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TESIS DOCTORAL

Topics in density forecast in stationary parametric univariate time series models

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Topics in density forecast in stationary parametric univariate time series models

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To my family!

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Abstract

In this thesis we study the computation and evaluation of density forecasts under model uncertainty in time series univariate models. First, we analyze the effects of uncertainty on density forecasts of linear univariate ARMA models. We consider three specific sources of uncertainty: parameter estimation, error distribution and lag order. For moderate sample sizes, as those usually encountered in practice, the most important source of uncertainty is the error distribution. We consider alternative procedures proposed to deal with each of these sources of uncertainty and compare their finite sample properties by Monte Carlo experiments. In particular, we analyze asymptotic, Bayesian and bootstrap procedures, including some very recent procedures which have not been previously compared in the literature. Second, we propose an extension of the Generalized Autocontour (G-ACR) tests of González-Rivera and Sun (2015) for one-step-ahead dynamic specifications of conditional densities *in-sample* and of forecast densities *out-of-sample*. The new tests are based on probability integral transforms (PITs) computed from bootstrap conditional densities that incorporate the parameter uncertainty without assuming any particular forecast error density. Consequently, the parametric specification of the conditional moments can be tested without relying on any particular error distribution. We show that the asymptotic distributions of the bootstrapped G-ACR (BG-ACR) tests are well approximated using standard asymptotic distributions. Furthermore, the proposed tests are easy to implement and are accompanied by graphical tools which provide suggestions about the potential misspecification. The results are illustrated by testing the dynamic specification of the Heterogenous autoregressive (HAR) model when fitted to the popular U.S. volatility index VIX.

Resumen

En esta tesis estudiamos la construcción y evaluación de densidades de previsión bajo incertidumbre de modelo en modelos de series temporales univariantes. Primero, analizamos los efectos de la incertidumbre en las densidades de previsión de modelos ARMA univariantes lineales. Consideramos tres fuentes específicas de incertidumbre: estimación de los parámetros, distribución de los errores y la orden del desfase. Para muestras de tamaño moderado, como aquellas que se encuentran normalmente en la práctica, la fuente más importante de incertidumbre es la de la distribución de los errores. Consideramos procedimientos alternativos propuestos para tratar cada una de esas fuentes de incertidumbre y comparamos sus propiedades para muestras finitas por medio de experimentos de Monte Carlo. En particular, analizamos procedimientos asintóticos, Bayesianos y de bootstrap, incluyendo algunos procedimientos muy recientes los cuales no han sido previamente comparados en la literatura. Segundo, proponemos una extensión del test Generalized Autocontour (G-ARC) de González-Rivera and Sun (2015) para las especificaciones dinámicas de un-paso-adelante de densidades condicionadas *in-sample* y densidades de predicción *out-of-sample*. Los nuevos tests están basados en la transformación de probabilidad integral (PITs) calculados por medio de densidades condicionadas de boostrap que incorporan la incertidumbre de parámetros sin asumir ninguna densidad particular del error de predicción. Como consecuencia, la especificacin paramétrica de los momentos condicionados puede ser testeada sin basarse en ninguna distribución particular del error. Demostramos que las distribuciones de los tests de boostrap G-ARC (BG-ACR) están bien aproximadas cuando usando distribuciones asintóticas estándar. Además, los tests propuestos son fáciles de implementar y están acompañados por herramientas gráficas, las cuáles proveen recomendaciones sobre la

posible mala especificación del modelo. Los resultados son ilustrados testeando la especificación dinámica del modelo autorregresivo hetereogéneo (HAR) cuando se ajusta al popular índice de volatilidad norteamericano VIX.

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Chapter 1

Introduction

Forecasting is of great importance for economic decision-making. Government institutions and regulatory authorities places considerable weight on the forecasts of major economic variables when taking policy decisions, and firms also rely on forecasting to manage their inventory and production. The literature has traditionally focused on production and evaluation of point forecasts, but recently, the focus has moved to obtain density forecasts of a variable of interest, since it represents a complete characterization of the uncertainty associated with the forecast, as opposed to a point forecast, which provides no information about the uncertainty of the prediction. The topic of forecast densities has received increasing attention in the economics and finance agenda; see Tay and Wallis (2000) for a survey. There is a clear need to use forecast intervals or forecast densities when setting macroeconomic policies and when managing financial risk in the insurance and banking institutions. A famous example of density forecasting in macroeconomics is the fan chart of inflation and gross domestic product (DGP) published quarterly by the Bank of England. The Bank of England publishes its forecasts of inflation and output growth as probability distributions, known as fan charts, rather than single forecasts, emphasizing the inevitable uncertainty around the outlook of the economy; see Clements and Smith (2000) for other examples of density forecasting in macroeconomics. Also, monetary policy decisions currently rely on judgemental probabilistic assessments; see, for example, Garratt et al.

(2014). In finance, one example of the interest of density forecasts is the construction of value at risks measures which are used to assess the amount of capital at risk from small probability events, such as catastrophes in insurance markets or monetary shocks that have large impact on interest rates; see Duffie and Pan (1997) and Berkowitz (2001) for further discussion. Another example of density forecasts within the finance context, is related with the maximization of the expected utility of an investor who is choosing an optimal asset allocation of stocks and bonds. In this case, there is a need to model the joint distribution of the assets; see Guidolin and Timmermann (2006).

With density forecasting spreading in applied econometrics and given its importance for taking economic and finance decisions, it is crucial to develop reliable techniques to construct and evaluate them. In this thesis, the focus is on the construction and evaluation of density forecasts based on time series models which are used to forecast the future evolution of a given variable observed during a particular period of time. Within the context of time series models, we only consider univariate and parametric models. Obviously, there is a large interest in multivariate and/or nonparametric forecasting. However, we need to reduce the context of the thesis within feasible bounds. This thesis contributes to the density forecasting literature in two different directions. First, in chapter 2, we analyze how different sources of uncertainty affect the construction of forecasting densities. The second contribution, in chapter 3, deals with the evaluation of forecasting densities.

1.1. Forecast densities and uncertainty

The standard theory of time series forecasting density is based on assuming that the model is known. Even assuming that there is a true model, it is rarely, if ever, the case that such model will be known a priori and there is no guarantee that it will be selected as the best fit to the data. Consequently, there is a question about how model uncertainty will affect the accuracy of forecasts; see, for example, Chatfield (1996, 2000). Chatfield (1996), for instance, describes the problems faced in the case of neglecting model uncertainty and shows with some examples that forecast intervals can be too narrow. Model uncertainty has also very important implications when forecasts are used in decision making process. For example, in the context of economic problems, Onatski and Stock (2002) and Onatski and Williams (2003) show that monetary policy may perform poorly when faced with a different error distribution or with slight variations of the model. Furthermore, neglecting model uncertainty can lead to big failures in risk management of derivatives; see, for example, Avramov (2006), Cont (2006), Schrimpf (2010) and Boucher et al. (2014). In the first part of this thesis, we shed light on the computation of density forecasts of univariate models ARMA models under model uncertainty, since this class of models has been wider used through the years as a forecasting tool, and is the basis of many fundamental ideas in time-series. We consider three sources of uncertainty: parameter, error distribution and lag order. Apart of studying the impact of the above uncertainties on the forecasts of ARMA models, we also provide a survey of all procedures of the literature developed with the aim of incorporating those uncertainties in the forecasts of ARMA models. For the parameter uncertainty, asymptotic methods are usually designed to incorporate it. However, they usually assume that the error distribution and lag order are known; see, for example, Yamamoto (1976) and Fuller and Hasza (1981). Alternatively, Hansen (2006) proposes an asymptotic procedure to construct forecast intervals that do not assume any particular error distribution. Bayesian methods, on their turn, incorporate naturally the parameter uncertainty through the construction of the parameter posterior distributions and they can also be designed to incorporate the lag order uncertainty. However, to be computationally feasible, Bayesian methods require to assume that the error distribution in known; see, for example, Monahan (1983), Le et al. (1996) and Ehlers and Brooks (2008). Alternatively, nonparametric Bayesian mixture procedures do not rely any error distribution assumption; see, for example, the proposal of Tang and Ghosal (2007). Finally, bootstrap procedures are able to incorporate all the uncertainties cited above; see for example, Kilian (1998a,b), Alonso et al. (2004, 2006) and Pascual et al. (2001, 2004) and Manzan and Zerom (2008), for instance, propose a non-parametric bootstrap procedure that does not assume any particular specification of the conditional moments.

1.2. Evaluating forecast densities

Apart of the importance of incorporating model uncertainty in forecasts, is the issue of how to test the correct specification of a conditional forecast density where model uncertainty is present. Appropriate tests should take into account that the forecast conditional distribution if often unknown and the specification of conditional moments is also unknown and has estimated parameters. Many tests available in the literature are based on testing a joint hypothesis of uniformity and independence of the probability integral transforms (PITs), which are applicable regardless of the particular users loss function. Among the tests, the most popular is due to Diebold et al. (1998); see also Berkowitz (2001) and Chen and Fan (2004) for extensions. However, none of these tests that check uniformity and independence take, into account parameter uncertainty. Instead of testing for independence and uniformity of PITS, González-Rivera et al. (2011) and González-Rivera and Yoldas (2012) propose autocontour (ACR) tests to evaluate the adequacy of conditional forecast densities. The ACR test, which can be applied to primitive series and model residuals. The ACR test explicitly accounts for parameter uncertainty. However, it assumes a parametric time-invariant function of the forecast density and cannot be implemented to multivariate forecast densities. To overcome these problems, González-Rivera and Sun (2015) propose the generalized autocontour (G-ACR) test, that is based on PITs instead of original observations or model residuals as in the case of the ACR test. Being based on autocontours, there is a graphical visualization aspect that is very helpful for guiding the modelling. Furthermore, it permits to focus on different areas of the conditional density in order to assess those regions of interest. However, like the other tests it still requires the specification of conditional density in order to compute the PITs, and there are applications in which the density does not have a closed form solution, as for example, multi-step predictive densities in non-linear or non-Gaussian models.

Therefore, in the third chapter, we propose an extension of the Generalized Autocontour (G-ACR) tests (Gonzalez-Rivera and Sun, 2015) for one-step-ahead dynamic specifications of conditional densities (in-sample) and of forecast densities (out-of-sample). The new tests are based on probability integral transforms (PITs) computed from bootstrap conditional densities that incorporate the parameter uncertainty without assuming any particular forecast error density. The new tests will allow us to focus on testing the parametric specification of the conditional moments without relying on any particular error distribution.

1.3. Organization of the thesis

The second chapter of this thesis studies the impact of model uncertainty on univariate ARMA models. As model uncertainty, we consider parameter, error distribution and lag order uncertainties. In order to deal with those uncertainties, we provide a complete study of all procedures that co-exist in literature developed with the aim of incorporating those uncertainties in the forecasts of ARMA models. In particular, we analyze asymptotic, Bayesian and bootstrap procedures, including some very recent procedures which have not been previously compared in the literature. We show the disadvantages and advantages of each procedure by the comparison of their finite sample performances in Monte Carlo simulations.

The third chapter of this thesis proposes an extension of the G-ACR test for dynamic specifications of a density model that do not rely on any particular assumption on the error distribution and take into account parameter uncertainty. Such test will be very useful in applications in which the density does not have a closed-form solution, as for example, multi-step predictive densities in non-linear or non-Gaussian models. Moreover, it provides a graphical device that suggests the potential misspecication of the fitted model, allowing to disentangle whether the misspecification comes from the functional form or from the assumed density. We show by Monte Carlo simulations that the proposed test has good finite properties.

Finally, in the fourth chapter, we provide a summary of the main contributions of the thesis along with lines of future research.

Chapter 2

Uncertainty and density forecasts of ARMA models: Comparison of asymptotic, Bayesian and bootstrap procedures

2.1. Introduction

The time series forecasting literature has traditionally focused on point forecasts. However, many aspects of the decision making process require making forecasts of an uncertain future. Consequently, forecasts ought to be probabilistic in nature, taking the form of probability distributions over future events; see, for example, Tay and Wallis (2000), Timmermann (2000), Greenspan (2004), Elliott and Timmermann (2008), Gneiting (2008) and Manzan and Zerom (2013) who discuss several issues related with density forecasts in economics and finance, and Chatfield (1993) and Christoffersen (1998), who stress the importance of interval forecasts for decision makers. Analytic construction of density forecasts has historically required restrictive and sometimes dubious assumptions, such as no parameter and/or model uncertainty and Gaussian innovations. However, in practice, any forecast model is an approximation to the data generating process (DGP); see the discussions by Wallis (1989), Onatski and Stock (2002) and Jordá et al. (2014). Furthermore, even if the model is correctly specified and time invariant, its parameters need to be estimated. Finally, density forecasts often rely on assumptions about the error distribution that might not be good approximations to the data distribution.

Model uncertainty may have important implications when forecasts are used in decision making processes; see Granger and Machina (2006). For example, in the context of economic problems, Draper (1995) shows that ignoring model uncertainty can seriously underestimate the uncertainty in forecasting oil prices, leading to forecast intervals that are too narrow. Onatski and Stock (2002) and Onatski and Williams (2003) show that monetary policy may perform poorly when faced with a different error distribution or with slight variations of the model. Onatski and Williams (2003) conclude that uncertainty about the parameters and the lag structure have the largest effects, whereas uncertainty about the serial correlation of the errors has minor effects. Brock et al. (2007) also explore ways to integrate model uncertainty into monetary policy evaluation. Finally, some spectacular failures in risk management have also emphasized the consequences of neglecting model uncertainty in the context of financial models; see, for example, Avramov (2006), Cont (2006), Schrimpf (2010) and Boucher et al. (2014).

In this chapter, we analyze the effects of uncertainty on the construction of densities in the context of forecast univariate ARMA models. We show that the most important distortions when constructing the densities using traditional methods appear in the context of short run forecasting when the forecast errors have a non-Normal distribution. We also compare the finite sample performance of the main alternative asymptotic, Bayesian and bootstrap procedures proposed to construct forecast densities that incorporate these uncertainties. Asymptotic methods are usually designed to incorporate the parameter uncertainty assuming a given error distribution and a given model specification; see, for example, Yamamoto (1976) and Fuller and Hasza (1981) for early references. More recently, Hansen (2006) proposes an asymptotic procedure to construct forecast intervals that does not rely on a particular assumption about the error distribution. In the context of Bayesian methods, several authors propose incorporating the parameter and lag order uncertainties using procedures based, for instance, on Bayesian Model Averaging or Reversible Jump Markov Chain Monte Carlo, which usually assume that the true model is within the model set considered; see Draper (1995) for an example of the use of Bayesian Model Averaging in economic problems. In order to be computationally feasible, Bayesian methods often assume a known error distribution, usually Gaussianity; see, for example, Monahan (1983), Le et al. (1996) and Ehlers and Brooks (2008). Alternatively, nonparametric Bayesian mixture procedures relax the distributional assumption; see, for instance, the proposal by Tang and Ghosal (2007). Nevertheless, Bayesian methods are often computationally intensive and time demanding. A competitive alternative to compute forecast densities that incorporate simultaneously the parameter, error distribution and lag order uncertainties is based on bootstrap procedures; see for example, Kilian (1998a,b), Alonso et al. (2004, 2006), Pascual et al. (2001, 2004) and Manzan and Zerom (2008). The latter authors propose a non-parametric bootstrap technique that does not assume any particular specification of the conditional moments.

We show that asymptotic methods provide reliable density forecasts only in large sample sizes and with known error distribution. On the other hand, Bayesian procedures provide very accurate density forecasts in small sample sizes, but require the correct error distribution and a large computational effort. It is also difficult to make them to take into account simultaneously all the uncertainties. As a simple alternative, the Bootstrap is able to provide reliable forecasts, regardless of the sample size and the error distribution.

The rest of the chapter is organized as follows. Section 2.2 introduces notation by describing the traditional construction of forecast densities and intervals in the context of univariate linear ARMA models. It also analyzes the effects of the uncertainties involved in the estimation of ARMA models on the forecast densities. Section 2.3 is devoted to the asymptotic, Bayesian and bootstrap procedures designed to incorporate these uncertainties in the forecasts of ARMA models, and finally, Section 2.4 concludes the chapter.

2.2. Forecast uncertainty in the context of univariate linear ARMA models

In this section, we introduce notation by describing the traditional procedure to construct forecast densities in the context of univariate linear ARMA models. The sources of uncertainty and their effects on forecast densities are also described.

2.2.1. Known error distribution, model specification and parameters

Consider the following ARMA(p, q) model

$$(1 - \phi_1 L - ... - \phi_p L^p) y_t = \mu + (1 - \theta_1 L - ... - \theta_q L^q)\varepsilon_t, \tag{2.1}$$

where y_t is the observation of the series of interest at time t, L is the lag operator, such that $L^i y_t = y_{t-i}$, for i=1,2,..., and ε_t is a strict white noise process with distribution F_ε and variance σ_ε^2. The polynomials $\phi(L) = (1 - \phi_1 L - ... - \phi_p L^p)$ and $\theta(L) = (1 - \theta_1 L - ... - \theta_q L^q)$ have all their roots outside the unit circle and no common roots between them. The autoregressive and moving average orders are p and q, respectively. Note that if ε_t is Gaussian, the polynomial $\theta(L)$ is not identifiable using second order moments, unless the invertibility assumption is imposed. However, for non-Gaussian errors, model (2.1) becomes identifiable on the basis of higher-order moments; see, for example, Breidt and Hsu (2005) and Hsu and Breidt (2009). The invertibility assumption in the non-Gaussian case is entirely artificial and removing it leads to a broad class of useful models. However, in this chapter, we assume invertibility.

If the loss function is quadratic[1] and the objective is to predict y_{T+h} given the information available at time T for $h > 0$, then the point forecast with minimum mean square forecast error (MSFE) is given by the conditional mean, denoted by $y_{T+h|T} = E(y_{T+h}|y_1, .., y_T)$; see Granger

[1]When the loss function is non-quadratic, constructing forecasts using the conditional mean is inappropriate, since the mean of the predictive distribution is not optimal as a point predictor; see Granger (1969), Christoffersen and Diebold (1997), Granger and Pesaran (2000a,b), Patton and Timmermann (2007a,b) and Gneiting (2011) for prediction problems involving asymmetric loss functions.

(1969). For the ARMA model in (2.1) with Gaussian errors and/or the MA parameters satisfying the invertibility condition, the conditional mean is a linear function of $\{y_1, ..., y_T\}$; see Rosenbaltt (2000).[2] Then, given the information observed up to time T and assuming that the errors are observable within the sample period and have a Gaussian distribution, the h-step-ahead forecast density of y_{T+h}, for $h = 1, 2, ...$, is given by

$$y_{T+h}|y_1, ..., y_T \sim N(y_{T+h|T}, MSFE(e_{T+h|T})), \tag{2.2}$$

where $y_{T+h|T}$ can be obtained recursively from

$$(1 - \phi_1 L - ... - \phi_p L^p)y_{T+h|T} = \mu + (1 - \theta_1 L - \ldots - \theta_q L^q)\varepsilon_{T+h|T}, \tag{2.3}$$

where $\varepsilon_{T+j|T} = 0$ for $j > 0$ and $\varepsilon_{T+j|T} = \varepsilon_{T+j}$ and $y_{T+j|T} = y_{T+j}$ for $j \leq 0$. $MSFE(e_{T+h|T})$ is the mean square forecast error of the h-step-ahead forecast error, $e_{T+h|T} = y_{T+h} - y_{T+h|T}$, which is given by

$$MSFE(e_{T+h|T}) = \sigma_\varepsilon^2 \sum_{i=0}^{h-1} \psi_i^2, \tag{2.4}$$

where ψ_0=1 and ψ_i, i=1,2,..., are the coefficients of the Wald representation of (2.1). The corresponding $(1 - \alpha)\%$ forecast intervals are given by

$$y_{T+h|T} \pm z_{\alpha/2}(MSFE(e_{T+h|T}))^{1/2}, \tag{2.5}$$

where $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard Normal distribution; see Granger et al. (1989) for a clear description of the construction of forecast intervals.

However, if the errors have a known but non-Gaussian distribution, then explicit expressions of the conditional forecast density can only be obtained for $h = 1$. When $h > 1$, there are not analytical expressions of the density. In this case, the forecast densities can be approximated by

[2] See Breidt and Hsu (2005) and Lanne et al. (2012) for forecasting in the context of non-invertible non-Gaussian MA models.

simulating $e_{T+h|T}$ using the true parameters and the forecast intervals are given by

$$\left[y_{T+h|T} + p_{\alpha/2}(MSFE(e_{T+h|T}))^{1/2}, y_{T+h|T} + p_{1-\alpha/2}(MSFE(e_{T+h|T}))^{1/2}\right], \tag{2.6}$$

where p_i is the ith percentile of the known error distribution when $h = 1$ and of the simulated distribution of $e_{T+h|T}$ when $h > 1$.

The construction of forecast densities described above requires the unrealistic assumption of a known forecast model, i.e, without parameter and/or lag order uncertainty and with a known error distribution. However, in practice, the forecast model is an approximation to the true DGP. Next, we revise the effects of neglecting the parameter, the error distribution and the lag order uncertainties on the construction of standard forecast densities and intervals.

2.2.2. Parameter uncertainty

Consider that the error distribution and the lag orders are known. When the parameters are unknown, the h-step-ahead forecast of y_{T+h} is obtained from equation (2.3) with the true parameters substituted by consistent estimates. In particular, in this chapter, we consider the Quasi-Maximum Likelihood (QML) estimator obtained by maximizing the Gaussian Likelihood. Hannan (1973) establishes the consistency and asymptotic Normality of the QML estimator when the model is stationary and invertible with finite second order moment and does not have a constant; see also Yao and Brockwell (1988) for a direct proof.[3] Recently, Bao (2016a) considers the ARMA model with a constant and derives a compact analytical representation of the asymptotic covariance matrix of the QML estimator. Note that, if there is not a MA part, the QML estimator reduces to Least Squares (LS). It is well known that in finite samples the LS estimator is biased; see, among others, Shaman and Stine (1988), Kiviet and Phillips (1994), Patterson (2000) and Ledolter (2009). For example, in an AR(1) model, the bias tends to shrink the LS estimator toward zero, with larger bias when the autoregressive coefficient is larger in absolute value. Ledolter

[3] Hsu and Breidt (2009) propose an exact ML estimator that does not require invertibility; see also Lii and Rosenblatt (1992, 1996), Huang and Pawitan (2000) and Gospodinov and Ng (2015) for alternative estimators.

(2009) shows that the effect of bias on point forecasts is small. However, when $h > 1$, it affects the coverage of forecast intervals since forecast intervals become too narrow. The coverage is improved when bias-adjusted estimates of the autoregressive parameter are used. Kim and Durmaz (2012), who describe alternative bias-correction procedures, show that substantial gains of correcting for bias can be obtained when the true AR model is very persistent and/or the forecast horizon is fairly short. The results about biases of the QML estimator when the model contains a MA component are much more scarce. Some examples regarding simple MA(1) models are Tanaka (1984) and Cordeiro and Klein (1994) who derive the approximated bias of the QML estimator under the data assumption of Normality. Other examples are Bao and Ullah (2007) that consider the case when the data is not Normal, but restrict it to a zero mean MA(1) model, and Demos and Kyriakopoulou (2013) that derive the bias of the QML estimator for a MA(1) model with a known or unknown intercept. More recently, Bao et al. (2014) derive the approximated bias of the QML estimator of the parameters in an invertible MA(1) model with a possible non-zero mean and non-Normal distributed errors. They show that the feasible multi-step-ahead forecasts are unbiased under any non-Normal distribution while the one-step-ahead forecast is unbiased under symmetric distributions. Finally, results for general ARMA(p,q) models are given by Bao (2016b).

In practice, the forecast density of y_{T+h} is obtained as in (2.2) with the unknown parameters involved in $y_{T+h|T}$ and $MSFE(e_{T+h|T})$ substituted by the corresponding QML estimates corrected by bias. Denote by $\hat{y}_{T+h|T}$ and $\widehat{MSFE}(e_{T+h|T})$ the point forecast and estimated MSFE, respectively. The latter is not the MSFE of $\hat{y}_{T+h|T}$, which is given by

$$MSFE(\hat{y}_{T+h|T}) = MSFE(e_{T+h|T}) + E_T[(y_{T+h|T} - \hat{y}_{T+h|T})^2], \tag{2.7}$$

where the last term, which is of order $O(T^{-1})$, depends on the mean square error (MSE) of the parameter estimator; see Fuller (1996). To illustrate the effect of the underestimation of the MSFE of $\hat{y}_{T+h|T}$ on the forecast densities, when it is obtained by $\widehat{MSFE}(e_{T+h|T})$, we carry out Monte

Carlo experiments based on $R = 1000$ replicates generated by two AR models. The first DGP is given by an AR(1) model with parameters $\mu = 0$, $\phi = 0.8$ and $\sigma^2_\varepsilon = 1$. The second DGP is a persistent AR(2) model with parameters $\mu = 0$, $\phi_1 = 0.6$, $\phi_2 = 0.3$ and $\sigma^2_\varepsilon = 1$. The disturbances are either Gaussian, Student-5 or $\chi^2_{(5)}$. For each replicate, the parameters are estimated by LS and corrected from bias. The bias correction is carried out using the procedure proposed by Orcutt and Winokur (1969) with the expression of Shaman and Stine (1988) and Stine and Shaman (1989) for the first order bias of the LS estimator of an AR model of known and finite order; see Patterson (2000) and Kim (2004) for implementations of this procedure.It is important to note that the bias correction can push estimates into the non-stationarity region, mainly when the model is highly persistent. Consequently, the stationarity correction proposed by Kilian (1998b) is implemented.[4] Then, the estimated conditional forecast densities are computed, for h =1, 6 and 12, as in (2.2) when the errors are Gaussian or by simulation when they are not Gaussian and h >1, using $\hat{y}_{T+h|T}$ and $\widehat{MSFE}(e_{T+h|T})$. These densities, denoted as EST, are constructed assuming that both the lag-order and the error distribution are known; see Table 2.1 for a summary of all procedures considered in this chapter to construct density and interval forecasts, their acronyms, properties and some references. We also obtain the corresponding 80% and 95% forecast intervals. Finally, for each replicate, we generate 1000 values of y_{T+h}, and construct their empirical forecast density and count how many of these values lie inside the EST intervals. Table 2.2 reports the Monte Carlo averages and standard deviations of the Mallows Distances (MD) proposed by Mallows (1972) between the empirical and EST h-step-ahead forecast densities when the DGP is the AR(2) model; see Czado and Munk (1998) and Levina and Bickel (2001) for some properties of the MD distance and Lopes et al. (2013) and Fresoli et al. (2015) for applications of the MD in the context of Gaussian and non-Gaussian VARFIMA(0,d,0) and VAR models, respectively.[5] It is shown that,

[4]Alternatively, Kim et al. (2010) propose a stationarity correction based on the stable spectral factorization of Poskitt and Salau (1993).

[5]The MD is computed as follows. Let $x_{(1)} \leq ... \leq x_{(N)}$ and $y_{(1)} \leq ... \leq y_{(N)}$ be ordered realizations of the random variables X and Y, with absolutely continuous distributions F and G, respectively. The MD between F and G is given by $MD(F,G) = \left(\frac{1}{N}\sum_{i=1}^{N}|x_{(i)} - y_{(i)}|^\alpha\right)^{1/\alpha}$. In this chapter we use $\alpha = 1$.

as expected, regardless of the error distribution, the MDs of EST decrease with the sample size and increase with the forecast horizon. Moreover, the averages and standard deviations of the distances have similar magnitudes for the different error distributions considered.

We also analyze the finite sample coverages of the forecast intervals obtained by EST. Table 2.3 reports the Monte Carlo average coverages of the EST forecast intervals when the nominal coverage is 80%. Note that, regardless of the distribution, if the sample size is T=50, the empirical coverages of EST are around 77%, slightly smaller than the nominal level. The undercoverage is slighly larger for non-Gaussian distributions. However, if the sample size is T=100 or larger, the coverage rates are very close to the nominal level. Consequently, the parameter uncertainty is not an important issue when constructing forecast intervals as far as the sample size is moderate or large.[6]

[6]The results for the AR(1) model and 95% nominal coverages are similar and are reported in Tables A.1 and A.3 of Appendix A.

Table 2.1: Procedures considered in the chapter.

Acronyms	Description	References
EST	Parameters estimated by QML. The error distribution and the lag order are known	
GAUS	Parameters estimated by QML. The error distribution is assumed to be Gaussian and the lag order is known	
$\text{GAUS}_{\text{aicc}}$	Parameters estimated by QML. The error distribution is assumed to be Gaussian and the lag order is estimated by the AICC criterion	
AEST	Parameters estimated by QML with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE The error distribution and the lag order are known	Fuller and Hasza (1981) and Fuller (1996)
AGAUS	Parameters estimated by QML with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE The error distribution is assumed to be Gaussian and the lag order is known	Fuller and Hasza (1981) and Fuller (1996)
$\text{AGAUS}_{\text{aicc}}$	Parameters estimated by QML with the MSFE of $\hat{y}_{T+h\|T}$ approximated by the AMSFE The error distribution is assumed to be Gaussin and the lag order is estimated by the AICC criterion	
SRA	Conditional forecast intervals that incorporate parameter and error distribution uncertainty	Hansen (2006)
BAYESN	Bayesian procedure to incorporate parameter uncertainty in the forecasts The errors are assumed to be Gaussian	Thompson and Miller (1986)
BAYEST	Bayesian procedure to incorporate parameter uncertainty in the forecasts The errors are assumed to be Student-ν	Thompson and Miller (1986)
BAYESL	Bayesian LASSO procedure. It incorporates parameter and lag order uncertainty in the forecasts	Schmidt and Makalic (2013)
BOOT	Bootstrap procedure. It incorporates parameter and error distribution uncertainty in the forecasts	Pascual et al. (2001, 2004)
BOOTEX	Sieve exogenous bootstrap procedure. It incorporates parameter, error distribution and lag order uncertainty	Alonso et al. (2004)
BOOTNP	Non-parametric bootstrap procedure	Manzan and Zerom (2008)

Table 2.2: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/GAUS	0.187	0.462	0.682	0.129	0.302	0.425	0.068	0.152	0.205
	(0.150)	(0.447)	(0.710)	(0.105)	(0.304)	(0.456)	(0.057)	(0.154)	(0.208)
$\text{GAUS}_{\text{aicc}}$	0.255	0.517	0.698	0.168	0.336	0.452	0.079	0.162	0.215
	(0.201)	(0.443)	(0.678)	(0.163)	(0.319)	(0.461)	(0.074)	(0.156)	(0.209)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.204	0.477	0.682	0.147	0.341	0.472	0.078	0.170	0.224
	(0.182)	(0.440)	(0.668)	(0.124)	(0.347)	(0.536)	(0.067)	(0.162)	(0.216)
GAUS	0.234	0.491	0.693	0.185	0.357	0.485	0.136	0.200	0.250
	(0.181)	(0.443)	(0.672)	(0.122)	(0.343)	(0.532)	(0.065)	(0.159)	(0.214)
$\text{GAUS}_{\text{aicc}}$	0.290	0.541	0.704	0.216	0.391	0.509	0.144	0.212	0.264
	(0.228)	(0.462)	(0.665)	(0.162)	(0.358)	(0.531)	(0.077)	(0.167)	(0.225)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.201	0.501	0.724	0.138	0.330	0.463	0.075	0.167	0.221
	(0.156)	(0.467)	(0.754)	(0.113)	(0.327)	(0.497)	(0.054)	(0.139)	(0.184)
GAUS	0.290	0.537	0.750	0.248	0.377	0.495	0.217	0.241	0.277
	(0.133)	(0.457)	(0.750)	(0.087)	(0.311)	(0.486)	(0.040)	(0.120)	(0.168)
$\text{GAUS}_{\text{aicc}}$	0.335	0.594	0.768	0.273	0.409	0.519	0.224	0.251	0.288
	(0.170)	(0.464)	(0.738)	(0.124)	(0.336)	(0.505)	(0.056)	(0.137)	(0.186)

Table 2.3: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/GAUS	78.06 (0.05)	10.84/11.09	77.39 (0.09)	11.09/11.52	77.55 (0.11)	10.99/11.46
	$\text{GAUS}_{\text{aicc}}$	76.60 (0.06)	11.69/11.71	77.38 (0.10)	11.18/11.44	76.96 (0.12)	11.37/11.67
100	EST	79.00 (0.04)	10.58/10.43	78.86 (0.06)	10.61/10.53	78.97 (0.08)	10.54/10.49
	$\text{GAUS}_{\text{aicc}}$	78.18 (0.05)	10.95/10.87	78.72 (0.07)	10.66/10.62	78.58 (0.08)	10.69/10.73
300	EST	79.66 (0.02)	10.18/10.17	79.78 (0.03)	10.2/10.02	79.82 (0.04)	10.16/10.02
	$\text{GAUS}_{\text{aicc}}$	79.47 (0.02)	10.26/10.26	79.66 (0.03)	10.23/10.11	79.72 (0.05)	10.2/10.08
	Student-5	h=1		h=6		h=12	
50	EST	76.99 (0.07)	11.70/11.31	76.28 (0.10)	12.07/11.64	76.87 (0.12)	11.76/11.37
	GAUS	81.33 (0.07)	9.50/9.16	78.02 (0.10)	11.19/10.79	78.06 (0.12)	11.15/10.78
	$\text{GAUS}_{\text{aicc}}$	79.74 (0.08)	10.21/10.04	77.62 (0.11)	11.16/11.22	77.30 (0.13)	11.33/11.36
100	EST	78.30 (0.05)	10.94/10.76	77.85 (0.07)	11.12/11.02	77.93 (0.09)	11.08/10.99
	GAUS	82.52 (0.05)	8.82/8.66	79.49 (0.07)	10.30/10.21	79.09 (0.09)	10.50/10.42
	$\text{GAUS}_{\text{aicc}}$	81.59 (0.06)	9.29/9.11	79.23 (0.08)	10.56/10.21	78.72 (0.09)	10.80/10.48
300	EST	79.53 (0.03)	10.22/10.26	79.50 (0.04)	10.20/10.3	79.58 (0.05)	10.14/10.28
	GAUS	83.65 (0.03)	8.15/8.19	81.11 (0.04)	9.40/9.49	80.68 (0.05)	9.58/9.74
	$\text{GAUS}_{\text{aicc}}$	83.35 (0.03)	8.29/8.35	80.90 (0.04)	9.49/9.61	80.47 (0.05)	9.67/9.84
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	77.31 (0.09)	11.68/11.01	76.45 (0.10)	11.74/11.80	76.45 (0.13)	11.66/11.88
	GAUS	82.52 (0.07)	5.74/11.72	77.51 (0.10)	10.04/12.44	77.15 (0.12)	10.42/12.42
	$\text{GAUS}_{\text{aicc}}$	80.61 (0.09)	7.13/12.25	77.23 (0.11)	10.20/12.57	76.35 (0.13)	10.83/12.81
100	EST	78.33 (0.07)	11.13/10.53	78.17 (0.07)	11.88/10.94	78.28 (0.09)	10.77/10.93
	GAUS	83.80 (0.05)	4.96/11.24	79.24 (0.07)	9.19/11.56	78.93 (0.09)	9.59/11.47
	$\text{GAUS}_{\text{aicc}}$	82.50 (0.07)	6.00/11.49	78.85 (0.08)	9.47/11.67	78.40 (0.09)	9.95/11.64
300	EST	79.53 (0.04)	10.32/10.13	79.48 (0.04)	10.33/10.18	79.59 (0.05)	10.23/10.17
	GAUS	85.29 (0.03)	3.86/10.84	80.57 (0.04)	8.62/10.80	80.25 (0.05)	9.05/10.69
	$\text{GAUS}_{\text{aicc}}$	84.85 (0.04)	4.23/10.92	80.33 (0.04)	8.81/10.85	80.03 (0.05)	9.21/10.75

2.2.3. Uncertainty about the error distribution

Traditional forecasting procedures in the context of linear time series models assume Gaussian forecast errors. However, often, the variables under analysis do not have a Gaussian distribution; see, for example, Li and McLeod (1988), Kilian (1998b) and Harvey and Newbold (2003) for departures from Gaussianity in the context of economic time series. Note that when the errors are non-Gaussian, it is not always clear which distribution should be assumed.

If the forecast densities and the corresponding intervals are constructed as in (2.2) and (2.5), the quantile of the Normal distribution could not be appropriate any longer. Denote by GAUS the forecast densities constructed as in (2.2) with the parameters substituted by their corresponding QML estimates corrected from bias. Note that, when the errors are Gaussian, the EST and GAUS procedures coincide. Table 2.2, which reports the Monte Carlo averages and standard deviations of the MD distances, shows that, when the errors are Student-5 and $\chi^2_{(5)}$, the distances are larger for the GAUS than for the EST densities, especially for asymmetric errors. Moreover, the difference between the distances of the EST and GAUS densities increases with the sample size and decreases with h. Note that when h=12 the MDs of the GAUS densities are very similar to those of the EST densities. For example, when the errors are $\chi^2_{(5)}$ and T=300, the increase in the average MD is $\frac{0.217-0.075}{0.075} = 189.33\%$ when $h = 1$ while the increase is $\frac{0.277-0.221}{0.221} = 25.34\%$ when $h = 12$. Therefore, it seems that assuming Normal forecast errors when they are non-normal has an important effect on the construction of forecast densities mainly when the sample size is large and the forecast horizon is small.

The Monte Carlo averages and standard deviations of the coverage rates of the GAUS forecast intervals are reported in Table 2.3, when the nominal coverage is 80%. In both cases, one-step-ahead GAUS intervals have average coverages that tend to overestimate the nominal level. The overcoverage is larger as T increases. Furthermore, when the errors follow a $\chi^2_{(5)}$ distribution, we observe that the coverage in the left tail is much smaller than that in the right tail. In accordance with the results in Table 2.2, these problems decrease when h increases, that is, the coverages tend to the nominal level, suggesting that the effect of assuming wrongly Normality is less important in the long term.

2.2.4. Uncertainty about the orders p and q of the ARMA process

Besides the uncertainty about the error distribution, when fitting an ARMA stationary model to a data set, the true orders of the underlying stochastic process are often unknown and should be determined. In practice, the model used for forecasting is chosen by using a selection criterion and forecasts are obtained conditional on the selected model which is considered as being the true one. The most popular selection criteria are the Akaike (1973) information criterion (AIC), its bias-corrected version (AICC) proposed by Hurvich and Tsai (1989, 1991), which penalizes larger models to counteract the overfitting nature of AIC, and the Bayesian information criterion (BIC) of Schwarz (1978); see Bhansali (1993) for a review of other selection procedures. Several authors study the effects of order misspecification on conditional forecasts. For instance, Tanaka and Maekawa (1984), assuming Gaussian errors, derive analytically the asymptotic MSFE when the forecasts are obtained from an AR(1) and the true model is an ARMA(1,1). For h=1, they derive expressions for the bias and the MSFE when the wrong model is assumed and conclude that, in this situation, the MSFE is underestimated. Davies and Newbold (1980) also show that although a MA(1) model can be approximated arbitrarily closely by an high order AR model, the finite sample effect of estimating additional parameters is that the forecast error variance increases.

Nevertheless, Chatfield (1996, 2000) warns about the forecast biases generated by formulating and fitting a model to the same data. He argues that those forecasts will be over-optimistic when the data-dependent model-selection process is ignored, leading to forecast intervals that are generally too narrow and fail to take into account the model uncertainty. In other words, it is expected that a model fitted to the same data used to formulate it will provide the best fit among the alternative models; see also Clements and Hendry (1998, 2001) for a detailed taxonomy of uncertainty applied to forecast errors in economic stationary and non-stationary time series.

In order to analyse the impact of the lag order uncertainty of an ARMA model on the density and interval forecasts, we carry out Monte Carlo experiments by generating replicates from the same AR(2) model described above. In each simulation, we assume an AR(p) model and select p using the AICC criterion with $p_{max} = T/10$ as recommended by Bhansali (1983). The parameters of the selected model are estimated by LS and corrected from bias, and the forecast densities and the corresponding forecast intervals are constructed assuming Gaussian errors. This procedure is denoted as $\text{GAUS}_{\text{aicc}}$.[7]

Table 2.2 provides the Monte Carlo MD averages and standard deviations of the $\text{GAUS}_{\text{aicc}}$ densities. We observe that, regardless of the error distribution, the distances between the true and $\text{GAUS}_{\text{aicc}}$ densities are larger than those obtained with the GAUS procedure and they decrease with the sample size, as expected, since the AICC criterion is asymptotically efficient. Furthermore, the differences between the GAUS and $\text{GAUS}_{\text{aicc}}$ distances decrease with the forecast horizon.

Analysing the Monte Carlo average coverages reported in Table 2.3, we observe that the coverages of the $\text{GAUS}_{\text{aicc}}$ intervals are similar to those of the EST and GAUS intervals when the errors are Gaussian and non-Gaussian, respectively.

2.3. Procedures to incorporate the forecast uncertainties of ARMA models

In the previous section, we have seen that the effects of parameter and lag-order uncertainties on the forecast densities are neglegible in moderate sample sizes. However, assuming wrongly Normality may generate important distortions mainly when forecasting in the short run. In this section, we revise the procedures proposed in the literature to incorporate the types of uncertainties described in the previous section and analyze their finite sample performance. We classify them in three categories: asymptotic, Bayesian and bootstrap procedures.

[7]Note that the bias correction procedure of Shaman and Stine (1988) and Stine and Shaman (1989), in the case of lag order misspecification, just holds when the order is overspecified.

2.3.1. Asymptotic methods

To correct the biases of the MSFE caused by parameter uncertainty, many authors propose using asymptotic approximations of the MSE of the QML estimator to compute the MSFE of $\hat{y}_{T+h|T}$ in (2.7). The derivation of the asymptotic MSFE (AMSFE) is usually based on assuming that the sample data used to estimate the parameters are statistically independent of the data used to construct the forecasts. Although Phillips (1979) points out that this assumption is quite unrealistic in practical situations, Maekawa (1987) shows that the AMSFE of AR(p) processes is the same regardless of whether the data used for parameter estimation is dependent on that used for forecasting. The expression of the AMSFE of AR(p) models has been derived by Fuller and Hasza (1981) who extend the results of Phillips (1979) for the AMSFE of AR(1) processes while Ansley and Kohn (1986) extend it to state-space models. As the general ARMA model can be formulated as a state-space model, the latter results also cover ARMA models as a special case. It is worth noting that the above results on the AMSFE have been derived in the context of Gaussian errors. Bao (2007) study the MSFE of the AR(1) model with non-Normal distributed errors and shows that it coincides with the unconditional AMSFE of Box and Jenkins (1970) and Yamamoto (1976). Bao and Zhang (2014) point out that results for AMSFE in the context of non-Normal data are not available for MA models.

In this chapter, we consider the conditional asymptotic approximation proposed by Fuller and Hasza (1981) and Fuller (1996). If the forecast errors are Gaussian, the conditional forecast density of y_{T+h} can be constructed as in (2.2) with $MSFE(e_{T+h|T})$ substituted by $AMSFE(\hat{y}_{T+h|T})$. Analogously, in the case of non-Gaussian errors, the distribution of $y_{T+1}|y_1, ..., y_T$ could be approximated by the distribution assumed for the error if h=1. For h >1, the forecast distribution of $y_{T+h}|y_1, ..., y_T$ could be simulated using the estimated parameters adjusted by bias as described above. The estimated AMSFE is denoted by $\widehat{AMSFE}$. Since the term associated to the parameter uncertainty in the AMSFE is of order T^{-1}, the impact of the parameter uncertainty to the MSFE of $\hat{y}_{T+h|T}$ is negligible when the sample size is relatively large. On the other hand, for a given sample size, the parameter uncertainty contribution increases with the forecast horizon.

The Monte Carlo results for the MD distances when the MSFE is replaced by the $\widehat{AMSFE}$ are approximately identical to those reported in Table 2.2. Table 2.5 reports the MC average coverages and standard deviations of the EST, GAUS and $\text{GAUS}_{\text{aicc}}$ intervals, computed with the MSFE substituted by the $\widehat{AMSFE}$ and denoted by AEST, ABJ and ABJ_{aicc}, respectively. The results show that, using the asymptotic correction, the coverages are only slighly larger than those reported in Table 2.3 without the correction. In general, when the coverage is bellow the nominal, we obtain coverages closer to the nominal. However, when the error distribution is non-Normal and the forecast density is assumed to be Normal, the overcoverage is even larger than that obtained without the asymptotic correction. Therefore, it seems that the asymptotic correction of the MSFE is not useful to obtain forecast intervals with better coverages. Furthermore, the computation of the AMSFE can become difficult in high order autoregressive or general ARMA models.

When constructing forecast intervals using the AMSFE, we need to assume a particular distribution for the errors. Alternatively, Hansen (2006) proposes the Simple Reference Adjustment (SRA) procedure to construct conditional asymptotic forecast intervals.[8] Unlike the asymptotic methods described above, the SRA procedure only requires i.i.d. errors, without relying on any particular assumption about the error distribution. The SRA intervals are based on direct forecast autoregressions whose forecast interval endpoints depend on the sample size and the empirical distribution of the residuals. In order to analyze the finite sample performance of the SRA procedure when constructing forecast intervals, consider again the same AR(2) model used in the previous Monte Carlo simulations. Table 2.5, which reports the Monte Carlo averages and standard deviations of the coverages of the SRA forecast intervals, implemented without estimating the lag order, shows that, regardless of the error distribution, the empirical coverages are close to the nominal when h =1, but they decrease substantially for $h = 6$ and 12. We also implement the SRA procedure after estimating the lag order and denote it by SRA_{aicc}. Comparing

[8]Note that the procedure proposed by Hansen (2006) does not allow the construction of forecast densities. Furthermore, there is no bias correction method available for direct forecast regressions.

the coverages of the $\mathrm{ABJ}_{\mathrm{aicc}}$ and $\mathrm{SRA}_{\mathrm{aicc}}$ densities, we observe that the latter only provides accurate coverages for h =1. The poor performance of the SRA forecast intervals in the long run may be due to the fact that SRA is based on direct forecasts rather on iterated forecasts, as the previous procedures are. Ing (2003) shows that when $\hat{p} > p$ is fixed, the relative performance of direct forecasts, in terms of mean square forecast error, deteriorates as the forecast horizon increases. Similar conclusions are found by Marcellino et al. (2006) who compares iterated and direct forecasts in macroeconomic time series. Therefore, it seems that the SRA intervals may only be applicable to sample sizes rather large and/or short horizons. For all procedures including the SRA, we have calculated the 95% interval coverages and the conclusions are similar.[9] However, we observe that for these two significance levels the SRA procedure often provides intervals with lengths that are unrealistically large.

[9]Results are reported in Tables A.6 and A.7 of Appendix A.

Table 2.4: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/AGAUS	0.187	0.464	0.688	0.130	0.304	0.429	0.068	0.152	0.207
	(0.150)	(0.447)	(0.711)	(0.105)	(0.304)	(0.457)	(0.057)	(0.154)	(0.209)
$AGAUS_{aicc}$	0.255	0.522	0.706	0.168	0.338	0.456	0.079	0.162	0.216
	(0.202)	(0.444)	(0.680)	(0.163)	(0.319)	(0.462)	(0.074)	(0.156)	(0.209)
BAYESN	0.193	0.406	0.561	0.139	0.280	0.363	0.092	0.177	0.225
	(0.148)	(0.348)	(0.616)	(0.094)	(0.214)	(0.303)	(0.057)	(0.132)	(0.174)
BAYESL	0.264	0.430	0.451	0.241	0.386	0.416	0.144	0.243	0.273
	(0.194)	(0.292)	(0.257)	(0.181)	(0.273)	(0.278)	(0.096)	(0.171)	(0.191)
BOOT	0.219	0.476	0.734	0.164	0.334	0.487	0.102	0.187	0.249
	(0.131)	(0.421)	(0.687)	(0.096)	(0.297)	(0.468)	(0.054)	(0.153	(0.215)
BOOTEX	0.251	0.494	0.710	0.190	0.351	0.494	0.110	0.197	0.259
	(0.158)	(0.404)	(0.651)	(0.132)	(0.296)	(0.450)	(0.065)	(0.154)	(0.214)
BOOTNP	0.483	0.812	0.974	0.404	0.623	0.755	0.306	0.411	0.494
	(0.277)	(0.447)	(0.552)	(0.247)	(0.379)	(0.454)	(0.219)	(0.275)	(0.315)
Panel B: Student-5		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.204	0.479	0.687	0.147	0.342	0.477	0.078	0.171	0.225
	(0.183)	(0.441)	(0.670)	(0.124)	(0.348)	(0.538)	(0.067)	(0.163)	(0.218)
AGAUS	0.236	0.493	0.699	0.187	0.360	0.491	0.137	0.201	0.252
	(0.182)	(0.443)	(0.673)	(0.122)	(0.343)	(0.534)	(0.066)	(0.161)	(0.217)
$AGAUS_{aicc}$	0.291	0.546	0.713	0.217	0.394	0.515	0.145	0.213	0.266
	(0.229)	(0.463)	(0.673)	(0.162)	(0.358)	(0.533)	(0.077)	(0.169)	(0.227)
BAYEST	0.197	0.376	0.513	0.146	0.287	0.375	0.097	0.178	0.221
	(0.144)	(0.300)	(0.477)	(0.094)	(0.221)	(0.322)	(0.052)	(0.118)	(0.149)
BAYESN	0.234	0.411	0.546	0.181	0.312	0.400	0.138	0.196	0.237
	(0.192)	(0.382)	(0.590)	(0.114)	(0.268)	(0.426)	(0.065)	(0.145)	(0.189)
BAYESL	0.305	0.424	0.443	0.263	0.391	0.413	0.180	0.262	0.287
	(0.244)	(0.332)	(0.287)	(0.184)	(0.283)	(0.278)	(0.095)	(0.187)	(0.217)
BOOT	0.240	0.485	0.721	0.183	0.369	0.532	0.113	0.198	0.258
	(0.160)	(0.415)	(0.652)	(0.111)	(0.333)	(0.538)	(0.054)	(0.150)	(0.213)
BOOTEX	0.270	0.505	0.706	0.211	0.390	0.533	0.122	0.208	0.266
	(0.187)	(0.410)	(0.637)	(0.139)	(0.330)	(0.509)	(0.063)	(0.152)	(0.214)
BOOTNP	0.512	0.805	0.968	0.418	0.645	0.809	0.330	0.436	0.527
	(0.311)	(0.429)	(0.507)	(0.315)	(0.419)	(0.551)	(0.210)	(0.309)	(0.413)
Panel C: $\chi^2_{(5)}$		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.202	0.503	0.730	0.138	0.331	0.466	0.075	0.167	0.223
	(0.156)	(0.468)	(0.755)	(0.113)	(0.328)	(0.498)	(0.054)	(0.139)	(0.185)
AGAUS	0.292	0.540	0.757	0.249	0.379	0.500	0.218	0.242	0.279
	(0.134)	(0.458)	(0.752)	(0.087)	(0.311)	(0.488)	(0.040)	(0.120)	(0.169)
$AGAUS_{aicc}$	0.336	0.598	0.777	0.274	0.411	0.524	0.225	0.252	0.290
	(0.170)	(0.468)	(0.756)	(0.124)	(0.337)	(0.507)	(0.056)	(0.137)	(0.187)
BAYESN	0.294	0.479	0.623	0.247	0.336	0.416	0.218	0.238	0.267
	(0.137)	(0.391)	(0.678)	(0.081)	(0.228)	(0.341)	(0.041)	(0.110)	(0.148)
BAYESL	0.341	0.486	0.489	0.303	0.423	0.439	0.246	0.296	0.311
	(0.172)	(0.327)	(0.298)	(0.128)	(0.258)	(0.263)	(0.072)	(0.157)	(0.184)
BOOT	0.229	0.507	0.767	0.167	0.355	0.515	0.103	0.191	0.252
	(0.142)	(0.440)	(0.730)	(0.103)	(0.323)	(0.512)	(0.050)	(0.136)	(0.192)
BOOTEX	0.261	0.529	0.745	0.194	0.376	0.518	0.111	0.200	0.260
	(0.152)	(0.431)	(0.709)	(0.127)	(0.322)	(0.496)	(0.067)	(0.144)	(0.200)
BOOTNP	0.496	0.782	0.951	0.403	0.623	0.780	0.336	0.414	0.497
	(0.284)	(0.435)	(0.545)	(0.253)	(0.393)	(0.498)	(0.221)	(0.257)	(0.309)

Table 2.5: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/AGAUS	78.67 (0.05)	10.54/10.79	78.05 (0.09)	10.76/11.18	78.23 (0.11)	10.65/11.13
	$\text{AGAUS}_{\text{aicc}}$	77.16 (0.06)	11.42/11.42	78.01 (0.10)	10.88/11.11	77.53 (0.12)	11.08/11.39
	SRA	79.84 (0.06)	8.98/11.18	72.32 (0.13)	12.24/15.44	64.80 (0.15)	15.69/19.51
	SRA_{aicc}	79.26 (0.07)	9.32/11.42	72.57 (0.13)	4.22 (1.25)	65.05 (0.16)	15.60/19.35
100	AEST	79.38 (0.04)	10.38/10.24	79.46 (0.06)	10.30/10.23	79.67 (0.08)	10.19/10.14
	$\text{AGAUS}_{\text{aicc}}$	78.61 (0.04)	10.74/10.65	79.33 (0.07)	10.36/10.31	79.26 (0.08)	10.35/10.39
	SRA	79.97 (0.04)	9.56/10.47	76.96 (0.08)	11.08/11.96	73.98 (0.11)	12.40/13.62
	SRA_{aicc}	79.40 (0.05)	9.77/10.83	76.42 (0.09)	11.29/12.29	73.17 (0.12)	12.74/14.08
300	AEST	79.80 (0.02)	10.11/10.09	80.05 (0.03)	10.07/9.88	80.13 (0.05)	9.99/9.87
	$\text{AGAUS}_{\text{aicc}}$	79.65 (0.02)	10.18/10.17	79.94 (0.03)	10.09/9.97	80.03 (0.05)	10.04/9.92
	SRA	79.97 (0.03)	9.86/10.16	79.19 (0.04)	10.37/10.44	78.63 (0.06)	10.64/10.73
	SRA_{aicc}	79.86 (0.03)	9.92/10.22	78.90 (0.04)	10.45/10.65	78.51 (0.06)	10.75/10.74
	Student-5	h=1		h=6		h=12	
50	AEST	77.55 (0.07)	11.42/11.03	76.88 (0.10)	11.76/11.36	77.48 (0.12)	11.44/11.08
	AGAUS	81.85 (0.06)	9.24/8.90	78.58 (0.10)	10.91/10.51	78.66 (0.12)	10.83/10.50
	$\text{AGAUS}_{\text{aicc}}$	80.24 (0.07)	9.98/9.78	78.13 (0.11)	10.91/10.96	77.80 (0.13)	11.08/11.11
	SRA	79.74 (0.07)	9.33/10.94	71.28 (0.13)	13.56/15.17	64.92 (0.16)	16.58/18.50
	SRA_{aicc}	79.25 (0.08)	9.46/11.29	71.45 (0.13)	13.13/15.42	65.27 (0.16)	16.28/18.44
100	AEST	78.62 (0.05)	10.77/10.61	78.40 (0.07)	10.86/10.74	78.61 (0.09)	10.75/10.64
	AGAUS	82.81 (0.05)	8.66/8.52	80.02 (0.07)	10.05/9.93	79.76 (0.09)	10.17/10.06
	$\text{AGAUS}_{\text{aicc}}$	81.94 (0.06)	9.11/8.95	79.82 (0.08)	10.27/9.91	79.38 (0.09)	10.48/10.14
	SRA	79.92 (0.05)	9.67/10.41	76.56 (0.09)	11.28/12.16	73.95 (0.11)	12.68/13.37
	SRA_{aicc}	79.28 (0.06)	10.01/10.71	76.31 (0.09)	11.44/12.25	73.25 (0.12)	12.93/13.82
300	AEST	79.64 (0.03)	10.16/10.20	79.75 (0.04)	10.08/10.16	79.86 (0.05)	10.00/10.14
	AGAUS	83.75 (0.03)	8.09/8.14	81.34 (0.04)	9.28/9.37	80.96 (0.05)	9.45/9.59
	$\text{AGAUS}_{\text{aicc}}$	83.50 (0.03)	8.23/8.27	81.17 (0.04)	9.36/9.47	80.79 (0.05)	9.53/9.68
	SRA	80.09 (0.03)	9.74/10.17	79.19 (0.04)	10.08/10.72	78.64 (0.06)	10.33/11.03
	SRA_{aicc}	79.97 (0.03)	9.79/10.24	79.00 (0.05)	10.29/10.71	78.38 (0.06)	10.57/11.05
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	78.01 (0.09)	11.20/10.77	77.10 (0.10)	11.35/11.53	77.07 (0.13)	11.27/11.65
	AGAUS	83.14 (0.07)	5.36/11.49	78.15 (0.10)	9.67/12.17	77.77 (0.13)	10.05/12.17
	$\text{AGAUS}_{\text{aicc}}$	81.17 (0.08)	6.77/12.05	77.79 (0.12)	9.86/12.34	76.88 (0.14)	10,52/12,60
	SRA	79.78 (0.08)	9.23/10.99	71.21 (0.13)	13.45/15.34	63.78 (0.16)	17.31/18.91
	SRA_{aicc}	78.98 (0.09)	9.76/11.26	71.33 (0.13)	13.58/15.10	64.16 (0.17)	16.93/18.92
100	AEST	78.77 (0.07)	10.82/10.40	78.78 (0.07)	10.52/10.69	78.98 (0.09)	10.37/10.64
	AGAUS	84.17 (0.05)	4.72/11.10	79.82 (0.07)	8.85/11.32	79.62 (0.09)	9.20/11.17
	$\text{AGAUS}_{\text{aicc}}$	82.96 (0.06)	5.70/11.35	79.46 (0.08)	9.11/11.42	79.08 (0.09)	9.57/11.35
	SRA	79.57 (0.05)	9.98/10.45	76.15 (0.08)	11.65/12.2	73.92 (0.11)	12.87/13.21
	SRA_{aicc}	78.98 (0.07)	10.48/10.54	76.04 (0.08)	11.82/12.14	73.25 (0.12)	13.10/13.65
300	AEST	79.71 (0.04)	10.20/10.08	79.78 (0.04)	10.15/10.07	79.91 (0.05)	10.04/10.03
	AGAUS	85.43 (0.03)	3.77/10.79	80.86 (0.04)	8.45/10.68	80.58 (0.05)	8.87/10.55
	$\text{AGAUS}_{\text{aicc}}$	85.05 (0.03)	4.09/10.85	80.65 (0.04)	8.62/10.71	80.39 (0.05)	9.01/10.59
	SRA	79.86 (0.03)	9.99/10.14	78.97 (0.05)	10.54/10.49	78.55 (0.06)	10.71/10.74
	SRA_{aicc}	79.71 (0.04)	10.11/10.18	78.68 (0.05)	10.75/10.57	78.28 (0.06)	10.90/10.82

2.3.2. Bayesian forecasts

One of the earliest references using Bayesian procedures to forecast in the context of time series models is Litterman (1979), who, in the context of Vector AR (VAR) models, describes the solution to the problem of overparameterization through the addition of instrumental information in the form of Bayesian prios. Later, Monahan (1983) constructs forecast densities that take into account parameter and lag order uncertainties using numerical integration techniques and restricting the analysis to models with no more than two parameters, that is, $p + q \leq 2$. Thompson and Miller (1986) overcome some of the computational difficulties and simulate future paths of time series for ARMA(p, q) models and h-steps-ahead forecasts, simulating from the predictive distribution rather than trying to obtain its analytical form. The Bayesian forecasting procedure of Thompson and Miller (1986) allows to assume other error distributions and they show, explicitly, how to construct forecast densities and intervals for ARMA models; see also Geweke and Whiteman (2006) for the principles of Bayesian forecasting. Later, Chib and Greenberg (1994) and Marriott et al. (1996) propose MCMC samples for ARMA models which enforce stationarity and invertibility, but they rely on Gaussian errors.

The Bayesian procedure of Thompson and Miller (1986) is illustrated by implementing it to construct forecast intervals for the AR(2) model considered previously. When the Bayesian procedure is implemented assuming Gaussian forecast errors, it is denoted as BAYESN while, if the errors are assumed to be Student-ν, it is denoted as BAYEST.[10] When Gaussianity is assumed, it is well known that any diffuse prior for ϕ and σ_ε^2 leads to Normal and Inverse Gamma posterior distributions, respectively. The joint and marginal posterior distributions are obtained using Gibbs sampler. Regarding the Student-ν case, we are not able to identify the posteriors of all parameters and therefore the Metropolis-Hasting algorithm is implemented. Following Sahu et al. (2003), we assume an exponential prior distribution with parameter 0.1 truncated in the region $\nu > 2$ for the degrees of freedom (ν) of the Student-ν.[11] We run 11000 iterations for the MCMC algorithms of BAYESN and BAYEST and save the last 1000 iterations to construct

[10]We did not consider $\chi^2_{(5)}$ errors since as far as we know there is not any proposal in the literature to deal with this distribution in the context of Bayesian forecasting.

