THE ROLE OF SEARCH FRICTIONS FOR OUTPUT AND INFLATION DYNAMICS: A BAYESIAN ASSESSMENT[*]

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Abstract

Search frictions in the goods market have proven to be a fruitful deviation from the fiction of a centralized Walrasian market providing a micro-foundation of the use of money as a medium of exchange. Moreover, persistent propagation of monetary shocks can arise in search-theoretic monetary models through the interaction of search-frictions in the goods and labor markets, and inventory holdings.

Here, a search-theoretic monetary DSGE model with capital and inventory investment is estimated, and its implications on output and inflation dynamics are contrasted with those of standard flexible price monetary models: a cash-in-advance and a portfolio adjustment cost model. Model estimation and comparison is conducted in a Bayesian way in order to account for possible model misspecification.

The search model can track inflation and output data better, as well as it dominates the other models in the ability to predict the autocorrelations of inflation and the persistent disinflation process after a technology shock. It generates a hump-shaped but not strong enough output response to a monetary shock. Current and near current correlations between output growth inflation are predicted well.

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1 Introduction

Starting from the models of Kiyotaki and Wright [11], [12] search theory has developed into the main paradigm of the microfoundation of money. Giving money an essential role, i.e., the use of money augments the set of allocations[1] this approach has become a useful tool for monetary theory. However, little quantitative analysis has been undertaken so far with this kind of models[2].

This paper addresses quantitatively the implications of search frictions in the goods market for inflation and output dynamics. I consider three different dimensions: Can a search-theoretic model track US output growth and inflation data better than other standard flexible price monetary models? Can it create more realistic contemporary and lagged correlations between output growth and inflation? And: How well do dynamic responses to shocks to money growth and technology match its empirical counterparts? Answering these questions helps us to assess whether modeling search frictions in the goods market has the potential to improve substantially the models to be used as laboratories to study the effects of monetary policy.

Search-theoretic monetary business cycle models explore the consequences of search frictions in the goods market for aggregate variables in business cycle frequencies, but are not tailored to fit the data. Thus, any version of this model class is probably highly misspecified, i.e. we cannot believe that any of these models comes close to the true data generating process (DGP). Obviously, one could try to enrich a search model with many additional features like habit formation, investment adjustment costs, sticky prices and wages, etc so as to deal with less misspecified models in the end. Apart from the big effort that would have to be undertaken the question arises what one can really learn from a comparison of highly complex models where many frictions interact with each other. To keep the models as simple as possible I follow Schorfheide [18] in applying a Bayesian methodology that allows to compare potentially misspecified models.

The model chosen from the class of search-theoretic monetary models (STM) is a full fledged business cycle model with search frictions in the goods

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[1]See e.g. Kocherlakota [13] and Wallace [22] on the issue of essentiality.

[2]In his lecture at the Canadian Economics Association Meetings (Hamilton 2005), published in Shi [20], Shi gives an overview of the literature, highlighting the quantitative contributions of Shi [19], Wang and Shi [23] , and Menner [15] and urging to conduct more quantitative analysis in the field of monetary search-theory.

and labor market[3]. The model includes capital formation and quadratic capital adjustment costs and is outlined and solved in Menner [15] [4]. There are several reasons for this choice. First, there are only few models of the search-theoretic literature capable to address macroeconomic issues. To study the effects of changes in money growth the early literature had to assume an upper bound in money holdings.[5] Shi [19] was the first to develop a tractable search-theoretic Dynamic Stochastic General Equilibrium (DSGE) model where prices are determined endogenously and money holdings are not bounded. His model exhibits a persistent mechanism to propagate monetary shocks that arises from the interaction of search-intensity and inventory investment but lacks the possibility of capital formation. Allowing for capital formation as in Menner [15] potentially helps the model to propagate shocks as it does in standard business cycle models. So, we want to allow the search model to make use of this feature in order to match the data. Furthermore, capital formation breaks the close link between employment and output present in a model with fixed capital[6]. Since we are interested in inflation and output dynamics we do not want to rely too heavily on outcomes of the labor market in determining output responses and hence use a model with capital formation.

Faig [7] has develloped recently an alternative model where the production sector is neoclassical and capital is accumulated by using the firm's own product as investment. The commerce sector is separated from the production sector. His model differs in many other details from the present model and the analysis centers on welfare implications of money growth accross different steady states. Transitional dynamics are not considered: he studies only monetary policies that keep the nominal interest rate constant. A higher nominal interest rate reduces the number of buyers and has ambiguous effects on the number of producers and hence output.

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[3]Search frictions in the labor market are a natural assumption in the presence of decentralized goods markets. However, in the model comparison below I also consider a version of the model with flexible labour markets.

[4]Without capital adjustment costs the calibrated model renders explosive dynamics. As shown later, by contrast, the estimation procedure can lead to parameters that imply stable dynamics also in the absence of capital adjustment costs.

[5]See Rupert et al. [17] for an excellent overview of the literature on search-theoretic monetary models before 2000.

[6]Log-linearizing the production function $y_t = n_t^{e_N} \bar{k}^{1-e_N}$ one sees immediately the proportionality between log-deviations of output $\hat{y}_t$ and employment $\hat{n}_t$: $\hat{y}_t = e_N \, \hat{n}_t$.

Recently, a different approach to avoid to assume bounded money holdings by Lagos and Wright [14] where agents alternate visiting decentralized and centralized goods markets has been used by many researchers. Although some of these Lagos-Wright type models also allow for capital formation, they assume instantaneous production, so there are no inventories. Together with the fact that all changes in money holdings in the decentralized markets are undone in the following centralized market, this presumably implies weak intertemporal links and probably a weak propagation of shocks[7].

The reader might ask why we do not compare the search model to models with sticky prices and/or sticky wages[8]. There are several answers to this. First, it is not that clear that costs of price adjustments on the firm level induce a considerable degree of price stickiness on the aggregate level. Golosov and Lucas [10] estimate the real effects of menu costs on the firm level to be very small. Therefore, one might want to step back and ask what aspects of a monetary economy lead to real effects of monetary surprises even when prices are flexible. Frictions in the goods market and in the asset market are candidates that are examined here. But even if one accepts the modelling device of sticky prices the comparison between the search-theoretic model and a sticky price model would be on unequal grounds[9]. Hence, from the class of flexible price models a cash-in-advance (CIA) model and a portfolio-adjustment cost (PAC) model are chosen. The former has as the only friction the constraint on the representative household to have enough money on hand to pay for the purchased goods, while the latter assumes, in addition, frictions in the portfolio adjustment.

We estimate the parameters of the models to be compared using Monte Carlo Markow Chain methods to draw from the posterior distribution. By comparing marginal data densities we find that the search-theoretic model tracks the time series of U.S. output growth and inflation better than the

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[7]Arouba and Wright [2] find a dichotomy between the real and monetary sector, while Arouba, Waller and Wright [1] in a very recent paper propose different variations where the monetary trades in the decentralized goods market have some influence on capital formation. A comparison of the present STM model with these type models is left for future research.

[8]Models with nominal rigidities are now widely used for policiy evaluation. See, for the most prominent models Christiano, Eichenbaum and Evans [6] and Smets and Wouters [21].

