Working Paper 11-22
Economic Series
May, 2011

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 916249875

Organizational Design of Multi-Product Multi-Market Firms

Joaquín Coleff*
Department of Economics
Universidad Carlos III de Madrid

May 2011

Abstract

In this paper, we seek to understand how a multi-product multi-market firm (for example, a multinational firm) designs its organizational structure and compensation scheme when its profitability is conditioned by how market information flows within the company. By modifying its organizational structure–centralizing or decentralizing decision making–and changing the weights of its compensation scheme, the firm can shape how information flows and is represented, changing the firm's profitability. We find that, when being multi-product (having to allocate a scarce resource between markets), the headquarters links the organizational design of decision rights between different product markets. The headquarters decentralizes decision rights in products with higher returns to product differentiation while it centralizes decision rights in products with lower returns to product differentiation. As centralization is complementary with product standardization and decentralization is complementary with product differentiation, the organizational design conditions the firm's market policy. The relation among product's decision rights remains even when the headquarters cannot control how local managers allocate resources in their own local divisions. Our results are robust to different generalizations. Our paper therefore, contributes to the literature on organizational design by analyzing the case of multi-product multi-market firms.

Keywords: Multinational, Multi-product, Organization Design, Resource Scarcity, Cheap Talk
JEL Classification: D2, D8, L2, G34

* I am indebted to Susanna Esteban for her support and helpful guide. I also thank Daniel García-González, Carlos Ponce, Antonio Cabrales, Marco Celentani, Marc Möller, Dolores de la Mata, Joel López Real and Jessica Bordón. All remaining errors are my own. Contact: jcoleff@eco.uc3m.es. Departamento de Economía, Universidad Carlos III de Madrid, Getafe (Madrid), Spain.

1 Introduction

In this paper, we seek to understand how a multi-product multi-market firm (for example, a multinational firm) designs its organizational structure and compensation scheme when its profitability is conditioned by how market information flows within the company. By modifying its organizational structure–centralizing or decentralizing decision making–and changing the weights of its compensation scheme, the firm can shape how information flows and is used, changing the firm's profitability. Our paper contributes to the literature on organizational design by analyzing the case of multi-product multi-market firms highlighting a relation on how decision rights are allocated within a firm.

In the model, a monopolist manufactures two different products to sell in two (country) markets. The products have independent demands (they are, for example, coffee and candy bars) and can be customized to meet the country demand's specificities. In each country, the firm has a manager who oversees demand information and who may have product-design decision rights if decision making has been decentralized.

The two country managers are under the umbrella of a headquarters' office (hereafter, HQ) that has the same objective function as the firm. Country demands are heterogeneous (for example, they have different demand elasticities) and thus country-level profits are maximized when the product's characteristics are tailored to the specificities of the country's demand. The country manager, however, only imperfectly observes his own country's demand heterogeneity, but can improve the quality of his signal by devoting more time and effort to information acquisition.

Managerial time, however, is a scarce resource that must be split between the two product markets the manager oversees. Time and effort are complementary. Effort determines the quality of the signal, while time lowers the cost of exerting it—that is, exerting the same amount of effort over a longer period of time is less costly to the manager. Nonetheless, the decisions in the two markets are not independent: as time is a limited resource, the additional time devoted to one market is not devoted to the other and this results in an increase in the latter market's cost of effort.

The firm must decide whether, and how much, to customize the products to the countries it sells to, adding hazelnut flavoring to its coffee or mixing crunchy rice puffs in its chocolate bars. This decision can be delegated to the country's manager, resulting in a decentralized structure, or be centralized in the HQ's office. Country managers are self-interested and their pays are endogenous to the firm. If their pay is simply a share of the firm's aggregate profits, we say their interests are aligned, while, instead, if they are an unequal average of the two countries' profits, we say their incentives are misaligned.

After observing their private signals, each manager acts upon his information. With centralized decision-making, the managers simultaneously send reports to the HQ's office. It is this office that, after receiving the two reports, decides the specificities of the products to manufacture. Instead, with decentralized decision-making, reports are exchanged between country managers, who then unilaterally determine the characteristics of their two products.

Obviously, when sending reports, the manager may be strategic, aiming to bias the characteristics of the products chosen by either the HQ or the other country's manager. Managers may misreport their observed signals to strategically bias product decisions. If the chosen products are identical across country markets, there are economies of scale in production and thus, cost savings. If, instead, the products are heterogeneous, the firm raises revenues as it is implementing third-degree price discrimination. Therefore, when misreporting, the manager seeks to have the two countries sell the same product, yet he wants this product be the ideal product in his own country. From the firm's point of view, having the manager misreport information is costly since it lowers the accuracy of the information transmitted, upon which other agents (HQ or other country) make product decisions. To align the manager's incentives, the firm can modify the manager's compensation, making it more or less aligned with the firm's profit, and/or decentralize decision-making.

To understand the workings of the model and gauge intuition for the results obtained in the described environment, it is useful to start with the existing literature. As our model builds upon this literature, it inherits some of its workings. Alonso, Dessein, and Matouschek (2008)[1] analyze the problem of a multi-market (two countries) single-product firm when the manager of each country perfectly, but privately observes information about true demand characteristics. As in our paper, there are economies of scale in homogenizing products and there are also gains from price discrimination when customizing the products to the respective country markets. Different from our model, however, are that information is exogenous and does not arise from exerting effort or allocating a scarce resource and that the compensation scheme is exogenously given.

Alonso, Dessein, and Matouschek (2008) show that, for compensation schemes that align incentives, the firm prefers to decentralize decisions, while for those that misalign them, the firm prefers to centralize them. To see this, consider the case of perfect alignment (country managers and HQ have the same objective function). Since there is perfect alignment, the maximization problems of all agents are the same and thus, regardless of who decides, the same decisions are made. As the misalignment increases, the weight on the manager's own country profit does too, the manager, then, starts having incentives to misrepresent information. In his report, the manager seeks to implement product characteristics that are close to his ideal product, as this makes his country's division profit grow. If the misalignment is small, the policy the country manager chooses with decentralization is similar to the one the HQ would choose, albeit the manager makes his choice with better information, and reduces the mismatch with the country's true ideal product. The manager bases his choice on his own observed information and the report that the other sends. Since the two managers have similar objective functions (the misalignment is small), there is little incentive to lie about the reports to each other and thus there is only a small bias in their choices. As the misalignment of incentives increases, however, decisions' biases increase under decentralization as objective functions now differ. Moreover communication under decentralization becomes too poor as they seek to effectively bias the other country's product implementation. As a result, the HQ prefers to centralize decisions and pursue lower production costs through product standardization. Although the managers also misreport information in their communication with the HQ the bias is smaller than it would be if the information were sent to the other manager. The HQ's objective function, being the sum of the two country profits, is closer to the country's objective function than the objective function of the other country and thus there are fewer incentives to lie.

Rantakari (forthcoming) can be viewed as adding a new trade-off to this single-product two-country environment. Now the manager does not perfectly observe the country's characteristics, instead, like in our paper, he obtains a signal that can be made more precise by exerting costly effort. As in Alonso, Dessein, and Matouschek (2008), if the objective functions are aligned, the manager has no incentive to lie when reporting the market's characteristic. However, because the cost of effort is only incurred

[1]See Rantakari (2008) for the case of asymmetric organizational design in the environment of two country single-product firm with private (perfectly observed) information.

by the manager and thus in his aligned objective function it carries more weight than the country's profit, he exerts less effort than optimal for the firm. With less effort, the manager obtains lower quality information, by choosing a product that does not coincide with the country's ideal which lowers profits. To give incentives to the manager to exert more effort and obtain higher quality information, the firm must compensate him by misaligning the objective function, but then the manager has incentives to misreport the signal, which, in turn, lowers profits. When the misalignment needed to induce effort is too large, the communication between countries is too imperfect, as their diverging objective functions lead them to strongly bias the reports, and this makes the firm prefer centralized decision-making.

In our paper, we extend this environment to make the firm multi-product and multi-market. Being multi-product may have multiple effects in the problem of the firm, here we focus on one: the country manager allocates a scarce resource—managerial time—to acquire information in the two markets he oversees. The product markets have independent demand yet differ in their returns to product differentiation, which makes the quality of information more valuable in one market than in the other.

Being multi-product and endogenously determining the allocation of the scarce resource qualitatively modify the findings. If time allocation between product markets were, instead, exogenous, the problem of the firm would be equivalent to the problem of two multi-market single-product firms and thus the results in Rantakari (forthcoming) would apply separately to each market. But, if the allocation of time is endogenous to the manager, as it is in our paper, the results are no longer an immediate generalization of Rantakari (forthcoming). The asymmetry of the returns to differentiation makes the firm want to shift resources (time and effort) to the market with higher returns to differentiation. To provide incentives for such a shift, the firm misaligns incentives, decentralizing decision-making in the high-return market, and aligning incentives and centralizing decision marking in the market with lower returns to differentiation. That is, to induce the correct shift of resources, the firm jointly modifies the return to effort and time in the two markets.

Nonetheless, this negatively correlated allocation of decision rights may not be optimal as it misses out on one effect in the market with lower return. If the difference in return between the two markets exists but is small, the cost of shifting resources away from the market with low return and of obtaining bad quality information in this market, is large. Low quality information means that product decisions with centralization are "incorrect" and yield low profitability although initially this market was almost as profitable as the other. To offset this effect, the firm modifies its decision to also decentralize the market with lower returns to differentiation so that more resources are devoted to it. That is, if the differences in returns to differentiation between markets are small, the firm prefers to decentralize the two markets. If, instead, the differences are large, the market with higher returns to differentiation is decentralized while the other is not.

It is worth noting that whether the firm prefers to decentralize the two markets depends critically on the manager's allocation of time not being verifiable. If the HQ was to monitor the allocation of time, the firm would not need to decentralize the market with lower returns to provide incentives. Instead, the firm could simply force the manager to devote more time to it. Then, with more time being devoted, the manager would obtain, and transmit, better information and this would, in turn, raise the profitability of product standardization and increase the returns from centralizing the market with lower returns.

Lastly, there are other strains of literature that our work relates to. Athey and Roberts (2001), Friebel and Raith (2010), and Dessein, Garicano, and Gertner (forthcoming) explore other environments but share with Rantakari (forthcoming) and our paper the trade-off between the choice of effort and the quality of decision making. Our paper also relates to the organizational design literature that uses a mechanism design approach, which is summarized in Mookherjee (2006). This literature analyzes the trade-offs between performance and incentives (not the allocation of decision rights) by assuming that contracts are complete, which implies that the revelation principle applies. In our framework, and in the one of Alonso, Dessein, and Matouschek (2008) and Rantakari (forthcoming), imposing completeness of contracts would imply that decision rights are always centralized, making the framework unsuitable to understand the problem of organizational design: contract incompleteness (i.e., pays that are contingent on realized profits) and decision-making allocation are inherent to the same problem.

The paper is organized as follows. In Section 2, we describe the model and solve it in Section 3. In Section 4 we extend the model to consider endogenous coordination needs and externalities in information acquisition, showing that our results are robust. In Section 5, we conclude.

2 Setup of the Model

A multi-division firm produces two products, A and B, which it sells in two different regional markets, 1 and 2. Each regional division is controlled by a local division manager who is in charge of obtaining information about demands' characteristics. We denote by θ_{ji} the demand characteristic of product j in region i, for $j \in \{A,B\}$ and $i \in \{1,2\}$.[2] Product demands are unrelated both by region and by product, which implies that demands' characteristics are independently distributed. We assume that θ_{ji} is uniformly distributed over the interval $[-\overline{\theta}_j, \overline{\theta}_j]$, where $\overline{\theta}_A$ and $\overline{\theta}_B$ are bounded and, without loss of generality, $\overline{\theta}_A \geq \overline{\theta}_B$.

We define a_{1A} as the type of product A the firm offers in region 1, where $a_{1A} \in \mathbb{R}$. We define a firm's product strategy for market A as a pair of actions (a_{1A}, a_{2A}).[3] For instance, given a strategy for product A, (a_{1A}, a_{2A}), and a taste for product A in region 1, θ_{1A}, profits derived from product A in region 1 are,

$$\Pi_{1A} = K - (a_{1A} - \theta_{1A})^2 - \beta(a_{1A} - a_{2A})^2.$$

The term K captures the maximum potential profits that the firm can obtain from product A.[4] The potential profits K and the actual profits Π_{1A} can differ for two reasons: 1) the firm does not achieve a good fit between product strategy and local demand characteristics in region 1, represented by the term $(a_{1A} - \theta_{1A})^2$; 2) the firm's strategy does not accomplish a good coordination in product A across regions, represented by $(a_{1A} - a_{2A})^2$. The more standard the product strategy, the lower the term $(a_{1A} - a_{2A})^2$, and the better the coordination across regions. Similarly, we define the profit for each regional division in each product as Π_{1B}, Π_{2A} and Π_{2B}.

[2]The parameter θ represents demand characteristics that can be used by the firm to increase its profits. For example, suppose a market with horizontal product differentiation and installed capacity, where preferences are single-peaked at θ, such that firm's profits increase as its product is closer to the bliss point θ.

[3]In this paper we use the concept of strategy to represent two different ideas. "Firm's strategy" is intended to represent the set of decisions that the firm as a whole takes and the objectives it pursues. "Players' strategies", in the sense of the game-theoretic literature, will refer to the mappings from histories into actions that every agent is entitled to choose.

[4]In an extension in Section 4.1, the term K is assumed to be an increasing function of the level of divisional integration. The level of integration represents the losses for not having a good coordination between product strategies. Some papers require that this level of divisional integration is a firm's choice variable and some papers do not. See Section 4 for a discussion.

The payoff of each local manager depends on the compensation scheme designed by the headquarters. For each product j, we denote by s_j the share of the division 2's profit in market of product j that is awarded to division 1 and $(1-s_j)$ the share of division 1's profits in market of product j that remains in division 1. The value of s_j ($\in [0,0.5]$) defines how much aligned the incentives of local managers are in terms of product j. For example, in one extreme case when $s_j = 0$, a local manager only cares about his own profits in market of product j; in the other extreme case when $s_j = 0.5$, a local manager cares equally about his own profits and the other manager's profits in market of product j, i.e., the manager receives half of the total firm's profits. The payoff of the local manager of division 1 in market j is

$$U_{j1} = (1-s_j)\Pi_{j1} + s_j\Pi_{j2} - C(e,t), \quad j \in \{A,B\},$$

where $C(e,t)$ is the cost of acquiring information of precision e when the manager allocates the amount t of resources, e.g., managerial time.

The headquarters chooses the organizational design of the firm, which is defined as an allocation of the decision rights, g, and as a compensation scheme, s for each product market, to maximize the expected sum of division payoffs. Decision rights can be centralized by the headquarters or decentralized to the local managers, $g \in \{C;D\}$, where C stands for centralization and D for decentralization. Under centralization of decision rights, each manager sends reports about demands' characteristics to the headquarters before it makes a decision of the product strategy. Under decentralization, local managers may communicate between themselves before taking a decision about the product to be sold in their own regions. This means that under decentralization the firm's product strategy results from the addition of two separate decisions.[5]

The problem of the headquarters choosing the optimal organization design would be simple if there were no agency problems. But, as communication is soft and non-verifiable, local managers act strategically to exaggerate their local information in their own interest. Following the literature starting from Alonso et al (2008), we model informal communication as a one-round cheap talk model (Crawford and Sobel, 1982).[6]

Moreover, information about demand characteristics is not perfectly observed by local managers. Instead, local managers observe an imperfect signal of demand characteristics with the following technology: the realization of the signal $\hat{\theta}$ equals the true value θ with probability $\sqrt{e}$ and equals a random draw from the distribution of θ with probability $1-\sqrt{e}$. The quality of this signal reflects the effort and resources local managers apply to learning about demand characteristics, and it is observable but non-verifiable to the organizational participants.

The cost of acquiring a signal of quality $\sqrt{e}$ exerting a level e of effort and allocating t resources is $C(e,t) \equiv \mu(t)C(e)\sigma^2$. Each local division has a budget of 1 of resources, i.e., $0 \le t \le 1$ and $t_A + t_B \le 1$

[5]Our framework is symmetric and we follow the literature to concentrate on symmetric structures for each market j ($j \in \{A,B\}$) which implies centralizing or decentralizing decision making. Defining s_{ji} the share of product j in region i the symmetric structure implies $s_{j1} = s_{j2} = s_j$. Similarly if g_{ji} is the allocation of decision right of product j in region i the symmetric structure implies $g_{j1} = g_{j2} = g_j$. For these results in asymmetric structures see Alonso et al (2008) and Rantakari (2008).

[6]Also see Geanakoplos and Milgrom (1991) for a reflection over information transmission within organizations: "we assume that only the surface content of a message like "produce 100 widgets" can be grasped costlessly; the subtler content, which depends on drawing an inference from the message using knowledge of the sender's decision rule, can be inferred only at a cost."

per local division, and effort is a free choice in $e \in [0,1]$. We assume $C(e) > 0$, $C'(e) > 0$, $C''(e) > 0$, $\mu(t) > 0$, $\mu'(t) < 0$, $\mu''(t) < 0$, and $\lim_{t \to 0} \mu(t) = +\infty$. We normalize the cost function to be proportional to product demand variability, σ^2, i.e., it is more difficult to find the information when it is disperse in a bigger interval. The function $C(e)$ provides the convexity of the cost of effort in learning about market characteristics. The function $\mu(t)$ scales the marginal cost of that effort. Exerting effort and allocating resources in collecting information in one market are complementary. The effort determines the precision of the signal, while resources assigned reduce the cost of this effort. For example, it is less costly for a manager to acquire an amount of information if this is exerted over a longer period of time.[7] We assume that $C(e) = -(e + \log(1-e))$ and $\mu(t) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $\overline{\mu} \in \mathbb{R}_+$ and $b \in [0,1]$.[8] We analyze two different cases concerning the allocation of resources. In our benchmark case, the headquarters controls how resources are allocated. In the second case, local managers are in charge of resource allocation.[9]

Finally, the timing of the model for the benchmark case where the headquarters controls resource allocation is as follows (Figure 1): first, the headquarters chooses the firm's structure (decision making and compensation scheme) for each product and an allocation of resources in each division; second, local managers simultaneously and independently choose how much effort, e, to devote to collect local information about each product taste; third, signals $\hat{\theta}_{ij}$ with precisions e_{ij} about the true values of θ_{ij} are observed; fourth, strategic communication takes place; fifth, products a_{ij} are chosen and, finally, payoffs are delivered. Figure 2.a shows a mix organizational design where the firm decentralizes the decision right of product A and centralizes the decision rights of product B. Manager 1 chooses a_{1A} and manager 2 chooses a_{2A} after they communicate with each other. The headquarters chooses a_{1B} and a_{2B} after communicating with country managers. Figure 2.b shows an organizational design where the firm decentralizes the decision rights of both products.