[11]Alternatively, as proposed by Jacquier et al. (2004), we also consider a truncated discrete uniform prior distribution for ν, so that $\nu \sim U[3, 40]$. Although using this prior we obtain similar MD distances, the coverage rates of the model with truncated exponential prior are closer to the nominal level. Consequently, the subsequent results are based on the truncated exponential prior.

the forecast densities and intervals. Table 2.4 reports the Monte Carlo averages and standard deviations of the MD distances between the Bayesian and the true forecast densities. These distances should be compared with those of EST densities reported in Table 2.2 as in both cases the lag order and error distribution are assumed to be known. We observe that, when the errors are Normal, the Bayesian distances are slightly larger for h =1. However, when the forecast horizon increase to h =6 and 12, the distances decrease. Note that their standard errors are also smaller. Similar results are obtained when the errors are Student-5. We also compute the Bayesian densities assuming Normality when the errors are truly Student-5 or $\chi^2_{(5)}$. In this case, the distances should be compared with those reported as GAUS in Table 2.2. Regardless of whether the true distribution is Student-5 or $\chi^2_{(5)}$, when the densities are constructed assuming Normality, the averages and standard deviations are almost identical to those obtained by the GAUS procedure when h=1. However, the averages and standard deviations are smaller for h=6 and 12.

Consider now the results for the coverages of the corresponding forecast intervals in Table 2.6. We observe that, if the true error distribution is known, the Bayesian procedure is able to provide coverages closer to the nominal level than those of the asymptotic methods. On the other hand, if we misspecify the error distribution when using the Bayesian procedure, we can have distorted coverages for 80% intervals in the short term, as happens to the GAUS and ABJ intervals. Furthermore, note that the overcoverage can be even larger than those of the GAUS intervals. Finally, when the true errors are $\chi^2_{(5)}$, the Bayesian intervals based on Gaussian errors are asymmetric when h=1.

Table 2.6: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BAYESN	79.70 (0.05)	10.10/10.2	79.10 (0.09)	10.29/10.62	78.35 (0.10)	10.68/10.97
	BAYESL	79.94 (0.06)	9.87/10.18	76.99 (0.09)	11.33/11.68	75.29 (0.11)	12.24/12.47
100	BAYESN	79.86 (0.04)	10.17/9.96	79.68 (0.06)	10.22/10.09	79.31 (0.07)	10.35/10.34
	BAYESL	79.72 (0.05)	10.28/10.01	78.09 (0.07)	11.02/10.89	76.72 (0.08)	11.64/11.64
300	BAYESN	79.81 (0.03)	10.10/10.08	79.98 (0.04)	10.05/9.967	79.88 (0.05)	10.10/10.02
	BAYESL	79.87 (0.03)	10.02/10.11	79.31 (0.04)	10.36/10.32	78.81 (0.05)	10.61/10.57
	Student-5	h=1		h=6		h=12	
50	BAYEST	80.12 (0.06)	9.96/9.91	78.38 (0.09)	10.83/10.79	77.82 (0.11)	11.11/11.07
	BAYESN	82.58 (0.06)	8.79/8.63	79.29 (0.09)	10.41/10.3	78.41 (0.11)	10.83/10.76
	BAYESL	82.46 (0.07)	8.56/8.97	77.36 (0.10)	11.26/11.38	75.59 (0.11)	12.21/12.20
100	BAYEST	80.12 (0.04)	10.03/9.85	79.01 (0.06)	10.55/10.44	78.62 (0.08)	10.72/10.65
	BAYESN	83.17 (0.05)	8.50/8.33	80.43 (0.07)	9.85/9.72	79.64 (0.08)	10.23/10.12
	BAYESL	83.09 (0.05)	8.45/8.46	79.23 (0.07)	10.49/10.28	77.66 (0.09)	11.24/11.10
300	BAYEST	80.14 (0.03)	9.89/9.96	79.87 (0.04)	10.06/10.07	79.59 (0.05)	10.16/10.25
	BAYESN	83.89 (0.03)	8.03/8.08	81.37 (0.04)	9.30/9.33	80.72 (0.05)	9.61/9.66
	BAYESL	83.76 (0.03)	8.10/8.14	80.78 (0.05)	9.64/9.58	79.77 (0.05)	10.13/10.10
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	83.77 (0.07)	5.06/11.17	79.28 (0.10)	9.04/11.68	78.12 (0.11)	9.90/11.97
	BAYESL	83.50 (0.08)	5.57/10.93	76.85 (0.10)	10.61/12.54	74.92 (0.11)	11.83/13.24
100	BAYESN	84.60 (0.05)	4.53/10.87	80.18 (0.07)	8.68/11.13	79.36 (0.08)	9.42/11.22
	BAYESL	83.94 (0.06)	5.32/10.73	78.42 (0.08)	9.91/11.66	77.00 (0.09)	10.87/12.14
300	BAYESN	85.37 (0.03)	3.89/10.74	80.83 (0.04)	8.47/10.7	80.29 (0.05)	9.02/10.70
	BAYESL	84.97 (0.04)	4.34/10.68	80.04 (0.05)	9.11/10.85	79.20 (0.06)	9.71/11.09

The Bayesian procedures described above assume that the error distribution is known. However, some Bayesian approaches are able to incorporate the error distribution uncertainty in their forecasts. They are based on nonparametric Bayesian mixture of models, but their main drawback is that they are intensive computationally and the construction of forecast intervals and densities is not straightforward; see Tang and Ghosal (2007) for applications in the context of autoregressive models.

Finally, some Bayesian procedures are designed to take into account the uncertainty about the lag order of ARMA models. For example, applications of Bayesian model averaging to AR processes are reported by Schervish and Tsay (1988) and Le et al. (1996). Other fixed-dimensional MCMC algorithms are proposed in Barnett et al. (1996, 1997) and Huerta and West (1999). In Barnett et al. (1996) the AR coefficients are reparameterized in terms of the partial correlation coefficients so that the AR model can be treated as a nested model and model-order selection is performed by associating a binary indicator variable with each coefficient, and using these to perform subset selection. Barnett et al. (1997) extend the procedure of Barnett et al. (1996) to ARMA models. On the other hand, Huerta and West (1999) define a prior structure directly on the roots of the AR characteristic polynomial and the model uncertainty is then naturally accounted for by allowing the roots to have zero moduli. Nevertheless, in these procedures, the maximum orders of the models are fixed and estimations are made on the saturated model, which may lead to a parameter space of a very large dimension and, consequently, the estimation becomes difficult. To avoid this problem, some authors propose to apply the Reversible Jump Markov Chain Monte Carlo (RJMCMC) algorithm proposed by Green (1995) which is a generalisation of the Metropolis Hasting algorithm that allows jumps between states of different dimensions. It can jointly estimate the orders p and q and the parameters ϕ, θ and σ_ε^2 of an ARMA model. The lag order uncertainty is accounted for explicitly in terms of the posterior distributions of p and q; see Troughton and Godsill (1998), Vermaak et al. (2004) and Ehlers and Brooks (2008) for applications to AR(p) models. Alternatively, Stephens (2000) proposes a procedure based on the simulation of a continuous time birth and death Markovian process between-model moves. Philippe (2006) adapts such algorithm to ARMA models and denotes it as the birth and death MCMC (BDMCMC) algorithm. Her choice is based on Brooks et al. (2003), whose numerical results favour the BDMCMC algorithm against the RJMCMC in terms of convergence assessment in the particular case of AR models. However, a comparison about forecast performance was not assessed.

A standard way to deal with the lag uncertainty in macroenometrics is to include a long set of lags and specifying that the larger the lag, the more likely, that the coefficient is to be close to zero. For example, one can specify that the jth lag has an independent normal distribution with zero mean and a standard deviation inversely proportional to j. The proportionality constant is a hyperparameter that can be estimated. These ideas lead Doan et al. (1984) to propose the so-called Minnesotan prior; see Karlsson (2013) for a survey. A related idea based on shrinkage has been proposed by Schmidt and Makalic (2013), who adapt the Bayesian LASSO to AR models. Their simulations show that their procedure performs well in terms of forecast errors when compared with a standard autoregresion order selection method and they suggest its extension to ARMA models. Nevertheless, it worth noting that the above Bayesian methods that incorporate the lag-order uncertainty rely on the Gaussian assumption of the errors and they are very intensive computationally.

The procedure of Schmidt and Makalic (2013), called BAYESL, is illustrated with Monte Carlo experiments, using $p_{max} = T/10$; see Tables 2.4 and 2.6 for the implementation of the BAYESL procedure. As well as for the BAYESN and BAYEST procedures, we run 11000 iterations and discard the first 10000. In Table 2.4 we observe that, when the errors are Normal, the distances are reduced with respect to $\text{GAUS}_{\text{aicc}}$ if T=50 and h=6 and 12. However, for T=100 and 300, the distances are larger. Similar results are obtained for the other two distributions considered. Looking at the Monte Carlo results of the interval coverages in Table 2.6 we observe that BAYESL generates coverages close to the nominal level for h=1, but as the forecast horizon increases, BAYESL underestimates the nominal level, regardless of the error distribution. Moreover, since BAYESL assumes Gaussianity, it presents distorted coverages as T increases for h=1 when the errors are non-Gaussian.

Finally, we can conclude that, unlike the asymptotic methods, the Bayesian methods are able to provide accurate forecast densities in moderate sample sizes and mainly in the short term when the true distribution is known. The drawback is that they may demand a large computing effort when the sample size is large. In our study, for example, the BAYEST and BAYESL procedures take approximately 18 and 39 hours, respectively, to compute the MD values and coverage rates of one Monte Carlo simulation of sample size $T = 300$; see Table 2.7 for a detailed time comparison between Bayesian and alternative procedures.

Table 2.7: Simulation time in hours of the most demanding procedures.

Procedure	T=50	T=100	T=300
BAYESN	0.13	0.17	0.41
BAYEST	3.16	6.12	18.59
BAYESL	7.44	13.70	39.35
BOOT	0.6	0.6	1.00
BOOTEX	0.83	0.83	1.23
BOOTNP	0.42	0.83	3.55

2.3.3. Bootstrap forecasts

A simple alternative to construct forecast densities that take into account the parameter, error distribution and lag-order uncertainties is based on bootstrap procedures. They are attractive because they use computationally simple algorithms. The original bootstrap procedure to obtain forecast densities is proposed by Thombs and Schucany (1990) in the context of AR(p) models to incorporate the parameter uncertainty. Extensions of their work include Masarotto (1990), Kabaila (1993), McCullough (1994), Breidt et al. (1995), Grigoletto (1998) and Kim (1999) among others. Pascual et al. (2001, 2004) propose an alternative procedure that does not require bootstrap re-sampling through the backward representation of the process and, consequently, it can be applied to models with moving-average components. The procedure by Pascual et al. (2001, 2004) is implemented by Clements and Taylor (2001) and Kim (2001) who apply the bootstrap-after-bootstrap of Kilian (1998a) in order to take into account the small sample bias of the parameter estimators to construct AR forecasts. Tanizaki et al. (2005) also provide a bootstrap bias-correction of the LS estimator of the parameters of AR(p) models.

In this chapter, we consider the bootstrap procedure proposed by Pascual et al. (2001, 2004) with the analytical parameter bias correction method described in Section 2.2 for an AR(p) model, whose advantage over the bootstrap bias correction of Kilian (1998a) is its computational efficiency; see Kim (2004) for the same bias correction procedure. This procedure is called BOOT. In the literature, we can find other alternatives to the bootstrap parameter bias correction. For instance, Clements and Kim (2007) show that when the process is near unit root or non-stationary, the parameter estimation proposed by Roy and Fuller (2001) performs better and is computationally cheaper. Another alternative is the grid bootstrap method of Gospodinov (2002), but it only applies to AR(1) models; see Kim and Durmaz (2012).

Analysing the Monte Carlo results of Table 2.4, we observe that when there is error distribution uncertainty, BOOT has lower MDs than GAUS and ABJ as T increases. Regarding the coverage rates, reported in Table 2.8, for T=50, the BOOT intervals already have coverages very close to the nominal levels for all forecast horizons, outperforming AEST and the Bayesian procedures that use the correct error distribution and lag-order. Note that the coverages BOOT do not decrease with the forecast horizon. This is a result of the implemented bias correction. The gain of bias correction can be substantial in small samples, when the AR root of the model is close to one and when the forecast horizon is larger; see Kim (2003, 2004).

Finally, the uncertainty associated with the lag order can be incorporated by using the endogenous lag-order bootstrap algorithm of Kilian (1998a), the sieve exogenous order bootstrap of Alonso et al. (2004) or the moving blocks bootstrap of Alonso et al. (2006). Clements and Kim (2007) show that incorporating the lag order selection has marginal small improvements when the true process is highly persistent. They also warn against the use of bootstrap techniques for highly persistent processes with non-Gaussian distributions.

We apply the sieve exogenous order bootstrap of Alonso et al. (2004) with the bias-correction procedure described in Section 2.2, denoted here as BOOTEX. This procedure is easier to implement than that of Alonso et al. (2006) and both procedures provide similar coverage results; see Alonso et al. (2006). Alonso et al. (2004) find in their Monte Carlo study that their proposal outperforms the endogenous lag-order bootstrap and provides consistent forecast intervals for ARMA processes.

Looking at the results reported in Tables 2.4 and 2.8, we observe that the BOOTEX procedure yields MDs and coverages very close to those obtained with BOOT, which assumes the correct lag-order, showing a clear advantage over the asymptotic methods that incorporate only the parameter variability in the forecasts, such as AEST, or also the error distribution in the forecast

Table 2.8: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BOOT	79.08 (0.06)	10.36/10.56	79.64 (0.08)	10.04/10.32	80.69 (0.10)	9.45/9.86
	BOOTEX	78.60 (0.06)	10.68/10.72	80.50 (0.09)	9.67/9.83	80.95 (0.11)	9.39/9.66
	BOOTNP	72.17 (0.17)	13.57/14.26	64.44 (0.15)	17.26/18.30	59.98 (0.15)	19.64/20.38
100	BOOT	79.29 (0.04)	10.37/10.34	80.06 (0.06)	9.97/9.96	80.76 (0.07)	9.62/9.62
	BOOTEX	79.12 (0.05)	10.46/10.42	80.46 (0.06)	9.81/9.72	80.82 (0.08)	9.60/9.58
	BOOTNP	75.77 (0.14)	12.43/11.80	69.50 (0.13)	15.23/15.28	66.63 (0.13)	16.60/16.77
300	BOOT	79.68 (0.03)	10.15/10.17	80.06 (0.04)	10.06/9.877	80.45 (0.05)	9.86/9.69
	BOOTEX	79.66 (0.03)	10.18/10.16	80.12 (0.04)	9.97/9.90	80.50 (0.05)	9.78/9.72
	BOOTNP	79.77 (0.10)	10.04/10.19	74.31 (0.08)	13.02/12.67	72.33 (0.07)	13.97/13.70
	Student-5	h=1		h=6		h=12	
50	BOOT	79.30 (0.06)	10.55/10.15	79.31 (0.09)	10.58/10.11	80.43 (0.11)	9.99/9.57
	BOOTEX	79.18 (0.07)	10.61/10.21	79.75 (0.10)	10.21/10.03	80.34 (0.12)	9.92/9.74
	BOOTNP	71.43 (0.18)	13.40/15.17	63.24 (0.15)	17.60/19.16	59.20 (0.15)	19.84/20.96
100	BOOT	79.62 (0.05)	10.36/10.02	79.66 (0.07)	10.20/10.14	80.24 (0.08)	9.86/9.89
	BOOTEX	79.36 (0.05)	10.47/10.17	79.88 (0.07)	10.16/9.95	80.36 (0.09)	9.92/9.71
	BOOTNP	76.26 (0.16)	12.09/11.65	68.19 (0.13)	16/15.81	65.00 (0.13)	17.36/17.65
300	BOOT	79.90 (0.03)	10.04/10.06	80.00 (0.04)	9.97/10.02	80.40 (0.05)	9.68/9.92
	BOOTEX	79.88 (0.03)	10.03/10.09	80.13 (0.04)	9.84/10.03	80.40 (0.05)	9.718/9.88
	BOOTNP	80.05 (0.12)	9.71/10.24	73.41 (0.09)	13.13/13.46	71.27 (0.09)	14.12/14.61
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	79.38 (0.08)	9.91/10.71	79.28 (0.09)	10.02/10.7	80.11 (0.11)	9.52/10.37
	BOOTEX	79.40 (0.09)	9.62/10.98	80.09 (0.10)	9.37/10.54	80.10 (0.12)	9.37/10.53
	BOOTNP	72.79 (0.17)	12.89/14.32	64.39 (0.14)	17.97/17.64	59.90 (0.14)	20.18/19.92
100	BOOT	79.43 (0.06)	10.21/10.37	79.65 (0.07)	9.86/10.48	80.35 (0.08)	9.49/10.16
	BOOTEX	79.25 (0.07)	10.36/10.39	79.93 (0.07)	9.72/10.35	80.34 (0.09)	9.55/10.11
	BOOTNP	76.43 (0.17)	10.44/13.13	68.80 (0.13)	15.63/15.58	65.53 (0.13)	17.33/17.14
300	BOOT	79.67 (0.04)	10.10/10.22	79.98 (0.04)	9.99/10.03	80.39 (0.05)	9.73/9.88
	BOOTEX	79.55 (0.04)	10.25/10.19	79.97 (0.04)	10.02/10.01	80.27 (0.05)	9.85/9.88
	BOOTNP	81.19 (0.12)	7.57/11.23	74.01 (0.08)	12.40/13.59	72.24 (0.08)	13.19/14.57

intervals as SRA, and over more complex methods that incorporate both parameter and lag-order uncertainties, as the Bayesian procedures. Moreover, it is worth noting that bootstrap procedures usually require less computational effort in comparison with Bayesian procedures. In our study, for example, the bootstrap procedure takes approximately 1 hour for computing the MD values and coverage rates of one Monte Carlo simulation when T=300, whereas the Bayesian procedures, as BAYEST and BAYESL, take more than 12 hours to compute the same measures.

Alternatively, we can use forecasting methods which are not model based, such as the nonparametric bootstrap of Manzan and Zerom (2008). Their method just requires that the time series under analysis follows a Markovian process. Consequently, it encompasses a wide range of relevant structures implied by various commonly used linear and non-linear models. Manzan and Zerom (2008) adapt the local bootstrap approach of Paparoditis and Politis (2001, 2002) to the context of out-of-sample forecast density estimation. For one-step-ahead forecasts, their proposed non-parametric procedure reduces to the well known conditional density estimator; see, for example, De Gooijer and Zerom (2003). The nonparametric bootstrap of Manzan and Zerom (2008) is denoted here as BOOTNP. It uses the nonparametric method of Diks and Manzan (2002) to select p, but it still depends on the choice of p_{max}, which we have considered as $p_{max} = T/10$. The simulation results show that BOOTNP provides the highest distances, and it is able to provide coverages close to the nominal only for T=300 and h=1; see Manzan and Zerom (2008) who also conclude that the performance of BOOTNP is only appropiate if T is large.

Given the good results of BOOTEX in comparison with the asymptotic and Bayesian procedures, we highlight the importance of considering resample methods for taking into account model uncertainty when constructing density and forecast intervals.

2.4. Conclusions

In this chapter, we compare alternative procedures to construct density and interval forecasts in the context of univariate ARMA models. We show that the most important source of uncertainty when constructing density forecasts for small forecast horizons is the error distribution. However, as the forecast horizon increases, the normal approximation of the density is more appropriate. Consequently, the asymptotic correction of the MSFE is not useful. Furthermore, it is only available for relatively simple ARMA models. The SRA procedure to construct asymptotic forecast intervals is sensible for small forecast horizons but does not work for large ones. Moreover, it requires large samples and small nominal coverages. Alternatively, Bayesian procedures are time consuming and computationally complicated when incorporating simultaneously parameter and lag order uncertainties without assuming a particular error distribution. Finally, bootstrap procedures seem to be a feasible alternative if the sample size is large and when the error distribution is unknown.

Chapter 3

A bootstrap approach for generalized autocontour testing

3.1. Introduction

Density forecasting is rapidly becoming a very active and important area of research in the analysis of economic and financial time series. The need to consider the full predictive density has long been recognized in the related literature; see Tay and Wallis (2000) for a survey. There are several reasons for this growing interest in density forecasting. First, complete probability distributions over outcomes provide helpful information for making economic decisions; see Granger and Pesaran (2000a,b). Second, density forecasts provide a characterization of forecast uncertainty which can be useful to central banks; see Britton et al. (1998) for the fan charts of the Bank of England and Alessi et al. (2014) for measures of economic uncertainty during the Global Financial Crisis of the Federal Reserve Bank of New York. Soyer and Hogarth (2012) also propose incorporating measures of uncertainty to avoid the illusion of predictability. Third, in the presence of non-normal forecast errors, even single forecast intervals may not provide an adequate summary of the expected future; see, for example, Lam and Veall (2002). Fourth, density forecasts are also important in the presence of realistic economic loss functions which cannot be reduced to the comparison of Mean Squared Forecast Errors of point forecasts; see Diebold and Mariano (2002) and Patton and Timmermann (2007b). Furthermore, in some applications, often the object of interest is a particular quantile of the forecast distribution as, for example, when forecasting the Value-at-Risk (VaR) of a given stock or portfolio; see Nieto and Ruiz (2016) for a recent survey on VaR forecasting. As a consequence, in an increasing number of empirical applications, forecast densities are obtained for macroeconomic and financial variables; see, for example, Fair (1980), for one of the first applications of computing probability forecasts using a macroeconomic model. Garratt et al. (2003), Giordani and Villani (2010), Jore et al. (2010), Clark (2011), Baumeister and Kilian (2012), Clark and Ravazzolo (2015) and Ravazzolo and Rothman (2016) are some more recent macroeconomic applications. The number of applications in the context of financial variables is very broad covering the construction of densities for both returns and volatilities; see, Andersen et al. (2003), Clements et al. (2008), Corradi et al. (2009), Maheu and McCurdy (2011) and Hallam and Olmo (2013, 2014) just to mention a few applications. Note that, in some of these applications, the forecasting densities are multivariate.

A problem often faced by forecasters is testing the correct specification of a conditional forecast density. Appropriate tests should take into account that the forecast conditional distribution is often unknown and the specification of conditional moments is also unknown and has estimated parameters. Furthermore, a useful test will indicate the source of rejection of a given forecasting model, that is, whether it is rejected because of the specification of the shape of the distribution or because of the specification of the conditional moments.

Many tests available in the literature are based on testing a joint hypothesis of uniformity and independence (i.i.d. U(0,1)) of the probability integral transforms (PITs), which are applicable regardless of the particular user's loss function. Among these tests, the most popular is due to Diebold et al. (1998); see also Berkowitz (2001) and Chen and Fan (2004) for extensions. Intuitively, the i.i.d. assumption of the PITs is related with the correct specification of the conditional moments, while the U(0,1) property characterizes the correct specification of the distribution. The PITs contain rich information on model misspecification which can be revealed by using their histogram and autocorrelogram as suggested by Diebold et al. (1998). However, none of these visual devices take into account the uncertainty associated with parameter estimation.

Furthermore, it is nontrivial to develop a formal test for the joint hypothesis of independence and uniformity of the PITs. The well-known Kolmogorov-Smirnov test, checks uniformity under the independence assumption rather than testing both properties jointly. Consequently, it would easily miss the non-independent alternatives when PITs have a marginal uniform distribution. Moreover, the Kolmogorov-Smirnov test does not take into account the impact of parameter estimation uncertainty on the asymptotic distribution of the statistic. To solve this problem, Bai (2003) proposes a Kolmogorov-Smirnov-type test based on a martingale transformation of the PITs whose asymptotic distribution is free from the impact of parameter estimation. However, the test proposed by Bai (2003) only checks uniformity and, consequently, it has no asymptotic unit power if the transformed PITs are uniform but not independent; see Corradi and Swanson (2006). Alternatively, Hong and Li (2005) propose a nonparametric-kernel-based test with power against violations of both independence and density functional form. Nevertheless, it depends on the choice of a bandwidth and, consequently, it is problematic how to choose it in an empirical context.

Instead of testing for independence and uniformity of PITs, González-Rivera et al. (2011) and González-Rivera and Yoldas (2012) propose autocontour (ACR) tests to evaluate the adequacy of conditional forecast densities. Relying on autocontours allows to obtain a graphical tool that can be very helpful for guiding the modelling. Moreover, it permits to focus on different areas of the conditional density in order to assess those regions of interest. The ACR test, which can be applied to both original series and model residuals, has several advantages: i) it has standard convergence rates and standard limiting distributions that deliver superior power; ii) it is computationally easy to implement as it is based on counting processes; iii) it does not require either a transformation of the original data or an assessment of the Kolmogorov goodness of fit; and iv) it explicitly accounts for parameter uncertainty. Yet, it assumes a parametric time-invariant function of the forecast density and it is complicated to implement to multivariate forecast densities. To overcome these problems, González-Rivera and Sun (2015) propose the generalized autocontour (G-ACR) test, that is based on PITs instead of original observations or residuals. In this way, the G-ACR test inherits the advantages of using PITs and of using autocontours. However it is still based on assuming a particular specification of the conditional density in order to compute the PITs. Therefore, when a given forecasting model is rejected, it is difficult to disentangle whether the rejection can be attributed to the assumed functional form of the error distribution or to the specification of the conditional moments. González-Rivera and Sun (2015) point out that the G-ACR tests are more powerful for detecting departures from the distributional assumption than for detecting misspecified dynamics. Furthermore, there are applications in which the density does not have a known closed-form solution, as for example, multi-step predictive densities in non-linear or non-Gaussian models.

In this chapter, we propose an extension of the G-ACR tests for dynamic specification of a density model (in-sample tests) and for evaluation of forecast densities (out-of-sample tests). Our contribution lies on computing the PITs from a bootstrapped conditional density so that no assumption on the functional form of the forecast error density is needed[1]. The only restrictions required on the error density are those needed to guarantee that the estimator of the parameters of the conditional moments is consistent and asymptotically Normal distributed. The bootstrap procedure allows for the incorporation of parameter uncertainty and can be extended to multivariate systems and multi-step forecasts. We show that the asymptotic distributions of the bootstrapped G-ACR (BG-ACR) tests are well approximated using standard asymptotic distributions. The proposed approach is very easy to implement and particularly useful to evaluate forecast densities when the error distribution is unknown. Furthermore, using graphical devices, the procedure allows the identification of the source of misspecification, namely, whether, it is the error distribution, or linear or non-linear dynamics.

The rest of the chapter is organized as follows. In section 3.2, we briefly describe the G-ACR test. Section 3.3 contains the main contribution of this chapter with the description of the new

[1]Bootstrapping was also proposed by Tsay (1992) for model checking because of its flexibility. The essence of the procedures proposed by Tsay (1992) is to obtain the empirical distribution of a specified functional via parametric bootstrap, which then serves to compare the corresponding functional quantity. The spirit of the procedure proposed in this chapter is very similar. However, differently from Tsay (1992), we do not assume known parameters or a known distributional assumption of the errors.

proposed BG-ACR tests. Their asymptotic properties and finite sample performance are also analyzed when implemented in-sample. Section 3.4 is devoted to analyzing their out-of-sample behavior. An empirical application to illustrate the advantages of the BG-ACR tests, when implemented to test for the adequacy of the Heterogeneous Autoregressive (HAR) model to obtain forecast densities of the VIX volatility index, is carried out in section 3.5. Finally, section 3.6 concludes.

3.2. The G-ACR test

In this section, we briefly describe the G-ACR test proposed by González-Rivera and Sun (2015).

Let $\{y_t\}_{t=1}^T$ denote the random process of interest with conditional density function $f_t(y_t|Y_{t-1})$, where $Y_{t-1} = (y_1, ..., y_{t-1})$ is the information set available up to time t-1. Observe that the random process y_t might enjoy of very general statistical properties, e.g. heterogeneity, dependence, etc. A conditional model is constructed by specifying a conditional mean, conditional variance or other conditional moments of interest, and making distributional assumptions on the functional form of $f_t(y_t|Y_{t-1})$. Based on the conditional model, the researcher might construct a density forecast denoted by $g_t(y_t|Y_{t-1})$ and obtain a sequence of PITs of $\{y_t\}_{t=1}^T$ w.r.t $g_t(y_t|Y_{t-1})$ as follows

$$u_t = \int_{-\infty}^{y_t} g_t(v_t|Y_{t-1})\, dv_t. \tag{3.1}$$

If $g_t(y_t|Y_{t-1})$ coincides with the true conditional density, $f_t(y_t|Y_{t-1})$, then the sequence of PITs, $\{u_t\}_{t=1}^T$, must be i.i.d. $U(0,1)$; see Rosenblatt (1952) and Diebold et al. (1998). Therefore, the null hypothesis $H_0 : g_t(y_t|Y_{t-1}) = f_t(y_t|Y_{t-1})$ is equivalent to the null hypothesis

$$H_0' : \{u_t\}_{t=1}^T \quad \text{is i.i.d.} \quad U(0,1). \tag{3.2}$$

Note that, if the forecast density coincides with the true DGP, then it is preferred by all forecasters regardless of their particular loss function; see Diebold et al. (1998) and Granger and Pesaran (2000a,b). In order to compute the PIT in equation (3.1), one needs to assume a particular distribution function for $g_t(y_t|Y_{t-1})$. Simple tests of independence and uniformity, such as, the Kolmogorov-Smirnov test suffer from the problems described in the introduction. Alternatively, González-Rivera and Sun (2015) propose using autocontours to evaluate the PITs.

Define G-ACR$_{k,\alpha_i}$ as the set of points in the plane (u_t, u_{t-k}) such that the square with $\sqrt{\alpha_i}$-side contains $\alpha_i\%$ of observations, i.e.,

$$\text{G-ACR}_{k,\alpha_i} = \{B(u_t, u_{t-k}) \subset \Re^2 | 0 \leq u_t \leq \sqrt{\alpha_i} \quad \text{and} \quad 0 \leq u_{t-k} \leq \sqrt{\alpha_i}, s.t. : u_t \times u_{t-k} \leq \alpha_i\}. \tag{3.3}$$

Define also the following indicator series I_t^{k,α_i}:

$$I_t^{k,\alpha_i} = \mathbf{1}((u_t, u_{t-k}) \in \text{G-ACR}_{k,\alpha_i}) = \mathbf{1}(0 \leq u_t \leq \sqrt{\alpha_i}, 0 \leq u_{t-k} \leq \sqrt{\alpha_i}). \tag{3.4}$$

If $g_t(y_t|Y_{t-1})$ is a consistent estimator of $f_t(y_t|Y_{t-1})$, then I_t^{k,α_i} is an asymptotically Bernoulli MA process whose order depends on k. The sample proportion of PIT pairs (u_t, u_{t-k}) within the G-ACR$_{k,\alpha_i}$ cube is given by

$$\widehat{\alpha}_i = \frac{\sum\limits_{t=k+1}^{T} I_t^{k,\alpha_i}}{T-k}. \tag{3.5}$$

Consider the statistic t_{k,α_i}, given by

$$t_{k,\alpha_i} = \frac{\sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)}{\sigma_{\alpha_i}}, \tag{3.6}$$

where $\sigma^2_{\alpha_i} = \alpha_i(1-\alpha_i) + 2\alpha_i^{3/2}(1-\alpha_i^{1/2})$. González-Rivera and Sun (2015) show that under the null hypothesis in (3.2) the t_{k,α_i} statistics in (3.6) is asymptotically standard Normal distributed.

The t-statistic in (3.6) is constructed for a single fixed autocontour, α_i, and a single fixed lag, k. However, it can be generalized to a set of lags and a fixed autocontour or to several autocontours with a fixed lag. In the first case, for a fixed autocontour α_i, define $L_{\alpha_i} = (\ell_{1,\alpha_i}, ..., \ell_{K,\alpha_i})'$ which is a $K \times 1$ stacked vector with element $\ell_{k,\alpha_i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$. Under H_0' in (3.2), $L_{\alpha_i}' \Lambda_{\alpha_i}^{-1} L_{\alpha_i}$ is asymptotically χ_K^2 distributed, where a typical element of the asymptotic covariance matrix, Λ_{α_i}, is given by:

$$\lambda_{j,k} = \begin{cases} \alpha_i(1-\alpha_i) + 2\alpha_i^{3/2}(1-\alpha_i^{1/2}), & j = k, \\ 4\alpha_i^{3/2}(1-\alpha_i^{1/2}), & j \neq k. \end{cases}$$

Alternatively, for a fixed lag k, define $C_k = (c_{k,1}, ..., c_{k,C})'$ which is a $C \times 1$ stacked vector with element $c_{k,i} = \sqrt{T-k}(\widehat{\alpha}_i - \alpha_i)$. Once more, under H_0' in (3.2), $C_k' \Omega_k^{-1} C_k$ has asymptotically a χ_C^2 distribution, where a typical element of the asymptotic covariance matrix, Ω_k, is given by:

$$\omega_{i,j} = \begin{cases} \alpha_i(1-\alpha_i) + 2\alpha_i^{3/2}(1-\alpha_i^{1/2}), & i = j, \\ \alpha_i(1-\alpha_j) + 2\alpha_i\alpha_j^{1/2}(1-\alpha_j^{1/2}), & i < j, \\ \alpha_j(1-\alpha_i) + 2\alpha_j\alpha_i^{1/2}(1-\alpha_i^{1/2}), & i > j. \end{cases}$$

If the researcher is interested in partial aspects of the densities, such as, a particular collection of quantiles, it is more informative to examine the L_{α_i} statistic, which incorporates information for all desired k lags. On the other hand, if he is interested in the whole distribution, C_k collects information on all C autocontours desired, given a fixed lag k.

The tests described above are based on a given known predictive density $g_t(y_t|\Omega_{t-1})$. However, in practice, the parameters associated with the moments of this density need to be estimated. González-Rivera and Sun (2015) analyze the effects of parameter estimation on the asymptotic distribution of t_{k,α_i} and, consequently, on L_{α_i} and C_k, and conclude that the corresponding adjustments to the asymptotic variance are model dependent, and consequently, difficult to calculate analytically. So as to overcome this drawback, they propose a fully parametric bootstrap procedure to approximate the asymptotic variance based on obtaining random extractions from the known error predictive density assumed under the null hypothesis.

The G-ACR tests described above can be implemented both in-sample and out-of-sample. González-Rivera and Sun (2015) show that when testing the out-of-sample specification, the importance of parameter uncertainty will depend on both the forecasting scheme and the size of the estimation sample (T) relative to the forecast sample (H). When, implementing the tests to out-of-sample forecast densities, the parameter uncertainty will distort their sizes as long as the proportion of the out-of-sample and in-sample sizes, H and T, respectively, is large. However, under the assumption of $\sqrt{T}$-consistent estimators, if $T \longrightarrow \infty$, $H \longrightarrow \infty$ and $H/T \longrightarrow 0$ as $T \longrightarrow \infty$, parameter uncertainty is asymptotic negligible, and no adjustment is needed for the test.

Finally, note that, if any of the G-ACR tests described above rejects the null hypothesis, there is not any indication about whether the rejection is due to an inadequate assumption about the error distribution or because the dynamics of the model are misspecified. González-Rivera and Sun (2015) point out that the G-ACR tests are more powerful for detecting departures from the distributional assumption than for detecting misspecified dynamics.

3.3. In-sample bootstrap BG-ACR tests

In this section, we propose a modification of the G-ACR test which allows testing for the specification of the conditional moments without making any particular assumption on the conditional distribution. We also justify heuristically the asymptotic distribution of the corresponding statistics and carry out Monte Carlo experiments to establish the finite sample performance of the new proposed tests.

3.3.1. Bootstrap predictive densities

Consider the following parametric model for the series of interest, y_t, $t = 1, ..., T$,

$$y_t = \mu_t + \sigma_t \varepsilon_t, \tag{3.7}$$

where μ_t and σ_t^2 are the conditional mean and variance of y_t, which are specified as parametric functions of Y_{t-1}. Finally, ε_t is a strict white noise process with distribution F_ε, such that $E(\varepsilon_t) = 0$

and $E(\varepsilon_t^2) = 1$. The parameters governing the conditional mean and variance need to be restricted to guarantee stationarity and the conditions required for their estimator to be consistent and asymptotically Normal. Asymptotic Normality of the parameter estimator is a requirement for the bootstrap to be asymptotically valid for the estimation of its sample distribution; see, for example, Hall and Yao (2003). Note that the asymptotic Normality of the estimator usually also depends on the distribution of the errors which should also be accordingly restricted.

A particular specification of (3.7) is following the popular AR(1)-GARCH(1,1) model which will be considered in this chapter to illustrate the proposed tests

$$\begin{aligned} y_t &= \mu + \phi y_{t-1} + a_t, \\ a_t &= \varepsilon_t \sigma_t, \\ \sigma_t^2 &= \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \tag{3.8}$$

where $|\phi| < 1$, $\alpha + \beta < 1$, $\omega > 0$ and $\alpha, \beta \geq 0$. These assumptions are required to guarantee the stationarity of y_t and the positiveness of the conditional variance.

In this chapter, we consider the Gaussian Quasi-Maximum Likelihood (QML) estimator of the parameters of the AR(1)-GARCH(1,1) model in (3.8) obtained by maximizing the Gaussian likelihood. Francq and Zakoian (2004) prove the strong consistency and asymptotic normality of the QML estimator of the ARMA-GARCH model under finite fourth order moment of the observed series.

It is worth noting that the procedure proposed in this chapter to obtain bootstrap in-sample conditional densities and the consequent BG-ACR statistics to evaluate them, can be applied to any other parametric specifications of the conditional mean and variance as far as a consistent and asymptotically Normal estimator of the parameters is available; see, for example, Mika and Saikkonen (2011) who prove the strong consistency and asymptotic normality of the Gaussian QML estimator allowing both the conditional mean and the conditional variance to be nonlinear.

Next, we describe the bootstrap algorithm proposed to obtain in-sample one-step-ahead bootstrap densities of y_t in the context of the AR(1)-GARCH(1,1) model in (3.8). The algorithm is based on the residual bootstrap algorithms of Pascual et al. (2004, 2006) for linear ARMA models and GARCH models, respectively.

In-sample bootstrap algorithm

Step 1 Obtain the residuals

Estimate the parameters of model in (3.8) by a two-step QML estimator: $\widehat{\mu}$, $\widehat{\phi}$, $\widehat{\omega}$, $\widehat{\alpha}$ and $\widehat{\beta}$. Obtain the residuals $\widehat{\varepsilon}_t = \frac{\widehat{a}_t}{\widehat{\sigma}_t}$, $t = 3, ..., T$, where

$$\widehat{a}_t = y_t - \widehat{\mu} - \widehat{\phi} y_{t-1} \tag{3.9}$$

and

$$\widehat{\sigma}_t^2 = \widehat{\omega} + \widehat{\alpha}\widehat{a}_{t-1}^2 + \widehat{\beta}\widehat{\sigma}_{t-1}^2, \tag{3.10}$$

with $\widehat{a}_2 = y_2 - \widehat{\mu} - \widehat{\phi} y_1$ and $\widehat{\sigma}_2^2 = \widehat{\omega}/(1 - \widehat{\alpha} - \widehat{\beta})$. Denote by $F_{\widehat{\varepsilon}}$ the empirical distribution of the centered and scaled residuals.

Step 2 Bootstrap replicates of parameter estimates

For $t = 3, ..., T$, obtain recursively a bootstrap replicate of y_t that mimics the dynamic dependence of the original series as follows

$$\sigma_t^{*2(b)} = \widehat{\omega} + \widehat{\alpha} a_{t-1}^{*2(b)} + \widehat{\beta}\sigma_{t-1}^{*2(b)}, \tag{3.11}$$

$$a_t^{*(b)} = \varepsilon_t^{*(b)} \sigma_t^{*(b)},$$

$$y_t^{*(b)} = \widehat{\mu} + \widehat{\phi} y_{t-1}^{*(b)} + a_t^{*(b)}, \tag{3.12}$$

where $a_2^{*(b)} = \widehat{a}_2$, $\sigma_2^{*2(b)} = \widehat{\sigma}_2^2$, $y_2^{*(b)} = y_2$ and $\varepsilon_t^{*(b)}$ are random extractions with replacement from $F_{\widehat{\varepsilon}}$. Estimate the parameters by QML using $\left\{ y_t^{*(b)} \right\}_{t=3}^{T}$, obtaining $\widehat{\mu}^{*(b)}$, $\widehat{\phi}^{*(b)}$, $\widehat{\omega}^{*(b)}$, $\widehat{\alpha}^{*(b)}$ and $\widehat{\beta}^{*(b)}$.

Step 3 Obtain in-sample bootstrap one-step-ahead predictive densities

For $t = 3, ..., T$, obtain in-sample one-step-ahead estimates of volatilities and observations as follows:

$$\sigma_t^{**2(b)} = \widehat{\omega}^{*(b)} + \widehat{\alpha}^{*(b)}(y_{t-1} - \widehat{\mu}^{*(b)} - \widehat{\phi}^{*(b)}y_{t-2})^2 + \widehat{\beta}^{*(b)}\sigma_{t-1}^{**2(b)}, \tag{3.13}$$

$$y_t^{**(b)} = \widehat{\mu}^{*(b)} + \widehat{\phi}^{*(b)}y_{t-1} + \sigma_t^{**(b)}\varepsilon_t^{*(b)}, \tag{3.14}$$

where $\sigma_2^{**2(b)} = \widehat{\omega}^{*(b)}/(1 - \widehat{\alpha}^{*(b)} - \widehat{\beta}^{*(b)})$ and $\varepsilon_t^{*(b)}$ are random extractions with replacement from $F_{\hat{\varepsilon}}$.

Step 4 Repeat steps 2 and 3 for $b = 1, ..., B^{(1)}$.

Note that in step 2, we obtain replicates of y_t^* which are not conditional on $\{y_1, ..., y_{t-1}\}$. In (3.11), σ_t^{*2} depends on a_{t-1}^{*2} while in (3.12) y_t^* depends on y_{t-1}^*. Therefore, independent replicates of the process are generated to estimate the parameters and to obtain an estimate of their sample distribution. However, in step 3, the bootstrap replicates, σ_t^{**2} and y_t^{**}, in (3.13) and (3.14), are obtained incorporating the parameter uncertainty through the bootstrap estimates of the parameters but always conditional on $\{y_1, ..., y_{t-1}\}$. In this way, at each moment of time, $t = 3, ..., T$, the above algorithm generates $B^{(1)}$ bootstrap replicates of y_t conditional on Y_{t-1}, which incorporate the parameter uncertainty and do not rely on any specific assumption about the distribution of ε_t. In order to decide the number of bootstrap replicates needed to obtain an appropriate estimate of the predictive density, one can implement the procedure proposed by Andrews and Buchinsky (2000). Note that the number of bootstrap replicates could be larger when dealing with non-linear GARCH errors than when the model is linear.

In-sample PITs can be easily computed as follows

$$u_t = \frac{1}{B^{(1)}} \sum_{b=1}^{B^{(1)}} \mathbf{1}(y_t^{**(b)} < y_t). \tag{3.15}$$

The corresponding indicators, I_t^{k,α_i}, and sample proportions, $\widehat{\alpha}_i$, can be computed as in (3.4) and (3.5), respectively. Finally, the t_{k,α_i}, L_{α_i} and C_k statistics can be calculated as explained above. In order to illustrate how the proposed procedure works, we have generated a time series of size T=5000 from the following homoscedastic AR(1) model:

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{3.16}$$

where $\phi = (0.5, 0.95)$ and ε_t is i.i.d. with either N(0,1) or centered and standardized Student-5 and $\chi^2_{(5)}$ distributions. In each case, an AR(1) model is fitted to the artificial series with the parameters estimated by QML. Then, the in-sample PITs are computed both assuming normal errors as in González-Rivera and Sun (2015) and implementing the bootstrap algorithm described above based on $B^{(1)} = 999$ replicates; see Pascual et al. (2004, 2006) for the same number of replicates and Horváth et al. (2004) for $B^{(1)} = 1499$. Figure 3.1 plots the autocontours for α_i=0.2 and 0.8 together with the pairs (u_t, u_{t-1}) for the model AR(1) with ϕ=0.5 and $\varepsilon_t \sim N(0,1)$ (first row); ϕ=0.5 and ε_t ∼Student-5 (second row); ϕ=0.5 and $\varepsilon_t \sim \chi^2_{(5)}$ (third row); and ϕ=0.95 and $\varepsilon_t \sim \chi^2_{(5)}$ (fourth row). First of all, note that when the PITs are computed using the bootstrap densities (first column of Figure 3.1), they are uniformly distributed on the surface regardless of the true error distribution of the underlying DGP. Therefore, they suggest that the fitted AR(1) model is adequate. However, when the PITs are computed as in the G-ACR procedure (second column of Figure 3.1), they are not uniformly distributed unless the errors are Gaussian. In this case, the model is rejected but there is not indication about whether it is rejected because of the specification of the conditional mean or because of the assumed error distribution.

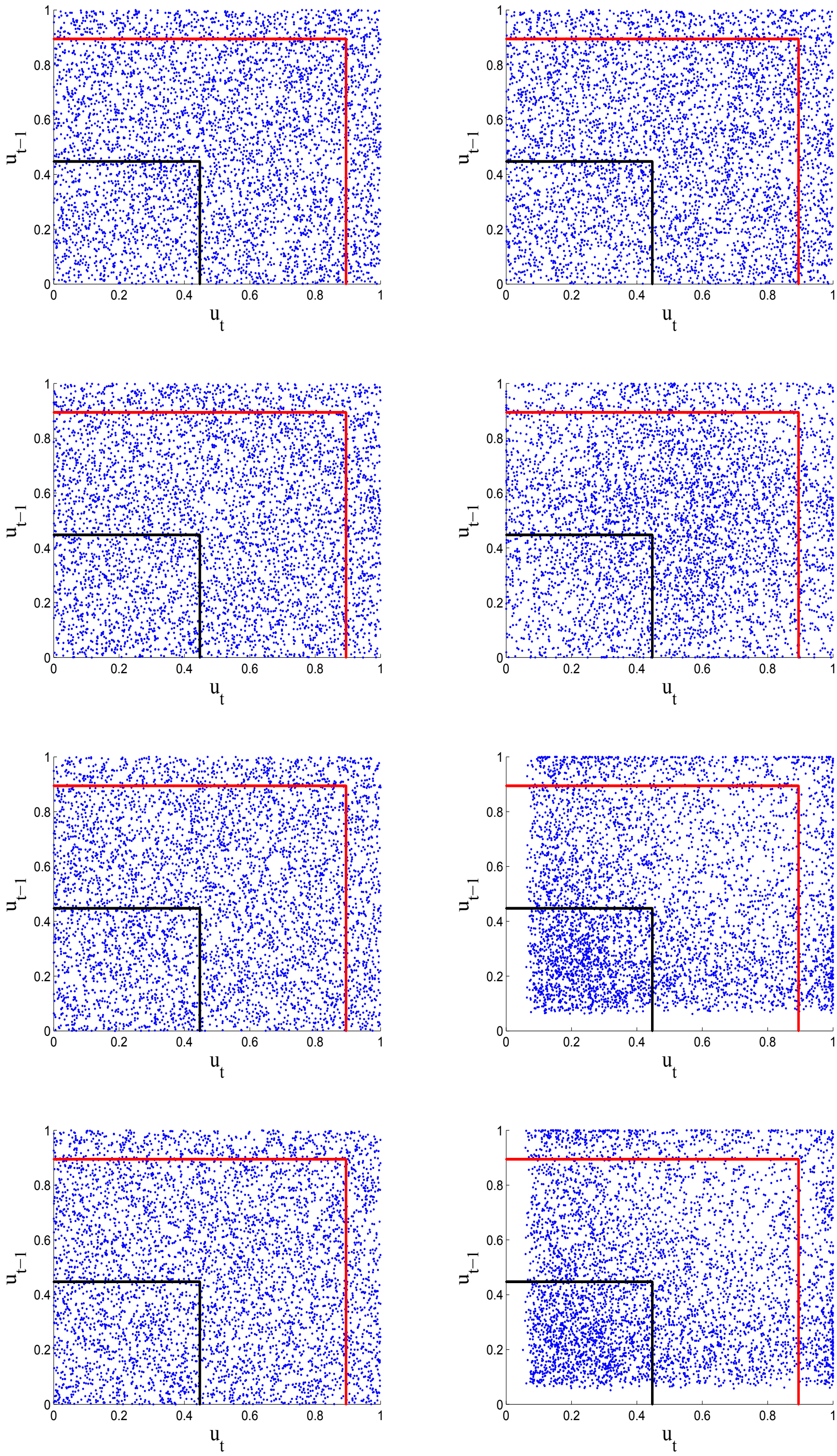


Figure 3.1: Univariate autocontours for the estimated AR(1) model with $T = 5000$. $ACR_{20\%,1}$ corresponds to the black box and the $ACR_{80\%,1}$ to the red box. The DGPs are the AR(1) model with: ϕ=0.5 and $\varepsilon_t \sim N(0,1)$ (first row); ϕ=0.5 and $\varepsilon_t \sim$ Student-5 (second row); ϕ=0.5 and $\varepsilon_t \sim \chi^2_{(5)}$ (third row); and ϕ=0.95 and $\varepsilon_t \sim \chi^2_{(5)}$ (fourth row). The PITs were computed using the bootstrap algorithm with $B^{(1)}$=1000 (first column), or assuming Gaussian errors (second column).

Consider now the following three DGPs:

$$y_t = 0.3y_{t-1} + 0.6y_{t-2} + \varepsilon_t. \tag{3.17}$$

$$y_t = \begin{cases} 0.5y_{t-1} + \varepsilon_t, & \text{for } \ t < T/2. \\ 1 + 0.5y_{t-1} + \varepsilon_t, & \text{for } \ t \geq T/2. \end{cases} \tag{3.18}$$

$$\begin{aligned} y_t &= 0.5y_{t-1} + \varepsilon_t \sigma_t. \\ \sigma_t^2 &= 0.05 + 0.5\varepsilon_{t-1}^2 \sigma_{t-1}^2 + 0.45\sigma_{t-1}^2, \end{aligned} \tag{3.19}$$

with ε_t being an independent white noise with either N(0,1) or centered and standardized Student-5 or $\chi^2_{(5)}$ distributions. As above, an AR(1) model is fitted to each of the simulated series and its parameters estimated by QML. Then the PITs are computed assuming Normal errors and using the bootstrap procedure. Figure 3.2 plots the autocontours for α_i=0.2 and 0.8 together with the pairs (u_t, u_{t-1}) when the DGP is the AR(2) model in (3.17) with $\chi^2_{(5)}$ errors (first row); the AR(1) model with structural break in the mean in (3.18) with $\varepsilon_t \sim \chi^2_{(5)}$ (second row); the GARCH model in (3.19) with Normal errors (third row); and the GARCH model in (3.19) with $\chi^2_{(5)}$ errors (fourth row). We can observe that, when the PITs are based on the bootstrap densities (first column of Figure 3.2), they suggest the source of the misspecification. In the first row, when the AR(1) model is fitted to the AR(2) series, we observe a linear relation between the PITs. In the second row, when the DGP is the AR(1) model with a break in the mean, the PITs do not show any particular linearity or non-linearity but they are concentrated on the top-right corner of the plot. Finally, when the DGP is the AR(1)-GARCH(1,1) model, we observe a non-linear relation between the PITs. Furthermore, in this last case, the autocontour plots are very similar regardless of the error distribution of the DGP. Comparing the bootstrap-based PITs with those obtained using the normal densities (second column of Figure 3.2), the rejection of the fitted models is also clear although there is not an obvious indication for its source.

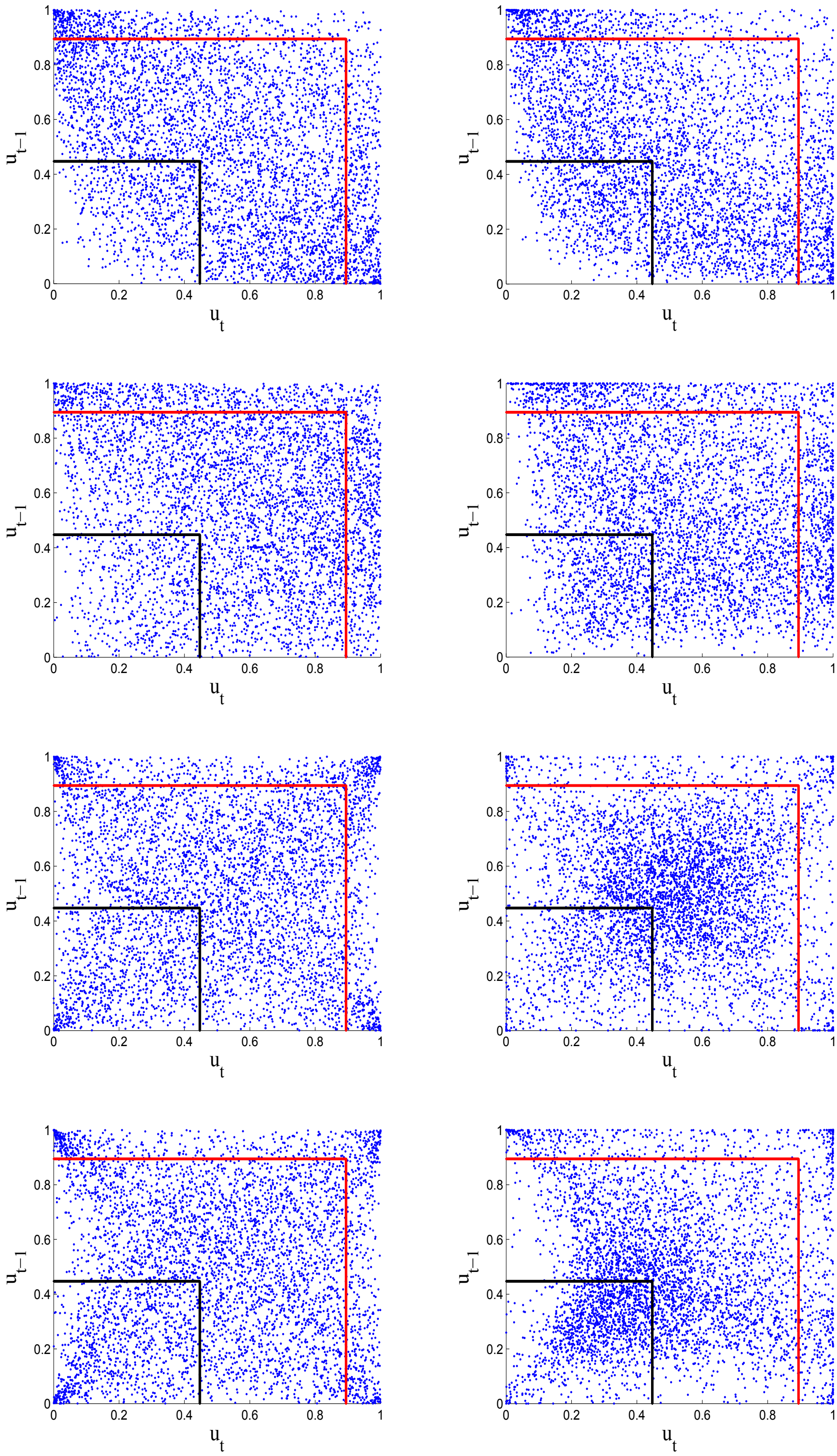


Figure 3.2: Univariate autocontours for estimated AR(1) model with T=5000. $ACR_{20\%,1}$ corresponds to the black box and the $ACR_{80\%,1}$ to the red box. The DGPs are: AR(2) with $\varepsilon_t \sim \chi^2_{(5)}$ (first row); AR(1) model with break in the mean with $\varepsilon_t \sim \chi^2_{(5)}$ (second row); AR(1)-GARCH(1,1) model with $\varepsilon_t \sim$ N(0,1) (third row); and AR(1)-GARCH(1,1) model with $\varepsilon_t \sim \chi^2_{(5)}$ (fourth row). The PITs were computed using the bootstrap algorithm with $B^{(1)}$=1000 (first column), or assuming Gaussian errors (second column).

The asymptotic distributions of the t_{k,α_i}, L_{α_i} and C_k statistics depend on the asymptotic validity of the residual bootstrap algorithm described above. The asymptotic validity of the residual bootstrap procedure when implemented to obtain predictive densities in the context of linear ARMA models, has been established by Pascual et al. (2004). However, as far as we know, there is not a formal proof of the validity of the algorithm to construct predictive densities in the context of nonlinear GARCH models. In order to show that the algorithm is asymptotically valid, one needs first to show that the bootstrap procedure in step 2 generates asymptotically valid estimates of the model parameters. When implemented in GARCH models, Hidalgo and Zaffaroni (2007) show the first order validity of $\widehat{\theta}^* = (\widehat{\omega}^*, \widehat{\alpha}^*, \widehat{\beta}^*)$ for an ARCH(∞) process characterized by a particular decay in the ARCH parameters.[2] If the bootstrap procedure were asymptotically valid for the estimation of the parameters, using the arguments in Pascual et al. (2004) and Reeves (2005), one can establish its validity for the predictive densities and consequently, the distribution of $\widehat{\alpha}_i$ should be as in (3.6) with the asymptotic variance corrected to take into account the parameter uncertainty.[3]

Following the suggestion of González-Rivera and Sun (2015), the variance of $\widehat{\alpha}_i$ is approximated using a bootstrap procedure. $B^{(2)}$ bootstrap replicates, $\{y_t^{*(b)}\}_{t=1}^T$ are generated as in (3.12) and $\widehat{\alpha}_i^{*(b)}$ is obtained using the bootstrap series as if they were the original series. The bootstrap variance of $\widehat{\alpha}_i$ is given by

$$\sigma_{\alpha_i}^{*2} = \frac{1}{B^{(2)} - 1} \sum_{b=1}^{B^{(2)}} \left(\widehat{\alpha}_i^{*(b)} - \frac{1}{B^{(2)}} \sum_{b=1}^{B^{(2)}} \widehat{\alpha}_i^{*(b)} \right)^2, \tag{3.20}$$

[2]Shimizu (2010, 2013, 2014) prove the consistency of the bootstrap QML estimators in the context of an AR(1)-ARCH(1) model. However, the residual bootstrap considered by Shimizu (2010, 2013, 2014) is not exactly the same as that considered in this chapter. All the trajectories share the same estimated conditional mean and variance when generating bootstrap replicates to estimate the parameters. It is important to point out that Corradi and Iglesias (2008) cast some doubts on the asymptotic validity of the residual bootstrap described in step 2. Alternatively, they show that a block bootstrap based on resampling the likelihood as proposed by Gonçalves and White (2004) is asymptotically valid. Therefore, in step 2 of the algorithm described above, one can consider using the block bootstrap instead of the residual bootstrap.

[3]Monte Carlo results on the size distortions of the t-statistic when the asymptotic variance is computed as in (3.6) are available in Table A.1 of Appendix B.

and the corresponding t-statistic is

$$t^*_{\alpha_i} = \frac{(\widehat{\alpha}_i - \alpha_i)}{\sigma^*_{\alpha_i}}, \tag{3.21}$$

which asymptotically has a N(0,1) distribution. In this chapter, results are based on $B^{(2)}$=500 bootstrap replicates to compute $\sigma^*_{\alpha_i}$; see González-Rivera and Sun (2015). Note that the number of replicates needed to estimate standard errors is smaller than that needed to estimate intervals; see Efron (1987).

Obviously, the variances and covariances of the portmanteau statistics can also be computed using the same arguments. In particular, a typical element of the covariance matrix of L_{α_i}, say $\lambda^*_{j,k}$, is obtained as follows:

$$\lambda^*_{j,k} = \begin{cases} \sigma^{2*}_{\alpha_i}, & \text{if } j = k, \\ \frac{1}{B^{(2)}-1} \sum\limits_{b=1}^{B^{(2)}} \left(\widehat{\alpha}^{*(b)}_{j,i} - \frac{1}{B^{(2)}} \sum\limits_{b=1}^{B^{(2)}} \widehat{\alpha}^{*(b)}_{j,i} \right) \left(\widehat{\alpha}^{*(b)}_{k,i} - \frac{1}{B^{(2)}} \sum\limits_{b=1}^{B^{(2)}} \widehat{\alpha}^{*(b)}_{k,i} \right), & \text{if } j \neq k. \end{cases} \tag{3.22}$$

Similarly, a typical element of the covariance matrix of C_k, say $\omega^*_{i,j}$, is obtained as follows:

$$\omega^*_{i,j} = \begin{cases} \sigma^{2*}_{\alpha_i}, & \text{if } i = j, \\ \frac{1}{B^{(2)}-1} \sum\limits_{b=1}^{B^{(2)}} \left(\widehat{\alpha}^{*(b)}_{k,i} - \frac{1}{B^{(2)}} \sum\limits_{b=1}^{B^{(2)}} \widehat{\alpha}^{*(b)}_{k,i} \right) \left(\widehat{\alpha}^{*(b)}_{k,j} - \frac{1}{B^{(2)}} \sum\limits_{b=1}^{B^{(2)}} \widehat{\alpha}^{*(b)}_{k,j} \right). & \text{if } i \neq j, \end{cases} \tag{3.23}$$

3.3.2. Monte Carlo experiments

In this section, we perform Monte Carlo simulations to assess the finite sample properties of the proposed statistics. For the size assessment, the DPG is a linear AR(1). We consider a model far from the non-stationary region and another one near the non-stationary region with different error distributions. For the power assessment, we consider linear and non-linear alternatives. The number of Monte Carlo replicates is $R = 1000$ and the sample size $T = 50, 100, 300, 1000$ and 5000. The number of bootstrap replicates is $B^{(1)} = 1000$, except if $T = 5000$, when we use $B^{(1)} = 2000$. Finally, the number of bootstrap replicates used to compute the variance of $\widehat{\alpha}_i$, L_{α_i} and C_k is $B^{(2)} = 500$.