[9]In principle one could introduce also some price stickiness in the search model. But this is a more elaborate task and seems to be worth while only after knowing that the search model can explain the data as least as good as other flexible price models.

portfolio adjustment cost model and the standard CIA model - coming close to VAR´s with 1 to 4 lags. Loss functions are used to compare the ability of the models to account for current, lagged and leading cross correlations of output growth and inflation. The expected loss, or risk, a researcher incurs when choosing the STM model is considerably lower for "lags" -1,0 and 1 than the ones he incurs when choosing one of the alternative models. However, when we look at 2 periods ahead and behind the STM model ranks least. Moreover, while the STM model improves slightly on the CIA model in replicating the dynamic responses to shocks to money growth and technology, it is the PAC that minimizes the loss in this dimension with considerable difference. The propagation mechanism of the STM model seems neither to be strong enough to replicate the persistence present in output, nor to compete with the imposed frictions on portfolio adjustment that turn out to be estimated to be strong. On the contrary, the STM model can predict well the persistant disinflation process after a technology shock and the autocorrelations up to lag 4 of inflation, which neither of the two other models can. Hence, search frictions in the goods market add a new propagation mechanism to the CIA model that behaves in some dimensions similar, but in some dimensions different to the mechanism created by frictions in the portfolio management of consumers.

The rest of the papers is organized as follows: In Section 1 the three models are outlined, and it is shown how they are solved, detrended and how the policy functions are transformed into state space form. Section 2 lays out the empirical strategy of Bayesian estimation and model evaluation. The results of the estimation process and the model comparison are presented in section 3. Section 4 concludes.

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2 The Models

In the following I will present the two models to be compared. Since the reader is probably less familiar with the search-theoretic monetary model than with the portfolio adjustment cost model I will explain the former in more detail and restrict myself to a short exposition of the latter.

2.1 The Search-Theoretic Monetary (STM) Model

2.1.1 The Economy and its Matching Process

In the model there are two search frictions: costly search for consumption and investment goods, as well as costly labor search. The economy is populated by a continuum of households with measure one, denoted by H. A continuum of goods with measure one, also denoted by H, can be produced with labor and fixed capital as inputs to production. Each good is storable only by its producer.Purchased investment goods can be installed as capital by incurring an installation cost, i.e. there exists a (quadratic) capital adjustment cost. Each household $h \in H$ produces good h and wants to consume a subset of goods different from its own product, and only goods from this subset can be used as capital for the production of good h. This induces a need for exchange before consumption / investment is possible. In the absence of a centralized market with a Walrasian auctioneer households have to search for trading partners with the desired goods. Generally, there will be no double-coincidence of wants. The literature following Kiyotaki and Wright [11],[12] established that in random search models under certain parametrizations fiat money gets valuable and is the only medium of exchange. To establish this in the present model would require a more detailed consideration of the exchange patterns. Instead, here it is simply assumed that fiat money is required in each transaction.

The matching in the goods market between sellers and buyers and in the labor market between producers and unemployed is assumed to be random. Hence, individual agents would face idiosyncratic risks: a priori, buyers do not know whether they can find the desired consumption/investment good and exchange it for the money and sellers do not know whether their product will be purchased. As a consequence money holdings, inventories of unsold goods and capital stocks would not be equally distributed among households and hence are individual state variables, as well as the employment status

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and the number of vacancies. To avoid the need of tracking the distributions of these individual state variables, it is assumed that the decision unit - the household - consists itself of a continuum of different agents. The members of the household share the purchased consumption-investment goods and regard the household's utility as the common objective. The household decides how much to consume and how much to invest. All the firms of a household are asigned the same amount of investment goods. Hence, after incurring the installation cost, all start the next period with the same capital stock. They also equally share workers and inventories. Finally, resource sharing of firms within a household allows the payment of wages regardless of whether the firms had a suitable match in the goods market. Under these assumptions the random matching process does not create idiosyncratic risk.

The household consists of five groups: one group of members enjoys leisure, the other four groups are active in markets: Entrepreneurs (set A_p with measure a_p), unemployed (A_u, measure u) workers (A_{nt}, measure: $a_p n_t$), and buyers (A_b, measure a_b). The values of a_p, u and a_b are assumed to be constant, while the number of workers $a_p n_t$ may vary over time. An entrepreneur consists of two agents: a producer and a seller. A producer in household h hires workers from other households to produce good h, which is sold by the seller. A worker inelastically supplies one unit of labor each period to other households' firms. A buyer searches with search intensity $s > 0$ to buy the household's desired good. The sellers search intensity is normalized to 1. In the following a hat on a variable indicates that the household takes this variable and all its future values as given when making the decisions at t.

The number of goods market matches is given by the matching function:

$$g\left(\hat{s}\right) \equiv z_1 \left(a_b \hat{s}\right)^\alpha \left(a_p\right)^{1-\alpha} = z a_b \hat{s}^\alpha, \qquad z \equiv z_1 \left(\frac{a_b}{a_p}\right)^{\alpha-1}.$$

Let $B = a_b/a_p$ be the buyers/sellers ratio. The matching rate per unit of search intensity is $g_b\left(\hat{s}\right) \equiv z\hat{s}^{\alpha-1}$, so that a buyer finds a desirable seller at a rate sg_b, and a seller meets a desirable buyer at a rate $g_s\left(\hat{s}\right) \equiv zB\hat{s}^\alpha$. Thus the measure of the set of buyers with suitable matches, A_{b*}, is $sg_b a_b$ and that of sellers with suitable matches, A_{p*}, is $g_s a_p$.

Each buyer j having found a seller $-j$ with his desired good exchanges $\hat{m}_t\left(j\right)$ units of money for $\hat{q}_t\left(-j\right)$ units of good $-j$, which implies a price of good $-j$ in this match of $\hat{P}_t\left(j\right) = \hat{m}_t\left(j\right)/\hat{q}_t\left(-j\right)$ and an average price of goods of $\hat{P}_t$.

Each producer j can create vacancies v_t with a cost of $\Upsilon(v_t)$. Unemployed workers have to search for a job and they do this supplying one unit of search effort inelastically. A worker supplies inelastically one unit of labor each period and receives a wage $\hat{W}$ in units of money. There is an exogenous constant job separation rate δ_n.

The matching function in the labor market is linearly homogeneous. The number of matches between firms and unemployed workers is given by $(a_p \hat{v})^A (u)^{1-A}$ and the number of matches per vacancy is $\mu(\hat{v}) \equiv (a_p \hat{v}/u)^{A-1}$.

2.1.2 The Household's Decisions

At the beginning of period t each household receives a lump sum monetary transfer τ_t from the central bank. The household distributes the available money M_t evenly among the buyers. Then the four active groups go to their respective markets and do not meet until the end of the period. At the end of the period the members of the household arrive at home carrying their trade receipts and residual balances and profits, respectively. They consume altogether the fraction of the bought goods who was dedicated for consumption and share the rest among the firms to increase each firm's capital stock. Also, goods inventories and employees are shared among the household's firms. Finally, the money not spent by the buyers, the wages earned and the profits are added to the money balance of the household for next period's shopping.

