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Juan Angel Lafuente

00-05

WORKING PAPERS

Universidad Carlos III de Madrid

Working Paper 00-5
Business Economics Series 5
January 2000

Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 6249608

OPTIMAL HEDGING UNDER DEPARTURES FROM THE COST-OF-CARRY VALUATION: EVIDENCE FROM THE SPANISH STOCK INDEX FUTURES MARKET

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Abstract

This paper provides an analytical discussion of the optimal hedge ratio when discrepancies between the futures trading price and its theoretical valuation according to the cost-of-carry model occurs. Under the assumption of a geometric Brownian motion for spot prices we model the mispricing by a new specific noise in the theoretical dynamic of futures market. Empirical evidence above the model is provided for the Spanish stock index futures. Ex-post simulations reveal that hedging effectiveness applying the estimated ratio is similar to the achieved with a systematic unitary hedge ratio, the optimal one when a mispricing does not appear. However, a small number of futures contracts is needed.

Key words: Hedge; Futures; Stock Index; GARCH; Mispricing.

JEL classification: C51, G11, G13.

*Universidad Carlos III de Madrid. Dept. Economía de la Empresa, C/ Madrid, 126, 28903 Getafe (Madrid), Spain. E-mail: jlafuent@emp.uc3m.es.

The autor is indebted to Alfonso Novales for many helpful comments and suggestions. Any remaining errors are my responsability.

1 Introduction

The Spanish stock index futures markets has become one of the fastest growing emerging futures markets in the world. Since its beginning at January 1992, the Ibex 35 futures contract is the highest trading activity in *Meff Renta Variable.* The acceptance of financial stock index futures contract is related with the hedging ability of this derivative instrument. In particular, through stock index futures market operations not it is not only possible to guarantee the profits obtained during a time period, but also to bound the losses on a given time period. These possibilities become especially relevant in the Spanish economy during the recent years, because the systematic decrease in interest rates as a consequence of the fiscal and monetary policies directed to achieve the European Union, caused a reallocation of private savings from riskless assets to stock exchange positions.

The relevant issue in a hedge operation is to determine the hedge ratio, which ratio provides the number of contracts that must be sold to counteract the opposite evolution in the spot prices, so that, the gains in one market must be offset by the losses in other. A biased estimation of the hedge ratio implies that the losses in one market are higher or lower than the profits in the other one. For example, if a hedger tries to anticipate a fall in the spot market applying a ratio with a negative bias, then the losses in the spot market can not be fully offset by the gains in the derivative market; contrary. if the spot index finally increases the return of the global position will be positive. This question is troublesome for a hedging strategy, because the aim of a hedge operation is to *convert* a position in the spot market in a riskless portfolio.

According to the cost-of-carry valuation (the most used forward pricing model), which assumes perfect markets and non stochastic interest rates and dividend yields, the theoretical price at time t of an index futures contract which matures at time T equals the opportunity cost of keeping a basket replicating the spot index from t to T, that is:

$$F^{*}_{t,T} = S_t \, e^{(r-d)\,(T-t)}, \tag{1}$$

where $F_{t,T}$ is the futures price, S_t is the index value and $(r-d)$ is the net cost of carry associated to the underlying stocks in the index, i.e., the riskless rate of return minus the dividends yield of the stocks in the index. Alternatively, the equation (1) can be expressed:

$$r_{s,t} = r_{f^*,t} + (r - d), \tag{2}$$

where $r_{s,t} = \ln\left(\frac{S_t}{S_{t-1}}\right)$ and $r_{f^*,t} = \ln\left(\frac{F^*_{t,T}}{F^*_{t-1,T}}\right)$, the spot and theoretical futures returns, respectively. Undeer the previous assumptions, the relationship in (2) implies that: a) the variance of return in the spot market equals the variance of return in the futures market, b) the contemporaneous rates of return of the underlying stock index and the futures contract are perfectly and positively correlated, and c) the non-contemporaneous rates of return are uncorrelated and no lead-lag relationship should appear. However, in the presence of market imperfections such as transactions costs, asymmetric information, capital requirements and short-selling restrictions there could be discrepancies between the traded futures price and its theoretical valuation according to the cost-of-carry model (see Mackinlay and Ramaswamy (1988), Lim (1992), Miller et al. (1994), Yadav and Pope (1990, 1994), and Büller and Kempf (1995), among others).

On the other hand, under market imperfections there may be a lead-lag relationship between spot and futures returns, as well as between volatilities. This way, there is a wealth of studies showing empirical evidence for the main stock index futures markets supporting the existence of lead-lag relationship between spot and futures returns, as well as between volatilities (see, for example, Stoll and Whaley (1990), Wahab and Lasghari (1997), Pizzi et al. (1998), Iihara et al. (1996), Koutmos and Tucker (1996), and Racine and Ackert (1998), among others). Under lead-lag relationships, it is possible to anticipate price movements and the risk level in one market from past information in the other market, a relevant question when using the futures contract as a hedge instrument for risky stock portfolios.

In this paper we propose a simple two period model with time changing optimal hedge ratios in which the presence of an arbitrage spread is taken into account. Assuming that spot prices evolve according with to a geometric brownian motion, we model mispricing by introducing a specific noise in the dynamics of the theoretical futures returns. The model shows that perfect positive correlation between spot and futures returns, which occurs when no specific noise in the futures market is taken into account, leads to a optimal hedge ratio equal to one. When return dynamics does not share a same noise, then even under perfect correlation between the common noise and the specific disturbance for the derivative market, the ratio will not be equal to one.

We also provide empirical evidence for this model for the period covering 20/12/93 to 20/12/96 in the Spanish stock index futures market. A bivariate error correction model with GARCH perturbations is used for estimate the conditional second moments of the market returns. Our model has the following characteristics: a) it incorporates the long-run equilibrium relationship between spot and futures prices, b) it takes into account the cross-markets interactions between returns and volatilities, c) it does not impose a constant conditional correlation coefficient in the matrix of second moments market returns, a significant difference with most of previous analysis (Park and Switzer (1995), Iihara et al. (1996), Koutmos and Tucker (1996), Racine and Ackert (1998), and Lien and Tse (1999), among others), and d) it captures the presence of an intraday U-shaped seasonal pattern for both spot and futures market volatility. Our modelization, representation and technique estimation allows to capture stochastically this intraday seasonal pattern for market volatilities, rather than trough deterministic variables, the most often way in the literature. We estimate taking hourly returns using the nearest to maturity contract. We recover estimates for the parameters of the theoretical model.

The empirical findings suggests that the derivative market has a specific noise, that is, that spot and futures market do not share a identical source of volatility, as a difference of the implied result under the cost of carry model. However, its correlation with the common market disturbance is small.

The simulations ex-post reveal that, when the estimated hedge ratio takes into account the presence of a specific noise in the futures market, the hedging effectiveness, in terms of variance reduction of the global return position, is similar to the one achieved with a constant unitary hedge ratio. However, a lower number of futures contracts is needed, which yields in an lower transaction costs. On the other hand, no significant improvement in hedging effectiveness is detected when the correlation between specific and common noise is explicitly taken into account.

The rest of the article is organized as follows. In the next section the optimal hedge ratio under departures from the cost of carry valuation is derived. Section 3 presents the econometric approach to estimate conditional second order moments for market returns, and also how to recover estimates for the theoretical parameters of the model. Section 4, we make ex-post simulations to investigate if taking into account departures from the theoretical cost-of-carry valuation, enhaces the hedging effectiveness of the futures contract.