For the case where managers can decide resource allocation (Figure 1), the timing changes in that resource allocation is made by local managers simultaneously with their choice of effort.

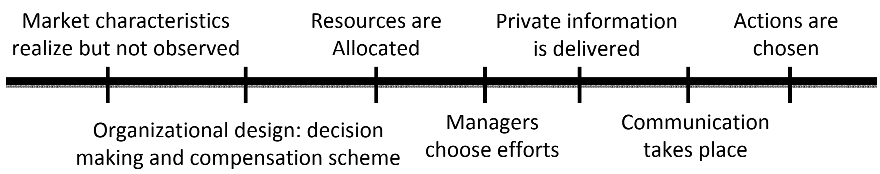
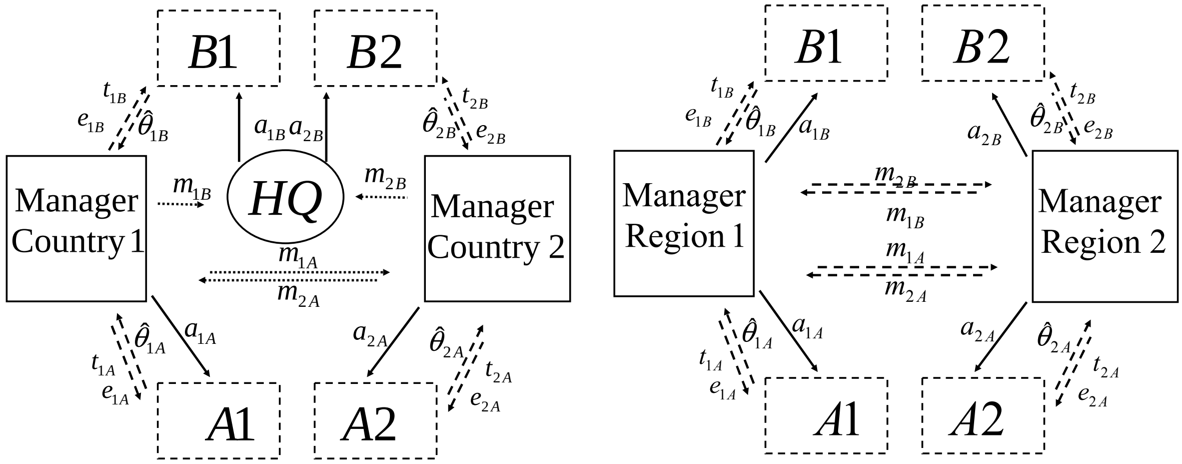


Figure 1: Timing.

[7]Resources and effort in one activity are complementary. Then, the efforts exerted to collect information about different products are substitutes. If managers allocate resources, our model can be interpreted as the multi-tasking model of Holmstron and Milgrom (1991) for substitute effort in the organizational design environment.

[8]These expressions capture all the general properties and contribute with simplicity in solving the model.

[9]We motivate the analysis of these two cases as follows: the headquarters may have calculated and allocated which is the optimal amount of resources in each local division for operational functioning (motivating our benchmark case). However, it could be argued that within each division, local managers administer how to distribute these resources for learning about market characteristics (motivating our second case). See Geanakoplos and Milgrom (1991): "To take advantage of the information processing potential of a group of managers, it is necessary to have the managers attend to different things. But these differences are themselves the major cause of failure of coordination among the several managers." In their model, "a chief executive allocates production targets, capital and other resources to division managers who in turn reallocate the budgeted items to their subordinates, etc. until the resources and targets reach the shops where production takes place." Nevertheless, delegating resource allocation may be based upon positive externalities in market learning. We adapt our model to this case in an extension.



(a) Mixed centralization and decentralization (b) Decentralization in both products' markets

Figure 2: Organizational Structure depending on decision making.

We focus on Pareto Efficient and Perfect Bayesian Nash equilibria. We derive the equilibria that maximize total expected profits by backward induction. Given the signals and the communication outcome, we find the best response functions. Then, anticipating these best response functions, local managers engage in optimal strategic communication. Anticipating the optimal strategic communication and actions, each local manager allocates resources and exerts effort to collect accurate local information. Finally, given optimal behavior, the headquarters chooses the optimal organizational design of the firm.

3 Equilibrium

To find the optimal structure we calculate the incentive scheme that maximizes total expected profits of the organization for the four possible allocation of decision rights, taking as given the best response of local managers in collecting, transmitting, and using information. Then, we compare which structure provides higher expected profits.

First we analyze how information is transmitted and used (Section 3.1). Second, we study how local managers acquire local information (Section 3.2). Third, we describe the resource allocation problem (Section 3.3). Finally, we characterize the organization structure that maximizes total profits (Section 3.4).

We focus on one product in sections 3.1 and 3.2, omitting product subindex j until section 3.3. We can omit the subindex because product A and B have unrelated demands and profit functions. They are related only because they share a common input, i.e., resource allocation.

With respect to the equilibrium concept, we focus on Pareto Efficient and Perfect Bayesian Nash equilibrium. In some stages, there are multiple equilibria that can be ranked from a pareto optimality perspective; therefore we concentrate on those equilibria that leave the agents with the maximum expected payoffs.[10] We assume that agents can coordinate over those equilibria when it is mutually beneficial.

[10]This criteria for selecting equilibria satisfies also the NITS condition of Chen, Kartik, and Sobel (2008).

3.1 Actions and Communication: Transmitting and Using Information

We proceed by backward induction and we analyze the communication and decision making stages for a given organizational design (allocation of decisions rights and a compensation scheme), and the amount of information acquired by local managers. Each local manager i has private information about market characteristic in his own region, θ_i. We solve how information is transmitted by managers and used by decision makers for one product. The solution is similar for both products, and follows from Alonso, Dessein and Matouscheck (2008).

Under centralization, the headquarters communicates with each local manager and forms beliefs about local demand characteristics, i.e., $E[\theta_1]$ and $E[\theta_2]$, and chooses the firm's product strategy solving

$$\max_{a_1,a_2} E[U_1+U_2] = E[\pi_1+\pi_2] = E[K(\beta)-(a_1-\theta_1)^2-(a_2-\theta_2)^2-2\beta(a_1-a_2)^2].$$

Under decentralization, local managers communicate between them, form beliefs about the other manager's action, i.e., manager 1 forms beliefs $E[a_2]$, and chooses his action solving

$$\max_{a_1} E[U_1] = E[(1-s)\pi_1+s\pi_2] = E[K(\beta)-(1-s)(a_1-\theta_1)^2-s(a_2-\theta_2)^2-\beta(a_1-a_2)^2].$$

The following proposition characterizes the optimal actions under centralization and decentralization.

Proposition 1. *1.a Conditioned on beliefs, the optimal actions under centralization are*

$$\begin{aligned}
a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{2\beta}{1+4\beta}E[\theta_2|m_2], \text{ and}\\
a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{1+2\beta}{1+4\beta}E[\theta_2|m_2].
\end{aligned}$$

1.b Conditioned on beliefs defined by $E_2[\theta_1]\equiv E_2[\theta_1|m_1]$ and $E_1[\theta_2]\equiv E_1[\theta_2|m_2]$, the optimal actions under decentralization are

$$\begin{aligned}
a_1^D(m_1,m_2,\theta_1) &= \frac{(1-s)}{(1-s+\beta)}\theta_1+\frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}E_2[\theta_1]+\frac{1-s+\beta}{1-s+2\beta}E_1[\theta_2]\right], \text{ and}\\
a_2^D(m_1,m_2,\theta_2) &= \frac{(1-s)}{(1-s+\beta)}\theta_2+\frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}E_1[\theta_2]+\frac{1-s+\beta}{1-s+2\beta}E_2[\theta_1]\right].
\end{aligned}$$

Proposition 1 is based on Alonso, Dessein, and Matouschek (2008) and Rantakari (forthcoming). Optimal decision making reveals that local managers cannot truthfully transmit the information acquired. Let $\overline{m}_i\equiv E_j[\theta_i|m_i]$ be the receiver j's expectation of θ_i after receiving the message m_i.[11] Each local manager has incentives to lie in order to improve the profits of his own division. The intrinsic incentives to lie under decentralization are $\overline{m}_1-\theta_1=\frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}\theta_1\equiv\omega_D\theta_1$ and under centralization are $\overline{m}_1-\theta_1=\frac{(1-2s)\beta}{1-s+\beta}\theta_1\equiv\omega_C\theta_1$. Given these incentives to lie, the only incentive compatible communication for the sender is, as described in Crawford and Sobel (1982), a partition of the state space. Recall that we

[11] After sending a message m_1, the receiver forms a posterior of $\overline{m}_1$ about θ_1. Communication is not perfect when the posterior $\overline{m}_1$ and the real value θ_1 differ.

have assumed that demand's characteristic of product j is uniformly distributed in $[-\overline{\theta}_j, \overline{\theta}_j]$. Then, we characterize the truthfully revealing partitions and communication equilibria.

Proposition 2. *Fix two positive integers N_1 and N_2, there exists at least one Perfect Bayesian Nash equilibrium characterized by the functions $(\upsilon_1(m_1|\theta_1), \upsilon_2(m_2|\theta_2), a_1(m_2, m_1, \theta_1), a_2(m_1, m_2, \theta_2), g_1(\theta_2|m_2), g_2(\theta_1|m_1))$. The communication rule $\upsilon_i(m_i|\theta_i)$, decision rule $a_i(m_i, m_{-i}, \theta_i)$, and beliefs $g_i(\theta_{-i}|m_{-i})$ satisfy:*

2 .a *$\upsilon_i(m_i|\theta_i)$ is uniform, with support on $[d_{i,h-1}, d_{i,h}]$ if $\theta_i \in [d_{i,h-1}, d_{i,h}]$.*

2 .b *$g_i(\theta_{-i}|m_{-i})$ is uniform, with support on $[d_{i,h-1}, d_{i,h}]$ if $m_{-i} \in [d_{i,h-1}, d_{i,h}]$.*

2 .c *The boundaries are defined by: i) $d_{i,h+1} - d_{i,h} = d_{i,h} - d_{i,h-1} + 4\frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta} d_{i,h}$ for $h = 1, ..., N_i - 1$ under decentralization; and ii) $d_{i,h+1} - d_{i,h} = d_{i,h} - d_{i,h-1} + 4\frac{(1-2s)\beta}{1-s+\beta} d_{i,h}$ for $h = 1, ..., N_i - 1$ under centralization.*

2 .d *The decision of each worker is defined by part 1.b of Proposition 1 under decentralization and by part 1.a of Proposition 1 under centralization.*

Taking the boundaries $d_0 = -\overline{\theta}$ and $d_N = \overline{\theta}$ of the space, the solution is defined by

$$d_h = \overline{\theta}\frac{x^h(1+y^N) - y^h(1+x^N)}{x^N - y^N} \quad 0 \leq h \leq N,$$

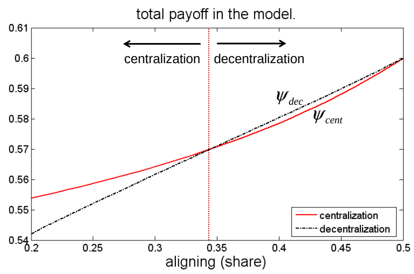
with $x = (1+2\omega) + \sqrt{(1+2\omega)^2 - 1}$ and $y = (1+2\omega) - \sqrt{(1+2\omega)^2 - 1}$, $\omega = \omega_D \equiv \frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}$ under decentralization, and $\omega = \omega_C \equiv \frac{(1-2s)\beta}{1-s+\beta}$ under centralization. Note that $xy = 1$, $x > 1$ and $y < 1$.

Proposition 2 is based on Alonso, Dessein, and Matouschek (2008), and Rantakari (forthcoming). Proposition 2 describes the communication equilibria with N-partition of the space $[-\overline{\theta}_j, \overline{\theta}_j]$. After communication takes place, the successful rate of communication is $E[\overline{m}_i^2] = V_i \sigma_j^2$, where V_i represents the proportion of the local information variance ($\sigma_j^2 \equiv \frac{\overline{\theta}_j^2}{3}$) that is communicated. The rate V_i is increasing in the number of partitions N. The equilibria with $+\infty$-partition are the ones that achieve the maximum expected payoffs for both managers. Onwards, we concentrate on these $+\infty$-partitions equilibria, which deliver the rate of transmission equal to $V_j = \frac{3+3\omega}{3+4\omega}$, with $\omega \in \{\omega_C, \omega_D\}$.[12]

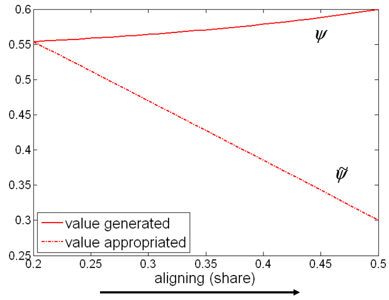
After information is acquired there are two costs in the communication and action stages. One cost is related with how well information is used, and is characterized by Λ in equation (1). Under centralization, the headquarters achieves the minimum cost, given that it maximizes total expected profits with the information available. Under decentralization, however, each local manager has a bias to the profit of his own division and does not internalize the externality that the decision about product strategy has on the other division.

The second cost is associated with information transmission and is characterized by $\Gamma(1-V)$ in equation (1). The factor V represents how well information is communicated between the ones who have the information and the ones who make decisions, i.e., how accurate communication is. The factor Γ

[12] These equilibria satisfy the Non Incentives To Separate (NITS) criteria of Chen, Kartik, and Sobel (2008) when the number of partitions is infinite. For finite partitions, it satisfies NITS if the number of partitions is odd.



(a) Centralization versus decentralization



(b) Value generated versus appropriated

Figure 3: Comparing payoffs under centralization and decentralization (a) and comparing the value generated and the value appropriated (b) by each manager .

represents how important this communication is for the expected profits, i.e., the value of communication accuracy. Under centralization there is more accurate communication than under decentralization because under the former the conflict between each manager and headquarters is lower, i.e., there is less incentive to exaggerate the private information. However, the value of that communication is also higher under centralization. The information that is not communicated under centralization is lost, in the sense that nobody can use that information for making decisions. Under decentralization, however, local managers use, at least, their own information for making their own decisions.

We identify these costs for each local manager under both centralization and decentralization. The expected profits of each division in each product are described in the following lemma. To avoid awkward notation we focus on local manager 1.

Lemma 1. *Under both structures the expected profit function for each local manager is characterized by*

$$E[\Pi_1] = K - \left[\underbrace{\Lambda_1 + \Gamma_{11}(1-V_1)}_{\textit{due to 1 information}} + \underbrace{\Lambda_1 + \Gamma_{21}(1-V_2)}_{\textit{due to 2 information}} \right] \sigma_\theta^2. \tag{1}$$

with the following expressions for centralization $V_C = \frac{3(1-s)(1+2\beta)}{(1-2s)\beta+3(1-s)(1+2\beta)}$, $\Lambda_{1C} = \frac{\beta}{1+4\beta}$, $\Gamma_{11C} = 1 - \Lambda_{1C}$, *and* $\Gamma_{12C} = -\Lambda_{1C}$. *For decentralization the values are* $V_D = \frac{3(1-s)(1-s+2\beta)}{(1-2s)(1-s+\beta)+3(1-s)(1-s+2\beta)}$, $\Lambda_{1D} = \frac{\beta[(1-s)^2+\beta]}{(1-s+2\beta)^2}$, $\Gamma_{11D} = \frac{\beta[(1-s)^2+\beta]}{(1-s+\beta)^2} - \Lambda_{1D}$, *and* $\Gamma_{12D} = \beta\frac{(1-s)^2}{(1-s+\beta)^2} - \Lambda_{1D}$.

Lemma 1 describes the situation under perfect information (the algebra for constructing these expressions are in Appendix 7). In brace we identify the value generated in region 1 by the information of manager 1 and manager 2.

Summarizing the results of this section, we have characterized the value of information under both centralization and decentralization. After information is acquired, it is transmitted and used through local managers to decision makers. Under both centralization and decentralization, the value of information increases in incentive alignment, s. Misaligning incentives (reducing s) reduces the value of information under both structures. This effect is higher under decentralization if $s \leq \underline{s}$. Hence, the value of information is higher under centralization if incentives are sufficiently misaligned, i.e., s low.[13] In Figure 3.a, we show the relation of the value generated in the communication and action stages under centralization (continuous line) and decentralization (dashed line) as a function of the compensation scheme s.

3.2 Acquiring Information

Let us assume now that information is imperfect and costly. Each manager invests an effort e in acquiring an imperfect signal $\hat{\theta}$ of the true value θ. The realization of the signal $\hat{\theta}$ equals the true value θ with probability $\sqrt{e}$ and a random draw from the same distribution of θ with probability $1-\sqrt{e}$. The higher the effort, the more accurate the signal. Following Rantakari (2010) we use e as a measure of the *quality of primary information* which has a cost $\mu(t)C(e)\sigma^2$ with $e \in [0,1]$ and $C(e) = -(e+\log(1-e))$.