Households decide at the beginning of each period about their consumption c_t, their total investment x_t and next period's total capital stock K_{t+1}, as well as on the amount of 'fiat' money in the next period M_{t+1}. We impose symmetry within a household, i.e. that each member of a group is assigned the same stocks of capital and money and the same decision rules. Thus, each buyer will receive $m_{t+1} = M_{t+1}/a_b$ units of money and each firm will have a capital stock $k_{t+1} = K_{t+1}/a_p$. Households choose the buyers' search intensity s_t, the number of vacancies for the firms v_t, and the inventory level and the amount of labor in each of their firms in period $t+1$, i_{t+1} and n_{t+1}. The quitting rate δ_n and the depreciation rates of inventories δ_i and capital δ_k are assumed to be constant. The individual firm's production technology has the form $f^i(n,k) = \Psi_t^{e_N} n_t^{e_N} k_t^{1-e_N}$, where $e_N < 1$. For convenience denote $f(n_t, K_t) \equiv f^i\left(n_t, \frac{K_t}{a_p}\right) = F_0 \Psi_t^{e_N} n_t^{e_N} K_t^{1-e_N}$ the individual firm's production function in terms of K, where $F_0 = a_p^{e_N - 1}$.

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In their decision households take the sequence of terms of trade and wages $\left\{\hat{q}_t, \hat{m}_t, \hat{W}_t\right\}_{t \geq 0}$ as given, as well as $\{M_0, K_0, i_0, n_0\}$. Since both buyers and sellers have a positive surplus from trade, it is optimal for households to choose $M_{t+1}, K_{t+1}, n_{t+1}$ and i_{t+1} such that in period $t+1$ every buyer carries the required amount of money $\hat{m}_{t+1}$ and that every seller has $\hat{q}_{t+1}$ units of good h to be sold. The assumptions $M_0 \geq \hat{m}_0 a_b$ and $i_0 + f(n_0, K_0) \geq \hat{q}_0$ ensure that buyers and sellers carry the necessary amounts of money and goods also in period 0.

Finally we have to specify preferences. We assume log utility in consumption, the disutility of working one unit of time is denoted by φ, the disutility of a buyer's search intensity is $\phi(s) = \varphi(0.5 \cdot s)^{1+1/e_\phi}$, and the disutility of maintaining a vacancy is $\Upsilon(v) = K_0 v^2$.

Households choose the sequence $\{c_t, x_t, s_t, v_t, M_{t+1}, K_{t+1}, i_{t+1}, n_{t+1}\}_{t \geq 0}$ to maximize their expected lifetime utility:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \mid A_n \mid \varphi - \mid A_b \mid \phi(s_t) - \mid A_p \mid \Upsilon(v_t) \right] \right\} \quad (\text{PH})$$

subject to:

$$c_t + x_t + \frac{b}{2} \left(\frac{x_t}{K_t} - \delta_k \right)^2 K_t \leq \mid A_{b_t^*} \mid \hat{q}_t \tag{1}$$

$$K_{t+1} \leq (1 - \delta_k) K_t + x_t \tag{2}$$

$$\frac{M_t}{a_b} \geq \hat{m}_t, \qquad on \ A_{bt^*} \tag{3}$$

$$i_t + f(n_t, K_t) \geq \hat{q}_t, \qquad on \ A_{pt^*} \tag{4}$$

$$\begin{aligned} M_t + \tau_t - &\mid A_{b_t^*} \mid \hat{m}_t + \mid A_{n_t} \mid \hat{P}_t \hat{W}_t \\ + &\mid A_{p_t^*} \mid \hat{m}_t - \mid A_p \mid \hat{P}_t \hat{W}_t n_t \geq M_{t+1}, \end{aligned} \tag{5}$$

$$\mid A_p \mid \left[(1 - \delta_n) n_t + v_t \mu_t - n_{t+1} \right] \geq 0 \tag{6}$$

$$(1 - \delta_i) \left[\mid A_p \mid \left[i_t + f(n_t, K_t) \right] - \mid A_{p_t^*} \mid \hat{q}_t \right] \geq \mid A_p \mid i_{t+1} \tag{7}$$

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The first constraint states that the household´s consumption and investment (plus quadratic investment cost) has to be bought by buyers which successfully meet a trading partner and have been endowed with sufficient money for the purchase of $\hat{q}_t$ each. The second one is the usual capital accumulation equation. The third condition represents a minimum money holdings constraint for each suitably matched buyer, while the third is a similar trading restriction for suitably matched sellers: each should have a sufficient stock of inventory and newly produced goods to satisfy the demand of the costumer. The law of motion of money balances states that money holdings at the beginning of period t+1 are no bigger than money holdings at the beginning of period t augmented by the monetary injection minus the money spent plus wages earned and cash receipts from firms. Next, a household can not allocate more workers of other households to its firms than those who worked in firms of the household in period t and have not quitted plus the newly hired workers. Inventories in period t+1 consist of the fraction of the excess supply of goods in period t that has not depreciated.

2.1.3   Solution of the model

Optimality conditions can be derived which together with the laws of motion for money balances, employment and inventories (5) - (7) determine the solution to this decision problem, once the terms of trade are specified and the equilibrium conditions are imposed[10]. The terms of trade are determined by Nash bargaining.

A symmetric search equilibrium is defined as a sequence of household's choices $\{\Gamma_{ht}\}_{t\geq 0}$, $\Gamma_{ht} \equiv (x_t, s_t, v_t, M_{t+1}, i_{t+1}, n_{t+1}, K_{t+1})_h$, expected quantities in trade $\left\{\hat{X}_t\right\}_{t\geq 0}$, $\hat{X}_t \equiv \left(\hat{m}_t, \hat{q}_t, \hat{W}_t\right)$, terms of trade $\{X_t\}_{t\geq 0}$ and expected average variables $(\hat{s}_t, \hat{v}_t)$, such that

(i) these variables are identical across households and relevant individuals;

(ii) given $\left\{\hat{X}_t\right\}_{t\geq 0}$ and (M_0, i_0, n_0, K_0), $\{\Gamma_{ht}\}_{t\geq 0}$ solves (PH) with $(s, v) = (\hat{s}, \hat{v})$;

(iii) X_t is a solution to the Nash bargaining process;

(iv) $\hat{X}_t = X_t \quad \forall\, t \geq 0$.

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[10]See Menner [15] for details.

2.2 The Portfolio Adjustment Cost (PAC) Model

Out of the class of flexible price monetary models I take as benchmark a cash-in-advance model with portfolio adjustment costs (PAC). The model is discussed in detail in Christiano [4], Christiano and Eichenbaum [5], and Nason and Cogley [16]. The model economy is populated by a representative household, a firm and a financial intermediary. At the beginning of period t, the household holds the entire money stock M_t of the economy and decides how much money to deposit as savings deposits D at the bank and how much to hold as cash Q. The household has to decide its split into liquid and illiquid assets before shocks are known. Or put it different, after the shocks are known the household decides on cash holdings and deposits for the next period: Q_{t+1} and $D_{t+1} = M_{t+1} - Q_{t+1}$. Cash does not pay interest but is needed to buy consumption goods. Deposits earn an interest rate r^d. The bank receives a monetary injection and lends it together with the deposits at rate r^f to the firm. Since the household cannot change its deposits after a surprise change in the monetary injection, the additional funds have to be absorbed by the firm, which in turn reduces the interest rate. Thus, this liquidity effect arises from the 'limited participation' of the household in the asset market. To render this effect more persistent Christiano and Eichenbaum [5] assume in addition to this limited particiaption setup that portfolio management is time consuming and therefore reduces utility by foregone leisure to the amount of:

$$\tilde{p}_t = \alpha_1 \left[\exp\left(\alpha_2 \left[\frac{Q_t}{Q_{t-1}} - m^* \right] \right) + \exp\left(-\alpha_2 \left[\frac{Q_t}{Q_{t-1}} - m^* \right] \right) - 2 \right] \quad (8)$$

After observing the shocks the household chooses consumption C_t, labour supply N_t^s, and next periods money stock M_{t+1}. The household receives wage payments $W_t N_t^s$ from the firm in the form of cash before consumption goods are purchased. The cash-in-advance constraint says that all consumption purchases must be paid for with cash at hand:

$$P_t C_t \leq Q_t + W_t N_t^s \quad (9)$$

At the end of the period the household gets back its saving deposits together with interest and receives the firm´s and the bank´s net cash inflow as dividends F_t and B_t, respectively.