Finally, section 5 summarizes and shows concluding remarks.

2 The Model

Let us assume that spot prices evolve according to a geometric brownian motion:

$$dS_t = \mu_{s,t} S_t dt + \sigma_{s,t} S_t dz_{1,t}, \tag{3}$$

where S_t is the index value, $\mu_{s,t}$ and $\sigma_{s,t}$ are the conditional mean and standard deviation of spot returns, and $dz_{1,t} = \varepsilon_{1t}\sqrt{dt}$, with ε_{1t} $i.i.d.\tilde{} N(0,1)$, a Wiener process. Taking into account the no arbitrage relationship between spot and futures prices (equation 1), the process for the evolution of the theoretical price of a futures contract can be obtained applying Îto's lemma:

$$dF^*_{t,T} = \mu_{f,t} F^*_{t,T} dt + \sigma_{s,t} F^*_{t,T} dz_{1,t}, \tag{4}$$

where $\mu_{f,t} = \mu_{s,t} - (r - d)$. Under perfect markets, the no arbitrage equilibrium relationship always holds, and the volatility of spot and futures returns must be the same. However, there is a wealth of studies showing systematic discrepancies between the traded futures price and its theoretical price according to the cost-of-carry valuation. In this situation, equation (4) might not be adequate to represent dynamic evolution of the traded futures price. We model such discrepancy by introducing a new specific noise in the derivative market:

$$dF_{t,T} = \mu_{f,t} F_{t,T} dt + \sigma_{s,t} F_{t,T} dz_{1,t} + \sigma_{N,t} F_{t,T} dz_{2,t}, \tag{5}$$

where $F_{t,T}$ is the traded futures price and $dz_{2,t} = \varepsilon_{2t}\sqrt{dt}$, with ε_{2t} $i.i.d.\tilde{} N(0,1)$. We do not impose any restriction on the correlation between the common or *general* noise (ε_{1t}) and the specific disturbance for the futures market (ε_{2t}), which we denote by $\rho_{12,t}$. Under equation (5) market returns do not necessarily exhibit perfect, positive and constant correlation. From equations (3) and (5) this correlation coefficient can be expressed:

$$\rho_{sf,t} = \frac{Cov_t\left(\frac{dS_t}{S_t dt}, \frac{dF_{t,T}}{F_{t,T} dt}\right)}{\sqrt{\left[Var_t\left(\frac{dS_t}{S_t dt}\right)\right]\left[Var_t\left(\frac{dF_{t,T}}{F_{t,T} dt}\right)\right]}} = \frac{\sigma^2_{s,t} + \rho_{12,t}\sigma_{s,t}\sigma_{N,t}}{\sqrt{\sigma^2_{s,t}\left(\sigma^2_{s,t} + \sigma^2_{N,t} + 2\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right)}} \tag{6}$$

Only wen $\sigma_{N,t} = 0$, that is, when the two markets share a identical noise, a perfect and positive correlation between market returns is observed.

It is assumed that the hedger holds a long spot position and intends to short futures to minimize the variance of the return for the hedged position over a certain temporal horizon. The hedge ratio is defined as the number of monetary units which must be allocated in a short futures position per monetary unit invested in the cash market. In a two period context the investor's hedging decision is to choose a hedge ratio that solves the following problem:

$$\begin{array}{ll} & Min\, Var_t \left(b_t \frac{dS_t}{S_t dt} - h_t \frac{dF_{t,T}}{F_{t,T} dt} \right) \\ & \{h_t\} \\ s.t. & \begin{array}{l} dS_t = \mu_{s,t} S_t dt + \sigma_{s,t} S_t dz_{1,t} \\ dF_{t,T} = \mu_{f,t} F_{t,T} dt + \sigma_{s,t} F_{t,T} dz_{1,t} + \sigma_{N,t} F_{t,T} dz_{2,t} \end{array} \end{array} \tag{7}$$

The first order condition leads to the following hedge ratio (see appendix 1):

$$\frac{h_t^*}{b_t} = \frac{\sigma_{s,t}^2 + \rho_{12,t}\sigma_{s,t}\sigma_{N,t}}{\sigma_{s,t}^2 + \sigma_{N,t}^2 + 2\rho_{12,t}\sigma_{s,t}\sigma_{N,t}}. \tag{8}$$

The second order condition ensures that the previous hedge ratio is optimun in order to minimization, since:

$$\frac{\partial^2 Var_t (h_t)}{\partial h_t^2} = 2\left(\sigma_{s,t}^2 + \sigma_{N,t}^2 + 2\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right) = 2Var_t\left(\frac{dF_{t,T}}{F_{t,T}dt}\right) > 0. \tag{9}$$

Proposition 1: The optimal hedge ratio is a function both of the relative proportion between the specific and common disturbance as well as of the conditional correlation between both noises.
Proof: Denominating $\delta_t = \frac{\sigma_{N,t}}{\sigma_{s,t}}$, equation (8) can be expressed:

$$\frac{h_t^*}{b_t} = \frac{1 + \rho_{12,t}\delta_t}{1 + \delta_t^2 + 2\rho_{12,t}\delta_t}. \tag{10}$$

The optimal solution is not determined only when $\rho_{12,t} = -1$ and $\delta_t = 1$. On this point of the parametric region the objective function always equals the variance of the unhedged position, regardless the applied ratio. To better understand the discontinuity of the function, let us to express the objective function as follows:

$$b_t\sigma^2_{s,t}\left[1+\left(\frac{h_t}{b_t}\right)^2\left(1+\delta_t^2+2\rho_{12,t}\delta_t\right)-2\left(\frac{h_t}{b_t}\right)\left(1+\rho_{12,t}\delta_t\right)\right]. \qquad (11)$$

When $\rho_{12,t} = -1$ and $\delta_t = 1$, them $1+\delta_t^2+2\rho_{12,t}\delta_t = 1+\rho_{12,t}\delta_t = 0$. In this case, the hedger has no choice since whatever the hedge ratio be, no reduction in the variance of the global position is achieved. When this situation arises, the optimal hedge ratio should be equal to zero.

Proposition 2: The optimal hedge ratio is lower than 1 under uncorrelation between the specific and common disturbance.

Proof: From equation (10), imposing $\rho_{12,t} = 0$ yields an optimal hedge ratio $\frac{h_t^*}{b_t} = \frac{1}{1+\delta_t^2}$, which is lower than one because $\delta_t^2 > 0$. The absence of correlation between the specific noise for the futures market and the *general* market noise is consistent with a scenario in which the futures markets is systematically more volatile than the spot market. Consequently, to cover a fluctuaction in the spot index level we need to sell futures in a lower proportion relative to the allocated resources in the long spot position.

Proposition 3: If $0 < \rho_{12,t} \leq 1$, the optimal hedge ratio is less than one.

Proof: When $\rho_{12,t} \in (0,1]$ the inequality $1+\rho_{12,t}\delta_t < 1+\delta_t^2+2\rho_{12,t}\delta_t$ always hold.

If both disturbances are positively correlated, the derivative market fluctuates with more intensity that the spot market, so that a less proportional amount of money is necessary to allocate in a short futures position for hedge the spot position.

Proposition 4: If $\delta_t = \frac{\sigma_{N,t}}{\sigma_{s,t}} \to 0$ the optimal hedge ratio converges to one, the optimal ratio when no discrepancies between the traded price of a futures contract and its valuation according to cost-of-carry model arise.

