \right]$$
(6)



Fig. 4 a) Calculated irradiated magnetoresistance vs the magnetic field B for frequencies of 300,400 and 500 GHz in the upper panel and 300,500 and 700 GHz in the lower one. Total  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations (red curves online).

In Fig, 3a we plot irradiated magnetoresistance vs B for the two cases of total  $R_{xx}$  (black color online) and  $R_{xx}$  without SdHO or  $R_{xx,RIRO}$  (red color online). For low B we can slightly observe the rise of beats. This effect is more clearly plotted in Fig. 3b where we exhibit calculated results of irradiated  $R_{xx}$  (black color online),  $R_{xx,RIRO}$  (red color on line), and the difference of both,  $\delta R_{xx}$  (blue color online), vs  $w/w_c$ . The  $R_{xx}$  curve has been shifted up for clarity in order to spot the presence of beats in  $R_{xx,RIRO}$ . The latter shows clear beats that are a signature of the strong interference produced by the simultaneous irradiation with two independent TH-frequencies. This theoretical prediction of a very rich interference  $R_{xx}$  profile with the presence of beats, would confirm in a future experiment the oscillating nature of the LS being driven by radiation.

In Fig. 4 we present the calculated irradiated magnetoresistance,  $R_{xx}$ , vs magnetic field *B* for simultaneous irradiation of three different sources of TH radiation of frequencies of 300 GHz, 400 GHz and 500 GHz in the upper panel

and 300 GHz, 500 GHz and 700 GHz in the lower one. We observe a more intense and complicated modulation of  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations, (red curves online). We expected this outcome but the more interesting result takes place in the lower panel at  $B \simeq (0.5 - 0.6)$  T. We observe that  $R_{xx} \rightarrow 0$  or approaches to ZRS with the appropriate combination of TH frequencies without increasing the radiation power.

# 4 Conclusions

In summary, we have reported on a theoretical work on magnetotransport under terahertz radiation with high mobility two-dimensional electron systems. We have focused on the interaction between the obtained radiation-induced magnetoresistance oscillations and the Shubnikov-de Haas oscillations. We study the strong modulation of the Shubnikov-de Haas oscillations at high enough TH radiation power. We have applied the radiation-driven electron orbits model and according to it the physical origin is the interference between the average advanced distance due to scattering between driven-Landau states, (radiation-induced resistance oscillations), and the available initial density of Landau states, (Shubnikov-de Haas oscillations). We have extended the theory to two or more frequencies of the TH band. Thus we have predicted an even stronger irradiated magnetoresistance profile with the rise of beats. This would confirm the oscillating nature of irradiated LS which will reveal a new way of interaction radiation-matter. Another remarkable results is that with the correct selection of TH frequencies due to the interference effect we can obtain regimes of  $R_{xx}$  close to ZRS.

# **5** Acknowledgments

This work is supported by the MINECO (Spain) under grant MAT2014-58241-P and ITN Grant 234970 (EU). GRUPO DE MATEMATICAS APLICADAS A LA MATERIA CONDENSADA, (UC3M), Unidad Asociada al CSIC.

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Keywords Magnetoresistance  $\cdot$  Terahertz  $\cdot$  two-dimensional electrons

## 1 Introduction

The effect of microwave-induced magnetoresistance  $(R_{xx})$  oscillations (MIRO)[1, 2] shows up in high mobility two-dimensional electron systems (2DES) when they are illuminated with microwaves (MW) at low temperature  $(T \sim 1K)$  and under low magnetic fields (B) perpendicular to the 2DES. This effect turns into zero resistance states (ZRS) at high enough radiation power (P). If MIRO and ZRS, can be qualified as striking, it is even more remarkable that after more than ten years of important experimental[3–25] and theoretical efforts [26–45], their physical origin still remains controversial and far from reaching a definite consensus among the scientists devoted to this field.

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**Fig. 1** Calculated irradiated magnetoresistance,  $R_{xx}$ , vs the magnetic field *B* for frequencies of 300 GHz and 400 GHz and T=1K. Both panels exhibit total  $R_{xx}$ , with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations, (red curves online).

