# SYNCHRONICITY BETWEEN MACROECONOMIC TIME SERIES: AN EXPLORATORY ANALYSIS

**Felipe M. Aparicio,
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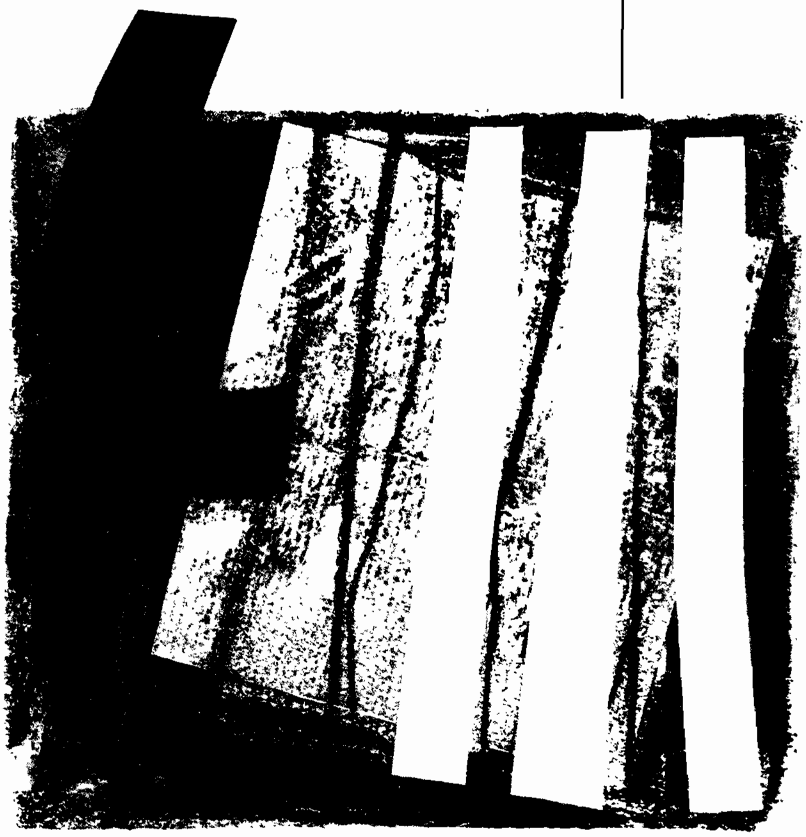

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**Abstract**

In this paper we analyse the performances of a model free cointegration testing device that we construct from functions of order statistics. First, we propose new exploratory techniques that consist in comparing the plots of the *range* and the *jump* sequences of the original series. These plots suggest alternative cointegration testing schemes. Here we focused on one of them, which involves two complementary test statistics. We report on some promising results obtained from Monte Carlo experiments as well as on some empirical applications of the new method to pairs of exchange-rates, and to gold and silver prices. Our study concludes that the proposed methodology is potentially robust to monotonic nonlinearities and serial correlation structure in the cointegration errors, and certain types of level shifts in the cointegrating relationship.

**Keywords:** Cointegration, nonlinearity, ranks, ranges, jumps.

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# SYNCHRONICITY BETWEEN MACROECONOMIC TIME SERIES: AN EXPLORATORY ANALYSIS

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ABSTRACT. In this paper we analyse the performances of a model free cointegration testing device that we construct from functions of order statistics. First, we propose new exploratory techniques that consist in comparing the plots of the *range* and the *jump* sequences of the original series. These plots suggest alternative cointegration testing schemes. Here we focused on one of them, which involves two complementary test statistics. We report on some promising results obtained from Monte Carlo experiments as well as on some empirical applications of the new method to pairs of exchange-rates, and to gold and silver prices. Our study concludes that the proposed methodology is potentially robust to monotonic nonlinearities and serial correlation structure in the cointegration errors, and certain types of level shifts in the cointegrating relationship.

## 1. INTRODUCTION

Processes which exhibit common trends or similar long waves in their sample paths are often called *cointegrated.* The concept of cointegration is inherently linear and originated in macroeconomics and finance (c.f.Granger (1981), Engle and Granger (1987)), where the theory suggests the presence of economic or institutional forces preventing two or more series to drift too far apart from each other. Take for example, those series as income and expenditure, the prices of a particular good in different markets, the interest rates in different countries, the velocity of circulation of money and short-run interest rates, etc. Cointegration relationships may also appear in engineering applications. For instance, between the outputs signals from different sensing or processing devices having a limited storage capacity or memory, and driven by a common persistent input flow (c.f. Aparicio (1995)).

Underlying the idea of cointegration is that of a *stochastic equilibrium* relationship (i.e. one which, apart from deterministic elements, holds on the average) between two cointegrated variables, $y_t, x_t$. A strict equilibrium exists when for some $\alpha \neq 0$, one has $y_t = \alpha x_t$. This unrealistic situation is

*Key words and phrases.* Cointegration, nonlinearity, ranks, ranges, jumps.
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replaced, in practice, by that of (linear) cointegration, in which the equilibrium error $z_t = x_t - \alpha x_t$ is different from zero but fluctuates around the mean much more frequently than the individual series.

So far, only few attempts to extend the concept of cointegration beyond the assumption of linearity in the relationship have been considered. This is essentially due to the fact that a general null hypothesis of cointegration encompassing nonlinear relationships is too wide to be tested. Notwithstanding, the possibility of nonlinear cointegrating relationships is real, and therefore it has prompted some interesting definitions and an ongoing research on the subject. The first of these attempts was due to Hallman (1990) and to Granger and Hallman (1991*a*). Following this, for a pair of series $y_t, x_t$ to have a cointegrating nonlinear *attractor*, there must be nonlinear measurable functions $f(.), g(.)$ such that $f(y_t)$ and $g(x_t)$ are both $I(d)$, $d > 0$, and $w_t = f(y_t) - g(x_t)$ is $\sim I(d_w)$, with $d_w < d$ (see also Granger and Terasvirta (1993)).

Assuming that $f$ and $g$ can be expanded as Taylor series up to some order $p \geq 2$ around the origin, we may write $w_t = c_0 + c_1 z_t + HOT(y_t, x_t)$, where $z_t = y_t - \alpha x_t$, and with $HOT(.,.)$ denoting higher-order terms. It follows that the linear approximation, $z_t$, to the true cointegration residuals differs from the latter by some higher-order terms which express that the *strength of attraction* onto the cointegration line $y_t = \alpha x_t$ varies with the levels of both series, $y_t$ and $x_t$.

As with linear cointegration. the case where $d_x = d_y = 1$, $d_z = 0$ and the cointegration residuals have finite variance is most important in practice, since it allows a straightforward interpretation in terms of equilibrium concepts. Figure 1 illustrates the case of nonlinear cointegration, with simulated nonlinear transformations of random walks.

The most general distributional results to unit-root testing were first obtained by Phillips (1987). Suppose $x_t$ has mean $\mu_t$, and let $\Delta(x_t - \mu_t) = \epsilon_t$, with $\Delta$ denoting the first differencing operator. The main assumption imposed to obtain the limit distribution of standard unit-root tests is the following, due to Herrndorf (1984):

**Assumption 1** (AS1). 1. $E(\epsilon_t) = 0$.
2. $\sup_t E(|\epsilon_t|^\gamma) < \infty$ *for some* $\gamma > 2$.
3. $0 < \lim_{N\to\infty} E\left(N^{-1}(\sum_{t=1}^{N} \epsilon_t)^2\right) < \infty$.
4. $\epsilon_t$ *is strong mixing, with mixing coefficients* $\alpha_i$ *satisfying* $\sum_{i=1}^{\infty} \alpha_i^{1-2/\gamma} < \infty$.

In a one-sided test for unit roots, the null hypothesis of a unit root in a series $x_t$, $H_0$ : $(1 - B)(x_t - \mu_t) = \epsilon_t$, is tested against the alternative $H_1$ : $(1 - \rho B)(x_t - \mu_t) = \epsilon_t$ with $|\rho| < 1$, where $\mu_t$ denotes the mean of $x_t$. The decision between $H_0$ and $H_1$ is based on the significance of the

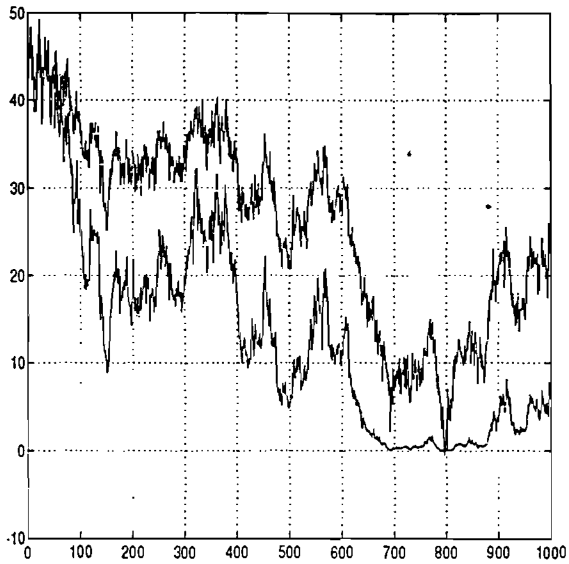



FIGURE 1. Two simulated nonlinearly cointegrated series. The upper series was obtained as $x_t = w_t + e_{x,t}$, while the lower one corresponds to $y_t = g(w_t) + e_{y,t}$, where $g(.)$ represents a third-order polynomial of its argument random walk variable $w_t$, and $\epsilon_{x,t}, \epsilon_{y,t}$ are independent*i.i.d.* sequences.

estimate of $\rho - 1$ in the regression:

$$\Delta(x_t - \mu_t) = (\rho - 1)(x_{t-1} - \mu_{t-1}) + \epsilon_t, \tag{1.1}$$

where $\Delta$ denotes the first differencing operator.