Proof: It is obvious from equation (11) that $\frac{h_t^*}{b_t} \to 1$ when $\delta_t = \frac{\sigma_{N,t}}{\sigma_{s,t}} \to 0$.

When the relative importance of specific noise in derivative market relative to the *general* market noise is negligible, the situation is similar to an scenario in where both markets share a common noise, so that the optimal hedge ratio is closely to the expected one under no departures from the cost of carry valuation.

When a negative relationship between both disturbances occurs the optimal hedge ratio depends on the relative market noise ratio. For example, if the correlation is perfect and negative, the optimal hedge ratio is $\frac{1}{1-\delta_t}$, wich is greater than one because $\delta_t > 0$.

Therefore, as expected, the incorporation of departures from the cost of carry valuation of the the futures contract enriches the hedging analysis. The model suggests that, if the futures markets has a specific noise and, consequently the spot and derivative market do not share an identical disturbance, the optimal "short futures position" requires a less proportional allocation relative to long spot position when the noises in both markets are uncorrelated or positively correlated. On the other hand, under a negative correlation between the specific and *general* noise, the optimal hedge ratio per unit long spot position might be above or below that unitary ratio, depending on the market noise ratio. Figures 1 and 2 (appendix 2) show the optimal hedge ratio as a function of one parameter (correlation or ratio between market noises) when fixing the other one. Figure 1 shows that, given a correlation between the noises, the optimal hedge ratio depends positively on the market noise ratio. On the other hand, Figure 2 shows that the lower absolute correlation be, a less ability to reduce variance of return of the hedged position. Indeed, Figure 2 reveals that under negative correlation there is no ability to reduce spot market volatility for a certain parametric interval of the market noise ratio. When this situation arise the optimal ratio is equal to zero.

3 Empirical Evidence

In this section we provide empirical evidence about the model proposed in the previous section for the period covering 20/12/93 to 20/12/96. The data used in this study are provided by MEFF RV (*Mercado Español de Futuros, Renta Variable*) for the period December 15, 1993-December 15, 1996. This period is interesting in three ways: a) By December 1993 the initial exponential growth of the Spanish stock index futures market had already ended, becoming a highly liquid market; b) along the period 1994-1997 the number of contracts negotiated stabilized around three millions per year (indeed the multiplier of the futures contract has show no change along this period, staying at 100 *ptas.* per basic point), and c) the sample period cover three different behaviors for the Spanish spot market, that can be summarized as follows: during the year 1994 the market basket value registered an annual lose proxy to the 7%; the year 1995 is characterized by high level price fluctuations, and finally the year 1996 has shown a systematic growth in the Ibex 35, being the annual yield proxy to the 40%. This way, the period analyzed can be considered as a representative sample of the market behavior.

The first set of data contains information concerning with the futures contract on *the* Ibex 35 index, that is: a) trading price, b) transaction hour, c) bid price, d) ask price, and e) accumulated negotiated volume (millions of *ptas.*) until the registered transaction. The second set of data consists of the minute by minute Ibex 35 index level, as well as the traded volume corresponding the 35 shares on the index. Since the nearest to maturity contract is systematically the most actively traded, only data for the nearby futures contracts is used. Therefore, we handle 36 futures contract along the sample.

Since the opening cash index is reflecting closing spot prices from the previous day, we remove the first hour trading interval for spot market[1]. Then, from 11:00 hours to 17:00 hours we select hourly market prices. Therefore, we have seven observations for spot and futures prices for each trading day.

An important source of bias to estimate second order conditional moments of spot and futures returns is the use of non-synchronous data. We remove this possibility by matching each futures price with the cash index value observed at the same minute. This way, we have two perfectly matched hourly price series. From hourly prices we generate the percentage return series for each market by taking the first difference of the natural logarithm of prices and after multiplying by 100, resulting seven hourly returns including the overnight and weekend returns. Finally we exclude overnight and weekend returns because they are measured over a longer time period. Consequently, the data set used in the analysis has six hourly returns per day. The number of trading days is equal to 743[2]. Overall, we have 4,458 return observations for each market.

Tables 1 to 3 present descriptive statistics for intraday hourly returns, as well as for the squared returns, in both markets. Table 1 shows the mean, standard deviation, skewness, kurtosis and autocorrelation functions for spot and futures market returns. As expected, the mean is similar in both

[1]The futures markety opnes at 10:45^{AM}. For the period December 1993 to November 1994, Fernández and Yzaguirre (1996) show that the 35 assets integrating the Ibex 35 are first negotiated, in avergage, after the 11:00^{AM}.

[2]We can not include data from: a) 02/14/95, b) 12/27/96, c) 05/27/96 and 07/29/96 because they were not avalaible from *Meff Renta Variable.*

markets, and the null hypothesis of a zero mean is not rejected in either case. There is a slight negative skewness in both markets, being more significant in the cash market, and empirical distributions of both intraday market returns show heavy tails, compared with the Normal distribution. The central cluster is sharper in the spot market. Both return series exhibit positive first order autocorrelation, that is, for each market the observed return in the previous hour anticipates a return of identical sign. However, only the first order autocorrelation coefficient for spot market is significant at the 5% level. This is consistent with the argument that infrequent trading of stocks in the index portfolio causes a larger inertia in the stock index (see, for example, Miller et al. (1994)).

Autocorrelation coefficients for the squared intraday returns are displayed in Table 2. Estimated coefficients for this function slowly decrease to zero revealing the existence of non-linear dependence in the return series, both in the spot and futures markets. Therefore, to analyze the intraday causal relationship between spot and futures markets the methodology representing the dynamics of market returns must take into account higher order dependence, possibly as a result of changing volatility over time. Interestingly enough, we observe that estimated coefficients for lags multiple of six are systematically positive and significant, being much higher than the rest. This structure suggests an intraday seasonal pattern in volatility in both markets, that is, the risk in the *opening* and *close* trading intervals is higher than in other trading periods.

3.1 Estimation of the conditional second moments of market returns

To estimate the conditional variance-covariance matrix of spot and futures returns, the previous analysis of autocorrelations and cross-correlations functions suggest that to represent the dynamics of intraday returns in both the spot and futures market a model should be used capturing a) the intermarket dependence between returns, b) the cross-interactions between volatilities, and c) the presence of an intraday seasonal pattern in spot and futures volatility.

We use an error correction[3] model with GARCH perturbations. Let $r_{s,t}$

[3]We test the cointegration hypothesis trough three test proposed in Engle and Granger (1987). The first contrast applies the Augmented Dickey-Fuller (1979) over the residu-

and $r_{f,t}$ the market returns, that is, $r_{s,t} = s_t - s_{t-1}$, and $r_{f,t} = f_t - f_{t-1}$, where s_t and f_t denote the logarithm of spot index and trading futures price respectively. The dynamics governing intraday market returns is described by the following equations:

$$\begin{pmatrix} r_{s,t} \\ r_{f,t} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} r_{s,t-1} \\ r_{f,t-1} \end{pmatrix} + \begin{pmatrix} \beta_s \\ \beta_f \end{pmatrix} (s_{t-1} - (\gamma_1 + \gamma_2 f_{t-1})) + \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix}, \quad (12)$$

with ε_t, the disturbance vector of innovations having a conditional distribution: $\varepsilon_t = \begin{pmatrix} \varepsilon_{s,t} & \varepsilon_{f,t} \end{pmatrix}' | \Omega_{t-1} \sim N(0, \Sigma_t)$, where Ω_{t-1} is the information set available at time $t-1$ and Σ_t is the conditional covariance matrix of returns. We include $(s_{t-1} - (\gamma_1 + \gamma_2 f_{t-1}))$, an error correction term incorporating the short-run adjusting device when deviations from the long- run equilibrium relationship appear.