We first describe the objective functions with imperfect signals, and then we point out the private incentives of managers and the headquarters to acquire information. Finally, we characterize how information is acquired.

Acquiring information of quality e_i by local manager i, with $i \in \{1,2\}$, with the corresponding cost of acquiring that information means that now the expected profits of division i in equation (1) becomes, (see Appendix 7)

$$E[\Pi_i] = K - \left[1 - e_i \underbrace{[1-\Lambda_i - \Gamma_{ii}(1-V_i)]}_{\text{due to } i \text{ information}} + e_k \underbrace{[\Lambda_i + \Gamma_{ki}(1-V_k)]}_{\text{due to } k \text{ information}} + \mu C(e_i)\right]\sigma_\theta^2, i = \{1,2\} \text{ and } k \neq i. \quad (2)$$

Since local managers do not internalize the externality that their own information generates on the other manager, we distinguish between the profit captured by each local manager, $E[\Pi_i]$, and the profit generated by each local manager, $E[\pi_i]$. Despite the fact that these values are not the same $\pi_i \lesseqgtr \Pi_i$, the aggregate profit captured equals the aggregate profit generated, i.e., $\sum_{i=1,2} \pi_i = \sum_{i=1,2} \Pi_i$. The profit captured by a local manager is represented in equation (2); however, the profit generated by a local manager is

$$E[\pi_i] = K - \left[1 - e_i \underbrace{[1-(\Lambda_i+\Lambda_k)-(\Gamma_{ii}+\Gamma_{ik})(1-V_i)]}_{=\psi_{ii} \text{ due to information of manager } i} + \mu C(e_i)\right]\sigma_\theta^2. \quad (3)$$

where ψ_{ii} represents the value generated by manager i with a signal of precision e_i. A local manager acquires information considering the effect that his own information has on his own payoff and not on the value that the information generates. For this reason there is an inefficiency in information acquisition.

[13]For extension 4.1 it is worth noting that, under both centralization and decentralization, the value of information is decreasing in local divisions' integration, β. The cutoff $\underline{s}(\beta)$ is increasing in β. Increasing local divisions' integration (increasing β) reduces the value of information under both structures but more under decentralization.

Given g and s the expected utility of each manager over a product is $E[U_i] = (1-s)E[\Pi_i] + sE[\Pi_k]$ or,

$$E[U_i] \quad = \quad K + \left[-1 + e_i\tilde{\psi}_{ii} + e_k\tilde{\psi}_{ki} - \mu C(e_i)\right]\sigma_\theta^2, \tag{4}$$

where $\tilde{\psi}_{ii}$ represents the value appropriated by manager i with a signal of precision e_i, $\tilde{\psi}_{ki}$ is the externality to manager i generated by manager k who has acquired a signal of precision e_k. The expressions are defined by $\tilde{\psi}_{ii} \equiv (1-s)[1-\Lambda_i - \Gamma_{ii}(1-V_i)] - s[\Lambda_k + \Gamma_{ik}(1-V_i)]$, $\tilde{\psi}_{ki} \equiv s[1-\Lambda_k - \Gamma_{kk}(1-V_k)] - (1-s)[\Lambda_i + \Gamma_{ki}(1-V_k)]$, $\tilde{\psi}_{kk} \equiv (1-s)[1-\Lambda_k - \Gamma_{kk}(1-V_k)] - s[\Lambda_i + \Gamma_{ki}(1-V_k)]$ and $\tilde{\psi}_{ik} \equiv s[1-\Lambda_i - \Gamma_{ii}(1-V_i)] - (1-s)[\Lambda_k + \Gamma_{ik}(1-V_i)]$. It is important to notice (Figure 3.b) that the value of the information captured by a local manager, $\tilde{\psi}(s,g)$, is decreasing in s under both centralization and decentralization. However, the value of information generated $\psi(s,g)$ is increasing in s under both centralization and decentralization. Because $\psi_i \neq \tilde{\psi}_{ii}$, the information acquired is not optimal. The following lemma describes the effort choice

Lemma 2. *Under both decentralization and centralization the effort choice is characterized as follows:*

2.a The optimal level of effort is given by $\psi = \mu C'(e^*)$.

2.b Each local manager chooses effort according to $\tilde{\psi} = \mu C'(\hat{e})$.

The comparative static implies that $\frac{\partial e}{\partial \mu} = -\frac{C'(e)}{\mu C''(e)} < 0$, $\frac{\partial e^*}{\partial s} > 0$ and $\frac{\partial \hat{e}}{\partial s} < 0$ under both centralization and decentralization, since $\frac{\partial \psi}{\partial s} > 0$ and $\frac{\partial \tilde{\psi}}{\partial s} < 0$. For the function $C(e) = -(e + \log(1-e))$, $\hat{e} = \frac{\tilde{\psi}}{\mu + \tilde{\psi}}$ and $e^* = \frac{\psi}{\mu + \psi}$.

The proof of Lemma 2 consists in solving the first order condition of the objective function respect to e. The objective function is defined by equation 3 in part 2.a of Lemma 2 and by equation 4 in part 2.b of Lemma 2. The information acquired by a local manager $\hat{e}$ is decreasing in his incentive alignment s because it depends on the perceived value of that information, $\tilde{\psi}$, while the optimal amount of information is increasing in incentive alignment, s, because it depends on the real value of information, ψ.

So far, we have described the incentives to exert effort for acquiring information, e, and how much value that information generates, $\psi(s)$. Under both structures, centralization and decentralization, local managers face similar trade-offs. A manager effort increases but the value generated decreases when local managers incentives are narrowed to their own division's profit. Under decentralization, however, the perceived value of information tends to be greater than under centralization.[14]

In this paper efforts are neither strategic complement nor strategic substitutes, since strategic effects cancel out due to the following assumptions: 1) independence of θ_{j1} and θ_{j2} (for $j \in \{A, B\}$); 2) messages $\overline{m}_1$ and $\overline{m}_2$ are unrelated in the communication game; and 3) the functional form of the profit function.[15] Relaxing these assumptions to capture the strategic interaction of the efforts appears a promising avenue for future research.

[14] $\tilde{\psi}$ is greater under decentralization except when β is excessively high and s excessively low. The advantage of decentralization is that a local manager uses his own information to adapt his product to his local market. Managers value more the information under decentralization except in some extreme circumstances where standardization is very important and compensation schemes are narrowed to local divisions' profits.

[15] Appendix 7 shows the form of the expected profit function under both structures.

3.3 Resource Allocation

Again subindex $j \in \{A,B\}$ stands for product A and product B respectively. Local managers have resources equal to 1 which are allocated to acquire information about different demands' tastes. The allocation of these resources determines the marginal cost of information, i.e., $\mu_A(t_A)$ and $\mu_B(t_B)$, where $t_A + t_B \leq 1$. The objective function of the headquarters is $\sum_{i=1,2} E[\pi_{iA}] + E[\pi_{iB}]$,

$$\sum_{i=1,2} \left\{ K - [1 - e_{iA}\psi_A + \mu_A C(e_{iA})]\, \sigma_A^2 \right\} + \left\{ K - [1 - e_{iB}\psi_B + \mu_B C(e_{iB})]\, \sigma_B^2 \right\}. \tag{5}$$

Note that $E[\pi_{iA}] + E[\pi_{iB}]$ is the value generated by local manager i within the firm. The objective function of a local manager in division i is $E[U_{iA}] + E[U_{iB}]$

$$\{K - \sigma_A^2[1 - e_{iA}\tilde{\psi}_{iiA} - e_{kA}\tilde{\psi}_{kiA} + \mu_A C(e_{iA})]\} + \{K - \sigma_B^2[1 - e_{iB}\tilde{\psi}_{iiB} - e_{kB}\tilde{\psi}_{kiB} + \mu_B C(e_{iB})]\}. \tag{6}$$

We analyze two situations. As a benchmark case we analyze the optimal resource allocation, i.e., how the headquarters allocates resources within each division. The headquarters chooses t_{Ai} for local manager i maximizing the expected profit in equation (34) subject to optimal choice of effort described in part 2.b of Lemma 2. In the second case, local managers allocate resources. Each manager chooses t_A and t_B maximizing his expected payoff defined in equation (38). In the following lemma we summarize how resource allocation is chosen.

Lemma 3. *The resource allocation t_A within each division is determined as follows:*

3.a The headquarters chooses t_A according to

$$-\frac{\partial \mu_A}{\partial t_A}\sigma_A^2 \left[C(e_A) - \frac{\partial e_A}{\partial \mu_A}[\psi_A - \mu_A C'(e_A)] \right] = -\frac{\partial \mu_B}{\partial t_B}\sigma_B^2 \left[C(e_B) - \frac{\partial e_B}{\partial \mu_B}[\psi_B - \mu_B C'(e_B)] \right]. \tag{7}$$

Replacing $\tilde{\psi} = \mu C'(e)$ we have $[\psi - \tilde{\psi}]$ as a measure of the moral hazard problem in each product market.

3.b A local manager chooses t_A according to[16]

$$-\frac{\partial \mu_A}{\partial t_A} C(e_{iA})\sigma_A^2 \quad = \quad -\frac{\partial \mu_B}{\partial t_B} C(e_{iB})\sigma_B^2. \tag{8}$$

We assume that $\mu(t)$ is sufficiently convex for an interior solution to exist.[17] A comparative static shows that more resources are allocated to learn about a product when more information is acquired, i.e., higher e, and when the product has higher returns to differentiation, i.e., higher σ^2. In effect, information acquisition and resource allocation are complementary. There are incentives to allocate more resources in markets where managers acquire more information, e.g., the higher e_A the higher t_A. Also more information is acquired in markets that receive more resources, the higher t_A the higher e_A.

As noted in the Section 3.2, the effort in information acquisition of local managers is not efficient, and thus their allocation of resources will also be distorted. Indeed, if the information is acquired effi-

[16] We apply envelope theorem to get this condition.

[17] The convexity of resource allocation outweighs the effort convexity problem and incentive alignment convexity problem.

ciently, the term $\frac{\partial e}{\partial \mu}[\psi - \mu C'(e)]$ disappears due to part 2.a of Lemma 2 and the resource allocation is always characterized by part 2.b of Lemma 3. The headquarters corrects this inefficiency in information acquisition through resource allocation as described in part 3.a of Lemma 3, or through organizational design, as is described below in the following section.

Whenever local managers allocate resources, they choose t_A considering the opportunity cost of those resources on their own expected utility. This is consistent with Geanakoplos and Milgrom (1991) who conclude "that managers at each level optimally focus attention only on those variables that determine the marginal productivity of resources and the marginal costs of production in the units under their command". As described above, the choice of t_A is only affected by the returns to differentiation σ^2 and the function $C(e)$. Hence, to reallocate resources in favor of market A, the headquarters not only can promote a more considerable effort in market A, increasing $C(e_A)$, but also can discourage effort in market B, reducing $C(e_B)$.

If $\sigma_A^2 = \sigma_B^2$, there is a symmetric allocation of resources and identical effort. Note from Lemma 3.b that the allocation of resources is symmetric only if $e_A = e_B$, and from Lemma 2.a we can have identical effort if resource allocation is symmetric.

The function $C(e) = -(e + \log(1-e))$ has a slope $C'(e) = \frac{e}{1-e}$. For the function $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$, with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$, the resource allocation choice is $t_A = \frac{1}{1+H^{\frac{1}{1+b}}}$ with[18]

$$H_1 \equiv \frac{C(e_{iB})}{C(e_{iA})}\frac{\sigma_B^2}{\sigma_A^2} \quad \text{and} \quad H_0 \equiv \frac{C(e_{iB}) - \frac{\partial e_B}{\partial \mu_B}[\psi_B - \mu_B C'(e_B)]}{C(e_{iA}) - \frac{\partial e_A}{\partial \mu_A}[\psi_A - \mu_A C'(e_A)]}\frac{\sigma_B^2}{\sigma_A^2}.$$

The term is H_1 when resource allocation is decided by each local manager and H_0 when resource allocation is chosen by the headquarters. The cost of acquiring information is $\mu_A(t_A) = \overline{\mu}0.5^b\left(1 + H^{\frac{1}{1+b}}\right)^b$. The following Lemma is crucial for our results.

Lemma 4. *Define t^* as the value that maximizes $[-\mu(t) - \mu(1-t)H]$. Assume $\mu(t,b) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ and $0 < H \leq 1$. Given $b_0 \in [0,1]$, the set of all functions with $b_1 \in [b_0, 1]$ satisfies the property that $\mu(t_0^*, b_0) \geq \mu(t_1^*, b_1)$.*

The proof of the Lemma 4 is in Section 6.1. This Lemma describes that if resource allocation is more important, then the marginal cost of acquiring information is lower in the market with higher variance. In other words, a higher b implies lower μ_A and higher μ_B if $\sigma_A^2 > \sigma_B^2$. Consequently, b represents the importance of resource allocation.

3.4 Optimal Structure

To find the optimal structure of the firm, we must evaluate the best response of local managers for each possible structure and compare the total expected payoff obtained under each possible combination of decision rights, i.e., centralization and decentralization in each product market. The problem is as follows

$$\max_{g,s} \sum_{i=1,2} \left\{K - [1 - e_{iA}\psi_A + \mu_A C(e_{iA})]\,\sigma_A^2\right\} + \left\{K - [1 - e_{iB}\psi_B + \mu_B C(e_{iB})]\,\sigma_B^2\right\}, \tag{9}$$

[18] With the properties $\mu'(t) = -b\overline{\mu}\left(\frac{0.5^b}{t^{1+b}}\right) < 0$, $\mu''(t) = b(1+b)\overline{\mu}\left(\frac{0.5^b}{t^{2+b}}\right) > 0$, $\frac{\partial \mu'(t)}{\partial b} < 0$, and $\frac{\partial \mu''(t)}{\partial b} > 0$.

subject to the optimal effort choice described in part 2.b of Lemma 2 and resource allocation choice described in part 3.a of Lemma 3, in the benchmark case where the headquarters chooses t, or part 3.b in Lemma 3, when local managers choose t. For simplicity, we assume $\sigma_B^2 \equiv 1$, such that the ratio of market variances is equal to the variance of product A, i.e., $\frac{\sigma_A^2}{\sigma_B^2} \equiv \sigma_A^2$, with $\sigma_A^2 \geq 1$ since $\overline{\theta}_A \geq \overline{\theta}_B$.[19] The first order conditions for the propositions of this section are in Appendix 7.

We first analyze the situation when resource allocation is not important, i.e., $b \to 0$, as an starting point to understand our benchmark case, where the headquarters allocates resources, and our first extension, where managers do. For this case, the optimal design for market A is independent of the optimal design for market B. In the following proposition we summarize the result.

Proposition 3. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \to 0$. There exists a threshold $\tilde{\mu}$ above which centralization outperforms decentralization and the choice of decision rights is independent of market of product A and market of product B.*

In Appendix 6.2 we develop a sketch of the formal proof in Rantakari (2010). We provide here some useful intuition for our next results. As explained in Section 3.1 and represented in Figure 3.a, centralization performs better than decentralization when the incentives of each local manager and the headquarters are sufficiently misaligned, i.e., s is low. The main trade off in designing the compensation scheme, i.e., in choosing s, is clearly observed in the first order condition respect to s,

$$\frac{\partial e}{\partial s}[\psi - \tilde{\psi}] + e\frac{\partial \psi}{\partial s} \quad = \quad 0. \tag{10}$$

The first term in the left hand side is negative, since $\frac{\partial e}{\partial s}$ is negative, and the second term is positive. When the manager's payoff depends more on total firm's profit, i.e., higher s, there is an increase in the value of the information acquired, $e\frac{\partial \psi}{\partial s}$, but also an increment in the value of acquiring further information $\frac{\partial e}{\partial s}[\psi - \tilde{\psi}]$, because ψ increases in s while $\tilde{\psi}$ decreases in s. When the cost of information is low, there is a lot of information acquisition, and the headquarters aligns managers' incentives with the firm's profits to increase the value of that information. In this case, decentralization performs better than centralization. When the cost of information is high, the headquarters prioritizes information acquisition, narrowing local managers incentives to their own division profit, and, eventually, the firm performs better under centralization.

The structure of the firm balances the moral hazard problem of suboptimal information acquisition with the suboptimal value generated by this information in the decision making process. When informational cost is low, the headquarters follows a strategy of product differentiation through decentralization, but when informational cost is high, it follows a strategy of product standardization through centralization.

Although it is not directly stated, we can infer by Proposition 3 that when centralization performs as well as decentralization, the compensation scheme under both structures differs, i.e., $s^C < s^D$. Assume that the informational cost is just the threshold $\tilde{\mu}$, and the headquarters is indifferent to centralizing or decentralizing decision making. This indifference between centralization and decentralization requires

[19]The absolute value of market variances matters if we endogenize β. For robustness, we describe the results in the case that the headquarters also chooses β in Section 4.1.

that $s \leq \underline{s}$.[20] Figure 3.a (Section 3.1) shows that when $s \leq \underline{s}$ the value of information is more sensitive to s under decentralization than under centralization. Hence, at the same informational cost, the optimal profit sharing s under centralization must be lower than the profit sharing under decentralization, i.e., $s^C < s^D$. This result arises because, once the headquarters provides incentives for information acquisition, centralization handles better, at the margin, the trade-off between acquiring and transmitting information. Then, it can foster more information acquisition through a lower s under centralization.

Summing up the result of Proposition 3, when $b \to 0$, resource allocation is not important, and the marginal cost of information is given by $\overline{\mu}$ in each market. The headquarters decentralizes decision making in both markets if $\overline{\mu} \leq \tilde{\mu}$, and centralizes them otherwise. Let us now see how the optimal structure changes as resource allocation becomes important.