A commonly used test statistic for testing $H_0$ is the $t$-ratio, $t_0$, based on the least-squares estimate of the parameter $\rho$ in (1.1)

$$t_0 = \frac{\hat{\rho} - 1}{\hat{\sigma}_0} \tag{1.2}$$

where

$$\hat{\sigma}_0 = \sqrt{N^{-1} \sum_{i=1}^{N} \hat{\epsilon}_i^{\,2} \left( \sum_{i=1}^{N} (x_{i-1} - \mu_{i-1})^2 \right)^{-1/2}} \tag{1.3}$$

is the standard deviation of $\hat{\rho} - 1$. The standard unit root test based on the $t$-ratio $t_0$ of regression (1.1), is commonly referred to as the *Dickey-Fuller* (DF) *test* Dickey and Fuller (1979). Dickey and Fuller also considered a way of palliating the effects of the autocorrelation in $\epsilon_t$ consists in "augmenting" the test by including sufficient lagged first differences $\Delta(x_{t-i} - \mu_{t-i})$, $i \geq 1$, in the right-hand side of (1.1), so as to remove as much as possible the serial correlation in $\epsilon_t$. This gave rise to the so called *Augmented Dickey-Fuller* (ADF) *test*. In small samples, the parametric correction implied by the ADF test procedure performs better than the previous non-parametric corrections. For a clear discussion about how to implement the unit-root test in practice, see Hamilton (1994) -chapter 17-.

By doing a mean value expansion of a general nonlinear transformation, Granger and Hallman (1991*b*) remarked that the residuals did not satisfy assumption AS1 in several cases -see also Escribano and Mira (1997)-. For example, if we let

$$x_t = f(y_t) \tag{1.4a}$$

$$y_t = y_{t-1} + \epsilon_t \tag{1.4b}$$

where $\epsilon_t$ represents an *i.i.d* sequence of zero–mean random variables, one could write

$$x_t = x_{t-1} + u_t \tag{1.5a}$$

$$u_t = f'(y_{t-1} + r_t)\epsilon_t \tag{1.5b}$$

with $r_t \in (x_{t-1}, x_t)$. Granger and Hallman (1991*b*) show that when using transformations such as

$$f(y_t) = y_t^2 \qquad f(y_t) = y_t^3$$

$$f(y_t) = sign(y_t) \qquad f(y_t) = \exp(y_t)$$

$$f(y_t) = \sin(y_t) \qquad f(y_t) = 1/y_t$$

in (1.4a), $u_t$ in (1.5b) does not satisfy Assumption 1.

Therefore unit-root tests are not invariant to nonlinear transformations of the variables, see Park and Phillips (1999). Granger and Hallman (1991*b*) proposed to use a Dickey-Fuller test based on the ranks of $x_t$[1] , since the ranks are robust to monotonic transformations. Hallman (1990) extended those results to test the null hypothesis of no cointegration on transformed series. However, the limit distribution of the unit-root test based on ranks was unknown. Breitung and Gourieroux (1997) obtained this limit distribution, but their test has some undesirable properties. Recently, Park and Phillips (1999) proposed an asymptotic theory for some types of nonlinear transformations of integrated time series. By using local time theory, they found that the rate of convergence of the limit distribution of cointegration tests based on the regression residuals depend on the properties of the nonlinear regression function. This underlines the intrinsic difficulty of cointegration tests that try to be robust to wide classes of nonlinearities.

[1]The rank of $x_i$ in the sample $X$ of size $n$ is defined as $r_{i,n}^{(x)} = \sum_{j=1}^{n} \mathbf{1}(x_i \geq x_j)$ -see for example David (1981)-.

Classical unit-root tests, like the ones proposed by Phillips (1987) and Dickey and Fuller (1979), are based on linear time series models, and most of the critical values are obtained under the assumption of Normality and independence of the errors. Our goal is to develop model free tests for cointegration that are robust to different deviations from the standard linear cointegration hypothesis.

## 2. Cointegration testing using the ranges

The objective of this section is twofold. First, based on a preliminary exploratory analysis, we propose a model free procedure for testing cointegration. Second, we analyse its behaviour on finite samples and show promising results as regards its robustness to different departures from the standard framework, such as monotonic nonlinearities or serial correlation in the cointegration errors. Our concern in this paper is mainly exploratory, and as such, we will deliberately skip any asymptotic analysis. Important questions such as the convergence rates and the limiting distributions of the standardised test statistics which we propose here are still under research, and will be addressed in a subsequent paper by the authors.

For our purpose, it may be interesting to see the cointegration property in terms of the following two conditions:

1. There are informational events that have a permanent effect on the levels of the series (in the linear case, this amounts to saying that the series are integrated).
2. The relevant informational events for either series occur simultaneously up to a constant delay, and their effects on their levels can be related.

Obviously, these conditions automatically raise the question of what should be considered as relevant informational events for a series. The definition that follows is somewhat arbitrary and is just taken for convenience. Herein, we regard as *permanent informational events* those inducing any outstanding trending behaviour (either deterministic or stochastic) in the levels of the series. The implication of this is that cointegration will basically consist in checking the synchronicity (up to a constant delay) of two sets of arrival times. In the sequel, we will made this characterisation operative by using some linear functions of order statistics called the *ranges* (see Aparicio and Escribano (1998)).

### 2.1. Exploratory analysis based on ranges.

The ranges are defined in terms of the *extremes* - Galambos (1984)-. For a sample of size $n$, $x_1, \cdots, x_n$, the order statistics of $x_t$ are given by the sequence $x_{1,n} \leq \cdots \leq x_{n,n}$, obtained after a permutation, of the indexes $\{1, \cdots, n\}$ such that $x_{i,n} \leq x_{i+j,n}$, $\forall j > 0$. The terms $x_{1,n} = \min\{x_1, \cdots, x_n\}$ and $x_{n,n} = \max\{x_1, \cdots, x_n\}$ are called the extremes, and the sequence of ranges for this $n$-size sample of $x_t$ is defined as $r_n^{(x)} = x_{n,n} - x_{1,n}$. Basically, a process defined by a sequence of ranges is an integrated *jump process*, where each jump corresponds to the arrival of a what we called a relevant informational event, which for us will be one which contributes either a new maximum or a new minimum level in the series.

Figure 2 show the range sequences $r_i^{(y)}$ and $r_i^{(x)}$ for pairs of linearly cointegrated, nonlinearly cointegrated (cubic nonlinearity), non-cointegrated, and $I(0)$ comoving time series. It can be seen that, for cointegrated series (either linear or nonlinear), the jumps occur at approximately the same instants, even though their amplitudes may not be related by a linear relationship (see figures 3 and 4). On the contrary, the jump sequences corresponding to the non-cointegrated series show no apparent relation between the arrival times of the two sets of jumps, and a similar behaviour is obtained when the series are $I(0)$ but comoving (figures 5 and 6).

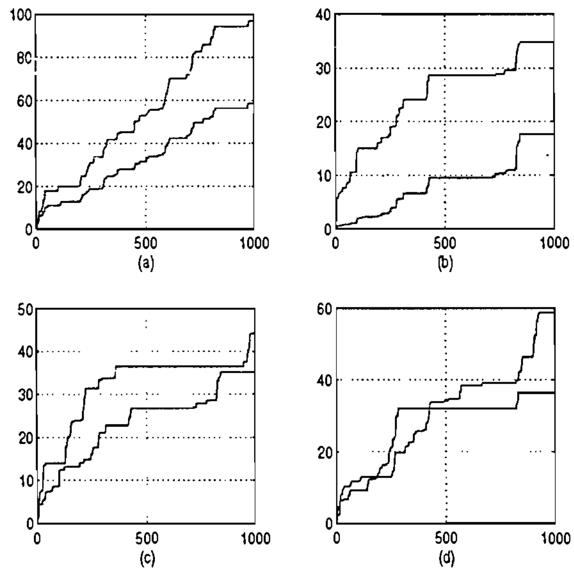



FIGURE 2. Cross-plots of the sequences of ranges for a pair of linearly, nonlinearly, non-cointegrated, and $I(0)$ comoving series: (a) linear cointegration, (b) nonlinear cointegration, (c) independent random walks, and (d) $I(0)$ comoving series.

Figure 7 shows the cross-plots of the range sequences for the four pair of series. It is apparent that cointegration implies a sort of continuity (synchronicity) in these plots. For the pairs of independent random walks and the pairs of $I(0)$ comoving series, the sequences of ranges evolve differently, which explain the discontinuities in the corresponding cross-plots. These discontinuities are more pronounced for pairs of independent random walks since the sample paths of the series consist of long strides, while for the $I(0)$ comoving series, the the different ways in which the range

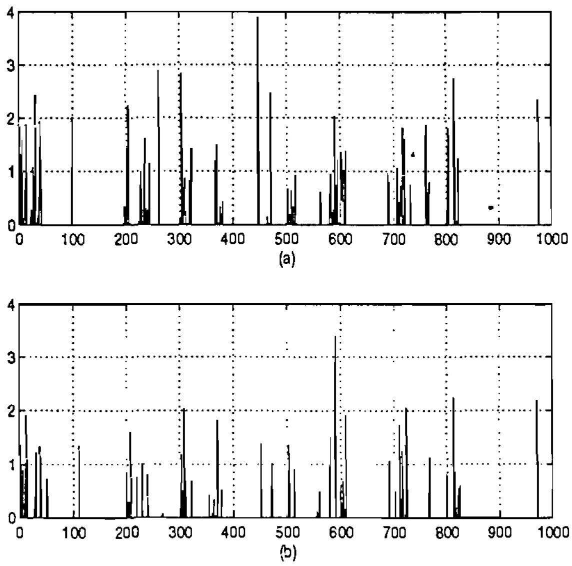



FIGURE 3. Sequences of jumps $\Delta r_i^{(y)}$ and $\Delta r_i^{(x)}$ for the linearly cointegrated series used in figure 2.

