TRADE LIBERALIZATION, COMPETITION AND GROWTH*

Omar Licandro (European University Institute)
Antonio Navas-Ruiz (Universidad Carlos III de Madrid and GREQAM)

Abstract

The aim of this paper is to understand whether international trade may enhance innovation and growth through an increase in competition. We develop a two-country endogenous growth model, both countries producing the same set of goods, with firm specific R&D and a continuum of oligopolistic sectors under Cournot competition. Since countries produce the same set of goods, trade openness makes markets more competitive, reducing prices and raising the incentives to innovate. More general, a reduction on trade barriers enhances growth by reducing domestic firms`market power.

Keywords: Trade openness, competition and growth, R&D
JEL Classification: F13, F43, O3

* We would like to thank J.A. Erce, C. García-Peñalosa, G. Impulitti, R. Marimon, M. Motta, C. Ponce, G. Ottaviano, and participants to the Vienna EEA-ESEM, Faro MMGD and ASSET 2006 meetings and the EUI-MWF workshop for useful comments. O. Licandro acknowledges the financial support of the Spanish Ministry of Education (SEJ2004-04579/ECON). E-mails: omar.licandro@eui.eu and antonio.navas@eui.eu.
1 Introduction

During the last two decades the volume of international trade has increased enormously, among developed countries in the 80’s and extending to developing countries in the 90s. This increase in trade volumes is contemporaneous with several attempts to create regional integration agreements, as for example the European Union and the MERCOSUR. There is a common belief that these changes have turned out into a larger more competitive environment, whose consequences for firm’s productivity and growth are still subject of controversy. While the EU and the OECD countries have carried out policies to stimulate competition, some voices alert about the impact of foreign competition on the productivity of domestic firms.

In this paper, we stress the positive impact on economic growth of the pro-competitive role of international trade. Our paper is motivated by recent empirical studies both at the firm and the industry level suggesting that the rise in competition coming from globalization has a no negligible positive impact on firm’s investments in innovation. For example, Harrison et al. (2006) evaluate the impact that different reforms carried out by the European Union under the Single Market Program have had in innovation intensity. They find that these policies have increased both competition, measured as a reduction in markups, and innovation intensity, measured as R&D expenditures over sales, leading to productivity growth in the manufacturing sector.

In models of endogenous growth, where technological progress is the result
of firms’ private decisions, international trade affects the incentives to innovate through different channels: The reallocative effects due to specialization can have an impact on growth if sectors present different scopes for technological progress (Grossman and Helpman, 1991); openness to trade promotes the exchange of knowledge leading to growth, the so-called knowledge spillovers (Rivera-Batiz and Romer, 1991, and Devereux and Lapham, 1994). However, in all these studies trade openness gives little space to competition, because of the monopolistically competitive nature of markets and the assumption that innovation is carried out by potential entrants. In Rivera-Batiz and Romer, for example, markups only depend on the elasticity of substitution among varieties; moreover, openness to trade increases the market size and the number of firms in the same proportion, leaving innovation rents unchanged. It is only under the existence of technological spillovers that innovation is fostered. Finally, their paper fails accounting for the empirical evidence cited above since it predicts constant average markups after trade liberalization and ignores, by assumption, the reaction in terms of R&D investments of incumbent firms.

More recently, the literature on competition and growth has developed models in which incumbents are allowed to upgrade their own technologies. The seminal work by Aghion et al. (2001) points out the escape from competition effect as an incentive to innovate in highly competitive environments. In a different paper Peretto (1999) considers an extension of Romer’s (1990) model, by adding cost-reduction innovations and strategic interaction among firms. A rise in product
market competition produces higher growth by reducing the number of firms, which increases markups and makes firms to innovate more. Therefore, there is a trade-off between competition and growth, since higher growth is associated with lower number of firms and higher markups. Peretto (2003) extends the previous exercise to trade openness, and shows that it reduces both the global number of firms and R&D costs—due to technological spillovers—rising the incentives to innovate.

