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ON THE TERM STRUCTURE OF INTERBANK INTEREST RATES: JUMP-DIFFUSION PROCESSES AND OPTION PRICING

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Abstract

In this paper we study the dynamic behavior of the term structure of Interbank interest rates and the pricing of options on interest rate sensitive securities. We posit a generalized single factor model with jumps to take into account external influences in the market. Daily data is used to test for jump effects. Qualitative examination of the linkage between Monetary Authorities interventions and jumps are studied. Pricing results suggests a systematic underpricing in bonds and call options if the jump component is not included. However, the pricing of put options on bonds presents indeterminacies.

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1. Introduction

This paper addresses the modelling of the term structure of Interbank interest rates and the pricing of options on interest rate sensitive securities. Traditional (one or more factor) models have so far assumed that interest rates evolve over time in a continuous way, see Duffie (1992, pp. 129-139). But there are some circumstances where this may not be a reasonable assumption. One interesting case is domestic Interbank Markets which are subject to exogenous interventions by the Monetary Authorities in their attempts to control the money supply. In this case those interventions may cause jump-like behavior in observed interest rates. This idea is similar to Merton's (1976) analysis of stock option pricing. Merton suggested that bursts of information are better depicted in price behavior as jumps. Thus, one may infer from Merton´s suggestion that the unexpected interventions by the Monetary Authorities are a set of signals to the market which convey information on money supply.

Of course many other reasons can affect interest rates in jump-like fashion, for instance supply or demand shocks and economic or political news. One of the targets of this paper is to deal with all those possible influences under the same umbrella, by positing a general enough model that can cope with these kinds of effects. Note also that another practical advantage of employing diffusion processes with superimposed discrete jumps is that we can take into account the "fat tails" usually found in the distribution of security prices.

The article is organized as follows. In Section 2 we present the theoretical background. Section 3 describes the econometric approach. Section 4 addresses the basic characteristics of our data sample. The empirical analysis is presented in Section 5. Section 6 analyzes the relationship between monetary authorities interventions and the jump-like behavior of interest rates. Section 7 discuss the pricing of bonds and options. Finally, Section 8 summarizes and concludes.

2. Theoretical Background

The basic framework in this paper is the single-factor model of interest rates, in the tradition of Vasicek (1977), and Cox, Ingersoll and Ross (1985a,b) among others. We generalize those models, following the suggestions by Das (1994a), who posits the addition of a jump component in the process followed by the state variable. The dynamics of the interest rate are given by the following jump-diffusion process:

$$dr = k(\theta - r)dt + \sigma r^{\tau} dz + J(\mu, \gamma^2) d\pi(h) \tag{1}$$

where, for the instantaneous riskless interest rate **r**, **k** is the coefficient of mean reversion, θ is the long run mean level of **r**, σ is the standard deviation of **r**, τ is the elasticity coefficient parameter, **dz** is a standard Gauss-Wiener process, **J** is the jump magnitude in **r** which has a Normal distribution with mean μ and variance γ^2 and **d**π**(h)** is a Poisson arrival process with a constant intensity parameter **h**. The jump and diffusion components on the interest rate process are assumed to be independent. Mean reversion (**k** > 0) ensures that **r** follows a stationary process.

Given the instantaneous interest rate **r** at period **t**, let **P[r,t,T]** represent the price of a riskless pure discount bond maturing at period **T**. From Ito's Lemma, the instantaneous rate of return on the bond is:

$$dP = (P_r dr + 0.5 P_{rr} (dr)^2 + P_t dt) \tag{2}$$

where subscripts denote partial derivatives. In perfect markets, the instantaneous expected rate of return for any asset can be written as the instantaneous riskless rate, **r**, plus a risk premium. Therefore the risk adjusted return on all zero coupon bonds must be the same. Assuming that the market price per unit of risk (λ(.)) for the bond is a general function that may depend on σ, r and τ, but not on T-t, and remembering that the jump and diffusion components in (1) are independent, the variance of changes in **r** is simply the sum of the variances of both components. The arbitrage-free pricing partial differential equation is as follows:

$$\begin{aligned} 0 = (k(\theta - r) - \lambda(\sigma, r, \tau))P_r + P_t + 0.5\sigma^2 r^{2\tau} P_{rr} - rP \\ + h\, E[P(r+J) - P(r)] \end{aligned} \tag{3}$$