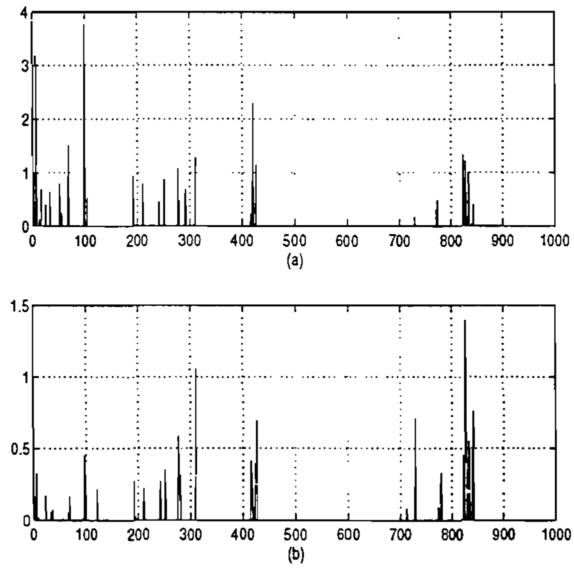



FIGURE 4. Sequences of jumps $\Delta r_i^{(y)}$ and $\Delta r_i^{(x)}$ for the nonlinearly cointegrated series used in figure 2.

series evolve are hidden by high-frequency fluctuations. As a consequence, the discontinuities in the last cross-plot are not so outstanding.

2.2. **Some nonparametric range statistics for cointegration testing.** As we mentioned before, our characterisation of cointegration requires that the following two conditions concerning the jumps (first differences of the ranges) are satisfied:

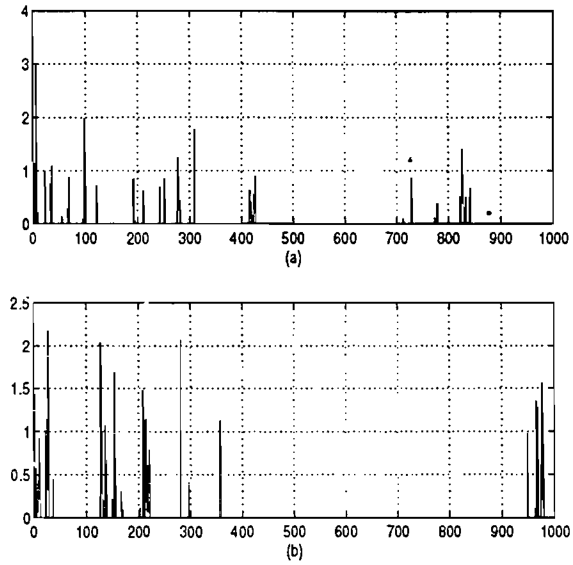



FIGURE 5. Sequences of jumps $\Delta r_i^{(y)}$ and $\Delta r_i^{(x)}$ for the independent random walks used in figure 2.

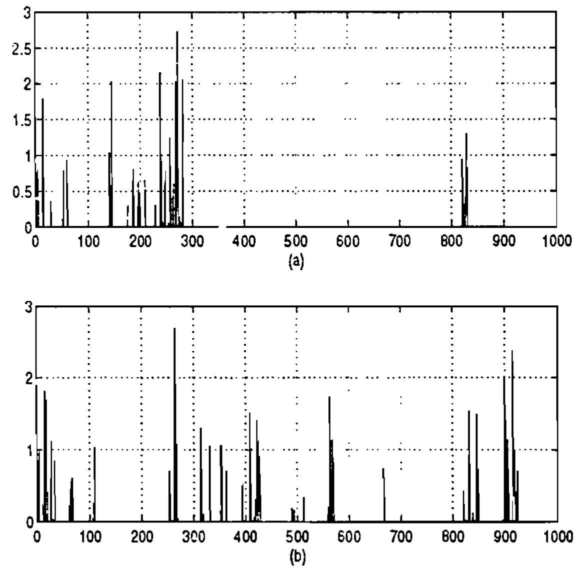



FIGURE 6. Sequences of jumps $\Delta r_i^{(y)}$ and $\Delta r_i^{(x)}$ for the pair of $I(0)$ comoving series used in figure 2.

1. For each series, the jumps are persistent. This means that the probability of occurrence of a new maximum or minimum is constant along time and is nonzero. That is, the series can fluctuate wildly around their mean and their levels are not stochastically bounded. Somehow, this requirement amounts to the long-memory property.
2. The jumps for both series occur at time instants that are equal up to a constant delay.

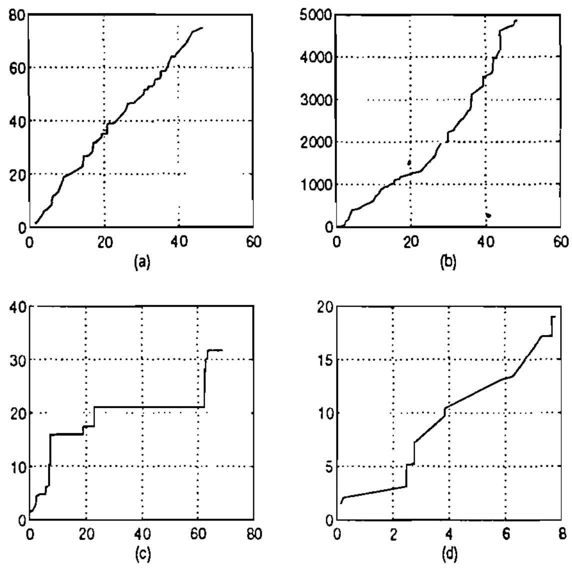



FIGURE 7. Cross-plots of the sequences of ranges for pairs of series that follow: (a) linear cointegration, (b) nonlinear cointegration, (c) independent random walks, and (d) $I(0)$ comoving series.

If either of these conditions are not satisfied then the series will not be cointegrated in the sense of our characterisation. For example, if for each series jumps clusters along orthogonal supports then these series will not be cointegrated. Further, to distinguish between linear and nonlinear cointegration, it would be enough to remark that while for linear cointegration, informational events having identical arrival times in both series will have approximately the same impact on their levels, for nonlinear or non-stationary cointegration these shocks may have quite different effects on each series.

Assuming that no series lags behind the other, then under cointegration, a cross-plot of the jumps for both series would show many points in the first quadrant, while for independent random walks or for $I(0)$ comoving series, the points in these plots would tend to lie very close to or along the non-negative half-axis. Therefore, one would be inclined to believe that the quality of fit of a regression line from the origin to the points in these plots would necessarily be less bad under cointegration than under non-cointegration. Figure 8 shows these cross-plots obtained from 100 replications of each pair of series. In order to summarise the information collected by the cross-plots of $\Delta r_t^{(y)}$ versus $\Delta r_t^{(x)}$ in a statistic, we remark that in the presence of a linear or a monotonic nonlinear cointegrating component in $x_t, y_t$ the sequences of ranges, $r_1^{(x)}, \cdots, r_n^{(x)}$ and $r_1^{(y)}, \cdots, r_n^{(y)}$, will be approximately proportional. We expect a similar behaviour from the sequences of jumps, $\Delta r_1^{(x)}, \cdots, \Delta r_n^{(x)}$ and $\Delta r_1^{(y)}, \cdots, \Delta r_n^{(y)}$. Thus, a non-parametric measure of linear cointegration could be provided by the following statistic, which provides a measure of the sample cross-correlation of the jump sequences

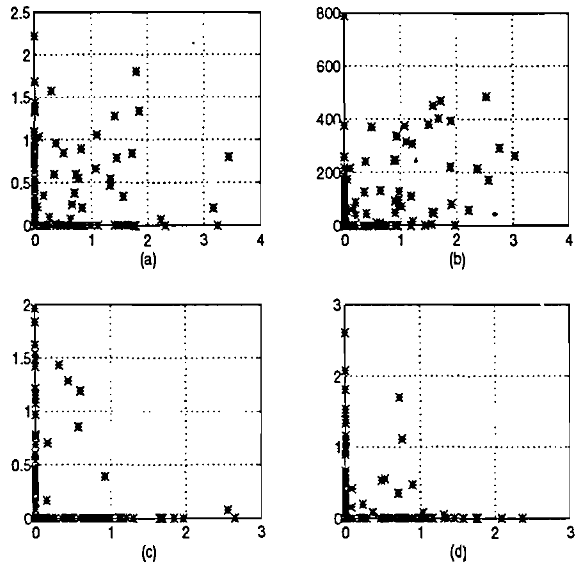



FIGURE 8. $\Delta r_i^{(y)}$ versus $\Delta r_i^{(x)}$ for pairs of: (a) linearly cointegrated series, (b) non-linearly cointegrated series (quadratic transformation), (c) non-cointegrated series (independent random walks), and (d) $I(0)$ linearly comoving series.