Proposition 4. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$. If resources are allocated efficiently, that is by the Headquarters, there exists a threshold in the ratio of returns to product differentiation $\tilde{\sigma}_{AB}$ above which the optimal design requires a mix structure with centralization in market of product B and decentralization in market of product A. Moreover, $\frac{\partial \tilde{\sigma}_{AB}}{\partial b} < 0$.*

Proof. The headquarters chooses t_A and t_B according to condition (7) in Lemma 3. We guarantee an interior solution in resource allocation when the function $\mu(t)$ is sufficiently convex. The allocation of t_A is increasing in the ratio σ_A^2, implying that $\mu_B(1-t_A)$ is increasing in σ_A^2, and $\mu_A(t_A)$ is decreasing in σ_A^2. The resource allocation, that is t_A, defines the costs' values μ_A and μ_B, and Proposition 3 applies in each market: if $\mu_j > \tilde{\mu}$, centralization outperforms decentralization, but if $\mu_j < \tilde{\mu}$, decentralization outperforms centralization in market of product j. The proof of $\frac{\partial \tilde{\sigma}_{AB}}{\partial b} < 0$ is in appendix 4. □

Proposition 4 has several implications for the optimal internal design of the firm and, consequently, the optimal firm's product strategy. First, the headquarters recognizes that the value of information is higher in markets with higher returns to differentiation than in markets with lower returns to differentiation. The reason is straightforward: the expected losses of a wrong product strategy are higher when consumers' tastes are more uncertain. Hence, the optimal resource allocation concentrates more resources in markets with higher returns to differentiation. This is shown in Lemma 3.a: the higher the ratio σ_A^2, the more resources the firm allocates to learn about product A and the fewer resources the firm allocates to learn about product B demand characteristics. Thus, there is higher informational cost and less information acquisition in market of product B, and lower informational cost and more information acquisition in market of product A.

Second, the headquarters recognizes that the information acquired by local managers is suboptimal in both markets. The suboptimal level of effort, however, is more important in a market with higher returns to differentiation. Through resource allocation, the headquarters reduces the effort cost and encourages more learning in the market with higher returns to differentiation.

As the ratio of the returns to differentiation between markets increases, more resources are allocated to market A and fewer to market B. The headquarters will, eventually, find it optimal to follow a strategy of product differentiation in market A and a strategy of product standardization in market B. To implement these strategies, the headquarters decentralizes decision making in market A, providing a compensation

[20] Since for $s > \underline{s}$, not only decentralization generates more value for a given effort e, i.e., $\psi^D > \psi^C$, but also local managers exert a higher effort, i.e., $e^D > e^C$ since $\tilde{\psi}^D > \tilde{\psi}^C$.

scheme that aligns local managers' incentives with the firm's profits. In market B, however, since the aim is to pursue standardization, it is better to centralize decision making with a compensation scheme that narrows local managers incentives to their own divisions' profit.

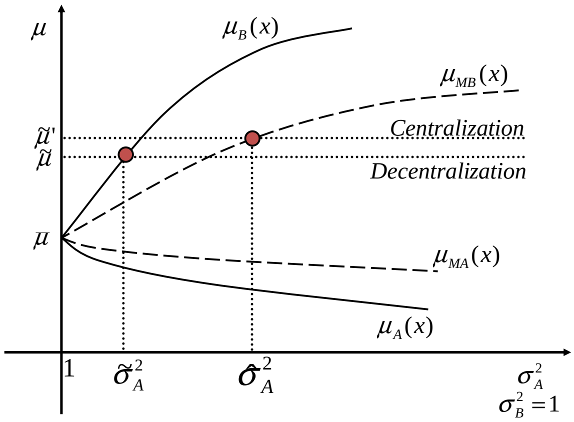


Figure 4: Optimal decision making as a function of the ratio in returns (we assume $\sigma_B^2 = 1$) and informational cost.

Figure 4 shows the relation between informational cost μ and returns to differentiation for each market when resources are allocated by the headquarters, for the particular case that $\overline{\mu} < \tilde{\mu}$. If $\overline{\mu} < \tilde{\mu}$, the headquarters decentralizes decision rights in both markets if $\sigma_A^2 < \tilde{\sigma}_A^2$ and chooses a mix decentralizing decisions about product A and centralizing decisions about product B otherwise. If $\overline{\mu} > \tilde{\mu}$, however, there exists $\breve{\sigma}_A^2$ such that the headquarters centralizes decision rights in both markets if $\sigma_A^2 > \breve{\sigma}_A^2$ and chooses a mix decentralizing decisions about product A and centralizing decisions about product B otherwise. Figure 5.a shows in blue the optimal compensation scheme as a function of the returns to product differentiation for a numerical example.[21]

Rantakari (forthcoming) describes a positive causal relation between returns to product differentiation (that he calls volatility) and decentralization. If decision rights are centralized in our model, an increment in product A's returns to differentiation can motivate the headquarters to decentralize the decision right of product A. Rantakari (forthcoming) drives this causality through a change in the needs for coordination β, while we drive this causality through a reallocation of resources.

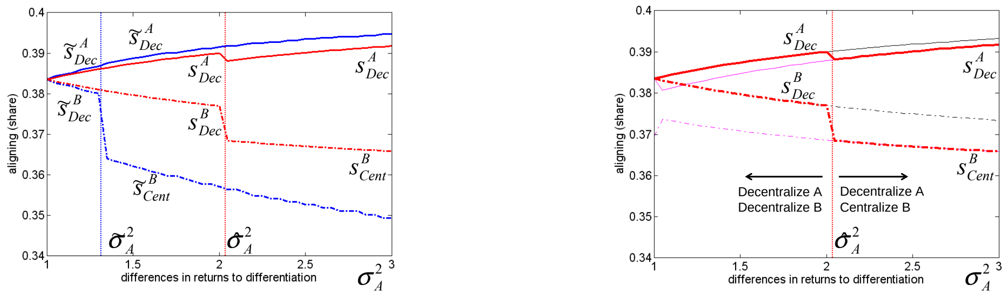
There is an ongoing discussion on the literature about how exogenous is the coordination need.[22] In any case, most authors agree that not all elements of the structure (e.g., contracts, decision rights, divisional integration) can be modified with the same ease and speed. An organization can revise the compensation scheme or its allocation of decision rights more often than its degree of integration which may require updating the equipment, logistic, and information technologies.[23] Our mechanism is more

[21] For this example $\beta = 4$, $\overline{\mu} = 0.65$, $b = 0.5$.

[22] Some authors argue that the headquarters is free to decide the degree of integration between the two different units (See Rantakari (2010)). Some other authors, however, believe that the need for coordination is an exogenous constraint given by technology, legal environment, culture, etc. (See Alonso, et al (2008) and Dessein, Garicano and Gertner (2010)).

[23] Eccles and Holland (1989) describe the case of Suchard when European Union merges most western european markets as

direct than Rantakari (forthcoming)'s one, and we can account for transitory decentralization. Nevertheless, our results are robust to endogenize the needs for coordination that we develop in an extension.



(a) Comparing contracts

(b) Contract when managers allocate resources

Figure 5: Optimal contract when the headquarters allocates resources (blue) and when local managers allocate resources (red).

In the following proposition we point out that our previous result is robust to the case where managers can decide or affect the allocation of resources. We show that, despite the headquarters can not control the allocation of resources, there exists a cutoff in the ratio of returns to differentiation above which the optimal structure combines decentralization and centralization. When the headquarters has no control over resources, she modifies the strategy of the firm to capture as much benefit as possible from resource allocation.

Proposition 5. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$. When local managers allocate resources, there exists a threshold for the ratio of market returns to product differentiation $\hat{\sigma}_A^2(\tilde{\mu})$ above which the optimal design requires a mixed structure with centralization in market of product B and decentralization in market of product A. Moreover, $\tilde{\sigma}_A^2 \leq \hat{\sigma}_A^2$.*

Proposition 5 claims that our result even though managers control the allocation of resources. However, there are two effects that make the relation less intensive, generating a less intensive relation between returns to differentiation and the mixed structure, i.e., $\tilde{\sigma}_A^2 \leq \hat{\sigma}_A^2$. Let's assume $\sigma_A^2 = \tilde{\sigma}_A^2$ and $\overline{\mu} < \tilde{\mu}$, which means that decentralization is optimal if markets have similar returns to differentiation, i.e., if ratio $\sigma_A^2 \sim 1$. Compared with Proposition 4 both effects favour decentralization in the market with low dispersion, i.e., they favour a strategy of product differentiation in market B.

a unique market. It took several years and lots of resources for the company to adapt the company to the new situation. Thomas (2011) also mentions how the market structure of western european markets modifies the needs for standardization. Procter & Gamble and Unilever reorganized their production after the pass of European Regulations in 1992. Firms have spent a lot of time and resources to launch programs to reduce the number of products and to centralize production in fewer plants, e.g., "path to growth" (Unilever in 2000), "Unilever 2010" (Unilever in 2004) and "Organization 2005" (Procter & Gamble in 1999). The program of Unilever included "a more streamlined brand portfolio, moving from 1600 brands in 1999 to a target number of 400 by the end of 2004". "P&G aimed to improve supply chain management of the proliferation of product, pricing, labeling, and packaging variations".

First, the headquarters would allocate more resources in the market with high returns to differentiation than local managers in fact do. Local managers allocate more resources to learn about the resources according to the condition in Lemma 3.b. The cost of acquiring information changes with the ratio of markets' return to product differentiation, i.e., t_A and μ_B increase with ratio σ_A^2, and μ_A decreases with σ_A^2. However, as local managers do not internalize the effect that resources can have in reducing the inefficiency in information acquisition, they allocate more resources in markets with low return to product differentiation. This favors decentralization in markets with low return to product differentiation.

Second, to correct the manager's misallocation of resources in favor of markets with high returns to differentiation, the headquarters changes the organizational design of the firm. Lemma 3.b describes how local managers allocate resources. The headquarters takes advantage of the complementarity between information acquisition and resource allocation to promote not only more information acquisition in those markets with high return to differentiation but also less information acquisition in markets with low return to differentiation. The headquarters implements this change through incentives alignment in markets with low return to differentiation and incentives misalignment in markets with high return to differentiation. These changes in structure, however, may also affect the firm's product strategy.

These two channels increase the likelihood that the headquarters chooses decentralization in market B, as a way to overcome the inefficiency in manager's resource allocation. The ratio of returns σ_A^2 must be higher for the mixed structure of decentralization and centralization to be optimal.

Summarizing, the externalities in information sharing generate suboptimal information acquisition. The headquarters alleviates this inefficiency allocating more resources to the markets where the problem is more serious, i.e., the markets with high return to product differentiation. However, when lacking control over resources, she uses the internal design of the firm, allocating decisions rights and modifying the compensation scheme to correct the effect of resource allocation. Hence, resource allocation and decision rights are imperfect substitutes in the sense that controlling resources leads to a mix strategy of centralization and decentralization, but lacking control derives in a decentralized organization.

By symmetry, we can analyze the implications of Proposition 5 for the case with $\overline{\mu} > \tilde{\mu}$, in which case the optimal organizational design when the ratio σ_A^2 is small requires centralizing decision making and a mix structure arises when the ratio σ_A^2 of the returns to differentiation is high.

Figure 5.b shows how the optimal contract depends on the ratio of returns to product differentiation in a numerical example. The aligning of incentives is higher in the market with higher returns. There are more returns to differentiation than in the other market. The difference in alignment increases with the ratio of returns to product differentiation. There is a threshold above which the organizational design changes, centralizing decisions about the product with lower returns. This change also affects the contract. The objectives functions depend more on the local division profits to motivate more information acquisition in both markets, however they are more misaligned in the centralized product.

Our results can be empirically identified in at least two ways. First, comparing multi-product multi-market firms with single-product multi-market firms and observing that there may be differences in their organizational design. This identification strategy is relevant since there are differences in organizational structure of firms operating in the same market that cannot be explained either by demand differences or by supply conditions or retailers environment. In this paper we offer a framework for differences that are born within internal organization.

A second way to identify our result is comparing multi-product multi-market firms along time. If there is a shock affecting the returns to product differentiation of some particular product, a firm can modify its organizational design to increase its profits. However, it may be difficult to observe this case because firms might modify informally their organizational design without changing formals procedures.[24]

4 Extensions

4.1 Coordination

In the previous section we discuss that in some papers the level of coordination among divisions is endogenous, while in some others is exogenous. Our results hold if we endogenize this choice. We extend results of Propositions 4 and 5 for the case that the headquarters chooses β. Figure 6.a shows the extension of Proposition 3, Figure 6.b of Proposition 4, and Figure 7 of Proposition 5.

The choice of β does not modify our previous results qualitatively, even when it does quantitatively. If the headquarters decides the optimal β for the structure of the organization, there is a negative relation between β and σ^2. There is a high risk of requiring high levels of coordination between local managers, as long as there is high uncertainty about demand characteristics, i.e., high σ^2. For high values of σ^2 the headquarters prefers to provide autonomy to local managers about what products to be offered in their markets and to follow a strategy of product differentiation, i.e., the higher σ^2 the lower β.

Analogously, the headquarters follows a standard product strategy in those markets with low returns to product differentiation. The effect of losing control over resources would also affect the decision of β. For details see Appendix 8.2. For the function $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$, there exists a cutoff $\tilde{\sigma}^2(\overline{\mu})$ and increasing relations $\tilde{\sigma}^2_{AM}(\sigma^2_B,\overline{\mu})$, $\tilde{\sigma}^2_A(\sigma^2_B,\overline{\mu})$, $\tilde{\sigma}^2_{BM}(\sigma^2_A,\overline{\mu})$ and $\tilde{\sigma}^2_B(\sigma^2_A,\overline{\mu})$ such that:

a- If resources are allocated efficiently (headquarters), decision rights over market j are centralized if $\sigma^2_j < \tilde{\sigma}^2_j(\sigma^2_{-j},\overline{\mu})$, for $j \in \{A,B\}$.

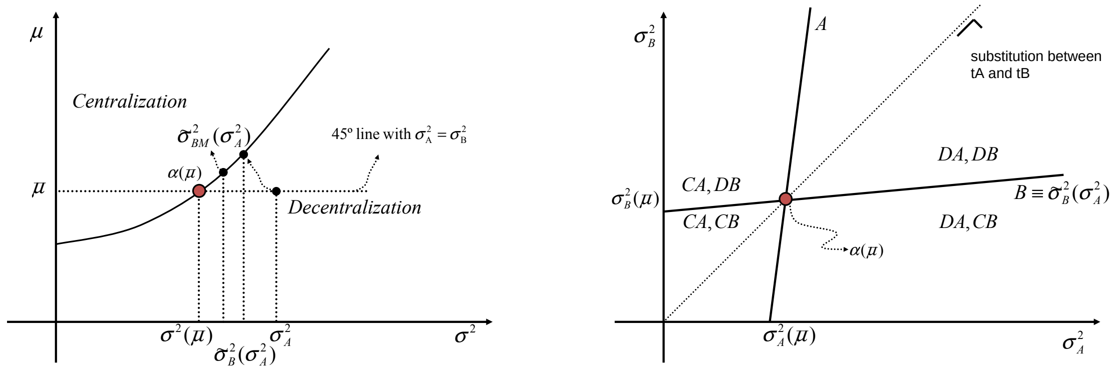
b- If resources are allocated by local managers, decision rights over market j are centralized if $\sigma^2_j < \tilde{\sigma}^2_{jM}(\sigma^2_{-j},\overline{\mu})$, for $j \in \{A,B\}$.

c- $\tilde{\sigma}^2_A(\sigma^2_B,\overline{\mu}) < \tilde{\sigma}^2_{AM}(\sigma^2_B,\overline{\mu})$ and $\tilde{\sigma}^2_B(\sigma^2_A,\overline{\mu}) < \tilde{\sigma}^2_{BM}(\sigma^2_A,\overline{\mu})$.

Centralization is chosen as an optimal structure if the return to product differentiation is sufficiently low, which is more likely for higher informational cost μ. This relation is shown in Figure 6.a. We remark now the effect of resource allocation on the headquarters decision over centralization and decentralization. When resource allocation becomes important, the headquarters promotes a lower cost in markets with high returns to differentiation. Not only the ratio of the returns to differentiation but also the level of the returns to differentiation matter in all markets to determine whether to centralize or decentralize decision rights.

[24]For instance, suppose a centralized organization with a formal procedure to introduce new products through the following procedure: a country manager writes a project suggesting a product which requires Headquarters' approval; however, the headquarters can commit (through reputation) to relax its approval requirements over those projects related to some particular products' lines or segments. This is interpreted as informal decentralization. I have been informally told by managers in international companies that this is a common proceeding.

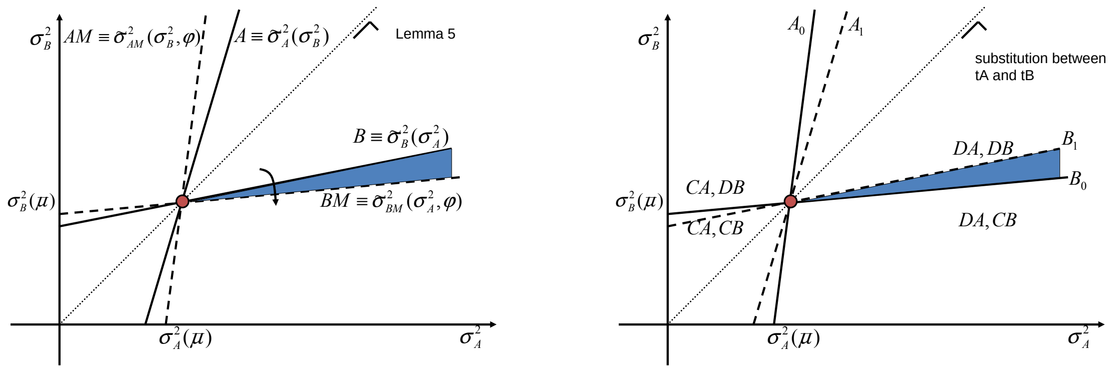
A low β fits better in a market with high return to differentiation, which also makes decentralization more attractive. On markets with high returns to differentiation, the cutoff ratio that makes the headquarters to centralize decision making in some markets increases. These thresholds are represented by increasing relations in Figure 6.b. As in our baseline analysis, these thresholds are higher when the headquarters lacks control over resources, which is shown in Figure 7.a. Relations are steeper when the resource allocation is more important in determining the information cost μ, as shown in Figure 7.b.



