THE MEASUREMENT OF STRUCTURAL AND EXCHANGE INCOME MOBILITY *

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Abstract
Chakravarty, Dutta and Weymark (1985) present axioms for an ethical index of relative income mobility in a two-period world. This paper suggests a decomposition of this index into i) an index of structural mobility, which captures the welfare effect of differences in the inequality of the cross-section income distributions; and ii) an index of exchange mobility, which captures the welfare impact of permutations or rank reversals between the first-and the secon-period income distributions. We propose a second decomposition in order to isolate the effect on mobility of mean incomes changes. The properties of all the income mobility concepts introduced in the paper do not require any new value judgements beyond the traditional ones.

Keywords: Income mobility; income inequality; rank reversals; income growth.

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I. INTRODUCTION

Compared to agrarian society, which has predominated most of historical times, our growth-oriented industrial society is presumed to be socially mobile and egalitarian. The recent availability of longitudinal data makes the measurement of such a central concept as mobility increasingly possible. In the words of the authors of a recent survey, the problem is that, compared with the neighboring area of inequality measurement, "the income mobility literature is still distressingly far from being unified on how to measure mobility and make mobility comparisons", Fields and Ok (2000, p. 586) –or FO for short, from here on.

Among the many issues that remain open, the main question we address in this paper is the distinction between structural and exchange mobility\(^1\). Since we want to study under what conditions structural, exchange and total mobility are socially desirable, we shall concentrate on ethical or normative mobility indexes which are capable of addressing this issue.

Ethical indices are derived from explicit social evaluation functions (SEF, for short). In a static context, the SEF is simply defined on the space of one-period income distributions. In the present dynamic context, what the SEF domain should be is not an obvious question. Given a decision in this regard, it is important to know whether in order to construct meaningful mobility measures we need SEFs which incorporate new value judgments beyond the traditional ones.

In their seminal contributions in this area, both Markandya (1982, 1984) and King (1983) restrict themselves to a two-period world and introduce novel SEFs which lead to new value judgements. In an intergenerational context with
two generations, Markandya (1982, 1984) identifies the social welfare with the welfare of the current generation. In turn, the welfare of the current generation is represented by a utilitarian SEF where the utility of any individual in this generation depends not only on his/her lifetime income, but also on the lifetime income of his/her parent. The link between income mobility and social welfare requires new value judgements on the nature and the strength of the inter-generational links in such a utility function. On the other hand, King (1983) proposes a two-period model where the SEF is defined on individual incomes during the second period and rank reversals between the two periods. Therefore, new value judgements about the welfare effects of rank reversals are required.

In this paper, we follow the ethical approach originally suggested in Chakravarty, Dutta and Weymark (1985), or CDW for short. They compare the actual time path of incomes received over a number of periods with a hypothetical benchmark which maintains constant over time the relative or absolute positions occupied by the individuals in the actual first-period income distribution. CDW also restrict themselves to a two-period model but, contrary to Markandya (1982, 1984) and King (1983)'s, their SEF is defined on aggregate incomes over the two periods and does not include any new value judgment beyond the traditional ones.

In this framework, we find it essential to distinguish between two types of rank reversals ignored in CDW: rank reversals between the first- and second-period income distributions, which we call permutations; and rank reversals between the first-period and the aggregate income distributions, which we call rerankings. The distinction can be illustrated by means of a pair of simple examples for an economy with two individuals. In both examples the first period income distribution is (2, 4). In example 1, the second period income distribution is (4, 3). Therefore, there is a permutation; but since the aggregate income
distribution is \( (6, 7) \), there is no reranking. In example 2, the second period income distribution is \( (7, 0) \), representing the same total income growth as before. The aggregate income distribution is now \( (9, 4) \), so there is both a permutation and a reranking.

Using this distinction, we offer two novel decompositions of CDW's mobility index. In the more basic decomposition, we express our income mobility index as the sum of two terms: the first one, which we call *structural or snapshot mobility*, captures the welfare effect of the change in income inequality between the aggregate and the completely immobile distribution, once all permutations have been eliminated. The second term, which we call *exchange or permutations mobility*, measures the welfare impact of permutations between the first- and the second-period income distributions, with or without rerankings between the initial and the aggregate income distributions.

We do not impose any value judgments either on permutations or rerankings. However, in the presence of permutations, we show that exchange mobility is always socially desirable. On the other hand, in the presence of rerankings, we show that there exists a second-period income distribution with less inequality which implies the same rate of income growth, the same income mobility, but no rerankings at all.

According to FO, "While most people feel that the notions of income inequality and income growth are largely independent concepts, it seems reasonable to view the movement aspect of mobility as closely related with income growth." (p. 563, emphasis in the original). Following up on this idea, we present a second decomposition of the CDW income mobility index into three terms. To begin with, we express our index as the sum of two terms: an *income transfer mobility index*, which captures the income mobility induced by changes in snapshot income inequality and permutations, holding the mean of the second-
period income distribution constant at the level of the first-period one; and an income growth mobility index, which is a residual capturing the income mobility induced by the differences in the mean of the two period income distributions. In a second step, the income transfer mobility index is decomposed into its structural and exchange mobility components. The paper explores the relation between the two decompositions.

The rest of the paper is organized in six sections. In section II we present the assumptions of the CDW model. Sections III and IV are devoted to the decomposition of the income mobility into structural and exchange mobility, and income transfer and income growth mobility, respectively. Section V compares our decompositions with the ideas put forth in the previous literature, while Section VI concludes.

II. THE MODEL

Among the approaches developed by economists for the study of economic and social mobility, it is useful to distinguish between two types. The first one considers explicitly the transition mechanism responsible for the time path of the variable of interest. Together with income, this variable may be any real-valued measure of socioeconomic position, like indexes of social or occupational status. The transition mechanism is often represented by a transition matrix which shows the fraction of the population which moves from one category to another in one time period. In this context, an index of mobility is defined as a real function on the set of transition matrices\(^{(2)}\).

The second approach, which we follow in this paper, is meant for a less abstract setting in which the variable of interest is income. Abstracting from the transition mechanism, we are simply concerned in a straightforward way with
the changes that can be observed in longitudinal data sets: changes in cross-
section or snapshot income inequality, changes in relative incomes or in absolute
income differences, and changes in mean incomes. In this context, an index of
mobility is defined as a real-valued function on the set of time paths of income
distributions. Indices of relative or absolute mobility are sensitive to changes in
relative incomes or in income differences, respectively(3).

Let there be $n \geq 2$ individuals, indexed by $i = 1, \ldots, n$, and let $D = \mathbb{R}_{++}^n$ be
the strictly positive orthant in $n$-dimensional Euclidean space. In a two period
world, let $x = (x^1, \ldots, x^n) \in D$ represent the income distribution of an $n$-person
society where individual $i$'s income level is $x^i$. Now assume that individual $i$'s
income has changed to $y^i$ in a given time interval. Following FO's terminology,
we say that $x$ has been transformed to $y = (y^1, \ldots, y^n) \in D$, and denote this so-called
distributional transformation by $x \rightarrow y$. Each individual $i$ is characterized by an
income stream $(x^i, y^i)$. Over the two periods, individual $i$ receives aggregate
income $z^i = x^i + y^i$. We refer to the distribution $z^i = (z^1, \ldots, z^n)$ as the aggregate
income distribution(4).