$\Delta r_t^{(x)}$ and $\Delta r_t^{(y)}$:

$$\rho_{x,y}^{(n)} = \frac{\sum_{i=2}^{n}\left(r_i^{(x)} - r_{i-1}^{(x)}\right)\left(r_i^{(y)} - r_{i-1}^{(y)}\right)}{\left(\sum_{i=2}^{n}\left(r_i^{(x)} - r_{i-1}^{(x)}\right)^2\right)^{1/2}\left(\sum_{i=2}^{n}\left(r_i^{(y)} - r_{i-1}^{(y)}\right)^2\right)^{1/2}}. \tag{2.1}$$

As it will be shown in the next section, the previous test statistic cannot discriminate properly between model model 1 of linear cointegration and model 3 of $I(0)$ linearly comoving series. An inspection of the cross-plots for the jump series reveals that the points in these plots tend to cluster at the origin for pairs of $I(0)$ comoving series, meaning that there are large time spells in which no relevant informational event appears for either series. Indeed, the very nature of the sample paths of $I(0)$ series entails that all relevant features of the series are captured in a comparatively small time interval, whereas the long strides of the sample paths of integrated series preclude this possibility. This explains why the quality of fit of a regression line from the origin to the points in the cross-plots cannot be distinguished from that obtained for pairs of linearly cointegrated series. This calls for a complementary nonparametric test statistic that takes into account these features in order to discriminate between pairs of cointegrated series and pairs of $I(0)$ series. Therefore we propose a second test statistic $R_{x,y}^{(n)}$, which we define as

$$R_{x,y}^{(n)} = \frac{J^+}{NJ}, \tag{2.2}$$

where $J^+$ denotes the number of points in the plots which occur on the positive half axes, and $NJ$ the number of points at the origin of these plots. In other words, $R_{x,y}^{(n)}$ measures the proportion of informational events that are only relevant to one series with respect to those which are not for either. The variable selected for the numerator in this ratio ensures that for pairs of independent random walks $R_{x,y}^{(n)}$ will take large values as compared to the cases of $I(0)$ series and of linear cointegration.

2.3. Monte Carlo simulations. In Table 1, the mean values and their standard deviations (given in brackets) for the jumps statistic $\rho_{x,y}^{(n)}$ is given for an experiment involving 5000 replications of cointegrated (linearly and nonlinearly -quadratic-) and non-cointegrated series of length $n = 1000$. The nonlinearly cointegrated series were obtained as in Figure 1, using a quadratic transformation of a common random walk component, plus an added independent white Gaussian noise. We also estimated the mean and standard deviation of the jumps statistic on a pair of linearly comoving $I(0)$ series generated with the following model:

$$x_t = 0.6x_{t-1} + e_{t,1}$$
$$y_t = 2.0x_t + e_{t,2},$$

where $e_{t,1}, e_{t,2}$ are independent *i.i.d.* sequences of Gaussian random variables (*r.v.'s*).
In the sequel, *LC* will stand for linear cointegration, *NLC* for nonlinear cointegration, and *IRW* for independent random walks.

| *Test statistic* | *LC* | *NLC (quadratic)* | *NC* | *I(0) Comoving* |
|---|---|---|---|---|
| $\rho_{x,y}^{(n)}$ | 0.2724 (0.0972) | 0.4650 (0.1003) | 0.0712 (0.0463) | 0.6580 (0.1234) |

TABLE 1. Mean values and standard deviations of the jump statistic $\rho_{x,y}^{(n)}$, evaluated on 5000 replications of linearly (LC) and nonlinearly (NLC) cointegrated, of independent random walks (IRW), and on a pair of $I(0)$ linearly comoving series, for a sample size of $n = 1000$.

Clearly, the case of independent random walks can be easily discriminated using this statistic in a unilateral test. Figure 9 shows the histogram plots of $\rho_{x,y}^{(n)}$ .

Under the null hypothesis $H_0$ of independent random walks we estimated the empirical density of $\rho_{x,y}^{(n)}$ by smoothing techniques. More specifically, we chose the Kernel Density Estimator with

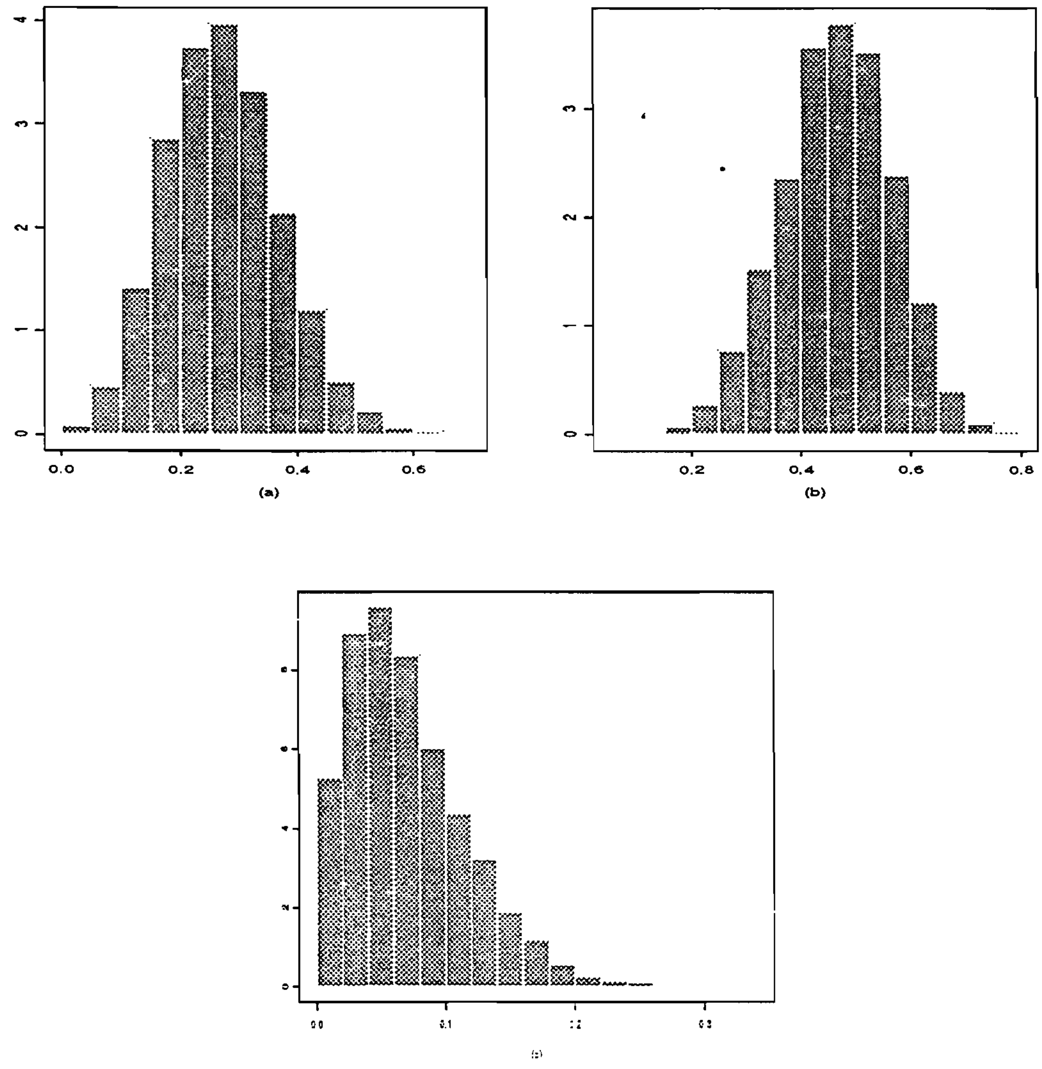



FIGURE 9. Histogram plots for $\rho_{x,y}^{(n)}$ where the frequencies are estimated from 5000 replications of sample size 1000 of: (a) linearly cointegrated series, (b) nonlinearly (quadratic) cointegrated series, and (c) non-cointegrated series (independent random walks).

the *Epachenikov* kernel function and bandwidth parameter $h = 0.005$ for different sample sizes ($n$ =100, 500, 1000) and for 5000 Monte Carlo simulated pairs of independent random walks with *i.i.d.* Normally distributed errors (Model 0). Figure 10 shows this density. We observe that, as $n$ tends to infinity, the shape of the estimated density is similar to that of a Chi-squared distribution. However, as the sample size increases, the critical values decrease quite fast and suggest that the asymptotic null distribution for $\rho_{x,y}^{(n)}$ could be degenerate. In this paper, we will not attempt to find out the proper convergence rate of this statistic. In spite of it, we analyse use this testing device on

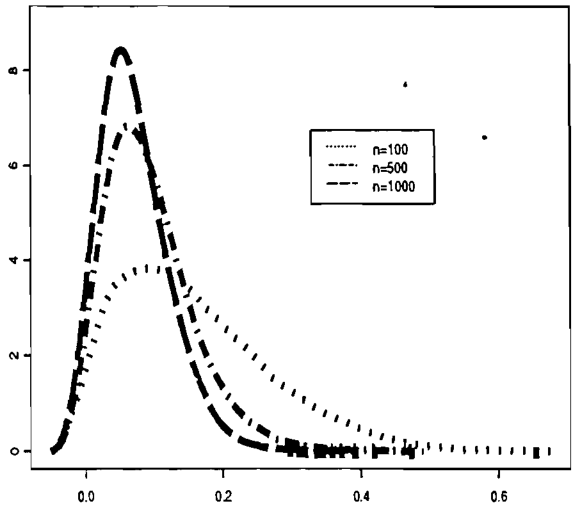



FIGURE 10. Kernel Density Estimator of $\rho_{x,y}^{(n)}$ of independent Random Walk series.

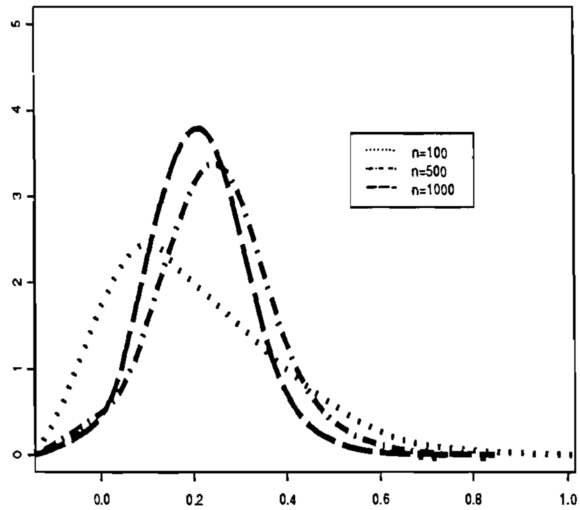



FIGURE 11. Kernel Density Estimator of $\rho_{x,y}^{(n)}$ of exponentially cointegrated series. The DGP's are given in equations (2.3a)–(2.3c)

finite samples by considering the empirical critical value, when fixing the sample size for different values.