In the beginning of period t after shocks are known the household chooses C_t, N_t^s, M_{t+1} and Q_{t+1} to maximize its discounted expected lifetime utility:

$$\max_{C_t, N_t^s, M_{t+1}, Q_{t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1-\phi) \ln(C_t) + \phi \ln(1 - N_t^s - \tilde{p}_t) \right\}$$

$$s.t. \quad P_t C_t \leq Q_t + W_t N_t^s$$

$$Q_t \leq M_t$$

$$M_{t+1} = (Q_t + W_t N_t^s - P_t C_t) + \left(1 + r_t^d\right)(M_t - Q_t) + F_t + B_t \quad (10)$$

The firm accumulates capital and hires labour services from the household and pays the wage bill out of the money borrowed from the bank. Then it produces using a Cobb-Douglas technology, and with the sales receipts it pays back the loan plus interest. Profits are paid as dividends to the household. Since the firm is owned by the household which values a unit of nominal dividends in terms of the consumption it buys next period its objective is to maximize the expected lifetime dividends discounted by date t+1 marginal utility of consumption. Hence the firm chooses next periods capital stock K_{t+1}, labour demand N_t^d, loans L_t, and dividends F_t to solve the problem:

$$\max_{L_t, N_t^d, F_t, K_{t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1} P_{t+1}} \right\}$$

$$s.t. \quad F_t \leq L_t + P_t \left[(\Psi_t N_t)^{e_N} K_t^{1-e_N} - x_t \right] - W_t N_t^d - L_t \left(1 + r_t^f\right)$$

$$K_{t+1} \leq (1 - \delta_k) K_t + x_t$$

$$W_t N_t^d \leq L_t \quad (11)$$

The financial intermediary is also owned by the household and solves:

$$\max_{B_t, L_t, D_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \frac{B_t}{C_{t+1} P_{t+1}} \right\}$$

$$s.t. \quad B_t \leq D_t + L_t \left(1 + r_t^f\right) - D_t \left(1 + r_t^d\right) - L_t + \tau_t,$$

$$L_t \leq D_t + \tau_t, \quad (12)$$

where $\tau_t = M_{t+1} - M_t$ is the monetary injection of the central bank.

Markets clear when $N_t^d = N_t^s$, $P_t C_t = M_t + \tau_t$, $(\Psi_t N_t)^{e_N} K_t^{1-e_N} = C_t + K_{t+1} - (1 - \delta_k) K_t$. In equilibrium also $r_t^f = r_t^d$ must hold.

2.3 The Cash-in-Advance (CIA) Model

For the purpose of model comparison it is convenient to use a version of the CIA model that can be generated from the PAC model by changing just two assumptions. First, there are no costs to adjust ones portfolio, i.e. $\tilde{p}_t = 0$. Second, there is no limited participation in asset markets because agents get to know the realisation of the money growth shock before they make their decision on deposits. This leads to the modified maximization problem of the household:

$$\max_{C_t, N_t^s, M_{t+1}, D_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1 - \phi) \ln (C_t) + \phi \ln (1 - N_t^s) \right\} \qquad (13)$$

subject to the same constraints as above, where Q_t has to be replaced by $M_t - D_t$. Since $\tilde{p}_t = 0$, the parameters α_1 and α_2 get obsolete.

2.4 Specification of Shocks and Detrending

The model economies are subjected to two exogenous shocks. The monetary injection takes place at the beginning of the period such that money growth follows an AR(1) process:

$$\ln \gamma_t = \rho_M \ln \gamma_{t-1} + \varepsilon_{M\,t}, \qquad \text{where } \gamma_t = M_{t+1}/M_t. \qquad (14)$$

The production technology is prone to a technology shock. Recall, the production function was assumed to be $f(n_t, K_t) = F_0 (\Psi_t n_t)^{e_N} K_t^{1-e_N}$ in the search-theoretic model and $f(N_t, K_t) = (\Psi_t N_t)^{e_N} K_t^{1-e_N}$ in the PAC model. We assume in both cases that labour augmenting technological progress follows a random-walk with drift:

$$\ln(\Psi_t) = \zeta + \ln(\Psi_{t-1}) + \varepsilon_{\Psi\,t}. \qquad (15)$$

The vector of innovations $\varepsilon_t = [\ \varepsilon_{\Psi\,t}\ ,\ \varepsilon_{M\,t}]'$ is assumed to be i.i.d. $N(0, \Sigma_e)$ with $\Sigma_e = diag(\sigma^2_{\Psi}, \sigma^2_M)$.

To get a stationary economy we detrend all real variables by deviding by Ψ_t. In the search model the multipliers are detrended by multiplying with Ψ_t. In the CIA and the PAC model, nominal variables and prices are transformed by $P_t = P_t \Psi_t / M_t$, Ξ_t / M_t, where $\Xi_t = [d_t, l_t, W_t]$. It can be shown that a steady state equilibrium exists in the detrended variables.

2.5 State-Space Representation

Collecting the observable variables of interest, namely output growth and inflation in a vector y_t we can represent the log-linearized equations defining equilibrium in state-space form by:

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\begin{aligned}
y_t &= \Upsilon_0 + \Upsilon_1\, s_t + \Upsilon_2\, \varepsilon_t \\
s_t &= \Phi_1 s_{t-1} + \Phi_2\, \varepsilon_t
\end{aligned}
\tag{16}
$$

where s_t is a vector of percentage deviations of detrended model variables from their respective steady state value.

The system matrices Υ_i and Φ_i are then nonlinear functions of the structural DSGE parameters θ, and the DSGE models generate a joint probability distribution for the data $Y_T = [y_{1,\dots}y_T]'$.

3 Empirical Strategy

3.1 Dealing with Model Misspecification

Our primary aim is to compare the empirical fit of different estimated DSGE models. Although their theoretical structure intends to capture various features of reality like capital formation, the use of money and several frictions, they are still highly stylized and we cannot claim that they are close to the true data generating processes (DGP) of our real world data. Schorfheide [18] proposed a Loss-function based Bayesian approach that allows to deal with this problem of misspecification and that will be used in the present paper: Using a highly-parametrized reference model that fits the data considerably well one can construct a combined DGP by averaging the models under consideration and the reference model. Deviations of model characteristics (e.g. second moments or impulse response functions) from the ones implied by the proposed DGP are then quantified via different loss functions.

3.2 Evaluation Procedure

Traditional Bayesian Model Comparison is based on the calculation of posterior odds ratios. Assigning prior probabilities to the models considered,

the posterior model probabilities can be calculated by

$$\pi_i = p(M_i/Y) = \frac{\pi_{i0}p(Y/M_i)}{\sum_i \pi_{i0}p(Y/M_i)},$$ (17)

where $p(Y/M_i)$ is the marginal data density

$$p(Y/M_i) = \int p(Y/\theta_{(i)}, M_i)p(\theta_{(i)}/M_i)d\theta_{(i)},$$ (18)

that is, the integral of the posterior (likelihood $p(Y/\theta_{(i)}, M_i)$ times the prior $p(\theta_{(i)}, M_i)$) over the parameter space. Since

$$\ln p(Y_T /M_i) = \sum_{t=1}^{T} \ln p(y_t/Y_{T-1}, M_i),$$ (19)

the log of the marginal data density can be interpreted as predictive score, i.e. as the one-step-ahead forecasting performance of model M_i.