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Fig. 2 Radiation power dependence of the calculated irradiated  $R_{xx}$  versus B, for increasing radiation power from to 0.1 mW to 6 mW. The subsequent curves are shifted up for clarity.

profile that approaches ZRS for a certain B. To carry out the study, we use the radiation-driven electron orbits model [26,27]. This model was proposed to study the effect of MIRO and ZRS [26,51–53], and according to it when a Hall bar is illuminated, the guiding centers of the irradiated Landau states (LS) perform a classical trajectory consisting in a harmonic motion along the direction of the current. Thus, the electron orbits move in phase and harmonically with each other at the radiation frequency, altering the scattering conditions and giving rise eventually to MIRO and, at higher P, ZRS.

# 2 Theoretical Model

In the theory of the radiation driven electron orbit model, the corresponding time-dependent Schrödinger equation can be exactly solved and the solution for the total wave function [26,51–53] reads:  $\Psi_n(x,t) \propto \phi_n(x - X_0 - x_{cl}(t),t)$ , where  $\phi_n$  is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function representing the LS is the same as the one of the standard quantum harmonic oscillator where the guiding center of the LS,  $X_0$  without radiation, is displaced by  $x_{cl}(t)$ .  $x_{cl}(t)$ is the classical solution of a negatively charged, forced and damped, harmonic oscillator[54,55],

$$x_{cl}(t) = \frac{-eE_o}{m^*\sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos(wt - \beta) = -A\cos(wt - \beta)$$
(1)

where  $E_0$  is the amplitude of the radiation electric field and  $\beta$  is a phase constant.  $\beta$  is the phase difference between the radiation-driven guiding center and the driving radiation.  $\gamma$  is a phenomenologically-introduced damping factor for the interaction of electrons with the lattice ions giving rise to the emission of acoustic phonons. For high-mobility 2DES[56],  $\beta \rightarrow \frac{\pi}{2}$ , then, the time-dependent guiding center is,  $X(t) = X_0 + x_{cl}(t) = X_0 - A \sin wt$ . This physically implies that the orbit guiding centers oscillate harmonically at the radiation frequency, w.

To calculate the longitudinal conductivity  $\sigma_{xx}$  in the 2DES we use the Boltzmann transport theory. With this theory and within the relaxation time approximation,  $\sigma_{xx}$  is given by the following equation [57,58]:

$$\sigma_{xx} = 2e^2 \int_0^\infty dE \rho_i(E) (\Delta X)^2 W_I\left(-\frac{df(E)}{dE}\right)$$
(2)

being E the energy and  $\rho_i(E)$  the density of initial LS.  $W_I$  is the remote charged impurity scattering rate, given, according to the Fermi's Golden Rule, by  $W_I = \frac{2\pi}{\hbar} |\langle \Psi_f | V_s | \Psi_i \rangle |^2 \delta(E_f - E_i)$ , where  $E_i$  and  $E_f$  are the energies of the initial and final LS.  $\Psi_i$  and  $\Psi_f$  are the wave functions corresponding to the initial and final LS respectively.  $V_s$  is the scattering potential for charged impurities[57],  $\Delta X$  is the average distance advanced by the electron between orbits in every scattering jump in the x direction and is given by[56],  $\Delta X = \Delta X^0 - A \sin(2\pi \frac{w}{w_c})$ .  $\Delta X^0$  is the advanced distance without radiation.

After some algebra we get to an expression for  $\sigma_{xx}$ [58–60]:

$$\sigma_{xx} = \frac{2e^2m^*}{\pi\hbar^2} (\Delta X)^2 W_I \left[ 1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi F}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right) \right]$$
(3)

where  $X_s = \frac{2\pi^2 k_B T}{\hbar w_c}$ ,  $\Gamma$  is the Landau level width and  $E_F$  the Fermi energy. To find the expression for  $R_{xx}$  we use the well-known tensorial relation  $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$ , where  $\sigma_{xy} \simeq \frac{n_e e}{B}$ ,  $n_e$  being the electron density, and  $\sigma_{xx} \ll \sigma_{xy}$ . Finally, the complete expression of  $R_{xx}$  reads:

$$R_{xx} = \frac{2e^2m^*}{\pi\hbar^2} \left(\frac{B}{n_e e}\right)^2 W_I \left[\Delta X^0 - A\sin\left(2\pi\frac{w}{w_c}\right)\right]^2 \left[1 + \frac{2X_s}{\sinh(X_s)}e^{-\frac{\pi F}{\hbar w_c}}\cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right]$$
(4)

With this expression we want to stand out the terms that are going to be responsible of the interference between radiation-induced resistance oscillations (RIRO), first bracket, and the SdHO, second bracket.