(a) Comparing μ and σ^2

(b) Comparing σ_A^2 and σ_B^2

Figure 6: Optimal structure depending on marginal informational cost, μ, and market returns to differentiation, σ^2.



(a) Firm's and Managers' cutoff

(b) Change in resource allocation importance

Figure 7: Optimal structure for change on the importance of resource allocation

4.2 Delegation

We assume that the headquarters delegates resource allocation on local division managers. We can extend our model to see that delegation may arise as the optimal's choice of the headquarters if there are externalities when learning about regional demands. For example, if learning about demand's taste of product B in region 1 can provide some additional information when learning about demand's taste of product A in region 1, the firm can prefer to merge two subdivisions in region 1. In this way, the firm designs the organization by regional divisions, as many international firms do. Let us call subdivisions $A1$ and $B1$ for the rest of this section.

Assume the following technology of the signal $\hat{\theta}_{1A}$: the signal equals the true value θ_{1A} with probability $\sqrt{e_{1A}+\alpha e_{1B}}$ and a random draw from the same distribution of θ_{1A} with probability $1-\sqrt{e_{1A}+\alpha e_{1B}}$, with $0<\alpha<1$, and $e\in[0,\frac{1}{1+\alpha}]$.[25] This technology of the signal generates an externality in information acquisition that is described as follows:

1- If subdivisions $A1$ and $B1$ are separated, local manager in division $A1$ collects information according to: $\tilde{\psi}_A(\beta,s,g)=\mu_A C'(\hat{e}_A)$.

2- If subdivisions $A1$ and $B1$ are integrated, local manager in division $A1$ collects information according to: $\tilde{\psi}_A(\beta,s,g)+\frac{\sigma_B^2}{\sigma_A^2}\alpha\tilde{\psi}_B(\beta,s,g)=\mu_A C'(\hat{e}_A)$.

3- If effort were contractible, the headquarters would ask the local manager to collect information according to: $\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B=\mu_A C'(e_A^*)$.

The net effect of this learning externality about market characteristics is intuitive. For the function $\mu=\overline{\mu}\left(\frac{0.5}{t}\right)^b$, the parameter b represents the discretion of local managers to reallocate resources, the higher the b the higher the managers' discretion. For a given value of α, integration is profitable if b is sufficiently low. There will be a cutoff on b such that delegation is optimal as long as b is below that value. As α increases, learning from products A and B becomes more complementary, which means that products are less rival. The inefficiency in resource allocation is reduced. Although there are some new insights, the main implications of this paper over the firm's optimal structure remain. To see a comparison of this problem with our previous model see Appendix 8.2.

5 Conclusion

In this paper, we analyze the internal design of a multi-product multi-division firm. We model the economic interactions of being multi-product that arise in an organization. We focus on the effect that the opportunity cost of resources has on decision making and compensation schemes.

Implementing a product differentiation policy on some products is more profitable than on other products. This relative profitability generates an opportunity cost of allocating scarce resources in each market. We show that the firm may prefer to centralize decision making in markets with lower returns to differentiation and decentralize decision making in markets with high returns to differentiation.

Empirically our result can be observed comparing the organizational design of multi-product multi-market firms with single-product multi-market firms. Alternatively, it can also be observed following dynamic organizational changes of a multi-product multi-market firm that suffers shocks which modify the relative returns to product differentiation.

[25] Introducing this change in the model does not modify the communication problem.

In this paper we have made a first move to extend the economic literature of organizational design into the analysis of multi-product firms. Much of this literature focuses on internal economic problems that frequently appear in international multi-product firms. Henceforth, an analysis of the multi-product multi-division firm is not only important but also necessary. We show that the allocation of decision rights and firm's strategy is not independent among different products. This is in line with Roberts (2004) who points out that "The structure [of the firm] does not follow strategy any more than strategy follows structure".

We have focused on the impact of the opportunity cost of resources on the internal organizational design. It is still necessary to carry out further work on analyzing other economic interactions that multinational firms face for being multi-product, e.g., interactions that arise on the demand side.

References

ALONSO, R., W. DESSEIN, AND N. MATOUSCHEK (2008): "When Does Coordination Require Centralization?," *American Economic Review*, 98, 145–179.

ATHEY, S., AND J. ROBERTS (2001): "Organizational Design: Decision Rights and Incentive Contracts," *American Economic Review Papers and Proceedings*.

CHEN, Y., N. KARTIK, AND J. SOBEL (2008): "Selecting Cheap Talk Equilibria," *Econometrica*, 76, 117–136.

CRAWFORD, V., AND J. SOBEL (1982): "Strategic Information transmission," *Econometrica*, 50, 1431–1452.

DESSEIN, W., L. GARICANO, AND R. GERTNER (forthcoming): "Organizing for Synergies," *American Economic Journal: Microeconomics*.

ECCLES, R., AND P. HOLLAND (1989): "Jacobs Suchard: Reorganizing for 1992," *Harvard Business School Case 489106*.

FRIEBEL, G., AND M. RAITH (2010): "Resource Allocation and Organizational Form," *American Economic Journal: Microeconomics*, 2, 1–33.

GEANAKOPLOS, J., AND P. MILGROM (1991): "A Theory of Hierarchies Based on Limited Managerial Attention," *Journal of Japanese and International Economies*, 5, 205–225.

HOLMSTRON, B., AND P. MILGROM (1991): "Multitask Principal Agent Analyses: Incentive Contracts, Asset Ownership and Job Design," *Journal of Law, Econ and Organizations*, 7, 24–52.

MOOKHERJEE, D. (2006): "Decentralization, Hierarchies, and Incentives: A Mechanism Design Perspective," *Journal of Economic Literature*, 44, 367–390.

RANTAKARI, H. (2008): "Governing Adaptation," *Review of Economic Studies*, 75, 1257–1285.

——— (forthcoming): "Organizational Design and Environmental Volatility," *Journal of Law, Economics and Organizations*.

ROBERTS, J. (2004): *The Modern Firm: Organizational Design for Performance and Growth*. Oxford University Press, New York, NY, US.

THOMAS, C. (2011): "Too Many Products: Decentralized Decision-Making in Multinational Firms," *American Economic Journal: Microeconomics*, 3, 280–306.

6 Appendix A: Proofs

6.1 Proof of Lemma 4

Proof. Without loss of generality, assume $\overline{\mu} = 1$, $\mu(t) = \left(\frac{0.5}{t}\right)^b$. Looking for t^* that maximizes $[-\mu(t) - \mu(1-t)H]$ we find that $t^* = \frac{1}{1+(H)^{\frac{1}{1+b}}}$ and $\mu(t^*) = 0.5^b \left[1 + (H)^{\frac{1}{1+b}}\right]^b$. Since $0 < H \leq 1$ then $t^* \geq 0.5$.

We need to prove that $\mu(t^*)$ is decreasing in b for $0 < b < 1$, i.e., $\frac{\partial \mu(t^*)}{\partial b} = \mu(t^*) \frac{\partial \log \mu(t^*)}{\partial b} < 0$. Taking logarithm and derivating respect to b we have $\log \mu(t^*) = b \log 0.5 + b \log \left[1 + (H)^{\frac{1}{1+b}}\right]$, and

$$\frac{d \log \mu(t^*)}{db} = \log 0.5 + \log \left[1 + (H)^{\frac{1}{1+b}}\right] - \frac{(H)^{\frac{1}{1+b}}}{\left[1 + (H)^{\frac{1}{1+b}}\right]} \frac{b}{(1+b)^2} \log(H). \tag{11}$$

Note in the last term of equation 11 that $1 - t^* \equiv \frac{(H)^{\frac{1}{1+b}}}{\left[1+(H)^{\frac{1}{1+b}}\right]}$ and that $1 - t^* < 0.5$. Working the last two terms, equation 11 can be expressed as $\log 0.5 + \log[H2]$, with $H2 \equiv \left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} + (H)^{\frac{1+bt^*}{(1+b)^2}}$. I need to prove that $H2 < 2$.[26] For the extreme case that $t^* = 0.5$ and $b = 1$ we have that $\left(\frac{1}{H}\right)^{\frac{1}{8}} + (H)^{\frac{3}{8}} \leq 2$ if $0.008 < H \leq 1$. Note that the expression $H2$ decreases as b decreases or t^* increases. For completing the proof, we check these derivatives respect to b and t^*,

$$\frac{\partial H2}{\partial b} = \left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} \log(\frac{1}{H})(1-t^*)\frac{1-b}{(1+b)^3} - \log(H)\frac{2-t^*+bt^*}{(1+b)^3}(H)^{\frac{1+bt^*}{(1+b)^2}} > 0.$$

Since $H \leq 1$, note that $\log(H) < 0$ and $\log(\frac{1}{H}) > 0$. And the derivative respect to t^* is,

$$\frac{\partial H2}{\partial t^*} = -\left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} \log(\frac{1}{H})\frac{b}{(1+b)^2} + \log(H)\frac{b}{(1+b)^2}(H)^{\frac{1+bt^*}{(1+b)^2}} < 0.$$

Finally, we check that the derivative $\frac{\partial \mu(t^*)}{\partial H} = \frac{b}{1+b} 0.5^b \left[1 + (H)^{\frac{1}{1+b}}\right]^{b-1} (H)^{\frac{-b}{1+b}} > 0$ which implies that if H decreases, then the cost $\mu(t^*)$ decreases and the cost $\mu(1-t^*)$ increases. The proof is complete. □

[26] Recall that $\log 0.5 + \log 2 = \log 1 = 0$.

6.2 Proof of Proposition 3

Sketch of the proof. (Based on Proposition 5 of Rantakari (2010)) Consider the optimal structure design for market A. We proceed in two steps: first determining the contract s under decentralization and under centralization, and second, comparing which structure generates higher profits. The first order condition respect to s shows a balance between less information which has more value and more information which has less value: $e_A \frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A}[\psi_A - \tilde{\psi}_A] = 0$. Since local managers appropriate the amount $\tilde{\psi}$ of the value generated, ψ, they acquire a suboptimal amount of information, e. However, this inefficiency can be partially corrected with the compensation scheme. A comparative static shows that incentives are narrowed to local manager's payoff when information is more expensive, i.e., $\frac{\partial s}{\partial \mu} < 0$. To have a good balance between the amount of information acquired and the value generated with this information, e and ψ respectively, the headquarters fosters information acquisition when it is expensive, even when this reduces the value generated by the information acquireds.

Evaluated in the optimal compensation scheme, decentralization outperforms centralization if the surplus value is greater, i.e., if $e_A \psi_A - \mu_A C(e_A)$ is higher under decentralization than under centralization. Recalling that: 1) e decreases and ψ increases when s increases; 2) s decreases when μ increases; 3) for high values of s, ψ and $\tilde{\psi}$ are higher under decentralization, and for low values of s, ψ is higher under centralization but $\tilde{\psi}$ is higher under decentralization; and finally 4) the information acquired increases with the value appropriated by local managers, i.e., the higher the $\tilde{\psi}$ the higher the e.

Low μ generates high e, which allows the firm to increase s and to decentralize. In words, low cost of information generates high amount of information, allowing the firm to increase the degree of incentives alignment and to decentralize decision rights. This result follows directly from section 3.1 (based on Alonso et al, 2008) and from section 3.2. □

6.3 Proof of Proposition 4

Proof. For the last part of Proposition 4, we prove that the cutoff $\tilde{\sigma}_{AB}$ decreases with b. For a given σ_{AB}, the higher the b the lower the μ_A and the higher the μ_B. Recall that t^* and μ_B depend on H and b and from Lemma 4 we have

$$\frac{\partial \mu_B(1-t^*)}{\partial b} = \mu_B(1-t^*)\frac{\partial \ln[\mu_B(1-t^*)]}{\partial b} > 0. \tag{12}$$

A higher b implies a higher μ_B. At $\tilde{\sigma}_{AB}$ the headquarters is indifferent between centralizing and decentralizing decision rights about product characteristics in market B. If b increases, μ_B increases and now the headquarters strictly prefers to centralize decision rights in market of product B. The cutoff $\tilde{\sigma}_{AB}$ decreases when b increases. □

6.4 Proof of Proposition 5

Proof. When local managers allocate resources, headquarters internalizes an additional effect of modifying the contract. Increasing s_A not only reduces the amount of effort e_A but also reduces the amount of resources that a local managers allocate to acquire information about product A, which indirectly also reduces the effort e_A. Moreover, a reduction in t_A fosters more information acquisition about product B.

If the headquarters allocates resources, t_A^*, the optimal s_A is defined by $\frac{\partial \psi_A}{\partial s_A} e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) = 0$. However, if local managers allocate resources the first order condition becomes:

$$\begin{aligned} \frac{\partial \psi_A}{\partial s_A} e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) &= -\frac{\partial t_A}{\partial s_A} \frac{\sigma_B^2}{\sigma_A^2} \left[\frac{\sigma_A^2}{\sigma_B^2} \frac{\partial e_A}{\partial t_A} [\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A} [\psi_B - \tilde{\psi}_B] \right], \\ \frac{\partial \psi_B}{\partial s_B} e_B + \frac{\partial e_B}{\partial s_B}(\psi_B - \tilde{\psi}_B) &= -\frac{\partial t_A}{\partial s_B} \left[\frac{\sigma_A^2}{\sigma_B^2} \frac{\partial e_A}{\partial t_A} [\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A} [\psi_B - \tilde{\psi}_B] \right]. \end{aligned}$$

With $\frac{\partial t_A}{\partial s_A} < 0$ and $\frac{\partial t_A}{\partial s_B} > 0$, and calling $\varphi \equiv \frac{\sigma_A^2}{\sigma_B^2} \frac{\partial e_A}{\partial t_A} [\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A} [\psi_B - \tilde{\psi}_B]$. These expressions separate the indirect effect of s_A on t_A from the direct effect of s_A on e_A (similarly for s_B). In Section 7.7.1 we describe that each manager reaction to an increase in σ_A^2 are $\frac{\partial e_A}{\partial \sigma_A^2} > 0$, $\frac{\partial e_B}{\partial \sigma_A^2} < 0$ and $\frac{\partial t_A}{\partial \sigma_A^2} > 0$. As σ_A^2 increases, more resources are allocated to product A; the HQ increases s_A to increase ψ_A and it reduces s_B to increase $\tilde{\psi}_B$ fostering higher effort in market B, i.e., higher e_B. Eventually, the headquarters chooses to centralize decision making in product B and decentralize decision making in product A.

However, comparing with the situation where headquarters allocates resources, the indirect effects make the cutoff $\hat{\sigma}_{AB}$ to be higher than $\tilde{\sigma}_{AB}$. To show that $\tilde{\sigma}_{AB} < \hat{\sigma}_{AB}$ we identify the two indirect effects that foster the headquarters to decentralize product B when σ_{AB} is around $\tilde{\sigma}_{AB}$. Let us assume that $\sigma_{AB} = \tilde{\sigma}_{AB}$. If this is the case and local managers allocate resources, the optimal organizational design, for the case where $\underline{\mu} < \tilde{\mu}$, requires to decentralize decision making in both products.

First, since local managers ignore the inefficiency in exerting effort, represented by $[\psi - \tilde{\psi}]$, they allocate more resources to market B than the headquarters would, i.e., $\hat{t}_A < t_A^*$. Given $\hat{t}_A < t_A^*$, we have that $\mu_A(\hat{t}_A) > \mu_A(t_A^*)$ and $\mu_B(1 - \hat{t}_A) < \mu_B(1 - t_A^*)$. Formally, in section 3.3 we show that the term H differs depending on whether headquarters or local managers allocate resources, being higher in the latter case. This difference in H comes from the fact that local managers ignore the inefficiency $[\psi - \tilde{\psi}]$, which affects directly the cost μ_B.

$$\frac{\partial \mu_B(1 - t^*)}{\partial H} = \frac{\partial}{\partial H} \left[\overline{\mu} 0.5^b \left(1 + H^{\frac{-1}{1+b}} \right)^b \right] < 0.$$

There is lower effort cost μ_B, when managers allocate resources. If information is cheaper in market B, the headquarters increases s_B in market B to increase the value generated by this information, which, at the same time, favors decentralization of decisions rights in market B.

Second, the headquarters recognizes the inefficient allocation of resources of local manager, and changes the organizational design to correct it. With this purpose, the headquarters increases s_B which also favors decentralization of decision rights in market B.