The posterior odds ratio is then the ratio of two posterior model probabilities. Note, that these odds do not change by the introduction of a reference model since its effect on the denominator in 17 cancels out when calculating the odds ratio. The model with the higher odds could be choosen as the model that better fits the data in the above mentioned sense. This corresponds to use a (0,1) loss function, that assigns a loss of 0 to the model with higher odds and 1 to the others. When dealing with potentially misspecified models this is probably not a good criterion, since it does not give the researcher a measure of how much he looses in choosing one misspecified model over another.

The proposal of Schorfheide [18] uses loss functions to quantify the deviations of some characteristics of the model with the ones obtained from the assumed combined DGP. His methodology is charactarized by 3 steps.

Step 1
Compute posterior distributions $p(\theta_{(i)}/Y, M_i)$ for the model parameters and calculate posterior model probabilities as in 17

Step 2
As the population characteristics φ are a function of the model parameters $\theta_{(i)}$ one can generate a posterior distribution of φ conditional on model M_i by drawing from the posterior distribution of $\theta_{(i)}$.

These posteriors $p(\varphi/Y, M_i)$ are then combined to the overall posterior of φ by the mixture

$$p(\varphi/Y) = \sum_{i=0}^{3} \pi_i p(\varphi/Y, M_i), \qquad (20)$$

where the weights are determined by the posterior model probabilities.

Step 3

Loss functions are introduced that penalize deviations of DSGE model predictions $\hat{\varphi}$ from population characteristics φ. The optimal predictor of φ - based only on model M_i - is

$$\hat{\varphi}_i = \arg\min{}_{\tilde{\varphi} \in \mathbb{R}^m} \int L(\varphi, \tilde{\varphi}) p(\varphi/Y) d\varphi. \qquad (21)$$

The three DSGE models are then judged according to the expected loss (risk) of the predictor $\hat{\varphi}_i$ under the overall posterior distribution $p(\varphi/Y)$:

$$R(\hat{\varphi}_i/Y) = \int L(\varphi, \hat{\varphi}_i) p(\varphi/Y) d\varphi. \qquad (22)$$

The posterior risk $R(\hat{\varphi}_i/Y)$ provides an absolute measure of how well model M_i predicts the population characteristic φ, while risk differences across DSGE models give us a relative measure of model adequacy, allowing a quantitative model comparison. We could, therefore, choose the model that minimizes the posterior risk.

Loss functions

1) Quadratic loss function

$$L_q(\varphi, \hat{\varphi}) = (\varphi - \hat{\varphi})'W(\varphi - \hat{\varphi}), \qquad (23)$$

where W is a positive definite $m \times m$ weight matrix. As shown in Schorfheide [18], the posterior risk then deponds only on the weighted distance between $\hat{\varphi}$ and the expectation of φ with respect to the overall posterior, $E[\varphi/Y]$, but not on higher moments of the posterior distribution[11].

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[11]In this paper I use an identity matrix as weight matrix, although one could give more or less importance to some of the characteristics in the vector φ, to mimic, for instance, the different importance RBC researchers give to certain second moments in their informal comparison of simulated and actual data.

2) L_p loss function

$$L_p(\varphi, \hat{\varphi}) = I\{p(\varphi/Y) > p(\hat{\varphi}/Y)\}, \qquad (24)$$

where I denotes the indicator function that is equal to one if $x > x_o$, and zero otherwise. This loss function penalizes point predictions that lie in regions of low posterior probability. If the posterior is unimodal, the expected L_p loss tells us how far the model prediction lies in the tails of the posterior distribution, similar as are doing usual p-values.

3) L_{χ^2} loss function

$$L_{\chi^2}(\varphi, \hat{\varphi}) = I\{C_{\chi^2}(\varphi/Y) < C_{\chi^2}(\hat{\varphi}/Y)\}, \qquad (25)$$

where

$$C_{\chi^2}(\varphi/Y) = (\varphi - E[\varphi/Y])'V_\varphi^{-1}(\varphi - E[\varphi/Y])\}, \qquad (26)$$

and V_φ is the posterior covariance of φ under $p(\varphi/Y)$

L_{χ^2} and L_p-loss are identical, if the posterior distribution of φ is Gaussian. In general, under the L_p-loss models are compared based on the height of the posterior density at $\hat{\varphi}$, while under L_{χ^2} the comparison is based on the weighted distance between $\hat{\varphi}_i$ and the posterior mean $E[\varphi/Y]$.

Optimal predictors:

The optimal predictor for L_q is the posterior mean of φ under model M_i, whereas for the other two loss functions $\hat{\varphi}_i$ depends on the shape of the posterior distribution. Since the predictor ought to be calculated only by information contained in $p(\varphi/Y, M_i)$, the latter replaces $p(\varphi/Y)$ in 21, and it follows that the optimal predictor $\hat{\varphi}_i$ for the L_p-loss is the posterior mode of $p(\varphi/Y, M_i)$ and for the L_{χ^2}-loss it is the posterior mean $E[\varphi/Y]$.

3.3 Specification of the Priors

Most priors for common parameters are taken from Schorfheide [18], for the rest of common parameters a wider prior distributions is assumed, s.t. the prior means used there and the calibrated values in Menner [15] are equally likely. Model-specific parameters of the STM model are centered around calibrated values. Table 1 provides a summary of the assumed prior distributions:

Table 1	Name	Range	Density	Mean	SE
All Models:	e_K	$[0,1]$	Beta	0.3560	(0.0200)
	β	$[0,1]$	Beta	0.9930	(0.0030)
	ζ	$\mathbb{R}$	Gaussian	0.0085	(0.0030)
	γ	$\mathbb{R}$	Gaussian	0.0100	(0.0025)
	ρ_M	$[0,1]$	Beta	0.6000	(0.2230)
	δ	$[0,1]$	Beta	0.0165	(0.0080)
	σ_A	$\mathbb{R}^+$	InvGamma	0.712 %*	(2.000*)
	σ_M	$\mathbb{R}^+$	InvGamma	0.600 %*	(2.000*)
CIA / PAC:	ϕ	$[0,1]$	Beta	0.6500	(0.0500)
Only PAC:	$\varkappa$	$\mathbb{R}^+$	Gamma	50.000	(20.000)
Only STM:	e_Φ	$\mathbb{R}^+$	Gamma	0.5000	(0.2500)
	α	$[0,1]$	Beta	0.5000	(0.2000)
	A	$[0,1]$	Beta	0.6000	(0.0500)
	B	$[0,1]$	Beta	0.5263	(0.0500)
	z	$[0,1]$	Uniform	0.5000	(0.2887)
	δ_n	$[0,1]$	Beta	0.0600	(0.0050)
	b	$[0,500]$	Uniform	250.00	(144.33)
	φ_0	$[0,100]$	Uniform	50.000	(28.877)
	Υ_o	$[0,1]$	Uniform	0.5000	(0.2887)
	σ	$[0,1]$	Beta	0.5000	(0.2000)
	δ_i	0.0072	fix	0.0072	(0.0000)
	u	0.0447	fix	0.0447	(0.0000)
	a_p	0.0069	fix	0.0069	(0.0000)

Notes: The parameter φ of the STM is determined from steady state conditions since n^* is normalized to 100. Note also, that $e_K = 1 - e_N$, and $\delta = \delta_k - \zeta$.