Studying the size

To investigate the size properties of the tests, we consider as DGP the AR(1) in equation (3.16). For each Monte Carlo replicate, we compute the proportions $\widehat{\alpha}_i$, for $k = 1, ..., 5$, and their bootstrap variances. Then, we compute the Monte Carlo averages and standard deviations of $\widehat{\alpha}_i$, together with the averages of the bootstrap standard deviations and the percentage of rejections of the null hypothesis when the nominal size of the test is 5%. Tables 3.1 and 3.2 report the Monte Carlo results for k=1 when $\phi = 0.5$ and the error is Gaussian and $\phi = 0.95$ and the errors are $\chi^2_{(5)}$, respectively. First of all, we observe that even for the smallest sample size of $T = 50$, the Monte Carlo averages of $\widehat{\alpha}_i$ are rather close to α_i and that the average of the bootstrap standard deviations is a good approximation to the Monte Carlo standard deviation of $\widehat{\alpha}_i$ for moderate sample sizes. However, note that for relatively small sample sizes, the bootstrap standard deviations tend to overestimate the empirical standard deviations of $\widehat{\alpha}_i$, mainly for the largest quantiles. Consequently, the size of the t_{1,α_i} statistic is smaller than the nominal. As the sample size increases, the percentage of rejections gets rather close to the 5% nominal level. The conclusions are quite similar for the close-to-unit-root model with $\chi^2_{(5)}$ errors.

We also analyze the finite sample performance of the two portmanteau tests. Table 3.3 reports the Monte Carlo percentage of rejections of $L^5_{\alpha_i}$ (adding up the information of the first five lags) and of C_1 (adding information of the thirteen quantiles previously considered) for the same two models considered in Tables 3.1 and 3.2. Looking at the results of $L^5_{\alpha_i}$, we observe that, regardless of the DGP considered, the Monte Carlo percentage of rejections is very close to the nominal size with a tendency to overreject for the largest quantiles. On the other hand, the results of C_1 show that it rejects less than the nominal size when the sample size is not large enough. However, if the sample size is large, the empirical size is larger than the nominal.

Table 3.1: Monte Carlo size results for t_{1,α_i}. The DGP is $y_t = 0.5y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$ and the nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.009	0.049	0.102	0.204	0.309	0.405	0.507	0.606	0.706	0.803	0.903	0.950	0.986
	std	(0.014)	(0.033)	(0.045)	(0.066)	(0.082)	(0.088)	(0.089)	(0.087)	(0.080)	(0.068)	(0.051)	(0.037)	(0.024)
	$\bar{\sigma}^*_{\alpha_i}$	0.015	0.034	0.050	0.071	0.085	0.093	0.096	0.094	0.088	0.077	0.060	0.049	0.034
	size	0.058	0.031	0.027	0.026	0.035	0.023	0.027	0.021	0.026	0.019	0.017	0.007	0.024
100														
	$\widehat{\alpha}_i$	0.009	0.050	0.101	0.202	0.304	0.403	0.501	0.600	0.703	0.801	0.901	0.951	0.989
	std	(0.009)	(0.022)	(0.032)	(0.044)	(0.054)	(0.058)	(0.061)	(0.057)	(0.051)	(0.043)	(0.031)	(0.023)	(0.013)
	$\bar{\sigma}^*_{\alpha_i}$	0.010	0.023	0.033	0.048	0.057	0.062	0.064	0.062	0.057	0.049	0.037	0.028	0.018
	size	0.042	0.032	0.041	0.031	0.036	0.028	0.034	0.031	0.023	0.025	0.016	0.016	0.009
300														
	$\widehat{\alpha}_i$	0.009	0.050	0.100	0.201	0.300	0.400	0.500	0.599	0.699	0.799	0.899	0.949	0.988
	std	(0.005)	(0.012)	(0.017)	(0.025)	(0.029)	(0.031)	(0.032)	(0.031)	(0.028)	(0.023)	(0.016)	(0.011)	(0.006)
	$\bar{\sigma}^*_{\alpha_i}$	0.006	0.013	0.018	0.026	0.031	0.033	0.034	0.033	0.030	0.025	0.018	0.013	0.008
	size	0.019	0.036	0.039	0.035	0.033	0.028	0.031	0.030	0.031	0.031	0.014	0.018	0.026
1000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.299	0.399	0.500	0.599	0.699	0.799	0.898	0.949	0.988
	std	(0.003)	(0.007)	(0.010)	(0.014)	(0.017)	(0.017)	(0.018)	(0.017)	(0.015)	(0.012)	(0.008)	(0.006)	(0.003)
	$\bar{\sigma}^*_{\alpha_i}$	0.003	0.007	0.010	0.014	0.016	0.017	0.018	0.017	0.015	0.013	0.009	0.006	0.003
	size	0.050	0.049	0.048	0.046	0.049	0.049	0.044	0.050	0.049	0.041	0.038	0.038	0.065
5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.799	0.900	0.950	0.989
	std	(0.001)	(0.003)	(0.004)	(0.006)	(0.007)	(0.008)	(0.008)	(0.007)	(0.006)	(0.005)	(0.004)	(0.002)	(0.001)
	$\bar{\sigma}^*_{\alpha_i}$	0.001	0.003	0.004	0.006	0.007	0.008	0.008	0.007	0.007	0.005	0.003	0.002	0.001
	size	0.044	0.070	0.052	0.046	0.048	0.054	0.052	0.055	0.039	0.057	0.052	0.044	0.127

Table 3.2: Monte Carlo size results for t_{1,α_i}. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \chi^2_{(5)}$ and the nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.015	0.058	0.109	0.209	0.307	0.407	0.504	0.603	0.705	0.804	0.901	0.950	0.984
	std	(0.022)	(0.048)	(0.067)	(0.089)	(0.098)	(0.102)	(0.097)	(0.094)	(0.081)	(0.064)	(0.043)	(0.034)	(0.023)
	$\bar{\sigma}^*_{\alpha_i}$	0.022	0.049	0.068	0.090	0.100	0.103	0.101	0.095	0.086	0.072	0.054	0.045	0.032
	size	0.061	0.046	0.026	0.022	0.015	0.023	0.016	0.025	0.013	0.012	0.001	0.004	0.009
100														
	$\widehat{\alpha}_i$	0.012	0.055	0.106	0.205	0.305	0.406	0.503	0.602	0.702	0.803	0.900	0.949	0.989
	std	(0.014)	(0.032)	(0.045)	(0.060)	(0.064)	(0.067)	(0.062)	(0.057)	(0.049)	(0.038)	(0.027)	(0.020)	(0.012)
	$\bar{\sigma}^*_{\alpha_i}$	0.015	0.033	0.046	0.060	0.066	0.067	0.065	0.060	0.053	0.043	0.032	0.025	0.018
	size	0.060	0.037	0.029	0.025	0.021	0.025	0.014	0.012	0.014	0.014	0.008	0.005	0.000
300														
	$\widehat{\alpha}_i$	0.011	0.052	0.102	0.202	0.303	0.402	0.502	0.601	0.701	0.800	0.899	0.949	0.988
	std	(0.007)	(0.017)	(0.024)	(0.030)	(0.033)	(0.032)	(0.032)	(0.028)	(0.024)	(0.018)	(0.013)	(0.009)	(0.006)
	$\bar{\sigma}^*_{\alpha_i}$	0.008	0.017	0.024	0.031	0.034	0.034	0.032	0.030	0.026	0.020	0.014	0.011	0.007
	size	0.044	0.036	0.033	0.039	0.034	0.022	0.032	0.024	0.031	0.017	0.026	0.018	0.011
1000														
	$\widehat{\alpha}_i$	0.011	0.051	0.101	0.201	0.301	0.401	0.501	0.600	0.700	0.800	0.899	0.949	0.988
	std	(0.004)	(0.009)	(0.012)	(0.016)	(0.017)	(0.017)	(0.016)	(0.015)	(0.012)	(0.009)	(0.006)	(0.004)	(0.003)
	$\bar{\sigma}^*_{\alpha_i}$	0.004	0.009	0.012	0.016	0.017	0.017	0.016	0.015	0.012	0.010	0.007	0.005	0.003
	size	0.054	0.048	0.046	0.051	0.039	0.040	0.042	0.048	0.045	0.034	0.037	0.043	0.101
5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.101	0.200	0.300	0.400	0.500	0.600	0.700	0.799	0.900	0.950	0.989
	std	(0.002)	(0.004)	(0.005)	(0.007)	(0.008)	(0.007)	(0.007)	(0.006)	(0.005)	(0.004)	(0.002)	(0.002)	(0.001)
	$\bar{\sigma}^*_{\alpha_i}$	0.002	0.004	0.005	0.007	0.007	0.007	0.007	0.006	0.005	0.004	0.002	0.002	0.001
	size	0.049	0.063	0.055	0.046	0.054	0.039	0.049	0.046	0.051	0.044	0.051	0.056	0.162

Table 3.3: Monte Carlo size results for $L^5_{\alpha_i}$ and C_1 statistics. The DGPs are: $y_t = 0.5y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0,1)$ (Panel A) and $y_t = 0.95y_{t-1} + \varepsilon_t, \varepsilon_t \sim \chi^2_{(5)}$ (Panel B). The nominal size is 5%.

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T							**Panel A**							
50	0.078	0.066	0.055	0.052	0.047	0.034	0.049	0.044	0.068	0.089	0.089	0.137	0.076	0.025
100	0.048	0.065	0.056	0.047	0.040	0.042	0.051	0.056	0.055	0.058	0.074	0.108	0.049	0.023
300	0.057	0.048	0.053	0.044	0.041	0.040	0.040	0.051	0.056	0.048	0.067	0.091	0.073	0.029
1000	0.050	0.051	0.047	0.045	0.042	0.042	0.049	0.048	0.051	0.046	0.062	0.061	0.179	0.054
5000	0.062	0.060	0.051	0.042	0.042	0.046	0.061	0.043	0.044	0.054	0.060	0.052	0.101	0.092
							Panel B							
50	0.121	0.091	0.073	0.043	0.036	0.033	0.048	0.059	0.064	0.063	0.081	0.107	0.058	0.023
100	0.098	0.065	0.053	0.045	0.038	0.041	0.054	0.048	0.051	0.038	0.091	0.115	0.027	0.008
300	0.083	0.051	0.057	0.032	0.047	0.045	0.041	0.040	0.036	0.050	0.060	0.095	0.075	0.023
1000	0.070	0.053	0.053	0.058	0.056	0.055	0.047	0.051	0.053	0.051	0.058	0.070	0.193	0.060
5000	0.063	0.051	0.051	0.048	0.043	0.044	0.044	0.034	0.050	0.062	0.051	0.048	0.120	0.089

Studying the power

With the purpose of studying the finite sample power of the tests, we generate replicates by the three models in equations (3.17), (3.18) and (3.19) and fit an AR(1) model. Under the null hypothesis, we test the correct specification of the AR(1) model without drift. The DGP in (3.17) allows investigating the power against departures from the independence hypothesis, while the DGP in (3.18) deals with the power against breaks in the conditional mean. Finally, the DGP in (3.19) permits to analyze power when the second moment is misspecified.

Tables 3.4 to 3.6 report the power results of t_{1,α_i}, for each of the three DGPs. Consider first the results reported in Tables 3.4 when the DGP is the AR(2) model. We observe that t_{1,α_i} has high power for the intermediate autocontours around the 10%-60% levels even when the sample size is moderately small, that is, $T = 100$. As the sample size increases, the power of t_{1,α_i} approaches one. Regarding the portmanteau tests, and in particular $L^5_{\alpha_i}$, we observe that its power is similar to the power of the t_{1,α_i} test, but its rejection rates are higher; see Panel A of Table 3.7. Furthermore, C_1 also shows high power, approaching one, even when $T = 300$. With respect to the DGP in (3.18), when corresponding to a break in the conditional mean, Table 3.5 shows that the power is also higher for the intermediate autocontours when the sample sizes are small. A remark to make is that, in this case, the t_{1,α_i} test is more powerful than the corresponding portmanteau tests of Panel B in Table 3.7. Finally, regarding the DGP in (3.19) when the series are generated by an AR(1)-GARCH(1,1) model, we observe in Tables 3.6 and 3.7 that the power of the t_{1,α_i} and $L^5_{\alpha_i}$ tests is higher in the extreme autocontours in comparison to the power reported by the intermediate autocontours, approaching one for $T = 5000$. The C_1 statistic also provides power close to one, but only in large sample sizes. Overall, the results suggest that larger sample sizes are needed to discriminate between the model under the null hypothesis and the GARCH model in (3.19).

Table 3.4: Monte Carlo power results for t_{1,α_i}. The DGP is the AR(2) model in (3.17) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.000	0.001	0.006	0.035	0.095	0.184	0.294	0.420	0.553	0.693	0.833	0.902	0.953
	std	(0.000)	(0.005)	(0.012)	(0.029)	(0.042)	(0.050)	(0.052)	(0.055)	(0.053)	(0.049)	(0.040)	(0.031)	(0.021)
	$\bar{\sigma}^*_{\alpha_i}$	0.017	0.041	0.060	0.085	0.100	0.109	0.112	0.109	0.100	0.085	0.063	0.049	0.032
	power	0.000	0.000	0.084	0.491	0.588	0.534	0.389	0.227	0.110	0.069	0.050	0.046	0.116
100														
	$\widehat{\alpha}_i$	0.000	0.002	0.008	0.044	0.111	0.205	0.320	0.446	0.577	0.712	0.848	0.916	0.970
	std	(0.001)	(0.004)	(0.010)	(0.022)	(0.030)	(0.035)	(0.038)	(0.037)	(0.035)	(0.032)	(0.025)	(0.020)	(0.014)
	$\bar{\sigma}^*_{\alpha_i}$	0.011	0.027	0.040	0.058	0.068	0.074	0.075	0.073	0.067	0.056	0.041	0.031	0.019
	power	0.000	0.268	0.830	0.947	0.961	0.925	0.831	0.636	0.354	0.195	0.080	0.075	0.060
300														
	$\widehat{\alpha}_i$	0.000	0.002	0.011	0.054	0.128	0.228	0.344	0.469	0.600	0.731	0.861	0.926	0.979
	std	(0.000)	(0.003)	(0.006)	(0.013)	(0.018)	(0.020)	(0.022)	(0.020)	(0.019)	(0.018)	(0.013)	(0.011)	(0.007)
	$\bar{\sigma}^*_{\alpha_i}$	0.006	0.014	0.021	0.029	0.034	0.037	0.038	0.036	0.033	0.027	0.019	0.014	0.008
	power	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.869	0.544	0.360	0.227
1000														
	$\widehat{\alpha}_i$	0.000	0.002	0.012	0.058	0.135	0.237	0.355	0.479	0.609	0.739	0.866	0.930	0.982
	std	(0.000)	(0.002)	(0.004)	(0.008)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.009)	(0.007)	(0.006)	(0.003)
	$\bar{\sigma}^*_{\alpha_i}$	0.003	0.007	0.011	0.015	0.017	0.018	0.018	0.018	0.016	0.013	0.009	0.006	0.003
	power	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.948	0.652
5000														
	$\widehat{\alpha}_i$	0.000	0.002	0.013	0.060	0.138	0.240	0.358	0.484	0.612	0.741	0.868	0.932	0.983
	std	(0.000)	(0.001)	(0.002)	(0.003)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.003)	(0.002)	(0.001)
	$\bar{\sigma}^*_{\alpha_i}$	0.001	0.003	0.005	0.006	0.007	0.008	0.008	0.007	0.007	0.005	0.004	0.002	0.001
	power	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999

Table 3.5: Monte Carlo power results for t_{1,α_i}. The DGP is the AR(1) model with break in the mean in (3.18) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.001	0.014	0.040	0.103	0.176	0.264	0.359	0.462	0.577	0.701	0.830	0.897	0.948
	std	(0.005)	(0.016)	(0.026)	(0.041)	(0.049)	(0.055)	(0.059)	(0.059)	(0.060)	(0.052)	(0.045)	(0.038)	(0.027)
	$\bar{\sigma}^*_{\alpha_i}$	0.016	0.037	0.053	0.076	0.090	0.099	0.102	0.099	0.092	0.079	0.061	0.049	0.033
	power	0.000	0.000	0.041	0.105	0.131	0.144	0.137	0.145	0.142	0.090	0.098	0.080	0.163
100														
	$\widehat{\alpha}_i$	0.002	0.017	0.043	0.109	0.184	0.273	0.372	0.477	0.589	0.712	0.840	0.909	0.967
	std	(0.004)	(0.013)	(0.019)	(0.027)	(0.033)	(0.039)	(0.042)	(0.041)	(0.039)	(0.036)	(0.029)	(0.026)	(0.017)
	$\bar{\sigma}^*_{\alpha_i}$	0.010	0.024	0.035	0.050	0.059	0.064	0.065	0.063	0.058	0.050	0.037	0.029	0.018
	power	0.001	0.107	0.284	0.417	0.488	0.516	0.486	0.460	0.441	0.374	0.258	0.212	0.136
300														
	$\widehat{\alpha}_i$	0.002	0.018	0.046	0.114	0.193	0.283	0.381	0.485	0.599	0.719	0.849	0.917	0.976
	std	(0.003)	(0.007)	(0.011)	(0.016)	(0.020)	(0.022)	(0.024)	(0.025)	(0.024)	(0.021)	(0.018)	(0.014)	(0.008)
	$\bar{\sigma}^*_{\alpha_i}$	0.006	0.013	0.019	0.027	0.032	0.034	0.035	0.033	0.030	0.025	0.018	0.013	0.008
	power	0.000	0.785	0.927	0.985	0.989	0.994	0.995	0.994	0.986	0.969	0.874	0.716	0.421
1000														
	$\widehat{\alpha}_i$	0.002	0.019	0.047	0.116	0.196	0.287	0.385	0.490	0.604	0.724	0.851	0.920	0.979
	std	(0.002)	(0.004)	(0.006)	(0.009)	(0.011)	(0.012)	(0.013)	(0.013)	(0.013)	(0.012)	(0.009)	(0.008)	(0.004)
	$\bar{\sigma}^*_{\alpha_i}$	0.003	0.007	0.010	0.014	0.017	0.018	0.018	0.017	0.015	0.013	0.009	0.006	0.003
	power	0.824	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.911
5000														
	$\widehat{\alpha}_i$	0.003	0.019	0.047	0.116	0.197	0.287	0.387	0.491	0.604	0.724	0.853	0.921	0.980
	std	(0.001)	(0.002)	(0.003)	(0.004)	(0.005)	(0.005)	(0.006)	(0.006)	(0.006)	(0.005)	(0.004)	(0.003)	(0.002)
	$\bar{\sigma}^*_{\alpha_i}$	0.001	0.003	0.004	0.006	0.007	0.008	0.008	0.007	0.007	0.005	0.004	0.002	0.001
	power	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3.6: Monte Carlo power results for t_{1,α_i}. The DGP is the AR(1)-GARCH(1,1) model in (3.19) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.018	0.063	0.111	0.209	0.312	0.418	0.522	0.622	0.719	0.815	0.908	0.953	0.991
	std	(0.019)	(0.039)	(0.056)	(0.078)	(0.089)	(0.096)	(0.093)	(0.086)	(0.077)	(0.061)	(0.041)	(0.031)	(0.019)
	$\bar{\sigma}^*_{\alpha_i}$	0.014	0.035	0.054	0.079	0.094	0.103	0.104	0.100	0.090	0.075	0.057	0.047	0.033
	power	0.204	0.073	0.048	0.034	0.022	0.025	0.019	0.012	0.014	0.010	0.006	0.006	0.009
100														
	$\widehat{\alpha}_i$	0.021	0.066	0.112	0.207	0.308	0.413	0.517	0.620	0.718	0.816	0.909	0.953	0.990
	std	(0.014)	(0.029)	(0.043)	(0.061)	(0.071)	(0.075)	(0.072)	(0.064)	(0.050)	(0.038)	(0.026)	(0.018)	(0.011)
	$\bar{\sigma}^*_{\alpha_i}$	0.010	0.025	0.039	0.058	0.069	0.074	0.074	0.069	0.060	0.047	0.033	0.025	0.018
	power	0.321	0.132	0.067	0.043	0.051	0.035	0.028	0.027	0.017	0.015	0.009	0.004	0.002
300														
	$\widehat{\alpha}_i$	0.023	0.066	0.112	0.206	0.306	0.410	0.516	0.618	0.718	0.816	0.908	0.953	0.989
	std	(0.009)	(0.022)	(0.031)	(0.041)	(0.046)	(0.048)	(0.045)	(0.039)	(0.031)	(0.021)	(0.012)	(0.008)	(0.005)
	$\bar{\sigma}^*_{\alpha_i}$	0.006	0.015	0.024	0.035	0.042	0.045	0.044	0.041	0.034	0.025	0.015	0.011	0.007
	power	0.542	0.232	0.094	0.052	0.045	0.044	0.038	0.043	0.040	0.051	0.031	0.006	0.006
1000														
	$\widehat{\alpha}_i$	0.024	0.066	0.111	0.204	0.303	0.408	0.514	0.616	0.717	0.814	0.908	0.953	0.989
	std	(0.006)	(0.011)	(0.016)	(0.023)	(0.026)	(0.027)	(0.026)	(0.021)	(0.017)	(0.012)	(0.006)	(0.004)	(0.002)
	$\bar{\sigma}^*_{\alpha_i}$	0.003	0.008	0.013	0.020	0.024	0.026	0.025	0.023	0.019	0.013	0.007	0.005	0.003
	power	0.929	0.494	0.152	0.061	0.048	0.051	0.066	0.070	0.105	0.159	0.145	0.070	0.031
5000														
	$\widehat{\alpha}_i$	0.024	0.066	0.110	0.202	0.302	0.406	0.512	0.615	0.716	0.814	0.908	0.954	0.989
	std	(0.003)	(0.007)	(0.010)	(0.014)	(0.016)	(0.017)	(0.016)	(0.015)	(0.012)	(0.007)	(0.003)	(0.002)	(0.001)
	$\bar{\sigma}^*_{\alpha_i}$	0.001	0.004	0.006	0.010	0.011	0.012	0.012	0.011	0.009	0.006	0.003	0.002	0.001
	power	0.999	0.925	0.419	0.119	0.098	0.113	0.197	0.330	0.482	0.692	0.724	0.476	0.110

Table 3.7: Monte Carlo power results for $L^5_{\alpha_i}$ and C_1 statistics. The DGPs are: the AR(2) model in (3.17) (Panel A), the AR(1) model with break in the mean in (3.18) (Panel B) and the AR(1)-GARCH(1,1) in (3.19) (Panel C). The nominal size is 5%.

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T							**Panel A**							
50	0.009	0.057	0.249	0.643	0.774	0.796	0.793	0.720	0.582	0.417	0.281	0.310	0.228	0.058
100	0.027	0.299	0.723	0.939	0.975	0.976	0.968	0.934	0.854	0.678	0.475	0.331	0.179	0.441
300	0.343	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	0.851	0.632	0.317	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.980	0.638	1.000
5000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000
							Panel B							
50	0.000	0.006	0.019	0.063	0.121	0.155	0.205	0.257	0.269	0.280	0.257	0.300	0.240	0.054
100	0.002	0.014	0.047	0.131	0.256	0.292	0.326	0.362	0.379	0.375	0.391	0.404	0.186	0.166
300	0.004	0.339	0.586	0.789	0.869	0.891	0.900	0.891	0.879	0.839	0.726	0.591	0.276	0.855
1000	0.743	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.984	0.676	1.000
5000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
							Panel C							
50	0.177	0.093	0.064	0.054	0.041	0.040	0.045	0.031	0.055	0.085	0.122	0.180	0.052	0.059
100	0.301	0.104	0.076	0.065	0.051	0.050	0.055	0.057	0.056	0.095	0.207	0.279	0.061	0.107
300	0.589	0.175	0.071	0.064	0.061	0.063	0.074	0.084	0.088	0.143	0.282	0.473	0.314	0.381
1000	0.935	0.366	0.144	0.090	0.088	0.091	0.106	0.161	0.238	0.331	0.520	0.653	0.886	0.907
5000	0.999	0.875	0.345	0.166	0.154	0.187	0.332	0.557	0.770	0.890	0.940	0.941	0.972	1.000

3.4. Out-of-sample one-step-ahead bootstrap BG-ACR tests

In this section, we extend the procedures and tests described above to one-step-ahead out-of-sample densities. Note that the in-sample bootstrap algorithm can be also applied to obtain bootstrap replicates of y_{T+1} by implementing equations (3.13) and (3.14) for $T+1$. Then, the corresponding PIT, $u_{T+1|T} = u_{T+1}$ and the indicator $I^{k,\alpha_i}_{T+1|T} = I^{k,\alpha_i}_{T+1}$ can be computed as in (3.15). In order to compute the proportion, it is necessary to obtain H one-step-ahead bootstrap forecast densities. If the parameters are not reestimated each time a new observation is available, then the in-sample algorithm can be implemented as described in section 3 with step 3 modified as follows:

Step 3′. Obtain bootstrap one-step-ahead out-of-sample forecast densities

For $h = 1, ..., H$, obtain out-of-sample one-step-ahead estimates of volatilities and observations as follows:

$$\begin{aligned} \sigma^{**2(b)}_{T+h|T+h-1} &= \widehat{\omega}^{*(b)} + \widehat{\alpha}^{*(b)}(y_{T+h-1} - \widehat{\mu}^{*(b)} - \widehat{\phi}^{*(b)} y_{T+h-2})^2 + \widehat{\beta}^{*(b)} \sigma^{**2(b)}_{T+h-1|T+h-2}, \\ y^{**(b)}_{T+h|T+h-1} &= \widehat{\mu}^{*(b)} + \widehat{\phi}^{*(b)} y_{T+h-1} + \sigma^{**(b)}_{T+h|T+h-1} \varepsilon^{*(b)}_{T+h}, \end{aligned} \tag{3.24}$$

where $\sigma^{**2(b)}_{T+1|T} = \sigma^{**2(b)}_{T+1}$ in (3.13) and $\varepsilon^{*(b)}_{T+1}$ are random extractions with replacement from $F_{\widehat{\varepsilon}}$.

At each moment $T+h$, $h = 1, ..., H$, we compute the out-sample one-step-ahead PITs as follows

$$u_{T+h|T+h-1} = \frac{1}{B^{(1)}} \sum_{b=1}^{B^{(1)}} \mathbf{1}(y^{**(b)}_{T+h|T+h-1} < y_{T+h}).$$

Using $\{u_{T+h|T+h-1}\}_{h=1}^{H}$, we compute the corresponding indicators, I^{k,α_i}_{T+h}, and the proportion

$$\widehat{\alpha}_i = \frac{\sum\limits_{h=k+1}^{H} I^{k,\alpha_i}_{T+h}}{H-k}.$$

Finally, the t-statistic is given by

$$t_{k,\alpha_i} = \frac{\sqrt{H-k}(\widehat{\alpha}_i - \alpha_i)}{\sigma_{\alpha_i}},$$

where $\sigma^2_{\alpha_i}$ is defined as (3.6). Note that $\sigma^2_{\alpha_i}$ can be estimated either as in expression (3.6) or by bootstrapping. As mentioned above, when testing the in-sample specification, ignoring parameter uncertainty may cause severe distortions in the size of the tests. However, when testing the out-of-sample specification, the importance of parameter uncertainty decreases as far $H/T \to 0$ when $T \to \infty$. Therefore, if H is small relative to T, one can compute the variance, $\sigma^2_{\alpha_i}$, by using the asymptotic expression.

As an illustration of the out-of-sample performance of the tests, $R = 1000$ replicates are generated by the AR(1) model in expression (3.16) with $\phi = 0.95$ and $\varepsilon_t \sim N(0,1)$. The model is estimated by OLS using T=50, 100, 300, 1000 and 5000 observations and H=50 and 500 out-of-sample one-step ahead densities and their corresponding PITs are obtained using the bootstrap procedure. The variance of $\widehat{\alpha}_i$ and the covariances in Λ_{α_i} and Ω_k are computed by bootstrapping.[4] Table 3.8 reports the Monte Carlo averages and standard deviations of $\widehat{\alpha}_i$ for k=1, together with the averages of the bootstrap standard deviations and the percentage of rejections of the null hypothesis for different autocontours when the nominal size of the test is fixed at 5%. Table 3.9 reports the size of the corresponding $L^5_{\alpha_i}$ and C_1 test statistics. Table 3.8 shows that the size of the t-test when computed for out-of-sample one-step-ahead densities is close to the nominal as far as T is relatively large and H/T is small. Obviously, increasing H improves the size properties of the test as far as the ratio H/T is still small; see the size results of the L_{α_i} and C_k tests which are reported in Table 3.9 for H=500.

[4]Results based on the asymptotic expression of the variances and covariances are very similar when H=50 and T=1000 (H/T=0.05) or T=5000 (H/T=0.01). When H=500, the results are similar if T=5000 (H/T=0.1). As mentioned above, in these cases, the parameter uncertainty is irrelevant. These results are reported in Tables A.6-A.8 of Appendix B.

Table 3.8: Monte Carlo size results for out-of-sample t_{1,α_i}. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 50$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.015	0.061	0.112	0.213	0.313	0.409	0.508	0.602	0.699	0.794	0.891	0.935	0.968
	std	(0.033)	(0.064)	(0.085)	(0.117)	(0.133)	(0.146)	(0.152)	(0.150)	(0.142)	(0.123)	(0.095)	(0.072)	(0.050)
	$\bar{\sigma}^*_{\alpha_i}$	0.032	0.065	0.088	0.117	0.135	0.144	0.148	0.146	0.138	0.123	0.096	0.076	0.052
	size	0.045	0.058	0.058	0.064	0.071	0.080	0.072	0.084	0.083	0.057	0.053	0.052	0.075
100														
	$\widehat{\alpha}_i$	0.013	0.058	0.109	0.209	0.305	0.404	0.502	0.600	0.697	0.795	0.893	0.940	0.980
	std	(0.021)	(0.048)	(0.069)	(0.093)	(0.112)	(0.121)	(0.124)	(0.121)	(0.115)	(0.104)	(0.077)	(0.059)	(0.033)
	$\bar{\sigma}^*_{\alpha_i}$	0.022	0.050	0.071	0.098	0.114	0.123	0.127	0.126	0.118	0.104	0.079	0.060	0.036
	size	0.049	0.052	0.050	0.050	0.060	0.060	0.061	0.053	0.049	0.048	0.044	0.059	0.070
300														
	$\widehat{\alpha}_i$	0.011	0.054	0.105	0.209	0.310	0.408	0.510	0.608	0.705	0.804	0.899	0.949	0.986
	std	(0.016)	(0.041)	(0.055)	(0.082)	(0.100)	(0.109)	(0.111)	(0.109)	(0.103)	(0.088)	(0.067)	(0.049)	(0.026)
	$\bar{\sigma}^*_{\alpha_i}$	0.017	0.040	0.058	0.081	0.096	0.104	0.108	0.106	0.100	0.088	0.067	0.049	0.026
	size	0.038	0.048	0.034	0.055	0.063	0.058	0.066	0.053	0.050	0.038	0.040	0.050	0.056
1000														
	$\widehat{\alpha}_i$	0.011	0.051	0.103	0.205	0.304	0.404	0.501	0.600	0.700	0.800	0.897	0.949	0.987
	std	(0.016)	(0.037)	(0.055)	(0.075)	(0.090)	(0.097)	(0.100)	(0.100)	(0.094)	(0.083)	(0.063)	(0.046)	(0.023)
	$\bar{\sigma}^*_{\alpha_i}$	0.016	0.037	0.054	0.075	0.088	0.096	0.100	0.099	0.093	0.082	0.062	0.046	0.023
	size	0.070	0.045	0.049	0.045	0.055	0.049	0.051	0.054	0.046	0.053	0.041	0.041	0.049
5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.099	0.202	0.300	0.398	0.502	0.603	0.701	0.800	0.896	0.947	0.987
	std	(0.014)	(0.035)	(0.051)	(0.072)	(0.085)	(0.093)	(0.097)	(0.097)	(0.090)	(0.078)	(0.058)	(0.043)	(0.023)
	$\bar{\sigma}^*_{\alpha_i}$	0.016	0.037	0.052	0.073	0.086	0.094	0.097	0.096	0.091	0.080	0.060	0.044	0.021
	size	0.050	0.043	0.043	0.036	0.048	0.051	0.053	0.057	0.044	0.052	0.025	0.032	0.048

Table 3.9: Monte Carlo size results for out-of-sample $L^5_{\alpha_i}$ and C_1 statistics. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5%, $H = 50$ (Panel A) and $H = 500$ (Panel B).

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T							**Panel A**							
50	0.116	0.113	0.096	0.083	0.070	0.072	0.090	0.091	0.108	0.108	0.121	0.117	0.147	0.086
100	0.100	0.075	0.084	0.050	0.039	0.051	0.062	0.080	0.107	0.105	0.116	0.136	0.094	0.078
300	0.120	0.080	0.068	0.059	0.066	0.073	0.069	0.070	0.077	0.091	0.118	0.139	0.087	0.071
1000	0.124	0.081	0.075	0.072	0.067	0.063	0.079	0.075	0.090	0.103	0.126	0.151	0.074	0.079
5000	0.093	0.077	0.054	0.058	0.053	0.062	0.063	0.062	0.073	0.079	0.116	0.161	0.090	0.064
							Panel B							
50	0.119	0.092	0.088	0.087	0.082	0.087	0.085	0.070	0.076	0.084	0.093	0.107	0.107	0.057
100	0.100	0.076	0.079	0.066	0.078	0.067	0.065	0.060	0.073	0.074	0.102	0.111	0.115	0.047
300	0.094	0.069	0.070	0.057	0.056	0.052	0.066	0.059	0.066	0.059	0.081	0.102	0.197	0.062
1000	0.074	0.063	0.059	0.055	0.060	0.059	0.065	0.056	0.075	0.068	0.081	0.090	0.164	0.065
5000	0.056	0.054	0.049	0.058	0.047	0.058	0.057	0.053	0.053	0.051	0.077	0.113	0.127	0.050

Finally, we study the finite sample power of the out-of-sample tests. With this purpose, R=1000 replicates are generated from the AR(2) model in (3.17) and the AR(1)-GARCH(1,1) model in (3.19). Under the null hypothesis, we consider an AR(1) process without drift. Table 3.10 reports the power of the t_{1,α_i} test when the DGP is the AR(2) model and H=500. In this case, the power increases when the information is accumulated either over several lags or over several quantiles; see the powers reported in Panel A of Table 3.12. The results corresponding to the AR(1)-GARCH(1,1) model are reported in Table 3.11 for H=500. In this case, we can observe that the power of the t_{1,α_i} test is very low except when α_i=0.01. Furthermore, Panel B of Table 3.12 shows that the power does not increase when accumulating information over different autocontours or over different lags. Accumulating information in this way seems to be of no help when dealing with non-linear misspecifications.

Table 3.10: Monte Carlo power results for out-of-sample t_{1,α_i}. The DGP is the AR(2) model in (3.17) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 500$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.000	0.002	0.008	0.038	0.100	0.193	0.305	0.430	0.559	0.692	0.829	0.898	0.948
	std	(0.001)	(0.003)	(0.008)	(0.021)	(0.038)	(0.059)	(0.079)	(0.095)	(0.102)	(0.098)	(0.083)	(0.068)	(0.049)
	$\bar{\sigma}^*_{\alpha_i}$	0.024	0.043	0.056	0.069	0.077	0.083	0.086	0.086	0.084	0.078	0.064	0.051	0.036
	power	0.000	0.030	0.450	0.764	0.809	0.745	0.653	0.526	0.418	0.327	0.234	0.212	0.211
100														
	$\widehat{\alpha}_i$	0.000	0.002	0.010	0.047	0.115	0.211	0.324	0.449	0.579	0.711	0.845	0.914	0.967
	std	(0.000)	(0.003)	(0.007)	(0.019)	(0.032)	(0.047)	(0.059)	(0.069)	(0.076)	(0.073)	(0.060)	(0.047)	(0.030)
	$\bar{\sigma}^*_{\alpha_i}$	0.012	0.026	0.037	0.049	0.056	0.061	0.064	0.064	0.063	0.057	0.045	0.035	0.021
	power	0.000	0.415	0.874	0.956	0.952	0.898	0.795	0.644	0.482	0.353	0.261	0.226	0.196
300														
	$\widehat{\alpha}_i$	0.000	0.002	0.011	0.055	0.129	0.229	0.344	0.469	0.599	0.730	0.859	0.925	0.978
	std	(0.000)	(0.002)	(0.006)	(0.015)	(0.023)	(0.033)	(0.040)	(0.046)	(0.049)	(0.047)	(0.038)	(0.028)	(0.015)
	$\bar{\sigma}^*_{\alpha_i}$	0.007	0.016	0.023	0.032	0.038	0.042	0.044	0.044	0.043	0.038	0.029	0.022	0.012
	power	0.000	0.999	1.000	1.000	1.000	0.991	0.951	0.832	0.665	0.444	0.310	0.244	0.194
1000														
	$\widehat{\alpha}_i$	0.000	0.002	0.012	0.058	0.134	0.237	0.355	0.482	0.610	0.739	0.866	0.930	0.982
	std	(0.000)	(0.002)	(0.006)	(0.012)	(0.019)	(0.026)	(0.033)	(0.038)	(0.039)	(0.036)	(0.029)	(0.022)	(0.011)
	$\bar{\sigma}^*_{\alpha_i}$	0.005	0.013	0.018	0.026	0.031	0.034	0.035	0.035	0.033	0.029	0.023	0.017	0.008
	power	0.149	1.000	1.000	1.000	1.000	1.000	0.985	0.898	0.736	0.521	0.349	0.258	0.205
5000														
	$\widehat{\alpha}_i$	0.000	0.002	0.012	0.059	0.139	0.239	0.358	0.484	0.614	0.741	0.869	0.933	0.985
	std	(0.000)	(0.002)	(0.005)	(0.012)	(0.018)	(0.023)	(0.028)	(0.033)	(0.033)	(0.030)	(0.025)	(0.018)	(0.009)
	$\bar{\sigma}^*_{\alpha_i}$	0.005	0.012	0.017	0.023	0.028	0.030	0.031	0.031	0.029	0.026	0.020	0.014	0.007
	power	0.813	1.000	1.000	1.000	1.000	1.000	1.000	0.961	0.802	0.608	0.385	0.246	0.187

Table 3.11: Monte Carlo power results for out-of-sample t_{1,α_i}. The DGP is the AR(1)-GARCH(1,1) model in (3.19) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 500$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
50														
	$\widehat{\alpha}_i$	0.030	0.070	0.115	0.213	0.323	0.432	0.537	0.630	0.718	0.800	0.875	0.912	0.943
	std	(0.022)	(0.036)	(0.047)	(0.063)	(0.082)	(0.098)	(0.109)	(0.114)	(0.110)	(0.100)	(0.083)	(0.069)	(0.054)
	$\bar{\sigma}^*_{\alpha_i}$	0.011	0.027	0.038	0.053	0.062	0.068	0.070	0.071	0.069	0.064	0.053	0.044	0.033
	power	0.388	0.183	0.119	0.090	0.118	0.148	0.198	0.219	0.243	0.248	0.193	0.228	0.289
100														
	$\widehat{\alpha}_i$	0.028	0.069	0.114	0.210	0.314	0.421	0.525	0.625	0.719	0.808	0.889	0.929	0.963
	std	(0.019)	(0.030)	(0.039)	(0.053)	(0.064)	(0.079)	(0.087)	(0.091)	(0.090)	(0.084)	(0.068)	(0.054)	(0.037)
	$\bar{\sigma}^*_{\alpha_i}$	0.009	0.021	0.032	0.045	0.052	0.057	0.058	0.057	0.054	0.049	0.040	0.031	0.020
	power	0.464	0.228	0.128	0.059	0.078	0.143	0.180	0.213	0.246	0.280	0.263	0.209	0.290
300														
	$\widehat{\alpha}_i$	0.026	0.067	0.113	0.207	0.308	0.413	0.519	0.619	0.718	0.811	0.899	0.943	0.977
	std	(0.015)	(0.025)	(0.032)	(0.040)	(0.048)	(0.058)	(0.066)	(0.069)	(0.069)	(0.065)	(0.052)	(0.040)	(0.024)
	$\bar{\sigma}^*_{\alpha_i}$	0.006	0.016	0.024	0.034	0.040	0.044	0.045	0.043	0.040	0.035	0.027	0.021	0.011
	power	0.551	0.281	0.135	0.066	0.065	0.120	0.169	0.214	0.265	0.305	0.314	0.300	0.271
1000														
	$\widehat{\alpha}_i$	0.025	0.067	0.112	0.205	0.305	0.410	0.515	0.617	0.716	0.813	0.905	0.950	0.985
	std	(0.012)	(0.019)	(0.024)	(0.030)	(0.035)	(0.041)	(0.046)	(0.049)	(0.051)	(0.049)	(0.040)	(0.031)	(0.017)
	$\bar{\sigma}^*_{\alpha_i}$	0.005	0.013	0.019	0.028	0.033	0.035	0.036	0.035	0.033	0.029	0.022	0.016	0.008
	power	0.587	0.317	0.151	0.059	0.073	0.097	0.127	0.198	0.238	0.286	0.298	0.304	0.206
5000														
	$\widehat{\alpha}_i$	0.024	0.066	0.110	0.202	0.303	0.408	0.513	0.617	0.716	0.814	0.908	0.954	0.989
	std	(0.012)	(0.018)	(0.022)	(0.026)	(0.031)	(0.036)	(0.041)	(0.045)	(0.047)	(0.045)	(0.036)	(0.027)	(0.013)
	$\bar{\sigma}^*_{\alpha_i}$	0.005	0.012	0.017	0.024	0.029	0.031	0.032	0.031	0.030	0.026	0.020	0.014	0.007
	power	0.632	0.330	0.148	0.065	0.066	0.087	0.133	0.189	0.244	0.302	0.320	0.316	0.124

Table 3.12: Monte Carlo size results for out-of-sample $L^5_{\alpha_i}$ and C_1 statistics. The DGPs are: the AR(2) model in (3.17) with $\varepsilon_t \sim N(0,1)$ and $H = 500$ (Panel A); and the AR(1)-GARCH(1,1) model with $\varepsilon_t \sim N(0,1)$ and $H = 500$ (Panel B). The nominal size is 5%.

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
							Panel A							
50	0.288	0.773	0.978	1.000	1.000	1.000	1.000	1.000	0.987	0.919	0.713	0.521	0.348	0.596
100	0.418	0.928	0.999	1.000	1.000	1.000	1.000	1.000	0.996	0.964	0.751	0.526	0.297	0.728
300	0.565	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.980	0.827	0.573	0.415	0.989
1000	0.691	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.986	0.832	0.602	0.385	1.000
5000	0.721	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.987	0.845	0.624	0.327	1.000
							Panel B							
50	0.371	0.150	0.111	0.096	0.122	0.128	0.157	0.189	0.223	0.259	0.377	0.448	0.528	0.385
100	0.445	0.185	0.122	0.087	0.092	0.118	0.153	0.186	0.242	0.297	0.378	0.500	0.558	0.495
300	0.525	0.242	0.110	0.088	0.095	0.108	0.132	0.164	0.252	0.341	0.441	0.491	0.599	0.569
1000	0.569	0.238	0.126	0.077	0.071	0.078	0.097	0.170	0.239	0.360	0.479	0.500	0.503	0.573
5000	0.602	0.277	0.115	0.064	0.071	0.079	0.113	0.161	0.231	0.322	0.484	0.551	0.412	0.600

3.5. Empirical application: modelling VIX

There is an increasing interest in modeling and forecasting the daily forward-looking market volatility index (VIX) from the Chicago Board Options Exchange (CBOE); see, for example, Whaley (2000, 2009), Engle and Gallo (2006), Fernandes et al. (2014), Diebold and Yilmaz (2015a) and Hassler et al. (2016). The VIX was originally computed as the weighted average of the implied volatilities from eight at-the-money call and put options of the S&P100 index with an average time to maturity of 30 days. In 2003, the VIX was entirely revised by changing the reference index to the S&P500 index, taking into account a wide range of strike prices for the same time to maturity and freeing its calculation from any specific option pricing model; see Whaley (2009) for a history of the VIX and Fernandes et al. (2014) for a detailed description of the VIX calculation. The VIX is important since it is a barometer of the overall market sentiment; see Whaley (2000, 2009) and Diebold and Yilmaz (2015b) who define it as a fear index. Furthermore, it reflects both the stock market uncertainty and the expected premium from selling stock market variance in a swap contract. Finally, there is an active market on VIX derivatives. The number of VIX futures contracts traded increased dramatically from about 1 million in 2007 to about 24 million in 2012 with the largest growth occurring after 2009, likely caused by the recent financial crisis; see, for example, Park (2016) and Song and Xiu (2016) for recent references on pricing VIX derivatives. The recent development of volatility-based derivative products generates an interest on predictive densities of volatility; see, for example, Corradi et al. (2009) who propose a feasible model free estimator of the conditional predictive density of integrated volatility based on subsampling. In the context of VIX, Konstantinidi and Skiadopoulos (2011) implement the bootstrap procedure of Pascual et al. (2004) to obtain forecast intervals for the VIX that are then used in a trading strategy. Konstantinidi et al. (2008) and Psaradellis and Sermpinis (2016) also compare several specifications of the VIX for trading purposes. It is commonly accepted that the VIX display long-memory; see, for example, Bandi and Perron (2006), Konstantinidi et al. (2008), Shimotsu (2012), Fernandes et al. (2014) and Hassler et al. (2016). Consequently, several authors propose variants of the simple and easy-to-estimate approximate long-memory HAR model of Corsi (2009) to represent and predict the VIX; see Fernandes et al. (2014), Caporin et al. (2016) and Psaradellis and Sermpinis (2016). The HAR model is given by

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_5 \bar{y}_{t-1:5} + \phi_{10} \bar{y}_{t-1:10} + \phi_{22} \bar{y}_{t-1:22} + \phi_{66} \bar{y}_{t-1:66} + \varepsilon_t, \tag{3.25}$$

where $\bar{y}_{t:i} = i^{-1} \sum_{j=0}^{i-1} y_{t-j}$ and ε_t is an independent white noise sequence.

In this section, we analyze the series of daily log-VIX index observed from January 2, 1990 to January 15, 2003 with a total of 5807 observations; see Fernandes et al. (2014) for an empirical analysis of the same series. Descriptive statistics of the full sample are reported in Table 3.13. We can observe that the skewness and kurtosis are not significantly different from the assumed values under Normality when using the correction proposed by Premaratne and Bera (2016). However, the Jarque-Bera test clearly rejects the Normality of log-VIX. Panel (a) of Figure 3.3 plots the time series of the log-VIX. With respect to the temporal dependence, panels (b) and (c) of Figure 3.3 plot the correlograms of log-VIX and its squares, respectively. The comparison of the correlations of log-VIX and (log-VIX)^2 suggests the presence of conditional heterocedasticity given that the correlations of squares are larger than those of the levels; see the values of the sample autocorrelations reported in Table 3.13. Fernandes et al. (2014) show that the null hypothesis of a unit-root is clearly rejected. Yet, they find strong evidence of long-memory. [5] The pure HAR model in equation (3.25) is fitted to the full sample with the estimated parameters reported in Table 3.14. Several residual diagnostics are reported in Table 3.13. We can observe that the distribution of the residuals is clearly non-Normal. Furthermore, the presence of conditional heterocedasticity is also more evident than when looking at the original log-VIX series. In-sample bootstrap conditional densities are computed as described in Section 3. Figure 3.4 plots kernel estimates of the bootstrap densities at different moments of the sample period. We can observe that not only the location of these densities changes over time. Although,

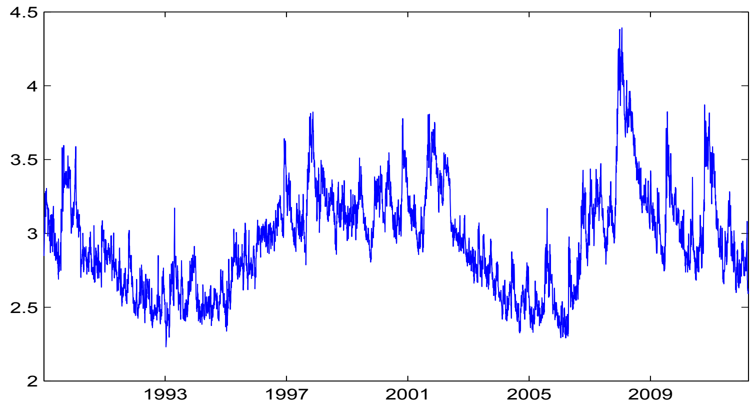
[5]Note that the unit-root tests carried out by Fernandes et al. (2014) do not take into account the presence of conditional heteroscedasticity. These tests should be modified as proposed by Choi (2015).

in general, there is a right skewness of the distribution, this skewness is more pronounced in some particular moments. Furthermore, we can also observe changes in the variance of the log-VIX. After computing the PITs, they are plotted in Figure 3.5, where we can observe that the PITs are not uniformly distributed. There is a concentration of PITs in the left and right top corners, suggesting that conditional heteroscedasticity has not been modeled when computing the conditional densities for log-VIX. For comparison purposes, we also plot in Figure 3.5, panel (b), the PITs computed by the procedure of González-Rivera and Sun (2015), where the errors are assumed to be Gaussian. We observe a concentration of points in the middle, suggesting that the HAR model is misspecified if Gaussian errors are assumed. However, as mentioned above, there is not indication of the source of misspecification.

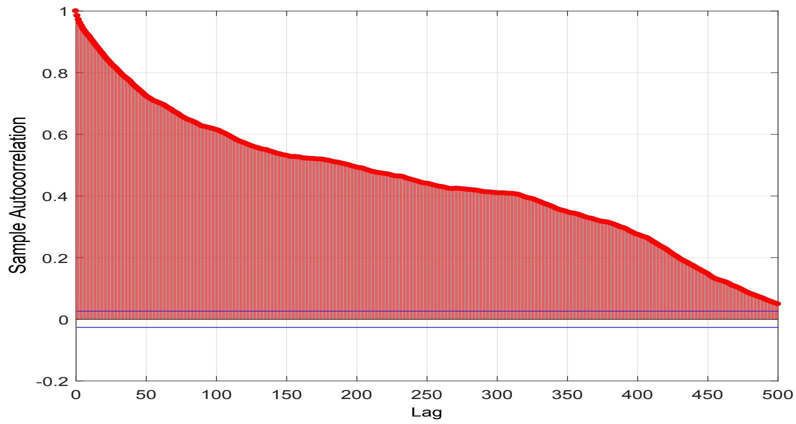
Table 3.13: Descriptive statistics

The first column corresponds to the summary statistics of the log-VIX series and the second and third to the residuals obtained with the HAR and HAR-GARCH models, respectively. ρ_k corresponds to the estimated sample autocorrelations of the log-VIX and ρ_k^2 to the sample correlations of its squares. k corresponds to the lag of the sample correlations. p-values are in parenthesis.

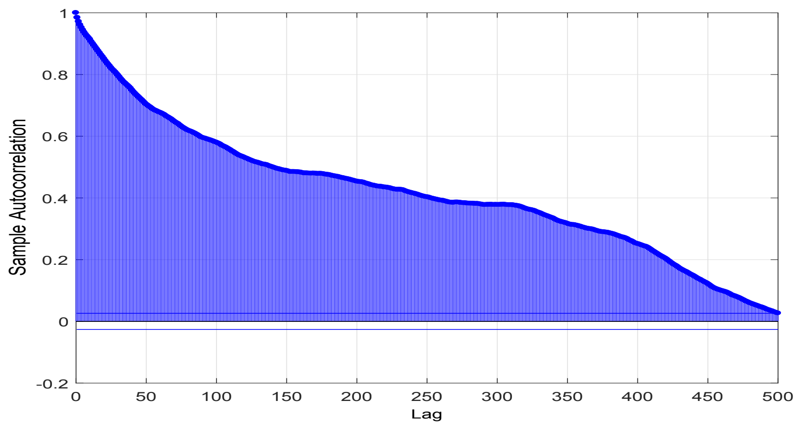
Sample statistics	Full sample	HAR model residuals	HAR-GARCH model residuals
Mean	2.953	0.000	0.001
Standard deviation	0.348	0.060	0.060
Skewness	0.539	0.915	0.946
	(0.000)	(0.000)	(0.000)
Kurtosis	3.288	7.481	7.523
	(0.000)	(0.000)	(0.000)
Jarque-Bera	300.683	5605.800	5749.000
	(0.000)	(0.001)	(0.001)
ρ_1	0.985	0.001	0.007
	(0.000)	(0.940)	(0.584)
ρ_{10}	0.916	0.052	0.044
	(0.000)	(0.016)	(0.057)
ρ_{100}	0.616	-0.002	0.014
	(0.000)	(0.000)	(0.000)
ρ_1^2	0.984	0.122	-0.003
	(0.000)	(0.000)	(0.830)
ρ_{10}^2	0.914	0.129	0.025
	(0.000)	(0.000)	(0.285)
ρ_{100}^2	0.582	-0.011	0.011
	(0.000)	(0.000)	(0.864)



(a)



(b)



(c)

Figure 3.3: Daily log-VIX is plotted in (a). In (b) and (c) are plotted the sample autocorrelations of the levels and squares of the log-VIX, respectively.

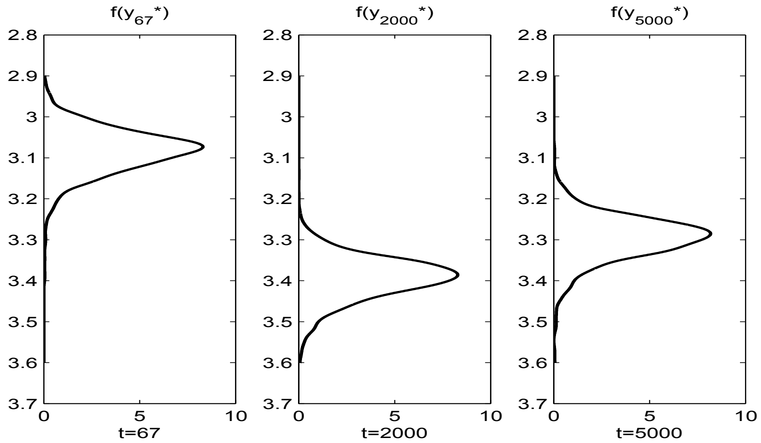


Figure 3.4: In-sample bootstrap one-step-ahead densities.

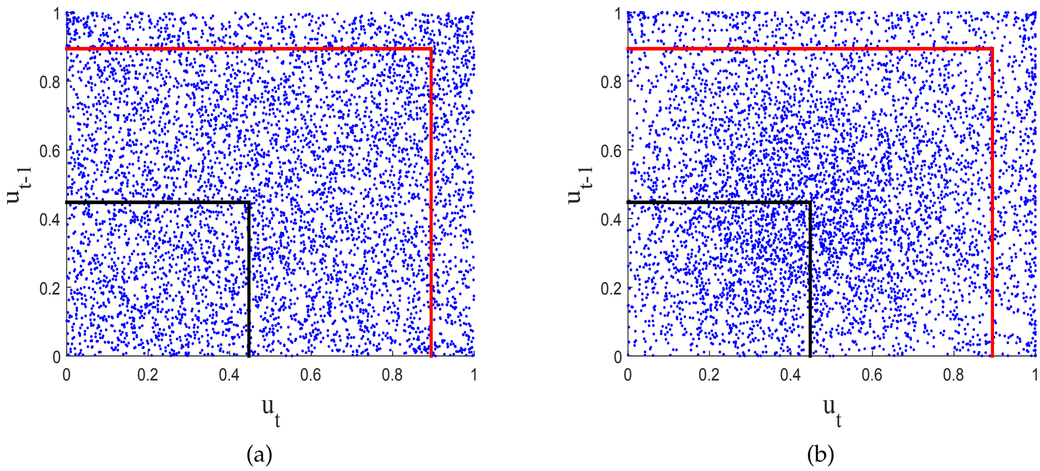


Figure 3.5: Univariate autocontours for the HAR model. In (a) the PITS are obtained with the bootstrap procedure described in Section 3.3 and in (b) they are obtained by the procedure of González-Rivera and Sun (2015) assuming Gaussian errors. $ACR_{20\%,1}$ corresponds to the black box and $ACR_{80\%,1}$ to the red box.

Table 3.14: Estimation results for the log-VIX index. t-statistics in parenthesis.

	HAR	HAR-GARCH
	Mean equation	
	Estimate	Estimate
$\widehat{\phi}_0$	0.024	0.029
	(3.325)	(4.369)
$\widehat{\phi}_1$	0.873	0.883
	(64.422)	(65.136)
$\widehat{\phi}_2$	-0.002	-0.009
	(-0.058)	(-0.344)
$\widehat{\phi}_3$	0.133	0.108
	(4.510)	(3.605)
$\widehat{\phi}_4$	-0.030	-0.013
	(-1.556)	(-0.710)
$\widehat{\phi}_5$	0.016	0.022
	(2.070)	(2.799)
	Variance equation	
		Estimate
$\widehat{\omega}$		2.784e-05
		(11.057)
$\widehat{\alpha}$		0.088
		(14.384)
$\widehat{\beta}$		0.834
		(71.694)

In order to test the null hypothesis about the adequacy of the HAR model to represent the conditional densities of the log-VIX, Panel A of Table 3.15 reports sample proportions, $\widehat{\alpha}_i$, and the in-sample tests t_{1,α_i}, $L^5_{\alpha_i}$ and C^{13}_1. We observe that most of the autocontours are rejected by the t_{1,α_i} and $L^5_{\alpha_i}$ test statistics. The C^{13}_1 test, which is computed adding information of all autocontours, rejects H_0 at 1% of significance. Therefore, as suggested in Figure 3.5, the basic HAR model is not appropriate to model the conditional densities of the daily log-VIX. Panel B of Table 3.15 also reports the corresponding tests obtained by the procedure of González-Rivera and Sun (2015), assuming Gaussian errors. We can see that the null is rejected by almost all the autocontours, with the statistics being much larger than when they are computed using the BG-ACR tests.

Table 3.15: Testing the models in-sample

The sample period runs from January 2, 1990 to January 15, 2003, including altogether 5807 observations. Panels A and C shows the tests obtained by the bootstrap procedure described in Section 3.3 and panels B and D presents the tests obtained by the procedure of González-Rivera and Sun (2015), where the errors are assumed to be Gaussian and the variance of the tests is computed by bootstrap. *, **, *** indicate that H_0 is rejected at 10%, 5% and 1% levels of significance, respectively.

	HAR model												
α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
	Panel A												
$\widehat{\alpha}_i$	0.009	0.052	0.105	0.203	0.309	0.412	0.512	0.610	0.705	0.803	0.899	0.950	0.989
$\lvert t_{1,\alpha_i}\rvert$	$\lvert 0.51\rvert$	$\lvert 1.16\rvert$	$\lvert 1.71\rvert$*	$\lvert 0.96\rvert$	$\lvert 2.79\rvert$***	$\lvert 3.71\rvert$***	$\lvert 4.08\rvert$***	$\lvert 3.86\rvert$***	$\lvert 2.31\rvert$**	$\lvert 1.42\rvert$	$\lvert 0.61\rvert$	$\lvert 0.15\rvert$	$\lvert 0.97\rvert$
$L^5_{\alpha_i}$	5.85	3.03	6.24	5.27	11.67**	18.51***	21.978***	16.84***	9.92*	4.83	11.34**	12.66**	26.26***
C^{13}_1	31.817***												
	Panel B												
$\widehat{\alpha}_i$	0.005	0.037	0.093	0.228	0.367	0.489	0.596	0.684	0.764	0.837	0.895	0.927	0.969
$\lvert t_{1,\alpha_i}\rvert$	$\lvert 3.93\rvert$***	$\lvert 5.02\rvert$***	$\lvert 2.08\rvert$**	$\lvert 6.30\rvert$***	$\lvert 12.69\rvert$***	$\lvert 16.98\rvert$***	$\lvert 17.85\rvert$***	$\lvert 16.02\rvert$***	$\lvert 13.51\rvert$***	$\lvert 7.87\rvert$***	$\lvert 1.21\rvert$	$\lvert 7.56\rvert$***	$\lvert 12.54\rvert$
$L^5_{\alpha_i}$	30.30***	73.09***	23.12***	51.01***	193.75***	321.82***	343.19***	284.29***	201.29***	71.28***	7.22	66.05***	198.36***
C^{13}_1	881.66***												
	HAR-GARCH model												
	Panel C												
$\widehat{\alpha}_i$	0.008	0.049	0.102	0.205	0.311	0.408	0.509	0.611	0.703	0.800	0.899	0.950	0.990
$\lvert t_{1,\alpha_i}\rvert$	$\lvert 1.85\rvert$*	$\lvert 0.23\rvert$	$\lvert 0.74\rvert$	$\lvert 1.45\rvert$	$\lvert 2.95\rvert$**	$\lvert 2.29\rvert$**	$\lvert 2.58\rvert$**	$\lvert 3.45\rvert$**	$\lvert 1.30\rvert$	$\lvert 0.18\rvert$	$\lvert 0.35\rvert$	$\lvert 0.16\rvert$	$\lvert 0.13\rvert$
$L^5_{\alpha_i}$	8.90	4.30	6.09	7.15	12.80**	8.31	9.92*	16.25***	5.12	2.93	8.90	7.08	5.85
C^{13}_1	22.86**												
	Panel D												
$\widehat{\alpha}_i$	0.003	0.037	0.099	0.234	0.363	0.480	0.577	0.666	0.749	0.818	0.893	0.930	0.971
$\lvert t_{1,\alpha_i}\rvert$	$\lvert 5.41\rvert$***	$\lvert 4.91\rvert$***	$\lvert 0.26\rvert$	$\lvert 7.44\rvert$***	$\lvert 11.79\rvert$***	$\lvert 14.05\rvert$***	$\lvert 12.88\rvert$***	$\lvert 11.15\rvert$***	$\lvert 8.93\rvert$***	$\lvert 3.62\rvert$***	$\lvert 1.77\rvert$*	$\lvert 6.37\rvert$***	$\lvert 10.70\rvert$***
$L^5_{\alpha_i}$	69.56***	51.28***	8.86	68.17***	152.47***	206.97***	186.68***	147.90***	89.45***	17.17***	10.74*	53.39***	142.98***
C^{13}_1	369.90***												

Based on the information of the tests and autocontours, we incorporate conditional heteroscedasticity and estimate the following HAR-GARCH model:

$$y_t = \phi_0 + \phi_1 y_{t-1}\bar{y}_{t-1:5} + \phi_{10}\bar{y}_{t-10:5} + \phi_{22}\bar{y}_{t-1:22} + \phi_{66}\bar{y}_{t-1:66} + \sigma_t\varepsilon_t, \qquad (3.26)$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2\sigma_{t-1}^2 + \beta\sigma_{t-1}^2.$$

Table 3.14 reports the estimation results. We can observe that the estimates of the parameters of the conditional mean are very similar. Although the standard errors are different, the conclusions on their significance is the same as in the homoscedastic model. Furthermore, the parameters of the conditional variance equation are significative with values similar to those encountered when the GARCH model is fitted to financial returns with $\widehat{\alpha}$ being rather small and $\widehat{\alpha}+\widehat{\beta}$ very close to one. The residual statistics are reported in Table 3.13, where we can observe that the sample autocorrelations of the squares of log-VIX are no longer significant. Finally, Table 3.15, panels C and D, reports the results for the BG-ACR and G-ACR tests, respectively. We can observe that when implementing the G-ACR, the HAR-GARCH model with Gaussian errors is still clearly rejected. On the other hand, the number of rejections when implementing the BG-ACR tests is much smaller than before. In particular, when looking at the results for the $L^5_{\alpha_i}$, the HAR-GARCH model is only rejected for autocontours 0.3, 0.5 and 0.6, while the basic HAR model is rejected for eight out-of thirteen quintiles. Therefore, including the conditional heteroscedasticity leads to a better specification if the purpose is to obtain accurate predictive densities in-sample.