This is the fundamental equation for the price of any zero coupon bond which has a value that depends solely on the instantaneous rate, **r**, and the time to maturity, **T-t**. With the boundary condition,

$$P[r, T, T] = 1.0 \tag{4}$$

Analytical solutions of (3) (if available) are usually obtained by positing that the functional form of the bond price is given by

$$P[r, t, T] = A[t, T] \exp[-B[t, T] r] \tag{5}$$

where

$$A[t,T] = f_A(\Phi) \quad , \quad B[t,T] = f_B(\Phi)$$
$$\Phi = (T-t, k, \theta, \sigma, \tau, h, \mu, \gamma^2, \lambda) \tag{6}$$

If it is not possible to find an analytical solution, numerical procedures may be used to approximate (3).

Explicit expressions for **A[t,T]** and **B[t,T]** have been reported for some particular cases. Ahn and Thompson (1988) studied the case of $\tau=0.5$ assuming a jump component equal to δ**dy** where δ is a negative constant and the intensity of **y** is taken to be π**r**, i.e. the jump arrival rate is proportional to the level of interest rate. Das (1994a) studied the cases $\tau=0.5$ and $\tau=0.0$ and parameterized both the size and sign of the jump component. To our knowledge, expressions for **A[t,T]** and **B[t,T]** for general values of τ have not been reported.

The valuation framework presented above, can be applied to other securities whose payoffs depend on interest rates, such as options and futures on bonds. Theoretical work on pricing interest rate sensitive securities for jump-diffusion process include Ahn and Thompson (1988), Naik and Lee (1990), Das (1994a) and Naik and Lee (1995). In those papers analytic models for bond and option prices are given. However, none of these models permits the pricing of American options. This is unfortunate given that almost all traded interest sensitive securities have American features. Furthermore, the pricing of "American-style" derivative securities usually requires numerical methods, either by Binomial trees or by finite-differencing methods, see Duffie(1992, Chap. 10). Recently, applications of numerical methods to jump-diffusion processes have been reported by Amin (1993) for the Binomial tree approach and by Das (1994b) for the finite-differencing approach. In this paper we follow the latter approach, using the Full Implicit Finite-Differencing (FIFD) method for bond and option pricing.

We now develop the procedure to solve equation (3) using the FIFD method. When using this method, careful specification of the boundary conditions is required. Since the state variable, **r**, varies in the range $[0, \infty)$, and the process requires backward recursion in time on a discrete time grid of the state variable, it is hard to establish a grid over this support. To deal with this problem we carry out the following transformation of variable

$$y = \frac{1}{1+\beta r} \quad , \ \beta > 0 \tag{7}$$

The new state variable, **y**, varies in the range $(0,1]$ and this makes the upper bound easy to establish. Using this transformation from **r** to **y** we obtain a transformed version of the Partial Differential Equation (3):

$$\begin{aligned} 0 = {} & P_y\left[\sigma^2\beta^{2-2\tau}y^{3-2\tau}(1-y)^{2\tau} - \beta y^2\left(k\left(\theta - \frac{1-y}{\beta y}\right) - \lambda(.)\right)\right] \\ & + P_{yy}\left[\frac{1}{2}\sigma^2\beta^{2-2\tau}y^{4-2\tau}(1-y)^{2\beta}\right] \\ & + P_t - \left(\frac{1-y}{\beta y}\right)P \\ & + \left[EP\left(\frac{1-y}{\beta y} + J\right) - P\left(\frac{1-y}{\beta y}\right)\right]h \end{aligned} \tag{8}$$

which can be written as

$$0 = P_y A + P_{yy} B + P_t - \left(\frac{1-y}{\beta y}\right)P + \left[EP\left(\frac{1-y}{\beta y} + J\right) - P\left(\frac{1-y}{\beta y}\right)\right]h \tag{9}$$

where

$$A = \left[\sigma^2\beta^{2-2\tau}y^{3-2\tau}(1-y)^{2\tau}-\beta y^2\left(k(\theta-\frac{1-y}{\beta y})-\lambda(.)\right)\right]$$
$$B = \left[\frac{1}{2}\sigma^2\beta^{2-2\tau}y^{4-2\tau}(1-y)^{2\beta}\right] \tag{10}$$