From standard notation, the second moments dynamics corresponding a GARCH(p,q) model can be represented as follows:

$$vech\Sigma_t = vech\bar{\Sigma} + \Theta_q(B)\, vech\left(\varepsilon_t \varepsilon_t'\right) + \Psi_p(B)\, vech\Sigma_t, \quad (13)$$

with $\Phi(0) = \Theta(0) = 0$, B is the backshift operator, ε_t is the innovations vector, $vech\Sigma_t = \begin{pmatrix} \sigma^2_{s,t} & \sigma_{sf,t} & \sigma^2_{f,t} \end{pmatrix}'$, and $vech\left(\varepsilon_t \varepsilon_t'\right) = \begin{pmatrix} \varepsilon_{s,t} & \varepsilon_{s,t}\varepsilon_{f,t} & \varepsilon_{f,t} \end{pmatrix}'$. However, we use an alternative VARMA (vectorial autoregressive moving average) representation for the previous equation. Consider the next trivariate stochastic vector:

$$\xi_t = vech\left(\varepsilon_t \varepsilon_t'\right) - vech\Sigma_t \quad (14)$$

Substituting equation (14) into equation (13) and rearranging:

$$\Gamma_r(B)\, vech\left(\varepsilon_t \varepsilon_t'\right) = vech\bar{\Sigma} + \Phi_p(B)\, \xi_t, \quad (15)$$

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als from cointegration equation. On the other hand, we use the contrast based on the augmented restricted and unrestricted vector autoregression representation. The results, which are not showed in the paper, provide empirical evidence supporting the presence of a common unit root between the natural logarithm of market prices.

where $\Gamma_r(B) = [I - (\Psi_q(B) + \Theta_p(B))]$, $r = \max\{p, q\}$, and $\Phi_p(B) = [I - \Psi_p(B)]$, that is, an ARMA$(r, p)$representation. According to equation (15) we posit a pure moving average process governing second order moments of intraday returns:

$$vech\left(\varepsilon_t \varepsilon_t^{'}\right) = vech\bar{\Sigma} + \left(\phi_1 B + \phi_2 B^6 + \phi_3 B^{12} + \phi_4 B^{18}\right)\xi_t. \tag{16}$$

If the moving average polinomial has no roots in the unit circle this representation captures a past depending squared innovations spanning a long period of time[4]. The following restrictions are introduced: a) matrices ϕ_2, ϕ_3, and ϕ_4 are diagonal, and b) denoting the (i, j) element in the matrix $\phi_1$by $\phi_{ij,}^1$ we impose: $\phi_{12}^1 = \phi_{21}^1 = \phi_{22}^1 = \phi_{23}^1 = \phi_{32}^1 = 0$. These restrictions are not relevant concerning the objectives of the paper, and they only pursuit to avoid numerical problems when estimating the model. An over parametrized model would produce numerical problems due to a lack of identification of all parameters in the model. We still permit cross-market interactions between volatilities trough the elements $\phi_{31}^1$and ϕ_{13}^1 in matrix ϕ_1. The intraday seasonal pattern in volatilities is captured by the diagonal elements in the matrices $\phi_j\,(j = 1, 2, 3)$ since it relates the conditional volatility at a given hour to that of previous days. The same appears to the conditional variance. This is a more general model than those in Park and Switzer (1995), Iihara et al. (1996), Koutmos and Tucker (1996), Racine and Ackert (1998), among others, for previous analysis of the main international stock index futures markets, since it not only allows for conditional covariances to change over time, but also it does not impose that the ratio between conditional covariances and conditional standard deviations be constant over time.

3.1.1 Model Estimation

Under the previous assumption of conditional Gaussian bivariate distribution for the vector of innovations, the log likelihood for the bivariate GARCH model can be written as follows:

[4]As we will see in the next subsection the estimated vectorial moving average model process can be represented as a infinite vectorial autoregressive model.

$$L(\theta) = -T\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\log|\Sigma_t| + \varepsilon_t\Sigma_t^{-1}\varepsilon_t^{'}\right] \qquad (17)$$

where θ is the parameter vector to be estimated, and $\varepsilon_t = (\varepsilon_{s,t}\,\varepsilon_{f,t})^{'}$. The log likelihood function is highly nonlinear in θ and a numerical maximization technique is required. We estimate by exact maximum likelihood with the $E4$ toolbox (to be used with Matlab)[5], which represents the model in the state space form. The optimization algorithm used is the so called BFGS (Broyden, Fletcher, Goldfarb and Shanno). For model estimation we adopt the following strategy: a) we first estimate by ordinary least squares the cointegration equation, incorporating the residuals as an exogenous vector in the model, and b) we fix the three elements in vector $vech\bar{\Sigma}$ using the estimated unconditional second order moments of market returns in the global sample. Therefore, when the numerical algorithm iterates it is not taking into account these three parameters. Overall, we have nineteen paramaters to estimate in the bivariate model.

Tables 4 and 5 shows the results of fitting the bivariate GARCH model to the intraday hourly index and futures returns. As we can expect the estimations concerning the error correction term have opposite signs. Taking into account that cointegration equation is previously estimated imposing unidirectional causal relationship from futures to spot market, we must expect that $\hat{\beta}_f > 0$ and $\hat{\beta}_s < 0$. While β_f is significant at the 5% level, we do not reject the hyphotesis that $\beta_s = 0$, suggesting that is the futures market the important part to play in order to realign prices between the no arbitrage spread.

Relative to markets interactions the model suggests an unidirectional causal relationship between both returns and volatilities, from the futures market to the spot market. Our results shows a seasonal pattern not only for spot and futures conditional volatilities, also for the conditional covariance betwwen spot and futures market returns. Table 6 reports the estimated spot and futures mean volatilities for each trading interval along the 743 trading sessions. An intraday U shaped curve for both volatilities is showed, suggesting that the *opening* and *close* trading periods have the higher volatility. This empirical finding are consistent with those in Chan et. al. (1991) and Daigler (1997).

[5]This toolbox has been developed in the *Departamento de Economía Cuantitativa, Universidad Complutense,* Madrid (Spain).

The average estimated conditional correlation coefficient is 0.87 (see table 8). This positive value reflects that innovations in both price processes have most often the same sign and, consequently, futures and spot prices fluctuactions have the same direction. However, the estimated correlation is below one, probably reflecting that the assumptions required for perfect correlation (no transactions costs and nonstochastic interest rates and dividend yields) are too restrictive.

3.1.2 Model Validation

We use three diagnosis test for the estimated GARCH model diagnostics. First, table 7 reports the Ljung-Box statistics for both the squares and the levels of standardized residual. They suggest that the bivariate error correction GARCH model susccesfully captures cross-markets interactions between the first and second moments of intraday hourly returns.