4 Results

4.1 Parameter estimates

Since the posteriors of the DSGE models do not belong to a well-known class of distributions, it is impossible to draws from the posterior directly. Instead we can only evaluate numerically the product of prior and likelihood. Hence a random walk Metropolis-Hastings algorithm, is used to generate draws from the posterior distributions. Technical details on how to generate draws and statistics from the VAR and DSGE posteriors are thoroughly explained in the appendix of Schorfheide [18]. In the following, I only note in what aspects I differed from his approach. Convergence of the Metropolis-Hastings algorithm could be achieved for the CIA and the PAC model very quickly, so I generated 90.000 draws from the posterior and discarded the first 10.000, while for the STM model I had to generate 300.000 draws, of which I discarded the first 220.000. Thus for each model we have 80.000 valid draws from the posterior parameter distribution The algorithm works as follows: At each iteration s, a candidate parameter vector $\vartheta_{(i)}$ is drawn from a jumping distribution $J_s(\vartheta_{(i)}/\theta_{(i)}^{s-1})$. Then the ratio r between the posterior at $\vartheta_{(i)}$ and the posterior at $\theta_{(i)}^{s-1}$ is calculated. The jump from $\theta_{(i)}^{s-1}$ to $\vartheta_{(i)}$ is accepted $(\theta_{(i)}^s = \vartheta_{(i)})$ with probability $\min(r, 1)$ and rejected $(\theta_{(i)}^s = \theta_{(i)}^{s-1})$ otherwise. For the CIA and PAC models I use the same jumping distribution as in Schorfheide [18], i.e. a Gaussian with mean $\theta_{(i)}^{s-1}$ and variance $c^2 \tilde{\Sigma}_{(i)}$, where $c = 0.2$ and $\tilde{\Sigma}_{(i)}$ is the inverse Hessian at the posterior mode. In the case of the STM model I choose a uniform distribution as jumping distribution. Since jumps are then bounded, it happens to be easier to achieve convergence of the Metropolis-Hastings algorithm where there are many parameters to estimate. The spread of the jump distribution was chosen parameter by parameter to achieve an average acceptance rate of about 40%, which has found to be a good choice for models with many parameters. Recursive mean plots and potential scale reduction factors (see Gelman et al. [8]) have been used to assess convergence. The potential scale reduction factors were less than 1.005 for all models. Therefore, we consider the number of draws as large enough to conduct inference. Posterior means and standard errors are calculated from the output of the Metropolis-Hastings algorithm and are shown in Table 2.

Table 2

	CIA Model		PAC Model		STM$_1$ Model		STM$_2$ Model	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE
e_K	0.4123	(0.0232)	0.4150	(0.0222)	0.3148	(0.0285)	0.3802	(0.0181)
β	0.9874	(0.0053)	0.9762	(0.0069)	0.9967	(0.0023)	0.9965	(0.0018)
ζ	0.0040	(0.0010)	0.0045	(0.0012)	0.0092	(0.0005)	0.0090	(0.0012)
γ	1.0136	(0.0016)	1.0132	(0.0017)	1.0099	(0.0010)	1.0092	(0.0021)
ρ_M	0.8557	(0.0341)	0.8500	(0.0481)	0.7975	(0.0236)	0.8068	(0.0210)
δ	0.0026	(0.0014)	0.0046	(0.0030)	0.0158	(0.0025)	0.0261	(0.0086)
σ_A	0.0130	(0.0009)	0.0159	(0.0010)	0.0141	(0.0010)	0.0169	(0.0010)
σ_M	0.0030	(0.0002)	0.0035	(0.0003)	0.0035	(0.0002)	0.0035	(0.0002)
ϕ	0.6827	(0.0518)	0.6676	(0.0514)				
$\varkappa$			64.872	(20.812)				
e_Φ					2.3198	(1.0214)	0.2395	(0.1084)
α					0.4619	(0.1035)	0.4791	(0.0934)
A					0.6050	(0.0505)	0.5465	(0.0592)
B					0.5118	(0.0493)	0.5073	(0.0509)
z					0.0620	(0.0233)	0.0441	(0.0195)
δ_n					0.0595	(0.0051)	0.0654	(0.0055)
b					255.86	(124.23)	0	0
φ_0					7.6992	(5.2328)	0.5000	0
Υ_o					0.0040	(0.0044)	1E-6	0
σ					0.5868	(0.1764)	0.0100	0

Notes: STM$_1$ is the complete search-theoretic monetary model featuring capital adjustment costs and search-frictions in the labor market. Model STM$_2$ is specified such that there are no capital adjustment costs ($b = 0$), and the labor market is very flexible due to extremely low hiring costs ($\Upsilon_0 = 1E$-6). With a low bargaining power of workers ($\sigma = 0.1$) wages correspond mainly to the marginal product of labor, as in Walrasian markets. Finally, φ_0 is normalized to 0.5 in order to avoid unreasonable low values for e_Φ.

We present the results of two different versions of the search-theoretic monetary model: STM_1 stands for the complete search-theoretic monetary model featuring capital adjustment costs and search-frictions in the labor market. Alternatively, model STM_2 is analyzed, where some parameters are fixed such that there are no capital adjustment costs and the labor market is very flexible due to extremely low hiring costs. This second model variant is considered for two reasons. First, as we can see in Table 2., many parameters of the full STM model are estimated very poorly. Especially the capital adjustment cost parameter b, and some variables related to the labor market, like the hiring cost Υ_0, and the bargaining power of workers σ. Second, once we decide to fix these parameters where the data is quite uninformative, it makes sense to use a parametrization that makes the models more comparable. Since the CIA and the PAC model do not feature capital adjustment costs, we can set $b = 0$ to shut down this feature in the STM model as well. In order to bring the STM model closer to the two competitors regarding the labor market, we can reduce substantially the frictions by setting $\Upsilon_0 = 1E\text{-}6$. A low bargaining power of workers makes wages correspond mainly to the marginal product of labor, as in Walrasian markets. Moreover, we normalize φ_0 to 0.5 in order to avoid unreasonable low values for e_Φ.

Consider first the estimation of the common model parameters. All of them are estimated quite precisely. For the CIA and PAC model the main difference to the results in Schorfheide [18], is that the mean of the parameters β and δ, for which the prior distribution has been widened, changes slightly. The discount factor is reduced, implying an annualized real interest rate of 5% and 10% respectively. The depreciation rate of capital δ increases and achieves more plausible values. The STM models' estimates for the real interest rate are much lower and much higher for δ, while the money growth rate and hence inflation is estimated lower. The autocorrelation of money shocks turn out somewhat smaller in the STM models, and there is also a small reduction in the capital share. The biggest difference is in the trend of technology growth which is estimated twice as high in the search models. Moreover, the data assigns a high portfolio adjustment cost parameter $\varkappa$ for the PAC model. Switching from STM_1 to STM_2 leaves the other search-model specific parameters almost untouched, only e_Φ adjusts itself to a lower value, and all parameters are estimated sufficiently precise. Hence apart from the calculation of the posterior model probabilities, where the results are also reported for the full search model, I will only consider model STM_2 in the model comparison below.