We also estimated the empirical density of $\rho_{x,y}^{(n)}$ for exponentially cointegrated variables, by Monte Carlo simulations as above with $h = 0.15$. The DGP was

$$w_t = w_{t-1} + e_{t,0} \tag{2.3a}$$

$$x_t = w_t + e_{t,1} \tag{2.3b}$$

$$y_t = 0.5 \ \exp w_t + e_{t,2} \tag{2.3c}$$

Figure 11 shows the estimated density for different sample sizes. We observe that as $n$ tends to infinity the estimated density approaches the shape a Normal distribution.

Under the null hypothesis $H_0$ of independent random walks (IRW), we computed the 5% right critical values of the empirical distribution of $\rho_{x,y}^{(n)}$ for different sample sizes ($n = 100, 224, 500, 1000, 2000, 5000$ and for 1000 simulated pairs of independent random walks with *i.i.d.* Normally distributed errors (*model 0*). The critical value corresponding to the sample size $n = 224$ is generated because it will be used later in one of the empirical applications of the test. The results are summarised in Table 2.

| *n=100* | *n=224* | *n=500* | *n=1000* | *n=2000* | *n=5000* |
|---|---|---|---|---|---|
| 0.39251 | 0.27330 | 0.22647 | 0.16781 | 0.1255 | 0.0828 |

TABLE 2. Simulated 5% right critical values of the empirical distribution of the test statistic $\rho_{x,y}^{(n)}$ under the null hypothesis.

First, we will show that cointegration can be defined by means of the synchronicity requirement when we allow for the appropriate time delay correction in the series. We consider 1000 replications of time series generated with the following linear model:

*Model 1a* (Linear Cointegration)

$$w_t = w_{t-1} + e_{t,0}$$

$$x_t = w_t + e_{t,1}$$

$$y_t = a \ w_{t-m} + e_{t,2}$$

where $e_{t,1}$ and $e_{t,2}$ represent independent *i.i.d.* sequences of *r.v.'s*. Table 3 shows the estimated power of the test based on $\rho_{x_t,y_t}^{(n)}$, the correlation coefficient between $x_t$ and $y_t$, for different sample sizes ($n = 100$, $n = 500$, and $n = 1000$) and for different values of the delay parameter $m$ ($m = 1$, 2, 5).

| $m\backslash n$ | $n$=*100* | $n$=*500* | $n$=*1000* |
|---|---|---|---|
| 1 | 0.123 | 0.324 | 0.598 |
| 2 | 0.073 | 0.177 | 0.333 |
| 5 | 0.038 | 0.071 | 0.163 |

TABLE 3. Power of the test based on $\rho^{(n)}_{x,y}$.

The results in Table 3 show that our test has lost power since we have misspecified the correct constant delay ($m$). However, if we consider the appropriate delay $m$ in the statistic, $\rho^{(n)}_{x_{t-m},y_t}$, the correlation between $x_{t-m}$ and $y_t$, for $m = 1, 2, 5$ we get back to the previously obtained power results. Table 4 shows the values of the estimated power of the test based on $\rho^{(n)}_{x_{t-m},y_t}$.

| $m\backslash n$ | $n$=*100* | $n$=*500* | $n$=*1000* |
|---|---|---|---|
| 1 | 0.50 | 0.78 | 0.90 |
| 2 | 0.45 | 0.74 | 0.89 |
| 5 | 0.48 | 0.85 | 0.95 |

TABLE 4. Power of the test based on $\rho^{(n)}_{x_{t-m},y}$.

Therefore, in general, synchronicity is a stronger requirement than cointegration. However, if we allow that the former property to take place at any constant delay, then cointegration is properly defined in terms of synchronicity (up to a constant delay).

Next, we computed the power and size of the unilateral test that uses the estimated critical values under different alternative models given below, for $a = 0.5$, $b = 0.6$, $c = 1000$ where $e_{t,0}, e_{t,1}, e_{t,2}$ are independent *i.i.d.* sequences of Gaussian *r.v.'s* with mean 0 and variance 1. Table 5 shows the estimated power for the fixed sample sizes $n = 100$, $n = 500$ and $n = 1000$.

*Model 1b* (Linear Cointegration)

$$\begin{aligned} w_t &= w_{t-1} + e_{t,0} \\ x_t &= w_t + e_{t,1} \\ y_t &= a\, w_t + e_{t,2}, \end{aligned}$$

*Model 2a* (Quadratic Cointegration)

$$
\begin{aligned}
w_t =& w_{t-1} + e_{t,0} \\
x_t =& w_t + e_{t,1} \\
y_t =& a\, w_t^2 + e_{t,2},
\end{aligned}
$$

*Model 2b* (Logarithmic Cointegration)

$$
\begin{aligned}
w_t =& w_{t-1} + e_{t,0} \\
x_t =& w_t + e_{t,1} \\
y_t =& a\ \log(w_t + c) + e_{t,2},
\end{aligned}
$$

*Model 2c* (Exponential Cointegration)

$$
\begin{aligned}
w_t =& w_{t-1} + e_{t,0} \\
x_t =& w_t + e_{t,1} \\
y_t =& a\ \exp(w_t/d) + e_{t,2},
\end{aligned}
$$

*Model 2d* (Exponential Cointegration)

$$
\begin{aligned}
w_t =& w_{t-1} + e_{t,0} \\
x_t =& w_t + e_{t,1} \\
y_t =& a\ \exp(w_t) + e_{t,2},
\end{aligned}
$$

*Model 3* ($I(0)$ comoving)

$$
\begin{aligned}
x_t =& b\, x_{t-1} + e_{t,1} \\
y_t =& 2.0\, x_t + e_{t,2},
\end{aligned}
$$

*Model 4* ($I(0)$ Independent)

$$
\begin{aligned}
x_t =& b\, x_{t-1} + e_{t,1} \\
y_t =& 0.8\, y_{t-1} + e_{t,2},
\end{aligned}
$$

*Model 5* ($I(0)$ and $I(1)$ Independent)

$$x_t = b\, x_{t-1} + e_{t,1}$$
$$y_t = y_{t-1} + e_{t,2},$$

| *Model* | *n=100* | *n=500* | *n=1000* |
|---|---|---|---|
| 1 | 0.481 | 0.774 | 0.890 |
| 2a | 0.899 | 0.991 | 0.998 |
| 2b | 0.069 | 0.098 | 0.123 |
| 2c | 0.058 | 0.074 | 0.097 |
| 2d | 0.538 | 0.649 | 0.794 |
| 3 | 0.979 | 0.992 | 0.999 |
| 4 | 0.114 | 0.255 | 0.423 |
| 5 | 0.076 | 0.080 | 0.098 |

TABLE 5. Estimated power for the test based on $\rho_{x,y}^{(n)}$ against different alternatives.

As expected, the results in Table 5 tell us that this test statistic cannot discriminate properly between model 1 (linear cointegration) and model 3 of $I(0)$ linearly comoving series. In fact, it yields significant values whenever the series are $I(0)$, as shown by the fact that the power of the test against pairs of independent $I(0)$ series is much larger than the estimated critical values under the null hypothesis, and increases rapidly with the sample size. Another drawback of this test is its varying power performance when different nonlinearities appear in the relationship. These results come in support of the findings in Park and Phillips (1999) by suggesting that there is no single convergence rate of the test statistic for all nonlinear transformations. A closer a look at Figures 10 and 11 reveal also certain discrepancies with the power results found for model 2*c* of exponential cointegration. These discrepancies arise from the existence of a linear regime and a more nonlinear regime for transformations such as the exponential and the logarithmic. Therefore, for the smaller values of the parameter $d$ in model 2*c*, the exponential transformation behaves approximately as linear, contrary to what happens for values of this parameter close to $d = 100$, as we used for the power estimation. Finally, the nonlinear cointegration experiment was also run for pairs of nonlinearly cointegrated time series with square-integrable transformations such as,

$$g(y_t) = \frac{1}{1+y_t^2} \tag{2.4a}$$

$$g(y_t) = \sin(y_t) \tag{2.4b}$$

$$g(y_t) = \cos(y_t) \tag{2.4c}$$

which are known to have a "stationarising" effect on the transformed series (since the variance of the transformed variable becomes bounded). As expected, in either of these cases, the test has no power.