Both effects favor to decentralize market of product B. At $\sigma_{AB} = \tilde{\sigma}_{AB}$ the headquarters strictly prefers to decentralize decision rights in market B. Then, it requires a higher value of σ_{AB} to centralize decisions rights in the market of product B. □

7 Appendix B: Solving the Problem

7.1 Close Forms

In this appendix we construct the close form solutions of the expressions of ψ and $\tilde{\psi}$ under centralization and decentralization that are the following:

$$\psi_c = \frac{(3(1-s))(1+2\beta)^2}{(1+4\beta)(\beta(1-2s)+(3(1-s))(2\beta+1))},$$

$$\psi_d = -(1-s)\frac{5s^3(1-2\beta)+s^2(5\beta+14(\beta^2-1))+s(16\beta+2\beta^2+13)-(4+6\beta^3+11\beta+12\beta^2)}{((1-s)+2\beta)(1-s+\beta)((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta))},$$

$$\tilde{\psi}_c = (3(1-s))\frac{(1+2\beta)((1-s)+\beta(3-4s)}{((1+4\beta)(\beta(1-2s)+(3(1-s))(2\beta+1))}, \text{ and}$$

$$\tilde{\psi}_d = (1-s)\frac{9\beta^2+11\beta+13s^2\beta+4-13s+14s^2-5s^3-12s\beta(2+\beta)}{((1-s)+2\beta)((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta)))}.$$

And its derivatives under centralization are:

$$\frac{\partial \psi^c}{\partial s} = \frac{3\beta(2\beta+1)^2}{(1+4\beta)(8\beta(1-s)-\beta+3(1-s))^2} \geq 0,$$

$$\frac{\partial \tilde{\psi}^c}{\partial s} = \frac{3(2\beta+1)(3(1-s)^2+\beta^2+2(1-s)\beta(10(1-s)-1)+8\beta^2(1-s)(4(1-s)-1))}{(1+4\beta)^2(8\beta(1-s)-\beta+3(1-s))^2} < 0. \quad (13)$$

And under decentralization are:

$$\frac{\partial \psi_d}{\partial s} = \frac{\beta^2((1-s)^3H_1+\beta(1-s)^2H_2+\beta^2(1-s)H_3+\beta^3H_4+12\beta^4)}{((1-s)+2\beta)^2(1-s+\beta)^2((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta))^2} \geq 0, \quad (14)$$

with $H_1 \equiv 160(1-s)^3-120(1-s)^2+27(1-s)-2$, $H_2 \equiv 680(1-s)^3-516(1-s)^2+1266(1-s)+10$, $H_3 \equiv 956(1-s)^3-744(1-s)^2+204(1-s)-16$ and $H_4 \equiv 448(1-s)^3-360(1-s)^2+102(1-s)-8$.

$$\frac{\partial \tilde{\psi}_d}{\partial s} = \frac{5(1-s)^5(5(1-s)-2)+6\beta(1-s)^4(30(1-s)-11)+(1-s)^4+6\beta^2(1-s)^3(69(1-s)-20)}{((1-s)+2\beta)^2(5(1-s)^2-(1-s)+8\beta(1-s)-\beta)^2}+$$
$$\frac{6(1-s)^3\beta+4\beta^3(1-s)^2(104(1-s)-23)+9(1-s)^2\beta^2+8(1-s)\beta^3+6\beta^4(32(1-s)^2-8(1-s)+1)}{((1-s)+2\beta)^2(5(1-s)^2-(1-s)+8\beta(1-s)-\beta)^2} < 0.$$

7.2 Decision Making: Using Information

In this section we build the expression in equation (1) for Centralization and Decentralization before communication outcome is introduced. To have the same terms we must replace the expression $E[\overline{m}^2] = [1-(1-V)]E[\theta^2]$.

We build the indirect function for $E[\Pi|m]$ given the equilibrium beliefs for $m \equiv E[\theta|m]$ and $E[\theta^2|m] \equiv m^2$. Remember that $\Pi_{1A} = K(\beta)-(a_{1A}-\theta_{1A})^2-\beta(a_{1A}-a_{2A})^2$.

Given the optimal policy for decision making and the information transmission process, the objective function of each division can be expressed as a function of σ_1^2, σ_2^2, $E[\theta_1^2|m]$ and $E[\theta_2^2|m]$. We show how to arrive to the optimal expressions.

7.2.1 Centralization

The optimal decision making under centralization are:

$$\begin{aligned}
a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{2\beta}{1+4\beta}E[\theta_2|m_2], \text{ and}\\
a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{1+2\beta}{1+4\beta}E[\theta_2|m_2].
\end{aligned}$$

Replacing them in the expected profit of division 1 we get the following terms:

$$\begin{aligned}
(a_1^C-\theta_1)^2 &= \left(E[\theta_1|m_1]-\theta_1+\frac{2\beta\left(E[\theta_2|m_2]-E[\theta_1|m_1]\right)}{1+4\beta}\right)^2,\\
&= (\overline{m}_1-\theta_1)^2+\frac{4\beta^2\left(\overline{m}_2-\overline{m}_1\right)^2}{(1+4\beta)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{(\overline{m}_2-\overline{m}_1)}{(1+4\beta)},\\
&= \left(\overline{m}_1^2-2\overline{m}_1\theta_1+\theta_1^2\right)+\frac{4\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{(1+4\beta)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{(\overline{m}_2-\overline{m}_1)}{(1+4\beta)}. \quad (15)\\
(a_1^C-a_2^C)^2 &= \left(\frac{E[\theta_1|m_1,m_2]-E[\theta_2|m_1,m_2]}{1+4\beta}\right)^2=\frac{\overline{m}_1^2-2\overline{m}_1\overline{m}_2+\overline{m}_2^2}{(1+4\beta)^2}. \quad (16)
\end{aligned}$$

Taking expectations we get that

$$\begin{aligned}
E[(a_1^C-\theta_1)^2] &= \left(E[\theta_1^2]-E[\overline{m}_1^2]\right)+\frac{4\beta^2\left(E[\overline{m}_2^2]+E[\overline{m}_1^2]\right)}{(1+4\beta)^2}, \quad (17)\\
E[(a_1^C-a_2^C)^2] &= \frac{E[\overline{m}_1^2]+E[\overline{m}_2^2]}{(1+4\beta)^2}. \quad (18)
\end{aligned}$$

Now, lets build the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively. Notice that taking ex-ante expectation the following terms are null: $E[\overline{m}_1\overline{m}_2]=0$, $E[(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$

$$E[\Pi_1] = \left(K(\beta)-\left[E[\theta_1^2]-\frac{1+3\beta}{(1+4\beta)}E[\overline{m}_1^2]+\frac{\beta}{(1+4\beta)}E[\overline{m}_2^2]\right]\right). \quad (19)$$

REMARK: Since $E[\overline{m}_1]=0$ and $E[\theta_1]=0$, then $E[\overline{m}_1^2]=VE[\theta_1^2]=E[\theta_1^2]-(1-V)E[\theta_1^2]$ The division payoff is $E[U_1]=(1-s)\Pi_1+s\Pi_2$.

7.2.2 Decentralization

The optimal decision making under decentralization are:

$$
\begin{aligned}
a_1^D(m_1,m_2,\theta_1) &= \frac{(1-s)}{(1-s+\beta)}\theta_1 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}\overline{m}_1 + \frac{1-s+\beta}{1-s+2\beta}\overline{m}_2\right], \text{ and} \\
a_2^D(m_1,m_2,\theta_2) &= \frac{(1-s)}{(1-s+\beta)}\theta_2 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}\overline{m}_2 + \frac{1-s+\beta}{1-s+2\beta}\overline{m}_1\right].
\end{aligned}
$$

Replacing it in the expected profit of division 1 we get the following terms:

$$
\begin{aligned}
(a_1^D-\theta_1)^2 &= \left(-\frac{\beta}{(1-s+\beta)}\theta_1 + \frac{\beta}{(1-s+\beta)}\overline{m}_1 + \frac{\beta}{1-s+2\beta}(\overline{m}_2-\overline{m}_1)\right)^2, \\
&= \frac{\beta^2}{(1-s+\beta)^2}(\overline{m}_1-\theta_1)^2 + \frac{\beta^2}{(1-s+2\beta)^2}(\overline{m}_2-\overline{m}_1)^2 + \frac{2\beta^2(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)}{(1-s+\beta)(1-s+2\beta)}, \\
&= \frac{\beta^2\left(\overline{m}_1^2-2\overline{m}_1\theta_1+\theta_1^2\right)}{(1-s+\beta)^2} + \frac{\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{(1-s+2\beta)^2} + \frac{2\beta^2(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)}{(1-s+\beta)(1-s+2\beta)}. \quad (20)
\end{aligned}
$$

$$
\begin{aligned}
(a_1^D-a_2^D)^2 &= \left(\frac{(1-s)(\theta_1-\theta_2)}{(1-s+\beta)} + \frac{\beta}{(1-s+\beta)}\frac{(1-s)(\overline{m}_2-\overline{m}_1)}{1-s+2\beta}\right)^2, \\
&= \frac{(1-s)^2}{(1-s+\beta)^2}(\theta_1^2+\theta_2^2-2\theta_1\theta_2) + \frac{\beta^2}{(1-s+\beta)^2}\frac{(1-s)^2(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_1\overline{m}_2)}{(1-s+2\beta)^2}, \\
&\quad + \frac{2\beta}{(1-s+\beta)^2}\frac{(1-s)^2}{1-s+2\beta}(\overline{m}_2\theta_1+\overline{m}_1\theta_2-\overline{m}_2\theta_2-\overline{m}_1\theta_1). \quad (21)
\end{aligned}
$$

Now, lets build the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively. Notice that taking ex-ante expectation the following terms are null: $E[\overline{m}_1\overline{m}_2]=0$,$E[\overline{m}_1\theta_2]=0$, $E[\overline{m}_2\theta_1]=0$,$E[\theta_1\theta_2]=0$, $E[(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$ By parts[27]

$$
\begin{aligned}
E[(a_1^D-\theta_1)^2] &= \frac{\beta^2E[\theta_1^2]}{(1-s+\beta)^2} + \frac{(1-s+\beta)^2-(1-s+2\beta)^2}{(1-s+\beta)^2(1-s+2\beta)^2}\beta^2E[\overline{m}_1^2] + \frac{\beta^2}{(1-s+2\beta)^2}E[\overline{m}_2^2], \\
&= \frac{\beta^2E[\theta_1^2]}{(1-s+\beta)^2} - \frac{2(1-s)+3\beta}{(1-s+\beta)^2(1-s+2\beta)^2}\beta^3E[\overline{m}_1^2] + \frac{\beta^2}{(1-s+2\beta)^2}E[\overline{m}_2^2].
\end{aligned}
$$

$$
E[(a_1^D-a_2^D)^2] = \frac{(1-s)^2}{(1-s+\beta)^2}(E[\theta_1^2]+E[\theta_2^2]) - \frac{[2(1-s)+3\beta]}{(1-s+\beta)^2}\frac{\beta(1-s)^2}{(1-s+2\beta)^2}(E[\overline{m}_2^2]+E[\overline{m}_1^2]).
$$

The expected profits $E[\Pi_1]$ are

$$
\left(K(\beta) - \left[\frac{\beta(1-s)^2}{(1-s+\beta)^2}E[\theta_2^2] + \beta\frac{\beta+(1-s)^2}{(1-s+\beta)^2}E[\theta_1^2] - \beta^2\frac{[2(1-s)+3\beta]}{(1-s+\beta)^2}\frac{\beta+(1-s)^2}{(1-s+2\beta)^2}E[\overline{m}_1^2] + \left(\beta\frac{\beta+(1-s)^2}{(1-s+2\beta)^2} - \frac{\beta(1-s)^2}{(1-s+\beta)^2}\right)E[\overline{m}_2^2]\right]\right). \quad (22)
$$

[27] In the profit function we have $E[(a_1^D-\theta_1)^2]+\beta E[(a_1^D-a_2^D)^2]$. Note that $\frac{(1-s+\beta)^2-(1-s+2\beta)^2}{(1-s+\beta)^2(1-s+2\beta)^2} = -\beta\frac{2(1-s)+3\beta}{(1-s+\beta)^2(1-s+2\beta)^2}$.

Since $E[\theta]=0$, $E[\theta^2]=V(\theta)=\frac{\overline{\theta}^2}{3}=\sigma^2$.

7.3 Strategic Communication: Transmission

7.3.1 Building $E[\overline{m}^2]$ for any Finite Partition.

I must find the payoff for any finite partition. The general formula for a N_j partition is:[28]

$$E[\overline{m}^2]=\frac{1}{2\overline{\theta}}\sum_{j\in N_j}\int_{d_{j-1}}^{d_j}\left(\frac{d_j+d_{j-1}}{2}\right)^2 d\theta. \tag{23}$$

By uniform distribution we get that $\int_{d_{j-1}}^{d_j}\left(\frac{d_j+d_{j-1}}{2}\right)^2 d\theta=(d_j-d_{j-1})\left(\frac{d_j+d_{j-1}}{2}\right)^2$.

From Proposition 2 we have $d_{j,i+1}-d_{j,i}=d_{j,i}-d_{j,i-1}+4bd_{j,i}$ with $b\equiv\frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}$ under decentralization and $b\equiv\frac{(1-2s)\beta}{1-s+\beta}$ under centralization. Also $d_i=\overline{\theta}\frac{x^i(1+y^N)-y^i(1+x^N)}{x^N-y^N}$ $\quad 0\leq i\leq N$ with $x=(1+2b)+\sqrt{(1+2b)^2-1}$ and $y=(1+2b)-\sqrt{(1+2b)^2-1}$. Property $xy=1$ and $x>1$ applies to both cases and are used all along the algebra. Replacing it in the expression above we have (Equation 27 in Alonso, Dessein and Matouscheck 2008).

$$\begin{aligned}E[\overline{m}^2] &= \frac{\overline{\theta}^2}{3}\left[\frac{(x^{3N_j}-1)(x-1)^2}{(x^{N_j}-1)^3(x^2+x+1)}-\frac{(x^{N_j}+1)^2(x+1)(1+y)}{x^{N_j}(x^{N_j}-y^{N_j})^2}\right],\\ &= \frac{\overline{\theta}^2}{3}\left[\frac{(x^{3N_j}-1)(x-1)^2}{(x^{N_j}-1)^3(x^2+x+1)}-\frac{x^{N_j}(x+1)^2}{x(x^{N_j}-1)^2}\right].\end{aligned} \tag{24}$$

For the case with infinite partitions we have

$$\lim_{N_j\to+\infty}E[\overline{m}^2]=\frac{\overline{\theta}^2}{4}\frac{(x+1)^2}{(x^2+x+1)}=\overline{\theta}^2\frac{1+b}{3+4b}=V\sigma^2=[1-(1-V)]\sigma^2. \tag{25}$$

If the sender is truly believed, he will report exaggerating the signal. For example, if $s=0$, a manager has incentives to misreport $\theta^R-\theta=\frac{\beta}{1+\beta}\theta$ under centralization and $\theta^R-\theta=\frac{1+\beta}{\beta}\theta$ under decentralization. For this case, his report is: $\theta^R\equiv\frac{(1+4\beta)(1+2\beta)}{(1+2\beta)^2+\beta}\theta=\frac{1+2\beta}{1+\beta}\theta$ under centralization and $\theta^R\equiv\frac{1+2\beta}{\beta}\theta$ under decentralization.

7.4 Imperfect Signals

Given the information acquired, specified by the effort e, each manager have a posterior about the true value. That is, given the realization of the signal $\hat{\theta}$ (that coincide with the true value of θ with probability $\sqrt{e}$), the manager's posterior is $\tilde{\theta}=E[\theta|\hat{\theta}]=\sqrt{e}\hat{\theta}$.[29] This posterior is the best guess that a manager has

[28]Note that $\overline{m}^2=\left(\frac{d_j+d_{j-1}}{2}\right)^2$ is the expected value given a particular signal. Then, $E[\overline{m}^2]$ is the ex-ante expected value in the expression.

[29]Note that we care about the mean of the posterior and not the posterior distribution. The posterior mean distribution is uniform in $[-\sqrt{e}\overline{\theta},\sqrt{e}\overline{\theta}]$.

over the true value θ, and he prefers to design a product that is closest to this estimation of consumers' tastes.

With the posterior, the headquarters has also inferences after she receives managers' reports that are $E[\tilde{\theta}_1^2|m]$ and $E[\tilde{\theta}_2^2|m]$ The optimal decision making under centralization are:

$$\begin{aligned} a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\tilde{\theta}_1|m_1]+\frac{2\beta}{1+4\beta}E[\tilde{\theta}_2|m_2], \text{ and} \\ a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\tilde{\theta}_1|m_1]+\frac{1+2\beta}{1+4\beta}E[\tilde{\theta}_2|m_2]. \end{aligned}$$

Then, replacing it in the expected profit of manager in country 1 we get the following terms:

$$\begin{aligned} (a_1^C-\theta_1)^2 &= \left(E[\tilde{\theta}_1|m_1]-\theta_1+\frac{2\beta\left(E[\tilde{\theta}_2|m_2]-E[\tilde{\theta}_1|m_1]\right)}{1+4\beta}\right)^2, \\ &= (\overline{m}_1-\theta_1)^2+\frac{4\beta^2(\overline{m}_2-\overline{m}_1)^2}{(1+4\beta)^2}+2\beta(\overline{m}_1-\theta_1)\frac{(\overline{m}_2-\overline{m}_1)}{(1+4\beta)}, \\ &= (\overline{m}_1-\theta_1)^2+\frac{4\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{(1+4\beta)^2}+2\beta(\overline{m}_1-\theta_1)\frac{(\overline{m}_2-\overline{m}_1)}{(1+4\beta)}. \end{aligned} \tag{26}$$

$$(a_1^C-a_2^C)^2 = \left(\frac{E[\tilde{\theta}_1|m_1,m_2]-E[\tilde{\theta}_2|m_1,m_2]}{1+4\beta}\right)^2=\frac{\overline{m}_1^2-2\overline{m}_1\overline{m}_2+\overline{m}_2^2}{(1+4\beta)^2}. \tag{27}$$

Notice that taking ex-ante expectation the following terms are null $E[\overline{m}_1\overline{m}_2]=0, E[(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$. Also, note that $\overline{m}^2=E[\tilde{\theta}_1]^2=e_1\hat{\theta}_1^2$ and then $E[\overline{m}_1^2]=e_1E[\hat{\theta}_1^2]=e_1V_1\frac{\overline{\theta}_1^2}{3}$. Taking expectations

$$E[(a_1^C-\theta_1)^2] = E[(\overline{m}_1-\theta_1)^2]+\frac{4\beta^2\left(E[\overline{m}_2^2]+E[\overline{m}_1^2]\right)}{(1+4\beta)^2}, \tag{28}$$

$$E[(a_1^C-a_2^C)^2] = \frac{E[\overline{m}_1^2]+E[\overline{m}_2^2]}{(1+4\beta)^2}. \tag{29}$$

Building the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively.

$$E[\Pi_1] = \left(K(\beta)-E[(\overline{m}_1-\theta_1)^2]+\frac{\beta}{(1+4\beta)}(E[\overline{m}_1^2]+E[\overline{m}_2^2])\right). \tag{30}$$

where the term $E[(\overline{m}_1-\theta_1)^2]$ is the difference between the real realization of θ and what the HQ believes of $\tilde{\theta}$ after receiving the report. Note that the message of the posterior is equivalent as a message of the signal, then $\overline{m}=\sqrt{e}E[\hat{\theta}|m]$.

$$\begin{aligned} E[(\overline{m}-\theta)^2] &= E[\left(\overline{m}-\sqrt{e}\hat{\theta}+\sqrt{e}\hat{\theta}-\theta\right)^2], \\ &= E[\left(\sqrt{e}E[\hat{\theta}|m]-\sqrt{e}\hat{\theta}\right)^2]+E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2]. \end{aligned}$$

with the first term being the communication accuracy of the signal, i.e.,$E[\left(\sqrt{e}E[\hat{\theta}|m]-\sqrt{e}\hat{\theta}\right)^2]=eVE[\theta^2]$.