As pointed out by FO, while it admittedly confines the analysis to only 2-
period paths of income distributions, this framework allows one to study both intra and intergenerational mobility measurement, depending on the length of the time period between the observation periods. In the intergenerational case, we may start by assuming that every parent has only one child. Then, $x^i$ and $y^i$
can be interpreted as the parent and the child incomes, respectively, and $z^i$ as
the dynastic or family income. In the intragenerational case, $x^i$ and $y^i$ can be
taken to be individual i's income while "young" and "old", respectively, and \( z_i \) his/her lifetime income. On the other hand, recall that the papers in the relevant literature, namely, King (1983) and Markandya (1982, 1984), also assume a two period world.

The ethical approach to measuring income mobility in CDW uses an intertemporal social evaluation function (SEF for short), \( v: D^2 \rightarrow R^1 \), where \( v(x, y) \) is the social welfare level associated with the distributional transformation \( x \rightarrow y \). The income mobility concept we wish to explore is the one embodied in a welfare comparison of the actual distributional transformation \( x \rightarrow y \), and a hypothetical benchmark \( x \rightarrow y_b \): the distributional transformation which would have resulted in the absence of mobility given the first period distribution \( x \). That is to say, mobility indices are obtained by comparing the actual level of social welfare \( v(x, y) \) with the level of social welfare \( v(x, y_b) \) which would have been obtained with the benchmark distributional transformation \( x \rightarrow y_b \).

To make this comparison operational, CDW make the following two fundamental assumptions referring to the notion of complete immobility in the relative case and the nature of the SEF, respectively.

**A. 1.** Let \( \mu(x) \) be the mean of any income distribution \( x \in D \). We say that the distributional transformation \( x \rightarrow y \) exhibits *complete relative immobility* if individual income shares are maintained through time equal to the income shares in the first period distribution \( x \), i.e., if \( y = y_b \), where \( y_b = (\mu(y)/\mu(x))x \), so that \( \mu(y_b) = \mu(y) \), and \( I(y_b) = I(x) \) for any index of relative income inequality \( I \). Consequently, the aggregate distribution for this benchmark transformation, denoted by \( z_{b'} \), has the same mean as \( z_a \), but gives each individual the same share
of actual aggregate income as they receive in period 1, that is, \(\mu(z_b) = \mu(z_a)\) and \(I(z_b) = I(x)^{(6)}\).

The second assumption requires that the only features of the distributional transformations \(x \rightarrow y\) and \(x \rightarrow y_b\) relevant for the welfare comparisons are their aggregate distributions \(z_a\) and \(z_b\). Formally:

A. 2. There exists a SEF \(W: D \rightarrow R^1\) such that, for all distributional transformations \(x \rightarrow y\), \(W(z_a) = v(x, y)\).

An income mobility index assigns a mobility value to each distributional transformation \(x \rightarrow y\), i.e. it is a function \(M: D^2 \rightarrow R^1\). CDW suggest the following index of income mobility in the relative case:

\[
M(x, y) = \frac{W(z_a) - W(z_b)}{W(z_b)}.
\]  

An immobile income structure is assigned a mobility value of zero\(^{(7)}\). Both periods are then reflected in the construction of the mobility indices, the first-period distribution through its effect on the aggregate benchmark distribution \(z_b\), and the second-period distribution through its effect on the actual aggregate distribution \(z_a\).

The next assumption, which is also taken from CDW, refers to the welfare evaluation of one-period income distributions.

A. 3. There exists a SEF defined on one-period incomes, \(W^*: D \rightarrow R^1\), and this function is the same as the two-period SEF \(W\), i.e. \(W^*(x) = W(x)\) and \(W^*(y) = W(y)\).

The identification of one-period evaluations with two-period ones is questionable, but it greatly simplifies our work\(^{(8)}\). The remaining properties of the income mobility index depend on additional assumptions on the SEF. For our analytical purposes, we only need that \(W\) can be expressed in terms of only two
statistics of the income distribution, the mean and an index of income inequality. However, for operational purposes it is convenient to specify the trade-off between efficiency and distributional considerations. Consequently, in the relative case we adopt the following assumption:

**A. 4.** For any income distribution $x \in D$, the SEF $W$ can be expressed as:

$$W(x) = \mu(x)(1 - I(x)).$$

Thus, social welfare is seen to be the product of the mean and an adjustment factor which varies inversely with an appropriate index of relative inequality $I^9$. In this case, the CDW income mobility index defined in (1) becomes

$$M(x, y) = \frac{I(x) - I(z_a)}{1 - I(x)} \quad (10). \quad (2)$$

In our view, using FO's terminology, the CBW income mobility index defined in equation (2) measures *mobility as movement*. However, contrary to descriptive income mobility indices, this ethical index allows us to determine whether the observed income movement is socially desirable. Consider the following two examples:

**E1:** $x = (2, 4) \rightarrow y = (4, 3); \ z_a = (6, 7)$

**E2:** $x = (2, 4) \rightarrow y = (2, 5); \ z_a = (4, 9)$.

The initial situation is the same in both examples, $x = (2, 4)$. Since $\mu(y) = 7/2$, the rate of income growth is also the same in E1 and E2. However, it is clear that $M(E1) = (I(2, 4) - I(6, 7))/(1 - I(2, 4)) > 0$, while $M(E2) = (I(2, 4) - I(4, 9))/(1 - I(2, 4)) < 0$. The reduction in income inequality in $z_a$ relative to $x$ causes $M(E1)$ to be positive, reflecting an increase in social welfare. The opposite situation causes $M(E2)$ to be negative, reflecting a social welfare loss.

Finally, in FO's terminology the CDW index defined in equation (2) is *weakly relative*, i.e. for any income transformation $x \rightarrow y$, $M(\lambda x, \lambda y) = M(x, y)$ for all $\lambda > 0$. The reason, of course, is that in the distributional transformation $\lambda x \rightarrow$
\[ \lambda y \] the aggregate income distribution is \( \lambda z_{a} \), where \( z_{a} = x + y \), and \( I(\lambda z_{a}) = I(z_{a})^{(11)} \).

III. STRUCTURAL AND EXCHANGE MOBILITY

III. 1. Definitions

Apparently, our income mobility index reflects welfare changes due solely to changes in income inequality from the initial to the final situation. One of the points of this paper is to clarify why this is not the case at all. Upon closer inspection, income changes in example E1 presented in the previous Section give rise to two effects: a change in cross-section or snapshot income inequality from \( I(1, 3) \) to \( I(4, 3) \); and a permutation of the ordering of individual incomes between the first- and the second-period income distributions - in distribution \( x \) individual 1 is poorer than 2, while in distribution \( y \) individual 1 is richer.

At this point, it is useful to consider a third example:

\[ E3: x = (2, 4) \rightarrow y = (5, 2); z_{a} = (7, 6). \]

Both the initial situation and the rate of income growth coincide with those of examples E1 and E2. Given the symmetry of \( I \), we have that \( I(6, 7) = I(7, 6) \). Therefore, we have that \( M(E3) = M(E1) \). The (important) novelty in relation to E1, is that in E3 there is both a permutation between the two snapshot distributions \( x \) and \( y \), and what we call a reranking between the first-period and the aggregate income distributions, \( x \) and \( z_{a} \), respectively.

Examples E1 and E3 suggest that our mobility index can be decomposed into two terms. One capturing the welfare change due to the change in inequality between the cross-section distributions \( x \) and \( y \) once all permutations have been removed, and a second one capturing the permutation effect with or without reranking between \( x \) and \( z_{a} \). Therefore, from a formal point of view what we
wish to achieve is a decomposition of the mobility index \( M(x, y) \) into a structural or snapshot mobility index, \( SM(x, y) \), and an exchange or permutations mobility component, \( EM(x, y) \).