The other tests provide empirical evidence supporting the null hypothesis of an a random data from an independent identical distribution. We first compute the correlation dimension, originally developed by Grassberger and Procaccia (1983). Given a time series, $\{x_t\}_{t=1}^{N}$ and defining a subsample $X_t^m = (x_t, x_{t+2,\cdots}, x_{t+m-1})$ the m-dimensional vector obtained by putting m consecutive observations together, the correlation dimension is defined as follows:

$$CD_m = \frac{\ln C_m(\epsilon_i) - \ln C_m(\epsilon_{i-1})}{\ln(\epsilon_i) - \ln(\epsilon_{i-1})}, \tag{18}$$

where $C_m(\epsilon)$ is the correlation integral:

$$C_m(\epsilon) = \frac{2}{N_m(N_m+1)} \sum_{t<s} I_\epsilon(X_t^m, X_s^m). \tag{19}$$

with $N_m = N + m - 1$, and $I_\epsilon(X_t^m, X_s^m)$is the following function:

$I_\epsilon(X_t^m, X_s^m) = 1$ *if* $\|X_t^m - X_s^m\| < \epsilon$

$I_\epsilon(X_t^m, X_s^m) = 0$ *in other case*,where $\|\|$ denotes the sup norm and ϵ is a sufficiently small number, usually fixed as a proportion of the standard deviation of the series. The correlation integral measures the probability that the distance between any two m-histories is less than ϵ. If the data are generated by a deterministic process, the correlation integral will be independent of m. Following Yan and Brorsen (1993) eleven values of ϵ are used: 0.9, 0.9^2, ..., 0.9^{11}. Five embedding dimensions (m) are used: 2, 4, 6, 8, 10. Table 8 reports the median of the ten estimates of CD_m for each value of m. Results suggests that the standardized residual can be considered random data rather than coming from a deterministic process.

Third, we compute the BDS statistic proposed by Brock et al. (1986) to test the null hypothesis of independent and identical distribution (*i.i.d.*). The BDS test is:

$$BDS_m(\epsilon, N_m) = (N_m)^{\frac{1}{2}} \frac{C_m(\epsilon, N_m) - [C_1(\epsilon, N)]^m}{\sigma_m(\epsilon, N_m)}, \tag{20}$$

where:

$$\sigma_m^2(\epsilon, N_m) = 4\left[K^m + 2\sum_{j=1}^{m-1} K^{m-j}C^{2j} + (m-1)^2C^{2m} - m^2KC^{2(m-1)}\right] \tag{21}$$

and:

$$C = C(\epsilon) = \int [F(z+\epsilon) - F(z-\epsilon)]\, dF(z) \tag{22}$$

$$K = K(\epsilon) = \int [F(z+\epsilon) - F(z-\epsilon)]^2\, dF(z) \tag{23}$$

where $F(\cdot)$ denotes the density function. Under the null hypothesis of *i.i.d.* $BDS_m(\epsilon, N_m) \to N(0,1)$ as $N_m \to \infty$. Large values would indicate strong evidence for nonlinearity in the data. Table 9 reports the BDS for the above five embedding dimensions. Two values of ϵ are used: $\frac{\epsilon}{std} = .25$ and $\frac{\epsilon}{std} = .5$. Empirical results (table 9) provide empirical supporting the null hypothesis of *i.i.d.* indicating no significant intertemporal dependence for standardized residual.

3.2 Estimates for the parameters of the theoretical model.

Let us denote by $\hat{x}_t$, $\hat{y}_t$ and z_t the estimated conditional variance and covariance for spot and futures market returns, respectively, that is: $Var_t\left(\frac{dS_t}{S_t dt}\right) = \sigma_{s,t}^2$ with $\hat{x}_t = \hat{\sigma}_{s,t}^2$. To recover estimations for $\sigma_{N,t}^2$ and $\rho_{12,t}$ we use the theoretical expressions for the conditional variance of futures returns and the contional covariance:

$$\hat{y}_t = \hat{x}_t + \hat{\sigma}_{N,t}^2 + 2\hat{x}_t^{\frac{1}{2}}\hat{\sigma}_{N,t}\hat{\rho}_{12,t} \tag{24}$$

$$\hat{z}_t = \hat{x}_t + \hat{x}_t^{\frac{1}{2}} \hat{\sigma}_{N,t} \hat{\rho}_{12,t} \tag{25}$$

From (25):

$$\hat{x}_t^{\frac{1}{2}} \hat{\rho}_{12,t} = \frac{\hat{z}_t - \hat{x}_t}{\hat{\sigma}_{N,t}} \tag{26}$$

Substituting equation (26) into equation (24) and rearranging:

$$\hat{\sigma}_{N,t}^2 = \hat{x}_t + \hat{y}_t - 2\hat{z}_t. \tag{27}$$

To recover estimates for the correlation between the two disturbances, substituting (27) into (26) and rearranging:

$$\hat{\rho}_{12,t} = \frac{\hat{z}_t - \hat{y}_t}{\hat{y}_t^{\frac{1}{2}} \sqrt{\hat{x}_t + \hat{y}_t - 2\hat{z}_t}}. \tag{28}$$

Finally, the optimal hedge ratio can be estimated from (10) by appliying $\hat{\delta}_t$ and $\hat{\rho}_{12,t}$.

Figures 3 to 5 provide the evolution for the estimated parameters $\hat{\sigma}_{s,t}^2$, $\hat{\sigma}_{N,t}^2$,and $\hat{\rho}_{12,t}$ respectively.

4 Simulated Hedging operations

To calibrate the effectiveness of the optimal hedge ratio we simulate a hedging operation assuming a daily revision of short position in the derivative market and using actual market data. We produce the ex-post simulated evolutions of returns for the hedged position for each of the trading hour: at 12:00, 13:00, 14:00, 15:00, 16:00 and 17:00 hours. Then, we measure hedging effectiveness as the proportion in the reduction of the standard deviation for the hedged position return relative to the standard deviation of market return (unhedged position), that is:

$$100 \times \frac{std\,(hedged\ return) - std\,(not\ hedged\ return)}{std\,(not\ hedged\ return)} \tag{29}$$

We compare the hedging effectiveness of the systematic unitary ratio and the optimal hedge ratio of the model, both taking into account the estimated correlation between the *general* and specific noise and also assuming no correlation. Tables 10, 11 and 12 show the empirical results (hedging effectiveness) for each trading hour and both for the global sample as well as for the following subsamples: a) 20/12/93 to 16/12/94, b) 16/12/94 to 12/15/95 and c) 12/18/95 to 12/20/96. Table 10 corresponds to the systematic unitary ratio. Results provided in table 11 concern with the estimated ratio taking into account the correlation between the *general* and the specific noise. Finally, table 12 show the results with the estimated regardless of this correlation. These tables also show the applied average hedge ratio, a relevant indicator of transaction costs.

Two relevant aspects are revealed from statistics provided in tables 10 to 12. First, the hedging effectiveness of the estimated ratios, both taking into account the correlation or ignoring it, is similar to the reached by systematically applying a unitary ratio. However, the number of futures contracts involved under the estimated hedge ratios is significantly lower, and as a consequence transaction costs and capital requirements are sensibly lower than when a *short-sighted* ratio is applied. Second, even though the hedging effectiveness concerning the restricted ratio (under the assumption of linear independence between the *general* and specific market disturbance) is always below to the reached considering a non-zero correlation, the difference is not very large. This can be explained by the characteristics of the second order moments returns estimated with the GARCH model. In the 75% of the global sample the estimated conditional volatility of the futures returns is greater than that corresponding to the spot market, wich is consistent with the implications of the theoretical model under uncorrelation between the specific and the *general* market noise.