4.2 Model Comparison

4.2.1 Posterior Model Probabilities

The first row of Table 3 shows the assumed prior model probabilities. Since we are ignorant about the best lag length to choose for the VAR, we use a mixture of lags 1 to 4. So, to each model a prior probability of 1/4 is assigned. The two versions of the search-theoretic model are analyzed alternatively. Again, STM_1 stands for the complete search-theoretic monetary model featuring capital adjustment costs and search-frictions in the labor market while in STM_2 there are no capital adjustment costs and the labor market is very flexible due to extremely low hiring cost.

Marginal data densities can only be calculated analytically for the VARs. Rows 3 and 4 show therefore two approximations of the marginal data density used in the literature. The Laplace Approximation uses the Hessian at the posterior mode to calculate a penalty on the value of the posterior at the mode, while the modified harmonic mean (Geweke [9]).is simulation-based. The two approximations give similar values, except for the case of the STM_2 model and in this case, since I base the further calculations on the harmonic mean approximation, the posterior probability will be lower than if I would have used the Laplace approximation.

We see that the VAR(3) has a posterior probability of nearly 100%. The other models contribute very little to the overall DGP. Nonetheless, we can compute the standard posterior odds with respect to the CIA model and we see that the latter outperforms the PAC model by a factor 90.000, but the search models do better. Their predictive score reaches nearly the one of the other VARs in the case of the full search model, and outperforms the one of the CIA model by 1E+11. But there odds are still far from the odds of the VAR(3). Finally, the PAC model performs slightly better in comparison to the analysis in Schorfheide [18], a result that is in line with the robustness analysis reported therein.

Table 3

	CIA	PAC	STM$_1$	STM$_2$	VAR(1)	VAR(2)	VAR(3)	VAR(4)
Prior Prob. $\pi_{i,0}$	1/4	1/4	(1/4)	(1/4)	1/16	1/16	1/16	1/16
Marg. Data Dens. ln $p(Y \mid M_i)$	N/A	N/A	N/A	N/A	1250.73	1253.01	1259.12	1249.95
Laplace Approximation	1195.94	1185.63	1247.90	1253.46	250.59	1252.77	1258.77	1249.46
Harmonic Mean	1196.59	1187.29	1246.41	1221.99	1250.73	1253.01	1259.10	1249.94
Posterior. Probability π_i	2.8E-27	2.6E-31	1.2E-05	3.1E-16	0.0002	0.0022	0.9974	0.0001
Posterior Odds π_i / π_1	1	9.1E-05	4.3E+21	1.1E+11	8.1E+22	7.9E+23	3.5E+26	3.2E+22

Notes: Marg. Data Dens. stands for the marginal data density, i.e. ln $p(Y_T \mid M_i)$, that is also approximated by Laplace Approximation and the simulation-based Harmonic Mean (Geweke [9]). CIA is the cash-in-advance model, PAC is the portfolio-adjustment-cost model, STM$_1$ is the complete search-theoretic-monetary model, while STM$_2$ is the search-theoretic-monetary model without capital adjustment costs and with flexible labor market. For the VARs, the harmonic mean approximations are based on 8,000 draws from the posterior parameter distribution, for the DSGE models it is based on 80,000 draws. In the latter case the estimated approximation error is about 0.5-0.6 (in log-units).

4.2.2 Comovement and Autocorrelation

Let's turn to the loss function analysis of second moments. Remember, that from now on we drop model STM_1 from our analysis. Consider first the cross-correlation of GDP growth and the inflation rate. Table 4 presents the results for these correlations up to 2 leads and 2 lags. The first two rows show the upper and lower bound of the 90% intervals of highest posterior density. Mode predictions of the CIA and PAC models of the contemporaneous correlation fall outside this interval, which is reflected in a very high L_p risk, whereas the STM model predicts the contemporaneous correlation of output growth and inflation very well. It does better also for 1 lag or 1 lead, but fails to hit the 90% interval for 2 lags. Here and in the case of 2 leads the ranking of the models is CIA, PAC and then STM.

Correlation (ΔGDP $_t$, Inflation $_{t+h}$)

Table 4	Model	h = - 2	h = - 1	h = 0	h = 1	h = 2
90% Interval (U)		0.0364	0.0565	-0.0488	0.0250	0.1014
90% Interval (L)		-0.2968	-0.2794	-0.3660	-0.3019	-0.2403
Mode Predection	CIA	0.0008	0.0018	-0.5741	-0.0283	0.0274
	PAC	0.0081	0.0216	-0.4897	0.0079	-0.0023
	STM	0.0971	-0.0315	-0.1361	-0.1704	-0.1571
Lp-risk	CIA	0.7265	0.7555	0.9737	0.5648	0.0000
	PAC	0.7572	0.8365	0.9445	0.7575	0.1452
	STM	0.9525	0.4644	0.2363	0.0000	0.4754

The STM model is even more successful if we look at the autocorrelations of inflation up to 4 lags as reported in Table 5. While the mode predictions of the other two models lie outside the 90% interval for all lags, the ones of the search model lie all inside. This is reflected in a L_p risk of roughly 1 for the CIA and PAC model and considerably lower L_p risk for the search model. Hence, with respect to unconditional moments the search model mostly outperforms its competitors.

Autocorrelation of Inflation: Corr (Inflation $_t$, Inflation $_{t-h}$)

Table 5	Model	h = 1	h = 2	h = 3	h = 4
90% Interval (U)		0.8877	0.8661	0.8376	0.7922
90% Interval (L)		0.6435	0.5863	0.5175	0.4301
Mode Predection	CIA	0.4210	0.3627	0.3138	0.2723
	PAC	0.4650	0.4009	0.3470	0.3016
	STM	0.8223	0.6661	0.5404	0.4404
Lp-risk	CIA	1.0000	1.0000	1.0000	0.9996
	PAC	0.9998	0.9999	1.0000	0.9979
	STM	0.5219	0.3811	0.8046	0.8633

4.2.3   Impulse Response Functions

In this subsection we compare impulse responses to a transitory and a permanent shock. In the VAR, they are identified via a standard long-run identification scheme as in Blanchard and Quah [3]. In the models, they correspond to a shock to money growth and technology[12]. Figure 1 plots the results. Dotted lines correspond to the 90% intervals of the impulse responses stemming from the assumed DGP the solid line is the corresponding mean response. The dash-dotted line represents the responses of the CIA model, the dashed line the ones of the PAC model and the dotted line with "+" shows the impulse responses of the search model.

A monetary shock does not induce strong output responses in the CIA model, and they go in the wrong direction. Assuming limited participation in asset markets and portfolio adjustment costs, as the PAC model does, is sufficient to get a hump shape output response. But also search-frictions in the goods market do the job, although not on impact and with less magnitude. Inflation does not show much persistence in the data after a transitory

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[12]We normalize the magnitude of the structural shocks by their long-run effects rather than by use of the estimated parameters σ_M and σ_A, that correspond to an estimation procedure that resulted in insignificant posterior probability. Thus, we consider a transitory (monetary) shock that increases the price level by 1% and a permanent (technology) shock that increases output by 1%.

shock, but this might depend on the identification scheme.While the CIA and the PAC models capture well the inflation response, the STM model shows a more persistent inflation response to a monetary shock than appear to be in the data.