The second term is the loss due to the lack of precision in the signal, i.e., $E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2]=(1-e)E[\theta^2]$.[30] Recall that V is the proportion of the variance communicated, i.e., how accurate the communication is. We prove now that the omitted term is equal to zero, i.e., $E[\left(\overline{m}-\sqrt{e}\hat{\theta}\right)\left(\sqrt{e}\hat{\theta}-\theta\right)]=0$.

Proof.

$$\begin{aligned}
E[\left(\overline{m}-\sqrt{e}\hat{\theta}\right)\left(\sqrt{e}\hat{\theta}-\theta\right)] &= E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)]-E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)], \\
&= E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)]-E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)].
\end{aligned}$$

The first term is not problematic $E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)]=0$, but the second term deserves a little more of attention to notice that $E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)]=E[e\hat{\theta}^2]-E[\sqrt{e}\hat{\theta}\theta]=0$. I prove that $E[e\hat{\theta}^2]=eE[\theta^2]=E[\sqrt{e}\hat{\theta}\theta]$:

$$\begin{aligned}
E[\sqrt{e}\hat{\theta}\theta]=E[\sqrt{e}\sqrt{e}\theta\theta+(1-\sqrt{e})\sqrt{e}x\theta]=E[e\theta^2+(1-\sqrt{e})\sqrt{e}x\theta]=, \\
eE[\theta^2]+(1-\sqrt{e})\sqrt{e}\underbrace{E[x\theta]}_{=0 \text{ independent}}=eE[\theta^2].
\end{aligned} \tag{31}$$

Now, we show the other part.

$$E[e\hat{\theta}^2]=E[\sqrt{e}e\theta^2+(1-\sqrt{e})ex^2]=\sqrt{e}eE[\theta^2]+(1-\sqrt{e})eE[x^2]=eE[\theta^2]. \tag{32}$$

The proof is complete. □

REMARK: Since $E[\overline{m}_1]=0$ and $E[\theta_1]=0$, then $E[\overline{m}_1^2]=e_1E[\hat{\theta}_1^2]=e_1V_1\frac{\overline{\theta}_1^2}{3}$

$$E[\Pi_1] = \left(K(\beta)-\frac{\overline{\theta}^2}{3}\left[1-\left(1-V_1\frac{1+3\beta}{(1+4\beta)}\right)e_1+e_2V_2\frac{\beta}{(1+4\beta)}\right]\right).$$

And the division payoff is $E[U_1]=(1-s)\Pi_1+s\Pi_2$. Adding up for both divisions

$$\begin{aligned}
E[\Pi_1]+E[\Pi_2] &= \left(2K(\beta)-\frac{\overline{\theta}^2}{3}\left[1-\left(1-V_1\frac{1+2\beta}{(1+4\beta)}\right)e_1-\left(1-V_2\frac{1+2\beta}{(1+4\beta)}\right)e_2\right]\right), \\
&= \left(2K(\beta)-\frac{\overline{\theta}^2}{3}\left[1-e_1\psi_1-e_2\psi_2\right]\right).
\end{aligned}$$

A similarly analysis accounts for the case under decentralization. For further details you can see Proposition 5 in Rantakari (2010).

[30] $E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2]=E[e\hat{\theta}^2]+E[\theta^2]-2E[\sqrt{e}\hat{\theta}\theta]=(1-e)E[\theta^2]$, which equation (31) proves that $E[e\hat{\theta}^2]=eE[\theta^2]$ and $E[\sqrt{e}\hat{\theta}\theta]=eE[\theta^2]$.

7.5 Indirect Profit Function before Communication and Decisions

Value created and value captured:

$$\begin{aligned}
\psi_1(s) &= 1 - \phi_{11}(s) - \phi_{12}(s), \\
\tilde{\psi}_{11}(s) &= (1-s)(1-\phi_{11}(s)) - s\phi_{12}(s), \\
\tilde{\psi}_{11}(s) &= (1-s)\psi_1(s) + (1-2s)\phi_{12}(s).
\end{aligned}$$

It is worth noting that from communication and decision making both $\phi_{11}(s)$ and $\phi_{12}(s)$ are decreasing in s. Under centralization the reason is that communication is becoming more precise. Under decentralization both decisions are less biased and communication is more precise. The externality $\phi_{12}(s)$ is almost always greater under decentralization.[31] This externality reflects the high responsiveness to local conditions (to increase revenues) which also reduces the coordination what is translated into higher production costs.

The expected profits for a particular product in division 1 are,

$$E[\Pi_1] = K(\beta) - \left[1 - e_1 \underbrace{[1 - \Lambda_1 - \Gamma_{11}(1-V_1)]}_{\text{due to 1 information}} + e_2 \underbrace{[\Lambda_1 + \Gamma_{21}(1-V_2)]}_{\text{due to 2 information}} + \mu C(e_1) \right] \sigma_\theta^2. \tag{33}$$

Expected value generated by division 1:

$$\begin{aligned}
E[\pi_{1A}] + E[\pi_{1B}] &= K_A - \sigma_A^2 + e_{1A}\psi_{1A}\sigma_A^2 - \mu_A(t_{1A})C(e_{1A})\sigma_A^2, \\
&+ K_B - \sigma_B^2 + e_{1B}\psi_{1B}\sigma_B^2 - \mu_B(1-t_{1A})C(e_{1B})\sigma_B^2.
\end{aligned}$$

Expected value appropriated by local manager in division 1:

$$\begin{aligned}
K - \sigma_A^2 + e_{1A}\tilde{\psi}_{11A}\sigma_A^2 + e_{2A}\tilde{\psi}_{21A}\sigma_A^2 - \mu_A(t_{1A})C(e_{1A})\sigma_A^2, \\
K - \sigma_B^2 + e_{1B}\tilde{\psi}_{11B}\sigma_B^2 + e_{2B}\tilde{\psi}_{21B}\sigma_B^2 - \mu_B(1-t_{1A})C(e_{1B})\sigma_B^2.
\end{aligned}$$

7.6 Headquarters Allocates Resources

7.6.1 Assumption

Sufficient Assumption for $\Pi_{t_A t_A} \leq 0$ (and when managers allocate t) is:

A1- $\mu(t)\mu''(t)C(e)C''(e) > C'(e)^2\mu'(t)^2$ for all t and e.

This assumption holds for functions $\mu(t) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in [0,1]$ and $\overline{\mu} \in \mathbb{R}_+$, and $C(e) = -(e + \log(1-e))$.

[31] Only if $s \to 0$ and $\beta \to +\infty$ the externality is a little greater under centralization.

7.6.2 Managers' Choices

Each manager solves:

$$\max_{e_{1A},e_{1B}} 2K-\sigma_A^2-\sigma_B^2+\left(e_{1A}\tilde{\psi}_{11A}+e_{2A}\tilde{\psi}_{21A}-\mu_A(t_{1A})C(e_{1A})\right)\sigma_A^2,$$
$$+\left(e_{1B}\tilde{\psi}_{11B}+e_{2B}\tilde{\psi}_{21B}-\mu_B(1-t_{1A})C(e_{1B})\right)\sigma_B^2.$$

The values of e_A and e_B are determined by $\tilde{\psi}\sigma^2-\mu(t)C'(e)\sigma^2=0$. The comparative static are: i- $\frac{\partial e}{\partial s}=\frac{\partial \tilde{\psi}}{\partial s}\frac{1}{\mu C''(e)}<0$; ii- $\frac{\partial e}{\partial t}=-\frac{\mu'(t)C'(e)}{\mu(t)C''(e)}>0$; $\frac{\partial e}{\partial \sigma^2}=0$; and $\frac{\partial e_A}{\partial s_B}=0$. Also

$$\frac{\partial^2 e}{\partial s\partial t}=-\frac{\partial \tilde{\psi}}{\partial s}\frac{1}{[\mu C''(e)]^2}\frac{\mu'(t)}{C''(e)}[C''(e)^2-C'(e)C'''(e)]>0 \qquad \text{if } e>0.5,$$
$$\frac{\partial^2 e}{\partial t^2}=-\frac{C'(e)C''(e)[\mu(t)\mu''(t)-\mu'(t)^2]-\mu'(t)^2\frac{C'(e)}{C''(e)}[C''(e)^2-C'''(e)C'(e)]}{[\mu C''(e)]^2}<0,$$
$$\frac{\partial^2 e}{\partial s^2}=\left[\frac{\partial^2 \tilde{\psi}}{\partial s^2}-\left(\frac{\partial \tilde{\psi}}{\partial s}\right)^2\frac{C'''(e)}{\mu C''(e)^2}\right]\frac{1}{\mu C''(e)}\gtreqless 0.$$

Where $\frac{\partial^2 e}{\partial s^2}\leq 0$ for μ sufficiently small. We can define $t_B=1-t_A$ and we have $\frac{\partial e_B}{\partial t_A}=\frac{\mu'(1-t_A)C'(e_B)}{\mu(1-t_A)C''(e_B)}<0$.

7.6.3 Headquarters' Design

The HQ solves:

$$\max_{s_A,s_B,t_A} E[\pi_{1A}]+E[\pi_{1B}]=K-\sigma_A^2+\left(e_{1A}\psi_{1A}-\mu_A(t_{1A})C(e_{1A})\right)\sigma_A^2,$$
$$+K-\sigma_B^2+\left(e_{1B}\psi_{1B}-\mu_B(1-t_{1A})C(e_{1B})\right)\sigma_B^2. \tag{34}$$

The first order conditions are,

$$\sigma_A^2\left[\frac{\partial \psi_A}{\partial s_A}e_A+\frac{\partial e_A}{\partial s_A}(\psi_A-\tilde{\psi}_A)\right]=0,$$
$$\sigma_B^2\left[\frac{\partial \psi_B}{\partial s_B}e_B+\frac{\partial e_B}{\partial s_B}(\psi_B-\tilde{\psi}_B)\right]=0,$$
$$-\mu'(t_A)C(e_A)\sigma_A^2+\frac{\partial e_A}{\partial t_A}(\psi_A-\tilde{\psi}_A)\sigma_A^2+\mu'(1-t_A)C(e_B)\sigma_B^2+\frac{\partial e_B}{\partial t_A}(\psi_B-\tilde{\psi}_B)\sigma_B^2=0.$$

The second order conditions are

$$\begin{pmatrix} \Pi_{s_As_A} & 0 & \frac{\partial \psi_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}+\frac{\partial^2 e_A}{\partial s_A\partial t_A}(\psi_A-\tilde{\psi}_A) \\ 0 & \Pi_{s_Bs_B} & \frac{\partial \psi_B}{\partial s_B}\frac{\partial e_B}{\partial t_A}+\frac{\partial^2 e_B}{\partial s_B\partial t_A}(\psi_B-\tilde{\psi}_B) \\ \sigma_A^2[\frac{\partial \psi_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}+\frac{\partial^2 e_A}{\partial s_A\partial t_A}(\psi_A-\tilde{\psi}_A)] & \sigma_B^2[\frac{\partial \psi_B}{\partial s_B}\frac{\partial e_B}{\partial t_A}+\frac{\partial^2 e_B}{\partial s_B\partial t_A}(\psi_B-\tilde{\psi}_B)] & \Pi_{t_At_A} \end{pmatrix}\begin{pmatrix} ds_A \\ ds_B \\ dt_A \end{pmatrix}.$$

Where $\Pi_{t_At_A} \equiv \sigma_A^2\left[-\mu_A''C_A - \mu_A'C_{1A}\frac{\partial e_A}{\partial t_A} + \frac{\partial^2 e_A}{\partial t_A^2}(\psi_A - \tilde{\psi}_A)\right] + \sigma_B^2\left[-\mu_B''C_B + \mu_B'C_{1B}\frac{\partial e_B}{\partial t_A} + \frac{\partial^2 e_B}{\partial t_A^2}(\psi_B - \tilde{\psi}_B)\right]$.[32] We can split $\Pi_{t_At_A}$ into two parts $\Pi_{t_At_A} \equiv \Pi^A_{t_At_A} + \Pi^B_{t_At_A}$. Also $\Pi_{s_As_A} \equiv \frac{\partial^2\psi_A}{\partial s_A^2}e_A + \frac{\partial\psi_A}{\partial s_A}\frac{\partial e_A}{\partial s_A} + \frac{\partial^2 e_A}{\partial s_A^2}(\psi_A - \tilde{\psi}_A) + \frac{\partial e_A}{\partial s_A}\frac{\partial(\psi_A - \tilde{\psi}_A)}{\partial s_A}$ (≤ 0 in our relevant domain). The determinant is defined by $|J| \equiv \Pi_{s_As_A}[\Pi_{s_Bs_B}\Pi_{t_At_A} - \Pi^2_{s_Bt_B}] - \Pi_{s_Bs_B}\Pi^2_{s_At_A}$ or

$|J| \equiv \Pi_{s_As_A}[\Pi_{s_Bs_B}\Pi^B_{t_At_A} - \Pi^2_{s_Bt_B}] + \Pi_{s_Bs_B}[\Pi_{s_As_A}\Pi^A_{t_At_A} - \Pi^2_{s_At_A}] < 0$

$$\begin{pmatrix} 0 \\ 0 \\ \mu_A'(t_A)C(e_A) - \frac{\partial e_A}{\partial t_A}(\psi_A - \tilde{\psi}_A) \end{pmatrix}(d\sigma_A^2) = \begin{pmatrix} 0 \\ 0 \\ -\Pi_{t_A\sigma_A^2} \end{pmatrix}(d\sigma_A^2).$$

Then, we have $\Pi_{s_As_A} \leq 0$, $\Pi_{s_Bs_B} \leq 0$, $\Pi_{t_At_A} \leq 0$ but $\Pi_{s_At_A} \geq 0$, $-\Pi_{t_A\sigma_A^2} \leq 0$ and $\Pi_{s_Bt_A} \leq 0$. I took common factor $\sigma_A^2\sigma_B^2$, then:

$$\frac{\partial s_A}{\partial \sigma_A^2} = \frac{-\Pi_{t_A\sigma_A^2}[-\Pi_{s_At_A}\Pi_{s_Bs_B}]}{|J|} > 0, \tag{35}$$

$$\frac{\partial t_A}{\partial \sigma_A^2} = \frac{-\Pi_{t_A\sigma_A^2}[\Pi_{s_As_A}\Pi_{s_Bs_B}]}{|J|} > 0, \tag{36}$$

$$\frac{\partial s_B}{\partial \sigma_A^2} = -\frac{-\Pi_{t_A\sigma_A^2}[\Pi_{s_Bt_A}\Pi_{s_As_A}]}{|J|} < 0. \tag{37}$$

Finally, the headquarters chooses to centralize or decentralize decision rights in each market considering the one that generates higher value, i.e., higher $[\psi_Ae_A - \mu_AC(e_A)]\sigma_A^2 + [\psi_Be_B - \mu_BC(e_B)]\sigma_B^2$. If $\overline{\mu}$ is sufficiently low and $\sigma_A^2 \sim \sigma_B^2$, the headquarters decentralizes decision rights about both products. If σ_A^2 increases, s_A and t_A increase and s_B decreases, making more likely that the firm prefers to centralize decision rights of product B (see equations (35), (36), and (37)). There is a cutoff $\tilde{\sigma}_A^2$ above which the headquarters decentralizes decision making about product A and centralizes decision making about product B.

7.7 Managers Allocate Resources

7.7.1 Managers' Choices

Each manager solves:

$$\max_{e_{1A},e_{1B},t_{1A}} 2K - \sigma_A^2 - \sigma_B^2 + (e_{1A}\tilde{\psi}_{11A} + e_{2A}\tilde{\psi}_{21A} - \mu_A(t_{1A})C(e_{1A}))\,\sigma_A^2,$$
$$+ (e_{1B}\tilde{\psi}_{11B} + e_{2B}\tilde{\psi}_{21B} - \mu_B(1 - t_{1A})C(e_{iB}))\,\sigma_B^2.$$

[32]Note that, replacing the expression of $\frac{\partial e_A}{\partial t_A}$, each term in brackets can be re-expressed as $\left[-\frac{1}{\mu_AC_{A11}}(\mu_A\mu_A''C_AC_{A11} - \mu_A'^2C_{1A}^2) + \frac{\partial^2 e_A}{\partial t_A^2}(\psi_A - \tilde{\psi}_A)\right] < 0$ guaranteeing that $\Pi_{t_At_A} \leq 0$.