The model in this paper is a two-country economy with R&D activities being undertaken by incumbents. Different from the literature cited above, both countries produce the same set of varieties, each variety being produced by $n$ firms in each country under Cournot competition. When countries open to trade, it is not the mass of varieties that changes but the number of firms competing in each variety. This approach has many important advantages for the study of the competitive effects of trade openness. Firstly, an increase in competition is modelled by an increase in the number of competitors offering the same product.¹ Note that in this framework, other measures of competition like markups, market shares or market concentration reduce when the number of firms increases.² Second, R&D is undertaken by incumbents and innovation is firm specific, implying that the return to innovation depends crucially on firm’s size. An increase in the number of firms has two opposite effects on firm’s size: A market share effect, by increasing

¹ In Aghion et al (2001), competition is measured by the elasticity of substitution between different varieties. However, as Koeninger and Licandro (2005) point out, the elasticity of substitution is an element of the environment reflecting preferences or technology. They claim that changes in the elasticity of substitution results on different efficient allocations, which may be confounded with the associated change in competition.

² An interesting discussion about the measurement of competition is in Motta (2004).
the number of competitors, it reduces individual market shares; and a competition effect, by increasing competition it has a negative effect on markups, increasing the size of the market. Third, trade openness affects growth through the competition effect only, since the reduction in the domestic market share suffered by local firms is compensated by their participation in the foreign market.\footnote{We do not consider technological spillovers, isolating the pure effect of competition on innovation.} Finally, it is important to say that this paper studies the case of economic integration among similar economies where openness to trade intensifies competition within the exiting industries rather than giving access to different goods produced abroad, something more frequent in cases of North-South trade.

The paper succeeds in obtaining a positive growth effect of trade openness as a consequence of the increase in innovation generated by a more competitive environment. Since the number of firms affect innovation non-linearly, the paper shows that gains from trade are larger the less competitive countries are in autarky, as firms would be more reactive in these environments. More generally, the paper shows that trade barriers reinforce domestic firms’ market power leading to a decrease in innovation and growth. This paper is related to the recent work by Neary, who successfully introduces oligopolistic elements into general equilibrium theory by assuming a continuum of sectors and an oligopolistic market within each sector. In this framework, the impact of trade liberalization on production and trade patterns is analyzed in Neary (2002), and the impact on wage inequality...
in Neary (2003), but still no work has explored its consequences for innovation and growth.

The paper is structured as follows. In section 2, we present the basic model in autarky and we try to understand what are the forces driving growth. In section 3, we allow for free trade in the case of two identical countries. Section 4 concludes.

2 Autarky

Consider an economy populated by a continuum of consumers of measure $L$, with instantaneous logarithmic preferences defined over two final consumption goods $X$ and $Y$,

$$\int_0^\infty (\ln C_t^x + \ln C_t^y)e^{-\rho t}dt, \quad \rho > 0,$$

where $C_t^x, C_t^y$ represent consumption levels. Good $Y$ is an homogeneous good.\(^4\) Good $X$ is a Dixit-Stiglitz composite good defined in a continuum of industries of measure $N$:

$$C_t^x = \left( \int_0^N x_{jt}^\alpha dj \right)^{\frac{1}{\alpha}}, \quad \alpha \in (0, 1),$$

where $x_{jt}$ represents consumption of good $j$. Each individual is endowed with one unit of labour at each point in time. In order to finance R&D activities, firms issue shares, $A_t$, which pay a rate of return $r_t$. Let us take the homogeneous good

\(^4\)The existence of a traditional good allows for the reallocation of labor to the R&D sector without necessarily reducing labor assigned to the composite good sector. A similar result would arrive under the assumption of an elastic labor supply as in Aghion et al (2001). When the effect of trade openness to employment is a key issue, this alternative would preferable.
as the numeraire (i.e. $p_t^Y = 1$). The representative consumer budget constraint is given by:
\[
\dot{A}_t = w_t + r_t A_t - \int_0^N p_{jt} x_{jt} dj - C^y_t, \quad A_0 > 0,
\]
where $w_t$ is the wage rate, and $p_{jt}$ is the price of good $j$.

Good Y is produced by a continuum of firms of measure one with technology:
\[
C^y_t = L^y_t, \quad (1)
\]
where $L^y_t$ represents labour allocated to this sector. Sector Y is competitive implying that $w_t = 1$.