3.6. Conclusions

In this chapter, we propose an extension of the G-ACR test of González-Rivera and Sun (2015) for dynamic specification of a density model (in-sample tests) and for evaluation of forecast densities (out-of-sample tests). Our contribution lies on computing the PITs from a bootstrapped conditional density so that no assumption on the functional form of the density is needed. Furthermore, the bootstrap procedure allows for the direct incorporation of parameter uncertainty. The proposed approach is particularly useful to evaluate forecast densities when the error distribution is unknown. Our proposed tests have size close to the nominal and are powerful for detecting departures from the assumed conditional density. To illustrate the usefulness of our approach, we extend the analysis of Fernandes et al. (2014) by evaluating the adequacy of conditional densities of the VIX daily market volatility index when computed fitting the HAR model. Our results suggest that conditional heteroscedasticity should be taken into account for an adequate construction of the conditional densities of the log-VIX.

In our research agenda, there are two direct extensions of the proposed BG-ACR tests that we plan to study. First, the extension of the proposed test to multi-step predictive densities is of great interest; see, for example, Jordà and Marcellino (2010), Staszewska-Bystrova (2011), Wolf and Wunderli (2015) and Jordá et al. (2014) for multistep forecasts based on bootstrap. Note that the functional form of multi-step predictive densities could be unknown or difficult to obtain even in cases where the one-step-ahead conditional density is known. Diebold et al. (1998) propose to partition the series of PITs into groups for which the iid uniformity is expected if the forecast densities were indeed correct. Analyzing this extension is left for future research.

Second, given that the tests considered in this chapter are based on the information contained in the vector of PITs which is condensed into an indicator, the tests proposed can be extended into a multivariate framework using the multivariate bootstrap procedures of Fresoli et al. (2015) and Fresoli and Ruiz (2016) for VARMA and multivariate GARCH models, respectively. It is also important to note that in a multivariate context, the PITs with respect to a multivariate conditional density are not longer independent and uniform even if the model is correctly specified; see, for example, Chen and Hong (2014). In the context of multivariate GARCH models, Bai and Chen (2008) propose evaluating the distribution by using the PITs of each individual component. However, this test may miss important information on the joint distribution and, in particular, may fail to detect misspecification in the joint dynamics.

Finally, the residual bootstrap implemented in this chapter to obtain one-step-ahead predictive densities can be modified in several directions. First, one can extended it to cope with lag-order uncertainty of the ARMA lags by implementing the procedures of Kilian (1998a) and Alonso et al. (2004, 2006). Another alternative is substituting the basic residual bootstrap implemented in this chapter to obtain the sample distribution of the parameters by the subsampling procedure proposed by Hall and Yao (2003). Alternatively, one can implement the block bootstrap based on resampling the likelihood proposed by Corradi and Iglesias (2008). Although, we do not expect the results to change qualitatively, the asymptotic validity of the bootstrap can be easier to prove in the case of GARCH errors.

Chapter 4

Conclusions and further research

Forecast densities has received increasing attention in the economics and finance literature because of its importance to economic decision-making. For instance, there is a clear need to use forecast intervals or forecast densities when setting macroeconomic policies and when managing financial risk in the insurance and banking institutions, and firms also rely on forecasting to manage their inventory and production. In practice, time series models are used to forecast the future evolution of a given variable observed during a particular period of time. However, the standard theory of time series forecasting density is based on assuming that the model is known. Even assuming that there is a true model, it is rarely, if ever, the case that such model will be known a priori and there is no guarantee that it will be selected as the best fit to the data. Thus there is usually considerable uncertainty as to which model should be used and apart of this is the question about how model uncertainty will affect the computation of forecasts and their accuracy. Therefore, appropriate procedures for constructing and testing forecast densities should take into account that the conditional forecast error distribution if often unknown and the specification of conditional moments is also unknown and has estimated parameters.

In this thesis we study the construction and evaluation of density forecasts of time series univariate models under model uncertainty. In the second chapter, we study the impact of model uncertainty on univariate ARMA models. As model uncertainty, we consider parameter, error distribution and lag order uncertainties. We provide a survey of all procedures that incorporate those uncertainties in the forecasts of ARMA models and compare their finite sample properties by Monte Carlo experiments by computing the Mallows Distance between the true and theoretical forecast densities and the coverage rates of each theoretical forecast interval. In particular, we analyze asymptotic, Bayesian and bootstrap procedures.

In the third chapter, we propose an extension of the Generalized Autocontour (G-ACR) tests (González-Rivera and Sun, 2015) for one-step-ahead dynamic specifications of conditional densities *in-sample* and of forecast densities *out-of-sample*. The new tests are based on probability integral transforms (PITs) computed from bootstrap conditional densities so that they incorporate the parameter uncertainty without making specific assumptions about the forecast error distribution and it can be extended to multivariate systems and multi-step forecasts. The only restrictions required on the error density are those that guarantee that the estimator of the parameters of the conditional moments is consistent and asymptotically Normal distributed. Furthermore, using graphical devices, our proposed bootstrap procedure allows the identification of the source of misspecification, namely, whether, it is the error distribution, or the linear or non-linear dynamics. We show by Monte Carlo simulations that the proposed test has good finite properties. The results are illustrated by testing the dynamic specification of the HAR model when fitted to the popular U.S. volatility index VIX.

The main contributions of this thesis can be summarized as follows:

- We find through Monte Carlos simulations that for for moderate sample sizes, the parameter and lag order uncertainties are not important. For small forecast horizons the most important source of uncertainty in univariate ARMA models is the error distribution. However, as the forecast horizon increases, the normal approximation of the density is more appropriate.

- We present the main alternative procedures proposed to construct forecast densities that incorporate the model uncertainties cited above, including some very recent procedures which have not been previously compared in the literature. We find that asymptotic methods are able to provide reliable density forecasts only in large sample sizes and with known error distribution; Bayesian methods are able to provide very accurate density forecasts in small sample sizes, but they require the correct error distribution and a large computational effort and is difficult to make them to incorporate all the uncertainties in the forecasts; and Bootstrap procedures are able to provide accurate forecasts, regardless the sample size and the error distribution. We find that Alonso et al. (2004)'s procedure (BOOTEX) is a simple and effective alternative to incorporate lag order and error distribution uncertainties. Given the good results of BOOTEX in comparison with the asymptotic and Bayesian procedures, we highlight the importance of considering resample methods for taking into account model uncertainty when constructing density and forecast intervals.

- We provide a easy to use test for one-step-ahead predictive densities (in-sample and out-of-sample) that do not rely on any particular assumption on the error distribution and take into account parameter uncertainty. Furthermore, it is accompanied by a graphical device to point the potential misspecification of the fitted model, where can disentangle whether the misspecificaion comes from the conditional moments or from the error distribution.

Along this survey, we encountered several gaps in the literature that could be the focus for further research. First, in relation to the second chapter of this thesis, the bias correction usually implemented to the ARMA parameters is based on a known and finite order. However, in practice, the order is also unknown and, consequently, the bias correction could not be appropriate. These corrections could also be important to be implemented in the context of bootstrap forecasts. It could be also interesting to study the effects of model uncertainty in VAR processes when constructing density forecasts, to extend the bootstrap procedure of Rupasinghe and Samaranayake (2012) to vector ARFIMA models and compare it with the Bayesian procedure proposed by Ravishanker and Ray (2002) and to study the effects of stationary transformation uncertainty on the forecasts of ARIMA models.

Finally, in relation to the third chapter, it would be interesting to extend our proposed bootstrap test (BC-ACR) to multi-step predictive densities and to incorporate not only parameter and error distribution uncertainties, but also the lag order uncertainty of the ARMA lags by implementing the procedures of Kilian (1998a) and Alonso et al. (2004, 2006). Finally, we could extend our tests into a multivariate framework using the multivariate bootstrap procedures of Fresoli et al. (2015) and Fresoli and Ruiz (2016) for VARMA and multivariate GARCH models, respectively.

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Appendix A

Appendix of Chapter 2

A.1. Conditional forecast densities estimated based on a simulated time series

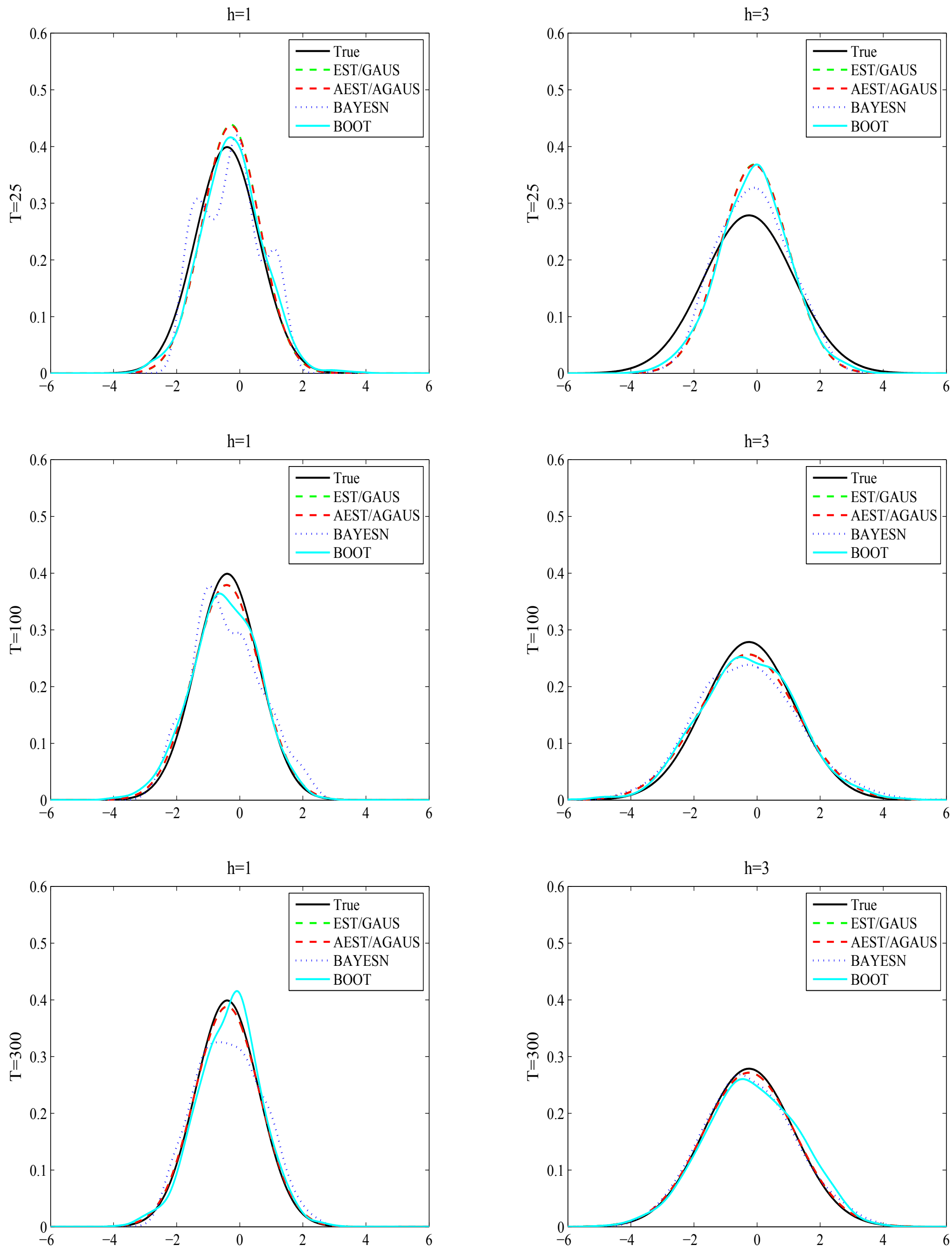


Figure A.1: Conditional forecast density of y_{T+h} for h=1 (left panels) and h=3 (right panels) generated by the model $y_t = 0.8y_{t-1} + \varepsilon_t$ with $\sigma_\varepsilon^2 = 1$, y_T=-0.5 and Gaussian errors, when T=25 (first row), T=100 (second row) and T=300 (third row), together with the densities estimated by EST, AEST, BAYESN and BOOT based on a simulated time series.

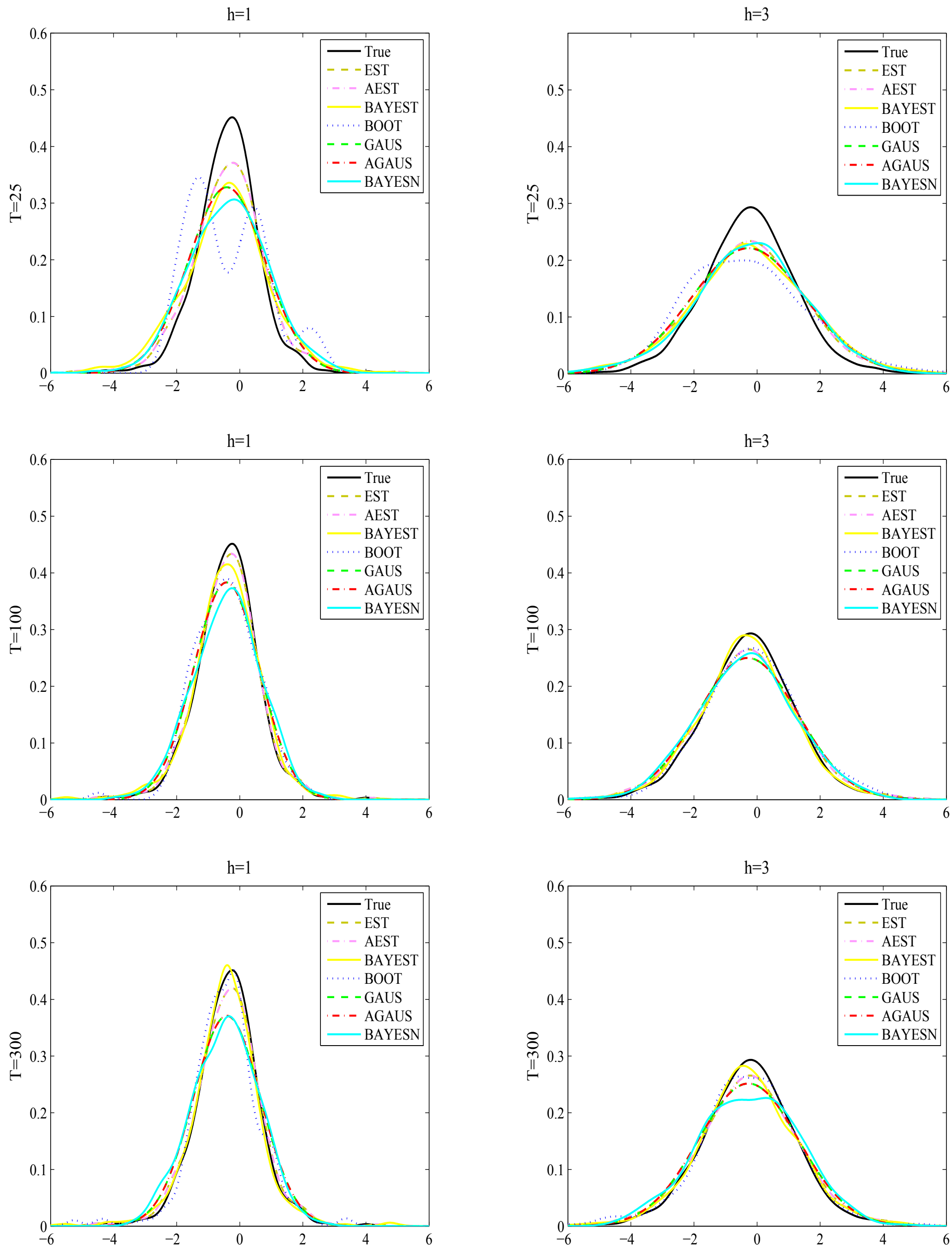


Figure A.2: Conditional forecast density of y_{T+h} for h=1 (left panels) and h=3 (right panels) generated by the model $y_t = 0.8y_{t-1} + \varepsilon_t$ with $\sigma_\varepsilon^2 = 1$, y_T=-0.5 and Student-5 errors rescaled to have unit variance, when T=25 (first row), T=100 (second row) and T=300 (third row), together with the densities estimated by EST, AEST, BAYEST, BOOT, GAUS, AGAUS and BAYESN based on a simulated time series.

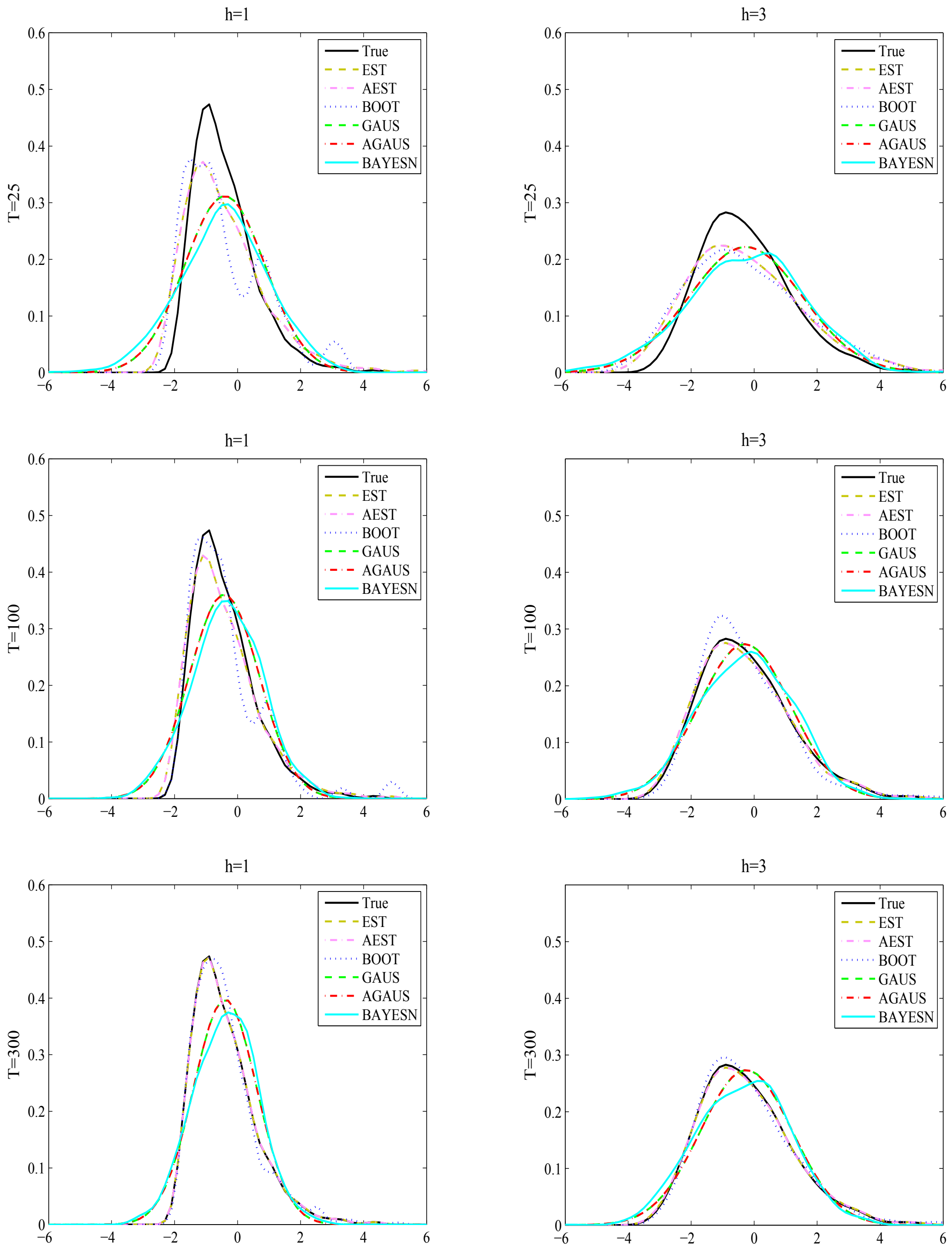


Figure A.3: Conditional forecast density of y_{T+h} for h=1 (left panels) and h=3 (right panels) generated by the model $y_t = 0.8y_{t-1} + \varepsilon_t$ with y_T=-0.5 and $\chi^2_{(5)}$ errors rescaled to have zero mean and unit variance, when T=25 (first row), T=100 (second row) and T=300 (third row), together with the densities estimated by EST, AEST, BOOT, GAUS, AGAUS and BAYESN based on a simulated time series.

A.2. Monte Carlo results for $y_t = 0.8y_{t-1} + \varepsilon_t$ with σ_ε^2=1

Table A.1: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.8y_{t-1} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/GAUS	0.140	0.332	0.343	0.095	0.210	0.200	0.053	0.114	0.107
	(0.123)	(0.338)	(0.373)	(0.074)	(0.183)	(0.170)	(0.044)	(0.095)	(0.081)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.151	0.342	0.361	0.108	0.237	0.233	0.061	0.130	0.126
	(0.124)	(0.345)	(0.388)	(0.086)	(0.209)	(0.206)	(0.050)	(0.105)	(0.097)
GAUS	0.188	0.361	0.377	0.155	0.260	0.257	0.128	0.168	0.165
	(0.122)	(0.342)	(0.387)	(0.084)	(0.205)	(0.204)	(0.053)	(0.103)	(0.097)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.145	0.332	0.342	0.101	0.215	0.211	0.056	0.118	0.115
	(0.117)	(0.351)	(0.396)	(0.073)	(0.166)	(0.159)	(0.038)	(0.083)	(0.081)
GAUS	0.253	0.384	0.390	0.228	0.280	0.272	0.211	0.210	0.203
	(0.093)	(0.336)	(0.380)	(0.055)	(0.150)	(0.144)	(0.031)	(0.068)	(0.066)

Table A.2: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.8y_{t-1} + \varepsilon_t$ with σ_ε^2=1.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/AGAUS	0.140	0.335	0.349	0.095	0.211	0.203	0.053	0.114	0.107
	(0.123)	(0.340)	(0.382)	(0.075)	(0.184)	(0.173)	(0.044)	(0.095)	(0.082)
BAYESN	0.153	0.299	0.334	0.111	0.215	0.218	0.080	0.147	0.144
	(0.121)	(0.244)	(0.295)	(0.071)	(0.149)	(0.146)	(0.043)	(0.085)	(0.074)
BOOT	0.190	0.390	0.460	0.136	0.250	0.258	0.092	0.154	0.153
	(0.110)	(0.375)	(0.455)	(0.065)	(0.200)	(0.201)	(0.040)	(0.095)	(0.082)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.151	0.345	0.366	0.108	0.238	0.235	0.061	0.130	0.126
	(0.125)	(0.350)	(0.399)	(0.086)	(0.211)	(0.209)	(0.050)	(0.106)	(0.098)
AGAUS	0.189	0.364	0.384	0.156	0.262	0.259	0.128	0.169	0.165
	(0.123)	(0.348)	(0.398)	(0.085)	(0.207)	(0.207)	(0.053)	(0.104)	(0.097)
BAYEST	0.155	0.295	0.326	0.119	0.228	0.230	0.083	0.144	0.144
	(0.108)	(0.237)	(0.294)	(0.068)	(0.154)	(0.160)	(0.038)	(0.076)	(0.066)
BAYESN	0.192	0.310	0.343	0.158	0.238	0.241	0.129	0.163	0.159
	(0.128)	(0.269)	(0.340)	(0.083)	(0.174)	(0.187)	(0.054)	(0.098)	(0.093)
BOOT	0.205	0.397	0.476	0.154	0.273	0.288	0.101	0.163	0.163
	(0.107)	(0.375)	(0.468)	(0.073)	(0.219)	(0.234)	(0.039)	(0.099)	(0.088)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.145	0.334	0.347	0.101	0.217	0.213	0.056	0.118	0.115
	(0.117)	(0.354)	(0.403)	(0.073)	(0.168)	(0.162)	(0.038)	(0.083)	(0.081)
AGAUS	0.254	0.388	0.395	0.229	0.282	0.273	0.212	0.210	0.203
	(0.094)	(0.340)	(0.387)	(0.056)	(0.153)	(0.148)	(0.031)	(0.068)	(0.066)
BAYESN	0.254	0.341	0.366	0.231	0.266	0.264	0.212	0.207	0.202
	(0.086)	(0.248)	(0.318)	(0.059)	(0.127)	(0.130)	(0.031)	(0.064)	(0.063)
BOOT	0.187	0.386	0.461	0.135	0.247	0.260	0.087	0.148	0.151
	(0.106)	(0.384)	(0.472)	(0.064)	(0.177)	(0.184)	(0.033)	(0.078)	(0.077)

Table A.3: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.8y_{t-1} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/GAUS	78.04 (0.05)	11.06/10.90	77.96 (0.09)	11.12/10.92	79.14 (0.10)	10.43/10.44
100	EST/GAUS	78.95 (0.04)	10.59/10.46	78.96 (0.06)	10.66/10.38	79.49 (0.07)	10.33/10.18
300	EST/GAUS	79.66 (0.02)	10.18/10.16	79.65 (0.04)	10.26/10.09	79.80 (0.04)	10.14/10.06
	Student-5	h=1		h=6		h=12	
50	EST	77.98 (0.06)	11.09/10.93	78.19 (0.09)	11.01/10.80	79.43 (0.10)	10.35/10.22
	GAUS	82.24 (0.06)	8.94/8.82	79.95 (0.09)	10.13/9.92	80.96 (0.10)	9.59/9.45
100	EST	79.02 (0.05)	10.63/10.34	79.20 (0.07)	10.60/10.21	79.87 (0.08)	10.17/9.96
	GAUS	83.19 (0.04)	8.52/8.29	80.94 (0.06)	9.72/9.34	81.39 (0.07)	9.41/9.19
300	EST	79.76 (0.03)	10.09/10.16	79.87 (0.04)	10.09/10.03	80.06 (0.05)	9.98/9.96
	GAUS	83.89 (0.03)	8.02/8.09	81.60 (0.04)	9.23/9.17	81.59 (0.04)	9.20/9.21
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	77.92 (0.08)	11.43/10.65	78.05 (0.09)	11.05/10.90	79.26 (0.10)	10.39/10.35
	GAUS	83.24 (0.07)	5.40/11.36	79.23 (0.09)	9.25/11.52	80.25 (0.10)	8.83/10.92
100	EST	78.75 (0.06)	11.01/10.24	79.02 (0.07)	10.68/10.30	79.68 (0.08)	10.22/10.10
	GAUS	84.38 (0.05)	4.66/10.96	80.24 (0.06)	8.83/10.93	80.74 (0.07)	8.60/10.66
300	EST	79.66 (0.04)	10.32/10.02	79.59 (0.04)	10.29/10.12	79.80 (0.05)	10.13/10.06
	GAUS	85.51 (0.03)	3.76/10.73	80.82 (0.04)	8.43/10.75	80.87 (0.04)	8.48/10.65
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	EST/GAUS	93.68 (0.03)	3.20/3.12	93.15 (0.06)	3.45/3.40	93.46 (0.06)	3.26/3.28
100	EST/GAUS	94.32 (0.02)	2.87/2.82	94.09 (0.03)	2.97/2.94	94.20 (0.04)	2.91/2.89
300	EST/GAUS	94.77 (0.01)	2.62/2.61	94.71 (0.02)	2.66/2.63	94.73 (0.02)	2.64/2.63
	Student-5	h=1		h=6		h=12	
50	EST	93.94 (0.03)	3.06/3.01	93.51 (0.05)	3.29/3.19	93.79 (0.05)	3.11/3.11
	GAUS	93.65 (0.03)	3.19/3.15	93.32 (0.05)	3.39/3.28	93.63 (0.05)	3.19/3.18
100	EST	94.48 (0.02)	2.79/2.73	94.30 (0.03)	2.91/2.79	94.40 (0.04)	2.82/2.78
	GAUS	94.21 (0.02)	2.93/2.86	94.12 (0.03)	3.00/2.87	94.25 (0.04)	2.89/2.85
300	EST	94.85 (0.01)	2.57/2.58	94.83 (0.02)	2.57/2.59	94.85 (0.02)	2.56/2.59
	GAUS	94.59 (0.01)	2.69/2.71	94.66 (0.02)	2.66/2.68	94.69 (0.02)	2.64/2.67
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	92.42 (0.06)	4.64/2.94	92.88 (0.06)	4.04/3.08	93.29 (0.06)	3.79/2.91
	GAUS	94.59 (0.02)	0.13/5.28	93.66 (0.05)	1.75/4.59	93.93 (0.05)	1.80/4.26
100	EST	93.44 (0.04)	3.86/2.70	93.88 (0.04)	3.37/2.75	94.07 (0.04)	3.22/2.71
	GAUS	95.00 (0.02)	0.01/4.99	94.54 (0.03)	1.28/4.19	94.60 (0.04)	1.37/4.03
300	EST	94.48 (0.02)	2.96/2.56	94.56 (0.02)	2.85/2.59	94.59 (0.03)	2.82/2.59
	GAUS	95.22 (0.01)	0.00/4.78	95.09 (0.02)	0.93/3.98	95.03 (0.02)	1.07/3.91

Table A.4: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.8y_{t-1} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/AGAUS	78.40 (0.05)	10.87/10.73	78.52 (0.09)	10.83/10.65	79.47 (0.10)	10.25/10.28
	SRA	79.53 (0.07)	9.11/11.36	72.00 (0.12)	12.48/15.52	68.85 (0.14)	13.87/17.28
	BAYESN	79.31 (0.05)	10.42/10.27	77.61 (0.08)	11.31/11.08	78.27 (0.09)	10.86/10.87
	BOOT	78.47 (0.06)	10.85/10.67	79.49 (0.09)	10.33/10.18	81.39 (0.10)	9.30/9.31
100	AEST/AGAUS	79.16 (0.04)	10.48/10.36	79.30 (0.06)	10.48/10.22	79.64 (0.07)	10.25/10.11
	SRA	79.73 (0.05)	9.60/10.67	76.46 (0.08)	11.26/12.28	75.92 (0.09)	11.57/12.50
	BAYESN	79.62 (0.04)	10.23/10.15	78.61 (0.06)	10.84/10.55	78.86 (0.07)	10.63/10.51
	BOOT	79.20 (0.04)	10.46/10.34	79.77 (0.06)	10.26/9.97	80.61 (0.07)	9.75/9.64
300	AEST/AGAUS	79.74 (0.02)	10.14/10.12	79.76 (0.04)	10.21/10.04	79.84 (0.04)	10.12/10.04
	SRA	79.90 (0.03)	9.79/10.31	78.90 (0.04)	10.43/10.67	78.92 (0.05)	10.48/10.60
	BAYESN	79.70 (0.03)	10.16/10.14	79.36 (0.04)	10.39/10.25	79.40 (0.04)	10.37/10.23
	BOOT	79.66 (0.03)	10.13/10.22	79.82 (0.04)	10.16/10.02	80.20 (0.04)	9.94/9.87
	Student-5	h=1		h=6		h=12	
50	AEST	78.27 (0.06)	10.94/10.78	78.70 (0.09)	10.76/10.54	79.72 (0.10)	10.21/10.07
	AGAUS	82.52 (0.06)	8.79/8.69	80.45 (0.09)	9.88/9.67	81.23 (0.10)	9.45/9.32
	SRA	80.01 (0.07)	8.94/11.05	73.27 (0.12)	12.27/14.47	69.31 (0.14)	14.03/16.65
	BAYEST	80.36 (0.05)	9.87/9.77	78.98 (0.08)	10.58/10.44	79.76 (0.09)	10.14/10.10
	BAYESN	83.19 (0.05)	8.48/8.33	79.58 (0.09)	10.35/10.08	80.21 (0.09)	9.97/9.83
	BOOT	79.42 (0.06)	10.32/10.26	80.61 (0.09)	9.74/9.64	82.51 (0.10)	8.75/8.74
100	AEST	79.20 (0.05)	10.54/10.26	79.52 (0.07)	10.43/10.05	80.00 (0.08)	10.10/9.90
	AGAUS	83.35 (0.04)	8.43/8.22	81.24 (0.06)	9.56/9.20	81.52 (0.07)	9.34/9.14
	SRA	80.13 (0.04)	9.63/10.24	77.15 (0.08)	10.97/11.89	76.35 (0.10)	11.60/12.04
	BAYEST	80.45 (0.04)	9.85/9.70	79.58 (0.06)	10.34/10.08	80.07 (0.07)	10.03/9.90
	BAYESN	83.70 (0.04)	8.28/8.02	80.61 (0.06)	9.84/9.55	80.86 (0.07)	9.66/9.48
	BOOT	79.71 (0.04)	10.25/10.04	80.67 (0.06)	9.87/9.45	81.54 (0.07)	9.36/9.10
300	AEST	79.82 (0.03)	10.06/10.13	79.98 (0.04)	10.05/9.98	80.09 (0.05)	9.97/9.94
	AGAUS	83.94 (0.03)	7.99/8.06	81.69 (0.04)	9.19/9.12	81.62 (0.04)	9.19/9.19
	SRA	79.97 (0.03)	9.80/10.23	79.23 (0.04)	10.25/10.52	79.14 (0.05)	10.44/10.42
	BAYEST	80.26 (0.03)	9.79/9.95	79.90 (0.04)	10.08/10.02	80.04 (0.04)	9.95/10.01
	BAYESN	83.95 (0.03)	8.01/8.04	81.36 (0.04)	9.35/9.29	81.24 (0.04)	9.38/9.38
	BOOT	79.83 (0.03)	10.07/10.10	80.41 (0.04)	9.85/9.73	80.63 (0.04)	9.70/9.67
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	78.32 (0.08)	11.16/10.52	78.64 (0.09)	10.71/10.65	79.59 (0.10)	10.21/10.19
	AGAUS	83.58 (0.07)	5.19/11.23	79.80 (0.09)	8.93/11.27	80.58 (0.10)	8.66/10.75
	SRA	79.90 (0.07)	9.23/10.86	72.73 (0.12)	12.58/14.69	69.13 (0.14)	14.22/16.65
	BAYESN	84.30 (0.06)	4.86/10.84	78.91 (0.08)	9.51/11.58	79.48 (0.09)	9.28/11.24
	BOOT	79.22 (0.07)	10.35/10.43	80.11 (0.09)	9.78/10.11	81.95 (0.10)	8.76/9.29
100	AEST	78.99 (0.06)	10.84/10.17	79.36 (0.07)	10.47/10.17	79.81 (0.08)	10.14/10.04
	AGAUS	84.60 (0.05)	4.52/10.88	80.56 (0.07)	8.64/10.80	80.87 (0.07)	8.53/10.60
	SRA	79.90 (0.05)	9.83/10.28	76.77 (0.08)	11.43/11.80	75.74 (0.09)	11.82/12.45
	BAYESN	84.87 (0.05)	4.43/10.71	79.82 (0.07)	9.07/11.10	80.10 (0.07)	9.00/10.90
	BOOT	79.37 (0.06)	10.51/10.12	80.11 (0.07)	9.99/9.89	81.08 (0.07)	9.39/9.53
300	AEST	79.74 (0.04)	10.26/9.99	79.68 (0.04)	10.23/10.08	79.83 (0.05)	10.12/10.05
	AGAUS	85.57 (0.03)	3.72/10.70	80.92 (0.04)	8.37/10.71	80.90 (0.05)	8.46/10.64
	SRA	80.15 (0.03)	9.87/9.99	78.98 (0.05)	10.43/10.59	78.82 (0.05)	10.46/10.72
	BAYESN	85.51 (0.03)	3.82/10.67	80.55 (0.04)	8.59/10.85	80.54 (0.05)	8.63/10.83
	BOOT	79.73 (0.04)	10.25/10.02	79.94 (0.04)	10.08/9.98	80.18 (0.05)	9.95/9.87

Table A.5: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.8y_{t-1} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	AEST/AGAUS	93.88 (0.03)	3.09/3.02	93.42 (0.06)	3.31/3.26	93.57 (0.06)	3.21/3.23
	SRA	92.17 (0.05)	2.79/5.04	85.02 (0.10)	6.07/8.91	82.76 (0.11)	8.22/9.02
	BAYESN	94.58 (0.03)	2.73/2.69	93.60 (0.05)	3.23/3.18	94.07 (0.06)	2.95/2.97
	BOOT	93.16 (0.04)	3.46/3.38	94.12 (0.05)	2.96/2.92	94.97 (0.06)	2.49/2.53
100	AEST/AGAUS	94.44 (0.02)	2.81/2.76	94.26 (0.03)	2.88/2.86	94.26 (0.04)	2.88/2.86
	SRA	94.09 (0.03)	2.36/3.55	91.46 (0.05)	3.55/4.98	89.61 (0.07)	4.51/5.87
	BAYESN	94.74 (0.02)	2.65/2.61	94.20 (0.03)	2.91/2.88	94.41 (0.04)	2.79/2.79
	BOOT	93.98 (0.03)	3.01/3.01	94.53 (0.03)	2.73/2.74	94.99 (0.04)	2.47/2.54
300	AEST/AGAUS	94.81 (0.01)	2.60/2.59	94.77 (0.02)	2.64/2.60	94.74 (0.02)	2.63/2.62
	SRA	94.74 (0.02)	2.42/2.85	94.12 (0.03)	2.76/3.12	93.60 (0.03)	3.06/3.34
	BAYESN	94.80 (0.01)	2.59/2.61	94.63 (0.02)	2.70/2.67	94.67 (0.02)	2.67/2.66
	BOOT	94.56 (0.02)	2.72/2.72	94.81 (0.02)	2.62/2.57	94.97 (0.02)	2.52/2.51
	Student-5	h=1		h=6		h=12	
50	AEST	94.07 (0.03)	2.99/2.94	93.75 (0.05)	3.19/3.07	93.89 (0.05)	3.05/3.06
	AGAUS	93.79 (0.03)	3.13/3.08	93.57 (0.05)	3.28/3.16	93.73 (0.05)	3.13/3.14
	SRA	92.99 (0.04)	2.42/4.59	85.95 (0.09)	5.77/8.27	82.94 (0.11)	8.45/8.61
	BAYEST	94.68 (0.02)	2.66/2.66	94.51 (0.04)	2.78/2.72	94.92 (0.04)	2.53/2.54
	BAYESN	94.31 (0.03)	2.89/2.80	93.70 (0.05)	3.21/3.09	94.16 (0.05)	2.91/2.92
	BOOT	93.47 (0.03)	3.33/3.20	94.31 (0.05)	2.87/2.82	95.16 (0.05)	2.44/2.41
100	AEST	94.55 (0.02)	2.76/2.69	94.44 (0.03)	2.83/2.72	94.46 (0.04)	2.79/2.75
	AGAUS	94.29 (0.02)	2.89/2.82	94.27 (0.03)	2.92/2.81	94.31 (0.04)	2.87/2.83
	SRA	94.59 (0.03)	2.26/3.15	91.64 (0.06)	3.71/4.66	89.81 (0.07)	4.61/5.58
	BAYEST	94.75 (0.02)	2.65/2.60	94.75 (0.03)	2.67/2.58	94.96 (0.03)	2.54/2.49
	BAYESN	94.54 (0.02)	2.76/2.70	94.27 (0.03)	2.93/2.80	94.50 (0.04)	2.75/2.75
	BOOT	94.23 (0.02)	2.95/2.82	94.74 (0.03)	2.71/2.55	95.14 (0.03)	2.48/2.38
300	AEST	94.88 (0.01)	2.55/2.57	94.88 (0.02)	2.55/2.57	94.86 (0.02)	2.56/2.58
	AGAUS	94.61 (0.01)	2.69/2.70	94.71 (0.02)	2.63/2.66	94.71 (0.02)	2.63/2.66
	SRA	94.92 (0.02)	2.35/2.73	94.38 (0.03)	2.63/2.99	93.9 (0.03)	2.91/3.19
	BAYEST	94.86 (0.01)	2.53/2.61	94.89 (0.02)	2.57/2.53	94.97 (0.02)	2.50/2.53
	BAYESN	94.62 (0.01)	2.68/2.70	94.64 (0.02)	2.68/2.68	94.65 (0.02)	2.66/2.69
	BOOT	94.70 (0.02)	2.65/2.64	94.97 (0.02)	2.53/2.50	95.08 (0.02)	2.45/2.46
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	92.69 (0.06)	4.43/2.87	93.20 (0.06)	3.84/2.95	93.43 (0.06)	3.72/2.85
	AEST	94.71 (0.02)	0.10/5.19	93.92 (0.05)	1.65/4.43	94.05 (0.05)	1.76/4.18
	SRA	92.59 (0.05)	3.06/4.35	86.00 (0.09)	5.88/8.12	83.22 (0.11)	8.57/8.21
	BAYESN	95.16 (0.02)	0.05/4.79	94.13 (0.04)	1.53/4.34	94.50 (0.05)	1.52/3.98
	BOOT	93.72 (0.04)	2.97/3.31	94.42 (0.05)	2.65/2.93	95.25 (0.05)	2.25/2.50
100	AEST	93.60 (0.04)	3.73/2.67	94.05 (0.04	3.26/2.69	94.12 (0.04)	3.19/2.69
	AEST	95.05 (0.02)	0.01/4.94	94.68 (0.03)	1.22/4.10	94.65 (0.04)	1.35/4.00
	SRA	94.33 (0.03)	2.57/3.09	91.60 (0.05)	3.92/4.48	89.49 (0.07)	4.96/5.55
	BAYESN	95.25 (0.02)	0.01/4.74	94.63 (0.03)	1.24/4.13	94.77 (0.03)	1.32/3.90
	BOOT	94.24 (0.03)	2.88/2.88	94.68 (0.04)	2.63/2.69	95.14 (0.04)	2.33/2.53
300	AEST	94.53 (0.02)	2.92/2.55	94.62 (0.02)	2.81/2.57	94.61 (0.03)	2.81/2.58
	AEST	95.23 (0.01)	0.00/4.76	95.14 (0.02)	0.91/3.95	95.04 (0.02)	1.06/3.89
	SRA	94.84 (0.02)	2.51/2.65	94.10 (0.03)	2.93/2.96	93.62 (0.03)	3.16/3.22
	BAYESN	95.23 (0.01)	0.00/4.77	95.01 (0.02)	0.98/4.01	95.00 (0.02)	1.08/3.92
	BOOT	94.58 (0.02)	2.74/2.68	94.79 (0.02)	2.62/2.59	94.89 (0.03)	2.55/2.56

A.3. Monte Carlo results for forecast intervals of 95% for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ^2_ε=1.

Table A.6: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	EST/GAUS	93.66 (0.03)	3.11/3.23	93.01 (0.05)	3.39/3.59	92.86 (0.07)	3.42/3.71
	$GAUS_{aicc}$	92.82 (0.04)	3.59/3.59	92.70 (0.06)	3.57/3.73	92.21 (0.07)	3.77/4.01
100	EST/GAUS	94.35 (0.02)	2.85/2.79	94.08 (0.03)	2.99/2.93	93.92 (0.04)	3.07/3.01
	$GAUS_{aicc}$	93.88 (0.03)	3.07/3.05	93.90 (0.04)	3.07/3.02	93.65 (0.05)	3.19/3.16
300	EST/GAUS	94.78 (0.01)	2.61/2.61	94.76 (0.02)	2.66/2.58	94.70 (0.02)	2.67/2.63
	$GAUS_{aicc}$	94.68 (0.01)	2.66/2.66	94.69 (0.02)	2.68/2.63	94.64 (0.03)	2.70/2.66
	Student-5	h=1		h=6		h=12	
50	EST	93.56 (0.03)	3.27/3.17	92.63 (0.06)	3.74/3.63	92.52 (0.07)	3.72/3.76
	GAUS	93.27 (0.03)	3.42/3.32	92.44 (0.06)	3.84/3.73	92.36 (0.07)	3.80/3.83
	$GAUS_{aicc}$	92.52 (0.04)	3.75/3.73	91.95 (0.07)	3.99/4.05	91.65 (0.08)	4.10/4.25
100	EST	94.20 (0.02)	2.92/2.89	93.66 (0.04)	3.20/3.14	93.47 (0.05)	3.29/3.25
	GAUS	93.92 (0.02)	3.05/3.03	93.47 (0.04)	3.29/3.23	93.33 (0.05)	3.35/3.32
	$GAUS_{aicc}$	93.48 (0.03)	3.29/3.23	93.29 (0.04)	3.45/3.26	93.09 (0.05)	3.54/3.37
300	EST	94.78 (0.01)	2.59/2.63	94.65 (0.02)	2.66/2.69	94.58 (0.03)	2.67/2.75
	GAUS	94.52 (0.01)	2.72/2.76	94.48 (0.02)	2.74/2.78	94.44 (0.03)	2.74/2.82
	$GAUS_{aicc}$	94.42 (0.01)	2.77/2.81	94.41 (0.02)	2.78/2.81	94.38 (0.03)	2.77/2.85
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	91.95 (0.07)	4.99/3.07	92.03 (0.07)	4.38/3.59	91.82 (0.08)	4.44/3.73
	GAUS	94.27 (0.03)	0.19/5.54	92.69 (0.06)	2.11/5.19	92.25 (0.08)	2.68/5.07
	$GAUS_{aicc}$	93.61 (0.04)	0.54/5.85	92.27 (0.07)	2.41/5.32	91.44 (0.09)	3.11/5.44
100	EST	93.03 (0.05)	4.18/2.79	93.47 (0.04)	3.49/3.03	93.35 (0.05)	3.57/3.08
	GAUS	94.79 (0.02)	0.04/5.16	94.01 (0.04)	1.51/4.48	93.70 (0.05)	2.02/4.28
	$GAUS_{aicc}$	94.46 (0.02)	0.21/5.33	93.73 (0.04)	1.68/4.59	93.35 (0.05)	2.22/4.42
300	EST	94.36 (0.03)	3.08/2.56	94.51 (0.02)	2.88/2.61	94.44 (0.03)	2.91/2.66
	GAUS	95.14 (0.01)	0.00/4.86	94.99 (0.02)	1.07/3.94	94.76 (0.02)	1.52/3.72
	$GAUS_{aicc}$	95.09 (0.01)	0.01/4.90	94.90 (0.02)	1.12/3.98	94.67 (0.03)	1.58/3.75

Table A.7: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	AEST/AGAUS	94.00 (0.03)	2.94/3.06	93.35 (0.05)	3.22/3.43	93.13 (0.07)	3.27/3.60
	$\text{AGAUS}_{\text{aicc}}$	93.16 (0.04)	3.42/3.42	93.01 (0.06)	3.41/3.58	92.41 (0.08)	3.65/3.94
	SRA	92.28 (0.04)	2.68/5.03	85.24 (0.10)	5.85/8.91	79.07 (0.13)	9.99/10.94
	SRA_{aicc}	91.86 (0.05)	2.95/5.18	85.44 (0.10)	5.87/8.69	79.10 (0.14)	10.04/10.86
100	AEST/AGAUS	94.56 (0.02)	2.74/2.69	94.40 (0.03)	2.83/2.77	94.27 (0.04)	2.89/2.84
	$\text{AGAUS}_{\text{aicc}}$	94.12 (0.03)	2.95/2.93	94.23 (0.04)	2.90/2.87	93.99 (0.05)	3.01/3.00
	SRA	94.21 (0.03)	2.35/3.43	91.50 (0.05)	3.73/4.77	87.33 (0.08)	5.57/7.10
	SRA_{aicc}	93.93 (0.03)	2.47/3.61	91.18 (0.06)	3.86/4.96	86.84 (0.09)	5.78/7.38
300	AEST/AGAUS	94.86 (0.01)	2.57/2.57	94.90 (0.02)	2.59/2.51	94.85 (0.02)	2.59/2.56
	$\text{AGAUS}_{\text{aicc}}$	94.77 (0.01)	2.61/2.61	94.83 (0.02)	2.61/2.56	94.80 (0.02)	2.62/2.58
	SRA	94.77 (0.01)	2.46/2.77	94.14 (0.02)	2.81/3.05	93.20 (0.03)	3.22/3.58
	SRA_{aicc}	94.70 (0.01)	2.49/2.81	93.97 (0.03)	2.86/3.17	93.13 (0.04)	3.29/3.58
	Student-5	h=1		h=6		h=12	
50	AEST	93.84 (0.03)	3.13/3.03	92.90 (0.06)	3.58/3.51	92.67 (0.08)	3.59/3.73
	AGAUS	93.55 (0.03)	3.28/3.17	92.71 (0.06)	3.68/3.61	92.53 (0.09)	3.67/3.80
	$\text{AGAUS}_{\text{aicc}}$	92.83 (0.04)	3.61/3.56	92.15 (0.07)	3.91/3.94	91.70 (0.10)	4.06/4.24
	SRA	92.69 (0.04)	2.55/4.75	84.17 (0.10)	6.86/8.97	79.06 (0.14)	10.86/10.08
	SRA_{aicc}	92.40 (0.05)	2.59/5.01	84.34 (0.11)	6.54/9.12	79.48 (0.13)	10.39/10.13
100	AEST	94.35 (0.02)	2.84/2.81	93.93 (0.04)	3.07/2.99	93.78 (0.05)	3.14/3.08
	AGAUS	94.07 (0.02)	2.97/2.96	93.75 (0.04)	3.16/3.09	93.65 (0.05)	3.20/3.15
	$\text{AGAUS}_{\text{aicc}}$	93.68 (0.03)	3.18/3.14	93.60 (0.04)	3.29/3.11	93.41 (0.05)	3.39/3.21
	SRA	94.46 (0.03)	2.24/3.30	91.05 (0.06)	3.97/4.98	87.03 (0.08)	5.99/6.97
	SRA_{aicc}	94.21 (0.03)	2.38/3.41	90.88 (0.06)	4.06/5.06	86.54 (0.09)	6.15/7.31
300	AEST	94.83 (0.01)	2.57/2.60	94.77 (0.02)	2.60/2.63	94.71 (0.03)	2.61/2.68
	AGAUS	94.57 (0.01)	2.70/2.73	94.61 (0.02)	2.68/2.71	94.59 (0.03)	2.67/2.74
	$\text{AGAUS}_{\text{aicc}}$	94.49 (0.01)	2.74/2.77	94.54 (0.02)	2.71/2.74	94.53 (0.03)	2.69/2.77
	SRA	94.85 (0.01)	2.39/2.75	94.25 (0.03)	2.69/3.06	93.26 (0.04)	3.06/3.68
	SRA_{aicc}	94.81 (0.02)	2.42/2.77	94.15 (0.03)	2.77/3.08	93.16 (0.04)	3.16/3.68
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	92.43 (0.06)	4.62/2.94	92.37 (0.07)	4.16/3.47	91.99 (0.09)	4.29/3.72
	AGAUS	94.49 (0.03)	0.15/5.36	93.00 (0.06)	1.97/5.03	92.38 (0.09)	2.61/5.01
	$\text{AGAUS}_{\text{aicc}}$	93.84 (0.04)	0.46/5.70	92.46 (0.08)	2.29/5.24	91.56 (0.10)	3.05/5.39
	SRA	92.58 (0.05)	3.00/4.42	84.21 (0.11)	6.77/9.02	77.85 (0.15)	11.82/10.32
	SRA_{aicc}	92.15 (0.06)	3.27/4.59	84.45 (0.11)	6.71/8.83	78.22 (0.15)	11.38/10.41
100	AEST	93.32 (0.05)	3.95/2.72	93.80 (0.04)	3.29/2.91	93.71 (0.05)	3.37/2.93
	AGAUS	94.91 (0.02)	0.03/5.05	94.30 (0.04)	1.38/4.32	94.03 (0.05)	1.87/4.09
	$\text{AGAUS}_{\text{aicc}}$	94.63 (0.02)	0.17/5.20	94.04 (0.04)	1.54/4.42	93.69 (0.05)	2.07/4.24
	SRA	94.08 (0.04)	2.77/3.15	90.99 (0.06)	4.09/4.92	86.97 (0.08)	6.11/6.92
	SRA_{aicc}	93.76 (0.04)	3.03/3.21	90.94 (0.06)	4.16/4.90	86.66 (0.09)	6.18/7.16
300	AEST	94.47 (0.03)	2.99/2.54	94.67 (0.02)	2.78/2.55	94.60 (0.03)	2.82/2.58
	AGAUS	95.18 (0.01)	0.00/4.82	95.11 (0.02)	1.02/3.86	94.90 (0.02)	1.46/3.64
	$\text{AGAUS}_{\text{aicc}}$	95.14 (0.01)	0.01/4.85	95.04 (0.02)	1.07/3.89	94.83 (0.03)	1.51/3.66
	SRA	94.81 (0.02)	2.58/2.61	94.12 (0.03)	3.01/2.87	93.07 (0.04)	3.51/3.42
	SRA_{aicc}	94.73 (0.02)	2.64/2.63	93.97 (0.03)	3.09/2.93	92.93 (0.04)	3.59/3.47

Table A.8: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BAYESN	94.79 (0.03)	2.56/2.64	94.37 (0.05)	2.79/2.84	94.15 (0.06)	2.85/2.99
	BAYESL	94.86 (0.03)	2.54/2.59	93.08 (0.06)	3.39/3.53	92.07 (0.07)	3.89/4.04
100	BAYESN	94.92 (0.02)	2.59/2.49	94.71 (0.03)	2.69/2.60	94.53 (0.04)	2.72/2.74
	BAYESL	94.78 (0.03)	2.67/2.54	93.76 (0.04)	3.15/3.09	93.00 (0.05)	3.49/3.51
300	BAYESN	94.91 (0.01)	2.54/2.55	94.93 (0.02)	2.59/2.48	94.92 (0.02)	2.56/2.52
	BAYESL	94.89 (0.01)	2.51/2.60	94.62 (0.02)	2.70/2.68	94.32 (0.03)	2.83/2.85
	Student-5	h=1		h=6		h=12	
50	BAYEST	94.54 (0.03)	2.73/2.72	94.00 (0.05)	3.03/2.96	93.94 (0.06)	3.03/3.03
	BAYESN	94.12 (0.03)	2.98/2.90	93.67 (0.05)	3.20/3.13	93.49 (0.06)	3.26/3.26
	BAYESL	94.25 (0.03)	2.81/2.93	92.50 (0.06)	3.73/3.77	91.54 (0.07)	4.17/4.29
100	BAYEST	94.60 (0.02)	2.71/2.68	94.42 (0.03)	2.82/2.76	94.28 (0.04)	2.88/2.84
	BAYESN	94.32 (0.02)	2.84/2.84	94.18 (0.04)	2.90/2.91	94.02 (0.05)	2.98/3.00
	BAYESL	94.41 (0.03)	2.79/2.79	93.49 (0.04)	3.28/3.23	92.83 (0.05)	3.56/3.61
300	BAYEST	94.80 (0.01)	2.59/2.60	94.82 (0.02)	2.59/2.59	94.81 (0.02)	2.55/2.64
	BAYESN	94.60 (0.01)	2.68/2.72	94.64 (0.02)	2.68/2.68	94.61 (0.03)	2.66/2.73
	BAYESL	94.62 (0.01)	2.67/2.71	94.41 (0.02)	2.81/2.78	94.12 (0.03)	2.91/2.98
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	94.91 (0.03)	0.15/4.94	94.17 (0.05)	1.54/4.29	93.68 (0.07)	2.04/4.28
	BAYESL	95.08 (0.03)	0.21/4.71	92.92 (0.06)	2.26/4.82	91.78 (0.07)	3.05/5.17
100	BAYESN	95.09 (0.02)	0.04/4.87	94.71 (0.03)	1.24/4.05	94.36 (0.04)	1.72/3.92
	BAYESL	95.16 (0.02)	0.16/4.69	93.88 (0.04)	1.80/4.32	93.01 (0.05)	2.50/4.49
300	BAYESN	95.21 (0.01)	0.00/4.78	95.11 (0.02)	1.03/3.86	94.88 (0.03)	1.46/3.66
	BAYESL	95.29 (0.01)	0.01/4.70	94.86 (0.02)	1.23/3.92	94.47 (0.03)	1.70/3.83

Table A.9: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BOOT	93.42 (0.04)	3.23/3.35	94.29 (0.04)	2.79/2.91	95.00 (0.05)	2.41/2.59
	BOOTEX	93.29 (0.04)	3.36/3.35	94.66 (0.05)	2.65/2.69	94.86 (0.06)	2.51/2.63
	BOOTNP	85.78 (0.14)	6.78/7.45	80.90 (0.12)	9.32/9.78	76.87 (0.13)	11.33/11.80
100	BOOT	94.14 (0.03)	2.94/2.92	94.76 (0.03)	2.67/2.57	95.05 (0.04)	2.49/2.45
	BOOTEX	94.11 (0.03)	2.93/2.96	94.92 (0.03)	2.55/2.53	95.09 (0.04)	2.46/2.44
	BOOTNP	89.42 (0.13)	5.35/5.24	86.50 (0.11)	6.87/6.63	84.26 (0.11)	8.09/7.64
300	BOOT	94.67 (0.02)	2.64/2.69	94.94 (0.02)	2.56/2.50	95.07 (0.02)	2.48/2.44
	BOOTEX	94.62 (0.02)	2.68/2.69	94.92 (0.02)	2.57/2.51	95.06 (0.02)	2.48/2.47
	BOOTNP	93.18 (0.09)	3.39/3.42	91.03 (0.06)	4.50/4.47	89.93 (0.05)	5.04/5.03
	Student-5	h=1		h=6		h=12	
50	BOOT	93.47 (0.04)	3.33/3.20	93.67 (0.05)	3.22/3.11	94.32 (0.06)	2.90/2.78
	BOOTEX	93.25 (0.04)	3.43/3.32	93.76 (0.06)	3.19/3.05	94.13 (0.06)	2.97/2.91
	BOOTNP	84.68 (0.16)	6.87/8.44	80.25 (0.13)	9.19/10.57	76.49 (0.14)	11.30/12.21
100	BOOT	94.09 (0.03)	3.02/2.89	94.23 (0.03)	2.92/2.85	94.52 (0.04)	2.72/2.77
	BOOTEX	93.98 (0.03)	3.07/2.94	94.37 (0.04)	2.86/2.77	94.61 (0.04)	2.72/2.67
	BOOTNP	89.15 (0.13)	5.59/5.25	86.32 (0.11)	6.94/6.74	83.69 (0.11)	8.12/8.19
300	BOOT	94.66 (0.02)	2.67/2.67	94.81 (0.02)	2.60/2.58	94.96 (0.02)	2.46/2.58
	BOOTEX	94.59 (0.02)	2.72/2.68	94.79 (0.02)	2.61/2.59	94.95 (0.02)	2.48/2.57
	BOOTNP	92.65 (0.10)	3.57/3.78	90.49 (0.08)	4.72/4.79	89.17 (0.08)	5.33/5.50
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	94.13 (0.05)	2.43/3.44	94.01 (0.06)	2.65/3.34	94.39 (0.07)	2.53/3.08
	BOOTEX	94.36 (0.05)	2.10/3.54	94.32 (0.06)	2.39/3.28	94.26 (0.07)	2.55/3.18
	BOOTNP	85.75 (0.15)	6.52/7.73	81.29 (0.12)	9.67/9.03	77.34 (0.12)	11.66/11.00
100	BOOT	94.41 (0.04)	2.56/3.03	94.47 (0.04)	2.61/2.93	94.81 (0.04)	2.42/2.77
	BOOTEX	94.42 (0.04)	2.49/3.08	94.62 (0.04)	2.44/2.93	94.78 (0.05)	2.47/2.75
	BOOTNP	89.56 (0.15)	4.20/6.24	86.17 (0.11)	7.38/6.45	83.51 (0.11)	8.88/7.61
300	BOOT	94.78 (0.02)	2.55/2.67	94.89 (0.02)	2.53/2.58	94.95 (0.03)	2.51/2.53
	BOOTEX	94.82 (0.03)	2.52/2.67	94.86 (0.02)	2.55/2.59	94.95 (0.03)	2.52/2.53
	BOOTNP	93.92 (0.08)	1.81/4.26	90.75 (0.06)	4.52/4.73	89.49 (0.06)	5.28/5.23