4.1 Summary and concluding remarks

A two periods hedging model is derived allowing for departures from the cost-of-carry valuation. Assuming a geometric Brownian motion for the dynamics of spot index, we model the mispricing by introducing a new specific noise in the dynamics of futures prices which can be correlated with the *general* market noise. The optimal hedge ratio depends on two factors: the relative size between the specific and common noise, and their corresponding correlation.

We provide empirical evidence about the model in the Spanish stock index futures market along the sample period from 20/12/93 to 20/12/96. A bivariate error correction model with GARCH perturbations is used to estimate the parameters of the theoretical model, and after, we recover estimates of the optimal hedge ratio.

The results of ex-post simulations reveals that applying a systematic unitary ratio, as predicted under no discrepancies between market futures price and its valuation according to the cost-of-carry model becomes more expensive. When taking into account the estimated correlation between the two market noises the variance reduction in the standard deviation for the hedged position return is similar to the reached with a static unitary ratio, but a lower number of futures contract is needed. Finally, a similar hedging effectiveness is achieved when ignoring the correlation between the specific and the *general* market noise, which is consistent with a derivative market more volatile than the spot market along the majority of the daily trading period.

References

[1] Bühler, W. and A. Kempf (1995), Dax index futures: mispricing and arbitrage in german markets, *Journal of Futures Markets* 7, 833-859.

[2] Brock, B.W., Dechert, W.D. and J. Scheinkman (1986), A test for independence based in the correlation dimension, *Working Paper,* University of Wisconsin.

[3] Chan K., Chan, K.C. and G.A. Karolyi (1991), Intraday volatility in the stock index and stock index futures markets, *Review of Financial Studies* 4, 657-684.

[4] Daigler R.T. (1997), Intraday futures volatility and theories of market behavior, *Journal of Futures Markets* 17, 45-74.

[5] Dickey, D.A. and W. Fuller (1979), Distribution of the estimators for autoregressive time series with unit root, *Journal of the American Statistical Association*, 74: 427-431.

[6] Engle, R.F. and C.W. Granger (1987), Cointegration and error correction: Representation, estimation and testing, *Econometrica* 55, 251-276.

[7] Fernández, P. and J. Yzaguirre (1996), Ibex 35. Análisis e investigaciones. Ediciones Internacionales Universitarias.

[8] Iihara Y., Kato, K. and T. Tokunaga (1996), Intraday returns dynamics between cash and the futures markets in Japan, *Journal of Futures Markets* 16, 147-162.

[9] Grassberger, P and I. Procaccia (1983), Measuring the strangeness of strange attractors, *Physica* 9D: 189-208.

[10] Koutmos G. and M. Tucker (1996), Temporal relationships and dynamics interactions between spot and futures stock markets, *Journal of Futures Markets* 16: 55-69.

[11] Mackinlay, A.C. and C. Ramaswamy (1988), Index futures arbitrage and the behavior of stock index futures prices, *Review of Financial Studies* 1, 137-158.

[12] Miller, M.H., Muthuswamy, J. and R.E. Whaley (1994), Mean reversion of standard & Poor's 500 index basis changes: Arbitrage-induced or statistical illusion?, *Journal of Finance* 49, 479-513.

[13] Lien, D. and Y.K. Tse (1999), Fractional cointegration and futures hedging, *Journal of Futures Markets* 19, 457-474.

[14] Lim, K. (1992), Arbitrage and price behavior of the Nikkei stock index futures, *Journal of Futures Markets* 12, 151-161.

[15] Park, T.H. and L.N. Switzer (1995), Bivariate Garch estimation of the optimal hedge ratios for stock index futures: A note, *Journal of Futures Markets* 15, 61-67.

[16] Pizzi, M.A., Economopoulos, A.J. and H.M. O'Neill (1998), An examination of the relationship between stock index cash and futures markets: A cointegration approach, *Journal of Futures Markets* 18, 297-305.

[17] Racine, M.D. and L.F. Ackert (1998), Time-varying volatility in Canadian and U.S. stock index and stock index futures markets: A multivariate analysis, *Working paper*, Federal Reserve Bank of Atlanta.

[18] Stoll, H.R. and R.E. Whaley (1990), The dynamics of stock index and stock index futures returns, *Journal of Financial and Quantitative Analysis* 25, 441-468.

[19] Wahab, M. and M. Lasghari (1993), Price dynamics and error correction in stock index and stock index futures markets: A cointegration approach, *Journal of Futures Markets* 13, 711-742.

[20] Yadav, P.K. and P.F. Pope (1990), Stock index futures arbitrage: International evidence, *Journal of Futures Markets* 10, 573-603.

[21] Yadav, P.K. and P.F. Pope (1994), Stock index futures mispricing: Profit opportunities or risk premia?, *Journal of Banking and Finance* 18, 921-953.

[22] Yang S.R. and W. Brorsen (1993), Nonlinear dynamics of daily futures prices: conditional heteroskedasticity or chaos?, *Journal of Futures Markets* 2, 175-191.

Appendix 1.

First order condition of the hedging model

Substituting the theoretical dynamic of spot and futures market returns into the objective function the problem can be expressed as follows:

$$\underset{\{h_t\}}{Min\,Var_t}\left[b_t\left(\mu_{s,t}+\sigma_{s,t}\frac{dz_{1,t}}{dt}\right)-h_t\left(\mu_{f,t}+\sigma_{s,t}\frac{dz_{1,t}}{dt}+\sigma_{N,t}\frac{dz_{2,t}}{dt}\right)\right] \qquad (A.1)$$

Applying the properties of the variance, and taking into account that $Var_t\left(\frac{dz_{1,t}}{dt}\right)=Var_t\left(\frac{dz_{2,t}}{dt}\right)=1$ and $Cov_t\left(\frac{dz_{1,t}}{dt},\frac{dz_{2,t}}{dt}\right)=\rho_{12,t}$, yields the following expression for the objective function:

$$b_t^2\sigma_{s,t}^2+h_t^2\left(\sigma_{s,t}^2+\sigma_{N,t}^2+2\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right)-2b_th_t\left(\sigma_{s,t}^2+\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right) \qquad (A.2)$$

Taking the derivative respect to h_t and equaling to zero:

$$2\left[h_t\left(\sigma_{s,t}^2+\sigma_{N,t}^2+2\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right)-b_t\left(\sigma_{s,t}^2+\rho_{12,t}\sigma_{s,t}\sigma_{N,t}\right)\right]=0 \qquad (A.3)$$

From (A.3), simplifying and rearranging yields equation (8).

Appendix 2. Statistical Tables.

Table 1. Summary statistics. Market returns

	Spot Market	Futures market
Mean	-0.0000	-0.0000
Standard deviation	0.0036	0.0030
Skewness	-0.7131	-0.0582
Kurtosis	10.7904	3.4209
$\rho\left(r_t, r_{t-k}\right)^{(a)}$		
$k = 1$	0.0758	0.0241
$k = 2$	0.0073	0.0029
$k = 3$	0.0542	0.0139
$k = 4$	0.0232	0.0460
$k = 5$	-0.0099	0.0181
$k = 6$	-0.0175	0.0028
$k = 7$	-0.0231	-0.0451
$k = 8$	-0.0216	-0.0121
$k = 9$	-0.0051	0.0001
$k = 10$	0.0144	0.0408
$k = 11$	0.0043	0.0110
$k = 12$	0.0250	0.0406
$k = 18$	0.0100	0.0296
$k = 24$	0.1390	-0.0040
Ljung-Box satistics[(b)]	73.41 (0.00)	54.39 (0.00)

Notes:[(a)] Autocorrelation function. The standard error for the autocorrelation coefficients can be aproximated by $\frac{1}{\sqrt{4,468}} \simeq 0.015$.[(b)] Ljung-Box test uses twenty four autocorrelation coefficients. P-value is in parentheses.