The procedure to solve equation (8) using the FIFD method involves a two-dimensional grid where we have the (transformed) state variable (y) on one axis and time (t) on the other. Let the variable i=1,2,...N index the state variable axis and the variable j=1,2,...T index the time axis on the grid where N and T are the number of points on each axis. We denote the price of a bond on the grid as $P_{i,j}$ and the value of the state variable as $y_{i,j}$. The distance between adjacent nodes on the i-axis is equal to **m**, and that between adjacent nodes on the j-axis is equal to **q**. Using this notation, we can write the differential equation (8) in difference equation form as follows:

$$0 = A_i\left[\frac{P_{i+1,j}-P_{i-1,j}}{2m}\right]+\left[\frac{P_{i,j+1}-P_{ij}}{q}\right]+B_i\left[\frac{P_{i+1,j}-2P_{ij}+P_{i-1,j}}{m^2}\right]$$
$$+ h\sum_{n=1}^{N} P_{nj} \times Prob\left[\frac{1-y_{nj}}{\beta y_{nj}} \mid \frac{1-y_{ij}}{\beta y_{ij}}\right] - hP_{ij} - \frac{1-y}{\beta y}P_{ij} \tag{11}$$
$$i=1,2...N\ ,\ j=1,2...T$$

The boundary conditions for pricing the bonds at maturity are simply

$$P\left(\frac{1-y}{\beta y}, T, T\right) = 1.0 \tag{12}$$

Rearranging equation (10) we can write

$$-\frac{P_{i,j+1}}{q} = P_{i+1,j}a_i + P_{ij}b_i + P_{i-1,j}c_i + h\sum_{n=1}^{N} P_{n,j} \ x \ Prob\left[\frac{1-y_{n,j}}{\beta y_{n,j}} \mid \frac{1-y_{ij}}{\beta y_{ij}}\right] \tag{13}$$

where

$$\begin{aligned} a_i &= \left[\frac{A_i}{2m}+\frac{B_i}{m^2}\right] \\ b_i &= \left[-\frac{1}{q}-\frac{2B_i}{m^2}-\frac{1-y}{\beta y}-h\right] \\ c_i &= \left[-\frac{A_i}{2m}+\frac{B_i}{m^2}\right] \end{aligned} \tag{14}$$

This system of N equations is solved by backward recursion, given the boundary conditions for the bond. The NxT-equations system in formulae (13) can be written in matrix form:

$$\begin{aligned} P_{j+1} &= XP_j \quad , \quad j = T-1,\ldots 1 \\ X &= -q(Q+Y) \end{aligned} \tag{15}$$

where Q is a N x N matrix containing the probabilities of jumping from any node P_{ij} to P_{nj}. P_{j+1} is an N x 1 vector and Y is a tridiagonal matrix where each row contains the coefficients a_i, b_i and c_i. Backward recursion is performed by computing the equation (15) from j=T-1 to j=1. For other interest rate derivative securities, which are functions of bond prices, appropriate boundary conditions can be imposed, and the prices can be computed off the grid. This approach allows almost all forms of path-independent valuation.

3. Econometric Framework

The model to be estimated for the dynamics of the interest rate is the following jump-diffusion process:

$$dr = k(\theta - r)dt + \sigma r^{\tau} dz + J(\mu, \gamma^2)d\pi(h) \tag{16}$$

We follow a two-step procedure. First we estimate the pure diffusion part of the model, setting **h**=0 in (16). Then we estimate both the jump´s location and size using a Likelihood Ratio test-type statistic. Finally we estimate jointly the full diffusion-jump model.