The first order conditions are

$$\begin{aligned}
\tilde{\psi}_A \sigma_A^2 - \mu_A(t_A) C_A' \sigma_A^2 &= 0, \\
\tilde{\psi}_B \sigma_B^2 - \mu_B(1-t_A) C_B' \sigma_B^2 &= 0, \\
-\mu_A'(t_A) C(e_A) \sigma_A^2 + \mu_B'(1-t_A) C(e_B) \sigma_B^2 &= 0.
\end{aligned}$$

Differentiating the first order conditions we have,

$$\begin{pmatrix} -\mu_A(t_A)C_A''\sigma_A^2 & 0 & -\mu_A'(t_A)C_A'\sigma_A^2 \\ 0 & -\mu_B(1-t_A)C_B''\sigma_B^2 & \mu_B'(1-t_A)C_B'\sigma_B^2 \\ -\mu_A'(t_A)C_A'\sigma_A^2 & \mu_B'(1-t_A)C_1(e_B)\sigma_B^2 & -(\mu_A''(t_A)C(e_A)\sigma_A^2 + \mu_B''(1-t_A)C(e_B)\sigma_B^2) \end{pmatrix} \begin{pmatrix} de_A \\ de_B \\ dt_A \end{pmatrix}.$$

$$\begin{pmatrix} 0 & -\frac{\partial \tilde{\psi}_A}{\partial s_A}\sigma_A^2 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial \tilde{\psi}_B}{\partial s_B}\sigma_B^2 \\ \mu_A'(t_A)C(e_A) & 0 & \mu_B'(t_B)C(e_B) & 0 \end{pmatrix} \begin{pmatrix} d\sigma_A^2 \\ ds_A \\ d\sigma_B^2 \\ ds_B \end{pmatrix}.$$

where

$$\begin{aligned}
|J| = -\mu_A(t_A)C_A''\sigma_A^2 \Big[\mu_B(1-t_A)C_B''\sigma_B^2 \Big(\mu_A''(t_A)C(e_A)\sigma_A^2 + \mu_B''(1-t_A)C(e_B)\sigma_B^2\Big) - [\mu_B'(1-t_A)C_B'\sigma_B^2]^2\Big] \\
+\mu_A'(t_A)C_A'\sigma_A^2\Big[\mu_A'(t_A)C_A'\sigma_A^2\mu_B(1-t_A)C_B''\sigma_B^2\Big]
\end{aligned}$$

Assumption $A1$, i.e., $\mu(t)\mu''(t)C(e)C''(e) > C'(e)^2\mu'(t)^2$ for all t and e, is a sufficient condition for $|J| < 0$. The comparatives static respect to s_A are

$$\frac{\partial e_A}{\partial s_A} = \frac{\partial \tilde{\psi}_A}{\partial s_A} \frac{\sigma_B^2\Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \mu_A'' C(e_A)\sigma_A^2}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A)\mu_A C_A'' - (\mu_A' C_A')^2\Big]\sigma_A^2} < 0.$$

$$\frac{\partial e_B}{\partial s_A} = -\frac{\partial \tilde{\psi}_A}{\partial s_A} \frac{\mu_A'(t_A)C_A'\mu_B'(1-t_A)C_B'\sigma_A^2}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A)\mu_A C_A'' - (\mu_A' C_A')^2\Big]\sigma_A^2} > 0.$$

$$\frac{\partial t_A}{\partial s_A} = -\frac{\partial \tilde{\psi}_A}{\partial s_A} \frac{\mu_A'(t_A)C_A'\mu_B(1-t_A)C_B''\sigma_A^2}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A)\mu_A C_A'' - (\mu_A' C_A')^2\Big]\sigma_A^2} < 0.$$

The comparatives static respect to σ_A^2 are

$$\frac{\partial e_A}{\partial \sigma_A^2} = \frac{\mu_A'(t_A)^2 C(e_A) C_A' \mu_B(1-t_A)C_B''}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A)\mu_A C_A'' - (\mu_A' C_A')^2\Big]\sigma_A^2} > 0.$$

$$\frac{\partial e_B}{\partial \sigma_A^2} = -\frac{\mu_A'(t_A)C(e_A)\mu_A(t_A)C_A''\mu_B'(1-t_A)C_1(e_B)}{\sigma_B^2\mu_A C_A''\left[\mu_B C_B''\mu_B''C(e_B)-[\mu_B'C_B']^2\right]+\mu_B C_B''\left[\mu_A''C(e_A)\mu_A C_A''-(\mu_A'C_A')^2\right]\sigma_A^2} < 0.$$

$$\frac{\partial t_A}{\partial \sigma_A^2} = -\frac{\mu_A'(t_A)C(e_A)\mu_A(t_A)C_A''\mu_B(1-t_A)C_B''}{\sigma_B^2\mu_A C_A''\left[\mu_B C_B''\mu_B''C(e_B)-[\mu_B'C_B']^2\right]+\mu_B C_B''\left[\mu_A''C(e_A)\mu_A C_A''-(\mu_A'C_A')^2\right]\sigma_A^2} > 0.$$

7.7.2 Headquarters' Design

the HQ's problem is $\max_{s_A,s_B} E[\pi_{1A}]+E[\pi_{1B}]$

$$\max_{s_A,s_B} K-\sigma_A^2+\left(e_{1A}\psi_{1A}-\mu_A(t_{1A})C(e_{1A})\right)\sigma_A^2+K-\sigma_B^2+\left(e_{1B}\psi_B-\mu_B(1-t_{1A})C(e_{1B})\right)\sigma_B^2. \tag{38}$$

the first order conditions are

$$\frac{\partial \psi_A}{\partial s_A}e_A+\frac{\partial e_A}{\partial s_A}(\psi_A-\tilde{\psi}_A)+\frac{\sigma_B^2}{\sigma_A^2}\frac{\partial e_B}{\partial s_A}(\psi_B-\tilde{\psi}_B)=0,$$
$$\frac{\partial \psi_B}{\partial s_B}e_B+\frac{\partial e_B}{\partial s_B}(\psi_B-\tilde{\psi}_B)+\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial s_B}(\psi_A-\tilde{\psi}_A)=0.$$

Notice that these first order conditions capture the effects of s_A on ψ_A and on e_A like when the headquarters allocate resources. However, it also captures the indirect effects of s_A on e_A and e_B through a change in t_A. This indirect effects are of second order magnitude and do not modify the main trade-off when choosing the organizational design.

The headquarters chooses to centralize or decentralize decision rights considering the sum of $[\psi_A e_A-\mu_A C(e_A)]\sigma_A^2+[\psi_B e_B-\mu_B C(e_B)]\sigma_B^2$. If $\overline{\mu}$ is sufficiently low and $\sigma_A^2\sim\sigma_B^2$, the headquarters decentralizes decision rights about both products. Since both products are similar in terms of returns to differentiation the headquarters follows a strategy of product differentiation in both products. If σ_A^2 increases, s_A and t_A increase and s_B decreases, making more likely that the firm prefers to centralize decision rights of product B. There is a cutoff $\hat{\sigma}_A^2$ above which the headquarters decentralizes decision making about product A and centralizes decision making about product B.

Note that the change in the organizational design that yields decentralization in product A and centralization in product B has an discrete jump in the optimal shares. The jump in s arises because the headquarters centralizes product B which non-locally reduces s_B; this also modifies $\psi_B-\tilde{\psi}_B$ and, consequently, provides incentives to reduce s_A and to increase s_B.

7.7.3 Redefining Effects

Given an efficient allocation of resources, t_A^*, the optimal s is defined by $\sigma_A^2[\frac{\partial\psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A)] = 0$. However, if local managers allocate resources the first order condition becomes:

$$\sigma_A^2[\frac{\partial\psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A)] + \frac{\partial t_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}[\psi_A\sigma_A^2 - \tilde{\psi}_A\sigma_A^2] + \frac{\partial t_A}{\partial s_A}\frac{\partial e_B}{\partial t_A}[\psi_B\sigma_B^2 - \tilde{\psi}_B\sigma_B^2],$$
$$+\frac{\partial t_A}{\partial s_A}[-\mu_A'(t_A)C_A(e_A)\sigma_A^2 + \mu_B'(1-t_A)C_B(e_B)\sigma_B^2] = 0.$$

with $\frac{\partial t_A}{\partial s_A} < 0$ and $\frac{\partial t_A}{\partial s_B} > 0$ leading to

$$\begin{aligned}
\frac{\partial\psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) &= -\frac{\partial t_A}{\partial s_A}\frac{\sigma_B^2}{\sigma_A^2}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big],\\
\frac{\partial\psi_B}{\partial s_B}e_B + \frac{\partial e_B}{\partial s_B}(\psi_B - \tilde{\psi}_B) &= -\frac{\partial t_A}{\partial s_B}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big].
\end{aligned}$$

calling $\varphi \equiv \frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]$. It is easier to separate the indirect effect of s_A and s_B on t_A from the direct effect of s_A on e_A and s_B on e_B. This alternative expression is used in the following section.

8 Appendix C: Extension

8.1 Extension in Section 4.1: Coordination

The first order conditions of the Headquarters problem are

$$-\frac{\partial\mu_A}{\partial t_A}\sigma_A^2\left[C(e_A) - \frac{\partial e_A}{\partial\mu_A}[\psi_A - \mu_A C'(e_A)]\right] = -\frac{\partial\mu_B}{\partial t_B}\sigma_B^2\left[C(e_B) - \frac{\partial e_B}{\partial\mu_B}[\psi_B - \mu_B C'(e_B)]\right], \tag{39}$$

$$\sigma_A^2\left[e_A\frac{\partial\psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A}[\psi_A - \mu_A C'(e_A)]\right] = 0, \tag{40}$$
$$\sigma_B^2\left[e_B\frac{\partial\psi_B}{\partial s_B} + \frac{\partial e_B}{\partial s_B}[\psi_B - \mu_B C'(e_B)]\right] = 0. \tag{41}$$

$$K'(\beta_A) - \sigma_A^2\left[e_A\frac{\partial\psi_A}{\partial\beta_A} + \frac{\partial e_A}{\partial\beta_A}[\psi_A - \mu_A C'(e_A)\right] = 0, \tag{42}$$
$$K'(\beta_B) - \sigma_B^2\left[e_B\frac{\partial\psi_B}{\partial\beta_B} + \frac{\partial e_B}{\partial\beta_B}[\psi_B - \mu_B C'(e_B)\right] = 0. \tag{43}$$

First, we could replace the optimal effort choice $\mu_A C'(e_A) = \tilde{\psi}$ into all first order conditions. Once t_A is chosen, the decisions of (s, β, g) are described by Rantakari (2010). Given the convexity of $\mu(t)$, the decision of t_A depends directly on who makes effort choice. In either case there exists increasing relations

$\tilde{\sigma}_B^2(\sigma_A^2)$ and $\tilde{\sigma}_A^2(\sigma_B^2)$ such that product A is centralized if $\sigma_A^2 < \tilde{\sigma}_A^2(\sigma_B^2)$ and decentralized if $\sigma_A^2 \geq \tilde{\sigma}_A^2(\sigma_B^2)$, and product B is centralized if $\sigma_B^2 < \tilde{\sigma}_B^2(\sigma_A^2)$ and decentralized if $\sigma_B^2 \geq \tilde{\sigma}_B^2(\sigma_A^2)$.

If local division managers control resources, the first order conditions are

$$\sigma_A^2 \left[e_A \frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A} [\psi_A - \mu_A C'(e_A)] \right] = -\frac{\partial t_A}{\partial s_A} \varphi, \tag{44}$$

$$\sigma_B^2 \left[e_B \frac{\partial \psi_B}{\partial s_B} + \frac{\partial e_B}{\partial s_B} [\psi_B - \mu_B C'(e_B)] \right] = -\frac{\partial t_A}{\partial s_B} \varphi. \tag{45}$$

$$K'(\beta_A) - \sigma_A^2 \left[e_A \frac{\partial \psi_A}{\partial \beta_A} + \frac{\partial e_A}{\partial \beta_A} [\psi_A - \mu_A C'(e_A) \right] = -\frac{\partial t_A}{\partial \beta_A} \varphi, \tag{46}$$

$$K'(\beta_B) - \sigma_B^2 \left[e_B \frac{\partial \psi_B}{\partial \beta_B} + \frac{\partial e_B}{\partial \beta_B} [\psi_B - \mu_B C'(e_B) \right] = -\frac{\partial t_A}{\partial \beta_B} \varphi, \tag{47}$$

with

$$\varphi \equiv \sigma_A^2 \frac{\partial \mu_A}{\partial t_A} \frac{\partial e_A}{\partial \mu_A} \left[\psi_A - \mu_A C'(e_A) \right] - \sigma_B^2 \frac{\partial \mu_B}{\partial t_B} \frac{\partial e_B}{\partial \mu_B} \left[\psi_B - \mu_B C'(e_B) \right]. \tag{48}$$

We can replace the optimal allocation of time chosen by managers into φ. There exists increasing relations $\tilde{\sigma}_{BM}^2(\sigma_A^2)$ and $\tilde{\sigma}_{AM}^2(\sigma_B^2)$ such that decision making about product A is centralized if $\sigma_A^2 < \tilde{\sigma}_{AM}^2(\sigma_B^2)$ and decentralized if $\sigma_A^2 \geq \tilde{\sigma}_{AM}^2(\sigma_B^2)$, and decision making about product B is centralized if $\sigma_B^2 < \tilde{\sigma}_{BM}^2(\sigma_A^2)$ and decentralized if $\sigma_B^2 \geq \tilde{\sigma}_{BM}^2(\sigma_A^2)$.

When $\sigma_A^2 > \sigma_B^2$ then $\varphi > 0$, and thus the firm prefers to allocate more resources in product A than implemented by local managers, i.e., $t_A < t_A^*$. The right hand side of equations (44) and (46) are positive and the right hand side of equations (45) and (47) are negative. The headquarters aligns incentive, s, and integrate divisions, β, to affect the resource allocation choice of local managers. When $\sigma_A^2 > \sigma_B^2$, the headquarters reduces incentive alignment of product A, i.e., ∇s_A, increases incentive alignment of product B, i.e., Δs_B, reduces integration of product A, i.e $\nabla \beta_A$, and increases integration of product B, i.e., $\Delta \beta_B$. In other words, local managers put too much resources in product B, because they underestimate the opportunity cost of resources. The headquarters finds less important to provide incentives for information acquisition in product B and then decentralization appears more profitable.

8.2 Extension in Section 4.2: Delegation

The headquarters objective function is:

$$E[\pi_{iA}] + E[\pi_{iB}] = \left\{ K_A - [1 - (e_A + \alpha e_B)\psi_A + \mu_A C(e_{iA})]\, \sigma_A^2 \right\} + \left\{ K_B - [1 - (e_B + \alpha e_A)\psi_B + \mu_B C(e_{iB})]\, \sigma_B^2 \right\}.$$

When the headquarters control resources, the first order conditions are

$$-\frac{\partial \mu_A}{\partial t_A} \sigma_A^2 \left[C(e_A) - \frac{\partial e_A}{\partial \mu_A} [\psi_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \psi_B - \mu_A C'(e_A)] \right] = -\frac{\partial \mu_B}{\partial t_B} \sigma_B^2 \left[C(e_B) - \frac{\partial e_B}{\partial \mu_B} [\psi_B + \frac{\sigma_A^2}{\sigma_B^2} \alpha \psi_A - \mu_B C'(e_B)] \right]. \tag{49}$$

$$\sigma_A^2 \left[(e_A + \alpha e_B) \frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A} [\psi_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \psi_B - \mu_A C'(e_A)] \right] = 0, \tag{50}$$

$$\sigma_B^2 \left[(e_B + \alpha e_A) \frac{\partial \psi_B}{\partial s_B} + \frac{\partial e_B}{\partial s_B} [\psi_B + \frac{\sigma_A^2}{\sigma_B^2} \alpha \psi_A - \mu_B C'(e_B)] \right] = 0. \tag{51}$$

Once t_A is chosen, the effort exerted by local managers is given by $\tilde{\psi}_A = \mu_A C'(\hat{e}_A)$. Note that the inefficiency in effort choice is given by $\psi_A - \tilde{\psi}_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \psi_B$.

If local division managers control resources, the first order conditions are

$$\sigma_A^2 \left[(e_A + \alpha e_B) \frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A} [\psi_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \psi_B - \mu_A C'(e_A)] \right] = -\frac{\partial t_A}{\partial s_A} \varphi, \tag{52}$$

$$\sigma_B^2 \left[(e_B + \alpha e_A) \frac{\partial \psi_B}{\partial s_B} + \frac{\partial e_B}{\partial s_B} [\psi_B + \frac{\sigma_A^2}{\sigma_B^2} \alpha \psi_A - \mu_B C'(e_B)] \right] = -\frac{\partial t_A}{\partial s_B} \varphi. \tag{53}$$

with

$$\varphi \equiv \sigma_A^2 \frac{\partial \mu_A}{\partial t_A} \frac{\partial e_A}{\partial \mu_A} \left[\psi_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \psi_B - \mu_A C'(e_A) \right] - \sigma_B^2 \frac{\partial \mu_B}{\partial t_B} \frac{\partial e_B}{\partial \mu_B} \left[\psi_B + \frac{\sigma_A^2}{\sigma_B^2} \alpha \psi_A - \mu_B C'(e_B) \right]. \tag{54}$$

However, the effort choice is now given by $\tilde{\psi}_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha \tilde{\psi}_B = \mu_A C'(\hat{e}_A)$, and the inefficiency in effort choice is given by $\psi_A - \tilde{\psi}_A + \frac{\sigma_B^2}{\sigma_A^2} \alpha (\psi_B - \tilde{\psi}_B)$. The headquarters faces a trade-off between allocating resources efficiently and internalizing the learning externality. If the firm is organized by regional divisions and resource allocation is delegated to local managers, there is an inefficiency in resource allocation but local managers internalize that learning about one product has a positive externality in learning about the other product.