The previous problems of the correlation range statistic led us to investigate the performances in finite samples of a testing device which combines the information given by the correlation statistic $\rho_{x,y}^{(n)}$ with that provided by the statistic $R_{x,y}^{(n)}$, defined in (2.2). The 5% critical values of the empirical distribution of $R_{x,y}^{(n)}$ were simulated from 1000 Monte Carlo replications of linear cointegration with *i.i.d.* Normally distributed errors, and for different fixed sample sizes ($n$ = 100, $n$ = 500 and $n$ = 1000). Table 6 shows the estimated left ($c_l$) and right ($c_r$) 5% critical values. The power of a test based on $R_{x,y}^{(n)}$ was estimated for the different models considered above, from 1000 replications of each, and for the sample sizes $n = 100, 224, 500, 1000$. For *model 3*, we analysed the power behaviour for different values of the AR(1) coefficient, $b$, going from 0.6 to 0.99. We also studied the power of $R_{x,y}^{(n)}$ against pairs of $I(0)$ monotonically nonlinearly comoving series using the new *model 6* defined below. Again, the critical values of this statistic decrease rapidly with the increasing

| *critical value* | *n=100* | *n=224* | *n=500* | *n=1000* |
|---|---|---|---|---|
| left ($c_l$) | 0.10714 | 0.0798 | 0.05161 | 0.03575 |
| right ($c_r$) | 0.39063 | 0.17740 | 0.15854 | 0.10544 |

TABLE 6. Simulated 5% left critical values and right critical values of the empirical distribution of the test statistic $R_{x,y}^{(n)}$ under the null hypothesis of linear cointegration (model 1).

sample size indicating that the test statistic needs to be corrected by the speed of convergence. For that, we must the asymptotic distribution of this test statistic, which is out of the scope of this paper. Furthermore, following the results in Park and Phillips (1999), the rate of convergence might depend on the particular nonlinear transformation. Therefore, we will restrict the analysis to the performance performance of our test statistic in for several fixed small samples.

*Model 6a* ($I(0)$, Quadratic Comoving)

$$x_t = 0.6x_{t-1} + e_{t,1}$$
$$y_t = 0.5x_t^2 + e_{t,2},$$

*Model 6b* ($I(0)$, Logarithmic Comoving)

$$x_t = 0.6x_{t-1} + e_{t,1}$$
$$y_t = \log(x_t + 1000) + e_{t,2},$$

*Model 6c* ($I(0)$, Exponential Comoving)

$$x_t = 0.6x_{t-1} + e_{t,1}$$
$$y_t = \exp(x_t/100) + e_{t,2},$$

Table 6 shows the estimated left ($c_l$) and right ($c_r$) 5% critical values for fixed sample sizes $n =$ 100, 224, 500, and 1000. The power of a test based on $R_{x,y}^{(n)}$ was estimated for different models from 1000 Monte Carlo replications. For *model 3*, we analysed the power behaviour for different values of the AR(1) coefficient, $b$, going from 0.6 to 0.99. We also studied the power of $R_{x,y}^{(n)}$ against pairs of $I(0)$ monotonically nonlinearly comoving series using *model 6*. The results are shown in Table 7.

The size results obtained for *model 2* (NLC) and the power of *model 6*, corresponding to pairs

| *Model* | *n=100* | *n=500* | *n=1000* |
|---|---|---|---|
| 2a ($< c_l$) | 0.113 | 0.123 | 0.215 |
| 2b ($< c_l$) | 0.018 | 0.032 | 0.053 |
| 2c ($< c_l$) | 0.024 | 0.023 | 0.052 |
| 3 ($b = 0.6$) ($< c_l$) | 0.681 | 0.993 | 1 |
| 3 ($b = 0.9$) ($< c_l$) | 0.597 | 0.982 | 1 |
| 3 ($b = 0.95$) ($< c_l$) | 0.431 | 0.938 | 0.999 |
| 3 ($b = 0.99$) ($< c_l$) | 0.359 | 0.676 | 0.857 |
| 4 ($< c_l$) | 0.098 | 0.805 | 0.975 |
| 6a ($< c_l$) | 0.477 | 0.994 | 1 |
| 6b ($< c_l$) | 0.164 | 0.909 | 0.995 |
| 6c ($< c_l$) | 0.300 | 0.896 | 0.961 |
| 5 ($< c_l$) | 0.015 | 0.013 | 0.048 |
| 0 ($> c_r$) | 0.637 | 0.736 | 0.838 |
| 0 ($< c_l$) | 0.000 | 0.000 | 0.000 |

TABLE 7. Estimated power for the test based on $R_{x,y}^{(n)}$ against different alternatives.

of $I(0)$ monotonically nonlinearly comoving series, reveal a remarkable robustness of the unilateral test that uses the simulated left critical values against monotonic nonlinearities in the relationship. Therefore by using $R_{x,y}^{(n)}$ in combination with $\rho_{x,y}^{(n)}$ (second and first stage of our test, respectively), it may be possible to distinguish between cointegrated (either linear or monotonically nonlinear) and pairs of $I(0)$ series on finite samples. The resulting diagnostics are shown in Table 8.

On the first hand, we remark that when the first test rejects the hypothesis of IRW but the second one maintains its null of linear cointegration, the overall device favours this latter hypothesis. On the other hand, when neither test rejects, the combined procedure cannot decide between a pair of nonlinearly cointegrated series and a pair $I(0)/I(1)$ of series. However, this is not contrary to intuition, since it is possible that a nonlinearity of a very high-order constrains the sample paths of the common $I(1)$ component in such a way that one of the series looks $I(0)$. This is exactly what we observed when square-integrable transformations -such as the ones analysed for $\rho_{x,y}^{(n)}$- define the cointegrating relationship. We finally study the size of the test under deviations from the

| $\rho_{x,y}^{(n)}$ | $R_{x,y}^{(n)}$ | *diagnostic* |
|---|---|---|
| rejects $H_0$ | rejects $H_0$ | SMC |
| rejects $H_0$ | holds $H_0$ | cointegration |
| holds $H_0$ | rejects $H_0$ | independence |
| holds $H_0$ | holds $H_0$ | NLC or pair of $I(0)/I(1)$ |

TABLE 8. Diagnostics when combining the testing procedures based on $\rho_{x,y}^{(n)}$ ($H_0$: IRW) and on $R_{x,y}^{(n)}$ ($H_0$: LC).

assumption of independence in the equilibrium errors. For example, when the errors $e_{t,2}$ in *model 1* (LC) are correlated. For our study, we considered AR(1) and MA(1) time series dependencies in the equilibrium errors. The autoregressive parameter $d$ was allowed to take the values $0.6, 0.9, 0.95$ and $0.99$ while the moving average parameter $\theta$ took the values $-0.8, -0.5, 0.5$, and, $0.8$. Formally, the models analysed were the following:

$$w_t = w_{t-1} + e_{t,0} \tag{2.5a}$$

$$x_t = w_t + e_{t,1} \tag{2.5b}$$

$$y_t = a\, w_t + e_{t,2} \tag{2.5c}$$

$$e_{t,2} = \varphi\, e_{t-1,2} + \epsilon_t \tag{2.5d}$$

$$e_{t,2} = \epsilon_t + \theta \epsilon_{t-1} \tag{2.5d'}$$

where $e_{t,0}, e_{t,1}, \epsilon_t$ are independent *i.i.d.* sequences of Gaussian *r.v.'s*. The results obtained in Tables 9 and 10 show that the size of $R_{x,y}^{(n)}$ (using the critical values from the left tail of its null distribution) against linear correlation structure in the errors has consistent values for moderate sample sizes.

| $\varphi\backslash n$ | *n=100* | *n=500* | *n=1000* |
|---|---|---|---|
| 0.6 | 0.12195 | 0.05353 | 0.05122 |
| 0.9 | 0.14286 | 0.06318 | 0.05421 |
| 0.95 | 0.15000 | 0.06652 | 0.04879 |
| 0.99 | 0.16883 | 0.07860 | 0.06354 |

TABLE 9. Size of the test based on $R_{x,y}^{(n)}$ when the equilibrium errors follow an AR(1) time series model.

| $\theta\backslash n$ | $n$=100 | $n$=500 | $n$=1000 |
|---|---|---|---|
| -0.8 | 0.034 | 0.046 | 0.037 |
| -0.5 | 0.032 | 0.053 | 0.032 |
| 0.5 | 0.020 | 0.044 | 0.032 |
| 0.8 | 0.021 | 0.037 | 0.026 |

TABLE 10. Size of the test based on $R_{x,y}^{(n)}$ when the equilibrium errors follow an MA(1) time series model.

## 3. Empirical applications

This section deals with some empirical case studies. We analyse different sets of financial time series to search for cointegration by using the proposed combination of statistics based on ranges. First, we consider three pairs of exchange-rates series and, finally, we analyse the relationship between the prices of gold and silver.

3.1. **Analysis of some exchange-rates series.** Consider the three exchange-rate series shown in Figure 12. These series represent the exchange rates of the US Dollar (EXRPD), the Deutsch Mark (EXRPM) and the Japanese Yen (EXRPY) (in units of 100 yens) against the Spanish Peseta. For this analysis, we used only the first 1000 daily observations from the series, starting from January the 1st. 1987.

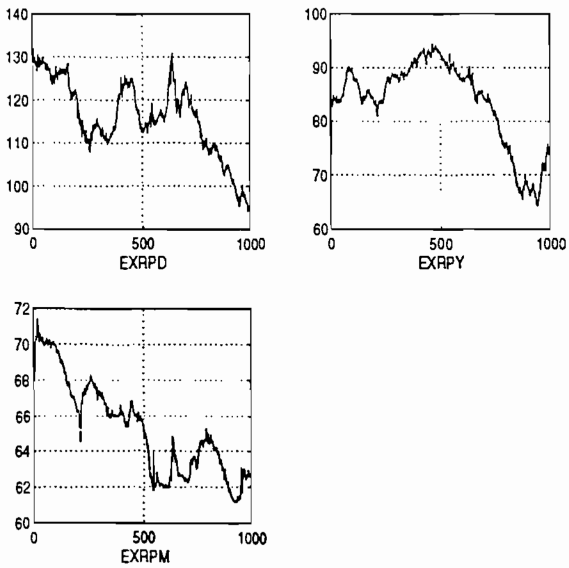



FIGURE 12. Daily foreign exchange-rate series from January 1987: EXRPD (Peseta/US Dollar), EXRPY (Peseta/100 Yens), EXRPM (Peseta/Deutsch Mark).

First of all, Dickey-Fuller tests allowing for an intercept and a trend was applied to our series. Using the critical values for $t_0$ given in Hamilton (1994), the unit root hypothesis could not be rejected at the 5% significance level for any of the three series, suggesting that they are $I(1)$.