For that purpose, it is important to retain the following terminology. Given a distributional transformation \( x \to y \), we will always consider that \( x \) is ordered according to the "less than or equal" relation. Whenever \( x \) and \( y \) are not equally ordered, we say that there has been some \textit{permutation} between them; whenever \( x \) and \( za = x + y \) are not equally ordered, we say that there has been some \textit{reranking} between them. Of course, any reranking between \( x \) and \( za \) implies some permutation between \( x \) and \( y \) (as in E3), but not the contrary (as in E1). Finally, given any distributional transformation \( x \to y \), define \( z_c = x + y' \), where \( y' \) is the second-period distribution \( y \) ordered as the initial distribution \( x \).

Armed with these concepts, we suggest the following decomposition of our mobility index:

\[
M(x, y) = SM(x, y) + EM(x, y),
\]

where

\[
SM(x, y) = \frac{W(z_c) - W(z_b)}{W(z_b)} = \frac{I(x) - I(z_c)}{1 - I(x)} \tag{5}
\]

\[
EM(x, y) = \frac{W(z_a) - W(z_c)}{W(z_b)} = \frac{I(z_c) - I(za)}{1 - I(x)}. \tag{6}
\]

We can view \( SM(x, y) \) as the income mobility associated with the distributional transformation \( x \to y' \) in which all the permutations between \( x \) and \( y \) have been eliminated, i.e. \( SM(x, y) = M(x, y') \). Then, exchange mobility is defined as a residual, i.e. \( EM(x, y) = M(x, y) - M(x, y') \). \(12\).

\section*{III. 2. Properties}

\textbf{Remark 1.} Since \( I(y') = I(y) \) and \( I(z_c) \in \{\min(I(x), I(y)), \max(I(x), I(y))\} \), we have that
\( SM(x, y) \geq 0 \iff I(x) \leq I(y). \) \hfill (7)

That is, the structural mobility index captures the welfare change due to the change in cross-section or snapshot inequality.

Consider the case in which there is no permutation between \(x\) and \(y\), so that \(y' = y\), and \(z_c = z_a\). As pointed out in FO, in King’s (1983) model there is no mobility. In our case, all mobility is structural mobility which, by (7), in general is different from zero.

In the presence of some permutation between \(x\) and \(y\), we can show that exchange mobility is always socially desirable.

**Theorem 1.** Let \(x \rightarrow y\) be a distributional transformation such that \(y' \neq y\) and \(z_c \neq z_a\), i.e. such that there is some permutation between \(x\) and \(y\). Then, \(EM(x, y) > 0\).

(See the proof in the Appendix).

If \(I(x) > I(y)\), then by (7) structural mobility is non-negative. Hence total mobility \(M(x, y)\) will be positive. An example of this situation is provided by E1, illustrated in Figure 1.

**Figure 1 around here**

When \(I(x) < I(y)\), the sign of \(M(x, y)\) depends on the relative strength of \(EM(x, y)\) and \(M(x, y)\). Consider the following example illustrated in Figure 2:

\(E4: x = (2, 4) \rightarrow y = (7, 0); z_a = (9, 4)\).

There is a reranking between \(x\) and \(z_a\) and, therefore, a permutation between \(x\) and \(y\) which causes \(EM(x, y) > 0\). On the other hand, since \(I(x) < I(y)\) we have that \(SM(x, y) < 0\). It turns out that the \(SM(x, y)\) is stronger than \(EM(x, y)\), so that \(M(x, y) < 0\).
In the presence of rerankings, we can show that there exists some reallocation of the second-period total income which gives rise to the same mobility but with no reranking at all. The elimination of rerankings does away with some or all permutations, causing exchange mobility to decrease or to disappear altogether. Total mobility remains constant because the new second-period income distribution has less inequality than the original one; a change that implies an increase in structural mobility which exactly offsets the reduction in exchange mobility. Formally, we have:

**Theorem 2.** Let \( x \rightarrow y \) be a distributional transformation so that there is some reranking between \( x \) and \( z_a \). Then, there exists some \( y^* \in \mathcal{D} \) with the following properties: (i) \( \mu(y^*) = \mu(y) \); (ii) \( M(x, y^*) = M(x, y) \); (iii) There is no reranking between \( x \) and \( z_a^* = x + y^* \); (iv) \( I(y^*) < I(y) \).

(See the proof in the Appendix).

If we are interested at all in the social welfare during the second period, then Theorem 2 ensures that, in the presence of rerankings, we can always increase the original second period welfare maintaining overall mobility constant. On the other hand, given a distributional transformation \( x \rightarrow y \) with some rerankings between \( x \) and \( z_{a'} \), Theorem 2 allows us to disentangle the part of exchange mobility \( EM(x, y) \) which is due solely to the rerankings and the part which is due to any remaining permutations between \( x \) and \( y \).

Consider the income transformation \( x \rightarrow y^* \) where \( y^* \) is the second-period income distribution found in Theorem 2. There are two cases. In the first case, there are no permutations between \( x \) and \( y^* \), so that all income mobility \( M(x, y^*) \) is structural mobility and the entire exchange mobility \( EM(x, y) \) can be attributed to the permutations induced by the original rerankings between \( x \) and
In the second case, after the removal of all rerankings between \(x\) and \(z_a\) there are still some permutations left between \(x\) and \(y^*\). In this case,

\[
M(x, y^*) = SM(x, y^*) + EM(x, y^*),
\]

where

\[
SM(x, y^*) = \frac{I(x) - I(z^*_c)}{1 - I(x)}
\]

\[
EM(x, y) = \frac{I(z^*_c) - I(z^*_a)}{1 - I(x)},
\]

\(z^*_a = x + y^*, z^*_c = x + y^{**}\), and \(y^{**}\) is the income distribution \(y^*\) ordered as \(x\). On the other hand,

\[
M(x, y) = SM(x, y) + EM(x, y),
\]

where

\[
SM(x, y) = \frac{I(x) - I(z_c)}{1 - I(x)}
\]

\[
EM(x, y) = \frac{I(z_c) - I(z_a)}{1 - I(x)},
\]

\(z_a = x + y, z_c = x + y', \) and \(y'\) is the income distribution \(y\) ordered as \(x\). By Theorem 2.iv, \(I(y^{**}) = I(y^*) < I(y) = I(y')\). Therefore, we must have that \(I(z^*_c) < I(z_c)\), so that \(SM(x, y^*) > SM(x, y)\). By Theorem 2.ii, we have that \(M(x, y^*) = M(x, y)\). Consequently, \(EM(x, y) > EM(x, y^*)\), and the expression \(EM(x, y) - EM(x, y^*)\) is a good measure of the income mobility due solely to the permutations between \(x\) and \(y\) induced by the rerankings between \(x\) and \(z_a\).

The final question in this Section refers to what happens when, given a distributional transformation \(x \rightarrow y\), we switch the roles of \(x\) and \(y\) and consider the distributional transformation \(y \rightarrow x\). It is clear that our income mobility index is sensitive to the choice of the base period. In our two period world this is not important because the choice of base period is naturally given to us both in the intergenerational and the intragenerational interpretations: the income distribution of the parent generation and the "young", respectively.
Nevertheless, it is interesting to know the consequences of the base period
reversal. It turns out that the result depends on the relationship between I(x) and
I(y). Formally, we have:

**Remark 2.** Let $x \rightarrow y$ be a distributional transformation with $z_a = x + y$.
Assume, without loss of generality, that there is some permutation between x
and y, and let $z_c = x + y'$, where $y'$ is income distribution y ordered as x. Define a
distributional transformation $y \rightarrow x$, where y is ordered according to the "less
than or equal" relation. Let $z_a^* = y + x$. Notice that $z_a^* = z_a$, so that $I(z_a^*) = I(z_a)$.
Let $z_c^* = y + x'$, where $x'$ is income distribution x ordered as y. Since $z_c^* = z_c$, we
have that:

$$SM(x, y) = \frac{I(x) - I(z_c)}{1 - I(x)} \leq SM(y, x) = \frac{I(y) - I(z_c)}{1 - I(y)} \makebox{ } (2)$$

Hence, $M(x, y) \leq M(y, x)$.