A.4. Monte Carlo results for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=4

Table A.10: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=4.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/GAUS	0.371	0.917	1.343	0.254	0.602	0.846	0.140	0.318	0.428
	(0.301)	(0.879)	(1.439)	(0.210)	(0.635)	(0.953)	(0.117)	(0.309)	(0.412)
$\text{GAUS}_{\text{aicc}}$	0.518	1.033	1.384	0.313	0.676	0.905	0.157	0.340	0.447
	(0.402)	(0.863)	(1.389)	(0.269)	(0.674)	(0.971)	(0.139)	(0.330)	(0.432)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.399	0.981	1.417	0.282	0.657	0.910	0.161	0.351	0.461
	(0.332)	(0.843)	(1.316)	(0.225)	(0.632)	(0.949)	(0.133)	(0.376)	(0.518)
GAUS	0.461	1.004	1.433	0.360	0.692	0.936	0.272	0.406	0.510
	(0.330)	(0.839)	(1.311)	(0.217)	(0.624)	(0.941)	(0.119)	(0.356)	(0.498)
$\text{GAUS}_{\text{aicc}}$	0.573	1.123	1.491	0.403	0.744	0.962	0.284	0.427	0.530
	(0.455)	(0.879)	(1.298)	(0.262)	(0.646)	(0.931)	(0.138)	(0.377)	(0.524)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.393	0.934	1.354	0.267	0.591	0.815	0.151	0.332	0.440
	(0.303)	(0.852)	(1.347)	(0.216)	(0.507)	(0.749)	(0.108)	(0.275)	(0.367)
GAUS	0.576	1.019	1.415	0.487	0.688	0.884	0.437	0.482	0.555
	(0.261)	(0.836)	(1.348)	(0.170)	(0.478)	(0.731)	(0.083)	(0.247)	(0.348)
$\text{GAUS}_{\text{aicc}}$	0.668	1.100	1.416	0.527	0.745	0.935	0.444	0.497	0.569
	(0.378)	(0.867)	(1.383)	(0.233)	(0.550)	(0.797)	(0.100)	(0.261)	(0.359)

Table A.11: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=4 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/GAUS	78.08 (0.05)	10.85/11.07	77.06 (0.09)	11.47/11.47	77.10 (0.12)	11.44/11.46
	$GAUS_{aicc}$	76.47 (0.06)	11.82/11.71	76.67 (0.11)	11.70/11.64	76.10 (0.13)	11.97/11.93
100	EST/GAUS	78.87 (0.04)	10.67/10.46	78.44 (0.06)	11.03/10.53	78.55 (0.08)	10.95/10.50
	$GAUS_{aicc}$	78.26 (0.04)	10.93/10.81	78.10 (0.07)	11.26/10.64	78.09 (0.09)	11.23/10.67
300	EST/GAUS	79.63 (0.02)	10.27/10.10	79.58 (0.03)	10.41/10.02	79.60 (0.05)	10.37/10.03
	$GAUS_{aicc}$	79.47 (0.02)	10.38/10.15	79.46 (0.04)	10.45/10.09	79.52 (0.05)	10.39/10.10
	Student-5	h=1		h=6		h=12	
50	EST	77.39 (0.07)	11.30/11.31	76.47 (0.10)	11.60/11.94	76.73 (0.12)	11.45/11.82
	GAUS	81.68 (0.06)	9.17/9.15	78.16 (0.09)	10.77/11.06	77.93 (0.11)	10.85/11.22
	$GAUS_{aicc}$	80.10 (0.08)	10.10/9.805	77.76 (0.11)	10.89/11.34	76.99 (0.13)	11.29/11.73
100	EST	78.50 (0.05)	10.74/10.76	78.00 (0.07)	11.04/10.96	78.08 (0.09)	10.99/10.93
	GAUS	82.74 (0.04)	8.62/8.646	79.64 (0.06)	10.22/10.14	79.23 (0.08)	10.41/10.36
	$GAUS_{aicc}$	82.03 (0.05)	8.92/9.05	79.30 (0.07)	10.32/10.38	78.77 (0.09)	10.62/10.61
300	EST	79.34 (0.03)	10.31/10.35	79.07 (0.04)	10.50/10.43	79.06 (0.05)	10.47/10.46
	GAUS	83.46 (0.03)	8.26/8.28	80.66 (0.04)	9.70/9.63	80.19 (0.05)	9.91/9.90
	$GAUS_{aicc}$	83.25 (0.03)	8.35/8.39	80.46 (0.04)	9.77/9.77	79.98 (0.05)	9.99/10.03
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	77.36 (0.09)	11.67/10.97	77.14 (0.09)	11.28/11.57	77.59 (0.11)	10.98/11.43
	GAUS	82.64 (0.07)	5.69/11.68	78.21 (0.09)	9.59/12.20	78.23 (0.11)	9.81/11.96
	$GAUS_{aicc}$	80.45 (0.09)	7.22/12.33	77.89 (0.11)	9.68/12.43	77.37 (0.12)	10.22/12.41
100	EST	78.55 (0.07)	10.87/10.58	78.66 (0.07)	10.48/10.86	79.05 (0.08)	10.19/10.76
	GAUS	83.95 (0.05)	4.77/11.28	79.68 (0.06)	8.83/11.49	79.65 (0.08)	9.06/11.29
	$GAUS_{aicc}$	82.83 (0.07)	5.59/11.58	79.24 (0.07)	8.99/11.76	79.01 (0.09)	9.36/11.63
300	EST	79.28 (0.04)	10.61/10.12	79.45 (0.04)	10.46/10.08	79.68 (0.05)	10.31/10.01
	GAUS	85.10 (0.03)	4.08/10.81	80.55 (0.04)	8.75/10.70	80.35 (0.05)	9.14/10.51
	$GAUS_{aicc}$	84.85 (0.03)	4.23/10.92	80.41 (0.04)	8.83/10.76	80.22 (0.05)	9.22/10.55
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	EST/GAUS	93.68 (0.03)	3.12/3.20	92.67 (0.06)	3.63/3.70	92.33 (0.08)	3.79/3.87
	$GAUS_{aicc}$	92.69 (0.04)	3.68/3.62	92.12 (0.07)	3.92/3.96	91.40 (0.09)	4.27/4.32
100	EST/GAUS	94.24 (0.02)	2.93/2.83	93.80 (0.04)	3.20/2.99	93.63 (0.05)	3.27/3.10
	$GAUS_{aicc}$	93.88 (0.02)	3.09/3.02	93.54 (0.04)	3.37/3.09	93.31 (0.05)	3.47/3.22
300	EST/GAUS	94.76 (0.01)	2.65/2.59	94.65 (0.02)	2.75/2.60	94.55 (0.03)	2.77/2.68
	$GAUS_{aicc}$	94.68 (0.01)	2.69/2.62	94.58 (0.02)	2.77/2.65	94.49 (0.03)	2.78/2.72
	Student-5	h=1		h=6		h=12	
50	EST	93.72 (0.03)	3.14/3.13	92.69 (0.06)	3.61/3.71	92.47 (0.07)	3.70/3.83
	GAUS	93.43 (0.03)	3.28/3.28	92.49 (0.06)	3.70/3.80	92.31 (0.07)	3.78/3.91
	$GAUS_{aicc}$	92.68 (0.04)	3.73/3.58	92.05 (0.06)	3.88/4.07	91.51 (0.08)	4.16/4.33
100	EST	94.31 (0.02)	2.82/2.87	93.75 (0.03)	3.15/3.10	93.58 (0.05)	3.24/3.19
	GAUS	94.04 (0.02)	2.95/3.00	93.56 (0.03)	3.25/3.19	93.44 (0.05)	3.30/3.26
	$GAUS_{aicc}$	93.70 (0.02)	3.11/3.19	93.28 (0.04)	3.36/3.36	93.05 (0.05)	3.49/3.45
300	EST	94.69 (0.01)	2.64/2.67	94.45 (0.02)	2.78/2.77	94.34 (0.03)	2.83/2.83
	GAUS	94.43 (0.01)	2.77/2.80	94.27 (0.02)	2.87/2.86	94.20 (0.03)	2.90/2.90
	$GAUS_{aicc}$	94.33 (0.01)	2.82/2.85	94.15 (0.02)	2.92/2.93	94.07 (0.03)	2.95/2.98
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	92.05 (0.06)	4.91/3.04	92.59 (0.06)	3.99/3.42	92.63 (0.07)	3.97/3.39
	GAUS	94.36 (0.03)	0.15/5.49	93.17 (0.05)	1.86/4.97	93.00 (0.07)	2.34/4.66
	$GAUS_{aicc}$	93.57 (0.04)	0.57/5.86	92.61 (0.07)	2.23/5.16	92.14 (0.08)	2.79/5.06
100	EST	93.24 (0.05)	3.98/2.78	93.78 (0.04)	3.27/2.95	93.78 (0.05)	3.26/2.96
	GAUS	94.75 (0.02)	0.06/5.18	94.23 (0.03)	1.38/4.39	94.06 (0.04)	1.84/4.10
	$GAUS_{aicc}$	94.42 (0.02)	0.20/5.38	93.88 (0.04)	1.52/4.59	93.61 (0.05)	2.03/4.37
300	EST	94.18 (0.03)	3.25/2.57	94.52 (0.02)	2.90/2.58	94.52 (0.03)	2.88/2.60
	GAUS	95.14 (0.01)	0.00/4.86	95.02 (0.02)	1.08/3.90	94.82 (0.03)	1.51/3.66
	$GAUS_{aicc}$	95.07 (0.01)	0.00/4.93	94.93 (0.02)	1.12/3.95	94.73 (0.03)	1.56/3.70

Table A.12: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with σ_ε^2=4.

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/AGAUS	0.372	0.920	1.354	0.255	0.605	0.854	0.140	0.319	0.429
	(0.301)	(0.878)	(1.456)	(0.210)	(0.635)	(0.955)	(0.117)	(0.309)	(0.413)
$\text{AGAUS}_{\text{aicc}}$	0.517	1.040	1.399	0.313	0.679	0.912	0.157	0.340	0.449
	(0.403)	(0.863)	(1.409)	(0.269)	(0.674)	(0.974)	(0.139)	(0.330)	(0.433)
BAYESN	0.379	0.767	1.057	0.274	0.562	0.750	0.182	0.362	0.463
	(0.280)	(0.638)	(1.103)	(0.196)	(0.502)	(0.763)	(0.114)	(0.271)	(0.348)
BAYESL	0.565	0.843	0.882	0.470	0.780	0.822	0.283	0.489	0.549
	(0.411)	(0.541)	(0.488)	(0.346)	(0.545)	(0.566)	(0.189)	(0.360)	(0.386)
BOOT	0.432	0.918	1.394	0.321	0.671	0.973	0.204	0.388	0.516
	(0.255)	(0.760)	(1.263)	(0.190)	(0.629)	(0.983)	(0.110)	(0.307)	(0.427)
BOOTEX	0.503	0.967	1.362	0.369	0.707	0.987	0.218	0.402	0.518
	(0.314)	(0.723)	(1.193)	(0.230)	(0.621)	(0.931)	(0.125)	(0.317)	(0.442)
BOOTNP	0.972	1.611	1.906	0.819	1.299	1.556	0.590	0.818	0.995
	(0.579)	(0.912)	(1.031)	(0.528)	(0.833)	(0.996)	(0.368)	(0.539)	(0.675)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.400	0.984	1.428	0.281	0.658	0.915	0.161	0.351	0.462
	(0.337)	(0.846)	(1.321)	(0.226)	(0.635)	(0.956)	(0.134)	(0.377)	(0.519)
AGAUS	0.466	1.009	1.446	0.363	0.695	0.943	0.274	0.407	0.512
	(0.336)	(0.842)	(1.315)	(0.217)	(0.626)	(0.946)	(0.120)	(0.356)	(0.499)
$\text{AGAUS}_{\text{aicc}}$	0.574	1.130	1.503	0.405	0.749	0.971	0.285	0.428	0.532
	(0.456)	(0.882)	(1.306)	(0.262)	(0.649)	(0.938)	(0.138)	(0.377)	(0.525)
BAYEST	0.387	0.787	1.077	0.286	0.561	0.732	0.194	0.355	0.446
	(0.297)	(0.605)	(0.993)	(0.176)	(0.437)	(0.614)	(0.106)	(0.276)	(0.369)
BAYESN	0.455	0.836	1.146	0.356	0.606	0.776	0.274	0.391	0.476
	(0.352)	(0.694)	(1.143)	(0.203)	(0.476)	(0.689)	(0.119)	(0.319)	(0.436)
BAYESL	0.603	0.913	0.943	0.511	0.798	0.822	0.361	0.505	0.558
	(0.508)	(0.671)	(0.559)	(0.318)	(0.553)	(0.550)	(0.185)	(0.375)	(0.439)
BOOT	0.478	0.998	1.508	0.355	0.704	1.007	0.230	0.406	0.533
	(0.303)	(0.793)	(1.286)	(0.197)	(0.610)	(0.974)	(0.121)	(0.365)	(0.530)
BOOTEX	0.532	1.041	1.471	0.393	0.750	1.012	0.244	0.426	0.547
	(0.370)	(0.785)	(1.248)	(0.234)	(0.613)	(0.929)	(0.134)	(0.388)	(0.564)
BOOTNP	1.080	1.717	2.029	0.818	1.251	1.541	0.680	0.869	1.070
	(0.725)	(0.954)	(1.061)	(0.506)	(0.734)	(0.935)	(0.428)	(0.642)	(0.888)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.394	0.939	1.367	0.266	0.594	0.822	0.151	0.333	0.443
	(0.303)	(0.852)	(1.349)	(0.216)	(0.508)	(0.754)	(0.108)	(0.276)	(0.369)
AGAUS	0.579	1.025	1.430	0.489	0.692	0.893	0.438	0.484	0.559
	(0.263)	(0.837)	(1.351)	(0.172)	(0.480)	(0.736)	(0.084)	(0.249)	(0.352)
$\text{AGAUS}_{\text{aicc}}$	0.669	1.112	1.439	0.529	0.750	0.945	0.445	0.499	0.573
	(0.379)	(0.884)	(1.450)	(0.233)	(0.552)	(0.803)	(0.100)	(0.263)	(0.363)
BAYESN	0.574	0.855	1.135	0.483	0.618	0.757	0.439	0.468	0.520
	(0.254)	(0.663)	(1.152)	(0.165)	(0.358)	(0.523)	(0.078)	(0.214)	(0.288)
BAYESL	0.706	0.918	0.929	0.621	0.815	0.848	0.496	0.576	0.604
	(0.363)	(0.597)	(0.520)	(0.280)	(0.532)	(0.531)	(0.152)	(0.310)	(0.371)
BOOT	0.450	0.956	1.463	0.321	0.649	0.933	0.201	0.378	0.499
	(0.272)	(0.800)	(1.305)	(0.204)	(0.514)	(0.803)	(0.098)	(0.271)	(0.378)
BOOTEX	0.527	1.002	1.411	0.367	0.690	0.947	0.218	0.405	0.525
	(0.344)	(0.817)	(1.341)	(0.247)	(0.545)	(0.825)	(0.123)	(0.292)	(0.397)
BOOTNP	1.002	1.585	1.907	0.797	1.234	1.524	0.663	0.841	1.036
	(0.606)	(0.887)	(1.025)	(0.497)	(0.746)	(0.899)	(0.439)	(0.591)	(0.747)

Table A.13: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=4 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/AGAUS	78.68 (0.05)	10.56/10.76	77.64 (0.10)	11.17/11.19	77.73 (0.12)	11.14/11.14
	$\text{AGAUS}_{\text{aicc}}$	77.03 (0.06)	11.54/11.43	77.22 (0.11)	11.43/11.35	76.58 (0.14)	11.75/11.67
	SRA	80.03 (0.06)	8.88/11.09	71.16 (0.13)	13.42/15.41	64.83 (0.15)	16.74/18.43
	SRAaic	79.36 (0.07)	9.22/11.42	71.47 (0.12)	13.40/15.14	64.86 (0.15)	16.66/18.48
100	AEST/AGAUS	79.25 (0.04)	10.48/10.27	79.05 (0.06)	10.73/10.23	79.25 (0.09)	10.60/10.16
	$\text{AGAUS}_{\text{aicc}}$	78.69 (0.04)	10.72/10.59	78.72 (0.07)	10.95/10.33	78.79 (0.09)	10.89/10.33
	SRA	79.89 (0.04)	9.65/10.45	76.46 (0.08)	11.32/12.22	73.70 (0.12)	12.83/13.47
	SRA_{aicc}	79.54 (0.05)	9.69/10.76	75.90 (0.09)	11.46/12.64	73.05 (0.12)	13.03/13.92
300	AEST/AGAUS	79.77 (0.02)	10.20/10.03	79.85 (0.03)	10.27/9.88	79.90 (0.05)	10.22/9.882
	$\text{AGAUS}_{\text{aicc}}$	79.64 (0.02)	10.30/10.06	79.74 (0.04)	10.31/9.95	79.83 (0.05)	10.22/9.94
	SRA	79.97 (0.03)	9.86/10.17	79.08 (0.04)	10.32/10.60	78.69 (0.06)	10.41/10.9
	SRA_{aicc}	79.85 (0.03)	9.95/10.20	78.81 (0.05)	10.45/10.74	78.50 (0.06)	10.47/11.02
	Student-5	h=1		h=6		h=12	
50	AEST	77.96 (0.07)	11.02/11.02	77.14 (0.10)	11.27/11.58	77.47 (0.12)	11.07/11.46
	AGAUS	82.22 (0.06)	8.90/8.88	78.83 (0.09)	10.44/10.73	78.66 (0.12)	10.48/10.86
	$\text{AGAUS}_{\text{aicc}}$	80.62 (0.07)	9.83/9.55	78.34 (0.11)	10.61/11.05	77.53 (0.13)	11/11.46
	SRA	80.09 (0.06)	8.68/11.23	71.54 (0.13)	12.62/15.84	64.04 (0.17)	15.94/20.02
	SRA_{aicc}	79.33 (0.08)	9.10/11.57	71.89 (0.13)	12.50/15.61	64.26 (0.17)	15.90/19.84
100	AEST	78.82 (0.05)	10.58/10.60	78.55 (0.07)	10.76/10.68	78.76 (0.09)	10.65/10.59
	AGAUS	83.04 (0.04)	8.47/8.49	80.20 (0.06)	9.95/9.85	79.88 (0.08)	10.09/10.03
	$\text{AGAUS}_{\text{aicc}}$	82.41 (0.04)	8.74/8.86	79.89 (0.07)	10.03/10.07	79.43 (0.09)	10.29/10.28
	SRA	79.99 (0.04)	9.44/10.57	76.64 (0.09)	11.00/12.36	74.05 (0.12)	11.99/13.96
	SRA_{aicc}	79.67 (0.05)	9.54/10.79	76.42 (0.09)	10.90/12.67	73.60 (0.12)	12.11/14.29
300	AEST	79.47 (0.03)	10.25/10.28	79.31 (0.04)	10.38/10.31	79.36 (0.05)	10.32/10.32
	AGAUS	83.57 (0.03)	8.21/8.22	80.91 (0.04)	9.58/9.51	80.48 (0.05)	9.77/9.75
	$\text{AGAUS}_{\text{aicc}}$	83.39 (0.03)	8.28/8.32	80.73 (0.04)	9.64/9.63	80.30 (0.05)	9.83/9.87
	SRA	79.84 (0.03)	9.89/10.26	78.74 (0.04)	10.41/10.85	78.27 (0.06)	10.68/11.05
	SRA_{aicc}	79.76 (0.03)	9.92/10.32	78.52 (0.05)	10.49/10.99	78.05 (0.06)	10.75/11.20
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	78.13 (0.08)	11.16/10.72	77.87 (0.09)	10.87/11.26	78.35 (0.12)	10.55/11.10
	AGAUS	83.31 (0.07)	5.28/11.42	78.90 (0.09)	9.20/11.89	78.98 (0.11)	9.39/11.63
	$\text{AGAUS}_{\text{aicc}}$	81.04 (0.09)	6.86/12.10	78.57 (0.11)	9.30/12.13	78.02 (0.13)	9.86/12.11
	SRA	79.97 (0.07)	9.40/10.63	71.45 (0.13)	13.94/14.61	64.55 (0.15)	17.02/18.43
	SRA_{aicc}	79.32 (0.09)	9.79/10.89	71.53 (0.13)	13.93/14.54	64.88 (0.16)	16.80/18.32
100	AEST	78.99 (0.07)	10.56/10.45	79.25 (0.07)	10.14/10.6	79.73 (0.08)	9.82/10.45
	AGAUS	84.33 (0.05)	4.52/11.15	80.25 (0.06)	8.51/11.23	80.32 (0.08)	8.70/10.98
	$\text{AGAUS}_{\text{aicc}}$	83.28 (0.06)	5.30/11.42	79.86 (0.07)	8.65/11.49	79.68 (0.09)	8.99/11.32
	SRA	79.88 (0.05)	9.70/10.42	76.41 (0.08)	11.38/12.21	73.81 (0.11)	12.69/13.50
	SRA_{aicc}	79.39 (0.06)	9.99/10.62	75.92 (0.09)	11.74/12.34	73.34 (0.12)	13.20/13.46
300	AEST	79.45 (0.04)	10.48/10.07	79.74 (0.04)	10.29/9.97	79.99 (0.05)	10.13/9.88
	AGAUS	85.25 (0.03)	3.98/10.76	80.82 (0.04)	8.59/10.59	80.67 (0.05)	8.95/10.38
	$\text{AGAUS}_{\text{aicc}}$	85.04 (0.03)	4.11/10.85	80.70 (0.04)	8.66/10.64	80.55 (0.05)	9.03/10.42
	SRA	79.75 (0.03)	10.18/10.07	78.87 (0.05)	10.55/10.58	78.48 (0.06)	10.78/10.74
	SRA_{aicc}	79.65 (0.04)	10.23/10.12	78.62 (0.05)	10.70/10.68	78.26 (0.06)	10.84/10.90

Table A.14: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=4 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	AEST/AGAUS	94.01 (0.03)	2.96/3.03	92.93 (0.07)	3.47/3.60	92.51 (0.09)	3.69/3.79
	$\text{AGAUS}_{\text{aicc}}$	93.04 (0.04)	3.51/3.45	92.31 (0.08)	3.81/3.88	91.43 (0.1)	4.28/4.29
	SRA	92.59 (0.04)	2.65/4.76	84.88 (0.10)	6.29/8.83	79.42 (0.13)	10.80/9.79
	SRA_{aicc}	92.17 (0.05)	2.85/4.98	85.18 (0.10)	6.27/8.56	79.41 (0.13)	10.67/9.92
100	AEST/AGAUS	94.44 (0.02)	2.83/2.73	94.14 (0.04)	3.04/2.83	93.98 (0.05)	3.10/2.92
	$\text{AGAUS}_{\text{aicc}}$	94.12 (0.02)	2.98/2.91	93.89 (0.04)	3.19/2.92	93.67 (0.05)	3.29/3.03
	SRA	94.22 (0.03)	2.38/3.39	91.24 (0.06)	3.95/4.80	87.06 (0.08)	5.87/7.07
	SRA_{aicc}	93.96 (0.03)	2.44/3.59	90.88 (0.06)	4.03/5.09	86.53 (0.09)	6.13/7.34
300	AEST/AGAUS	94.83 (0.01)	2.61/2.56	94.80 (0.02)	2.68/2.53	94.71 (0.02)	2.68/2.61
	$\text{AGAUS}_{\text{aicc}}$	94.77 (0.01)	2.65/2.58	94.73 (0.02)	2.69/2.57	94.66 (0.03)	2.69/2.64
	SRA	94.79 (0.02)	2.47/2.74	94.15 (0.03)	2.71/3.13	93.18 (0.04)	3.16/3.66
	SRA_{aicc}	94.75 (0.02)	2.49/2.75	94.01 (0.03)	2.77/3.22	93.06 (0.04)	3.19/3.75
	Student-5	h=1		h=6		h=12	
50	AEST	94.02 (0.03)	2.99/2.99	93.05 (0.05)	3.43/3.52	92.77 (0.07)	3.54/3.69
	AGAUS	93.73 (0.03)	3.13/3.141	92.86 (0.06)	3.53/3.62	92.62 (0.07)	3.61/3.77
	$\text{AGAUS}_{\text{aicc}}$	93.00 (0.04)	3.57/3.44	92.35 (0.07)	3.73/3.92	91.65 (0.09)	4.06/4.29
	SRA	93.13 (0.04)	2.28/4.59	85.18 (0.10)	6.00/8.82	78.80 (0.14)	10.25/10.95
	SRA_{aicc}	92.85 (0.05)	2.39/4.76	85.40 (0.10)	5.85/8.75	79.02 (0.14)	10.14/10.85
100	AEST	94.45 (0.02)	2.75/2.79	94.03 (0.03)	3.01/2.95	93.91 (0.04)	3.07/3.02
	AGAUS	94.18 (0.02)	2.89/2.93	93.86 (0.03)	3.10/3.04	93.78 (0.05)	3.14/3.09
	$\text{AGAUS}_{\text{aicc}}$	93.89 (0.02)	3.02/3.09	93.61 (0.04)	3.21/3.18	93.40 (0.05)	3.33/3.27
	SRA	94.70 (0.03)	2.06/3.24	91.29 (0.05)	3.76/4.95	87.01 (0.08)	5.61/7.38
	SRA_{aicc}	94.56 (0.03)	2.09/3.36	91.16 (0.06)	3.68/5.16	86.75 (0.09)	5.72/7.53
300	AEST	94.74 (0.01)	2.62/2.64	94.58 (0.02)	2.72/2.71	94.49 (0.03)	2.76/2.75
	AGAUS	94.48 (0.01)	2.75/2.77	94.40 (0.02)	2.80/2.79	94.35 (0.03)	2.83/2.82
	$\text{AGAUS}_{\text{aicc}}$	94.40 (0.01)	2.78/2.81	94.29 (0.02)	2.85/2.86	94.24 (0.03)	2.87/2.88
	SRA	94.87 (0.02)	2.39/2.74	94.20 (0.03)	2.74/3.06	93.10 (0.04)	3.23/3.67
	SRA_{aicc}	94.84 (0.02)	2.40/2.76	94.05 (0.03)	2.79/3.16	93.00 (0.04)	3.25/3.75
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	92.57 (0.06)	4.51/2.91	92.98 (0.06)	3.76/3.26	92.95 (0.08)	3.80/3.25
	AGAUS	94.59 (0.03)	0.10/5.31	93.50 (0.05)	1.74/4.763	93.27 (0.07)	2.25/4.48
	$\text{AGAUS}_{\text{aicc}}$	93.80 (0.04)	0.52/5.68	92.90 (0.07)	2.10/4.99	92.37 (0.09)	2.69/4.94
	SRA	92.63 (0.05)	3.06/4.31	85.33 (0.09)	6.59/8.08	79.38 (0.13)	11.33/9.29
	SRA_{aicc}	92.17 (0.06)	3.36/4.48	85.51 (0.10)	6.52/7.97	79.66 (0.13)	11.07/9.27
100	AEST	93.53 (0.05)	3.75/2.72	94.09 (0.04)	3.09/2.82	94.12 (0.05)	3.08/2.80
	AGAUS	94.88 (0.02)	0.04/5.08	94.51 (0.03)	1.27/4.22	94.37 (0.04)	1.72/3.92
	$\text{AGAUS}_{\text{aicc}}$	94.58 (0.02)	0.16/5.25	94.19 (0.04)	1.39/4.41	93.93 (0.05)	1.88/4.18
	SRA	94.32 (0.04)	2.57/3.11	91.25 (0.05)	3.87/4.89	86.97 (0.08)	5.98/7.05
	SRA_{aicc}	94.10 (0.05)	2.71/3.19	90.93 (0.06)	4.15/4.92	86.62 (0.09)	6.33/7.05
300	AEST	94.30 (0.03)	3.16/2.55	94.68 (0.02)	2.79/2.52	94.67 (0.03)	2.79/2.54
	AGAUS	95.17 (0.01)	0.00/4.83	95.14 (0.02)	1.02/3.84	94.97 (0.03)	1.44/3.58
	$\text{AGAUS}_{\text{aicc}}$	95.12 (0.01)	0.00/4.88	95.06 (0.02)	1.06/3.87	94.90 (0.03)	1.49/3.61
	SRA	94.58 (0.02)	2.76/2.65	94.07 (0.03)	2.97/2.95	93.11 (0.04)	3.42/3.47
	SRA_{aicc}	94.55 (0.02)	2.77/2.68	93.87 (0.03)	3.09/3.04	92.98 (0.04)	3.45/3.57

Table A.15: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=4 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BAYESN	79.48 (0.05)	10.20/10.32	78.05 (0.08)	11.03/10.92	77.23 (0.10)	11.46/11.32
	BAYESL	79.58 (0.06)	10.41/10.01	76.26 (0.09)	12.06/11.69	74.58 (0.11)	12.86/12.57
100	BAYESN	79.68 (0.04)	10.27/10.05	79.07 (0.06)	10.68/10.26	78.71 (0.08)	10.84/10.44
	BAYESL	79.65 (0.05)	10.25/10.10	77.60 (0.07)	11.42/10.98	76.34 (0.09)	12.06/11.60
300	BAYESN	79.81 (0.03)	10.13/10.06	79.51 (0.03)	10.37/10.12	79.26 (0.05)	10.54/10.20
	BAYESL	79.91 (0.03)	10.05/10.04	79.35 (0.04)	10.29/10.35	78.69 (0.05)	10.69/10.63
	Student-5	h=1		h=6		h=12	
50	BAYEST	80.27 (0.06)	9.84/9.89	78.62 (0.09)	10.58/10.8	77.99 (0.11)	10.88/11.13
	BAYESN	82.93 (0.06)	8.54/8.54	79.18 (0.09)	10.29/10.53	78.19 (0.11)	10.80/11.01
	BAYESL	83.06 (0.06)	8.55/8.39	77.72 (0.10)	11.06/11.22	75.77 (0.11)	12.06/12.17
100	BAYEST	80.26 (0.04)	9.82/9.92	79.10 (0.06)	10.50/10.40	78.64 (0.08)	10.68/10.67
	BAYESN	83.42 (0.04)	8.25/8.32	80.20 (0.06)	9.94/9.85	79.42 (0.08)	10.33/10.26
	BAYESL	83.31 (0.05)	8.33/8.35	78.86 (0.08)	10.45/10.70	77.36 (0.09)	11.30/11.35
300	BAYEST	80.01 (0.03)	9.96/10.03	79.46 (0.04)	10.30/10.24	79.24 (0.05)	10.37/10.39
	BAYESN	83.63 (0.03)	8.18/8.19	80.65 (0.04)	9.73/9.61	79.86 (0.05)	10.08/10.05
	BAYESL	83.64 (0.03)	8.18/8.18	80.41 (0.04)	9.74/9.85	79.38 (0.05)	10.30/10.31
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	83.80 (0.07)	5.14/11.06	78.93 (0.09)	9.37/11.69	78.24 (0.1)	9.94/11.82
	BAYESL	83.24 (0.08)	5.89/10.87	77.23 (0.10)	10.54/12.22	75.39 (0.11)	11.76/12.85
100	BAYESN	84.70 (0.05)	4.41/10.89	80.09 (0.06)	8.75/11.16	79.49 (0.08)	9.29/11.22
	BAYESL	83.92 (0.06)	5.33/10.75	78.43 (0.08)	9.91/11.67	77.13 (0.09)	10.89/11.99
300	BAYESN	85.26 (0.04)	4.05/10.69	80.50 (0.04)	8.78/10.72	79.99 (0.05)	9.35/10.66
	BAYESL	84.89 (0.04)	4.44/10.66	80.17 (0.05)	9.08/10.75	79.37 (0.06)	9.67/10.96
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BAYESN	94.64 (0.03)	2.65/2.71	93.95 (0.05)	3.02/3.03	93.65 (0.06)	3.18/3.17
	BAYESL	94.69 (0.03)	2.75/2.56	92.75 (0.06)	3.69/3.56	91.61 (0.07)	4.25/4.14
100	BAYESN	94.83 (0.02)	2.64/2.53	94.45 (0.03)	2.87/2.68	94.31 (0.04)	2.90/2.79
	BAYESL	94.83 (0.02)	2.61/2.57	93.56 (0.04)	3.32/3.12	92.82 (0.05)	3.69/3.48
300	BAYESN	94.83 (0.01)	2.60/2.57	94.69 (0.02)	2.72/2.59	94.57 (0.03)	2.75/2.68
	BAYESL	94.91 (0.02)	2.55/2.54	94.60 (0.02)	2.71/2.69	94.19 (0.03)	2.92/2.89
	Student-5	h=1		h=6		h=12	
50	BAYEST	94.61 (0.03)	2.69/2.69	94.28 (0.05)	2.82/2.89	94.16 (0.06)	2.89/2.95
	BAYESN	94.25 (0.03)	2.85/2.89	93.61 (0.05)	3.15/3.24	93.40 (0.06)	3.29/3.31
	BAYESL	94.45 (0.03)	2.79/2.77	92.63 (0.06)	3.66/3.71	91.60 (0.07)	4.15/4.25
100	BAYEST	94.69 (0.02)	2.64/2.68	94.48 (0.03)	2.79/2.73	94.40 (0.04)	2.82/2.78
	BAYESN	94.45 (0.02)	2.76/2.79	94.09 (0.03)	2.98/2.93	94.02 (0.04)	2.99/2.99
	BAYESL	94.52 (0.02)	2.71/2.77	93.38 (0.04)	3.29/3.33	92.75 (0.05)	3.62/3.64
300	BAYEST	94.75 (0.01)	2.59/2.65	94.61 (0.02)	2.69/2.69	94.59 (0.02)	2.69/2.71
	BAYESN	94.51 (0.01)	2.74/2.76	94.27 (0.02)	2.88/2.84	94.22 (0.03)	2.91/2.88
	BAYESL	94.57 (0.01)	2.70/2.73	94.28 (0.02)	2.85/2.87	93.94 (0.03)	3.03/3.04
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	94.99 (0.02)	0.10/4.91	94.21 (0.05)	1.50/4.29	94.00 (0.06)	1.87/4.12
	BAYESL	95.10 (0.03)	0.24/4.67	93.09 (0.05)	2.22/4.68	92.09 (0.07)	3.02/4.89
100	BAYESN	95.11 (0.02)	0.03/4.87	94.68 (0.03)	1.24/4.08	94.56 (0.04)	1.61/3.83
	BAYESL	95.20 (0.02)	0.11/4.69	93.86 (0.04)	1.79/4.34	93.16 (0.05)	2.46/4.39
300	BAYESN	95.21 (0.01)	0.00/4.79	95.05 (0.02)	1.07/3.88	94.82 (0.03)	1.52/3.66
	BAYESL	95.28 (0.01)	0.01/4.72	94.83 (0.02)	1.23/3.93	94.44 (0.03)	1.71/3.84

Table A.16: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ_ε^2=4 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BOOT	79.09 (0.06)	10.41/10.51	79.47 (0.08)	10.29/10.23	80.50 (0.10)	9.77/9.72
	BOOTEX	78.61 (0.06)	10.78/10.61	79.90 (0.10)	10.12/9.97	80.21 (0.11)	9.95/9.83
	BOOTNP	72.59 (0.16)	13.51/13.90	64.46 (0.14)	17.84/17.70	60.13 (0.14)	19.90/19.97
100	BOOT	79.31 (0.04)	10.44/10.25	79.78 (0.06)	10.32/9.89	80.43 (0.08)	9.97/9.60
	BOOTEX	79.28 (0.05)	10.42/10.30	80.09 (0.07)	10.15/9.76	80.57 (0.08)	9.88/9.55
	BOOTNP	75.52 (0.15)	12.52/11.97	68.89 (0.13)	15.78/15.33	66.01 (0.13)	17.15/16.85
300	BOOT	79.75 (0.03)	10.16/10.09	79.96 (0.04)	10.21/9.83	80.22 (0.05)	10.02/9.76
	BOOTEX	79.66 (0.03)	10.25/10.09	79.97 (0.04)	10.22/9.81	80.25 (0.05)	10.05/9.70
	BOOTNP	80.17 (0.09)	9.88/9.95	74.27 (0.08)	12.88/12.85	72.27 (0.08)	14.03/13.7
	Student-5	h=1		h=6		h=12	
50	BOOT	79.45 (0.06)	10.23/10.32	79.63 (0.09)	10.09/10.28	80.39 (0.10)	9.67/9.94
	BOOTEX	79.45 (0.06)	10.37/10.18	80.20 (0.10)	9.72/10.08	80.44 (0.12)	9.55/10.01
	BOOTNP	71.05 (0.19)	13.87/15.08	63.10 (0.15)	17.51/19.39	59.09 (0.15)	19.61/21.30
100	BOOT	79.71 (0.04)	10.20/10.09	79.81 (0.06)	10.14/10.05	80.39 (0.08)	9.90/9.70
	BOOTEX	79.74 (0.04)	10.16/10.11	80.06 (0.07)	9.97/9.97	80.34 (0.08)	9.92/9.75
	BOOTNP	76.24 (0.15)	11.95/11.81	68.91 (0.13)	15.48/15.60	65.66 (0.13)	16.98/17.35
300	BOOT	79.62 (0.03)	10.19/10.19	79.63 (0.04)	10.17/10.20	79.89 (0.05)	10.08/10.02
	BOOTEX	79.70 (0.03)	10.15/10.15	79.67 (0.04)	10.13/10.21	79.79 (0.05)	10.03/10.18
	BOOTNP	79.51 (0.12)	10.34/10.15	73.10 (0.10)	13.63/13.27	70.77 (0.10)	14.71/14.52
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	79.67 (0.07)	9.81/10.51	79.91 (0.08)	9.52/10.57	81.04 (0.10)	8.92/10.04
	BOOTEX	79.47 (0.09)	9.76/10.77	80.54 (0.10)	9.14/10.32	80.99 (0.12)	8.93/10.08
	BOOTNP	72.32 (0.19)	12.75/14.93	64.19 (0.15)	18.52/17.29	59.93 (0.14)	20.77/19.30
100	BOOT	79.71 (0.06)	9.97/10.32	80.17 (0.06)	9.45/10.38	80.99 (0.08)	8.93/10.08
	BOOTEX	79.47 (0.07)	9.99/10.53	80.31 (0.07)	9.33/10.36	80.83 (0.09)	9.04/10.13
	BOOTNP	77.64 (0.15)	9.76/12.60	68.90 (0.12)	15.44/15.67	66.17 (0.12)	16.50/17.34
300	BOOT	79.56 (0.04)	10.35/10.09	79.94 (0.04)	10.14/9.92	80.40 (0.05)	9.85/9.75
	BOOTEX	79.72 (0.04)	10.22/10.06	79.96 (0.04)	10.05/9.98	80.31 (0.05)	9.89/9.80
	BOOTNP	81.27 (0.13)	7.21/11.52	73.85 (0.09)	12.48/13.67	72.05 (0.09)	13.33/14.62

Table A.17: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ and σ^2_ε=4 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BOOT	93.55 (0.03)	3.21/3.24	94.23 (0.05)	2.89/2.89	94.65 (0.06)	2.67/2.67
	BOOTEX	93.27 (0.04)	3.38/3.35	94.31 (0.05)	2.88/2.82	94.38 (0.06)	2.79/2.83
	BOOTNP	85.77 (0.14)	6.75/7.48	81.29 (0.12)	9.28/9.43	77.56 (0.12)	11.24/11.20
100	BOOT	94.08 (0.03)	3.00/2.92	94.54 (0.03)	2.82/2.64	94.83 (0.04)	2.64/2.53
	BOOTEX	94.03 (0.03)	3.01/2.96	94.71 (0.04)	2.73/2.55	94.93 (0.04)	2.60/2.46
	BOOTNP	89.65 (0.12)	5.30/5.04	86.46 (0.10)	6.94/6.60	84.05 (0.10)	8.08/7.86
300	BOOT	94.68 (0.02)	2.71/2.61	94.85 (0.02)	2.66/2.49	94.95 (0.02)	2.55/2.49
	BOOTEX	94.66 (0.02)	2.71/2.63	94.85 (0.02)	2.68/2.47	94.98 (0.02)	2.55/2.47
	BOOTNP	93.76 (0.07)	3.05/3.19	91.20 (0.05)	4.35/4.45	89.81 (0.05)	5.11/5.08
	Student-5	h=1		h=6		h=12	
50	BOOT	93.63 (0.03)	3.17/3.20	93.77 (0.05)	3.13/3.10	94.32 (0.05)	2.85/2.83
	BOOTEX	93.46 (0.04)	3.33/3.21	94.06 (0.05)	2.98/2.95	94.26 (0.06)	2.84/2.89
	BOOTNP	83.68 (0.17)	7.75/8.57	80.21 (0.13)	9.58/10.21	76.30 (0.14)	11.52/12.18
100	BOOT	94.26 (0.02)	2.85/2.88	94.32 (0.03)	2.85/2.83	94.70 (0.04)	2.69/2.61
	BOOTEX	94.22 (0.02)	2.87/2.91	94.33 (0.04)	2.84/2.83	94.57 (0.04)	2.73/2.69
	BOOTNP	88.98 (0.12)	5.49/5.53	86.72 (0.10)	6.55/6.73	83.93 (0.10)	7.87/8.20
300	BOOT	94.61 (0.02)	2.68/2.71	94.61 (0.02)	2.71/2.68	94.71 (0.03)	2.65/2.64
	BOOTEX	94.63 (0.02)	2.68/2.69	94.58 (0.02)	2.71/2.72	94.65 (0.03)	2.66/2.69
	BOOTNP	92.33 (0.10)	3.78/3.88	90.24 (0.08)	4.97/4.79	88.91 (0.08)	5.62/5.47
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	94.21 (0.04)	2.37/3.43	94.39 (0.05)	2.42/3.18	95.03 (0.05)	2.15/2.82
	BOOTEX	94.11 (0.05)	2.28/3.61	94.55 (0.06)	2.32/3.13	94.70 (0.07)	2.37/2.93
	BOOTNP	85.44 (0.17)	6.62/7.94	80.97 (0.12)	10.26/8.77	76.96 (0.13)	12.54/10.50
100	BOOT	94.58 (0.03)	2.40/3.02	94.71 (0.03)	2.40/2.89	95.13 (0.04)	2.23/2.64
	BOOTEX	94.67 (0.04)	2.20/3.12	94.73 (0.04)	2.36/2.91	95.06 (0.05)	2.21/2.73
	BOOTNP	90.15 (0.14)	3.84/6.01	86.39 (0.10)	6.94/6.67	83.96 (0.10)	8.19/7.85
300	BOOT	94.60 (0.02)	2.69/2.71	94.81 (0.02)	2.59/2.59	95.000 (0.03)	2.48/2.51
	BOOTEX	94.68 (0.02)	2.55/2.77	94.82 (0.02)	2.57/2.61	94.96 (0.03)	2.51/2.53
	BOOTNP	93.54 (0.10)	1.87/4.59	90.74 (0.07)	4.48/4.78	89.42 (0.07)	5.32/5.25

A.5. Monte Carlo results for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ with σ^2_ε=1.

Table A.18: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ with σ^2_ε=1

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/GAUS	0.181	0.885	1.610	0.134	0.712	1.295	0.078	0.413	0.752
	(0.146)	(0.790)	(1.580)	(0.095)	(0.566)	(1.062)	(0.067)	(0.432)	(0.814)
$\text{GAUS}_{\text{aicc}}$	0.402	1.149	1.938	0.202	0.803	1.385	0.082	0.409	0.737
	(0.303)	(0.907)	(1.900)	(0.209)	(0.628)	(1.126)	(0.078)	(0.422)	(0.791)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.204	0.983	1.786	0.151	0.739	1.326	0.094	0.467	0.826
	(0.162)	(0.864)	(1.787)	(0.134)	(0.583)	(1.115)	(0.099)	(0.483)	(0.905)
GAUS	0.236	1.002	1.801	0.191	0.763	1.349	0.149	0.515	0.874
	(0.159)	(0.857)	(1.777)	(0.129)	(0.574)	(1.101)	(0.092)	(0.465)	(0.880)
$\text{GAUS}_{\text{aicc}}$	0.435	1.259	2.150	0.245	0.841	1.421	0.149	0.506	0.851
	(0.338)	(1.041)	(2.227)	(0.241)	(0.614)	(1.141)	(0.094)	(0.453)	(0.851)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.193	0.956	1.751	0.139	0.702	1.274	0.084	0.432	0.769
	(0.150)	(0.879)	(1.839)	(0.100)	(0.576)	(1.121)	(0.068)	(0.403)	(0.764)
GAUS	0.286	1.022	1.801	0.247	0.784	1.341	0.221	0.550	0.864
	(0.130)	(0.858)	(1.820)	(0.082)	(0.544)	(1.090)	(0.050)	(0.370)	(0.738)
$\text{GAUS}_{\text{aicc}}$	0.456	1.269	2.141	0.301	0.858	1.419	0.221	0.542	0.845
	(0.288)	(1.010)	(2.285)	(0.182)	(0.585)	(1.132)	(0.050)	(0.359)	(0.710)

Table A.19: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ and σ^2_ε=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/GAUS	78.66 (0.05)	10.71/10.64	76.27 (0.09)	11.86/11.87	74.80 (0.13)	12.58/12.62
	$GAUS_{aicc}$	76.68 (0.08)	11.83/11.48	66.95 (0.11)	16.73/16.32	65.36 (0.14)	17.51/17.12
100	EST/GAUS	79.23 (0.04)	10.44/10.34	77.82 (0.06)	11.16/11.02	76.80 (0.08)	11.74/11.46
	$GAUS_{aicc}$	78.11 (0.05)	10.99/10.90	75.69 (0.09)	12.22/12.09	74.63 (0.11)	12.81/12.56
300	EST/GAUS	79.68 (0.02)	10.20/10.12	79.04 (0.03)	10.51/10.45	78.59 (0.05)	10.72/10.68
	$GAUS_{aicc}$	79.47 (0.02)	10.31/10.22	79.05 (0.03)	10.49/10.46	78.57 (0.05)	10.72/10.71
	Student-5	h=1		h=6		h=12	
50	EST	77.94 (0.07)	11.25/10.81	76.13 (0.10)	12.09/11.78	74.89 (0.13)	12.64/12.47
	GAUS	82.19 (0.06)	9.11/8.71	77.55 (0.10)	11.38/11.07	75.74 (0.13)	12.21/12.04
	$GAUS_{aicc}$	79.75 (0.10)	10.80/9.46	67.76 (0.12)	16.18/16.06	65.51 (0.14)	17.15/17.35
100	EST	78.73 (0.05)	10.49/10.78	77.84 (0.07)	10.86/11.29	77.06 (0.09)	11.17/11.77
	GAUS	82.93 (0.05)	8.43/8.65	79.25 (0.07)	10.18/10.57	77.92 (0.09)	10.75/11.33
	$GAUS_{aicc}$	81.77 (0.07)	9.13/9.09	77.08 (0.09)	11.32/11.6	75.62 (0.11)	11.92/12.46
300	EST	79.54 (0.03)	10.21/10.24	79.22 (0.04)	10.49/10.29	78.86 (0.05)	10.68/10.47
	GAUS	83.72 (0.03)	8.12/8.162	80.64 (0.04)	9.77/9.59	79.67 (0.05)	10.28/10.05
	$GAUS_{aicc}$	83.58 (0.03)	8.19/8.24	80.57 (0.04)	9.79/9.64	79.58 (0.05)	10.30/10.12
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	77.84 (0.08)	11.35/10.81	76.20 (0.10)	11.91/11.89	74.87 (0.13)	12.41/12.72
	GAUS	83.17 (0.07)	5.29/11.53	77.07 (0.10)	10.40/12.53	75.30 (0.13)	11.46/13.24
	$GAUS_{aicc}$	80.04 (0.10)	7.77/12.19	67.29 (0.12)	16.03/16.67	65.39 (0.15)	16.96/17.66
100	EST	79.01 (0.06)	10.56/10.44	78.12 (0.06)	10.84/11.04	77.22 (0.09)	11.18/11.6
	GAUS	84.52 (0.05)	4.34/11.14	78.94 (0.06)	9.38/11.69	77.60 (0.09)	10.27/12.13
	$GAUS_{aicc}$	82.99 (0.06)	5.43/11.58	76.61 (0.09)	10.72/12.67	75.19 (0.11)	11.63/13.19
300	EST	79.76 (0.04)	10.07/10.17	79.35 (0.04)	10.26/10.4	79.07 (0.05)	10.39/10.55
	GAUS	85.42 (0.03)	3.71/10.87	80.18 (0.04)	8.78/11.03	79.46 (0.05)	9.47/11.07
	$GAUS_{aicc}$	85.26 (0.03)	3.81/10.93	80.12 (0.04)	8.83/11.05	79.36 (0.05)	9.53/11.11
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	EST/GAUS	94.03 (0.03)	2.99/2.97	92.24 (0.06)	3.89/3.87	90.92 (0.09)	4.56/4.52
	$GAUS_{aicc}$	92.87 (0.05)	3.63/3.49	85.46 (0.09)	7.43/7.11	83.78 (0.12)	8.29/7.93
100	EST/GAUS	94.47 (0.02)	2.78/2.75	93.51 (0.04)	3.28/3.20	92.65 (0.06)	3.71/3.63
	$GAUS_{aicc}$	93.84 (0.03)	3.12/3.03	91.87 (0.06)	4.12/4.01	90.92 (0.08)	4.60/4.48
300	EST/GAUS	94.81 (0.01)	2.60/2.59	94.44 (0.02)	2.79/2.76	94.09 (0.03)	2.97/2.95
	$GAUS_{aicc}$	94.70 (0.01)	2.66/2.64	94.44 (0.02)	2.79/2.77	94.06 (0.03)	2.98/2.96
	Student-5	h=1		h=6		h=12	
50	EST	93.99 (0.03)	3.09/2.92	92.38 (0.06)	3.96/3.66	91.29 (0.09)	4.46/4.25
	GAUS	93.71 (0.03)	3.24/3.05	92.16 (0.06)	4.07/3.77	91.13 (0.09)	4.54/4.33
	$GAUS_{aicc}$	92.86 (0.06)	3.89/3.25	85.35 (0.10)	7.38/7.27	83.59 (0.13)	8.12/8.29
100	EST	94.39 (0.03)	2.75/2.85	93.61 (0.04)	3.16/3.23	92.97 (0.06)	3.42/3.61
	GAUS	94.12 (0.03)	2.89/2.98	93.41 (0.04)	3.26/3.33	92.82 (0.06)	3.49/3.68
	$GAUS_{aicc}$	93.58 (0.05)	3.22/3.21	91.82 (0.06)	4.09/4.09	91.01 (0.08)	4.43/4.56
300	EST	94.84 (0.01)	2.58/2.58	94.55 (0.02)	2.77/2.68	94.30 (0.03)	2.89/2.81
	GAUS	94.58 (0.01)	2.71/2.70	94.37 (0.02)	2.86/2.77	94.17 (0.03)	2.96/2.87
	$GAUS_{aicc}$	94.52 (0.01)	2.74/2.74	94.33 (0.02)	2.87/2.79	94.10 (0.03)	2.99/2.91
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	92.47 (0.06)	4.51/3.01	91.97 (0.07)	4.22/3.80	90.90 (0.10)	4.63/4.46
	GAUS	94.45 (0.03)	0.13/5.42	92.54 (0.07)	2.21/5.25	91.15 (0.10)	3.21/5.64
	$GAUS_{aicc}$	93.87 (0.05)	0.45/5.68	85.70 (0.10)	5.64/8.65	83.79 (0.13)	6.89/9.32
100	EST	93.70 (0.04)	3.56/2.74	93.53 (0.04)	3.34/3.13	92.87 (0.06)	3.63/3.50
	GAUS	94.88 (0.02)	0.01/5.10	93.95 (0.04)	1.53/4.51	93.06 (0.06)	2.37/4.57
	$GAUS_{aicc}$	94.55 (0.03)	0.10/5.35	92.29 (0.06)	2.39/5.32	91.25 (0.07)	3.29/5.46
300	EST	94.48 (0.03)	2.92/2.60	94.51 (0.02)	2.80/2.69	94.26 (0.03)	2.92/2.82
	GAUS	95.10 (0.01)	0.00/4.90	94.90 (0.02)	1.14/3.96	94.48 (0.03)	1.73/3.79
	$GAUS_{aicc}$	95.06 (0.01)	0.00/4.94	94.87 (0.02)	1.16/3.98	94.42 (0.03)	1.76/3.82

Table A.20: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ with σ_ε^2=1.