Each good $j$ in $X$ is produced by $n$ firms in an oligopolistic environment. A firm $i$ in $j$ produces using technology (let us omit the subscript $j$ for simplicity)
\[
q_{it} = z_{it} L^x_{it}, \quad (2)
\]
where $z_{it}$ is the stock of knowledge, which is assumed to be firm-specific. Firms in $X$ can also invest in R&D activities leading to a reduction in marginal production costs. The R&D technology is
\[
\dot{z}_{it} = (L^z_{it})^\gamma z_{it}, \quad \gamma \in (0, 1), \quad (3)
\]
where $L^*_i$ represents labor allocated to R&D.\footnote{Since we are focusing on the effects of a pure increase in competition on growth, no technological spillovers are assumed consistently with Aitken and Harrison (1999) findings of little or no evidence of technological spillovers coming from international trade.}

At any point in time firms in $j$ decide the quantity to supply and the optimal allocation of workers to both activities, physical production and R&킬, taking into consideration other firms’ strategies. This game belongs to the family of differential games, or repeated games defined in continuous time, in which past actions affect current payoffs. Two different concepts of perfect Markov Nash equilibria have been proposed in the literature, the open-loop and the closed-loop Nash equilibrium. This paper focuses in open-loop Nash equilibria (OLNE), mainly for two reasons. Firstly, for simplicity, since under certain assumptions standard optimal control theory techniques can be applied in order to find OLNE, and secondly, because in models without uncertainty every OLNE is a closed-loop Nash equilibrium, (CLNE).\footnote{In a model without uncertainty the information sets, at time $t$ and at time $t+s$, relevant to take the optimal decisions in $t+s$ are the same, therefore every open-loop is also a closed-loop Nash equilibrium. See Fehrstmann and Mueller (1984).}

In an open-loop Nash equilibrium firms decide at time $t = 0$ the optimal path of strategies taking other firms’ path strategies as given. In this sense an open-loop Nash equilibrium is equivalent to a static Nash equilibrium where the possible strategies are time-paths of actions and the payoffs associated are infinite sum of payoffs.

Let $a_i = [q^i_T, L^i_T]$ for $T \geq t$ be firm’s $i$ strategy, where $[q^i_T, L^i_T]$ are the time-paths of output and R&D workers, and let us call $\Omega_i$, the set of strategies of firm $i$. Let $V_i$ be the value of firm $i$ when the $v$ firms in the market, $n \geq 2$, play strategies...
$A_n = [a_1, a_2, \ldots, a_n]$.

**Definition 1** At time $t$, $A_n = [a_i, a_{-i}]$ is an open loop Nash equilibrium if

$$V_i[A_n] \geq V_i[A'_n] \geq 0,$$

where $A'_n = [a'_i, a_{-i}]$, $\forall a'_i \in \Omega_i$.

This condition implies that the optimal time path of strategies $a_i$ maximizes the value of firm $i$ taking as given other firms’ strategies, $(a_{-i})$, and that the firm value has to be non-negative.

### 2.1 Solving for the autarkic equilibrium

Consumers solve the standard optimal control problem defined above. The optimal conditions are

\[ E^x_t = E^y_t = E_t, \quad (4) \]
\[ \frac{\dot{E}_t}{E_t} = r_t - \rho, \quad (5) \]
\[ p_{jt} = \left( \frac{L E_t}{x_{jt}} \right)^{1-\alpha} P_t, \quad (6) \]

where $E^x_t, E^y_t$ are individual expenditures in goods $X, Y$, respectively, i.e.,

\[ E^x_t = \int_0^N p_{jt} x_{jt} dj \]
and $E^y_t = C^y_t$. In the following, we use the notation $E_t$ to refer to both. The price index of the composite good $X$ is given by

$$P_t = \left( \int_0^N p_{jt}^{\alpha} \, dj \right)^{\alpha-1}.$$

Firm $i$ producing good $j$ solves the problem:

$$V_{is} = \max \int_s^\infty R_{s,t} (p_{jt} - z_{it}^{-1}) q_{it} - L_{i,t}^z \, dt, \quad \text{s.t.}$$

where $R_{s,t} = e^{-\int_s^t r_s \, d\tau}$ is the usual market discount factor. Deriving first order conditions, rearranging terms and applying symmetry, we get:

$$q_t = \theta z_t l E_t, \quad \text{(8)}$$

$$1 = \gamma v_t (L_t^z)^{\gamma^{-1}} z_t, \quad \text{(9)}$$

$$\frac{z_t^{-2} q_t}{v_t} + (L_t^z)^{\gamma} = -\frac{\dot{v}}{v} + r_t, \quad \text{(10)}$$
where $v_t$ is the costate associated with variable $z_t$ and $\theta \equiv \frac{n^{-1+\alpha}}{n}$ is the inverse of the markup rate. Note that equilibrium is symmetric under the assumption that the initial stock of knowledge is equal for all firms in all sectors, i.e. $z_{i0} = z_0, \forall i$. As it can be seen in the last term of equation (8), the relevant scale is the number of workers per firm, $l = \frac{L}{nN}$.

The left hand side of condition (10) is the marginal gain of accumulating one more unit of knowledge, and it can be decomposed in two parts: the first consisting of the reduction in marginal production costs, which are proportional to the quantity supplied, and the second representing learning by doing in research. Notice that the benefit of a cost-reduction innovation depends on the quantity produced, since it determines the amount of saved resources following such a reduction.

Given that the quantity produced determines the innovation effort, the way in which quantities are decided is fundamental for growth. This is in equation (8). In particular, we are interested in understanding the effect of a change in the number of firms on the incentives to innovate. In our model, an increase in the number of firms generates two different, opposite forces. On the one hand, the market share of each firm reduces, which can be seen in the last term of condition (8), since $l = \frac{L}{nN}$. This is the size effect or the market share effect. On the other hand, the markup $\frac{1}{\theta}$ depends negatively on the perceived elasticity of demand $\frac{n}{1-\alpha}$. Consequently, an increase in the number of firms has a positive effect on quantities by increasing the inverse of the markup, represented by the first term on the right hand side of (8). This is the competition effect.
The labor market clearing condition is

\[ nN(L^x_t + L^z_t) + L^y_t = L. \] (11)

The financial market-clearing condition implies that the aggregate asset demand \( LA_t \) is equal to the stock market value of firms:

\[ LA_t = nNV_t. \] (12)

Finally, let us impose the market-clearing condition in sector \( Y \):

\[ LE^y_t = L^y_t. \] (13)

### 2.2 Balanced growth path

A Balanced Growth Path (BGP) is an equilibrium path in which variables \( L^x_t, L^z_t, L^y_t, r_t, E_t, E^y_t, q_t \), are constant and \( q_t, z_t, v_t, p_t \) grow at a constant rate. The following proposition shows that it exists and is unique.\(^7\)

**Proposition 2** An interior BGP exists and is unique

**Proof.** Combining (3), (8), (9) and (10), under \( \dot{L}^z_t = 0 \), we get

\[ \theta \gamma (L^z)^{\gamma - 1} tE = \rho. \]

\(^7\)In Appendix B, we also show that the economy jumps to its BGP at the initial time.
Substituting the latter equation, (2), (4), (8), and (13) into the labor market-clearing condition (11), we get

\[ f(L^z) = \left( \frac{1 + \theta}{\theta} \right) \frac{\rho}{\gamma} (L^z)^{1-\gamma} + L^z = l. \]  

(14)

Since \( f(.) \) is monotonically increasing, and satisfies the limit conditions \( \lim_{x \to 0} f(x) = 0 \) and \( \lim_{x \to l} f(x) > l \), existence and uniqueness derive directly from the intermediate value theorem.