**IV. THE ROLE OF INCOME GROWTH**

As we saw in Section II, the income mobility index defined in equation
(2) measures mobility as movement. Since it is a weakly relative index, a
distributional transformation $x \rightarrow y$ which involves only a change in scale, with
$y = \lambda x$ for some $\lambda > 0$, causes no mobility, i. e., $M(x, \lambda x) = 0$. Therefore, $M(x, y) \neq 0$
implies that either $I(x) \neq I(y)$ or that there are some permutations between x and
y. In this case, we can take into account FO's observation that it is reasonable to
view the movement aspect of mobility as closely related to positive or negative-income growth, i. e., to differences between $\mu(y)$ and $\mu(x)$. 
To any distributional transformation $x \rightarrow y$ with $\mu(x) = \mu(y)$, let us associate another distributional transformation $x \rightarrow u$ where $u$ is the income distribution defined by $u = (\mu(x)/\mu(y))y$. Therefore, $\mu(u) = \mu(x)$ and $I(u) = I(y)$. Notice also that the set of permutations between $x$ and $y$ is equivalent to the set of permutations between $x$ and $u$: if for a pair of individuals $i$ and $j$ we have that $x^i > x^j$, then $y^i < y^j$ if and only if $u^i = (\mu(x)/\mu(y))y^i < u^j = (\mu(x)/\mu(y))y^j$.

Given a distributional transformation $x \rightarrow y$, we can view the income distribution $u$ as arising from $x$ through income transfers among the individuals, which lead to income inequality $I(u) = I(y)$ and the same set of permutations between $x$ and $y$. Let $v_a = x + u$ be the aggregate income distribution in distributional transformation $x \rightarrow u$, and let $v_b = 2x$, so that $\mu(v_b) = \mu(v_a) = 2\mu(x)$ and $I(v_b) = I(x)$. The income mobility associated with the distributional transformation $x \rightarrow u$, which we call the *income transfer mobility index* $M(x, u)$, is equal to:

$$M(x, u) = \frac{W(v_a) - W(v_b)}{W(v_b)} = \frac{I(x) - I(v_a)}{1 - I(x)}.$$ 

If there is any permutation between $x$ and $u$, then let $u'$ be the income distribution $u$ ordered as $x$, and let $v_c = x + u'$. Then we have:

$$M(x, u) = SM(x, u) + EM(x, u),$$

where

$$SM(x, u) = \frac{I(x) - I(v_c)}{1 - I(x)},$$

$$EM(x, u) = \frac{I(v_c) - I(v_a)}{1 - I(x)}.$$ 

Of course, in view of Remark 1, $SM(x, u) \geq 0 \iff I(x) \geq I(u) = I(y)$; while, in view of Theorem 1, $EM(x, u) > 0$.

The differences in income mobility between distributional transformations $x \rightarrow y$ and $x \rightarrow u$ are solely due to differences between $\mu(u) =
\( \mu(x) \) and \( \mu(y) \). If we define the *income growth mobility index* \( \text{GRM}(x, y, u) = M(x, y) - M(x, u) \), then we can write:

\[
M(x, y) = SM(x, u) + EM(x, u) + \text{GRM}(x, y, u). \tag{8}
\]

Expression (8) indicates that our income mobility index \( M(x, y) \) can be decomposed into three terms: \( SM(x, u) \), which captures the structural mobility due to differences in cross-section income inequality once permutations between \( x \) and \( u \) (or \( x \) and \( y \)) have been removed, holding mean income constant; \( EM(x, u) \), which captures the exchange mobility due to the permutations between \( x \) and \( u \) (or \( x \) and \( y \)), holding mean income constant; and \( \text{GRM}(x, y, u) \), which captures the income mobility due to the income growth from \( \mu(u) = \mu(x) \) and \( \mu(y) \) in the distributional transformation \( x \rightarrow y \). (14)

What relationships can be established between the concepts involved in the two decompositions presented in this paper? In so far as our income mobility index captures mobility as movement, in a descriptive context one may expect that \( M(x, y) \gtrless M(x, u) \), or what is the same, \( \text{GRM}(x, y, u) \gtrless 0 \), depending on whether \( \mu(y) \gtrless \mu(x) \). As we can see in the following result, in our normative context this is only the case under certain circumstances.

**Theorem 3.** Let \( x \rightarrow y \) be a distributional transformation with \( \mu(y) \neq \mu(x) \). Consider the following three cases.

1. There are no permutations between \( x \) and \( y \).
   - (a) If \( I(y) \leq I(x) \), then \( M(x, y) \gtrless M(x, u) \Leftrightarrow \text{GRM}(x, y, u) \gtrless 0 \Leftrightarrow \mu(y) \gtrless \mu(x) \).
   - (b) If \( I(y) > I(x) \), then \( M(x, y) \lessgtr M(x, u) \Leftrightarrow \text{GRM}(x, y, u) \lessgtr 0 \Leftrightarrow \mu(y) \gtrless \mu(x) \).

2. There are some permutations between \( x \) and \( y \), but no rerankings between \( x \) and \( v_a = x + u \). Consider the following two sub-cases.
(a) Assume that $M(x, y)$ attains its upper bound, i.e., assume that $I(v_a) = 0$ so that $M(x, y) = I(x)/[1 - I(x)]$. Then $M(x, y) < M(x, u) \iff \text{GRM}(x, y, u) < 0$ regardless of the relationship between $\mu(y)$ and $\mu(x)$.

(b) Assume that $M(x, y) < I(x)/[1 - I(x)]$. In this case:

(i) $M(x, y) < M(x, u) \iff \text{GRM}(x, y, u) < 0 \iff \mu(y) < \mu(x)$;

(ii) We can have $0 < M(x, y) < M(x, u)$ and $\text{GRM}(x, y, u) < 0$, in spite of the fact that $\mu(y) > \mu(x)$.

(3) There are some rerankings between $x$ and $v_a$. Independently of the sign of $M(x, u)$, we can have $M(x, y) < M(x, u)$ with $\mu(y) > \mu(x)$, and $M(x, y) > M(x, u)$ with $\mu(y) < \mu(x)$.

(See the proof in the Appendix).

The relationship between $M(x, y)$ and $M(x, u)$ as a function as $\mu(y)$ and $\mu(x)$ that could have been expected in a descriptive context, only obtains in cases 1.a, 2.a with $\mu(y) < \mu(x)$, and 2.b.i. In case 1.b, the problem arises when we have $M(x, y) < 0$, a situation absent in a purely descriptive context where income mobility is always non-negative. As can be seen in the proof, in cases 2.a with $\mu(y) > \mu(x)$, 2.b.ii and 3 the problem arises when we have rerankings between $x$ and $z_a$, with or without rerankings between $x$ and $v_a$.

V. COMPARISON WITH THE LITERATURE

In this Section, we will compare our two decompositions with the discussion of the issues in the previous literature.