The pure diffusion is estimated using the discrete time technology of Chan *et al.* (1992), based on an iterated version of Hansen´s GMM. The econometric specification is:

$$\begin{gathered} r_t - r_{t-1} = a + br_{t-1} + \epsilon_t \\ E[\epsilon_t] = 0 \qquad E[\epsilon_t^2] = \sigma^2 r_{t-1}^{2\tau} \end{gathered} \tag{17}$$

so that

$$k = -b \qquad \theta = -\frac{a}{b} \tag{18}$$

Given the parameter vector $\Omega=(\alpha, \beta, \sigma, \tau)$ and the residuals ϵ_t in (17), let the moment vector $f_t(\Omega)$ be

$$f_t(\Omega) = \begin{bmatrix} \epsilon_t \\ \epsilon_t r_{t-1} \\ \epsilon_t^2 - \sigma^2 r_{t-1}^{2\tau} \\ (\epsilon_t^2 - \sigma^2 r_{t-1}^{2\tau})\, r_{t-1} \end{bmatrix} \tag{19}$$

Under the null hypothesis, if the restrictions implied by (16) are true, $E[f_t(\Omega)]=0$. We replace $E[f_t(\Omega)]$ with its sample counterpart $g_T(\Omega)$, using T observations,

$$g_T(\Omega) = \frac{1}{T}\sum_{t=1}^{T} f_t(\Omega) \tag{20}$$

Then, the GMM estimator is:

$$\Omega_0 = argmin\ J_T(\Omega) \tag{21}$$

where

$$J_T(\Omega) = g_T(\Omega)' V g_T(\Omega) \tag{22}$$

and V is an appropriate weighting matrix. To deal with possible residual autocorrelation and heteroskedasticity we employ the Newey-West corrected covariance matrix for the GMM model and then iterate till convergence.

To test the overidentifying restrictions of the model we use the chi-square test. The quantity $TJ_T(\Omega)$ is distributed χ^2 with degrees of freedom equal to the number of moment conditions less the number of parameters estimated.

Using the estimated values of the diffusion parameters, we estimate the jump locations using an approach based on the ideas of Aase and Guttorp (1987). Essentially the procedure is to compute a selected criterion function for each observation, assuming no jumps (restricted) and compare it with its value assuming one jump (unrestricted). In our case, the criterion function is the quadratic form $J_T(\Omega)$. We compute for each observation the test statistic

$$R = T[J_T(\Omega_0) - J_T(\Omega_1)] \tag{23}$$

This test is asymptotically distributed χ^2 with one degree of freedom, and can be interpreted as the normalized difference of the restricted $J_T(\Omega_0)$ and unrestricted $J_T(\Omega_1)$ objective functions. This procedure provides the location and size of the jumps with their sign.

Therefore, at each point in time, we know whether a jump ocurred or not, as well as its sign. Thus, at each t, we can write the conditional expectation and variance of the change in the interest rate depending on the event at that time. Using a dummy variable D_i, $i=1,2..T$, which takes values $D_i=1$ if there is a jump and $D_i=0$ otherwise, we obtain the moments for the estimation as follows:

$$\begin{gathered} \epsilon_t = r_t - r_{t-1} - (a + br_{t-1}) - dD_t \\ E[\epsilon_t] = 0 \\ E[\epsilon_t^2] - \sigma^2 r_{t-1} - (dD_t)^2 = 0 \end{gathered} \tag{24}$$

where we use as a starting point the parameters estimated for the pure diffusion process.

4. Data Characteristics

In this section we address the basic characteristics of our data sample. The instantaneous riskless interest rate is approximated by daily overnight Spain Interbank offer rates[1]. The data was obtained from the Research Department, Bank of Spain, and consists of annualized rates. Daily data spans the period from January, 1, 1988 to March, 10, 1994. The number of observations is 1534. Figure 1 shows the Overnight rate. Note the periodic "drops" in the overnight rate as well as the significant increase in the volatility associated with the turbulence in the European Monetary System. It is worth mentioning that in the period from September, 1992 to May, 1993 the peseta was devaluated three times.

Table I provides summary statistics of the interest rate (**r**) as well as the changes in interest rate (**dr**). The unconditional average interest rate is 13% and its standard deviation is around 180 basis points. The mean change in interest rates is slightly negative and its volatility is about 33 basis points. The excess kurtosis in the distribution of changes in interest rates indicate the presence of fat tails in the interest rate distribution. The autocorrelation coefficients of the interest rate (see Table II) are close to unity and decay quite slowly. The autocorrelation coefficients of the changes in interest rate are small and negative. Therefore, mean reversion in interest rate is suggested in our sample.