**2.3 Output growth**

In this economy, production in sectors \( Y \) and \( X \) do not grow at the same rate. Consistent with national accounts, let us define growth by the mean of a Divisia index, meaning that the growth rate of real output is equal to the growth rate of both final sectors weighted by the share of each sector on nominal output. Since the homogeneous sector is not growing, and preferences are logarithmic, the growth rate of output is

\[ g = \frac{1}{2} \frac{\dot{q}}{q} = \frac{1}{2} \frac{\dot{z}}{z} = \frac{1}{2} (L^z)^\gamma. \]

Technical progress only affects sector \( X \), making the growth rate depend on the amount of labor allocated to research in this sector.

\( \theta \) is the inverse of the markup and may be seen as a measure of the degree of competition. By differentiating (14), the growth rate can be easily shown to be increasing in \( \theta \). This is what we have referred before as the competition effect.
There is a positive relation between the degree of competition and the perceived elasticity of demand, which depends positively on both the number of firms $n$ and the elasticity of substitution $\alpha$. As we have commented before, an increase on $\theta$ leads firms to increase the quantity produced. Given that innovation can be exploited in a large number of units, firms increases innovation too. This result is the opposite to that found in monopolistic competitive models, where a rise in the elasticity of substitution decreases the markup and reduces the innovation rate. When incumbents carry out process innovation, the scale of operation becomes an important determinant of R&D decisions. The rise in the perceived elasticity of demand increases the quantity supplied and therefore the return to innovation.

3 Free trade

Let us assume that countries are identical. Since both economies are equal in factor endowments and initial stocks of knowledge no pattern of specialization from trade is observed and all the gains from trade comes from an increase in competition.

Let us assume that transportation costs are of the iceberg type; precisely, $(1+\tau)$ units of the product must be shipped in order to serve 1 unit abroad, where $\tau > 0$ is the percentage of total production that disappears in the process of shipping. Notice that for foreign firms selling in the domestic market, the markup in autarky has to be larger than the transportation costs, meaning that there is trade iff $\tau < \frac{1}{\eta}$. Let us assume it in the next.
Under international trade, firms are able to serve both markets so some clarification about the notation must be made. Let us define the quantity \( q^h_{ct} \) as the quantity supplied by a firm located in country \( h \) to market \( c \), where \( c, h \in \{A, B\} \). That is \( q^A_{Bt} \) is the quantity supplied by the B-firm to the A-market. Whenever only one superscript appears it indicates that the variable is defined for that economy, that is, \( E^x_A \) would be the expenditure assigned to \( X \) by households located in country \( A \).

Under symmetry, first order conditions for country \( A \) under free trade are:

\[
\left( \frac{IE^{xA}_t}{q^A_{At} + q^A_{Bt}} \right)^{1-\alpha} P^A_t \left( \frac{(n - (1 - \alpha)) q^A_{At} + n q^A_{Bt}}{n(q^A_{At} + q^A_{Bt})} \right) = (z^A_t)^{-1}, \tag{15}
\]

\[
\left( \frac{IE^{xB}_t}{q^B_{Bt} + q^B_{At}} \right)^{1-\alpha} P^B_t \left( \frac{(n - (1 - \alpha)) q^B_{At} + n q^B_{Bt}}{n(q^B_{At} + q^B_{Bt})} \right) = (z^A_t)^{-1}(1 + \tau), \tag{16}
\]

\[
1 = \gamma v^A_t (L^A_t)^\gamma z^A_t, \tag{17}
\]

\[
\frac{(z^A_t)^{-2} (q^A_{At} + (1 + \tau)q^B_{Bt})}{v^A_t} + (L^A_t)^\gamma = \frac{-v^A_t}{v^A_t} + r_t. \tag{18}
\]

Conditions (17), (18) are identical to conditions (9), (10) except from the fact that in (18), when computing the return on innovation, firms take into account quantities supplied to both markets. Conditions (15), (16) determine the optimal quantities supplied in each market and are analogous to condition (8), but one for each market. Notice that firms do not supply the same quantities to both market. B-firms solve an identical problem and their first order conditions are equal to those of country \( A \) but changing the subscripts and the superscripts, from \( B \) to \( A \).
and vice versa.