We should begin with the ideas advanced in the following classical quotation from Markandya (1982), pp. 307-8: “within the sociological literature a distinction is made between changes in mobility that can be attributed to the
increased availability of positions in higher social classes and those changes that can be attributed to an increased intergenerational movement among social classes, for a given distribution of positions among these classes” (Emphasis in the original). Based on this quotation, FO review the difficulties involved in an operational interpretation of these concepts. In particular, to illustrate the concept of exchange and structural mobility FO offer the following examples, pp. 564-565:

EVI: \[ x = (1, 2, 3) \rightarrow y = (3, 2, 1); z_a = (4, 4, 4) \]

EVII: \[ x = (1, 2, 3) \rightarrow y = (2, 1, 3); z_a = (3, 3, 6) \]

EVIII: \[ x = (1, 2, 3) \rightarrow y = (1, 2, 6); z_a = (2, 4, 9) \]

EIX: \[ x = (1, 2, 3) \rightarrow y = (2, 3, 6); z_a = (3, 5, 9). \]

FO state that the structure of the economy in EVI and EVII did not change because the cross-sectional distributions have the same income inequality and the same mean. Therefore, they claim that process VI depicts more income mobility solely because of the differences in exchange mobility. On the other hand, with respect to distributional transformations VIII and IX, FO state that “since the income ranks of the individuals are preserved, and since nobody’s income is decreased, it may be reasonable to say that there is no exchange mobility in these processes”. Moreover, “to the extent that IX is more mobile than VIII, the difference is due entirely to differences in structural mobility.”

The main difference between this –admittedly partial and suggestive–analysis and ours is that, according to FO, for an income distribution \( y \) to have the same structure as \( x \), we must have both \( I(y) = I(x) \) and \( \mu(y) = \mu(x) \), as in examples EVI and EVII. Correspondingly, structural mobility comes from differences between \( I(y) \) and \( I(x) \), as well as between \( \mu(y) \) and \( \mu(x) \) –as in their examples EVIII and EIX. In our case, two income distributions have the same
structure only if \( I(y) = I(x) \), whatever the relationship between \( \mu(y) \) and \( \mu(x) \). The implications of this fundamental difference are as follows.

As far as exchange mobility is concerned, we agree with FO that in EVI and EVIII all income mobility is exchange mobility. However, in our view FO restrict their analysis to a limited set of permutations between \( y \) and \( x \). In particular, they ignore cases in which \( \mu(y) \) can diverge from \( \mu(x) \), as in the following examples:

\[
\text{E6: } \ x = (1, 2, 3) \rightarrow y = (6, 4, 2); \ z_a = (7, 6, 5)
\]

\[
\text{E7: } \ x = (1, 2, 3) \rightarrow y = (2, 2/3, 4/3); \ z_a = (3, 8/3, 13/3).
\]

Distribution \( x \) has been chosen as in EVI and EVIII, while \( I(y) = I(x) \) so that all income mobility is again exchange mobility in our sense. The differences are that in E6, \( \mu(y) = 6 > \mu(x) = 2 \), while in E7, \( \mu(y) = 4/3 < \mu(x) = 2 \). Since in EVI, \( I(z_a) = I(4, 4, 4) = 0 \), income mobility reaches its upper bound, \( I(x)/\{1 - I(x)\} \). The ordering of EVII, E6 and E7 would depend on the relationship between \( I(3, 3, 6), I(7, 6, 5) \) and \( I(3, 8/3, 13/3) \) for an appropriate income inequality index.

As far as structural mobility, the following two examples help to understand the differences between the two approaches:

\[
\text{E8: } \ x = (1, 2, 3) \rightarrow y = (8/11, 12/11, 24/11)
\]

\[
\text{E9: } \ x = (1, 2, 3) \rightarrow y = (1.5, 2, 2.5).
\]

Since in EVIII, EIX, E8 and E9 there are no permutations, in all cases all income mobility is structural mobility. If we compare EVIII and E8, we find that since \( I(y) = I(8/11, 12/11, 24/11) = I(2, 3, 6) \), income mobility due to structural change in our sense is the same in both examples. Thus, in spite of the fact that in E8 everybody's income is decreased and \( \mu(y) = 4/3 < \mu(x) = 2 \), we have that \( M(E8) = M(EVIII) \). In E9, \( \mu(y) = \mu(x) = 2 \) and income mobility can be attributed to the
increased income of the poorest individual, rather than in the higher social classes as in Markandya’s quotation and FO examples VIII and IX.

In the second place, it should be noticed that FO present no examples of a distributional transformation which exhibits both structural and exchange mobility in their sense. Consider, for example, the transformation

\[ \text{E10: } x = (1, 2, 3) \rightarrow y = (9, 0, 3); z_a = (10, 2, 6). \]

In order to explore the impact on income mobility of the class of permutations between \( x \) and \( y \) to which they restrict their analysis, the difficulty, of course, is how to build from \( y \) an income distribution with the same income inequality and the same mean income as \( x \). In our case, what we do is construct an income distribution \( y' \) with the same inequality as \( y \) and in which all permutations between \( x \) and \( y \) have been removed, i.e. we choose \( y' \) to be the income distribution \( y \) ordered as \( x \). Then we define structural mobility as the income mobility associated to the distributional transformation \( x \rightarrow y' \), that is, \( SM(x, y) = M(x, y') = \frac{I(x) - I(z_a)}{1 - I(x)} \); in E10, \( z_c = x + y' = (1, 5, 12) \). Our measure of exchange mobility is the difference between \( M(x, y) \) and \( M(x, y') \), i.e. \( EM(x, y) = \frac{I(z_c) - I(z_a)}{1 - I(x)} \).

If one wants to isolate the effect of the changes in the mean in a distributional transformation \( x \rightarrow y \), our proposal is to use our second decomposition in which income mobility is seen to arise from three sources: changes in cross-section income inequality and permutations between \( x \) and \( y \), holding the mean income constant at the level of \( \mu(x) \); and changes in mean incomes which get captured by the term \( GRM(x, y, u) \).

In the third place, FO’s analysis refers to a descriptive view of income mobility that is silent on the conditions under which structural, exchange or total mobility is socially desirable. In particular, it appears that the distinction between
what we call permutations and rerankings has no role in a descriptive view of income mobility\(^{(15)}\). Consequently, from this position it is impossible to appreciate the potential deleterious effect on social welfare of "too much" rerankings (see E4 and Theorem 2). Finally, as we have seen in the previous Section, whether one adopts a descriptive or a normative approach influences our views on how income mobility varies with changes in \(\mu(y)\).

VI. CONCLUSIONS

Until recently, we could only evaluate an economy's performance over time with individual income data drawn from two (or more) independent population samples in different time periods. It is quite obvious that the usual comparison in inequality or social welfare terms of two snapshot income distributions permits only a partial evaluation of the dynamic economic process. A convincing criticism is conveniently summarized in the following quotation from Karcher et al. (1995): "...since the individuals' positions on the cardinal income scale rarely remain unchanged over time, an increase in a snapshot measure of inequality is clearly consistent with there having been a significant amount of equalizing mobility over time. Only if all individuals' earnings remain constant from period to period will a measure of inequality or welfare give the same result irrespective of the length of the accounting period".

CDW's modeling of the dynamic evaluation problem can be viewed as an attempt to confront this criticism. Conceptually, they suggest that an income mobility index can be obtained by comparing the social welfare of the actual distributional transformation with the social welfare of a hypothetical distributional transformation which exhibits no mobility. Operationally, they assume that the social welfare of any distributional transformation can be
identified with the social welfare of the aggregate income distribution. In particular, under the type of SEF they adopt, what matters at the social level is the inequality of the aggregate income distribution—a fundamental idea which the previous quotation points out too.