## 3 Results and discussion.

The simulations that we present in this paper are based on a set of parameters given by  $m^* = 0.067$  for the GaAs effective mass,  $n_e = 3 \times 10^{11} cm^{-2}$  for the electron density, T = 1K for the temperature and  $\Gamma = 10^{-5}$  eV for the LL



Fig. 3 a) Calculated irradiated magnetoresistance,  $R_{xx}$ , vs the magnetic field *B* for frequencies of 300 GHz and 400 GHz. Total  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations or  $R_{xx,RIRO}$ , (red curves online). b) Same frequencies as in a), calculated irradiated  $R_{xx}$  (black color online),  $R_{xx}$  without SdHO, ( $R_{xx,RIRO}$ ) (red color on line), and the difference of both,  $\delta R_{xx}$  (blue color online) vs  $w/w_c$ . Total  $R_{xx}$  curve is shifted up for clarity. It is observed the rise of beats, more clearly in the case of  $R_{xx,RIRO}$ .

width. For the latter we have considered a constant value, i.e., independent of B, that is a rather good approximation for low values of B, as in our case.

Figure 1 exhibits irradiated  $R_{xx}$  vs B for 300 GHz in the upper panel and 400 GHz in the lower panel. For both panels we represent the total  $R_{xx}$ (black curves online) and the averaged out  $R_{xx}$  (without SdHO, red curves online), in order to stand out only the effect of RIRO. Then, we can see intense RIRO in the TH regime that clearly fulfill for both frequencies the periodicity in  $B^{-1}$  and the 1/4-cycle phase shift of the oscillations minima,  $(w/w_c =$ 

5/4, 9/4, 13/4...). Besides, it is interesting to observe with the TH regime, how the radiation induced oscillations overlap with the more rapidly varying with the magnetic field SdHO giving rise to a strong modulation of the latter. This modulation is explained, according to our model, by the interference effect between the harmonic terms showing up in Eq. 4. Thus, this effect is mainly dependent on the radiation frequency and on the Fermi energy or electron density.

In Fig. 2, we present the P dependence of the TH irradiated  $R_{xx}$  versus B for increasing P, from to 0.1 mW to 6 mW. We observe that RIRO's amplitude increases with P. This can be straightforward explained according to our model since the radiation electric field  $E_0$  shows up in the numerator of the amplitude of RIRO, A. The most interesting effect can be observed around B = 0.6 T. In this region we obtain the evolution of SdHO as a function of increasing P. As in experiment[49,50], the SdHO vanish as  $R_{xx}$  tends to zero. In other words, we obtain the suppression of SdHO in the region of radiation-induced zero resistance states. According to our model this is because this region corresponds to a situation where the advanced distance  $\Delta X \to 0$ , making smaller and smaller the obtained  $R_{xx}$ , including resistance background and SdHO.

We consider now the magnetoresistance of the 2DES irradiated by two different sources with different frequencies of the TH band. In the framework of our model is relatively simple to extend the theory to two or more different radiation frequencies[61–63]. This is a consequence of the application of the superposition principle. The latter is, in turn, a consequence of the *linear* characteristic of the equation of the driven and damped classical oscillator. Then if we add two or more radiation sources to the system the displacement of the driven LL guiding center will be equal to the sum of the corresponding individual solutions. We would expect for  $R_{xx}$  a much richer and more complex profile since we would have more independent sources of interference. And the more frequencies are included the more elaborated profile would be obtained. It is straightforward to get to an expression for  $R_{xx}$  in the case of a multifrequency scenario:

$$R_{xx} \propto \left[\Delta X^0 - \sum_{i} \left[A \sin\left(2\pi \frac{w_i}{w_c}\right)\right]\right]^2 \left[1 + \frac{2X_s}{\sinh(X_s)}e^{-\frac{\pi\Gamma}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right]$$
(5)