In order to complete the definition of an equilibrium allocation, market clearing conditions need to be added:

\[ nN(L_i^x + L_i^y) + L_i^y = L, \quad h = \{A, B\}, \]  \hspace{1cm} (19)

\[ LA_i^h = n Nv_i^h, \quad h = \{A, B\}, \]  \hspace{1cm} (20)

\[ L(E_t^y + E_{t}^y) = L_t^y + L_t^y. \]  \hspace{1cm} (21)

### 3.1 Balanced growth path

A balanced growth path for this economy is a symmetric equilibrium in which variables \( L_i^x, L_i^y, L_i^z, E_t, E_{t}^y, r_t, q_t^y \) are constant and variables \( z_t, q_{A_t}^A = q_{B_t}^B, q_{A_t}^B = q_{B_t}^A, v_t, p_t \) grow at a common constant rate (we have omitted some supraindexes).

**Proposition 3** Under \( \tau < \frac{1-\alpha}{n-1+\alpha} \), a balanced growth path exists and is unique.

**Proof.** See Appendix A. \( \blacksquare \)

As shown in the appendix, equilibrium conditions can be reduced to the following equation in \( L^z \)

\[ f^*(L^z) = \left( \frac{1 + \theta^*}{\theta^*} \right) \frac{\rho}{\gamma} (L^z)^{1-\gamma} + L^z = l, \]  \hspace{1cm} (22)
which is in fact the same equation than in autarky but with $\theta^*$ given by:

$$
\theta^* = \frac{(2n - 1 + \alpha)(2(1 - \alpha)(1 + \tau) + \tau^2(1 - \alpha - n))}{n(2 + \tau)^2(1 - \alpha)}.
$$

(23)

The question is whether the growth rate of technological progress is higher under free trade than under autarky or in another terms, whether $\theta^* > \theta$.

**Proposition 4** Under $\tau < \frac{1-\alpha}{n-1+\alpha}$, $\theta^* > \theta$.(The growth rate under free trade is always higher than in autarky)

**Proof.** See Appendix A. ■

Trade openness has no effect on the r.h.s. of equation (22) because neither local resources, nor the local number of producers changes. In other words, the increase in the number of competitors in each country has no size effect, since firms are at the same time selling in both countries. However, the increase in the number of competitor has an effect through competition. In the extreme case $\tau = 0$, the markup takes the same functional form as in autarky, but with $2n$ instead of $n$ as the number of competitors. A reduction in markups puts the competition effect at work as already explained in the previous section. Even if firms are selling less in their domestic market than under autarky, the global quantity they supply is larger, because of the competition effect. Therefore, openness to international trade leads to more innovation and growth. Proposition 4 shows that the competition effect also works under trade frictions. It is important to notice that this result is not driven by any scale effect, since the number of workers per firm $l$
is equal in both cases, under autarky and trade openness. Now, we proceed to
discuss some comparative statics.

Transportation costs are a barrier for foreign competitors reinforcing the market
power of domestic firms and making the competition effect less effective, as shown
in the proposition below.

**Proposition 5** An increase in transportation costs has a negative impact on the
rate of innovation

**Proof.** It can be easily shown by differentiating (23) with respect to $\tau$. ■

Finally, the difference between R&D investment in both regimes, autarky and
free trade, is small when the number of firms is large. This is due to the fact
that $n$ has a non-linear impact on produced quantities; while for a small number
of firms the reduction in quantities due to free trade is important, for a large
number of firms it has a very small impact. For example, the gains from trade
completely vanish when competition in autarky is at the largest value compatible
with positive trade. Remind that there is trade if and only if $\tau < \frac{1 - \alpha}{n - 1 + \alpha}$, or
equivalently iff $n < \frac{1 + \tau}{\tau} (1 - \alpha)$. It is easy to see that $\theta^* = \theta$ if $n = \frac{1 + \tau}{\tau} (1 - \alpha)$.
The fundamental reason is that firms’ response to the increase in competition due
to trade openness is strong when the autarky level of competition is low, while in a
competitive autarky economy, the response of firms to an increase in competition
is quite small since they already have very little market power.
4 Conclusions

This paper develops an endogenous growth model with firm specific innovations, Cournot competition on a continuum of oligopolistic markets and free trade between identical economies. It shows that international trade induces growth in participant countries through an increase in competition. The paper differs from the literature by constructing an environment where openness to trade generates a pure increase in competition and makes firms to innovate more to profit from the associated increase in market size. This research reinforces the view that at least for the case of developed countries trade openness enhances innovation and growth through an increase in competition.