[1] The interest rate data is computed as the average rate for all transactions on a specified term in a given day.

5. Empirical Analysis

This section presents the estimation of equation (17). First we estimate the pure diffusion model, which nests eight interest rate models (see Table III) derived from the restrictions on the parameters a, b and τ in eq. (17). In a second stage, we use the estimated values of the diffusion parameters to obtain the location, size and sign of the jumps.

5.1 Modelling Pure Diffusion Processes

Table IV presents the estimation results obtained for the pure diffusion processes. We estimate the unrestricted diffusion process derived from the equation (16) as well as eight restricted models derived through restrictions on the parameters of this model. The χ^2 tests for goodness-of-fit indicate that the Brennan-Schwartz, Cox-Ingersoll-Ross-85 and Vasicek models exhibit the closest to zero GMM minimized criterion values. The lowest χ^2 value corresponds to the Brennan-Schwartz model, which assume the highest value for τ among these, and all three models have χ^2 values smaller than 0.8. In these models a single parameter, τ, is restricted. As no restrictions are imposed on the parameters a and b, all of them show mean reversion. The Dothan and Cox-Ingersoll-Ross-80 models which assume that the parameters a and b are null, fit less well the data but none can be rejected at the 90% confidence level. The Black-Scholes and Merton models have χ^2 values in excess than 5 and can be rejected at the 90% confidence level. The Cox model, which assumes that the parameter **a** is equal to zero, can be rejected at the 95% confidence level.

Note that models which imply mean reversion have the lowest GMM criterion χ^2 values. On the other hand, models which assume that the parameters a and/or b are null - that is, there is no mean reversion in interest rates - have high χ^2 values and are therefore not acceptable.

The parameters estimates of the unconstrained model, which are very similar to the estimates of the Brennan-Schwartz model, show that the parameters a and b are different from zero and, hence, there is evidence of mean reversion in interest rates. Another feature of this model is that the estimated value for the parameter τ is 0.96. Therefore, the conditional volatility of the process is very sensitive to the level of the interest rate. This value is higher than the values assumed by the most common models as the Vasicek or Cox-Ingersoll-Ross-85 models.

To obtain more information on the performance of the alternative pure diffusion models, we test their in-sample forecasting power in relation to the level and volatility of interest rates. First we use the fitted values for equation (17) to compute the time series of conditional expected interest rates changes and conditional variances for the unrestricted and the eight restricted models. Then, we compute the R_j^2 ($j = 1, 2$) statistics. These two values are reported in the two last columns of Table IV and show (for each model) the proportion of the total variation in the ex post interest rates changes or squared interest rates changes that can be explained by the conditional expected interest rates changes and conditional volatility measures, respectively.

The R_1^2 value is the measure related to the actual interest rate changes. The unrestricted and Brennan-Schwartz models have the best explanatory performance and are closely followed by the Cox-Ingersoll-Ross-85 and Vasicek models. The remaining models have no explanatory power. In the case of the R_2^2 statistic, which measures the degree of the model's explanatory power of the volatility of the interest rate changes, the highest value corresponds also to the unconstrained and the Brennan-Schwartz models, which are followed by the Cox-Ingersoll-Ross-85, Cox and Black-Scholes models. Therefore, these two measures, which indicate the predictive power of the models, provide a classification of the alternative models which is very similar to the one obtained when parameters of the pure diffusion models were estimated. Given the previous results, we choose the Brennan-Schwartz process as a tentative model for the pure diffusion part of the Interbank interest rate.

5.2. Modelling Jumps

Once we have estimated the pure diffusion models, we use those results to estimate the jumps location. After applying the econometric procedure described in Section 3, we find 77 jumps in the sample. Figure 2 plots the time series of the interest rate and the location of the jumps. The summary statistics of the jumps are reported in Table V. The mean jump size is about 7 basis points and its volatility (measured by the standard deviation) is around 140 basis points. The arrival frequency of the jumps is 5.02% and, therefore, there is approximately one jump per month. The distribution of jump sizes is shown in Figures 3 and 4. Separating the jumps by their sign, there are 37 positive jumps while the remaining 40 are negative. The mean of the negative jumps is 100 basis points and the mean of the positive jumps is 123 basis points. The distribution of the negative jumps has a lower variance than the distribution of the positive jumps.