Panel A: Gaussian		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/AGAUS	0.182	0.892	1.655	0.135	0.713	1.298	0.078	0.414	0.754
	(0.146)	(0.829)	(1.811)	(0.095)	(0.565)	(1.065)	(0.067)	(0.432)	(0.813)
BAYESN	0.198	0.933	1.663	0.140	0.679	1.156	0.094	0.421	0.705
	(0.152)	(0.837)	(1.755)	(0.096)	(0.578)	(1.061)	(0.054)	(0.334)	(0.605)
BAYESL	0.263	0.947	1.392	0.236	0.866	1.228	0.154	0.538	0.789
	(0.198)	(0.680)	(0.897)	(0.162)	(0.622)	(0.857)	(0.096)	(0.377)	(0.558)
BOOT	0.206	0.830	1.515	0.153	0.672	1.198	0.109	0.465	0.807
	(0.126)	(0.728)	(1.493)	(0.080)	(0.492)	(0.943)	(0.060)	(0.394)	(0.751)
BOOTEX	0.280	1.059	1.826	0.206	0.814	1.379	0.138	0.543	0.903
	(0.193)	(0.906)	(1.789)	(0.131)	(0.615)	(1.040)	(0.083)	(0.418)	(0.752)
BOOTNP	1.114	1.978	2.757	1.249	1.852	2.545	1.455	1.616	2.154
	(0.704)	(0.918)	(1.028)	(0.754)	(0.985)	(1.090)	(0.761)	(0.993)	(1.092)
Panel B: Student-5		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.206	0.995	1.843	0.152	0.741	1.331	0.094	0.470	0.831
	(0.163)	(0.926)	(2.092)	(0.134)	(0.583)	(1.124)	(0.100)	(0.485)	(0.909)
AGAUS	0.239	1.016	1.863	0.193	0.766	1.354	0.150	0.518	0.880
	(0.161)	(0.924)	(2.099)	(0.130)	(0.574)	(1.110)	(0.092)	(0.467)	(0.884)
BAYEST	0.200	0.924	1.658	0.145	0.629	1.075	0.101	0.406	0.669
	(0.152)	(0.807)	(1.718)	(0.103)	(0.506)	(0.966)	(0.068)	(0.301)	(0.566)
BAYESN	0.241	1.013	1.821	0.185	0.688	1.160	0.144	0.446	0.722
	(0.169)	(0.921)	(2.009)	(0.129)	(0.569)	(1.085)	(0.091)	(0.398)	(0.709)
BAYESL	0.299	0.974	1.439	0.281	0.892	1.247	0.191	0.597	0.864
	(0.225)	(0.726)	(0.942)	(0.198)	(0.611)	(0.809)	(0.130)	(0.467)	(0.679)
BOOT	0.223	0.917	1.682	0.174	0.695	1.232	0.123	0.499	0.867
	(0.143)	(0.810)	(1.719)	(0.119)	(0.510)	(0.988)	(0.078)	(0.417)	(0.811)
BOOTEX	0.297	1.126	1.964	0.225	0.851	1.431	0.157	0.585	0.968
	(0.220)	(1.054)	(1.970)	(0.162)	(0.642)	(1.131)	(0.117)	(0.485)	(0.831)
BOOTNP	1.116	2.025	2.835	1.225	1.834	2.556	1.483	1.630	2.167
	(0.780)	(0.961)	(1.101)	(0.777)	(0.996)	(1.131)	(0.934)	(1.080)	(1.163)
Panel C: $\chi^2_{(5)}$		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.194	0.970	1.825	0.139	0.703	1.279	0.085	0.434	0.774
	(0.150)	(0.960)	(2.220)	(0.101)	(0.575)	(1.136)	(0.068)	(0.403)	(0.763)
AGAUS	0.287	1.035	1.875	0.248	0.786	1.347	0.222	0.552	0.869
	(0.129)	(0.940)	(2.205)	(0.082)	(0.545)	(1.110)	(0.050)	(0.370)	(0.738)
BAYESN	0.293	1.036	1.820	0.247	0.726	1.191	0.219	0.491	0.727
	(0.141)	(0.938)	(2.155)	(0.080)	(0.551)	(1.094)	(0.044)	(0.292)	(0.552)
BAYESL	0.339	1.026	1.455	0.315	0.903	1.255	0.254	0.625	0.864
	(0.181)	(0.701)	(0.948)	(0.151)	(0.584)	(0.807)	(0.081)	(0.376)	(0.582)
BOOT	0.215	0.907	1.666	0.156	0.662	1.189	0.109	0.465	0.813
	(0.134)	(0.845)	(1.791)	(0.086)	(0.501)	(0.996)	(0.061)	(0.372)	(0.720)
BOOTEX	0.288	1.134	1.967	0.214	0.797	1.369	0.140	0.546	0.915
	(0.207)	(1.029)	(2.068)	(0.138)	(0.591)	(1.103)	(0.085)	(0.417)	(0.757)
BOOTNP	1.100	1.973	2.791	1.211	1.814	2.534	1.474	1.600	2.153
	(0.708)	(0.906)	(1.039)	(0.720)	(0.975)	(1.108)	(0.767)	(1.011)	(1.102)

Table A.21: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/AGAUS	78.91 (0.06)	10.57/10.52	75.64 (0.12)	12.18/12.18	73.45 (0.18)	13.31/13.24
	SRA_{aicc}	78.55 (0.08)	9.58/11.87	69.38 (0.16)	14.18/16.44	8.48 (2.73)	19.57/22.40
100	AEST/AGAUS	79.53 (0.04)	10.29/10.18	78.12 (0.06)	11.03/10.85	77.14 (0.09)	11.61/11.24
	SRA_{aicc}	78.75 (0.05)	10.17/11.08	74.55 (0.11)	12.15/13.3	8.55 (1.70)	14.18/15.36
300	AEST/AGAUS	79.84 (0.02)	10.12/10.05	79.34 (0.03)	10.36/10.3	79.07 (0.05)	10.47/10.45
	SRA_{aicc}	79.45 (0.03)	10.22/10.33	78.40 (0.05)	10.90/10.7	8.58 (0.81)	11.42/11.29
	Student-5	h=1		h=6		h=12	
50	AEST	78.09 (0.08)	11.14/10.77	75.38 (0.13)	12.38/12.23	73.77 (0.17)	13.13/13.11
	AGAUS	82.29 (0.07)	9.03/8.68	76.77 (0.13)	11.68/11.55	74.57 (0.17)	12.72/12.70
	SRA_{aicc}	78.85 (0.09)	9.61/11.54	70.74 (0.15)	13.57/15.69	60.60 (0.20)	18.48/20.92
100	AEST	78.95 (0.05)	10.39/10.66	78.05 (0.07)	10.77/11.18	77.31 (0.10)	11.03/11.66
	AGAUS	83.13 (0.05)	8.33/8.54	79.47 (0.07)	10.07/10.46	78.15 (0.10)	10.62/11.22
	SRA_{aicc}	79.16 (0.06)	9.76/11.08	75.08 (0.11)	11.75/13.17	8.49 (2.32)	13.52/15.28
300	AEST	79.70 (0.03)	10.14/10.16	79.48 (0.04)	10.36/10.16	79.29 (0.05)	10.47/10.24
	AGAUS	83.86 (0.03)	8.06/8.08	80.90 (0.04)	9.64/9.46	80.11 (0.05)	10.07/9.82
	SRA_{aicc}	79.51 (0.03)	9.99/10.51	78.65 (0.05)	10.55/10.79	8.42 (0.90)	11.16/11.23
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	77.98 (0.09)	11.27/10.75	75.14 (0.14)	12.51/12.35	73.28 (0.19)	13.31/13.41
	AGAUS	83.25 (0.08)	5.28/11.46	75.97 (0.14)	11.06/12.97	73.67 (0.19)	12.42/13.9
	SRA_{aicc}	78.32 (0.10)	10.18/11.50	70.97 (0.16)	14.14/14.88	8.78 (3.64)	18.83/20.42
100	AEST	79.31 (0.06)	10.35/10.34	78.37 (0.07)	10.71/10.92	77.53 (0.09)	11.03/11.44
	AGAUS	84.75 (0.05)	4.21/11.04	79.18 (0.07)	9.26/11.56	77.91 (0.10)	10.12/11.97
	SRA_{aicc}	78.56 (0.08)	10.52/10.92	75.26 (0.10)	12.31/12.43	8.50 (1.73)	14.27/14.04
300	AEST	79.95 (0.04)	9.94/10.11	79.64 (0.04)	10.08/10.28	79.53 (0.05)	10.13/10.34
	AGAUS	85.58 (0.03)	3.61/10.81	80.47 (0.04)	8.62/10.91	79.93 (0.05)	9.21/10.86
	SRA_{aicc}	79.53 (0.05)	10.22/10.25	78.68 (0.05)	10.72/10.6	8.53 (0.89)	11.02/10.85

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	AEST/AGAUS	94.12 (0.03)	2.94/2.94	91.19 (0.11)	4.41/4.404	88.47 (0.18)	5.83/5.70
	SRA_{aicc}	91.51 (0.06)	3.13/5.37	81.39 (0.14)	8.37/10.24	69.17 (0.19)	15.30/15.53
100	AEST/AGAUS	94.62 (0.02)	2.71/2.67	93.66 (0.04)	3.25/3.10	92.69 (0.07)	3.78/3.52
	SRA_{aicc}	93.65 (0.03)	2.69/3.66	89.68 (0.08)	4.64/5.68	82.90 (0.13)	7.81/9.29
300	AEST/AGAUS	94.89 (0.01)	2.56/2.55	94.59 (0.02)	2.72/2.69	94.35 (0.03)	2.82/2.82
	SRA_{aicc}	94.53 (0.02)	2.58/2.89	93.78 (0.03)	3.05/3.17	92.35 (0.05)	3.76/3.89
	Student-5	h=1		h=6		h=12	
50	AEST	93.96 (0.04)	3.12/2.92	91.09 (0.13)	4.51/4.40	89.17 (0.17)	5.42/5.40
	AGAUS	93.68 (0.04)	3.26/3.06	90.89 (0.13)	4.61/4.50	89.02 (0.17)	5.49/5.48
	SRA_{aicc}	92.20 (0.06)	2.69/5.10	82.18 (0.12)	8.06/9.75	70.35 (0.19)	14.57/15.08
100	AEST	94.49 (0.03)	2.72/2.79	93.68 (0.05)	3.14/3.18	92.97 (0.07)	3.43/3.61
	AGAUS	94.23 (0.03)	2.85/2.92	93.48 (0.05)	3.24/3.28	92.83 (0.07)	3.49/3.68
	SRA_{aicc}	94.09 (0.04)	2.31/3.60	89.43 (0.09)	4.88/5.70	83.12 (0.13)	7.66/9.22
300	AEST	94.90 (0.01)	2.55/2.55	94.68 (0.02)	2.71/2.61	94.52 (0.03)	2.79/2.69
	AGAUS	94.65 (0.01)	2.68/2.67	94.50 (0.02)	2.79/2.70	94.39 (0.03)	2.86/2.75
	SRA_{aicc}	94.72 (0.02)	2.47/2.80	93.85 (0.04)	2.92/3.23	92.68 (0.05)	3.58/3.74
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	92.48 (0.07)	4.52/3.00	90.37 (0.14)	5.09/4.53	88.01 (0.20)	6.19/5.79
	AGAUS	94.41 (0.04)	0.20/5.39	90.91 (0.14)	3.16/5.92	88.23 (0.20)	4.94/6.83
	SRA_{aicc}	91.81 (0.07)	3.41/4.78	82.22 (0.13)	8.07/9.70	70.74 (0.19)	14.39/14.87
100	AEST	93.86 (0.04)	3.44/2.70	93.60 (0.05)	3.32/3.07	92.87 (0.08)	3.67/3.47
	AGAUS	94.95 (0.02)	0.01/5.04	94.00 (0.04)	1.57/4.43	93.03 (0.08)	2.46/4.51
	SRA_{aicc}	93.68 (0.05)	2.87/3.45	90.16 (0.07)	4.72/5.12	83.92 (0.12)	7.60/8.47
300	AEST	94.59 (0.03)	2.83/2.57	94.67 (0.02)	2.70/2.62	94.52 (0.03)	2.77/2.71
	AGAUS	95.14 (0.01)	0.00/4.86	95.03 (0.02)	1.08/3.88	94.71 (0.03)	1.63/3.66
	SRA_{aicc}	94.51 (0.03)	2.69/2.79	93.94 (0.03)	3.16/2.90	92.66 (0.05)	3.74/3.59

Table A.22: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BAYESN	80.02 (0.05)	10.00/9.98	76.35 (0.10)	11.74/11.91	74.41 (0.13)	12.72/12.87
	BAYESL	80.12 (0.06)	9.79/10.10	74.76 (0.10)	12.43/12.82	72.16 (0.12)	13.64/14.20
100	BAYESN	79.97 (0.04)	10.00/10.03	77.77 (0.06)	11.15/11.08	76.45 (0.09)	11.78/11.76
	BAYESL	79.88 (0.04)	10.11/10.01	76.26 (0.08)	11.61/12.13	74.69 (0.10)	12.32/13.00
300	BAYESN	79.91 (0.03)	10.11/9.97	78.96 (0.04)	10.59/10.45	78.36 (0.05)	10.85/10.79
	BAYESL	79.87 (0.03)	10.21/9.92	78.72 (0.04)	10.92/10.36	78.02 (0.06)	11.20/10.78
	Student-5	h=1		h=6		h=12	
50	BAYEST	80.92 (0.06)	9.66/9.41	78.06 (0.09)	11.06/10.88	76.72 (0.12)	11.65/11.63
	BAYESN	83.30 (0.06)	8.46/8.23	78.18 (0.10)	11.02/10.80	76.06 (0.12)	12.06/11.89
	BAYESL	83.31 (0.07)	8.52/8.17	76.44 (0.10)	11.98/11.59	73.75 (0.12)	13.19/13.06
100	BAYEST	80.50 (0.04)	9.67/9.84	78.67 (0.06)	10.47/10.85	77.69 (0.08)	10.93/11.38
	BAYESN	83.52 (0.05)	8.14/8.34	79.31 (0.07)	10.21/10.48	77.58 (0.09)	11.03/11.39
	BAYESL	82.99 (0.06)	8.44/8.57	77.44 (0.08)	11.15/11.41	75.64 (0.10)	12.04/12.32
300	BAYEST	80.28 (0.03)	9.79/9.92	79.58 (0.04)	10.23/10.19	79.17 (0.05)	10.47/10.36
	BAYESN	83.92 (0.03)	8.01/8.07	80.64 (0.04)	9.69/9.67	79.50 (0.05)	10.28/10.22
	BAYESL	83.68 (0.03)	8.02/8.30	80.11 (0.05)	9.86/10.02	79.03 (0.06)	10.48/10.48
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	84.10 (0.07)	4.87/11.03	77.44 (0.10)	10.16/12.40	75.23 (0.13)	11.40/13.37
	BAYESL	83.85 (0.08)	5.30/10.85	75.51 (0.10)	11.52/12.97	73.06 (0.12)	12.91/14.03
100	BAYESN	84.95 (0.05)	4.21/10.84	78.76 (0.07)	9.53/11.71	77.06 (0.09)	10.64/12.3
	BAYESL	84.05 (0.06)	5.16/10.79	77.11 (0.08)	10.59/12.29	75.46 (0.10)	11.78/12.76
300	BAYESN	85.50 (0.03)	3.73/10.77	80.18 (0.04)	8.83/11.00	79.22 (0.05)	9.62/11.15
	BAYESL	84.90 (0.04)	4.43/10.67	79.61 (0.05)	9.31/11.07	78.83 (0.06)	9.96/11.21
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BAYESN	94.96 (0.03)	2.53/2.51	92.99 (0.06)	3.48/3.52	91.85 (0.08)	4.04/4.10
	BAYESL	95.03 (0.03)	2.41/2.56	91.93 (0.07)	4.00/4.06	90.21 (0.09)	4.80/4.99
100	BAYESN	94.92 (0.02)	2.55/2.53	93.72 (0.04)	3.20/3.08	92.93 (0.06)	3.58/3.49
	BAYESL	94.89 (0.02)	2.58/2.53	92.90 (0.05)	3.47/3.63	91.87 (0.06)	3.89/4.24
300	BAYESN	94.92 (0.01)	2.55/2.52	94.44 (0.02)	2.79/2.76	94.09 (0.03)	2.95/2.95
	BAYESL	94.86 (0.01)	2.62/2.52	94.24 (0.02)	3.00/2.75	93.87 (0.03)	3.15/2.98
	Student-5	h=1		h=6		h=12	
50	BAYEST	94.83 (0.03)	2.62/2.55	93.89 (0.05)	3.14/2.98	93.29 (0.07)	3.41/3.30
	BAYESN	94.44 (0.03)	2.82/2.74	93.04 (0.06)	3.59/3.36	92.33 (0.07)	3.94/3.73
	BAYESL	94.61 (0.03)	2.74/2.65	92.26 (0.06)	3.96/3.78	90.87 (0.08)	4.61/4.52
100	BAYEST	94.75 (0.02)	2.60/2.65	94.22 (0.04)	2.84/2.93	93.75 (0.05)	3.04/3.20
	BAYESN	94.49 (0.02)	2.71/2.79	93.64 (0.04)	3.17/3.19	93.02 (0.05)	3.45/3.54
	BAYESL	94.40 (0.03)	2.76/2.84	92.69 (0.05)	3.64/3.67	91.89 (0.06)	3.97/4.13
300	BAYEST	94.89 (0.01)	2.55/2.56	94.72 (0.02)	2.66/2.62	94.52 (0.03)	2.75/2.73
	BAYESN	94.66 (0.01)	2.68/2.66	94.38 (0.02)	2.82/2.80	94.18 (0.03)	2.94/2.89
	BAYESL	94.64 (0.02)	2.61/2.75	94.15 (0.03)	2.93/2.92	93.92 (0.03)	3.00/3.07
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	94.92 (0.03)	0.17/4.90	93.23 (0.06)	2.00/4.77	92.22 (0.08)	2.80/4.97
	BAYESL	95.11 (0.03)	0.23/4.66	92.23 (0.06)	2.73/5.04	90.73 (0.08)	3.81/5.46
100	BAYESN	95.15 (0.02)	0.01/4.84	94.09 (0.04)	1.54/4.37	93.21 (0.05)	2.35/4.44
	BAYESL	95.22 (0.02)	0.08/4.69	93.29 (0.04)	2.08/4.63	92.33 (0.06)	2.97/4.69
300	BAYESN	95.17 (0.01)	0.00/4.83	94.90 (0.02)	1.17/3.94	94.48 (0.03)	1.76/3.76
	BAYESL	95.23 (0.01)	0.01/4.76	94.64 (0.02)	1.38/3.98	94.27 (0.03)	1.92/3.80

Table A.23: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BOOT	78.41 (0.06)	10.74/10.85	77.21 (0.09)	11.43/11.36	76.76 (0.12)	11.64/11.61
	BOOTEX	78.40 (0.07)	10.74/10.86	75.90 (0.12)	11.99/12.11	75.37 (0.15)	12.17/12.45
	BOOTNP	69.12 (0.26)	15.32/15.56	48.52 (0.17)	25.90/25.58	41.43 (0.15)	29.52/29.05
100	BOOT	79.10 (0.04)	10.43/10.47	78.60 (0.06)	10.73/10.67	78.50 (0.08)	10.84/10.67
	BOOTEX	79.08 (0.05)	10.49/10.43	78.66 (0.08)	10.69/10.65	78.62 (0.10)	10.75/10.62
	BOOTNP	74.65 (0.26)	12.36/12.99	54.87 (0.17)	22.34/22.78	48.00 (0.15)	25.83/26.17
300	BOOT	79.55 (0.03)	10.24/10.21	79.45 (0.03)	10.33/10.22	79.42 (0.05)	10.30/10.28
	BOOTEX	79.53 (0.03)	10.36/10.11	79.25 (0.04)	10.54/10.21	79.40 (0.06)	10.44/10.16
	BOOTNP	82.15 (0.23)	9.45/8.39	62.35 (0.16)	19.02/18.64	56.13 (0.15)	22.16/21.71
	Student-5	h=1		h=6		h=12	
50	BOOT	79.26 (0.06)	10.51/10.23	77.87 (0.09)	11.2/10.93	77.38 (0.12)	11.38/11.24
	BOOTEX	79.37 (0.08)	10.4/10.23	76.25 (0.12)	11.86/11.89	75.66 (0.15)	12.04/12.3
	BOOTNP	67.51 (0.28)	17/15.48	48.10 (0.18)	26.64/25.26	40.50 (0.16)	30.38/29.12
100	BOOT	79.52 (0.05)	10.19/10.29	79.13 (0.06)	10.32/10.55	79.04 (0.08)	10.27/10.70
	BOOTEX	79.62 (0.05)	10.15/10.23	78.40 (0.08)	10.60/11	78.06 (0.11)	10.73/11.21
	BOOTNP	74.51 (0.27)	13.45/12.04	54.82 (0.18)	22.84/22.34	47.96 (0.16)	26.02/26.02
300	BOOT	79.78 (0.03)	10.12/10.10	79.79 (0.04)	10.26/9.95	79.78 (0.05)	10.28/9.93
	BOOTEX	79.78 (0.03)	10.01/10.22	79.44 (0.05)	10.26/10.29	79.52 (0.06)	10.32/10.17
	BOOTNP	82.70 (0.24)	8.14/9.16	63.20 (0.16)	17.81/18.98	56.65 (0.15)	21.26/22.09
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	78.64 (0.07)	10.53/10.82	77.53 (0.10)	10.92/11.55	77.02 (0.12)	11.03/11.95
	BOOTEX	78.92 (0.09)	10.11/10.98	75.89 (0.12)	11.87/12.24	75.54 (0.15)	11.85/12.61
	BOOTNP	70.00 (0.28)	15.83/14.17	48.44 (0.18)	26.86/24.7	40.52 (0.16)	30.90/28.59
100	BOOT	79.40 (0.05)	10.14/10.45	79.15 (0.06)	10.12/10.73	79.04 (0.08)	10.13/10.84
	BOOTEX	79.69 (0.07)	9.67/10.64	78.75 (0.08)	10.35/10.90	78.71 (0.10)	10.41/10.88
	BOOTNP	74.10 (0.28)	13.57/12.33	53.57 (0.18)	24.10/22.33	46.91 (0.16)	27.27/25.81
300	BOOT	79.82 (0.04)	9.94/10.24	79.82 (0.04)	9.92/10.26	79.84 (0.05)	9.92/10.24
	BOOTEX	79.72 (0.05)	10.10/10.18	79.50 (0.05)	10.22/10.28	79.73 (0.06)	10.03/10.24
	BOOTNP	82.36 (0.25)	8.69/8.94	63.10 (0.16)	18.21/18.69	56.55 (0.15)	21.24/22.21

Table A.24: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverage of 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BOOT	93.27 (0.04)	3.41/3.32	93.15 (0.06)	3.44/3.41	92.76 (0.08)	3.62/3.61
	BOOTEX	93.31 (0.04)	3.40/3.29	92.27 (0.08)	3.89/3.83	91.87 (0.10)	4.13/4.00
	BOOTNP	79.72 (0.26)	9.79/10.48	64.38 (0.18)	17.87/17.76	56.68 (0.16)	21.75/21.57
100	BOOT	94.08 (0.03)	2.97/2.94	94.09 (0.03)	2.98/2.93	93.94 (0.05)	3.04/3.01
	BOOTEX	93.99 (0.03)	3.07/2.93	94.04 (0.04)	2.99/2.97	94.01 (0.06)	3.01/2.98
	BOOTNP	86.76 (0.22)	6.16/7.08	72.84 (0.17)	13.29/13.88	66.14 (0.16)	16.63/17.23
300	BOOT	94.62 (0.02)	2.69/2.69	94.62 (0.02)	2.71/2.67	94.56 (0.03)	2.72/2.72
	BOOTEX	94.58 (0.02)	2.72/2.69	94.51 (0.02)	2.80/2.68	94.58 (0.03)	2.77/2.65
	BOOTNP	93.20 (0.17)	3.74/3.06	81.59 (0.15)	9.57/8.84	75.85 (0.15)	12.42/11.72
	Student-5	h=1		h=6		h=12	
50	BOOT	93.51 (0.04)	3.31/3.18	92.77 (0.06)	3.72/3.51	92.63 (0.08)	3.72/3.65
	BOOTEX	93.38 (0.04)	3.34/3.28	91.98 (0.08)	4.07/3.95	91.74 (0.10)	4.08/4.18
	BOOTNP	78.68 (0.27)	10.85/10.48	63.63 (0.19)	18.97/17.4	55.63 (0.17)	23.05/21.32
100	BOOT	94.15 (0.03)	2.90/2.95	93.90 (0.04)	3.05/3.04	93.85 (0.05)	3.03/3.11
	BOOTEX	94.13 (0.03)	2.86/3.00	93.46 (0.05)	3.22/3.32	93.36 (0.07)	3.26/3.38
	BOOTNP	85.66 (0.24)	7.83/6.51	72.17 (0.18)	14.12/13.71	65.16 (0.17)	17.52/17.31
300	BOOT	94.66 (0.02)	2.69/2.65	94.61 (0.02)	2.77/2.62	94.61 (0.03)	2.76/2.62
	BOOTEX	94.57 (0.02)	2.71/2.71	94.46 (0.03)	2.79/2.75	94.55 (0.03)	2.75/2.71
	BOOTNP	92.84 (0.18)	3.11/4.05	82.00 (0.15)	8.44/9.55	76.49 (0.15)	11.32/12.18
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	94.02 (0.04)	2.39/3.59	93.13 (0.06)	2.97/3.91	92.79 (0.09)	3.10/4.11
	BOOTEX	94.34 (0.05)	2.07/3.59	92.38 (0.08)	3.30/4.31	91.96 (0.11)	3.50/4.53
	BOOTNP	79.76 (0.27)	10.78/9.46	64.03 (0.2)	18.29/17.69	55.77 (0.18)	22.49/21.74
100	BOOT	94.72 (0.03)	2.22/3.06	94.27 (0.03)	2.62/3.11	94.18 (0.05)	2.67/3.15
	BOOTEX	94.98 (0.03)	1.85/3.17	94.12 (0.05)	2.65/3.23	94.07 (0.06)	2.77/3.16
	BOOTNP	85.05 (0.25)	8.02/6.92	71.71 (0.19)	14.53/13.76	64.67 (0.18)	18.07/17.26
300	BOOT	94.80 (0.02)	2.41/2.79	94.77 (0.02)	2.51/2.72	94.76 (0.03)	2.52/2.72
	BOOTEX	95.02 (0.03)	2.24/2.73	94.67 (0.03)	2.58/2.75	94.71 (0.03)	2.54/2.76
	BOOTNP	92.44 (0.18)	4.12/3.44	82.16 (0.15)	8.79/9.05	76.64 (0.14)	11.38/11.98

A.6. Monte Carlo results for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with σ^2_ε=1.

Table A.25: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with σ^2_ε=1

Panel A: Gaussian	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST/GAUS	0.189	0.412	0.662	0.125	0.307	0.501	0.072	0.183	0.299
	(0.167)	(0.352)	(0.633)	(0.105)	(0.265)	(0.458)	(0.060)	(0.175)	(0.300)
$\text{GAUS}_{\text{aicc}}$	0.384	0.812	1.039	0.232	0.524	0.720	0.073	0.184	0.298
	(0.271)	(0.408)	(0.566)	(0.234)	(0.464)	(0.608)	(0.062)	(0.175)	(0.297)
Panel B: Student-5	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.199	0.429	0.678	0.134	0.322	0.522	0.080	0.195	0.304
	(0.170)	(0.376)	(0.671)	(0.105)	(0.277)	(0.497)	(0.057)	(0.161)	(0.282)
GAUS	0.233	0.434	0.681	0.175	0.329	0.526	0.137	0.209	0.318
	(0.165)	(0.376)	(0.671)	(0.099)	(0.274)	(0.493)	(0.055)	(0.158)	(0.278)
$\text{GAUS}_{\text{aicc}}$	0.404	0.832	1.058	0.258	0.519	0.717	0.139	0.212	0.320
	(0.296)	(0.466)	(0.616)	(0.236)	(0.429)	(0.566)	(0.062)	(0.165)	(0.282)
Panel C: $\chi^2_{(5)}$	T=50			T=100			T=300		
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
EST	0.193	0.435	0.703	0.134	0.320	0.515	0.079	0.194	0.303
	(0.149)	(0.380)	(0.726)	(0.106)	(0.255)	(0.439)	(0.060)	(0.164)	(0.284)
GAUS	0.285	0.455	0.716	0.247	0.349	0.534	0.219	0.237	0.334
	(0.126)	(0.373)	(0.721)	(0.086)	(0.248)	(0.435)	(0.045)	(0.154)	(0.276)
$\text{GAUS}_{\text{aicc}}$	0.421	0.820	1.057	0.328	0.556	0.747	0.220	0.237	0.332
	(0.223)	(0.451)	(0.627)	(0.211)	(0.426)	(0.565)	(0.048)	(0.155)	(0.271)

Table A.26: Monte Carlo averages and standard errors (in parenthesis) of coverages of the estimated forecast intervals for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	EST/GAUS	78.20 (0.06)	10.79/11.02	75.36 (0.10)	12.17/12.47	73.19 (0.14)	13.24/13.57
	$GAUS_{aicc}$	76.80 (0.08)	11.14/12.06	82.37 (0.14)	8.52/9.11	79.12 (0.17)	10.24/10.64
100	EST/GAUS	79.29 (0.04)	10.27/10.44	77.83 (0.07)	11.02/11.15	76.68 (0.10)	11.57/11.75
	$GAUS_{aicc}$	77.98 (0.06)	10.82/11.20	78.77 (0.11)	10.35/10.88	76.53 (0.13)	11.46/12.01
300	EST/GAUS	79.88 (0.02)	10.13/9.99	79.26 (0.04)	10.60/10.15	78.75 (0.05)	10.90/10.35
	$GAUS_{aicc}$	79.71 (0.02)	10.20/10.09	79.17 (0.04)	10.63/10.20	78.66 (0.05)	10.93/10.41
	Student-5	h=1		h=6		h=12	
50	EST	77.41 (0.07)	11.20/11.39	74.48 (0.12)	12.85/12.67	72.52 (0.15)	13.97/13.51
	GAUS	81.71 (0.07)	9.07/9.22	76.65 (0.11)	11.76/11.59	73.97 (0.15)	13.23/12.79
	$GAUS_{aicc}$	79.62 (0.10)	10.17/10.21	82.76 (0.14)	8.45/8.78	79.63 (0.17)	9.98/10.39
100	EST	78.52 (0.05)	10.72/10.76	76.95 (0.08)	11.63/11.41	75.78 (0.11)	12.28/11.93
	GAUS	82.77 (0.04)	8.61/8.62	78.95 (0.08)	10.62/10.43	77.04 (0.10)	11.64/11.31
	$GAUS_{aicc}$	81.55 (0.06)	9.06/9.38	80.27 (0.11)	9.76/9.97	77.65 (0.13)	11.11/11.24
300	EST	79.44 (0.03)	10.28/10.28	78.94 (0.04)	10.57/10.49	78.56 (0.05)	10.77/10.67
	GAUS	83.60 (0.03)	8.20/8.201	80.84 (0.04)	9.62/9.53	79.71 (0.05)	10.19/10.10
	$GAUS_{aicc}$	83.45 (0.03)	8.27/8.28	80.77 (0.04)	9.65/9.58	79.63 (0.05)	10.22/10.15
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	78.03 (0.09)	11.30/10.67	75.38 (0.12)	12.44/12.18	73.21 (0.15)	13.30/13.50
	GAUS	83.20 (0.08)	5.43/11.37	76.91 (0.11)	10.31/12.78	74.06 (0.15)	11.94/14.00
	$GAUS_{aicc}$	80.39 (0.11)	7.85/11.76	83.08 (0.14)	6.94/9.99	79.60 (0.17)	8.76/11.64
100	EST	79.11 (0.07)	10.61/10.28	77.91 (0.08)	11.13/10.97	76.73 (0.10)	11.61/11.66
	GAUS	84.55 (0.05)	4.48/10.97	79.30 (0.08)	9.13/11.57	77.49 (0.10)	10.34/12.17
	$GAUS_{aicc}$	82.16 (0.09)	6.59/11.25	80.32 (0.11)	8.89/10.78	77.86 (0.13)	10.33/11.81
300	EST	79.86 (0.04)	10.05/10.09	79.33 (0.04)	10.34/10.33	78.82 (0.06)	10.62/10.56
	GAUS	85.56 (0.03)	3.66/10.78	80.73 (0.04)	8.35/10.91	79.53 (0.05)	9.41/11.06
	$GAUS_{aicc}$	85.36 (0.03)	3.81/10.84	80.64 (0.04)	8.42/10.94	79.46 (0.05)	9.46/11.08
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	EST/GAUS	93.70 (0.04)	3.09/3.21	91.42 (0.07)	4.16/4.41	89.37 (0.11)	5.12/5.51
	$GAUS_{aicc}$	92.88 (0.05)	3.37/3.75	94.26 (0.08)	2.61/3.13	91.73 (0.10)	3.87/4.39
100	EST/GAUS	94.52 (0.02)	2.71/2.77	93.38 (0.04)	3.29/3.32	92.40 (0.07)	3.77/3.83
	$GAUS_{aicc}$	93.79 (0.03)	3.06/3.16	93.40 (0.06)	3.20/3.39	91.82 (0.08)	3.94/4.24
300	EST/GAUS	94.90 (0.01)	2.56/2.53	94.49 (0.02)	2.84/2.67	94.17 (0.03)	3.01/2.81
	$GAUS_{aicc}$	94.82 (0.01)	2.60/2.58	94.44 (0.02)	2.86/2.69	94.11 (0.03)	3.04/2.85
	Student-5	h=1		h=6		h=12	
50	EST	93.75 (0.03)	3.09/3.16	91.46 (0.07)	4.26/4.28	89.43 (0.11)	5.29/5.27
	GAUS	93.45 (0.03)	3.24/3.31	91.26 (0.07)	4.36/4.38	89.27 (0.11)	5.38/5.35
	$GAUS_{aicc}$	92.70 (0.05)	3.62/3.67	93.99 (0.08)	2.94/3.06	91.75 (0.11)	3.97/4.28
100	EST	94.32 (0.02)	2.82/2.85	93.18 (0.05)	3.48/3.34	92.17 (0.07)	3.97/3.86
	GAUS	94.04 (0.02)	2.97/2.99	93.01 (0.05)	3.56/3.42	92.04 (0.07)	4.04/3.92
	$GAUS_{aicc}$	93.57 (0.04)	3.11/3.33	93.46 (0.06)	3.25/3.29	91.96 (0.08)	3.99/4.04
300	EST	94.77 (0.01)	2.61/2.62	94.43 (0.02)	2.79/2.77	94.12 (0.03)	2.93/2.96
	GAUS	94.51 (0.01)	2.74/2.75	94.28 (0.02)	2.88/2.85	93.99 (0.03)	2.99/3.02
	$GAUS_{aicc}$	94.44 (0.01)	2.77/2.79	94.24 (0.02)	2.89/2.87	93.93 (0.03)	3.01/3.06
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	EST	92.36 (0.07)	4.69/2.95	90.92 (0.09)	5.25/3.83	89.06 (0.12)	6.03/4.90
	GAUS	94.43 (0.03)	0.23/5.34	91.92 (0.08)	2.45/5.63	89.62 (0.11)	3.93/6.45
	$GAUS_{aicc}$	93.61 (0.05)	1.04/5.36	94.30 (0.08)	1.87/3.83	91.81 (0.11)	3.14/5.05
100	EST	93.60 (0.05)	3.72/2.68	93.09 (0.06)	3.83/3.09	92.15 (0.07)	4.27/3.58
	GAUS	94.95 (0.02)	0.04/5.01	93.84 (0.05)	1.49/4.67	92.59 (0.07)	2.52/4.89
	$GAUS_{aicc}$	94.26 (0.04)	0.63/5.11	94.02 (0.06)	1.75/4.23	92.35 (0.08)	2.90/4.75
300	EST	94.58 (0.02)	2.87/2.56	94.41 (0.02)	2.92/2.67	94.11 (0.03)	3.05/2.83
	GAUS	95.16 (0.01)	0.00/4.84	94.95 (0.02)	0.92/4.13	94.42 (0.03)	1.61/3.97
	$GAUS_{aicc}$	95.12 (0.01)	0.00/4.88	94.91 (0.02)	0.94/4.15	94.38 (0.03)	1.64/3.98

Table A.27: Monte Carlo averages and standard deviations (in parenthesis) of MD distances between the estimated and true forecast densities for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with σ_ε^2=1.

Panel A: Gaussian		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST/AGAUS	0.190	0.417	0.683	0.126	0.308	0.503	0.072	0.183	0.300
	(0.167)	(0.362)	(0.706)	(0.105)	(0.265)	(0.462)	(0.060)	(0.174)	(0.300)
BAYESN	0.218	0.466	0.692	0.149	0.330	0.487	0.094	0.194	0.286
	(0.176)	(0.349)	(0.611)	(0.111)	(0.273)	(0.437)	(0.058)	(0.157)	(0.250)
BAYESL	0.290	0.471	0.605	0.250	0.412	0.524	0.151	0.246	0.329
	(0.212)	(0.328)	(0.375)	(0.177)	(0.314)	(0.400)	(0.101)	(0.185)	(0.261)
BOOT	0.226	0.391	0.618	0.158	0.295	0.466	0.104	0.203	0.316
	(0.153)	(0.333)	(0.615)	(0.094)	(0.241)	(0.414)	(0.054)	(0.157)	(0.273)
BOOTEX	0.280	0.497	0.715	0.203	0.374	0.556	0.137	0.231	0.349
	(0.185)	(0.328)	(0.497)	(0.132)	(0.267)	(0.391)	(0.084)	(0.175)	(0.280)
BOOTNP	0.520	0.850	0.995	0.449	0.768	0.925	0.340	0.545	0.719
	(0.294)	(0.503)	(0.533)	(0.261)	(0.468)	(0.517)	(0.204)	(0.360)	(0.447)
Panel B: Student-5		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.200	0.433	0.701	0.134	0.323	0.524	0.080	0.195	0.304
	(0.171)	(0.389)	(0.772)	(0.105)	(0.277)	(0.499)	(0.057)	(0.161)	(0.282)
AGAUS	0.235	0.439	0.705	0.176	0.330	0.529	0.138	0.210	0.319
	(0.167)	(0.389)	(0.773)	(0.100)	(0.275)	(0.496)	(0.055)	(0.158)	(0.278)
BAYEST	0.214	0.443	0.648	0.146	0.301	0.450	0.098	0.176	0.254
	(0.150)	(0.333)	(0.572)	(0.087)	(0.233)	(0.395)	(0.048)	(0.129)	(0.201)
BAYESN	0.250	0.474	0.695	0.180	0.333	0.491	0.137	0.194	0.276
	(0.181)	(0.385)	(0.659)	(0.108)	(0.284)	(0.487)	(0.053)	(0.139)	(0.219)
BAYESL	0.321	0.485	0.612	0.266	0.398	0.507	0.185	0.237	0.311
	(0.239)	(0.369)	(0.417)	(0.174)	(0.293)	(0.353)	(0.099)	(0.179)	(0.241)
BOOT	0.241	0.404	0.633	0.169	0.305	0.481	0.113	0.203	0.312
	(0.156)	(0.358)	(0.655)	(0.091)	(0.251)	(0.459)	(0.050)	(0.150)	(0.263)
BOOTEX	0.294	0.513	0.737	0.216	0.372	0.556	0.144	0.231	0.343
	(0.206)	(0.368)	(0.545)	(0.138)	(0.273)	(0.424)	(0.085)	(0.166)	(0.266)
BOOTNP	0.535	0.827	0.991	0.453	0.714	0.896	0.356	0.504	0.668
	(0.311)	(0.477)	(0.504)	(0.309)	(0.438)	(0.501)	(0.224)	(0.325)	(0.409)
Panel C: $\chi^2_{(5)}$		T=50			T=100			T=300	
	h=1	h=6	h=12	h=1	h=6	h=12	h=1	h=6	h=12
AEST	0.194	0.439	0.722	0.134	0.321	0.518	0.079	0.195	0.305
	(0.149)	(0.389)	(0.800)	(0.106)	(0.255)	(0.440)	(0.060)	(0.164)	(0.284)
AGAUS	0.286	0.459	0.735	0.248	0.350	0.537	0.220	0.238	0.336
	(0.127)	(0.383)	(0.797)	(0.086)	(0.247)	(0.436)	(0.046)	(0.155)	(0.276)
BAYESN	0.297	0.497	0.734	0.252	0.357	0.504	0.219	0.232	0.310
	(0.136)	(0.376)	(0.715)	(0.088)	(0.253)	(0.412)	(0.044)	(0.143)	(0.233)
BAYESL	0.353	0.490	0.621	0.329	0.437	0.554	0.257	0.283	0.360
	(0.184)	(0.335)	(0.398)	(0.160)	(0.283)	(0.366)	(0.084)	(0.192)	(0.271)
BOOT	0.232	0.412	0.666	0.159	0.301	0.479	0.105	0.202	0.313
	(0.149)	(0.377)	(0.728)	(0.098)	(0.231)	(0.402)	(0.054)	(0.154)	(0.264)
BOOTEX	0.282	0.515	0.756	0.212	0.375	0.556	0.142	0.233	0.351
	(0.172)	(0.340)	(0.538)	(0.121)	(0.250)	(0.379)	(0.085)	(0.173)	(0.269)
BOOTNP	0.518	0.823	0.984	0.447	0.701	0.878	0.355	0.521	0.687
	(0.292)	(0.481)	(0.499)	(0.262)	(0.430)	(0.494)	(0.237)	(0.360)	(0.449)

Table A.28: Monte Carlo averages and standard errors (in parenthesis) of the forecast intervals constructed by the asymptotic procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ and σ^2_ε=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	AEST/AGAUS	78.48 (0.06)	10.65/10.87	74.91 (0.12)	12.37/12.72	71.66 (0.19)	13.95/14.39
	SRA_{aicc}	78.62 (0.07)	9.51/11.87	72.33 (0.12)	12.86/14.81	65.71 (0.16)	16.48/17.81
100	AEST/AGAUS	79.47 (0.04)	10.18/10.35	78.02 (0.07)	10.93/11.05	76.83 (0.11)	11.52/11.65
	SRA_{aicc}	79.17 (0.05)	9.68/11.15	75.62 (0.08)	11.68/12.70	72.10 (0.12)	13.50/14.40
300	AEST/AGAUS	79.95 (0.02)	10.09/9.96	79.46 (0.04)	10.49/10.05	79.11 (0.05)	10.72/10.17
	SRA_{aicc}	79.73 (0.03)	10.02/10.26	78.43 (0.04)	10.85/10.72	77.67 (0.07)	11.28/11.05
	Student-5	h=1		h=6		h=12	
50	AEST	77.66 (0.07)	11.08/11.26	74.04 (0.14)	13.12/12.84	71.43 (0.19)	14.71/13.86
	AGAUS	81.92 (0.07)	8.96/9.11	76.17 (0.14)	12.04/11.78	72.80 (0.18)	14.01/13.18
	SRA_{aicc}	78.63 (0.08)	9.70/11.67	73.14 (0.12)	12.79/14.06	65.31 (0.16)	16.69/18
100	AEST	78.67 (0.05)	10.64/10.68	77.07 (0.08)	11.57/11.36	75.80 (0.12)	12.25/11.95
	AGAUS	82.90 (0.04)	8.54/8.56	79.05 (0.08)	10.56/10.39	77.04 (0.12)	11.62/11.35
	SRA_{aicc}	78.79 (0.05)	9.93/11.28	75.58 (0.09)	12.04/12.38	72.44 (0.13)	13.50/14.06
300	AEST	79.50 (0.03)	10.26/10.25	79.11 (0.04)	10.49/10.40	78.87 (0.05)	10.62/10.51
	AGAUS	83.65 (0.03)	8.17/8.18	81.00 (0.04)	9.54/9.45	80.03 (0.05)	10.03/9.94
	SRA_{aicc}	79.47 (0.03)	10.13/10.39	78.50 (0.05)	10.61/10.89	77.88 (0.07)	11.09/11.03
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	78.32 (0.09)	11.10/10.57	74.96 (0.14)	12.63/12.41	71.69 (0.20)	14.02/14.29
	AGAUS	83.44 (0.08)	5.27/11.28	76.47 (0.13)	10.54/13	72.51 (0.19)	12.73/14.76
	SRA_{aicc}	78.49 (0.10)	10.50/11.00	72.93 (0.13)	12.86/14.21	65.22 (0.17)	16.09/18.69
100	AEST	79.31 (0.07)	10.47/10.22	78.09 (0.08)	11.01/10.90	76.88 (0.11)	11.51/11.61
	AGAUS	84.71 (0.05)	4.38/10.91	79.48 (0.08)	9.03/11.49	77.64 (0.11)	10.25/12.11
	SRA_{aicc}	79.19 (0.08)	9.97/10.83	76.25 (0.09)	11.57/12.19	72.97 (0.13)	13.31/13.72
300	AEST	79.95 (0.04)	9.98/10.07	79.55 (0.04)	10.21/10.23	79.20 (0.05)	10.41/10.39
	AGAUS	85.63 (0.03)	3.61/10.76	80.95 (0.04)	8.23/10.83	79.92 (0.05)	9.18/10.91
	SRA_{aicc}	79.42 (0.05)	10.32/10.27	78.50 (0.05)	10.89/10.62	77.74 (0.07)	11.25/11.01
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	AEST/AGAUS	93.85 (0.04)	3.02/3.13	90.67 (0.11)	4.51/4.82	86.79 (0.19)	6.29/6.92
	SRA_{aicc}	91.71 (0.05)	2.80/5.48	86.28 (0.09)	5.54/8.18	79.91 (0.14)	10.44/9.65
100	AEST/AGAUS	94.62 (0.02)	2.66/2.72	93.43 (0.05)	3.27/3.29	92.24 (0.08)	3.88/3.87
	SRA_{aicc}	93.77 (0.03)	2.52/3.71	90.98 (0.06)	3.99/5.02	85.99 (0.09)	6.42/7.59
300	AEST/AGAUS	94.94 (0.01)	2.55/2.52	94.60 (0.02)	2.79/2.61	94.37 (0.03)	2.92/2.72
	SRA_{aicc}	94.63 (0.02)	2.54/2.83	93.79 (0.03)	3.05/3.16	92.61 (0.05)	3.68/3.71
	Student-5	h=1		h=6		h=12	
50	AEST	93.86 (0.03)	3.04/3.10	90.53 (0.12)	4.77/4.70	87.38 (0.18)	6.61/6.00
	AGAUS	93.56 (0.03)	3.18/3.25	90.33 (0.12)	4.87/4.80	87.24 (0.18)	6.69/6.08
	SRA_{aicc}	92.29 (0.05)	2.74/4.97	86.77 (0.10)	5.52/7.70	80.40 (0.14)	10.31/9.29
100	AEST	94.38 (0.02)	2.79/2.82	93.16 (0.05)	3.47/3.37	91.80 (0.10)	4.08/4.13
	AGAUS	94.11 (0.02)	2.94/2.96	92.99 (0.05)	3.55/3.46	91.66 (0.10)	4.15/4.19
	SRA_{aicc}	94.14 (0.03)	2.44/3.42	91.05 (0.06)	4.02/4.93	86.52 (0.10)	6.17/7.31
300	AEST	94.79 (0.01)	2.60/2.61	94.51 (0.02)	2.76/2.73	94.27 (0.03)	2.85/2.88
	AGAUS	94.53 (0.01)	2.73/2.74	94.36 (0.02)	2.83/2.80	94.15 (0.03)	2.91/2.94
	SRA_{aicc}	94.68 (0.02)	2.50/2.82	93.95 (0.03)	2.93/3.12	92.88 (0.04)	3.49/3.63
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	AEST	92.55 (0.07)	4.55/2.91	90.21 (0.11)	5.60/4.18	86.44 (0.20)	7.19/6.36
	AGAUS	94.50 (0.03)	0.22/5.27	91.17 (0.11)	2.86/5.97	86.98 (0.19)	5.24/7.78
	SRA_{aicc}	91.94 (0.07)	3.54/4.53	86.91 (0.10)	5.50/7.58	80.12 (0.15)	10.36/9.52
100	AEST	93.72 (0.05)	3.63/2.65	93.14 (0.06)	3.80/3.06	92.01 (0.09)	4.32/3.67
	AGAUS	94.99 (0.02)	0.04/4.97	93.86 (0.05)	1.50/4.64	92.42 (0.09)	2.63/4.95
	SRA_{aicc}	93.81 (0.05)	2.93/3.26	91.29 (0.06)	4.09/4.63	86.60 (0.10)	6.26/7.14
300	AEST	94.63 (0.02)	2.83/2.54	94.52 (0.02)	2.84/2.63	94.32 (0.03)	2.93/2.75
	AGAUS	95.18 (0.01)	0.00/4.82	95.04 (0.02)	0.88/4.07	94.60 (0.03)	1.53/3.86
	SRA_{aicc}	94.48 (0.03)	2.81/2.72	93.97 (0.03)	3.07/2.96	92.84 (0.04)	3.65/3.51

Table A.29: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals constructed by the Bayesian procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverages of 80% and 95%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BAYESN	78.94 (0.06)	10.31/10.75	78.35 (0.10)	10.66/10.99	76.38 (0.12)	11.72/11.90
	BAYESL	79.84 (0.06)	9.81/10.35	78.15 (0.09)	10.73/11.12	75.23 (0.11)	12.25/12.52
100	BAYESN	79.49 (0.04)	10.21/10.30	79.17 (0.07)	10.40/10.43	77.92 (0.09)	11.05/11.03
	BAYESL	79.81 (0.05)	9.93/10.26	78.76 (0.08)	10.55/10.69	76.59 (0.10)	11.66/11.75
300	BAYESN	79.87 (0.03)	10.14/9.99	79.70 (0.04)	10.35/9.95	79.05 (0.05)	10.62/10.33
	BAYESL	80.11 (0.03)	9.93/9.96	79.85 (0.04)	10.12/10.03	78.91 (0.06)	10.62/10.47
	Student-5	h=1		h=6		h=12	
50	BAYEST	79.50 (0.06)	10.20/10.30	78.61 (0.10)	10.61/10.79	77.01 (0.12)	11.42/11.57
	BAYESN	82.28 (0.06)	8.81/8.91	79.30 (0.10)	10.32/10.39	76.94 (0.13)	11.58/11.48
	BAYESL	82.75 (0.07)	8.38/8.87	79.25 (0.10)	10.12/10.62	76.18 (0.12)	11.69/12.13
100	BAYEST	79.85 (0.04)	10.08/10.06	79.26 (0.06)	10.37/10.37	78.14 (0.09)	10.97/10.89
	BAYESN	82.89 (0.04)	8.49/8.62	80.28 (0.07)	9.86/9.86	78.40 (0.09)	10.91/10.69
	BAYESL	83.11 (0.05)	8.27/8.61	80.18 (0.07)	9.85/9.97	77.64 (0.09)	11.00/11.35
300	BAYEST	80.00 (0.03)	10.00/9.99	79.60 (0.04)	10.23/10.17	79.28 (0.05)	10.31/10.41
	BAYESN	83.61 (0.03)	8.18/8.21	81.16 (0.04)	9.41/9.43	80.12 (0.05)	9.91/9.97
	BAYESL	83.54 (0.03)	8.16/8.29	81.43 (0.04)	9.21/9.36	79.86 (0.06)	9.97/10.17
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	83.66 (0.07)	5.19/11.15	79.74 (0.11)	8.63/11.63	77.37 (0.13)	10.09/12.54
	BAYESL	83.51 (0.08)	5.81/10.68	79.35 (0.09)	9.27/11.38	76.11 (0.11)	11.14/12.76
100	BAYESN	84.46 (0.05)	4.65/10.89	80.53 (0.08)	8.54/10.92	78.62 (0.10)	9.81/11.58
	BAYESL	83.88 (0.06)	5.32/10.79	79.79 (0.08)	9.31/10.89	77.24 (0.10)	10.89/11.87
300	BAYESN	85.43 (0.03)	3.79/10.78	81.10 (0.04)	8.17/10.74	79.76 (0.05)	9.30/10.94
	BAYESL	84.96 (0.04)	4.35/10.69	81.12 (0.05)	8.19/10.69	79.46 (0.06)	9.48/11.05
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BAYESN	94.40 (0.03)	2.70/2.90	94.04 (0.06)	2.85/3.10	92.94 (0.08)	3.38/3.67
	BAYESL	94.86 (0.03)	2.47/2.67	93.86 (0.05)	2.98/3.16	91.96 (0.07)	3.91/4.13
100	BAYESN	94.72 (0.02)	2.63/2.65	94.44 (0.04)	2.79/2.77	93.67 (0.06)	3.16/3.16
	BAYESL	94.88 (0.02)	2.55/2.56	94.16 (0.04)	2.92/2.93	92.87 (0.06)	3.54/3.59
300	BAYESN	94.89 (0.01)	2.56/2.54	94.81 (0.02)	2.65/2.53	94.46 (0.03)	2.84/2.70
	BAYESL	95.00 (0.01)	2.49/2.51	94.86 (0.02)	2.60/2.54	94.31 (0.03)	2.87/2.81
	Student-5	h=1		h=6		h=12	
50	BAYEST	94.40 (0.03)	2.81/2.79	94.36 (0.05)	2.78/2.86	93.64 (0.07)	3.11/3.25
	BAYESN	93.98 (0.03)	2.98/3.03	93.68 (0.05)	3.14/3.18	92.71 (0.08)	3.61/3.68
	BAYESL	94.39 (0.03)	2.71/2.90	93.60 (0.05)	3.11/3.28	91.93 (0.07)	3.94/4.13
100	BAYEST	94.55 (0.02)	2.70/2.75	94.64 (0.03)	2.69/2.66	94.08 (0.05)	2.93/2.98
	BAYESN	94.21 (0.02)	2.89/2.90	94.13 (0.04)	2.95/2.92	93.43 (0.05)	3.34/3.23
	BAYESL	94.45 (0.02)	2.74/2.81	94.04 (0.04)	2.96/3.00	92.83 (0.06)	3.49/3.67
300	BAYEST	94.80 (0.01)	2.59/2.60	94.77 (0.02)	2.62/2.61	94.59 (0.03)	2.68/2.72
	BAYESN	94.53 (0.01)	2.73/2.74	94.50 (0.02)	2.75/2.75	94.29 (0.03)	2.81/2.90
	BAYESL	94.56 (0.01)	2.69/2.74	94.66 (0.02)	2.63/2.71	94.21 (0.03)	2.84/2.94
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BAYESN	94.85 (0.03)	0.16/4.99	94.25 (0.05)	1.50/4.24	93.17 (0.07)	2.31/4.52
	BAYESL	95.13 (0.03)	0.31/4.56	94.13 (0.05)	1.79/4.08	92.34 (0.07)	2.95/4.71
100	BAYESN	95.05 (0.02)	0.05/4.90	94.72 (0.04)	1.25/4.03	93.76 (0.06)	2.10/4.14
	BAYESL	95.22 (0.02)	0.14/4.64	94.45 (0.04)	1.61/3.94	93.00 (0.06)	2.66/4.34
300	BAYESN	95.18 (0.01)	0.00/4.82	95.16 (0.02)	0.88/3.96	94.62 (0.03)	1.56/3.82
	BAYESL	95.23 (0.02)	0.02/4.76	95.10 (0.02)	0.99/3.90	94.49 (0.03)	1.68/3.83

Table A.30: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverage of 80%.

		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	80%	10%/10%	80%	10%/10%	80%	10%/10%
50	BOOT	77.60 (0.06)	10.96/11.43	76.80 (0.10)	11.45/11.75	75.64 (0.13)	12.01/12.35
	BOOTEX	78.51 (0.07)	10.47/11.03	82.96 (0.09)	8.33/8.71	82.80 (0.12)	8.41/8.79
	BOOTNP	70.47 (0.16)	14.79/14.74	63.74 (0.13)	18.56/17.70	57.55 (0.12)	21.78/20.68
100	BOOT	78.76 (0.04)	10.52/10.73	78.71 (0.07)	10.63/10.67	78.43 (0.09)	10.72/10.85
	BOOTEX	79.26 (0.04)	10.10/10.64	82.46 (0.07)	8.61/8.92	82.64 (0.09)	8.51/8.85
	BOOTNP	73.09 (0.16)	13.22/13.69	68.52 (0.14)	15.81/15.67	63.47 (0.14)	18.47/18.06
300	BOOT	79.68 (0.03)	10.26/10.06	79.72 (0.04)	10.36/9.914	79.68 (0.05)	10.40/9.93
	BOOTEX	79.77 (0.03)	10.19/10.04	81.39 (0.04)	9.49/9.13	81.77 (0.05)	9.34/8.89
	BOOTNP	79.45 (0.11)	10.28/10.27	74.83 (0.10)	12.83/12.34	70.59 (0.11)	15.17/14.25
	Student-5	h=1		h=6		h=12	
50	BOOT	78.34 (0.06)	10.79/10.87	77.04 (0.11)	11.56/11.4	75.82 (0.14)	12.24/11.94
	BOOTEX	78.94 (0.07)	10.51/10.54	82.83 (0.10)	8.68/8.49	82.50 (0.13)	8.85/8.65
	BOOTNP	69.82 (0.18)	14.96/15.21	62.63 (0.14)	18.75/18.62	56.07 (0.14)	22.12/21.80
100	BOOT	78.86 (0.05)	10.60/10.53	78.36 (0.08)	10.96/10.68	77.96 (0.10)	11.21/10.83
	BOOTEX	79.46 (0.05)	10.17/10.36	82.10 (0.07)	9.02/8.88	82.23 (0.09)	8.93/8.83
	BOOTNP	73.50 (0.17)	13.09/13.41	67.81 (0.15)	16.23/15.96	62.19 (0.14)	19.28/18.53
300	BOOT	79.60 (0.03)	10.19/10.20	79.62 (0.04)	10.24/10.15	79.69 (0.05)	10.24/10.07
	BOOTEX	79.73 (0.03)	10.10/10.17	81.27 (0.04)	9.35/9.38	81.75 (0.05)	9.07/9.18
	BOOTNP	79.89 (0.11)	10.02/10.09	74.55 (0.11)	13.00/12.45	70.42 (0.11)	15.17/14.42
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	78.08 (0.08)	11.05/10.87	77.21 (0.11)	11.28/11.51	75.91 (0.14)	11.69/12.41
	BOOTEX	79.45 (0.09)	9.86/10.69	83.87 (0.10)	7.00/9.13	83.25 (0.12)	7.29/9.46
	BOOTNP	70.03 (0.18)	15.32/14.66	62.98 (0.14)	19.49/17.53	56.49 (0.14)	22.75/20.76
100	BOOT	78.84 (0.06)	10.71/10.46	79.02 (0.08)	10.44/10.54	78.68 (0.10)	10.46/10.86
	BOOTEX	80.06 (0.07)	9.39/10.55	82.71 (0.07)	8.15/9.13	82.69 (0.09)	8.23/9.08
	BOOTNP	74.03 (0.17)	11.95/14.02	68.60 (0.14)	16.04/15.36	63.29 (0.14)	18.94/17.77
300	BOOT	79.56 (0.04)	10.19/10.25	79.93 (0.04)	9.99/10.08	79.80 (0.05)	10.07/10.13
	BOOTEX	79.92 (0.05)	9.88/10.20	81.66 (0.05)	8.85/9.49	81.93 (0.06)	8.75/9.32
	BOOTNP	79.67 (0.14)	8.59/11.74	73.92 (0.12)	13.05/13.03	69.82 (0.12)	15.52/14.66

Table A.31: Monte Carlo averages and standard errors (in parenthesis) of forecast intervals by the bootstrap procedures for model $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ and σ_ε^2=1 with nominal coverage of 95%.

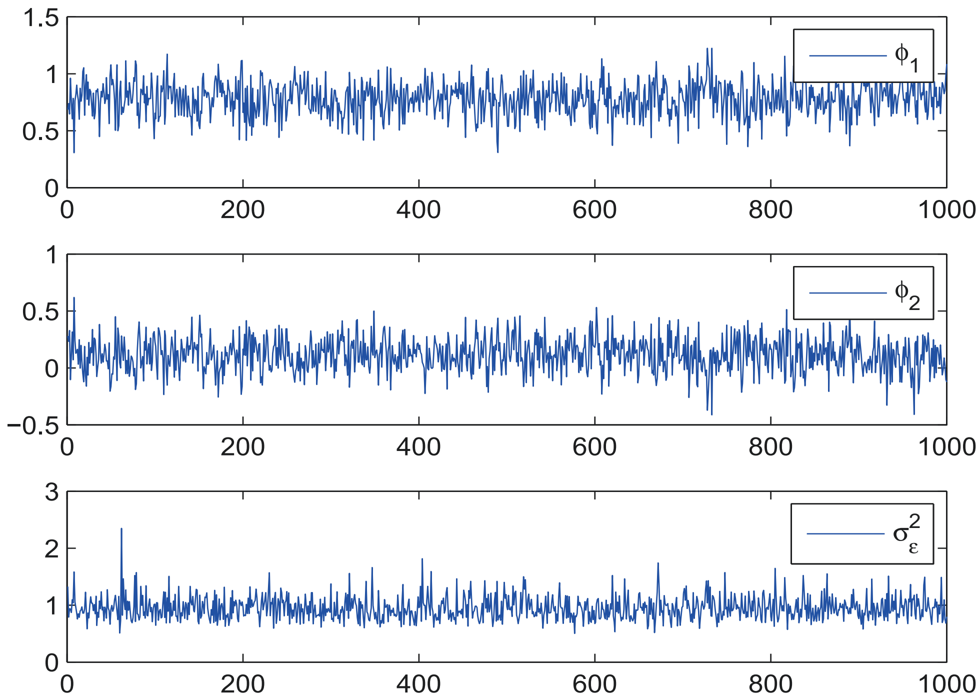
		h=1		h=6		h=12	
Sample size	Method	Coverage	Coverage below/above	Coverage	Coverage below/above	Coverage	Coverage below/above
	Gaussian	95%	2.5%/2.5%	95%	2.5%/2.5%	95%	2.5%/2.5%
50	BOOT	92.77 (0.04)	3.53/3.71	92.75 (0.07)	3.52/3.73	92.06 (0.10)	3.83/4.11
	BOOTEX	93.28 (0.04)	3.23/3.49	95.79 (0.05)	1.96/2.25	95.30 (0.07)	2.18/2.51
	BOOTNP	84.20 (0.14)	8.05/7.75	80.24 (0.11)	10.01/9.76	74.34 (0.12)	13.11/12.55
100	BOOT	93.81 (0.03)	3.08/3.12	94.05 (0.04)	2.98/2.97	93.85 (0.06)	3.05/3.11
	BOOTEX	94.09 (0.03)	2.89/3.02	95.79 (0.04)	2.06/2.15	95.74 (0.05)	2.04/2.22
	BOOTNP	87.50 (0.14)	6.20/6.29	84.97 (0.12)	7.67/7.36	80.70 (0.12)	9.78/9.52
300	BOOT	94.60 (0.02)	2.73/2.67	94.75 (0.02)	2.70/2.55	94.80 (0.03)	2.69/2.51
	BOOTEX	94.67 (0.02)	2.71/2.62	95.54 (0.02)	2.30/2.16	95.70 (0.03)	2.20/2.09
	BOOTNP	93.03 (0.08)	3.57/3.40	91.03 (0.08)	4.64/4.32	88.14 (0.09)	6.11/5.75
	Student-5	h=1		h=6		h=12	
50	BOOT	92.99 (0.04)	3.48/3.53	92.61 (0.07)	3.71/3.69	91.85 (0.11)	4.08/4.08
	BOOTEX	93.32 (0.04)	3.32/3.36	95.15 (0.05)	2.49/2.37	95.00 (0.07)	2.52/2.48
	BOOTNP	83.14 (0.16)	8.43/8.44	80.08 (0.12)	10.18/9.75	73.91 (0.13)	13.29/12.81
100	BOOT	93.82 (0.03)	3.18/3.00	93.69 (0.05)	3.26/3.05	93.43 (0.06)	3.34/3.23
	BOOTEX	94.03 (0.03)	3.06/2.91	95.38 (0.03)	2.39/2.23	95.45 (0.04)	2.31/2.25
	BOOTNP	87.30 (0.15)	5.90/6.79	85.04 (0.12)	7.67/7.28	80.21 (0.13)	10.19/9.60
300	BOOT	94.58 (0.02)	2.73/2.69	94.63 (0.02)	2.76/2.61	94.62 (0.03)	2.73/2.65
	BOOTEX	94.58 (0.02)	2.69/2.72	95.34 (0.02)	2.35/2.31	95.51 (0.02)	2.23/2.26
	BOOTNP	92.59 (0.08)	3.64/3.77	90.89 (0.08)	4.70/4.41	88.26 (0.09)	5.97/5.78
	$\chi^2_{(5)}$	h=1		h=6		h=12	
50	BOOT	93.43 (0.05)	3.00/3.57	93.03 (0.08)	3.22/3.75	92.19 (0.10)	3.52/4.29
	BOOTEX	94.48 (0.05)	2.00/3.51	95.90 (0.05)	1.55/2.56	95.40 (0.07)	1.92/2.68
	BOOTNP	83.09 (0.16)	8.83/8.07	79.40 (0.13)	11.71/8.89	73.72 (0.13)	14.84/11.43
100	BOOT	94.04 (0.04)	2.94/3.01	94.26 (0.05)	2.76/2.98	93.93 (0.06)	2.89/3.18
	BOOTEX	94.97 (0.04)	1.97/3.06	95.87 (0.04)	1.75/2.38	95.62 (0.05)	1.98/2.39
	BOOTNP	87.95 (0.14)	5.05/6.99	85.23 (0.12)	8.06/6.71	80.62 (0.12)	10.64/8.74
300	BOOT	94.62 (0.02)	2.67/2.72	94.81 (0.02)	2.54/2.66	94.82 (0.03)	2.53/2.64
	BOOTEX	95.06 (0.03)	2.20/2.73	95.68 (0.02)	1.96/2.36	95.79 (0.03)	1.93/2.28
	BOOTNP	92.68 (0.12)	2.53/4.79	90.27 (0.10)	5.11/4.62	87.47 (0.10)	6.86/5.67

A.7. Convergence diagnosis of BAYESN

A.7.1. DGP: $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with $\varepsilon \sim N(0,1)$.

Figure A.4: Time series plot of the parameters of model BAYESN. Burning=1000.

T=50



T=300

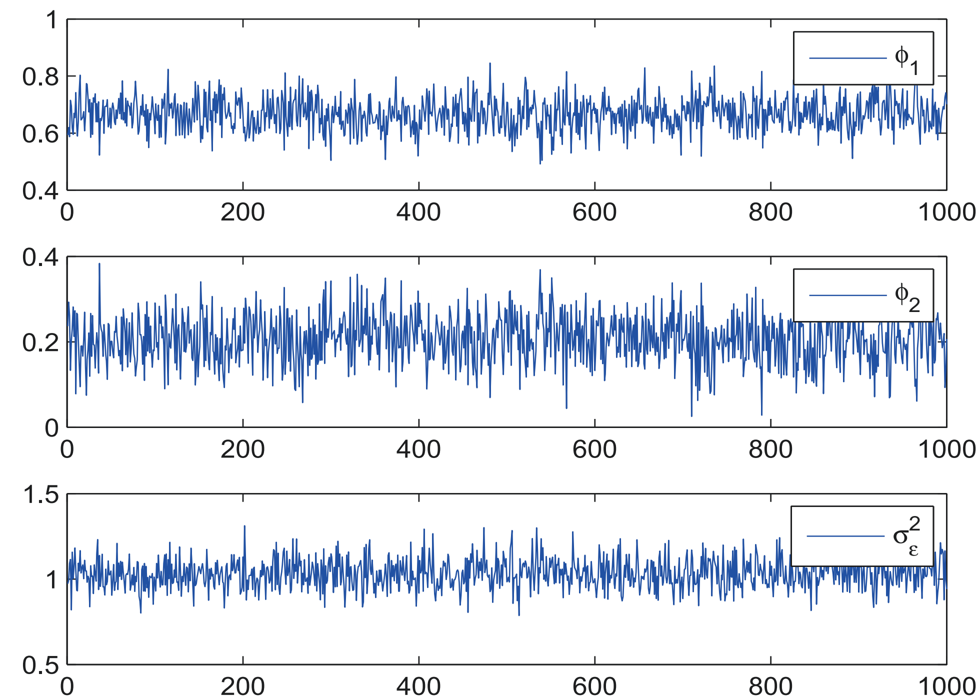
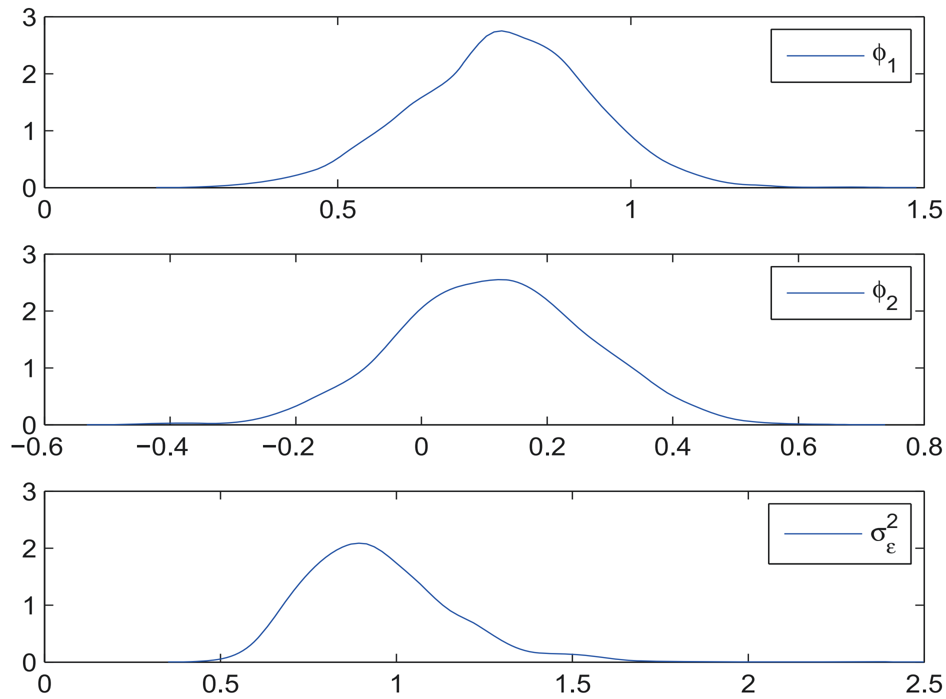


Figure A.5: Kernel density of the parameters of model BAYESN. Burning=1000.