Next, we run an *Augmented Dickey-Fuller* (ADF) test (the conventional DF test was augmented with one or three lags of the first differences of the series) on the cointegrating regression residuals of each of the three pairs of series. The values obtained for the $t_0$ test statistic are shown in Table 11. Using the 5% critical value of $t_0$ from Mackinnon (1991), we could not reject the null hypothesis

| *EXRPY/EXRPM* | *EXRPY/EXRPD* | *EXRPM/EXRPD* |
|---|---|---|
| -0.71 | -1.96 | -1.48 |

TABLE 11. Values of the ADF test applied the the levels of the three pairs of exchange rate series.

of no linear cointegration except for the pair (EXRPY,EXRPD), since the 5% critical value is -1.93.

The range sequences for the three pairs of series are shown superimposed on each other in Figures 13, 14 and 15, while their crossplots appear in Figure 20.

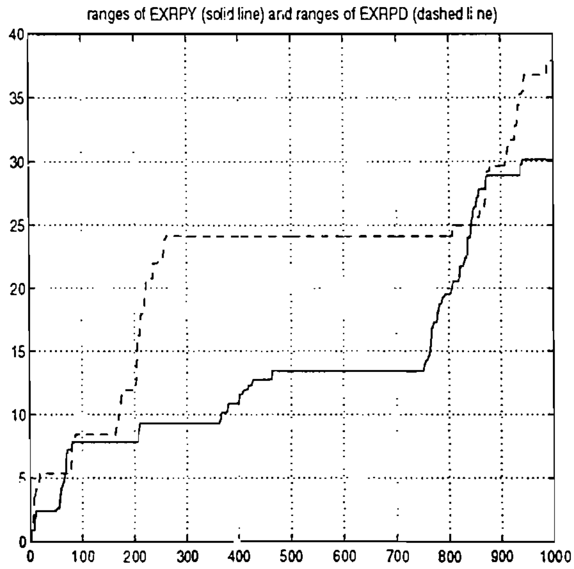



FIGURE 13. Range sequences for the pair (EXRPY,EXRPD).

The jump sequences are shown in Figures 18, 17 and 19 for the pairs (EXRPY,EXRPM), (EXRPY,EXRPD), and (EXRPM,EXRPD), respectively, and the corresponding crossplots appear in Figure 16.

As we pointed in a previous section, the relative way in which the jumps cluster along the horizontal axis in these plots tells us how likely it is the cointegration hypothesis. In our case, the evidence

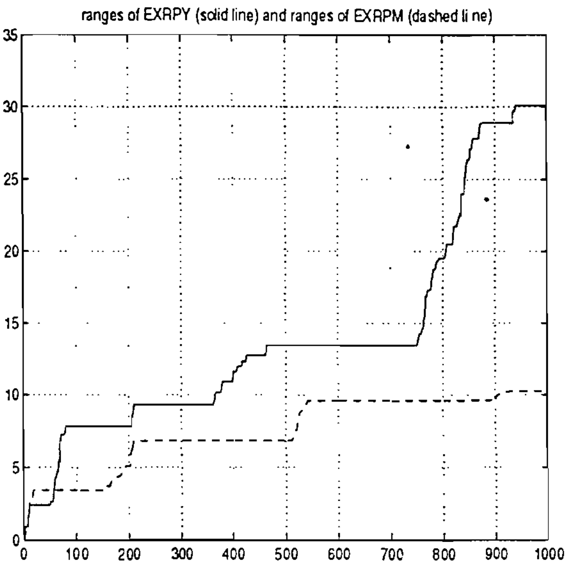



FIGURE 14. Range sequences for the pair (EXRPY,EXRPM).

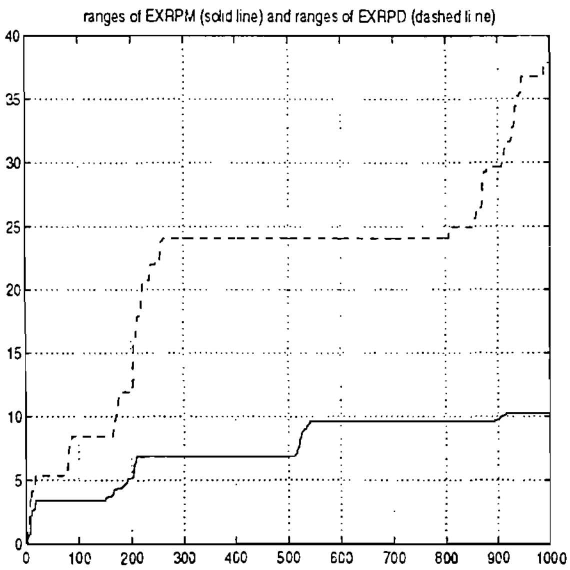



FIGURE 15. Range sequences for the pair (EXRPM,EXRPD).

of cointegration is comparatively weak for the pair (EXRPY,EXRPM), since no synchronicity (not even up to a time delay constant) between the two sets of arrival times can be observed in the figures (the two jump series have almost no overlapping support except around $t = 0$ and $t = 200$). For the pairs (EXRPY,EXRPD) and (EXRPM,EXRPD), most of the jumps in the two sets are synchronous or close to it. However, at some time spells, some jumps appear for one series while

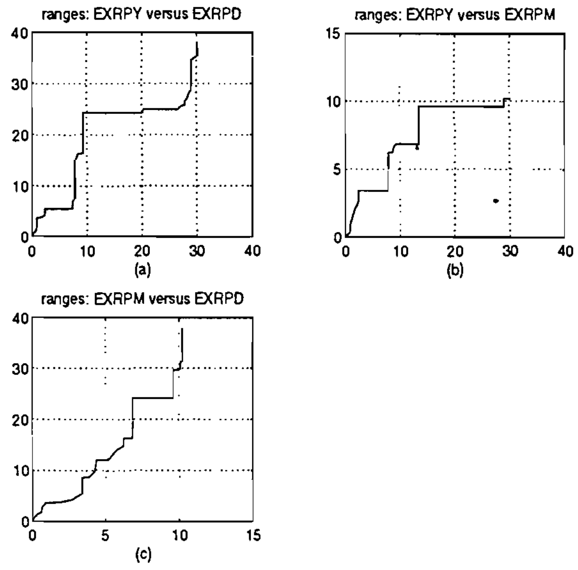



FIGURE 16. Crossplot of the range sequences for the three pair of series.

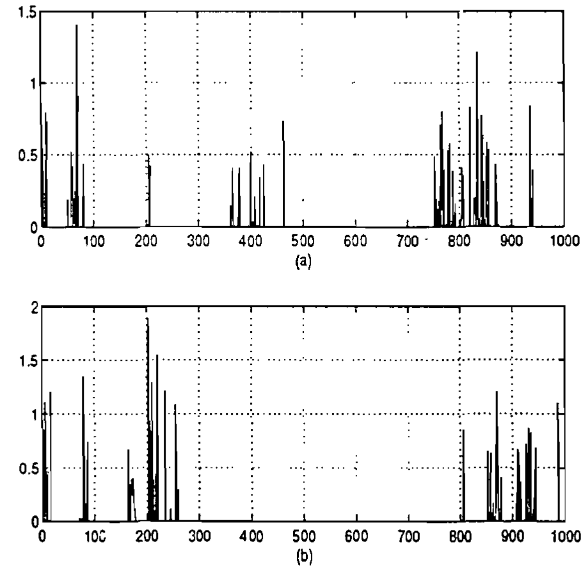



FIGURE 17. Jump sequence for the pair (EXRPY,EXRPD).

not for the other, thus suggesting that cointegration may hold in either in a nonlinear or a non constant way.

The values obtained with the range correlation statistic $\rho_{x,y}^{(n)}$, using the critical values obtained at the 5% significance level and for a sample size of $n = 1000$, are given in Table 12. From the range correlation statistic we get that only EXRPD and EXRPM are cointegrated since 0.23 is larger than 0.167. Also, Table 13 shows the values taken by the complementary range test statistic $R_{x,y}^{(n)}$. The results of Table 13 support the hypothesis that the series are cointegrated since in all of

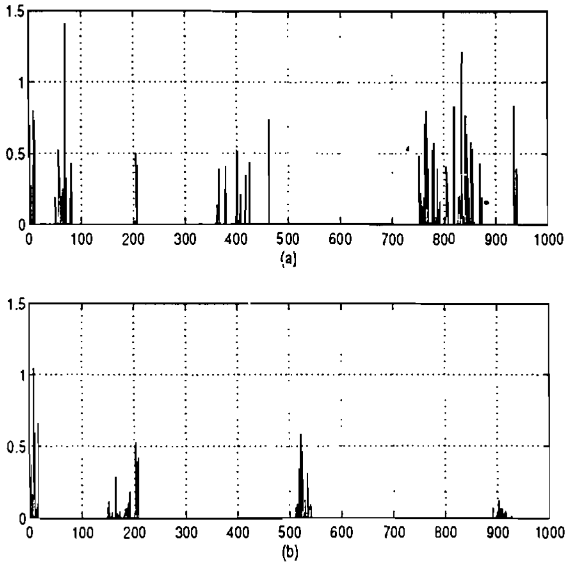



FIGURE 18. Jump sequence for the pair (EXRPY,EXRPM).