Once the location of the jumps is known, we are able to include this information in our model by means of dummy variables. This is done by estimating the two models in equations (25)-(26) and (27)-(28).

The first model includes one dummy variable that indicates the moment when a jump ocurred:

$$\begin{aligned} r_t - r_{t-1} &= a + br_{t-1} + dD_t + \epsilon_t \\ E[\epsilon_t] = 0 \qquad E[\epsilon_t^2] &= \sigma^2 r_{t-1}^2 \end{aligned} \tag{25}$$

where

$$D_t = \begin{cases} 1 & \text{if there is a jump} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

We also estimate a second model with two dummy variables which distinguish between positive and negative jumps:

$$\begin{aligned} r_t - r_{t-1} &= a + br_{t-1} + d^+D_t^+ + d^-D_t^- + \epsilon_t \\ E[\epsilon_t] = 0 \qquad E[\epsilon_t^2] &= \sigma^2 r_{t-1}^2 \end{aligned} \tag{27}$$

where

$$\begin{aligned} D_t^+ &= \begin{cases} 1 & \text{if there is a positive jump} \\ 0 & \text{otherwise} \end{cases} \\ D_t^- &= \begin{cases} 1 & \text{if there is a negative jump} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{28}$$

The parameter estimates for the two jump-Brennan-Schwartz models are shown in Table VI.

The a and b coefficients are both significantly different from zero pointing out that mean reversion is present in our sample, even after taking account of jumps. The coefficient associated with the jump dummy variable is not different from zero. When different effects are allowed for positive and negative jumps the coefficients are statistically significant. The degree of fit of the model with jumps is similar to the model without jumps, as measured by the R^2. However, when separate variables are included for positive and negative jumps the degree of fit increases substantially. Therefore we conclude that taking into account the presence of a jump process increases the model´s explanatory power.

6. Monetary Authorities Interventions

In this section we investigate the extent to which interventions by the Bank of Spain (Spain's Monetary Authority) are responsible for at least part of the jump-like behavior of interest rate time series. The Bank of Spain (BS) uses a "target" interest rate (Tipo de Intervencion, TI henceforth) in open market transactions. Periodically, the BS makes interventions in the Interbank market, offering money for lending or borrowing at the TI rate. Data for the TI rate was obtained from the Research Department of the Bank of Spain. Short term rates tend to track the TI rate rather closely. This can be noticed from the regression results in Table VII. The regression of the short rate on the contemporaneous TI rate is highly significant. Also the TI one-day lag is found to be significant. This reflect the fact that BS sets the TI rate after the start of the interbank trading session. The results in Table VII suggest that about 50% of the total variance in the short rate is explained by its relationship with the TI rate.

We compare a change in the BS target rate with the occurrence of a jump as derived in the jump-diffusion model. If both occur together, we assume that the jump was caused by the BS intervention. There were a total of 77 jumps in the interest rate and 160 BS target rate changes. Jumps and changes in the BS target rate coincide on 22 days. The summary statistics for these jumps and the associated changes in the BS target rate are reported in Table VIII. The mean jump size is near zero and the jumps volatility, indicated by their standard deviation is about 160 basis points. Therefore, this restricted set of 22 jumps is more volatile than the whole set of 77 jumps. In a similar way to the whole set of jumps, the highest jump sizes occur in the last twenty months of our sample period.

In the 22 days where BS actions and jumps occurred together, there were 10 positive and 12 negative jumps. Therefore, in this restricted set of jumps, the proportion of positive and negative jumps is similar to the proportion we found in the whole sample. Their average, in the restricted set of jumps, are greater than the values obtained in the whole sample and, similarly, the positive jumps mean value is greater than the (absolute value of the) mean value of negative jumps, which is in excess of 110 basis points.