Table 2. Autocorrelations functions. Squared returns

	Spot Market	Futures market
$\rho\left(r_t^2, r_{t-k}^2\right)$		
$k = 1$	0.0874	0.1435
$k = 2$	-0.0028	0.0760
$k = 3$	-0.0056	0.0261
$k = 4$	0.0255	0.0732
$k = 5$	0.1015	0.1001
$k = 6$	0.2506	0.1982
$k = 7$	0.0299	0.0630
$k = 8$	-0.0028	0.0397
$k = 9$	-0.0135	0.0389
$k = 10$	0.0028	0.0315
$k = 11$	0.0241	0.0820
$k = 12$	0.1341	0.1464
$k = 18$	0.1697	0.1183
$k = 24$	0.1591	0.1325
Ljung-Box satistics[(b)]	724.08 (0.00)	800.83 (0.00)

Notes:[(a)]Autocorrelation function.The standard error for the autocorrelation coefficients can be aproximated by $\frac{1}{\sqrt{4,468}} \simeq 0.015$. [(b)]Ljung-Box test uses twenty four autocorrelation coefficients. P-value is in parentheses.

Table 3. Cross correlations functions

	Returns	Squared returns
	$\rho\left(r_{s,t}, r_{f,t-k}\right)^{(*)}$	$\rho\left(r_{s,t}^2, r_{f,t-k}^2\right)$
$k=-24$	0.0062	0.0617
$k=-18$	0.0321	0.0618
$k=-12$	0.0415	0.0881
$k=-11$	0.0003	0.1191
$k=-10$	0.0353	0.0740
$k=-9$	-0.0051	0.0105
$k=-8$	-0.0066	0.0307
$k=-7$	-0.0281	0.0215
$k=-6$	-0.0081	0.1194
$k=-5$	-0.0179	0.2490
$k=-4$	0.0144	0.0527
$k=-3$	0.0281	0.0201
$k=-2$	0.0123	0.0195
$k=-1$	0.0309	0.1037
$k=0$	0.6708	0.3457
$k=1$	0.1275	0.1724
$k=2$	0.0198	0.0333
$k=3$	0.0358	0.0295
$k=4$	0.0255	0.0388
$k=5$	0.0314	0.0421
$k=6$	0.0102	0.0958
$k=7$	-0.0301	0.1187
$k=8$	-0.0392	0.0307
$k=9$	0.0007	0.0245
$k=10$	0.0117	0.0012
$k=11$	0.0149	0.0048
$k=12$	0.0268	0.0569
$k=18$	0.0232	0.0754
$k=24$	0.0094	0.0632

Notes: $^{(*)}r_{s,t}$ and $r_{f,t-k}$ denotes spot and futures returns in period t and $t-k$ respectively. The standard error for the cross correlation coefficients can be aproximated by $\frac{1}{\sqrt{4,468}} \simeq 0.015$.

Table 4. Maximum likelihood estimation. Mean equation

Dependent variable			
coefficient[*]	Spot return	coefficient	Futures return
α_{11}	0.061 (0.017)	α_{21}	-0.006 (0.014)
α_{12}	-0.106 (0.021)	α_{22}	-0.032 (0.018)
β_s	-0.078 (0.014)	β_f	0.012 (0.012)

Note:[*]In parenthesis are the estimated standard errors.

Table 5. Maximum likelihood estimation. Variance equation

Dependent variable					
coeff.[*]	$\varepsilon_{s,t}$	coeff.	$\varepsilon_{f,t}$	coeff.	$\varepsilon_{s,t}\,\varepsilon_{f,t}$
ϕ_{11}^{1} [**]	0.002 (0.004)	ϕ_{31}^{1}	0.004 (0.003)	ϕ_{22}^{2}	0.078 (0.008)
ϕ_{13}^{1}	0.065 (0.011)	ϕ_{33}^{1}	0.036 (0.008)	ϕ_{22}^{3}	0.012 (0.007)
ϕ_{11}^{2}	0.136 (0.004)	ϕ_{11}^{2}	0.036 (0.011)	ϕ_{22}^{4}	0.007 (0.005)
ϕ_{11}^{3}	0.077 (0.007)	ϕ_{11}^{3}	0.051 (0.008)		
ϕ_{11}^{4}	0.068 (0.004)	ϕ_{11}^{4}	0.035 (0.005)		

Note:[*]In parenthesis are the estimated standard errors.
[**]ϕ_{ij}^{r} denotes the i-rows j-colums element in the matrix ϕ_r (r=1,2,3,4).

Table 6. Intraday estimated GARH model mean volatility

Trading hour intervals	Spot market	Futures market
11:00 - 12:00	0.091[*]	0.160
12:00 - 13:00	0.083	0.089
13:00 - 14:00	0.081	0.084
14:00 - 15:00	0.083	0.086
15:00 - 16:00	0.086	0.090
16:00 - 17:00	0.101	0.112

Note:[*]Each mean is computed from the 743 daily estimated GARCH volatilities at the upper hour interval.

Table 7. Model diagnostics

Ljung-Box Q statistics$^{(*)}$	Spot market	Futures market
Standardized residuals	31.23 (0.40)	39.15 (0.12)
Squares standardized residuals	31.90 (0.37)	37.83 (0.15)

Note:$^{(*)}$The number of lags equals 30. P-value is in parenthesis.

Table 8. Correlation dimension estimates$^{(*)}$

	$m=2$	$m=4$	$m=6$	$m=8$	$m=10$
Spot market	0.146	0.157	0.484	0.046	0.100
Futures market	0.188	0.055	0.208	0.163	0.114

Note:$^{(*)}$ For each embedding dimension, eleven values of epsilon are used: 0.9, 0.9^2, ..., 0.9^{11}. The average correlation dimension from the ten estimated one is reported.

Table 9. Model diagnostics. BDS$^{(*)}$ test.

$\frac{\epsilon}{std}=0.25$	Spot market	Futures market
$m=2$	0.288	-1.514
$m=4$	-0.051	0.067
$m=6$	0.389	0.411
$m=8$	1.255	1.023
$m=10$	1.718	1.279
$\frac{\epsilon}{std}=0.5$		
$m=2$	-0.054	-0.422
$m=4$	-0.100	-0.205
$m=6$	-0.065	-0.336
$m=8$	-0.0371	-0.398
$m=10$	-0.030	-0.490

Note:$^{(*)}$Under the null of *i.i.d.* the distribution converges to a standard Normal density.

Table 10. Results of simulated hedging operations. Unitary ratio.

	Hedging efectiveness[1]				Average hedge ratio			
Futures	Sample				Sample			
Revision	Global	S1[a]	S2[b]	S3[c]	Global	S1[a]	S2[b]	S3[c]
12:00	7.92%	7.20%	7.15%	20.79%	1.00	1.00	1.00	1.00
13:00	45.62%	47.22%	46.56%	39.62%	1.00	1.00	1.00	1.00
14:00	43.43%	41.13%	46.80%	44.26%	1.00	1.00	1.00	1.00
15:00	39.31%	33.46%	36.44%	52.64%	1.00	1.00	1.00	1.00
16:00	45.29%	45.36%	40.29%	52.70%	1.00	1.00	1.00	1.00
17:00	58.43%	59.73%	58.50%	54.44%	1.00	1.00	1.00	1.00

(1) Hedging effectiveness is defined as the proportion of reduction in the standard deviation of the hedged position relative to the market volatility (standard deviationn of the unhedged position).