In Fig. 3 we exhibit calculated magnetoresistance for the irradiation by two simultaneous sources of different frequencies: 300 and 400 GHz. For this specific case  $R_{xx}$  can be expressed as:

$$R_{xx} \propto \left[ \Delta X^0 - \left[ A_1 \sin \left( 2\pi \frac{w_1}{w_c} \right) + A_2 \sin \left( 2\pi \frac{w_2}{w_c} \right) \right] \right]^2 \\ \times \left[ 1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi F}{\hbar w_c}} \cos \left( \frac{2\pi E_F}{\hbar w_c} \right) \right]$$
(6)



**Fig. 4** a) Calculated irradiated magnetoresistance vs the magnetic field *B* for frequencies of 300,400 and 500 GHz in the upper panel and 300,500 and 700 GHz in the lower one. Total  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations (red curves online).

In Fig, 3a we plot irradiated magnetoresistance vs B for the two cases of total  $R_{xx}$  (black color online) and  $R_{xx}$  without SdHO or  $R_{xx,RIRO}$  (red color online). For low B we can slightly observe the rise of beats. This effect is more clearly plotted in Fig. 3b where we exhibit calculated results of irradiated  $R_{xx}$  (black color online),  $R_{xx,RIRO}$  (red color on line), and the difference of both,  $\delta R_{xx}$  (blue color online), vs  $w/w_c$ . The  $R_{xx}$  curve has been shifted up for clarity in order to spot the presence of beats in  $R_{xx,RIRO}$ . The latter shows clear beats that are a signature of the strong interference produced by the simultaneous irradiation with two independent TH-frequencies. This theoretical prediction of a very rich interference  $R_{xx}$  profile with the presence of beats, would confirm in a future experiment the oscillating nature of the LS being driven by radiation.

In Fig. 4 we present the calculated irradiated magnetoresistance,  $R_{xx}$ , vs magnetic field *B* for simultaneous irradiation of three different sources of TH radiation of frequencies of 300 GHz, 400 GHz and 500 GHz in the upper panel

and 300 GHz, 500 GHz and 700 GHz in the lower one. We observe a more intense and complicated modulation of  $R_{xx}$  with radiation-induced resistance oscillations and Shubnikov-de Haas (SdHO) oscillations, (black curves online), and  $R_{xx}$  without Shubnikov-de Haas (SdHO) oscillations, (red curves online). We expected this outcome but the more interesting result takes place in the lower panel at  $B \simeq (0.5 - 0.6)$  T. We observe that  $R_{xx} \rightarrow 0$  or approaches to ZRS with the appropriate combination of TH frequencies without increasing the radiation power.

# 4 Conclusions

In summary, we have reported on a theoretical work on magnetotransport under terahertz radiation with high mobility two-dimensional electron systems. We have focused on the interaction between the obtained radiation-induced magnetoresistance oscillations and the Shubnikov-de Haas oscillations. We study the strong modulation of the Shubnikov-de Haas oscillations at high enough TH radiation power. We have applied the radiation-driven electron orbits model and according to it the physical origin is the interference between the average advanced distance due to scattering between driven-Landau states, (radiation-induced resistance oscillations), and the available initial density of Landau states, (Shubnikov-de Haas oscillations). We have extended the theory to two or more frequencies of the TH band. Thus we have predicted an even stronger irradiated magnetoresistance profile with the rise of beats. This would confirm the oscillating nature of irradiated LS which will reveal a new way of interaction radiation-matter. Another remarkable results is that with the correct selection of TH frequencies due to the interference effect we can obtain regimes of  $R_{xx}$  close to ZRS.

# Acknowledgments

This work is supported by the MINECO (Spain) under grant MAT2014-58241-P and ITN Grant 234970 (EU). GRUPO DE MATEMATICAS APLICADAS A LA MATERIA CONDENSADA, (UC3M), Unidad Asociada al CSIC.

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Supplementary Material

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