T=50



T=300

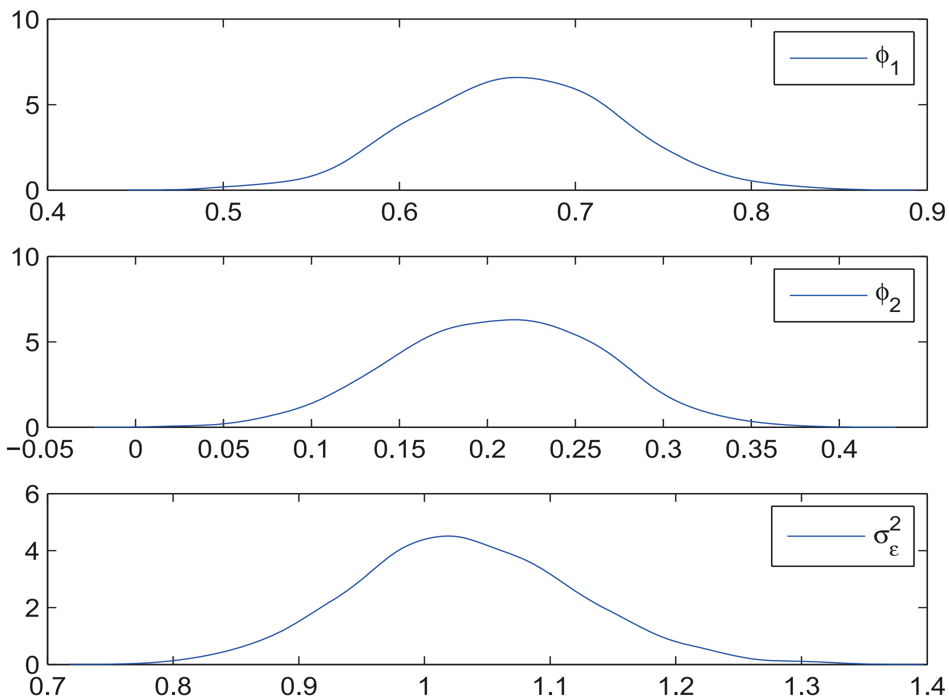
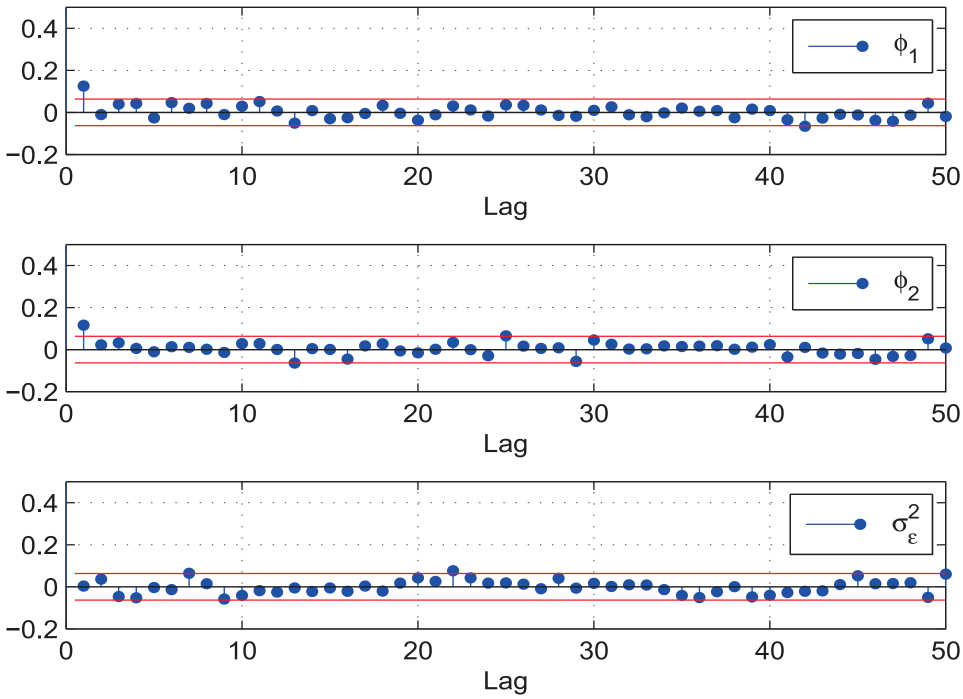
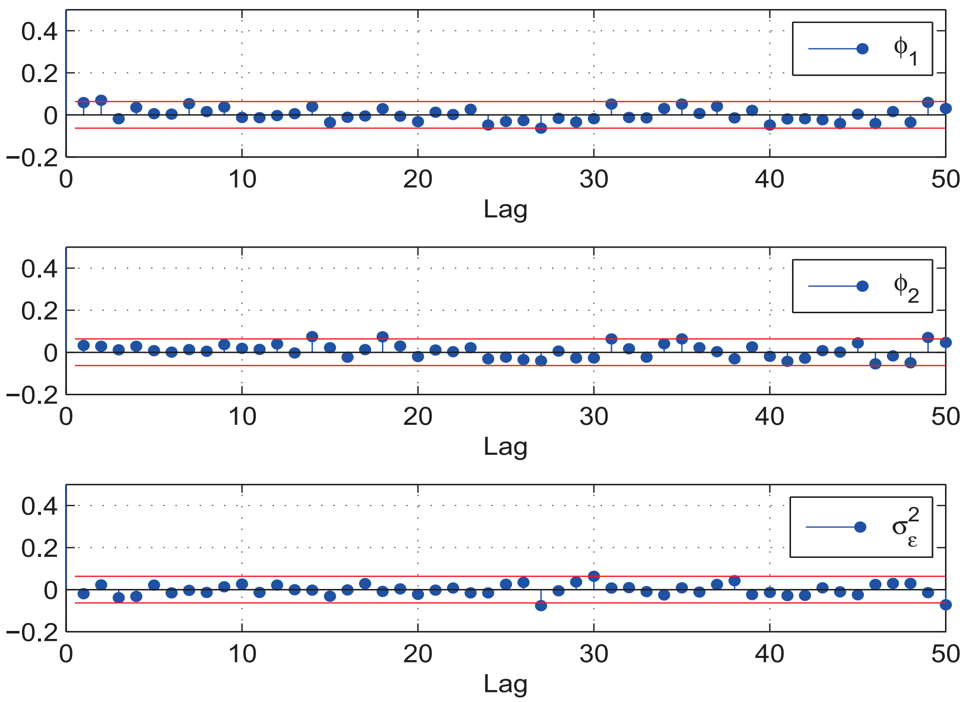


Figure A.6: Autocorrelation function of the parameters of model BAYESN. Burning=1000.

T=50



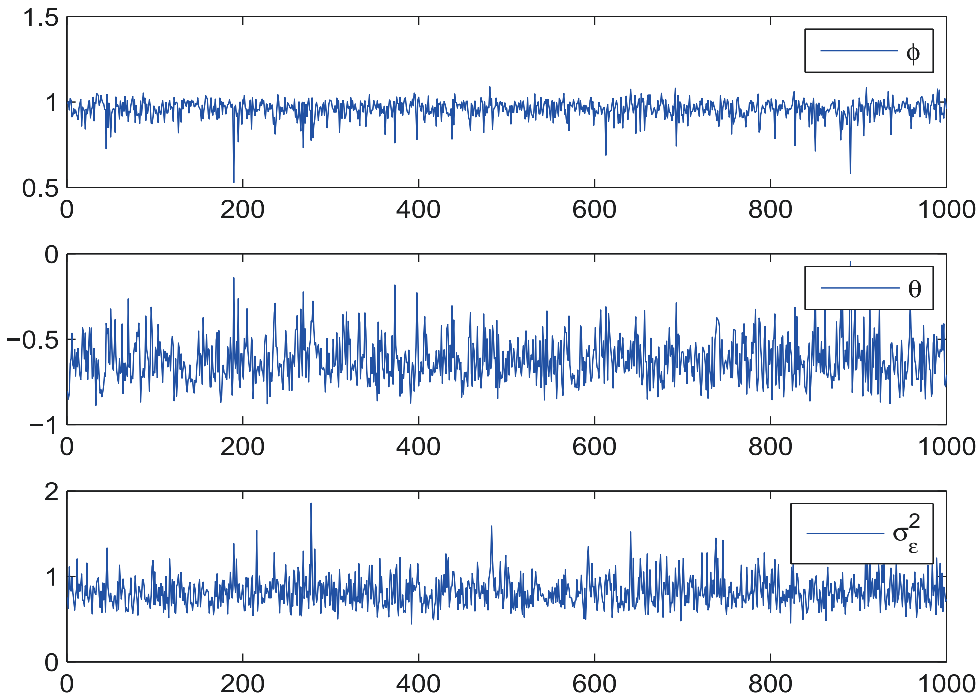
T=300



A.7.2. DGP: $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with $\varepsilon \sim N(0,1)$.

Figure A.7: Time series plot of the parameters of model BAYESN. Burning=1000.

T=50



T=300

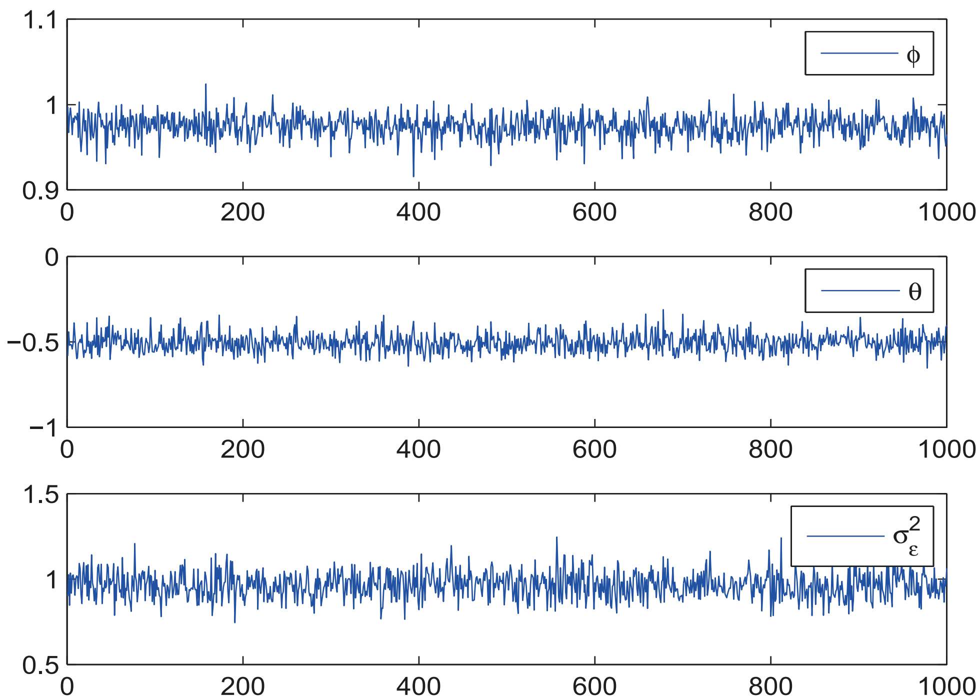
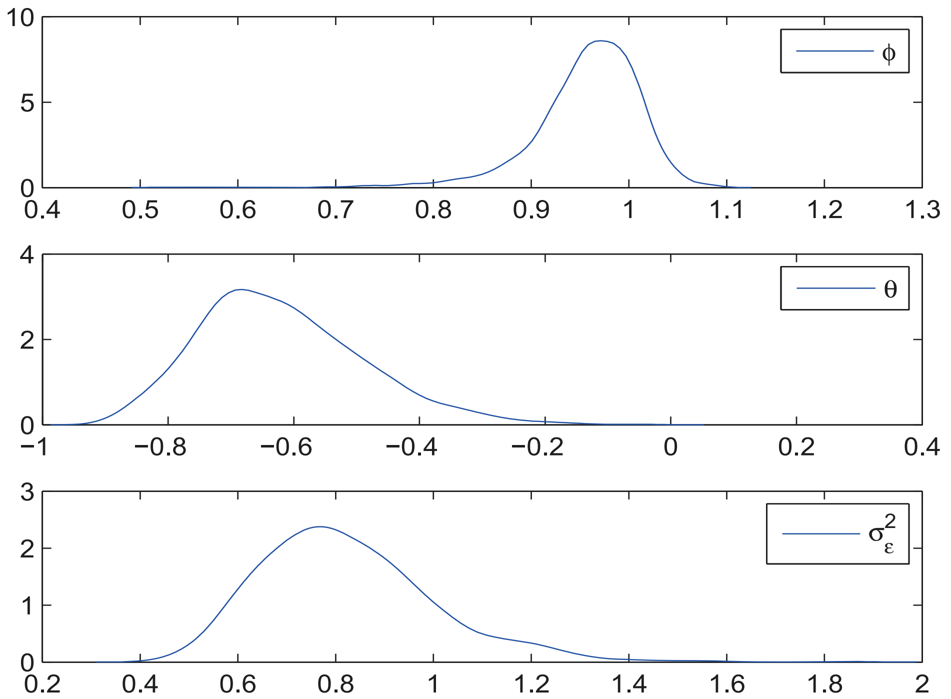


Figure A.8: Kernel density of the parameters of model BAYESN. Burning=1000.

T=50



T=300

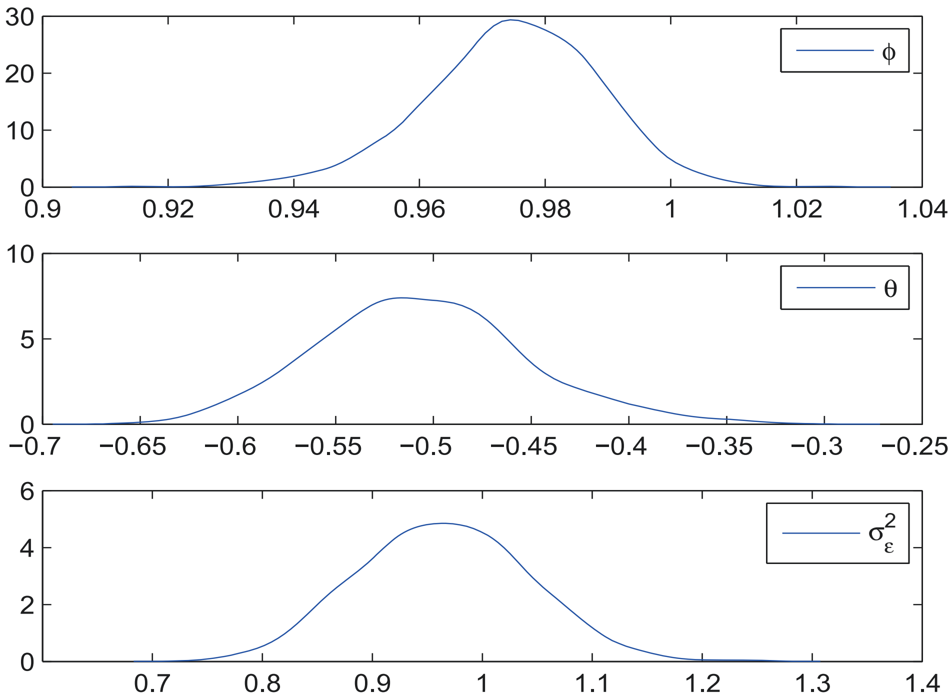
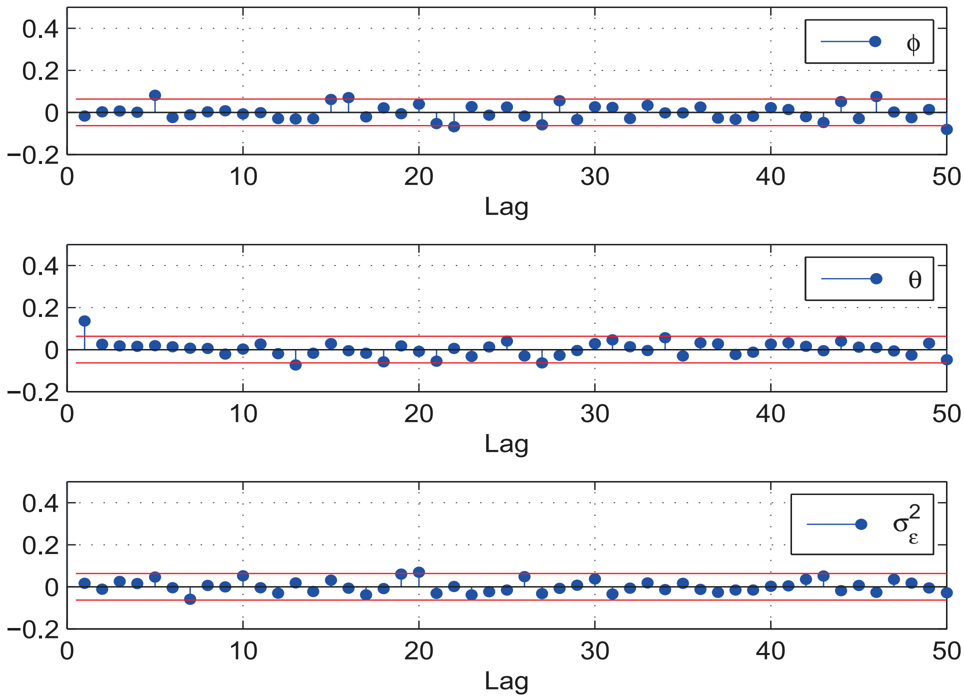
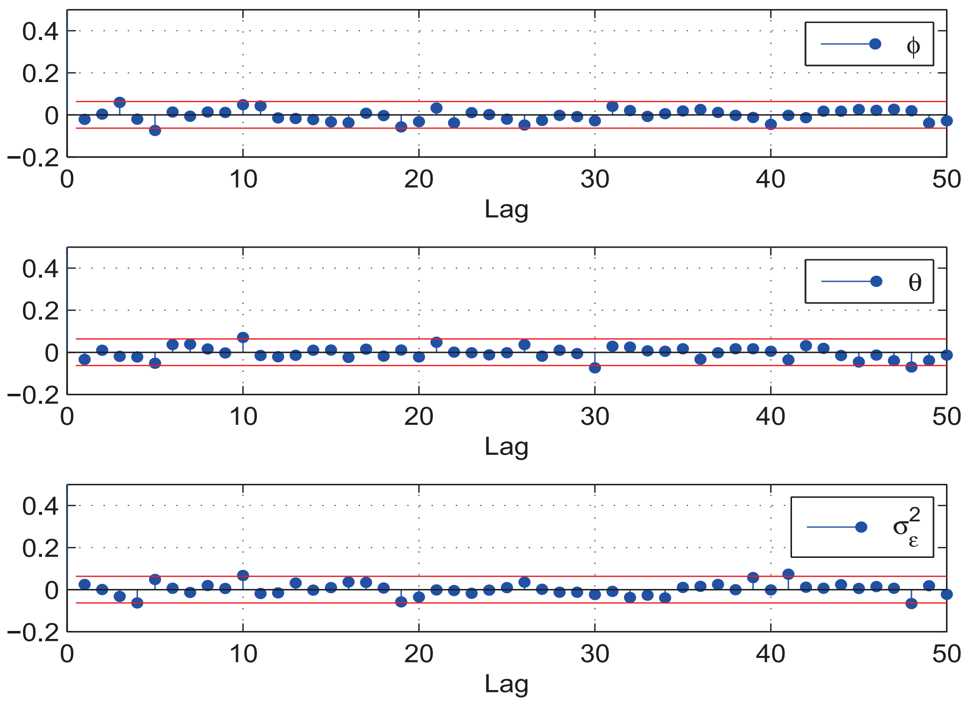


Figure A.9: Autocorrelation function of the parameters of model BAYESN. Burning=1000.

T=50



T=300

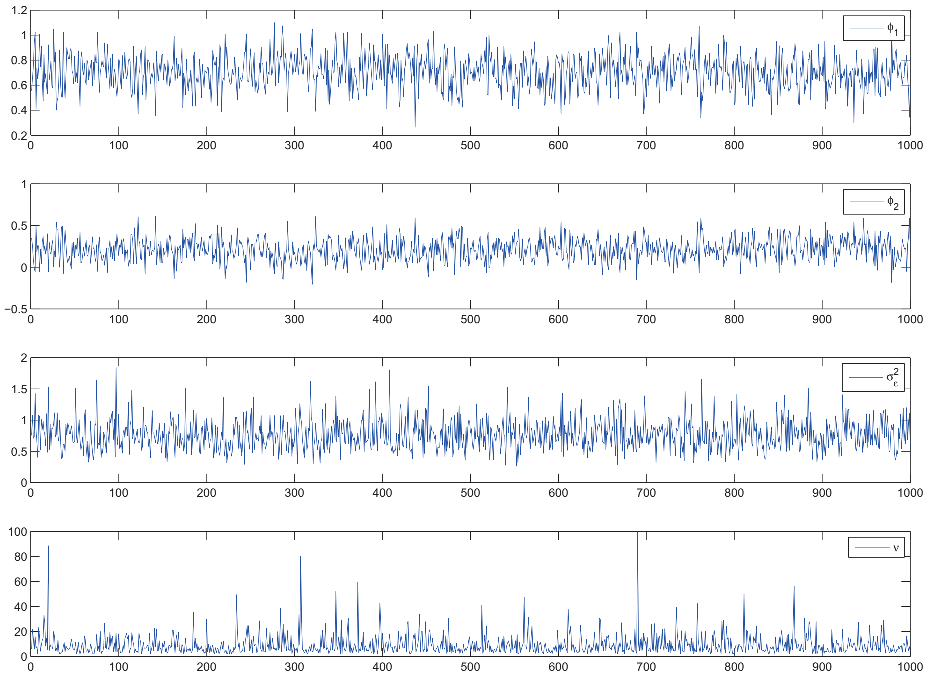


A.8. Convergence diagnosis of model BAYEST.

A.8.1. DGP: $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with $\varepsilon \sim Student - 5$.

Figure A.10: Time series plot of the parameters of model BAYEST. Burning=1000.

T=50



T=300

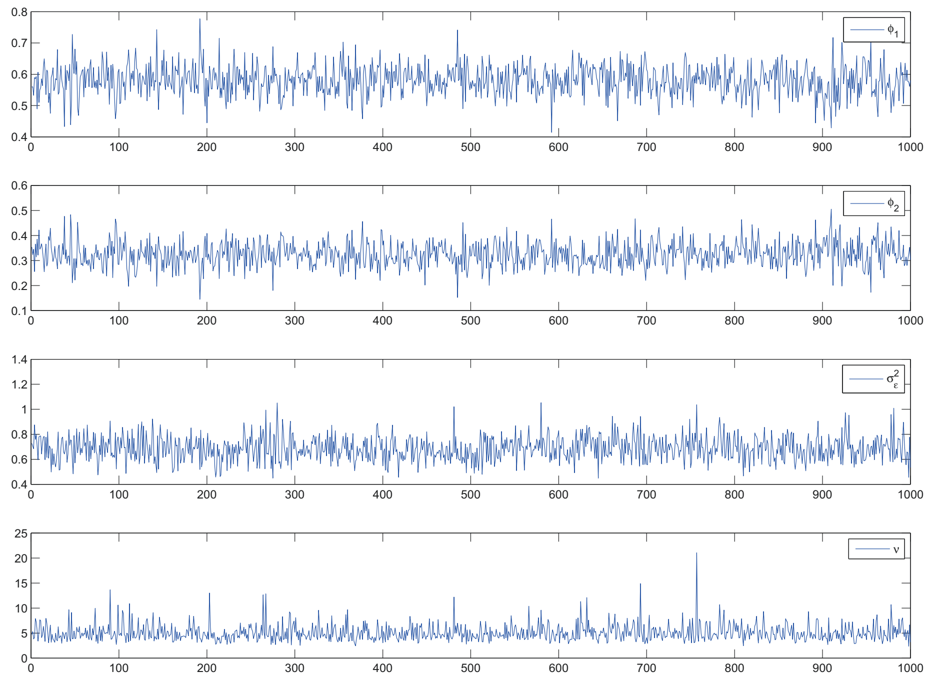
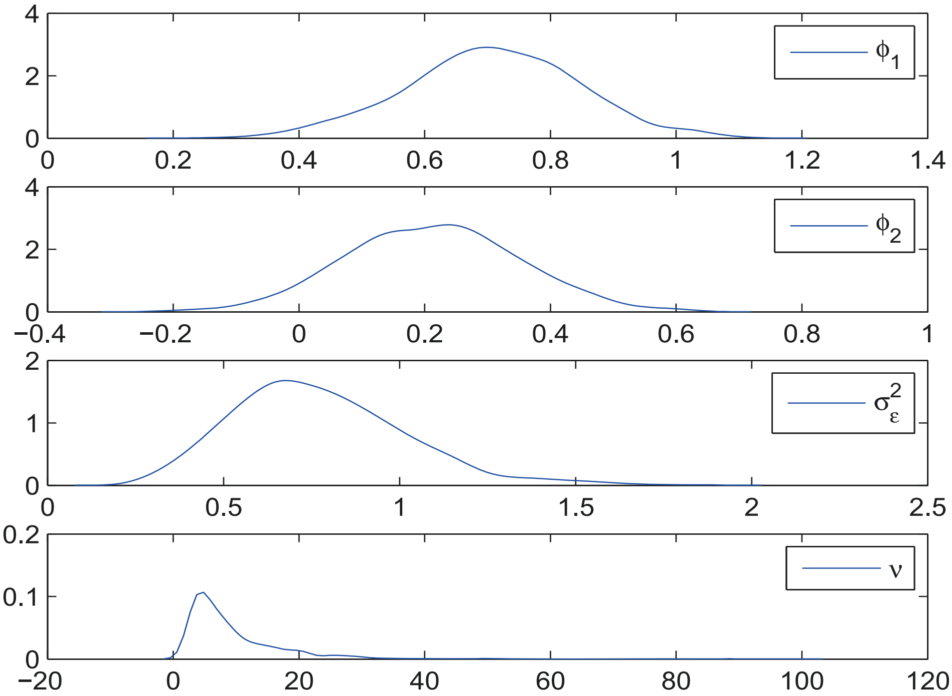


Figure A.11: Kernel density of the parameters of model BAYEST. Burning=1000.

T=50



T=300

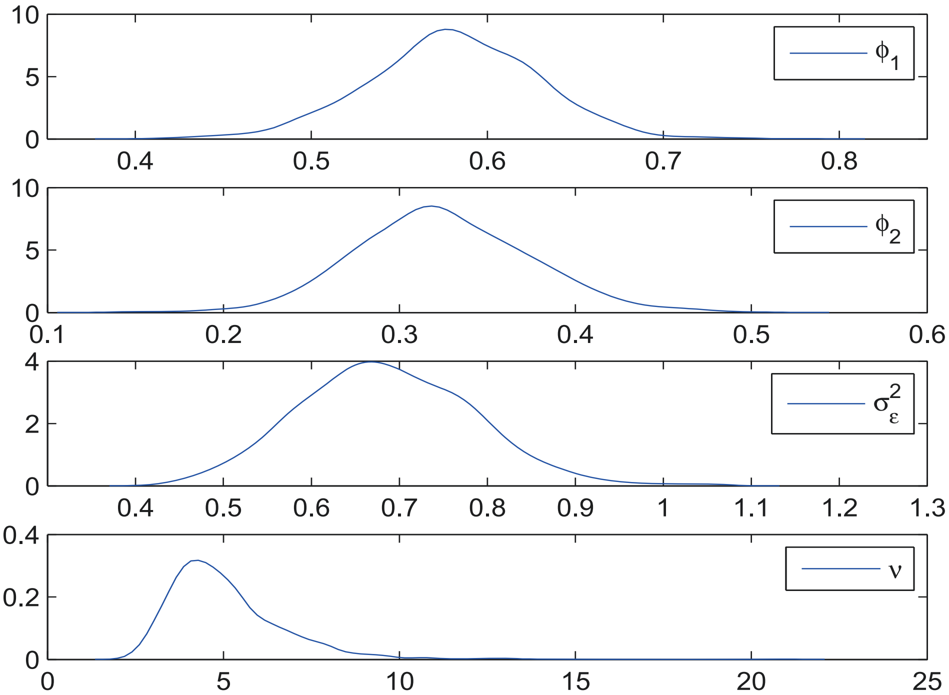
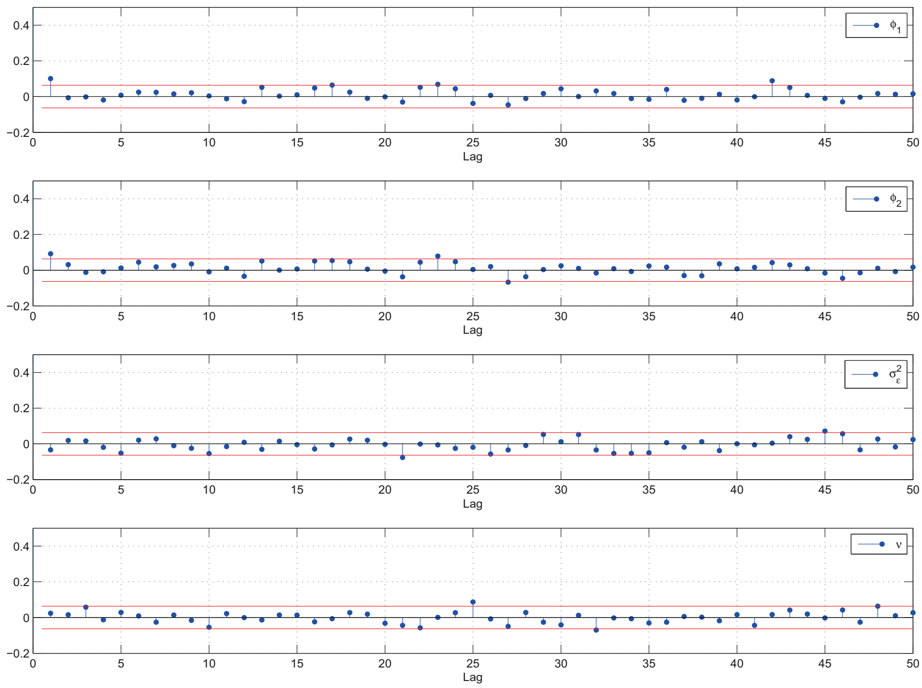
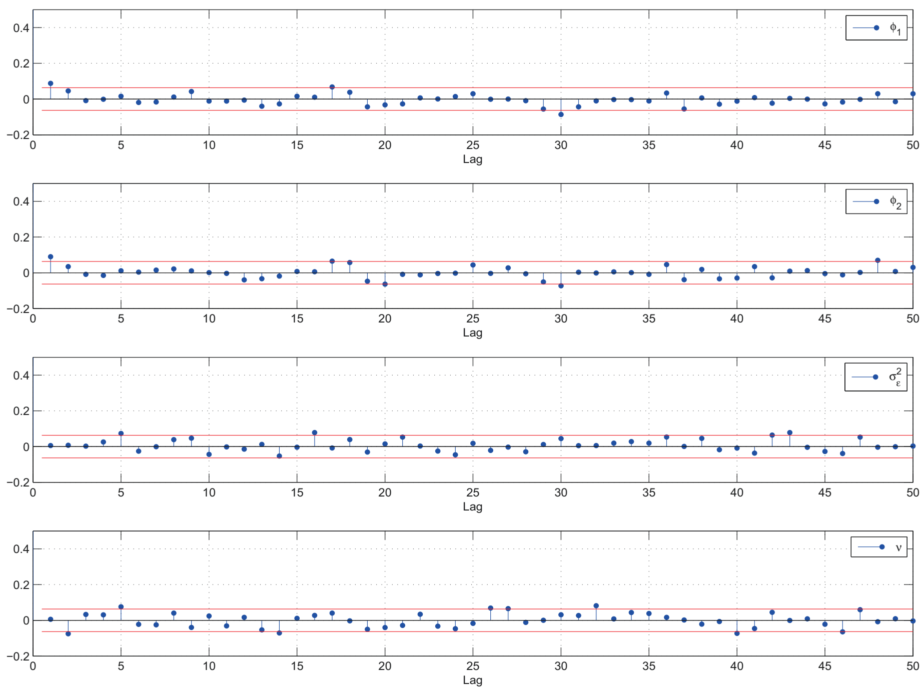


Figure A.12: Autocorrelation function of the parameters of model BAYEST. Burning=1000.

T=50



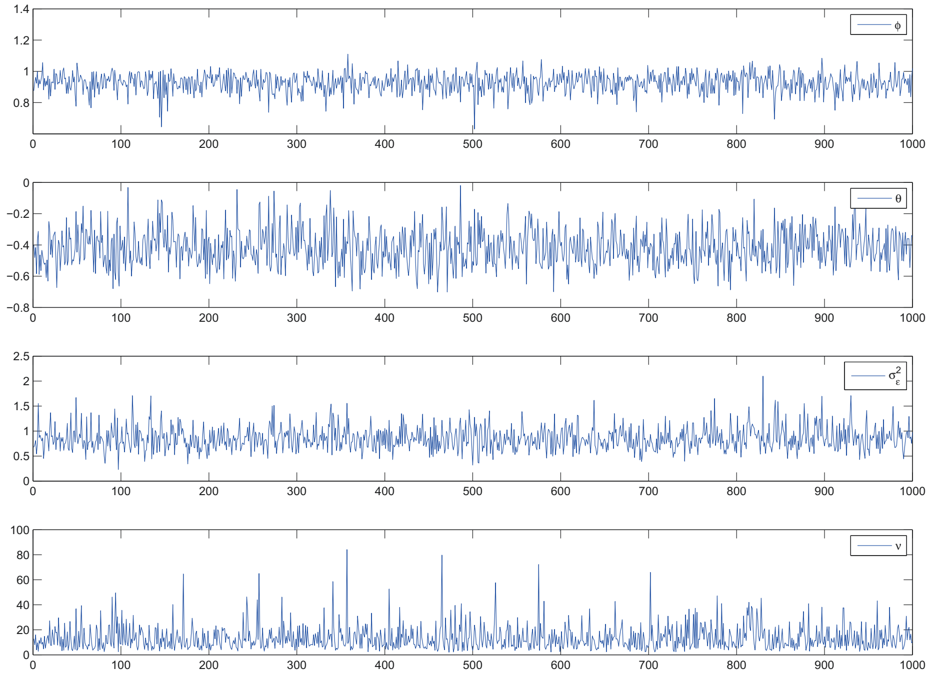
T=300



A.8.2. DGP: $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with $\varepsilon \sim Student - 5$.

Figure A.13: Time series plot of the parameters of model BAYEST. Burning=1000.

T=50



T=300

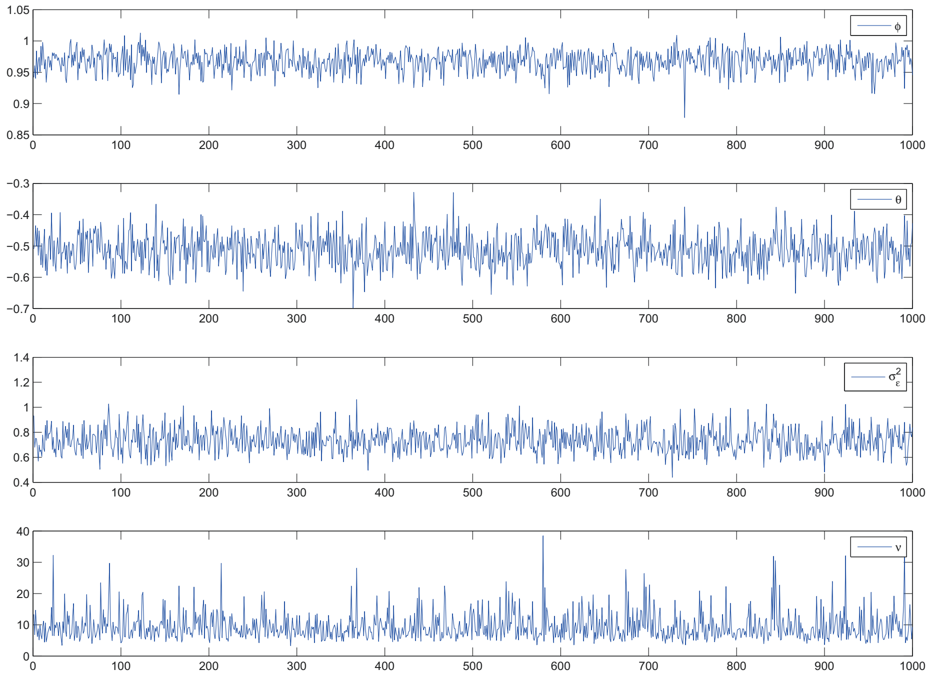
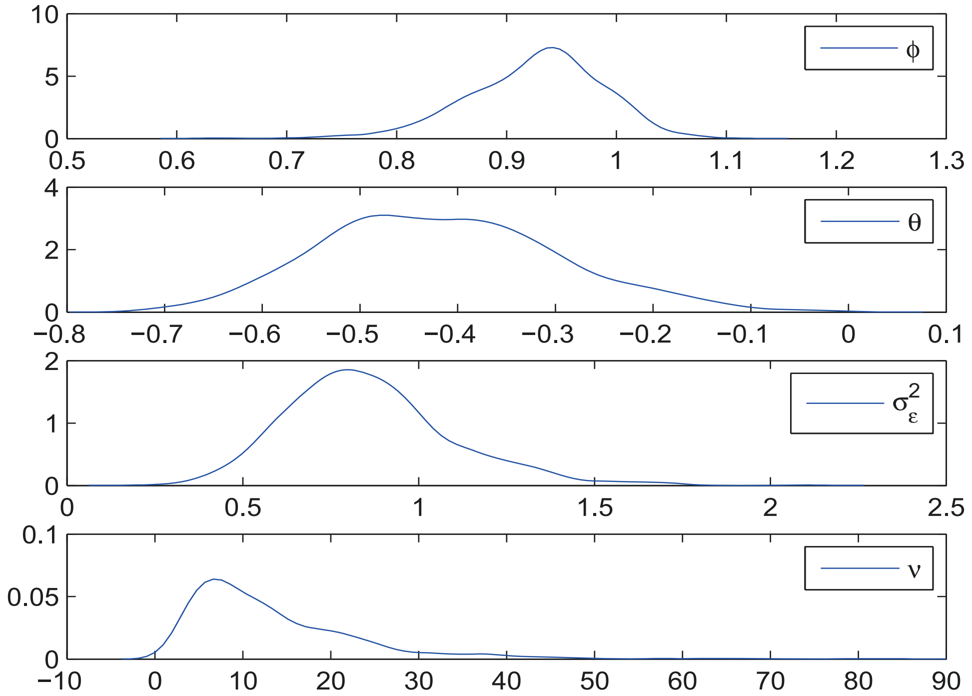


Figure A.14: Kernel density of the parameters of model BAYEST. Burning=1000.

T=50



T=300

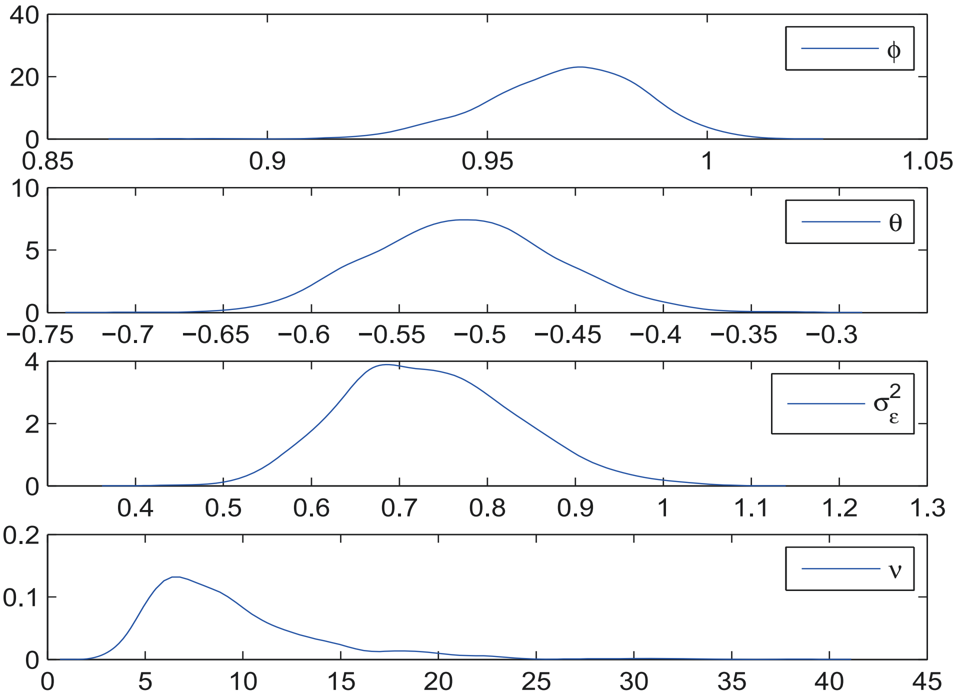
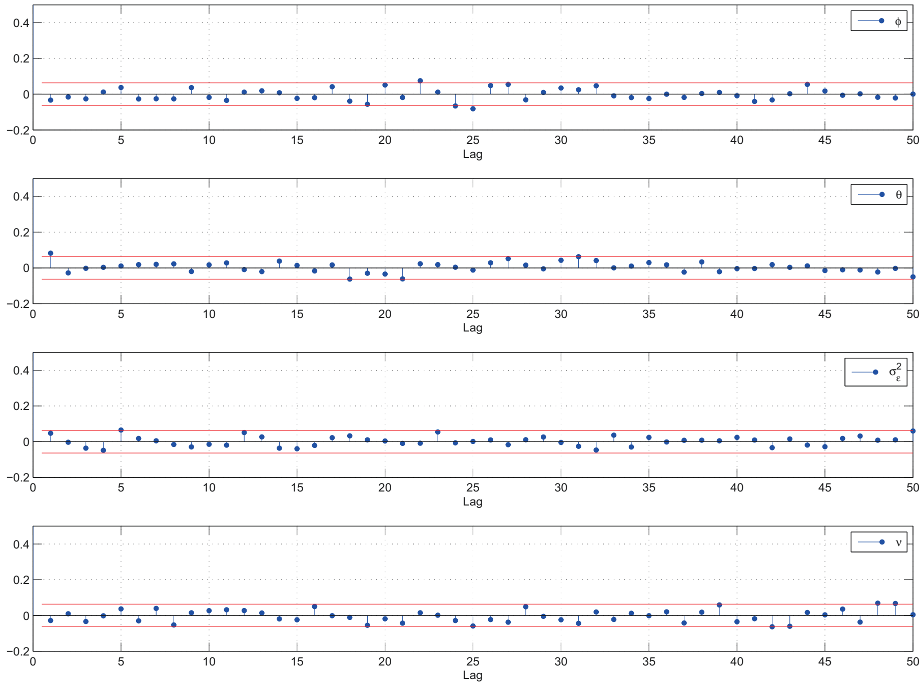
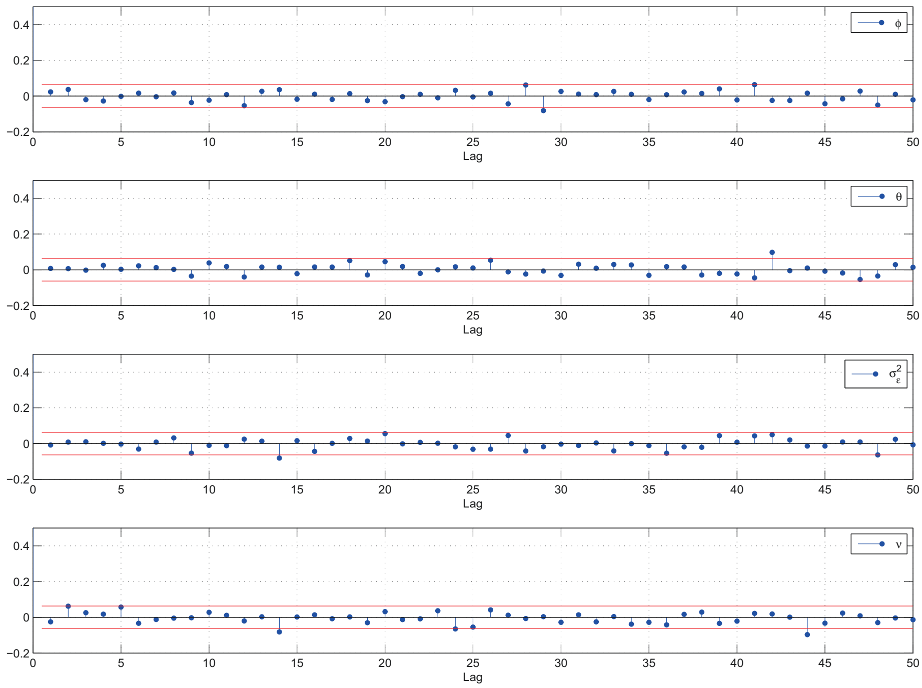


Figure A.15: Autocorrelation function of the parameters of model BAYEST. Burning=1000.

T=50



T=300



A.9. Convergence diagnosis: model BAYESL.

A.9.1. DGP: $y_t = 0.6y_{t-1} + 0.3y_{t-2} + \varepsilon_t$ with $\varepsilon \sim N(0, 1)$.

Figure A.16: Time series plot of the parameters of model BAYESL. T=50 and burning=1000.

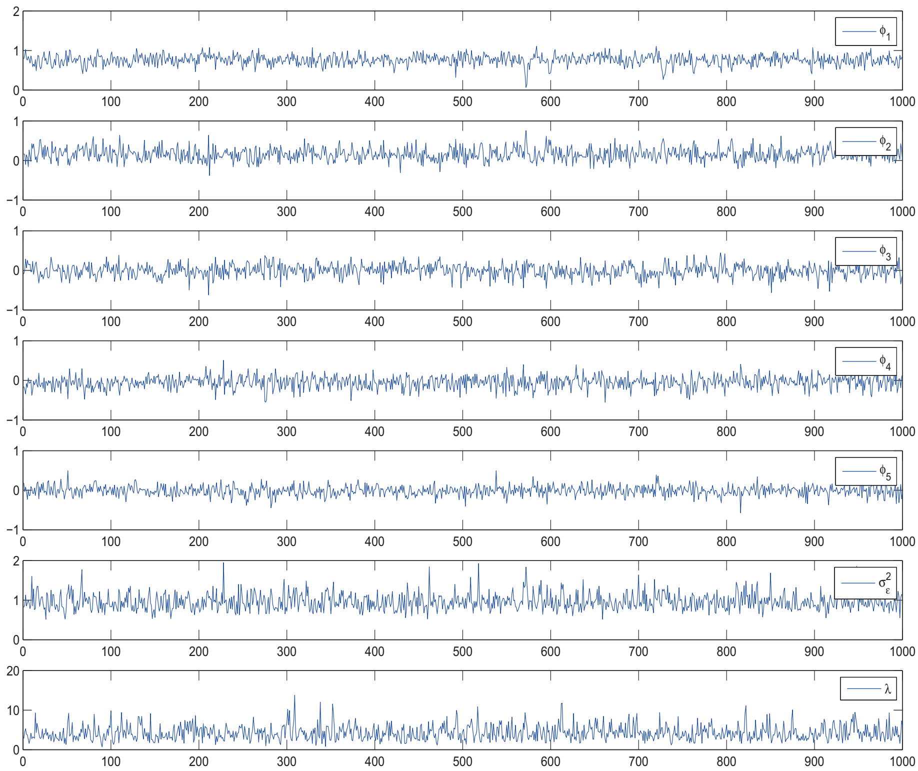


Figure A.17: Kernel density of the parameters of model BAYESL. T=50 and burning=1000.

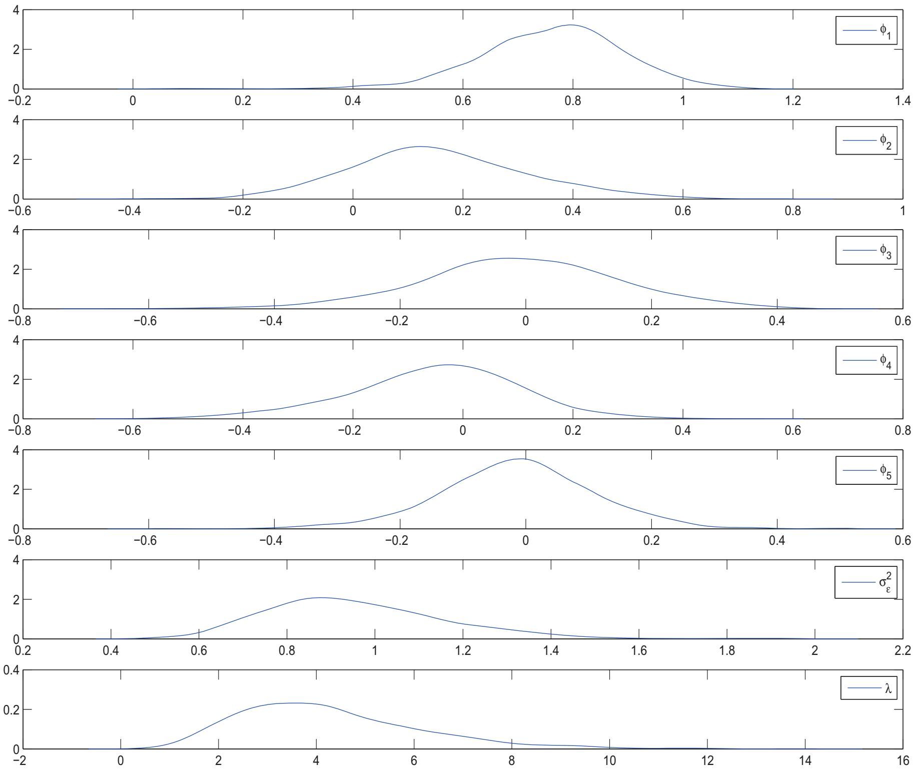


Figure A.18: Autocorrelation function of the parameters of model BAYESL. T=50 and burning=1000.

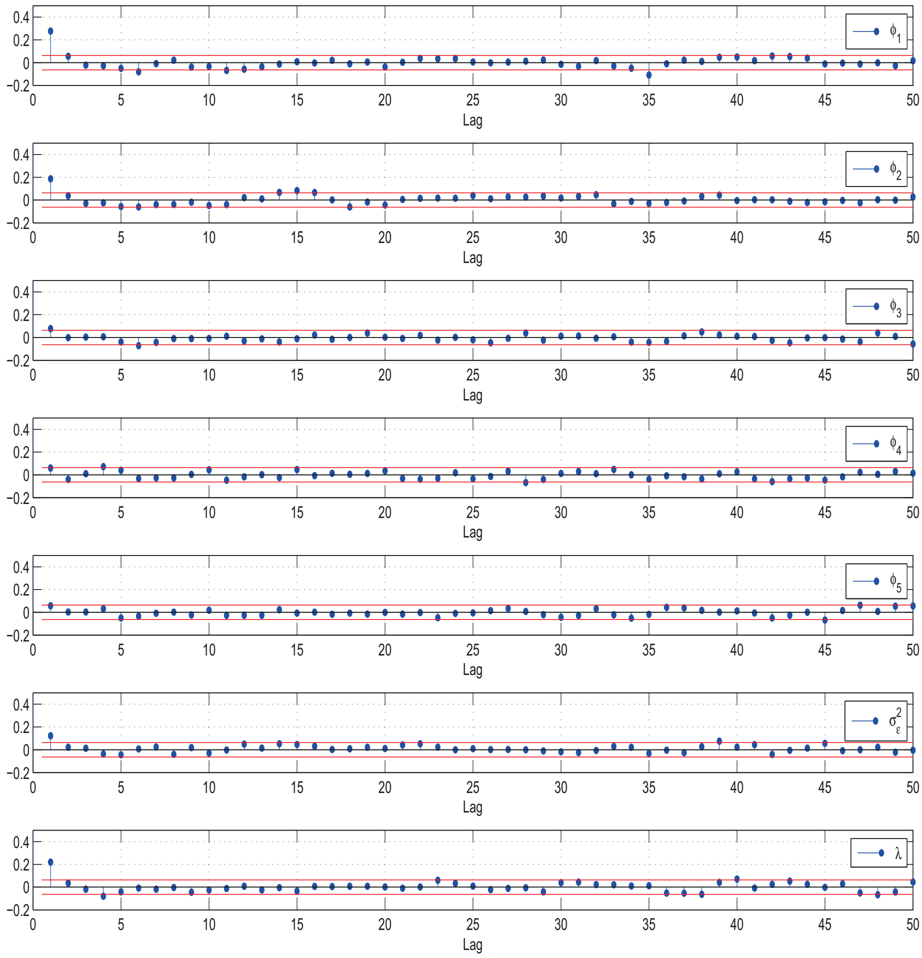
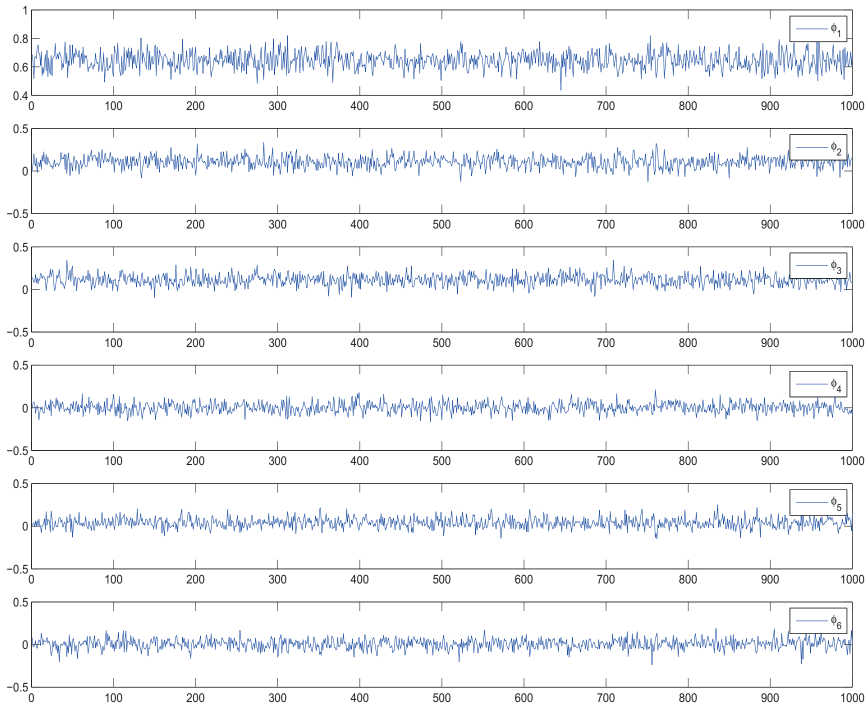


Figure A.19: Time series plot of the parameters of model BAYESL. T=300 and burning=1000.