Turning to the output effects of a permanent shock we see a large 90% interval, and the response of the STM model is close to the upper bound for various periods, while the other models' responses lie moreless symmetrically at some distance to the mean output response: the CIA (PAC) model over-(under-)predicts the mean response, but still doing better than the search model. The latter, however, has a strong advantage in predicting the inflation response to a permanent shock. After few periods it resembles the mean response, while inflation in the other models goes back to steady state rapidly.

Figure1

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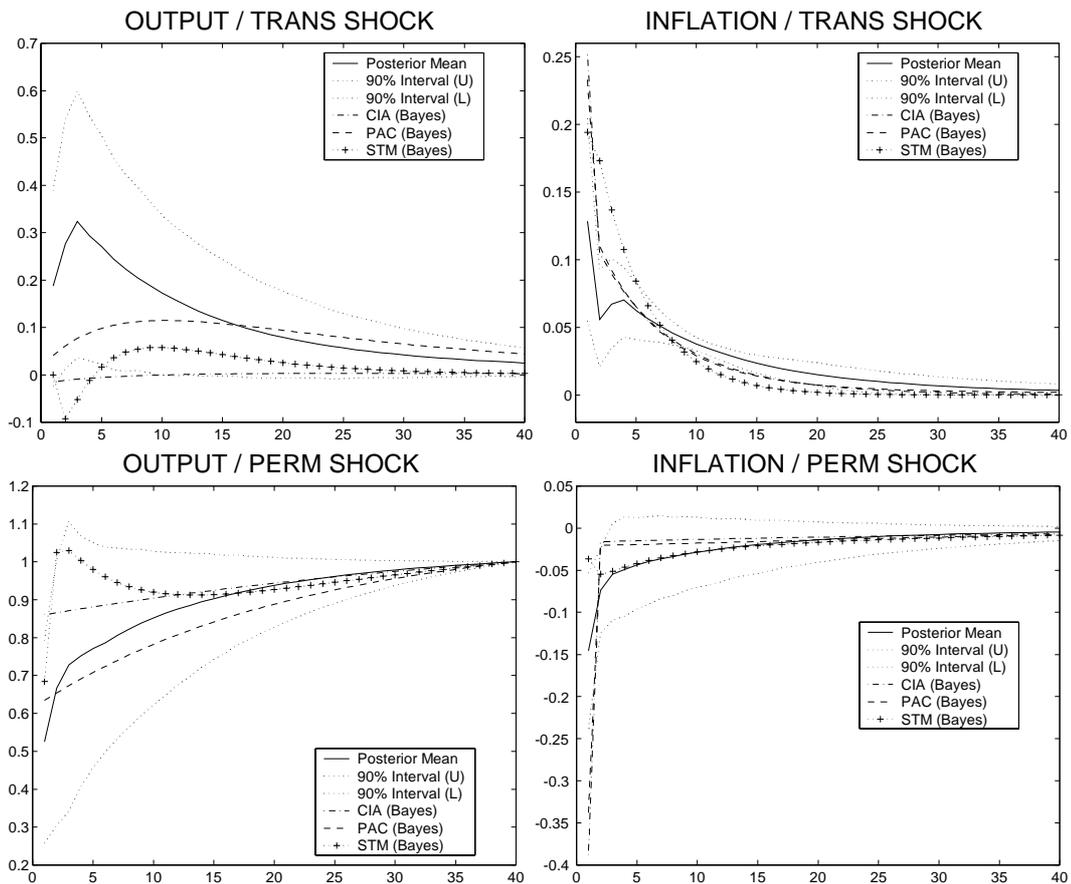
To quantify the ability of the models to predict dynamic responses we turn again to the loss function analysis. Table 6 presents the L_q risk and the $L\varkappa^2$-risk, together with the $C\varkappa^2$-statistic used for the calculation of the latter for the four different impulse responses (lags 1 to 12 jointly). The weighting matrix W in the calculation of the L_q risk is the 12×12 identity matrix scaled by the factor 1/12. The L_q and the $L\varkappa^2$ statistics confirm the visual impression from Figure 1. Looking at the first column we see that the STM model improves slightly on the CIA model but is poorer than the PAC model in predicting the impulse response of output to a monetary shock when using the L_q criterion. Things are different considering the $L\varkappa^2$-risk. The STM does much worse, and in contrast to the result in Schorfheide [18] the CIA model performs slightly better than the PAC. This result seems to be sensitive to the precision of the calculation of the inverted Hessian at the mode. With respect to responses of inflation to a money shock both criteria give the same ranking: the PAC model dominates the CIA model, which outperforms the search model. Output effects of technology shocks give again mixed results. While with L_q risk we have the same ranking as before, with $L\varkappa^2$ loss the STM model ranks between PAC and CIA models. A striking feature of column 4 is the large losses the latter models incur when looking at the ability to predict inflation responses to a technology shock. Here, the STM model clearly outperforms its competitors.

Table 6	Model	d ln GDP/d ε_M	d Δln P/d ε_M	d ln GDP/d ε_A	d Δln P/d ε_A
Lq-risk	CIA	0.0556	0.0016	0.0192	0.0057
	PAC	0.0214	0.0012	0.0046	0.0037
	STM	0.0514	0.0021	0.0357	0.0010
C$\varkappa^2$	CIA	4.7692	17.557	37.300	126.967
	PAC	4.7750	14.231	14.956	87.291
	STM	33.116	90.436	28.765	11.383
L$\varkappa^2$-risk	CIA	0.4122	0.8449	0.9545	0.9943
	PAC	0.4127	0.7924	0.8208	0.9887
	STM	0.9417	0.9899	0.9330	0.7225

5 Conclusion

This paper presents the results of a Bayesian model comparison to provide a quantitative assessment of the role of search-frictions in the goods market. Both, the complete STM model with and without capital adjustment costs and labor search frictions, outperform their two competitor models by their predictive score measured by the marginal data density. One alternative model considered is a standard cash-in-advance model on which the search model improves on in nearly all of the considered dimensions. Search in the goods market adds a propagation mechanism that results in hump shaped output responses to a monetary shock, and that generates a persistent disinflation after a technology shock. Contemporaneous and lagged and leaded correlations of inflation and output growth can be predicted considerably better, although not so for longer lags and leads. Finally, the search model predicts very well the autocorrelations of inflation, while the CIA model can not. Thus, search frictions in the goods market do make a difference.

The additional frictions imposed on the portfolio choice of the consumers in the PAC model act also as a mechanism to propagate monetary shocks persistently - at least with respect to output. Its response to a monetary shock is more pronounced and more persistant than the response of output in the STM model. But the PAC model cannot predict the persistant disinflation process after a technology shock. Together with the CIA model it is more able to trace the response of inflation to a monetary shock. The predictions of all models for the output response to technology shocks lie well within the wide 90% intervalls, so we can hardly discriminate amongst them. The PAC model shares with the CIA model the failure to predict the autocorrelations of inflation and the contemporaneous correlations of inflation and output growth. So, with respect to the question whether the frictions in the goods market or the frictions in the asset market provide a better model to predict characteristics of the data, this analysis cannot be decisive.

Given the poor estimation of the complete search-theoretic model, further attempts to estimate the model on the basis of more data series and more structural shocks seem to be a precondition for further empirical analysis with this search-theoretic monetary business cycle model. Once a good estimation of the parameters is achieved one can confront the model with many more business cycle stylized facts and see what features of the model contribute to what extent to the ability to predict certain characteristics. The results of the calibrated model look promising to justify this further effort.

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