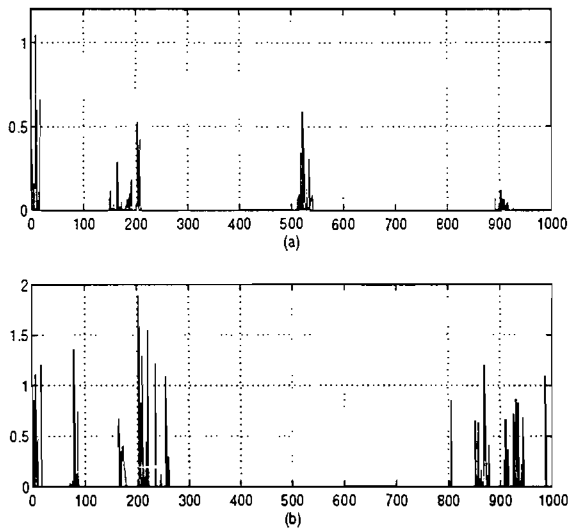
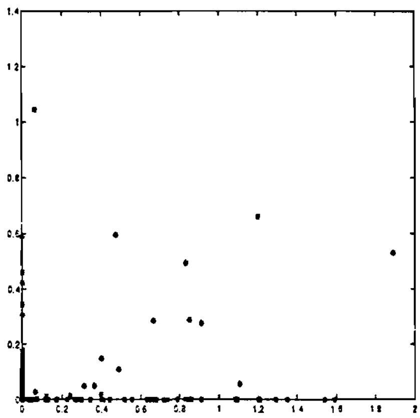


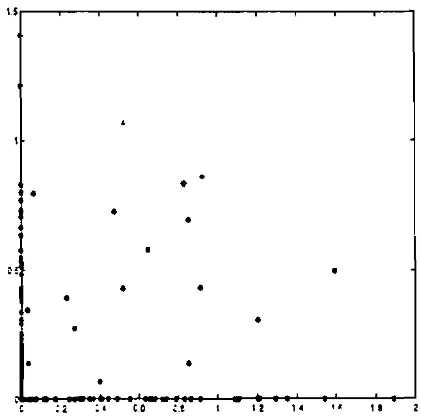

FIGURE 19. Jump sequence for the pair (EXRPM,EXRPD).

the cases the value of $R_{x,y}^{(n)}$ is larger that the 5% C.V.=0.035. Therefore, our analysis contradicts the results obtained in the unit root analysis based on DF tests. One possible explanation is the existence of level shifts or nonlinearities in the cointegration relationship.

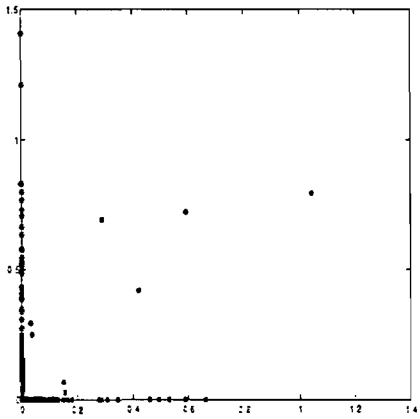
3.2. **Analysis of gold and silver prices.** Gold and silver have been actively traded for thousands of years and remain important closely observed markets. Here, following Escribano and Granger (1998), monthly prices are analysed since the end of 1971 until 1996. We are interested in testing the



(a) EXRPY vs. EXRPM



(b) EXRPY vs. EXRPD



(c) EXRPM vs. EXRPD

FIGURE 20. Crossplots of the jump sequences.

| *EXRPY/EXRPM* | *EXRPY/EXRPD* | *EXRPM/EXRPD* |
|---|---|---|
| 0.037 | 0.152 | 0.231 |

TABLE 12. Values taken by $\rho_{x,y}^{(n)}$ on the three pairs of exchange-rate time series, for $n = 1000$. 5% C.V.= 0.167

| *EXRPY/EXRPM* | *EXRPY/EXRPD* | *EXRPM/EXRPD* |
|---|---|---|
| 0.080 | 0.074 | 0.075 |

TABLE 13. Values taken by $R_{x,y}^{(n)}$ on the four pairs of financial time series, for $n = 1000$. 5% C.V.= 0.035

existence of contemporaneous relationships between the prices of the two commodities in log levels. The economic interpretation of the situation is not simple, since gold and silver have both distinct commercial uses for which there are no substitutes, thus suggesting that the two markets should be separated. Nevertheless, they do act elsewhere as quite close substitutes, as jewelry investments are often used to reduce certain types of risks (for instance, high inflation risks) in portfolios. These prices are determined in clearly speculative markets and therefore their behaviour should be captured by unit-root time series models. The unit root is supported by the Dickey–Fuller type tests using from 1 to 6 lags of the dependent variable and including constant and constant and trend variables in the regression equation.

There is, however, a feature of this data that makes it particularly interesting, which is the widely known and well-documented *bubble* in silver prices from roughly June 1979 to March 1980. A plot of the evolution of these prices in logarithms is shown in Figure 21, together with a crossplot of these series which shows the structural splitting in the relationship at the time of the bubble. It is seen that both gold and silver prices do increase considerably during and around the bubble, but the shift in gold price is less remarkable.

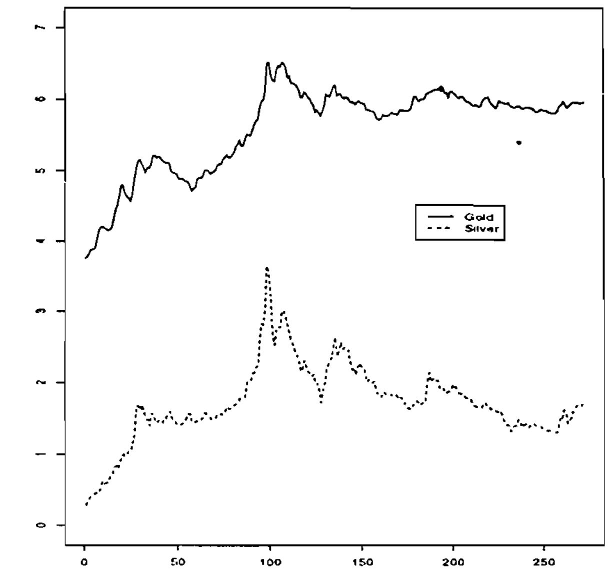
The final objective now is to investigate the effect of the bubble on testing for the existence of a long-run relationship (cointegration). The results in Escribano and Granger (1998) support several empirical features:

1. Evidence of integration ($I(1)$) appears in the whole sample, both in levels and in the logarithms of the series.
2. The intercept-dummies (level shifts in the intercept. introduced to explain the bubble period and its impact on the post-bubble period) greatly strengthen the evidence of cointegration.

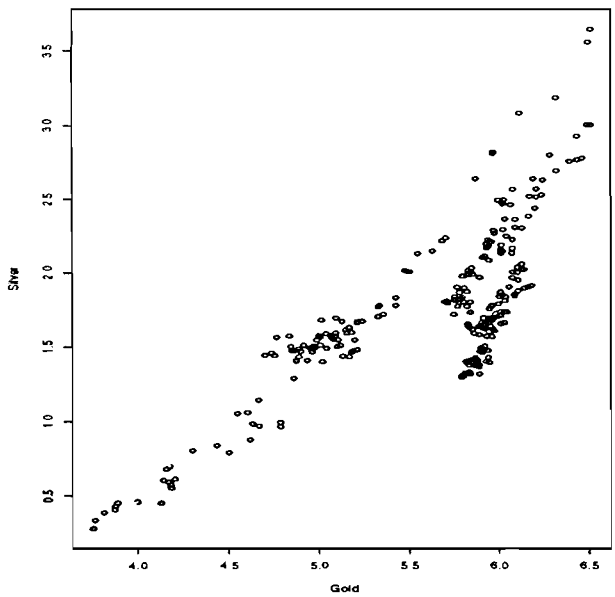
When testing the null hypothesis using the DF test on the residuals (see, Engle and Granger (1987)) with one lag we get $t_0 = -2.23$ and therefore we cannot reject $H_0$, at the 5% significance since the critical value is -3.7. However, if we consider the two level shifts that occur in the cointegration relationship we get $t_0 = -4.54$, which rejects the null of no cointegration[2].

Testing for cointegration between the log prices of gold and silver with statistics based on the ranges is interesting since those procedures do not require prior estimation of the cointegrating relationship and could indicate departures from linearity. On the available data sample (of length $n = 224$), the values obtained for $\rho_{x,y}^{(n)}$ and for $R_{x,y}^{(n)}$ (with $x_t, y_t$ representing now the logarithms of the

[2]There is a large literature on the effects of structural breaks on unit root and cointegration tests. See, for example, the references in Arranz and Escribano (2000).




(a)




(b)

FIGURE 21. Logarithms of the gold and silver prices and crossplots of the series

gold and the silver price series) were 0.98 (5% C.V.=0.27) and 0.095 (5% C.V.=0.08), respectively. These findings support the rejection of $H_0$ of independent random walks and non-rejection of the linear cointegration hypothesis (between the log prices) respectively at the 5% significance level, and therefore the price series are cointegrated in logarithms.

## 4. Conclusion

In this paper we propose using some linear functions of the order statistics from a pair of series as a starting point for obtaining a model free bivariate cointegration testing device for finite samples. The plots of the sequences of first differences of the *ranges* suggest a new characterisation of the cointegration property where the relevant feature is the synchronicity of the arrival times of what we call here *relevant informational events.* That is, a pair of series will be non-cointegrated when the sequences of first differences of ranges (the so-called *jump sequences*) have orthogonal supports, thereby implying a lack of synchronicity between the two sets of jumps. The comparison of the behaviour of these jump sequences led us to propose two complementary test statistics that, when used in combination, can help us in discriminating between the cases of cointegration, independence, comovements, and pairs of series with different memory properties.

The analysis of the asymptotic properties of these statistics is beyond the scope of this paper. However, our analysis on finite samples suggest that the proposed methodology is robust to different departures from the classical linear cointegration hypothesis with uncorrelated cointegration errors.

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