By restricting the analysis to identical economies, the present paper may be seen as a contribution to the understanding of the growth effect of regional integration agreement among similar countries, as it is the case of France and Germany in the European Union and to some extend Brazil and Argentina in the MERCOSUR. A natural extension will be the study of economies with different initial conditions (i.e. different technological levels) or different factor endowments. This would make possible the study of the interaction between developed and developing economies, as it is the case of Mexico and the US in NAFTA or the accession of Ireland and Spain to the EU. Differences in the initial stock of knowledge determine the initial differences in marginal costs and market shares; differences in market size depend upon differences in factor endowments. The innovation path
of both economies will be determined by how these two forces interact.

As a complement to that extension we can explore how different trade policies affect the results. The model will predict that unilateral trade policies will reduce growth in the liberalizing country since the increase in competition coming from this policy is offset by the creation of an artificial comparative advantage to foreign firms. However, preferential trade liberalization agreements, will enhance growth in the liberalizing countries reducing growth in the protectionist country due to the fact that, the reduction of trade barriers between the two countries increases competitiveness of these firms in both economies with respect to firms in a third country.\(^8\)

Another interesting extension would allow for sectorial differences. In this case, it would possible to identify sectors having larger gains from trade. Considering, for simplicity, intersectorial independence, we suspect that the less competitive sectors will have larger gains from trade.

References


\(^8\) Preferential trade liberalization reduces trade barriers between two economies, but it does not alter the symmetry properties of firms in both countries. In this case market shares of the firms of both liberalizing countries increase fostering innovation and creating a comparative advantage over a third protectionist country. These results are complementary to those derived in Melitz and Ottaviano (2005) in which unilateral and preferential agreements were having similar effects on industry aggregate productivity in a static framework.


**A Free Trade**

**Proposition 3** Under \( \tau < \frac{1-\alpha}{n-1+\alpha} \), a balanced growth path exists and is unique.

**Proof.** Under symmetry, \( z^A_t = z^B_t = z_t \), \( q^A_{At} = q^B_{Bt} = q_t \) and \( q^B_{At} = q^A_{Bt} = \dot{q}_t \), for all.
Taking condition (15) for both countries, we get

\[
\frac{(n - 1 + \alpha)q_t + n\tilde{q}_t}{nq_t + (n - 1 + \alpha)\tilde{q}_t} = \left( \frac{1}{1 + \tau} \right),
\]

implying

\[
\tilde{q}_t = \frac{(1 - \alpha)(1 + \tau) - n\tau}{1 - \alpha + n\tau}q_t,
\]

which requires

\[
\tau < \frac{1 - \alpha}{n - 1 + \alpha}
\]

to \( q \) and \( \tilde{q} \) be simultaneously positive.

Under symmetry, \( E_i^A = E_i^B = E_t, P_i^A = P_i^B = P_t \) and \( p_{jt} = p_t \) for all \( j \) and \( t \), implying that the inverse demand function (6) for any variety produced in any country becomes

\[
p_t = \left( \frac{LE}{n(q_t + \tilde{q}_t)} \right)^{1-\alpha} P_t = \frac{LE}{q_t + \tilde{q}_t},
\]

the last equality follows from the definition of the price index \( P \). Substituting the latter condition and (24) in (15) and rearranging terms, it follows

\[
q_t = \left( \frac{(1 - \alpha + n\tau)(2n - 1 + \alpha)}{n(2 + \tau)^2(1 - \alpha)} \right) z_t LE
\]

\[
\tilde{q}_t = \left( \frac{((1 - \alpha)(1 + \tau) - n\tau)(2n - 1 + \alpha)}{n(2 + \tau)^2(1 - \alpha)} \right) z_t LE.
\]