We build a window of 5 days around the BS target rate change in order to check if the market was able to anticipate (or reverse) a BS move. If we find that the change in the BS target rate is preceded by a jump in the market in the prior week, this may be interpreted as anticipation. The results are displayed graphically in Figure 5. In this figure, the x-axis represents the number of days by which the BS intervention precedes the jump (a negative value indicates the number of days between a jump and a posterior BS rate change). The highest peaks of this figure correspond to the central value (number of days in which a BS action and a jump occur simultaneously) and to the extreme values (days in which a BS action and a jump are separated by five days). Another interesting feature of this figure is its symmetry.

Therefore, we may say that some, but not all, jumps are related with the monetary authorities´ interventions but perhaps there are also other factors (e.g. exchange rate shocks) that should be taken into account if we want to explain the occurrence of jumps in our sample. In conclusion, it seems that the analysis indicates that about one third of jump-like shocks are coincident with BS actions. Moreover, it is not very clear whether the market was able to either anticipate or reverse a BS move systematically.

7. Pricing Derivatives

For illustrative purposes, we implement the algorithm described in section 2 to price bonds with maturities of 3 months and 1 year and also a 3 month option on a 1 year bond[2]. The options, puts and calls, are priced under pure-diffusion and jump-diffusion assumptions. We price both European and American Options. The following parameter values are used:

$$\begin{array}{llll} a = 0.149, & b = -0.008, & \sigma = 0.0088, & \lambda(.) = 0 \\ \mu = 0, & \gamma = 1.392, & h = 12.3, & \tau = 1.0 \end{array} \qquad (29)$$

The strike price is 0.95. The parameters in (29) are those of the model estimated for the

[2] Many other pricing simulations for bonds and options were performed. The bond pricing simulation was performed for maturities ranging from 1 year to 10 years. The option pricing simulation was performed for 3, 6 and 9 months options on bonds of different maturities. Detailed results are available on request.

overnight interest rate which follows a jump-enhanced Brennan-Schwartz process.

Table IX contains the results. Given that the pricing results for European and American call options are identical, we report both jointly. There are some interesting facts. The price of a one year bond is higher under the jump-diffusion model. This is the well-known "asymmetric" effect, caused by the asymmetry of the bond pricing function. This feature of high prices will increase with the duration of the bond. Also, options will increase in value because the jumps induce "fat tails" into the distribution of bond prices. This is the "time value" effect. In the case of calls, both effects, asymmetric and time value, result in the option increase in value. However, in the case of puts, the asymmetric effect reduces the option value whereas the time value effect increases its value. The trade-off between these two forces leaves the jump-diffusion value of the put indeterminate in comparison with the pure-diffusion case.

8. Conclusion

This paper has presented a single-factor model for the term structure of the Interbank interest rate, when the instantaneous rate follows a jump-diffusion process. The empirical implementation suggests that jump-diffusions better explain interest rate behavior than pure-diffusion models. Some economic implications of jump activity are explored with an analysis of changes in the Bank of Spain target rate. As a result, some, but not all, jumps are found to be related with Central Bank interventions. Additionally, we price European-style and American-style interest rate contingent claims (bond and options) using the finite-differencing approach, enhanced to deal with partial differential equations derived in a jump-diffusion model. The existence of jumps affects bonds and call options very much like in the case of stocks. We find underpricing if the jumps are not taken into account. However the put options pricing present some indeterminacies.

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Table I. Interest Rates: Descriptive Statistics

This table provides summary statistics of the overnight Spain Interbank interest rate (r_t) as well as the changes in this interest rate ($r_t - r_{t-1}$). Means, standard deviations, skewness coefficients and excess kurtosis are computed from January 1988 through March 1994. Raw data is in percentage terms.

Variables	Number of Observations	Mean	Standard Deviation	Skewness Coefficient	Excess Kurtosis
r_t	1534	13.003	1.8363	-0.424	-0.496
$r_t - r_{t-1}$	1533	-0.0035	0.3368	3.0023	98.507

Table II. Interest Rates: Correlation Structure

This table shows correlation coefficients of order j, denoted by ρ_j, of the overnight Spain Interbank interest rate (r_t) as well as the changes in this interest rate ($r_t - r_{t-1}$). These coefficients are computed from January 1988 through March 1994. N denotes the number of observations. Raw data is in percentage terms.