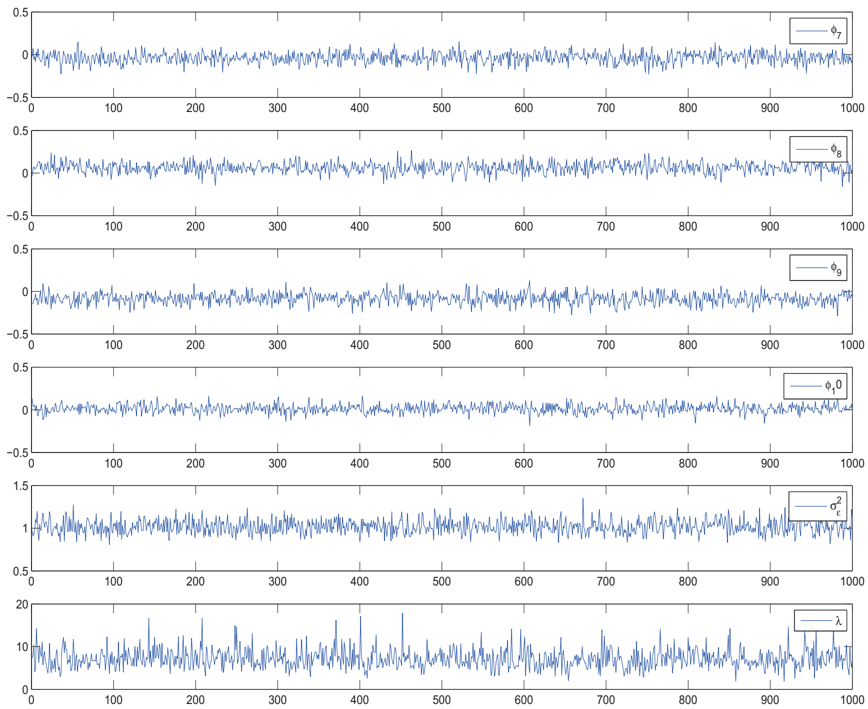
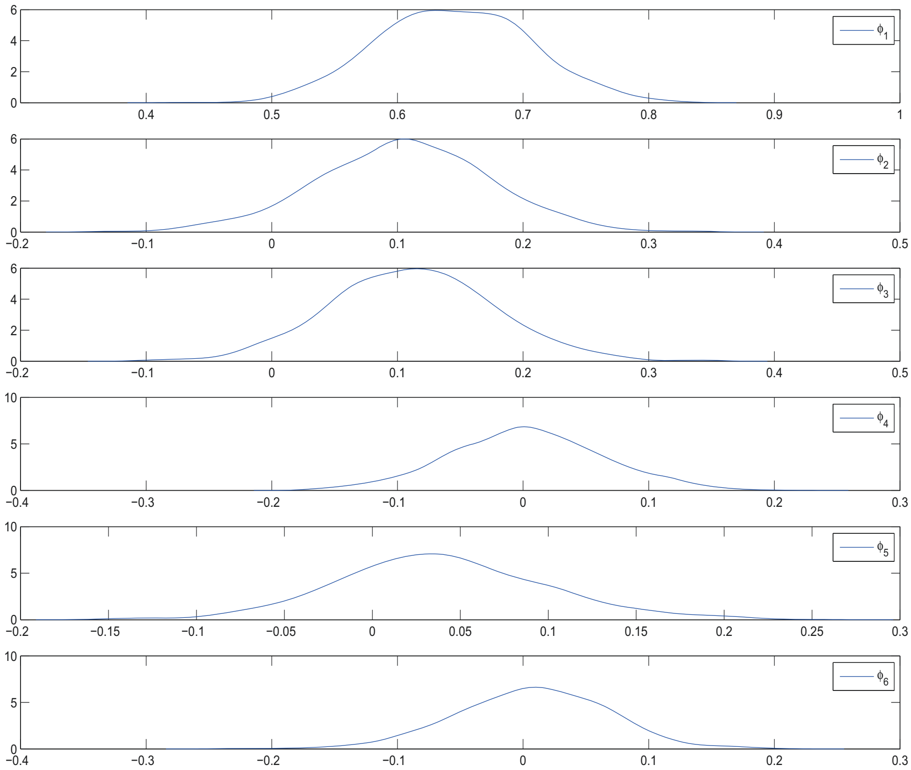


Figure A.20: Kernel density of the parameters of model BAYESL. T=300 and burning=1000.



c

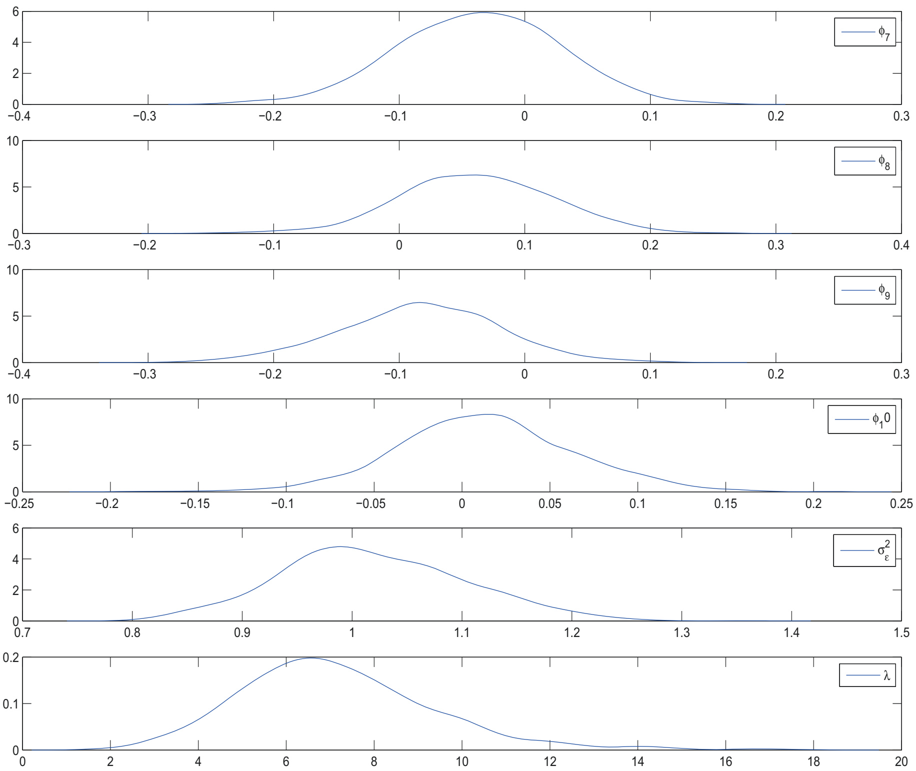
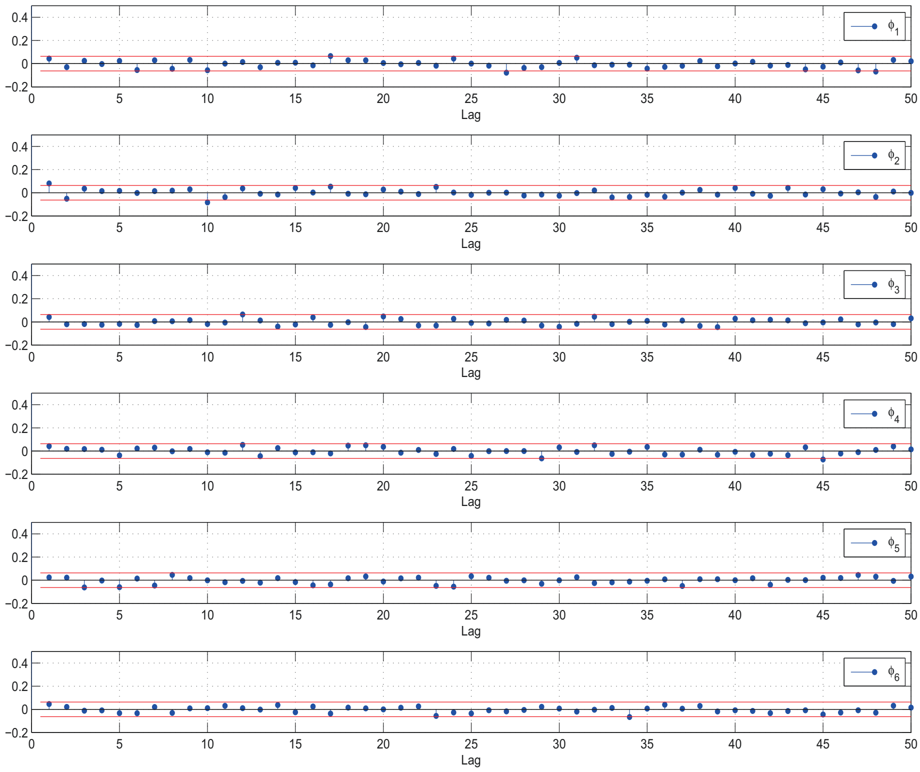
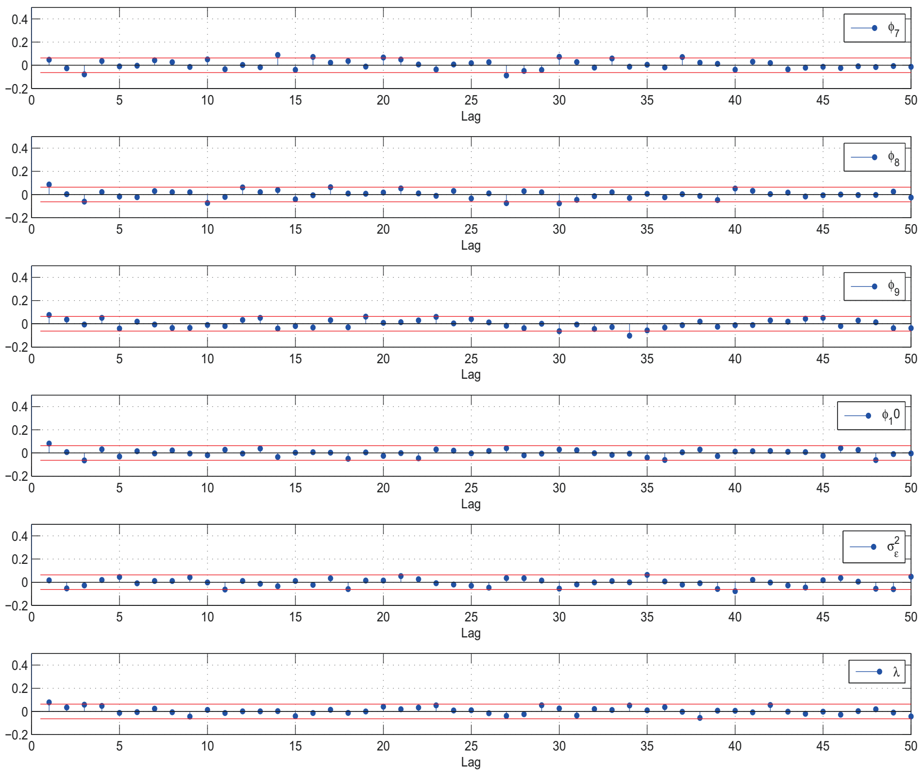


Figure A.21: Autocorrelation function of the parameters of model BAYESL. T=300 and burning=1000.



c



A.9.2. DGP: $y_t = 0.98y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$ with $\varepsilon \sim N(0,1)$.

Figure A.22: Time series plot of the parameters of model BAYESL. T=50 and burning=1000.

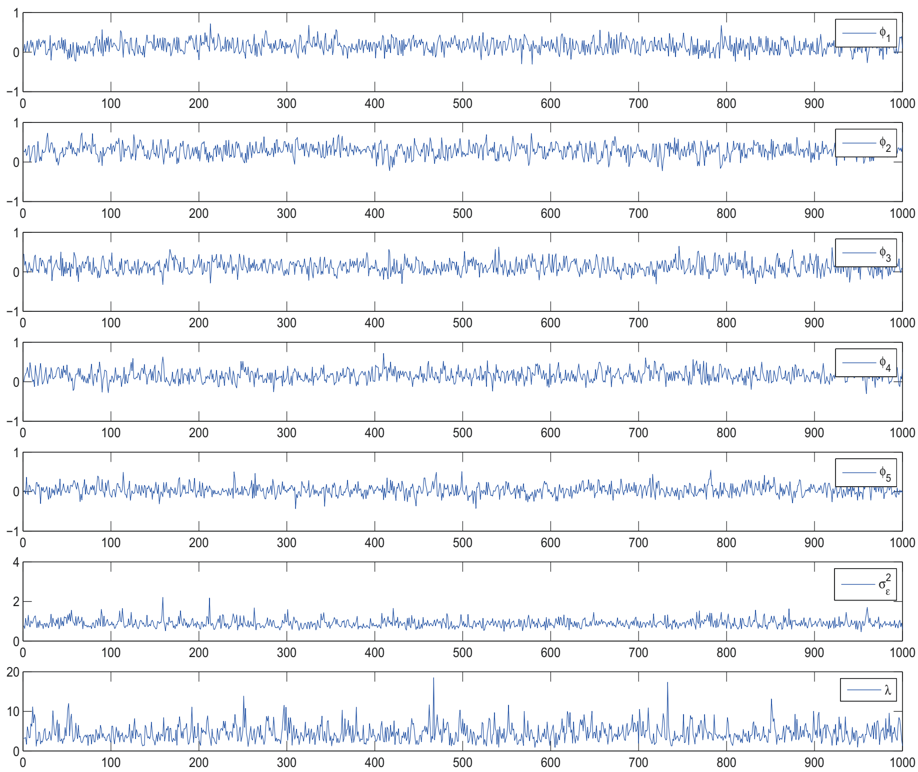


Figure A.23: Kernel density of the parameters of model BAYESL. T=50 and burning=1000.

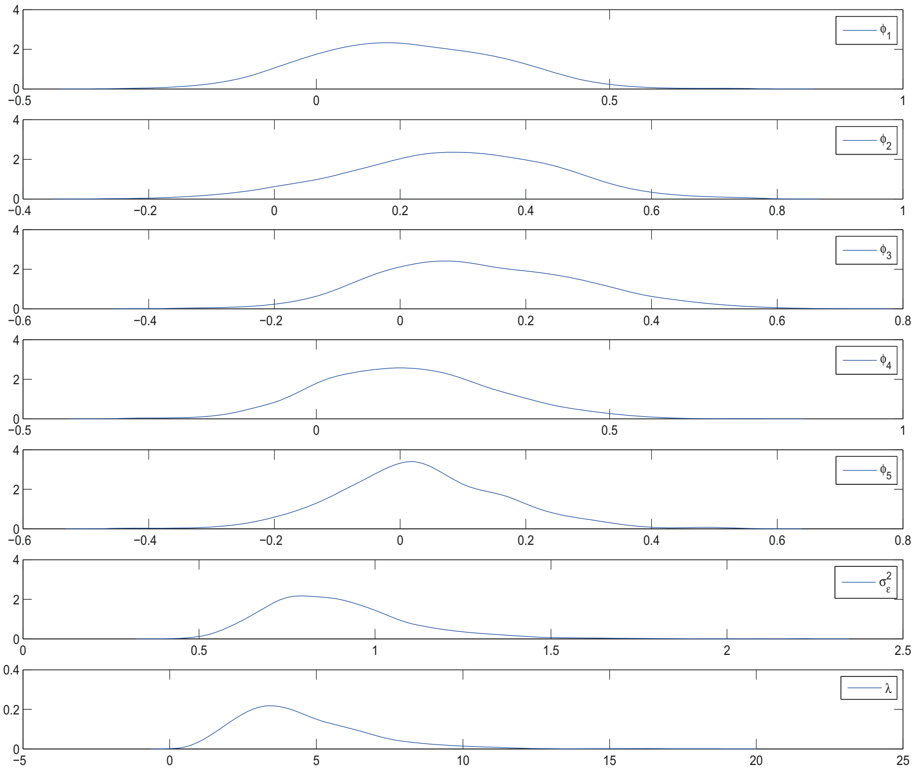


Figure A.24: Autocorrelation function of the parameters of model BAYESL. T=50 and burning=1000.

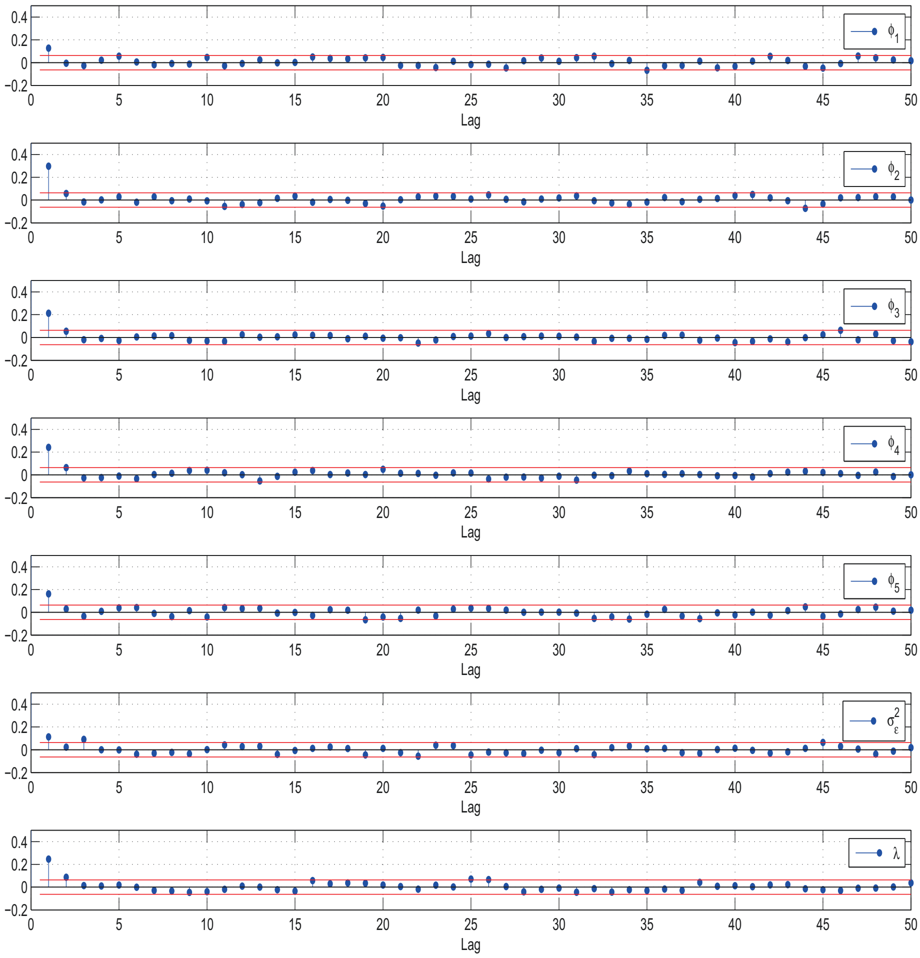
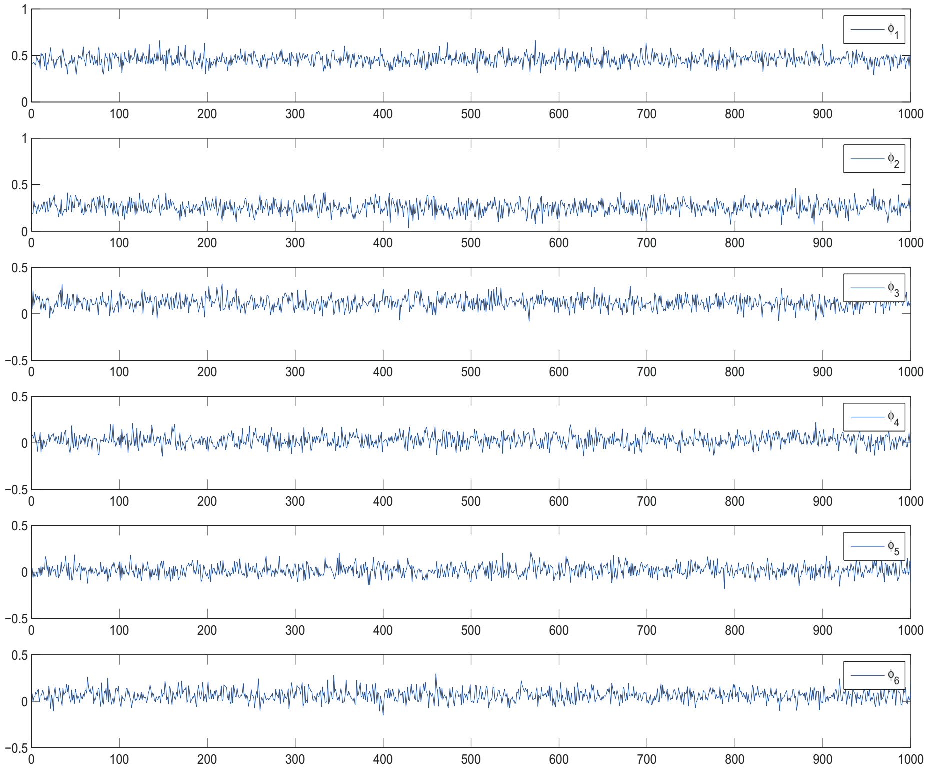


Figure A.25: Time series plot of the parameters of model BAYESL. T=300 and burning=1000.



c

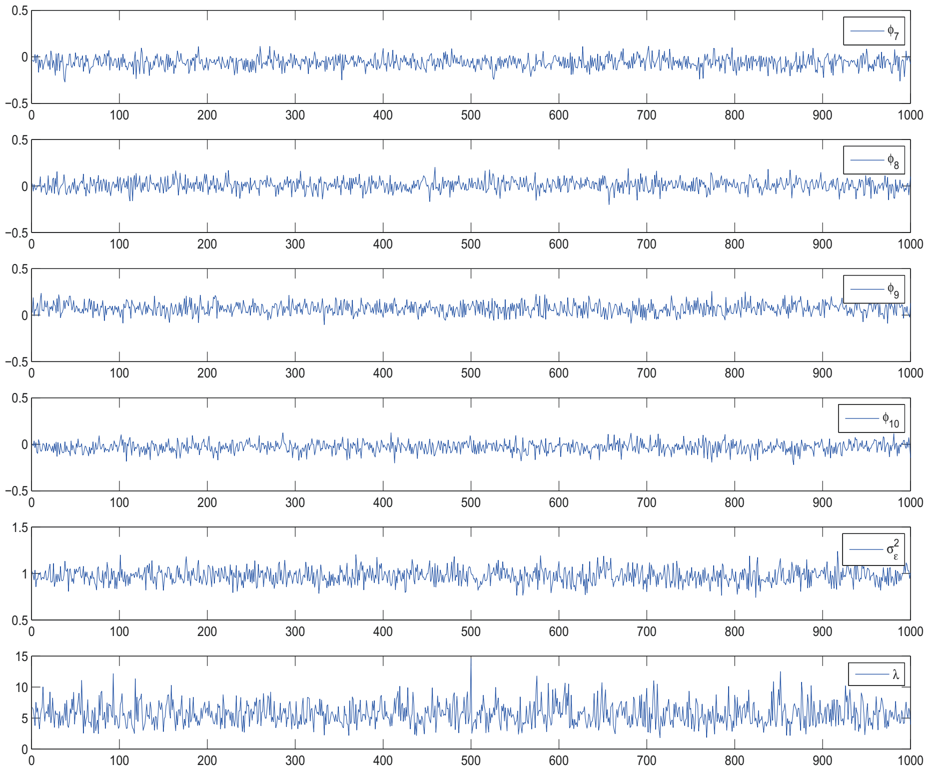
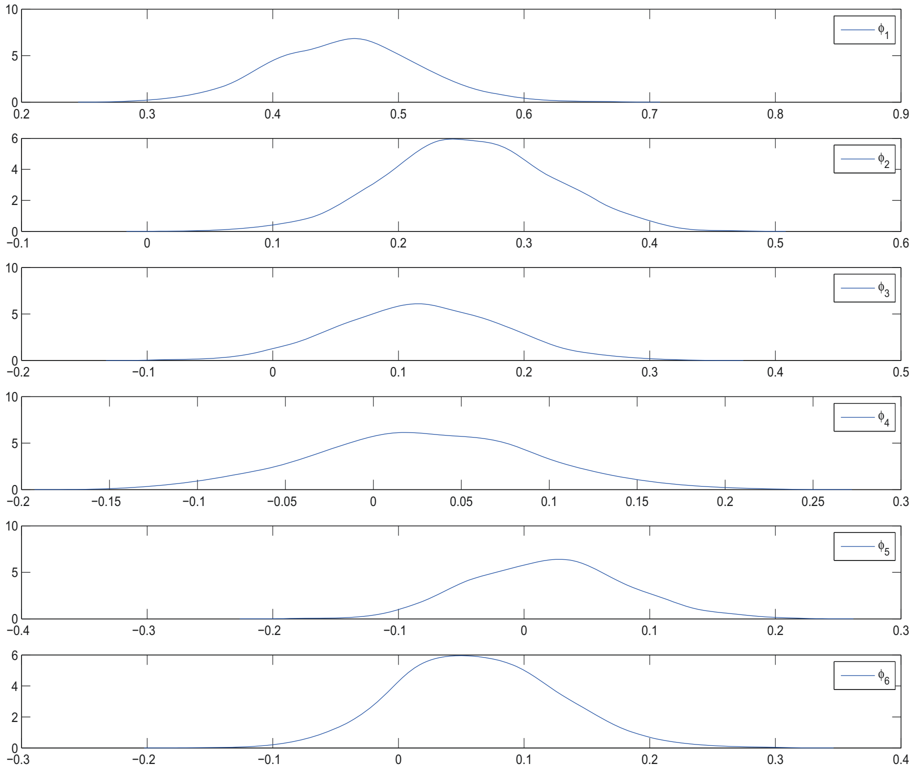


Figure A.26: Kernel density of the parameters of model BAYESL. T=300 and burning=1000.



c

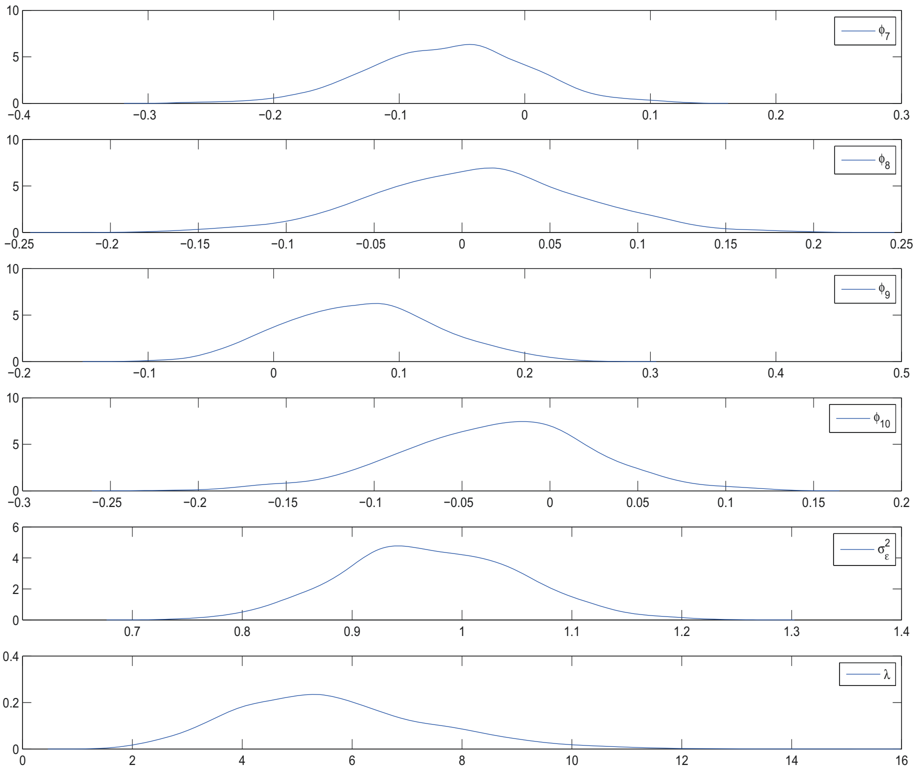
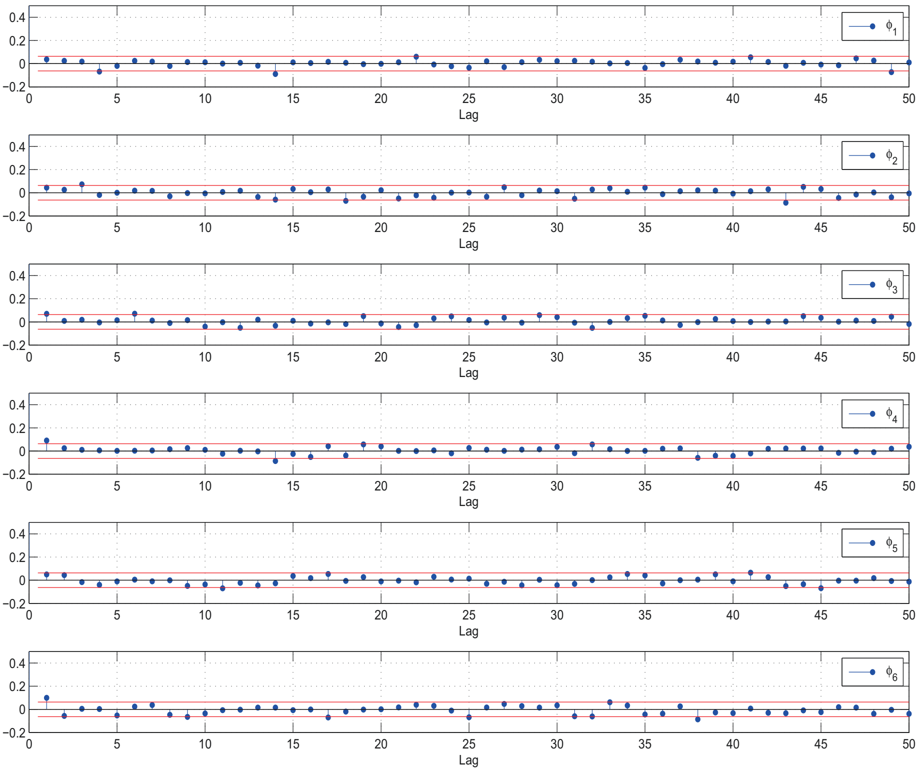
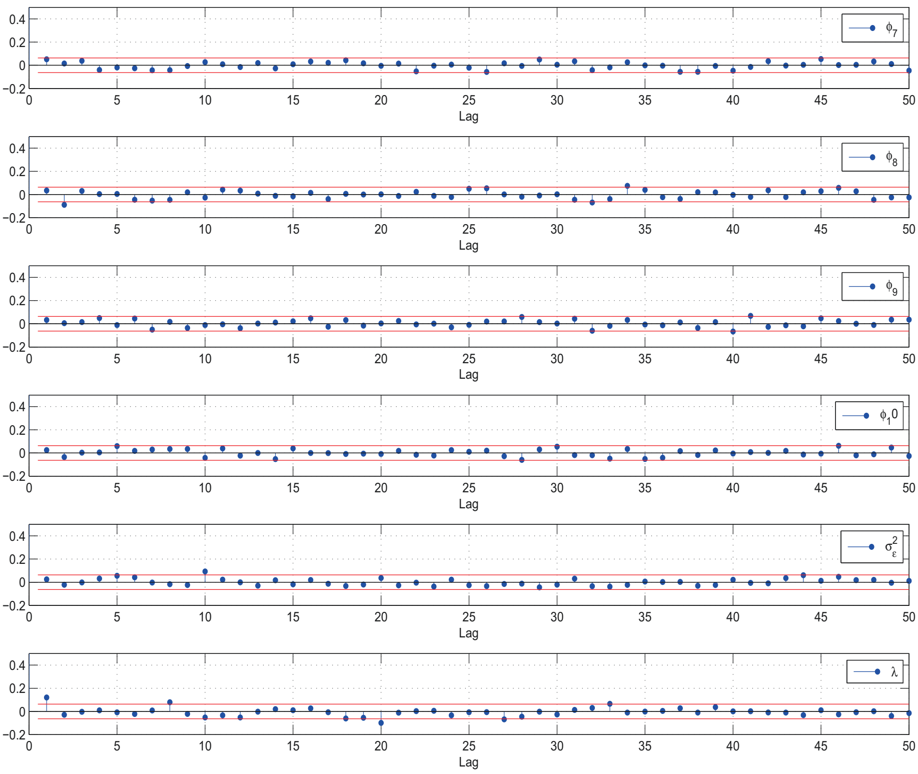


Figure A.27: Autocorrelation function of the parameters of model BAYESL. T=300 and burning=1000.



c



Appendix B

Appendix of Chapter 3

Table A.1: Monte Carlo size results for t_{1,α_i} using the asymptotic variance in (3.6). The DGP is $y_t = 0.5y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$ and the nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.009	0.049	0.102	0.203	0.308	0.406	0.506	0.605	0.706	0.803	0.902	0.950	0.986
	std	(0.014)	(0.033)	(0.045)	(0.066)	(0.082)	(0.089)	(0.091)	(0.087)	(0.080)	(0.066)	(0.051)	(0.037)	(0.024)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.068	0.039	0.026	0.020	0.036	0.048	0.028	0.035	0.025	0.021	0.018	0.014	0.046
T=100														
	$\widehat{\alpha}_i$	0.009	0.050	0.102	0.201	0.303	0.404	0.502	0.600	0.704	0.802	0.901	0.950	0.989
	std	(0.009)	(0.022)	(0.032)	(0.044)	(0.054)	(0.058)	(0.061)	(0.058)	(0.051)	(0.043)	(0.032)	(0.023)	(0.013)
	σ_{k,α_i}	0.011	0.026	0.037	0.051	0.060	0.066	0.068	0.067	0.064	0.056	0.042	0.031	0.014
	size	0.006	0.022	0.026	0.022	0.035	0.032	0.028	0.020	0.018	0.017	0.016	0.013	0.066
T=300														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.201	0.300	0.400	0.500	0.599	0.699	0.799	0.899	0.949	0.988
	std	(0.005)	(0.012)	(0.018)	(0.025)	(0.029)	(0.031)	(0.032)	(0.031)	(0.027)	(0.023)	(0.016)	(0.011)	(0.006)
	σ_{k,α_i}	0.006	0.015	0.021	0.029	0.035	0.038	0.039	0.039	0.037	0.032	0.024	0.018	0.008
	size	0.021	0.011	0.017	0.021	0.025	0.014	0.023	0.010	0.010	0.008	0.003	0.007	0.031
T=1000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.299	0.399	0.500	0.599	0.699	0.799	0.899	0.949	0.988
	std	(0.003)	(0.007)	(0.010)	(0.014)	(0.016)	(0.017)	(0.018)	(0.017)	(0.015)	(0.012)	(0.009)	(0.006)	(0.003)
	σ_{k,α_i}	0.003	0.008	0.012	0.016	0.019	0.021	0.021	0.021	0.020	0.018	0.013	0.010	0.004
	size	0.030	0.014	0.022	0.021	0.018	0.014	0.024	0.016	0.006	0.005	0.003	0.001	0.010
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.699	0.800	0.899	0.949	0.988
	std	(0.001)	(0.003)	(0.004)	(0.006)	(0.007)	(0.008)	(0.008)	(0.007)	(0.007)	(0.005)	(0.004)	(0.002)	(0.001)
	σ_{k,α_i}	0.002	0.004	0.005	0.007	0.008	0.009	0.010	0.009	0.009	0.008	0.006	0.004	0.002
	size	0.018	0.020	0.018	0.017	0.010	0.016	0.014	0.010	0.009	0.002	0.002	0.001	0.066

Table A.2: Monte Carlo size results for t_{k,α_i}. The DGP is $y_t = 0.5y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$. $B^{(1)}$=2000 for all T and the nominal size is 5%.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.009	0.049	0.102	0.203	0.309	0.404	0.507	0.606	0.706	0.803	0.904	0.951	0.986
	std	(0.014)	(0.033)	(0.046)	(0.066)	(0.081)	(0.088)	(0.090)	(0.087)	(0.079)	(0.067)	(0.049)	(0.036)	(0.024)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.015	0.034	0.050	0.071	0.085	0.093	0.096	0.094	0.088	0.077	0.060	0.048	0.033
	size	0.057	0.031	0.027	0.024	0.032	0.028	0.024	0.020	0.024	0.017	0.008	0.009	0.025
T=100														
	$\widehat{\alpha}_i$	0.009	0.050	0.101	0.201	0.304	0.403	0.502	0.601	0.704	0.802	0.901	0.952	0.990
	std	(0.009)	(0.022)	(0.032)	(0.044)	(0.054)	(0.058)	(0.061)	(0.057)	(0.050)	(0.043)	(0.031)	(0.023)	(0.013)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.010	0.023	0.033	0.048	0.057	0.062	0.064	0.062	0.057	0.049	0.036	0.028	0.018
	size	0.041	0.032	0.038	0.032	0.042	0.035	0.030	0.034	0.019	0.025	0.014	0.015	0.005
T=300														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.201	0.300	0.400	0.500	0.599	0.700	0.799	0.900	0.950	0.989
	std	(0.005)	(0.012)	(0.017)	(0.024)	(0.029)	(0.032)	(0.032)	(0.031)	(0.027)	(0.023)	(0.016)	(0.011)	(0.006)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.006	0.013	0.018	0.026	0.031	0.033	0.034	0.033	0.030	0.025	0.018	0.013	0.007
	size	0.018	0.031	0.036	0.030	0.031	0.035	0.034	0.026	0.034	0.030	0.021	0.018	0.017
T=1000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.299	0.399	0.500	0.599	0.700	0.799	0.900	0.949	0.989
	std	(0.003)	(0.007)	(0.010)	(0.014)	(0.016)	(0.017)	(0.018)	(0.017)	(0.015)	(0.012)	(0.008)	(0.005)	(0.003)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.003	0.007	0.010	0.014	0.016	0.017	0.018	0.017	0.015	0.012	0.008	0.006	0.003
	size	0.057	0.049	0.038	0.047	0.037	0.055	0.049	0.055	0.046	0.047	0.035	0.028	0.039
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.799	0.900	0.950	0.989
	std	(0.001)	(0.003)	(0.004)	(0.006)	(0.007)	(0.008)	(0.008)	(0.007)	(0.006)	(0.005)	(0.004)	(0.002)	(0.001)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.001	0.003	0.004	0.006	0.007	0.008	0.008	0.007	0.007	0.005	0.003	0.002	0.001
	size	0.044	0.070	0.052	0.046	0.048	0.054	0.052	0.055	0.039	0.057	0.052	0.044	0.127

Table A.3: Monte Carlo size results for t_{1,α_i}. The DGP is $y_t = 0.95 y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$ and the nominal size is 5%

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.011	0.054	0.106	0.211	0.309	0.412	0.511	0.612	0.712	0.809	0.905	0.950	0.984
	std	(0.016)	(0.037)	(0.057)	(0.083)	(0.096)	(0.105)	(0.107)	(0.104)	(0.094)	(0.080)	(0.057)	(0.043)	(0.026)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.017	0.039	0.058	0.084	0.099	0.108	0.111	0.109	0.101	0.087	0.066	0.051	0.034
	size	0.061	0.024	0.029	0.023	0.021	0.020	0.015	0.014	0.015	0.007	0.003	0.012	0.019
T=100														
	$\widehat{\alpha}_i$	0.010	0.051	0.102	0.204	0.305	0.404	0.503	0.604	0.704	0.804	0.902	0.949	0.988
	std	(0.010)	(0.025)	(0.036)	(0.052)	(0.061)	(0.066)	(0.068)	(0.067)	(0.061)	(0.048)	(0.035)	(0.025)	(0.014)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.011	0.026	0.039	0.055	0.065	0.071	0.072	0.071	0.065	0.055	0.041	0.031	0.019
	size	0.038	0.031	0.031	0.024	0.020	0.009	0.011	0.013	0.012	0.008	0.010	0.005	0.006
T=300														
	$\widehat{\alpha}_i$	0.010	0.050	0.101	0.202	0.302	0.401	0.502	0.601	0.701	0.801	0.900	0.949	0.988
	std	(0.006)	(0.013)	(0.020)	(0.027)	(0.032)	(0.034)	(0.035)	(0.033)	(0.030)	(0.024)	(0.017)	(0.012)	(0.006)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.006	0.014	0.020	0.028	0.033	0.036	0.036	0.035	0.032	0.026	0.019	0.014	0.008
	size	0.029	0.033	0.035	0.031	0.031	0.034	0.026	0.025	0.024	0.017	0.022	0.017	0.016
T=1000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.201	0.301	0.401	0.501	0.600	0.700	0.800	0.899	0.949	0.988
	std	(0.003)	(0.007)	(0.010)	(0.014)	(0.017)	(0.018)	(0.018)	(0.017)	(0.015)	(0.012)	(0.008)	(0.006)	(0.003)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.003	0.007	0.010	0.014	0.017	0.018	0.018	0.017	0.016	0.013	0.009	0.006	0.003
	size	0.055	0.044	0.040	0.050	0.037	0.042	0.038	0.030	0.039	0.041	0.037	0.032	0.051
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.300	0.399	0.500	0.600	0.700	0.799	0.900	0.950	0.989
	std	0.001	0.003	0.005	0.006	0.007	0.008	0.008	0.008	0.007	0.005	0.003	0.002	0.001
	$\bar{\sigma}^*_{k,\alpha_i}$	0.001	0.003	0.005	0.006	0.007	0.008	0.008	0.007	0.007	0.005	0.003	0.002	0.001
	size	0.054	0.051	0.052	0.055	0.050	0.058	0.054	0.062	0.055	0.057	0.058	0.036	0.141

Table A.4: Monte Carlo size results for t_{1,α_i}. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$, with ε_t ∼Student-5 and the nominal size is 5%

T		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.010	0.053	0.107	0.213	0.316	0.418	0.515	0.613	0.712	0.811	0.907	0.951	0.985
	std	(0.015)	(0.038)	(0.062)	(0.091)	(0.110)	(0.119)	(0.120)	(0.111)	(0.099)	(0.077)	(0.050)	(0.036)	(0.023)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.041	0.062	0.092	0.110	0.119	0.121	0.115	0.102	0.084	0.061	0.048	0.033
	size	0.053	0.035	0.038	0.019	0.020	0.016	0.017	0.012	0.014	0.008	0.004	0.004	0.012
T=100														
	$\widehat{\alpha}_i$	0.010	0.052	0.104	0.207	0.307	0.406	0.506	0.607	0.705	0.804	0.901	0.951	0.989
	std	(0.010)	(0.026)	(0.039)	(0.059)	(0.071)	(0.075)	(0.075)	(0.068)	(0.061)	(0.047)	(0.030)	(0.021)	(0.012)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.011	0.027	0.042	0.062	0.074	0.079	0.079	0.075	0.066	0.053	0.036	0.027	0.018
	size	0.048	0.032	0.026	0.022	0.017	0.015	0.009	0.004	0.010	0.012	0.008	0.004	0.003
T=300														
	$\widehat{\alpha}_i$	0.010	0.051	0.101	0.203	0.303	0.403	0.503	0.603	0.702	0.802	0.900	0.950	0.988
	std	(0.006)	(0.014)	(0.020)	(0.031)	(0.036)	(0.038)	(0.038)	(0.035)	(0.031)	(0.023)	(0.014)	(0.009)	(0.005)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.006	0.014	0.022	0.033	0.038	0.041	0.041	0.038	0.033	0.025	0.016	0.011	0.007
	size	0.037	0.041	0.026	0.028	0.028	0.014	0.021	0.022	0.019	0.017	0.016	0.015	0.011
T=1000														
	$\widehat{\alpha}_i$	0.010	0.050	0.101	0.201	0.300	0.400	0.501	0.600	0.700	0.800	0.899	0.949	0.988
	std	(0.003)	(0.007)	(0.011)	(0.016)	(0.019)	(0.020)	(0.020)	(0.018)	(0.015)	(0.012)	(0.007)	(0.005)	(0.003)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.003	0.008	0.011	0.017	0.020	0.021	0.021	0.019	0.016	0.012	0.008	0.005	0.003
	size	0.038	0.043	0.039	0.037	0.040	0.040	0.034	0.038	0.032	0.039	0.029	0.038	0.091
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.799	0.900	0.950	0.989
	std	0.001	0.003	0.005	0.007	0.008	0.009	0.009	0.008	0.007	0.005	0.003	0.002	0.001
	$\bar{\sigma}^*_{k,\alpha_i}$	0.001	0.003	0.005	0.007	0.009	0.009	0.009	0.008	0.007	0.005	0.003	0.002	0.001
	size	0.052	0.043	0.039	0.050	0.036	0.039	0.039	0.043	0.035	0.040	0.038	0.030	0.174

Table A.5: Monte Carlo size results for $L^5_{\alpha_i}$ and C_1 statistics. The DGPs are: $y_t = 0.5y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$ (Panel A), where we set $B^{(1)}$=2000 for all T; $y_t = 0.95y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$ (Panel B); $y_t = 0.95y_{t-1} + \varepsilon_t$, ε_t ∼Student-5 (Panel C). The nominal size is 5%. .

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T	Panel A													
50	0.076	0.069	0.051	0.050	0.047	0.035	0.047	0.044	0.067	0.079	0.073	0.105	0.071	0.030
100	0.048	0.057	0.042	0.046	0.041	0.044	0.050	0.049	0.050	0.058	0.065	0.107	0.047	0.019
300	0.055	0.044	0.042	0.048	0.046	0.044	0.041	0.046	0.052	0.051	0.071	0.091	0.063	0.017
1000	0.043	0.051	0.041	0.037	0.042	0.043	0.046	0.046	0.046	0.051	0.053	0.068	0.150	0.026
5000	0.062	0.060	0.051	0.042	0.042	0.046	0.061	0.043	0.044	0.054	0.060	0.052	0.101	0.092
T	Panel B													
50	0.071	0.071	0.053	0.045	0.049	0.035	0.044	0.052	0.080	0.081	0.081	0.114	0.070	0.023
100	0.049	0.067	0.055	0.051	0.033	0.035	0.053	0.056	0.046	0.061	0.077	0.109	0.061	0.017
300	0.050	0.042	0.053	0.047	0.049	0.048	0.039	0.049	0.062	0.042	0.068	0.091	0.078	0.031
1000	0.044	0.060	0.040	0.049	0.039	0.041	0.055	0.049	0.051	0.053	0.064	0.071	0.178	0.052
5000	0.062	0.056	0.045	0.069	0.057	0.064	0.062	0.055	0.054	0.053	0.052	0.052	0.130	0.104
T	Panel C													
50	0.085	0.075	0.073	0.043	0.039	0.052	0.063	0.065	0.069	0.079	0.078	0.109	0.053	0.012
100	0.07	0.067	0.055	0.051	0.045	0.045	0.044	0.052	0.055	0.065	0.076	0.119	0.039	0.019
300	0.053	0.059	0.043	0.056	0.046	0.046	0.043	0.037	0.049	0.045	0.055	0.092	0.09	0.017
1000	0.046	0.05	0.038	0.043	0.041	0.039	0.04	0.042	0.043	0.042	0.056	0.073	0.189	0.062
5000	0.055	0.047	0.039	0.056	0.047	0.053	0.058	0.071	0.048	0.042	0.055	0.056	0.116	0.094

Table A.6: Monte Carlo size results for t_{1,α_i} using the asymptotic variance. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 50$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.015	0.060	0.112	0.213	0.312	0.410	0.508	0.603	0.698	0.795	0.890	0.935	0.968
	std	(0.033)	(0.064)	(0.086)	(0.116)	(0.133)	(0.145)	(0.151)	(0.149)	(0.142)	(0.123)	(0.096)	(0.072)	(0.050)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.163	0.141	0.137	0.165	0.191	0.196	0.220	0.214	0.216	0.214	0.129	0.141	0.191
T=100														
	$\widehat{\alpha}_i$	0.012	0.058	0.110	0.209	0.305	0.404	0.502	0.599	0.698	0.794	0.894	0.941	0.980
	std	(0.020)	(0.047)	(0.068)	(0.095)	(0.112)	(0.120)	(0.124)	(0.122)	(0.116)	(0.104)	(0.077)	(0.059)	(0.033)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.138	0.116	0.106	0.112	0.140	0.118	0.136	0.127	0.134	0.144	0.080	0.110	0.110
T=300														
	$\widehat{\alpha}_i$	0.011	0.054	0.106	0.209	0.310	0.409	0.508	0.608	0.706	0.803	0.899	0.948	0.985
	std	(0.017)	(0.040)	(0.055)	(0.083)	(0.099)	(0.108)	(0.110)	(0.108)	(0.102)	(0.087)	(0.067)	(0.049)	(0.026)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.111	0.091	0.070	0.082	0.103	0.090	0.096	0.082	0.083	0.081	0.058	0.058	0.059
T=1000														
	$\widehat{\alpha}_i$	0.011	0.051	0.104	0.205	0.304	0.402	0.501	0.598	0.699	0.799	0.898	0.949	0.987
	std	(0.016)	(0.038)	(0.054)	(0.075)	(0.090)	(0.097)	(0.101)	(0.101)	(0.094)	(0.083)	(0.063)	(0.046)	(0.023)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.117	0.065	0.063	0.051	0.072	0.058	0.067	0.072	0.061	0.068	0.044	0.046	0.050
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.099	0.201	0.300	0.399	0.502	0.603	0.701	0.800	0.896	0.947	0.988
	std	(0.014)	(0.035)	(0.051)	(0.071)	(0.085)	(0.093)	(0.097)	(0.097)	(0.089)	(0.078)	(0.059)	(0.043)	(0.023)
	σ_{k,α_i}	0.015	0.036	0.052	0.073	0.086	0.093	0.097	0.096	0.091	0.080	0.060	0.044	0.020
	size	0.077	0.056	0.039	0.037	0.055	0.061	0.054	0.062	0.052	0.057	0.031	0.038	0.045

Table A.7: Monte Carlo size results for t_{1,α_i} using the asymptotic variance. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 500$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.016	0.063	0.117	0.218	0.316	0.413	0.511	0.605	0.700	0.795	0.889	0.935	0.969
	std	(0.016)	(0.031)	(0.042)	(0.054)	(0.063)	(0.067)	(0.068)	(0.067)	(0.066)	(0.062)	(0.053)	(0.043)	(0.030)
	σ_{k,α_i}	0.005	0.011	0.016	0.023	0.027	0.029	0.030	0.030	0.028	0.025	0.019	0.014	0.006
	size	0.321	0.340	0.378	0.371	0.392	0.379	0.382	0.380	0.404	0.428	0.456	0.485	0.517
T=100														
	$\widehat{\alpha}_i$	0.013	0.057	0.109	0.211	0.309	0.408	0.507	0.604	0.700	0.797	0.893	0.942	0.980
	std	(0.009)	(0.021)	(0.030)	(0.042)	(0.050)	(0.053)	(0.055)	(0.056)	(0.054)	(0.050)	(0.042)	(0.033)	(0.020)
	σ_{k,α_i}	0.005	0.011	0.016	0.023	0.027	0.029	0.030	0.030	0.028	0.025	0.019	0.014	0.006
	size	0.205	0.228	0.252	0.264	0.284	0.268	0.273	0.280	0.280	0.312	0.347	0.344	0.351
T=300														
	$\widehat{\alpha}_i$	0.011	0.052	0.103	0.205	0.303	0.405	0.504	0.603	0.701	0.800	0.898	0.947	0.985
	std	(0.006)	(0.015)	(0.021)	(0.030)	(0.036)	(0.038)	(0.041)	(0.041)	(0.041)	(0.036)	(0.028)	(0.021)	(0.012)
	σ_{k,α_i}	0.005	0.011	0.016	0.023	0.027	0.029	0.030	0.030	0.028	0.025	0.019	0.014	0.006
	size	0.129	0.108	0.126	0.133	0.145	0.124	0.127	0.156	0.143	0.154	0.178	0.175	0.231
T=1000														
	$\widehat{\alpha}_i$	0.011	0.051	0.101	0.201	0.300	0.398	0.499	0.598	0.698	0.798	0.898	0.947	0.987
	std	(0.005)	(0.013)	(0.019)	(0.026)	(0.029)	(0.033)	(0.034)	(0.035)	(0.033)	(0.030)	(0.023)	(0.017)	(0.008)
	σ_{k,α_i}	0.005	0.011	0.016	0.023	0.027	0.029	0.030	0.030	0.028	0.025	0.019	0.014	0.006
	size	0.075	0.075	0.089	0.086	0.069	0.084	0.078	0.105	0.097	0.104	0.104	0.101	0.138
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.099	0.201	0.301	0.401	0.501	0.601	0.701	0.800	0.899	0.949	0.989
	std	(0.005)	(0.011)	(0.017)	(0.023)	(0.028)	(0.031)	(0.033)	(0.031)	(0.028)	(0.025)	(0.019)	(0.014)	(0.007)
	σ_{k,α_i}	0.005	0.011	0.016	0.023	0.027	0.029	0.030	0.030	0.028	0.025	0.019	0.014	0.006
	size	0.055	0.039	0.053	0.048	0.057	0.073	0.057	0.068	0.042	0.055	0.052	0.051	0.061

Table A.8: Monte Carlo size results for $L^5_{\alpha_i}$ and C_1 statistics, using the asymptotic covariances . The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5%, $H = 50$ (Panel A) and $H = 500$ (Panel B)

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T							Panel A							
50	0.185	0.154	0.121	0.107	0.123	0.132	0.145	0.162	0.184	0.207	0.294	0.337	0.226	0.372
100	0.145	0.103	0.103	0.076	0.059	0.073	0.091	0.127	0.160	0.203	0.258	0.310	0.146	0.233
300	0.136	0.095	0.079	0.067	0.074	0.075	0.084	0.097	0.113	0.159	0.239	0.295	0.105	0.129
1000	0.131	0.076	0.088	0.069	0.066	0.074	0.095	0.101	0.117	0.169	0.229	0.281	0.084	0.086
5000	0.097	0.073	0.049	0.054	0.050	0.066	0.072	0.088	0.104	0.136	0.231	0.279	0.079	0.074
T							Panel B							
50	0.339	0.316	0.293	0.292	0.293	0.291	0.289	0.280	0.313	0.333	0.392	0.433	0.512	0.96
100	0.237	0.201	0.207	0.201	0.193	0.197	0.196	0.193	0.222	0.258	0.272	0.344	0.351	0.862
300	0.140	0.104	0.100	0.098	0.098	0.103	0.097	0.105	0.132	0.116	0.157	0.236	0.236	0.549
1000	0.089	0.065	0.072	0.059	0.077	0.071	0.080	0.080	0.089	0.088	0.115	0.168	0.170	0.212
5000	0.057	0.047	0.045	0.048	0.050	0.057	0.056	0.054	0.051	0.060	0.082	0.155	0.119	0.072

Table A.9: Monte Carlo size results for t_{1,α_i}. The DGP is $y_t = 0.95y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 500$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.017	0.063	0.117	0.218	0.316	0.413	0.510	0.606	0.700	0.796	0.890	0.935	0.969
	std	(0.016)	(0.031)	(0.042)	(0.055)	(0.063)	(0.066)	(0.068)	(0.067)	(0.066)	(0.062)	(0.053)	(0.043)	(0.030)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.026	0.044	0.056	0.068	0.076	0.081	0.084	0.086	0.084	0.079	0.066	0.054	0.039
	size	0.061	0.074	0.072	0.078	0.074	0.058	0.053	0.040	0.034	0.037	0.033	0.047	0.066
T=100														
	$\widehat{\alpha}_i$	0.013	0.057	0.109	0.211	0.309	0.409	0.507	0.604	0.700	0.797	0.893	0.942	0.980
	std	(0.009)	(0.021)	(0.030)	(0.042)	(0.050)	(0.053)	(0.054)	(0.055)	(0.055)	(0.050)	(0.042)	(0.033)	(0.020)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.011	0.025	0.034	0.045	0.052	0.057	0.060	0.061	0.059	0.055	0.044	0.034	0.022
	size	0.044	0.054	0.054	0.066	0.071	0.061	0.058	0.047	0.041	0.042	0.042	0.060	0.073
T=300														
	$\widehat{\alpha}_i$	0.011	0.052	0.103	0.204	0.303	0.405	0.505	0.603	0.701	0.800	0.898	0.947	0.985
	std	(0.007)	(0.015)	(0.021)	(0.030)	(0.035)	(0.038)	(0.041)	(0.041)	(0.040)	(0.036)	(0.028)	(0.021)	(0.012)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.006	0.015	0.022	0.031	0.036	0.040	0.042	0.042	0.041	0.037	0.029	0.022	0.012
	size	0.057	0.051	0.054	0.056	0.051	0.047	0.047	0.054	0.047	0.054	0.040	0.045	0.074
T=1000														
	$\widehat{\alpha}_i$	0.011	0.051	0.101	0.201	0.300	0.399	0.499	0.598	0.698	0.799	0.898	0.948	0.987
	std	(0.005)	(0.013)	(0.018)	(0.026)	(0.029)	(0.033)	(0.035)	(0.035)	(0.034)	(0.030)	(0.023)	(0.017)	(0.008)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.005	0.012	0.018	0.025	0.030	0.033	0.034	0.034	0.033	0.029	0.022	0.017	0.008
	size	0.054	0.062	0.057	0.063	0.049	0.055	0.049	0.058	0.059	0.056	0.061	0.064	0.074
T=5000														
	$\widehat{\alpha}_i$	0.010	0.050	0.099	0.200	0.301	0.400	0.501	0.601	0.700	0.799	0.899	0.949	0.989
	std	(0.005)	(0.011)	(0.017)	(0.023)	(0.028)	(0.031)	(0.032)	(0.031)	(0.029)	(0.026)	(0.019)	(0.014)	(0.007)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.005	0.012	0.017	0.023	0.027	0.030	0.031	0.031	0.029	0.026	0.020	0.014	0.007
	size	0.046	0.035	0.040	0.043	0.060	0.064	0.055	0.062	0.041	0.047	0.044	0.047	0.071

Table A.10: Monte Carlo power results for t_{1,α_i}. The DGP is the AR(2) model in (3.17) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 50$

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.000	0.002	0.008	0.039	0.101	0.197	0.310	0.437	0.566	0.699	0.833	0.900	0.951
	std	(0.002)	(0.006)	(0.015)	(0.036)	(0.063)	(0.090)	(0.117)	(0.133)	(0.140)	(0.133)	(0.112)	(0.094)	(0.068)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.034	0.068	0.093	0.123	0.141	0.152	0.156	0.154	0.145	0.128	0.100	0.078	0.053
	power	0.000	0.000	0.000	0.036	0.176	0.196	0.183	0.161	0.158	0.135	0.142	0.135	0.141
T=100														
	$\widehat{\alpha}_i$	0.000	0.002	0.010	0.048	0.116	0.213	0.328	0.453	0.584	0.710	0.845	0.914	0.967
	std	(0.001)	(0.007)	(0.016)	(0.038)	(0.061)	(0.082)	(0.106)	(0.123)	(0.127)	(0.121)	(0.099)	(0.076)	(0.046)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.023	0.053	0.075	0.103	0.120	0.130	0.134	0.132	0.124	0.108	0.082	0.062	0.037
	power	0.000	0.000	0.000	0.130	0.230	0.231	0.220	0.184	0.168	0.168	0.146	0.123	0.155
T=300														
	$\widehat{\alpha}_i$	0.000	0.002	0.011	0.055	0.130	0.229	0.343	0.467	0.601	0.731	0.858	0.924	0.979
	std	(0.001)	(0.007)	(0.016)	(0.037)	(0.055)	(0.075)	(0.092)	(0.103)	(0.109)	(0.099)	(0.081)	(0.062)	(0.032)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.017	0.041	0.059	0.083	0.098	0.107	0.111	0.109	0.103	0.090	0.068	0.050	0.026
	power	0.000	0.000	0.000	0.340	0.371	0.326	0.272	0.211	0.167	0.140	0.153	0.133	0.109
T=1000														
	$\widehat{\alpha}_i$	0.000	0.003	0.012	0.056	0.133	0.240	0.355	0.481	0.607	0.738	0.865	0.929	0.983
	std	(0.001)	(0.007)	(0.017)	(0.036)	(0.056)	(0.079)	(0.094)	(0.104)	(0.104)	(0.099)	(0.078)	(0.058)	(0.028)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.038	0.054	0.076	0.089	0.097	0.101	0.100	0.094	0.083	0.062	0.046	0.023
	power	0.000	0.000	0.032	0.478	0.482	0.360	0.290	0.223	0.215	0.168	0.123	0.114	0.076
T=5000														
	$\widehat{\alpha}_i$	0.000	0.002	0.012	0.058	0.138	0.241	0.359	0.488	0.618	0.746	0.873	0.936	0.985
	std	(0.001)	(0.007)	(0.016)	(0.036)	(0.056)	(0.073)	(0.090)	(0.100)	(0.103)	(0.094)	(0.073)	(0.055)	(0.027)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.037	0.052	0.073	0.086	0.094	0.097	0.096	0.091	0.080	0.060	0.044	0.021
	power	0.000	0.000	0.143	0.503	0.470	0.375	0.310	0.229	0.190	0.152	0.123	0.094	0.066

Table A.11: Monte Carlo power results for t_{1,α_i}. The DGP is the AR(1)-GARCH(1,1) model in (3.19) with $\varepsilon_t \sim N(0,1)$. The nominal size is 5% and $H = 50$.

T	α_i	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.99
T=50														
	$\widehat{\alpha}_i$	0.029	0.071	0.117	0.213	0.323	0.434	0.537	0.630	0.719	0.800	0.876	0.914	0.945
	std	(0.040)	(0.061)	(0.075)	(0.099)	(0.121)	(0.140)	(0.150)	(0.156)	(0.156)	(0.145)	(0.124)	(0.106)	(0.086)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.021	0.047	0.066	0.090	0.105	0.114	0.118	0.117	0.111	0.100	0.080	0.064	0.047
	power	0.205	0.133	0.086	0.066	0.093	0.113	0.127	0.161	0.196	0.186	0.148	0.176	0.202
T=100														
	$\widehat{\alpha}_i$	0.029	0.071	0.118	0.212	0.315	0.423	0.524	0.624	0.715	0.806	0.889	0.929	0.965
	std	(0.042)	(0.060)	(0.071)	(0.092)	(0.106)	(0.124)	(0.139)	(0.148)	(0.149)	(0.138)	(0.116)	(0.094)	(0.069)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.018	0.043	0.061	0.085	0.098	0.107	0.110	0.108	0.102	0.090	0.071	0.054	0.033
	power	0.216	0.141	0.094	0.063	0.070	0.087	0.116	0.163	0.210	0.202	0.146	0.159	0.185
T=300														
	$\widehat{\alpha}_i$	0.025	0.066	0.111	0.208	0.312	0.414	0.520	0.621	0.721	0.817	0.905	0.946	0.979
	std	(0.036)	(0.058)	(0.068)	(0.085)	(0.097)	(0.115)	(0.128)	(0.136)	(0.139)	(0.131)	(0.108)	(0.087)	(0.056)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.039	0.056	0.078	0.092	0.100	0.102	0.101	0.095	0.083	0.064	0.048	0.026
	power	0.199	0.127	0.077	0.067	0.061	0.082	0.111	0.156	0.190	0.191	0.114	0.120	0.113
T=1000														
	$\widehat{\alpha}_i$	0.023	0.067	0.113	0.205	0.307	0.412	0.518	0.620	0.720	0.818	0.911	0.953	0.986
	std	(0.033)	(0.052)	(0.065)	(0.078)	(0.093)	(0.107)	(0.118)	(0.128)	(0.129)	(0.120)	(0.097)	(0.076)	(0.040)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.037	0.053	0.074	0.088	0.095	0.098	0.097	0.092	0.081	0.061	0.045	0.023
	power	0.215	0.151	0.096	0.065	0.060	0.087	0.113	0.157	0.180	0.193	0.103	0.111	0.079
T=5000														
	$\widehat{\alpha}_i$	0.025	0.066	0.109	0.203	0.302	0.407	0.511	0.614	0.715	0.813	0.909	0.956	0.989
	std	(0.036)	(0.053)	(0.063)	(0.075)	(0.089)	(0.105)	(0.120)	(0.132)	(0.133)	(0.124)	(0.098)	(0.074)	(0.039)
	$\bar{\sigma}^*_{k,\alpha_i}$	0.016	0.037	0.052	0.073	0.086	0.094	0.097	0.096	0.090	0.080	0.060	0.044	0.021
	power	0.254	0.141	0.090	0.052	0.061	0.082	0.122	0.160	0.185	0.205	0.110	0.100	0.070

Table A.12: Monte Carlo power results for $L^5_{\alpha_i}$ and C_1 statistics. The DGPs are: the AR(2) model in (3.17) with $\varepsilon_t \sim N(0,1)$ and $H = 50$ (Panel A); and the AR(1)-GARCH(1,1) model with $\varepsilon_t \sim N(0,1)$ and $H = 50$ (Panel B). The nominal size is 5%.

	$L^5_{0.01}$	$L^5_{0.05}$	$L^5_{0.1}$	$L^5_{0.2}$	$L^5_{0.3}$	$L^5_{0.4}$	$L^5_{0.5}$	$L^5_{0.6}$	$L^5_{0.7}$	$L^5_{0.8}$	$L^5_{0.9}$	$L^5_{0.95}$	$L^5_{0.99}$	C^{13}_1
T							Panel A							
50	0.071	0.149	0.293	0.582	0.745	0.784	0.733	0.64	0.534	0.412	0.299	0.235	0.240	0.151
100	0.106	0.194	0.351	0.641	0.767	0.766	0.703	0.611	0.514	0.423	0.313	0.279	0.196	0.161
300	0.119	0.253	0.403	0.673	0.755	0.755	0.692	0.623	0.496	0.393	0.294	0.287	0.120	0.137
1000	0.137	0.259	0.446	0.706	0.758	0.733	0.668	0.568	0.479	0.377	0.293	0.299	0.120	0.164
5000	0.122	0.247	0.416	0.646	0.749	0.738	0.664	0.544	0.442	0.344	0.258	0.270	0.096	0.168
T							Panel B							
50	0.253	0.149	0.088	0.063	0.055	0.066	0.076	0.100	0.125	0.182	0.249	0.281	0.298	0.291
100	0.287	0.155	0.093	0.062	0.055	0.063	0.091	0.106	0.148	0.188	0.260	0.292	0.226	0.291
300	0.259	0.136	0.087	0.070	0.061	0.060	0.072	0.097	0.129	0.162	0.217	0.239	0.124	0.261
1000	0.261	0.144	0.083	0.049	0.045	0.061	0.063	0.097	0.116	0.157	0.207	0.215	0.093	0.249
5000	0.258	0.154	0.091	0.048	0.043	0.065	0.071	0.097	0.126	0.166	0.208	0.230	0.087	0.240