There are several observable factors in longitudinal data sets which affect the inequality of the aggregate income distribution. What we have done in this paper is to develop the idea that, within CDW’s framework, it is useful to start by concentrating on changes in cross-section or snapshot income inequality, and changes in income shares (or absolute income differences). Within the limits of a two-period model, we have shown how to decompose CDW’s income mobility index into two indexes of structural and exchange mobility which capture, respectively, the welfare effect of these two types of income changes. In so doing, we have shown the relevance of distinguishing between two kinds of rank reversals: permutations between the first- and second-period income distributions, and rerankings between the initial and the aggregate or final situation.

This decomposition should help in the interpretation of observable data. Take, for instance, the well-known fact that in many OCDE countries—notably, the U.K and the U.S.—income inequality increased considerably during the 1980s\(^{(16)}\). Some may fear that the rich are getting richer while the poor are getting poorer, which should cause a decrease in social welfare; while some may share Friedman (1962)'s belief that “...capitalism undermines status and induces social mobility”\(^{(17)}\) offseting the increase in snapshot income inequality. In the language of the theory, we can be certain that structural mobility in those countries during this period is negative. The first group of people fears that during this period exchange mobility might be small, causing total income mobility to be negative;
while Friedman's position can be interpreted as the belief that, whenever necessary, under capitalism exchange mobility would come to the rescue to ensure that the change in social welfare resulting from income changes over time would always be positive.

If, in addition, one wants to isolate the effect on mobility of mean incomes changes, we then propose a second decomposition in which income mobility is seen to arise from three sources: changes in cross-section income inequality and permutations, holding the mean income of the second-period income distribution constant at the level of the first-period one; and changes in mean incomes which appear as a residual in an income growth mobility term.

From a conceptual point of view, we should emphasize that our results on the properties of the different income mobility concepts introduced along the paper, have been obtained in a framework which, contrary to the seminal contributions by King (1983) and Markandya (1982, 1984), does not involve any new value judgments beyond the traditional ones. In particular, we do not assume that either permutations or rerankings have any positive value.

We believe that in situations where there typically are individual rank reversals, our approach is immediately applicable. Consider, for instance, the following three problems characterized by two stages in a static context. In the first place, CDW point out that, in their study of tax and benefit progressivity, Blackorby and Donaldson (1984) compare the actual after-tax income distribution with the distribution which would have resulted if taxes had been raised proportionally. Reinterpreting $z_a$ as after-tax income, and $z_b$ as the hypothetical distribution arising from proportional taxation, CDW's income mobility index becomes Blackorby and Donaldson's index of relative tax progressivity (18). Think also of the possibility of interpreting $z_a$ as household income, and $z_b$ as the
distribution that results from either only considering household head incomes versus the incomes earned by other household members, or only considering all market incomes as opposed to public transfers.

Whether the decompositions developed in this paper might work in other income mobility models is an open question left for future research. In any case, we should mention that the greatest limitation of our approach is surely the restriction to a two period world. The extension to a truly multiperiod context must start with a model of how to evaluate, from an ethical point of view, multiperiod individual income streams. On the other hand, if the present two period model were to be naively extended to three or more periods, we know that the results would depend on the decision about the reference period. Therefore, what is also needed is an appropriate suggestion for the notion of an immobile distributional transformation in a multiperiod context.
APPENDIX

Theorem 1. Let \( x \rightarrow y \) be a distributional transformation where there is some permutation between \( x \) and \( y \). Then \( EM(x, y) > 0 \).

Proof:

Recall that \( x = (x^1, ..., x^n) \) is ordered so that \( x^1 \leq x^2 \ldots \leq x^n \), and let \( y' \) be the vector \( y \) ordered as \( x \). That there is some permutation between \( x \) and \( y \), means that \( y \neq y' \). Let \( 1 \leq i \) be the first individual for whom \( y^i > y'^i \). There must be some individual \( j > i \) for whom \( y^j = y'^j \). Thus, \( x^i < x^j \) and \( y^i > y^j \).

Let us interchange the ranks of individuals \( i \) and \( j \) in vector \( y \), i.e. let us define \( y_1 \) such that \( y^i_1 = y^i, y^j_1 = y^j, \) and \( y^k_1 = y^k \) for all \( k \neq i, j \). Define \( z_{a1} = x + y_1 \). Since \( \mu(y_1) = \mu(y), \) \( \mu(z_{a1}) = \mu(z_a) \). Note that \( z^i_1 = x^i + y^i, z^j_1 = x^j + y^j, \) and \( z^k_1 = z^k_a \) for all \( k \neq i, j \). Since \( z^i_a - z^i_{a1} = x^i - x^i < 0 \) and \( z^j_a - z^j_{a1} = x^j - x^j > 0 \), we have that

\[
z^i_a < z^i_{a1} \text{ and } z^j_a > z^j_{a1}. \tag{1}
\]

At the same time, \( z^i_a - z^i_{a1} = y^i - y^i > 0 \) and \( z^j_a - z^j_{a1} = y^j - y^j < 0 \), which implies that

\[
z^i_a > z^i_{a1} \text{ and } z^j_a < z^j_{a1}. \tag{2}
\]

There are two cases. Given that \( x^i < x^j \), if there is a reranking involving individuals \( i \) and \( j \), i.e. if \( z^i_a > z^j_a \), then by (1) we have:

\[
z^i_{a1} < z^j_a < z^i_a < z^j_{a1}.
\]

Otherwise, i.e. if \( z^i_a \leq z^j_a \), then by (2) we have:

\[
z^i_{a1} < z^i_a \leq z^j_a < z^j_{a1}.
\]

In both cases, the aggregate income distribution \( z_a \) can be obtained from \( z_{a1} \) via a progressive transfer. Therefore, \( I(z_a) < I(z_{a1}) \) for all well behaved relative or absolute inequality indexes \( I \).

Recall that \( EM(x, y) = \{I(z_c) - I(z_a)\}/\{1 - I(x)\}, \) where \( z_c = x + y' \). If \( y_1 = y' \), so that \( z_{a1} = z_{c'} \), then we are done. Otherwise, there must be some individual \( r > i \) for whom \( y^r_1 > y'^r \). In that case, there must be some individual \( s > r \) for whom \( y^s_1 \)
Thus, \( x^r < x^s \) and \( y^r_1 > y^s_1 \). Let us interchange the ranks of individuals \( r \) and \( s \) in vector \( y^r_1 \), i.e. let us define \( y^r_2 \) such that \( y^r_2 = y^s_1, y^s_2 = y^r_1, \) and \( y^r_k = y^s_k \) for all \( k \neq r, s \). Define \( z_{a2} = x + y_2 \). By an argument analogous to the previous one, we conclude that \( I(z_{a1}) < I(z_{a2}) \). If \( y_2 = y' \), so that \( z_{a2} = z_{c'} \), then we are done. Otherwise, we proceed in the same manner until we show that \( I(z_a) < I(z_c) \), i.e. \( EM(x,y) > 0 \).

Q.E.D.

Theorem 2. Let \( x \rightarrow y \) be a distributional transformation where there is some reranking between \( x \) and \( z_{a'} \). Then, there exists some \( y^* \in D \) with the following properties: i) \( \mu(y^*) = \mu(y) \); ii) \( M(x,y^*) = M(x,y) \); iii) There is no reranking between \( x \) and \( z_a^* = x + y^* \); iv) \( I(y^*) < I(y_2) \).