At the balanced growth path, \( r_t = \rho \) from (5), and \( \frac{1}{z} = (L^z)^\tau \) from (3). From (17),
(18), (25) and (26), we obtain:

\[
\gamma (L^z)^{\gamma -1} \theta^* l E = \rho
\]

where, by analogy with the autarky case,

\[
\theta^* = \frac{(2n - 1 + \alpha)(2(1 - \alpha)(1 + \tau) + \tau^2(1 - \alpha - n))}{n(2 + \tau)^2(1 - \alpha)}.
\]

From the labor market clearing condition (19),

\[
L^x + L^z + \frac{L^y}{nN} = l.
\]

From (21) and (4), \(L^y = LE\); from (2), \(q + (1 + \tau) \dot{q} = z L^x\). Substitution \(q\) and \(\dot{q}\)
by their expressions in (25) and (26), we get

\[
\mu(L^z) = \frac{1 + \theta^*}{\theta^*} \frac{\rho}{\gamma} (L^z)^{1 - \gamma} + L^z = l,
\]

i.e., is the same equation as in the autarky model but with \(\theta^*\) instead of \(\theta\). Interiority and uniqueness of the solution is therefore ensured by looking at the autarkic balanced growth path proof.

Proposition 4 Under \(\tau < \frac{1-\alpha}{n-1+\alpha}\), \(\theta^* > \theta\)
Proof. From the definition of $\theta^*$ and $\theta$,

$$
\theta^* - \theta = \frac{(2n - 1 + \alpha)(2(1 - \alpha)(1 + \tau) + \tau^2(1 - \alpha - n))}{n(2 + \tau)^2(1 - \alpha)} - \frac{n - 1 + \alpha}{n}
$$

$$
= \frac{(1 - \alpha)^2(1 + \tau) + \tau^2 n(1 - \alpha - n)}{n(2 + \tau)^2(1 - \alpha)}.
$$

It can be easily shown that the r.h.s. is decreasing in $\tau$, with $\theta^* - \theta = 0$ when $\tau$ is at its maximum value $\frac{1-\alpha}{n-1+\alpha}$. □

B Stability analysis under autarky

Let us combine equations (2), (8) and (11) to get

$$
L^z = \theta l E
$$

$$
\frac{1 + \theta}{\theta} L^z + L^z = l,
$$

implying

$$
E = \frac{l - L^z}{(1 + \theta) l}.
$$

By logdifferentiation,

$$
\frac{\dot{E}}{E} = -\frac{L^z \dot{L}^z}{l - L^z \dot{L}^z}.
$$
From (5),

\[ r = \rho - \frac{L^z \dot{L}^z}{l - L^z L^z}. \]  \hspace{1cm} (27)

Adding (3) and (10), we get

\[ \dot{z} + \frac{\dot{v}}{v} = r - \frac{q}{z} \frac{1}{zv} = r - \gamma \theta l (L^z)^{\gamma - 1} E = r - \frac{\theta \gamma}{1 + \theta} (L^z)^{\gamma - 1} (l - L^z). \]  \hspace{1cm} (28)

The second equality emerges after substituting \( \xi \) and \( \frac{1}{zv} \) by their expressions in (8) and (9), respectively, and the last one after substituting the expression for \( E \) computed just above. Differentiating (9)

\[ \dot{z} + \frac{\dot{v}}{v} = (1 - \gamma) \frac{\dot{L}^z}{L^z}. \]

Substituting it and (27) in (28), we get

\[ \frac{\dot{L}^z}{L^z} = \left( \rho - \frac{\theta \gamma}{1 + \theta} (L^z)^{\gamma - 1} (l - L^z) \right) \left( \frac{l - L^z}{(1 - \gamma) l + \gamma L^z} \right). \]

The sign of the second term in the r.h.s. is positive since \( L^z \leq l \). The unique interior steady state, let us denote it by \( L^* \), is obtained by equalizing the first element of the r.h.s. to zero (condition (14)). As it can be easily seen, \( \dot{L}^z < 0 \) for \( L^z \in (0, L^*) \) and \( \dot{L}^z > 0 \) for \( L^z \in (L^*, l) \), implying that the interior steady state is unstable. Consequently, the only rational expectation equilibrium is \( L_t^z = L^* \) for all \( t \geq 0 \).