(a) Subsample 1: 20/12/93 - 16/12/94.

(b) Subsample 2: 16/12/94 to 12/15/95.

(c) Subsample 3: 12/18/95 to 12/20/96.

Table 11. Results of simulated hedging operations. Unrestricted Ratio[2].

	Hedging efectiveness[1]				Average hedge ratio			
Futures	Sample				Sample			
Revision	Global	S1[a]	S2[b]	S3[c]	Global	S1[a]	S2[b]	S3[c]
12:00	5.27%	3.59%	6.97%	18.88%	0.53	0.36	0.49	0.74
13:00	43.53%	43.82%	74.42%	44.88%	0.77	0.74	0.76	0.81
14:00	47.03%	44.09%	51.32%	48.03%	0.80	0.77	0.80	0.82
15:00	42.22%	37.58%	63.96%	53.61%	0.80	0.79	0.80	0.82
16:00	45.99%	42.93%	80.98%	54.67%	0.79	0.77	0.80	0.80
17:00	54.51%	54.88%	53.16%	55.24%	0.77	0.74	0.77	0.79

(1) Hedging effectiveness is defined as the proportion of reduction in the standard deviation of the hedged position relative to the market volatility (standard deviation of the unhedged position).

(2) Estimated ratio taking into account the correlation between the specific and *general* market noise.

(a) Subsample 1: 20/12/93 - 16/12/94.

(b) Subsample 2: 16/12/94 to 12/15/95.

(c) Subsample 3: 12/18/95 to 12/20/96.

Table 12. Results of simulated hedging operations. Restricted Ratio[2].								
	Hedging efectiveness[1]				Mean hedge ratio			
Futures	Sample				Sample			
Revision	Global	S1[a]	S2[b]	S3[c]	Global	S1[a]	S2[b]	S3[c]
12:00	4.52%	2.92%	6.02%	17.34%	0.42	0.41	0.28	0.58
13:00	37.52%	36.71%	56.62%	43.31%	0.59	0.55	0.57	0.64
14:00	42.71%	39.77%	46.33%	44.72%	0.62	0.59	0.62	0.65
15:00	38.92%	35.32%	55.29%	48.15%	0.61	0.58	0.62	0.64
16:00	41.80%	38.63%	71.91%	48.67%	0.60	0.57	0.60	0.62
17:00	44.89%	45.31%	42.52%	47.10%	0.56	0.52	0.57	0.60

(1) Hedging effectiveness is defined as the proportion of reduction in the standard deviation of the hedged position relative to the market volatility (standard deviation of the unhedged position).

(2) Estimated ratio under uncorrelation between the specific and *general* market noise.

(a) Subsample 1: 20/12/93 - 16/12/94.

(b) Subsample 2: 16/12/94 to 12/15/95.

(c) Subsample 3: 12/18/95 to 12/20/96.

Figures

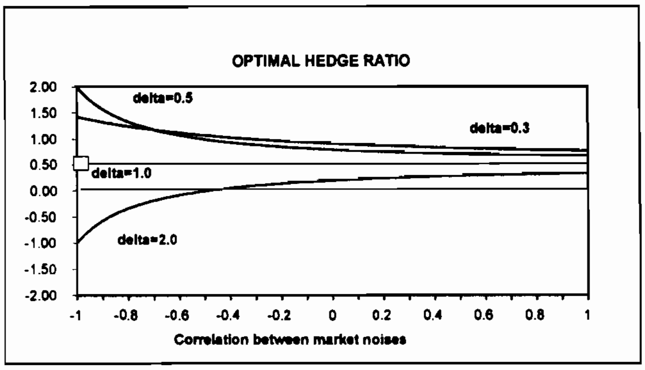

Figure 1.

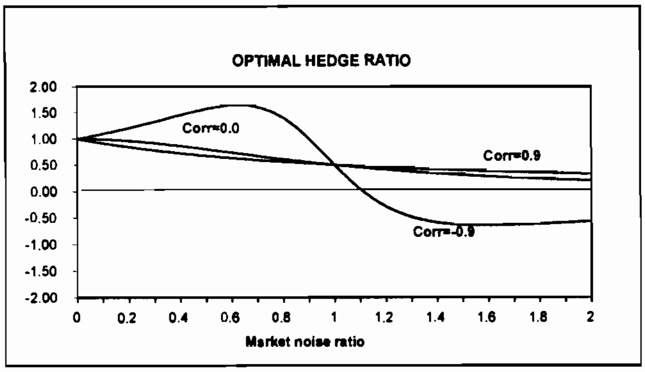

Figure 2.

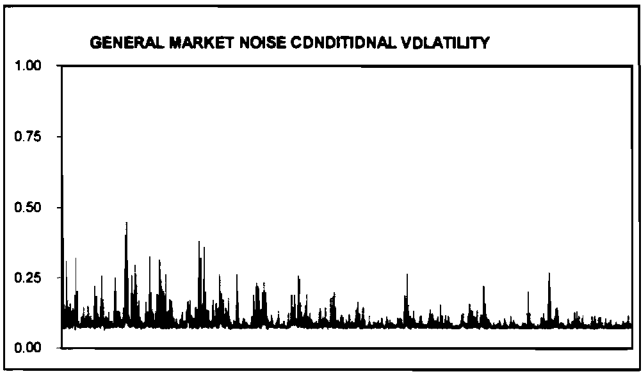

Figure 3.

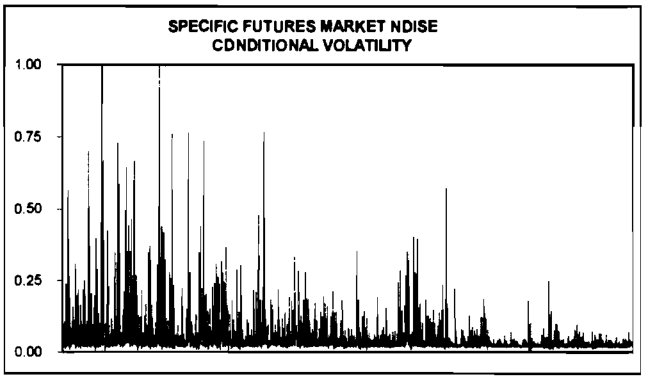

Figure 4.

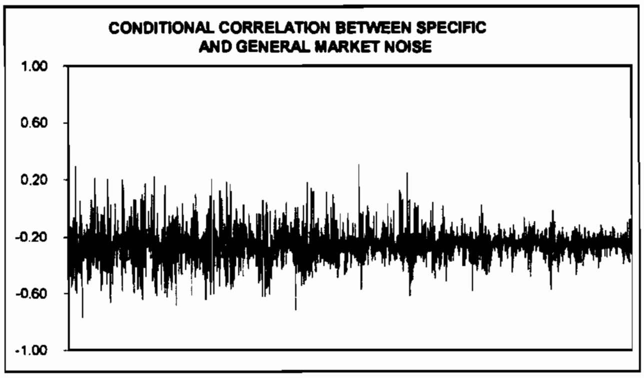
CONDITIONAL CORRELATION BETWEEN SPECIFIC AND GENERAL MARKET NOISE
1.00
0.60
0.20
-0.20
-0.60
-1.00

Figure 5.