Proof: Let \( z_{a'} \) be the vector \( z_a \) ordered as \( x \). Define \( y^* = z_{a'} - x \). If \( z^{i'}_a = z^i_a \), then \( y^{i*} = z^{i'}_a - x^i = y^i > 0 \). If \( z^{i'}_a \neq z^i_a \), say \( z^{i'}_a < z^i_a \), then there must be some other individual \( j > i \) such that \( z^{i'}_a = z^j_a = x^j + y^j \). Since \( x^i \leq x^j \), \( y^{i*} = x^j + y^j - x^i > 0 \). Thus, \( y^* \in D \), so that \( x \rightarrow y \) is a distributional transformation with \( z_a^* = x + y^* = z_{a'} \).

Since \( \mu(z_a^*) = \mu(z_{a'}) = \mu(z_a) \), we have that \( \mu(y^*) = \mu(y) \), which is condition (i). Since \( I(z_a^*) = I(z_{a'}) = I(z_a) \), we have

\[
M(x,y^*) = \frac{I(x) - I(z_a^*)}{I(x) - I(z_a)} = \frac{I(x) - I(z_a)}{I(x) - I(x)} = M(x,y),
\]

which is condition (ii). Since \( z_a^* = z_{a'} \) and \( z_{a'} \) is ordered as \( x \), there is no reranking between \( x \) and \( z_a^* \), which is condition (iii).

That there is some reranking between \( x \) and \( z_a \) means that \( z_a^* \neq z_{a'} \). Let \( 1 \leq i \) be the first individual for whom \( z^{i'}_a > z^i_a \). There must be some individual \( j > i \) for whom \( z^j_a = z^{i'}_a \). Let us interchange the ranks of individuals \( i \) and \( j \) in vector \( z_a \), i.e. let us define \( v_1 \) such that \( v^i_1 = z^j_a, v^j_1 = z^i_a, \) and \( v^k_1 = z^k_a \) for all \( k \neq i, j \). Clearly, \( \mu(v_1) = \mu(z_a) \). Define \( y_1 = v_1 - x \). Thus, \( y^i_1 = z^j_a - x^i, y^j_1 = z^i_a - x^j, y^k_1 = z^k_a - x^k = y^k \) for all \( k \neq i, j \), and \( \mu(y_1) = \mu(v_1) - \mu(x) = \mu(y) \).
Note that $z_a^i > z_a^j$ if and only if $x^i + y^i > x^j + y^j$, or $y^i - y^j < x^j - x^i$. On the other hand, $v_1^i = z_a^i > v_1^j = z_a^j$ if and only if $x^j + y_1^j > x^i + y_1^i$, or $y_1^i - y_1^j < x^j - x^i$. Therefore, $y_1^i - y_1^j < y^i - y^j$. Taking into account that $y_1^k = y^k$ for all $k \neq i, j$, and $\mu(y_1) = \mu(y)$, this implies that $y_1$ can be obtained from $y$ via a progressive transfer. Therefore, $I(y_1) < I(y)$ for all well behaved relative or absolute inequality indexes $I$.

If $v_1 = z_a'$, then we are done because $y_1 = y^*$. Otherwise, there must exist some individual $r > i$ for which $v_1^r > z_a^r$, as well as some other $s > r$ for whom $v_1^s = z_a^s$. Let us interchange the ranks of individuals $r$ and $s$ in vector $v_1$, i.e. let us define $v_2$ such that $v_2^i = v_1^s$, $v_1^s = v_1^r$, and $v_1^k = v_1^k$ for all $k \neq i, j$. Define $y_2 = v_2 - x$. By an argument analogous to the previous one, we conclude that $I(y_2) < I(y_1)$. If $v_2 = z_a'$, so that $y_2 = y^*$, then we are done. Otherwise, we proceed in the same manner until we show that $I(y^*) < I(y)$, which is point iv) in the Theorem.

Q.E.D.

Theorem 3. Let $x \to y$ be a distributional transformation with $\mu(y) \neq \mu(x)$. Recall that

$$SM(x, y) = \frac{I(x) - I(z_c)}{1 - I(x)}$$

$$SM(x, u) = \frac{I(x) - I(v_c)}{1 - I(x)},$$

where $z_c = x + y'$, $v_c = x + y'$, and $y'$ and $u'$ are the income distributions $y$ and $u$ ordered as $x$. It is easy to see that

$$SM(x, y) \geq SM(x, u) \Leftrightarrow \mu(y) \geq \mu(x).$$

(5)

As in case (1) of the theorem, assume that there are no permutations between $x$ and $y$. In this case, $M(x, y) = SM(x, y)$ and $M(x, u) = SM(x, u)$. If $I(y') = I(u') \leq I(x)$, as in case 1.a, then in view of (5):

$$M(x, y) \geq M(x, u) > 0 \Leftrightarrow GRM(x, y, u) \geq 0 \Leftrightarrow \mu(y) \geq \mu(x).$$

If, as in case 1.b, $I(y') = I(u') > I(x)$, then in view of (5):

$$M(x, y) \leq M(x, u) < 0 \Leftrightarrow GRM(x, y, u) \leq 0 \Leftrightarrow \mu(y) \geq \mu(x).$$

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As in case (2), assume that there are some permutations between x and y, but no rerankings between x and \( v_a = x + u \). Assume also, as in case 2.a, that \( M(x, y) \) attains its upper bound, i.e. \( I(v_a) = 0 \) and \( M(x, y) = I(x)/\{1 - I(x)\} \). Then, by the definition of an upper bound, \( M(x, y) < M(x, u) \iff GRM(x, y, u) < 0 \) regardless of the relationship between \( \mu(y) \) and \( \mu(x) \).

As in case 2.b, assume that \( M(x, y) < I(x)/\{1 - I(x)\} \). Notice that if \( \mu(y) \leq \mu(x) \), as in case 2.b.i, then there is no reranking between x and \( z_a = x + y \). Otherwise, i.e., if there is a pair of individuals i, j such that \( x^i > x^j \), \( y^i < y^j \), and \( x^i + y^i > x^j + y^j \), then \( x^i + (\mu(x)/\mu(y)) y^i > x^j + (\mu(x)/\mu(y)) y^j \), i.e. \( x^i + u^i > x^j + u^j \), a contradiction with the assumption that there are no rerankings between x and \( v_a \). Consider the case in which \( I(y) = I(u) < I(x) \). Since \( z_a = x + y \) and \( v_a = x + u \), \( I(z_a) > I(v_a) \) for all y with \( \mu(y) < \mu(x) \). Therefore, in this case: \( 0 < M(x, y) < M(x, u) \iff GRM(x, y, u) < 0 \iff \mu(y) < \mu(x) \).

If \( \mu(y) > \mu(x) \), as in case 2.b.ii, then \( SM(x, y) > SM(x, u) \) as we saw in case 1.a. However, in this case there can be rerankings between x and \( z_a' \) which may cause \( EM(x, y) \) to be sufficiently smaller than \( EM(x, u) \) and \( M(x, y) < M(x, u) \). This is illustrated in the following example, in which there is a permutation between x and y, no reranking between x and \( v_a \), but a reranking between x and \( z_a' \):

\[
\begin{align*}
x &= (1, 5) \rightarrow y = (40, 20), \quad z_a = (41, 25); \\
x &= (1, 5) \rightarrow u = (4, 2), \quad v_a = (5, 7).
\end{align*}
\]

It is clear that \( I(x) = I(1, 5) > I(z_a) = I(41, 25) > I(v_a) = I(5, 7) \), so that \( 0 < M(x, y) < M(x, u) \). Therefore, \( GRM(x, y, u) < 0 \), in spite of the fact that \( \mu(y) = 30 > \mu(x) = 3 \).

Assume, as in case 3, that there are some rerankings between x and \( v_a' \). Consider the following two examples, in which \( M(x, u) = 0 \).

\[
\begin{align*}
x &= (1, 2) \rightarrow y = (12, 0), \quad z_a = (13, 2); \\
x &= (1, 2) \rightarrow y' = (2.5, 0), \quad z_a' = (3.5, 2); \\
x &= (1, 2) \rightarrow u = (3, 0), \quad v_a = (4, 2).
\end{align*}
\]

In the first example, \( I(x) = I(1, 2) < I(z_a) = I(13, 2) \) and \( I(x) = I(1, 2) = I(v_a) = I(4, 2) \). Therefore, \( M(x, y) < M(x, u) = 0 \), in spite of the fact that \( \mu(y) = 6 > \mu(x) = 1.5 \).

In the second example, \( I(x) = I(1, 2) < I(z_a') = I(3.5, 2) \). Therefore, \( M(x, y) > M(x, u) = 0 \).
\( u \) = 0 in spite of the fact that \( \mu(y) = 1.25 < \mu(x) = 1.5 \). Finally, consider the following two examples, in which \( M(x, u) < 0 \):

\[
\begin{align*}
\text{Example 1:} & \quad x = (3, 4) \rightarrow y = (24, 4), z_a = (27, 8); \\
\text{Example 2:} & \quad x = (3, 4) \rightarrow y' = (1, 1/6), z'_a = (4, 25/6);
\end{align*}
\]

\[
\begin{align*}
\text{Example 1:} & \quad x = (3, 4) \rightarrow u = (6, 1), v_a = (9, 5).
\end{align*}
\]

In both examples, \( I(x) = I(3, 4) < I(v_a) = I(9, 5) \), so that \( M(x, u) < 0 \). In the first example, \( I(z_a) = I(27, 8) > I(v_a) = I(9, 5) \), so that \( M(x, y) < M(x, u) = 0 \) in spite of the fact that \( \mu(y) = 17.5 > \mu(x) = 3.5 \). In the second example, \( I(x) = I(3, 4) > I(z'_a) = I(4, 25/6) \), so that \( M(x, y) > M(x, u) \) in spite of the fact that \( \mu(y) = 7/6 < \mu(x) = 3.5 \).

Q.E.D.
(1) As FO conclude, "All in all, the present literature on income mobility falls short of providing an exact, robust decomposition of total mobility into its basic sources" (p. 565). For another valuable discussion of the difficulties involved in modeling structural and exchange mobility, see Shorrocks (1993).

(2) For the strengths and limitations of this approach, as well as references, see Sections 2.5, 4.3 and 5 in Fields and Ok (2000).

(3) For descriptive measures in this framework, see inter alia the relative indices suggested by Shorrocks (1978b) and Cowell (1985), and the absolute indices proposed by Berrebi and Silber (1983) and Fields and Ok (1996).

(4) For other aggregation schemes, see, for instance, Shorrocks (1978a), Maasoumi and Zandvakili (1989, 1990) -based on Maasoumi (1986)- as well as the criticism of them by Dardadoni (1990). For another approach to the construction of lifetime income, see Cowell (1979).

(5) In the individualistic tradition of welfare economics, we want the welfare of every individual to count in the definition of social welfare. In the normative approach to income mobility where the SEF is defined on the set of transition matrices, some of this information is lost. From this point of view, it is an advantage to work with distributional transformations where all individual incomes are explicitly considered.

(6) In the absolute case, the benchmark distributional transformation $x \rightarrow y_b$ would be chosen to be absolutely immobile, i.e. income differences would be preserved through time.

(7) In the absolute case, we would have $M_A(x, y) = W(z_a) - W(z_b)$.

(8) Shorrocks (1978a) justifies $A.3$ as a direct application in the intertemporal context of the population replication axiom, usually assumed in income distribution theory in order to compare the income inequality of populations of different size.

(9) For example, CDW assume that $W$ is homothetic. In this case, it is well known that we can write $W(x) = \mu(x)[1 - I^{AKS}(x)]$, where $I^{AKS}$ is the relative inequality index obtained according to the Atkinson-Kolm-Sen procedure which uses the notion of an equally distributed income. Alternatively, because of its good additive separability properties in Ruiz-Castillo (2000) we use $W(x) = \mu(x)[1 - I_c(x)]$, where $I_c$ for $c = 1, 2$, stand for the following members of the general entropy family of income inequality indices: the first inequality index originally suggested by Theil, and an ordinal transformation of the coefficient of variation, respectively.

(10) In the absolute case, the SEF can be expressed in terms of the mean and a translatable index of absolute inequality $I_A$. In particular, we may assume that
\[ W(x) = \mu(x) - I_A(x), \] in which case the absolute income mobility index defined in note (7) becomes \( M_A(x, y) = I_A(x) - I_A(z_a). \) Because of its good normative and additive separability properties, the best choice in this case is the Kolm-Pollak SEF—see Blackorby and Donaldson (1980).

(11) It is easy to see that, given a distributional transformation \( x \rightarrow y, M(\lambda x, \alpha y) \neq M(x, y) \) for different \( \lambda \) and \( \alpha > 0 \), i.e. in FO's terminology our mobility index is not strongly relative or intertemporally scale invariant. On the other hand, in the absolute case the income mobility index defined in note (10) is weakly absolute, i.e. \( M_A(x + \delta, y + \delta) = M_A(x, y) \) for all vectors \( \delta = (\delta, \ldots, \delta) \) whose elements are all equal.

(12) In this sense, our approach follows definition II in Markandya (1984). On the other hand, in the absolute case we will simply have \( M_A(x, y) = SM_A(x, y) + EM_A(x, y) \), where \( SM_A(x, y) = I_A(x) - I_A(z_c) \), and \( EM_A(x, y) = I_A(z_c) - I_A(z_a) \).

(13) The terminology to differentiate between "income transfer" and "income growth" mobility, has been adapted from Fields and Ok (1996), where an analogous distinction between total movement due to the transfer of income and total movement due to economic growth (or to economic contraction) is drawn for the purpose of decomposing a descriptive and absolute income mobility index into two terms.

(14) In the absolute case, we simply have \( M_A(x, y) = SM_A(x, u) + EM_A(x, u) + GRM(x, y, u) \), where \( SM_A(x, u) = I_A(x) - I_A(v_c) \), \( EM_A(x, u) = I_A(v_c) - I_A(v_a) \), \( GRM(x, y, u) = M_A(x, y) - M_A(x, u) \), and \( M_A(x, u) = I_A(x) - I_A(v_a) \).

(15) As a matter of fact, in FO's examples EVI and EVII there are permutations between \( x \) and \( y \) but no rerankings between \( x \) and \( z_a \).


(17) The full quotation is as follows: "Consider two societies that have the same distribution of annual income. In one there is great mobility and change so that the position of particular families in the income hierarchy varies from year to year. In the other, there is great rigidity so that each family stays in the same position year after year. The one kind of inequality is a sign of dynamic change, social mobility, equality of opportunity; the other, of a status society. The confusion of these two kinds of inequality is particularly important precisely because competitive free-enterprise capitalism tends to substitute the one for the other...capitalism undermines status and induces social mobility".

(18) For an analysis of the structural and exchange income mobility induced by the income tax, as well as the measurement of horizontal inequality, see Section III in Ruiz-Castillo (1997).
REFERENCES


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Structural mobility = $I(x) - I(z_c) > C$

Exchange mobility = $I(z_c) - I(z_a) > C$

Mobility = $I(x) - I(z_a) > C$

FIGURE 1
Structural mobility = \( I(x) - I(z_c) < 0 \)

Exchange mobility = \( I(z_c) - I(z_a) > 0 \)

Mobility = \( I(x) - I(z_a) < 0 \)

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