Variables	N	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
r_t	1534	0.9809	0.9656	0.9542	0.9450	0.9366	0.9287
$r_t - r_{t-1}$	1533	-0.113	-0.114	-0.067	-0.009	-0.013	0.052

Table III. Alternative Pure Diffusion Processes

This table shows the alternative pure diffusion models that reflect the dynamics of the interest rate. These processes derive from restrictions on the parameters a, b and τ in the system of equations

$$
\begin{aligned}
r_t - r_{t-1} &= a + b r_{t-1} + \epsilon_t \\
E[\epsilon_t] = 0 \qquad E[\epsilon_t^2] &= \sigma^2 r_{t-1}^{2\tau}
\end{aligned}
\tag{17}
$$

Model	a	b	τ
Merton (1973)	---	0	0
Vasicek (1977)	---	---	0
Cox, Ingersoll and Ross (1985b)	---	---	0.5
Dothan (1978)	0	0	1
Black and Scholes (1973)	0	---	1
Brennan-Schwartz (1980)	---	---	1
Cox, Ingersoll and Ross (1980)	0	0	1.5
Cox (1975)	0	---	---
Unrestricted	---	---	---

Table IV. Estimates of the Alternative Pure Diffusion Models

This table provides the parameter estimates (with t-statistics in parentheses) of unrestricted and restricted alternative pure diffusion models for Spain Interbank interest rates. GMM minimized criterion (χ^2) values, with p-values in parentheses and associated degrees of freedom (d.f.) are also reported. The sample period is from January 1988 to March 1994. The R_j^2 statistics are computed as the proportion of the total variation of the actual interest rate changes (j=1) and their volatility (j=2) explained by the respective predictive values for each model. The parameters are estimated by means of the Generalized Method of Moments applied to the estimation of equation (16).

Model	a	b	σ^2	τ	χ^2	d.f.	R_1^2	R_2^2
Merton (1973)	0.00119 (0.181)	0	0.06305 (2.98)	0	5.30 (0.0706)	2	-0.0002	0
Vasicek (1977)	0.1721 (2.15)	-0.0132 (-2.15)	0.1113 (3.5)	0	0.7911 (0.3737)	1	0.0062	0
Cox, Ingersoll and Ross (1985b)	0.1834 (2.26)	-0.0145 (-2.28)	0.000001 (5.6)	0.5	0.3161 (0.5739)	1	0.0063	0.0009
Dothan (1978)	0	0	0.000452 (6.43)	1	5.23 (0.1557)	3	-0.0001	0.0006
Black and Scholes (1973)	0	-0.00008 (-0.18)	0.00044 (5.8)	1	5.216 (0.0736)	2	0.0	0.0007
Brennan-Schwartz (1980)	0.1874 (2.3)	-0.0146 (-2.3)	0.000657 (3.72)	1	0.0068 (0.9342)	1	0.0064	0.0014
Cox, Ingersoll and Ross (1980)	0	0	0.00957 (5.04)	1.5	5.308 (0.1506)	3	0.0	0.0006
Cox (1975)	0	-0.0001 (-0.17)	0.0016 (0.09)	0.85 (0.73)	5.213 (0.0224)	1	0.0	0.0009
Unrestricted	0.1874 (2.2)	-0.014 (-2.3)	0.000911 (3.4)	0.96 (29)	-----	---	0.0064	0.0014

Table V. Jumps Size: Descriptive Statistics

This table provides summary statistics of the jumps located in the interest rates series from January 1988 through March 1994. We report statistics for the following variables: $JUMP_t$, which denotes jumps size, $JUMPPOS_t$, which includes the size of positive jumps, and $JUMPNEG_t$, which includes the size of negative jumps. N denotes, for each type, the number of located jumps.

Variables	Number of Jumps	Mean	Standard Deviation	Skewness Coefficient	Excess Kurtosis
$JUMP_t$	77	0.07037	1.392555	0.7515	3.90682
$JUMPPOS_t$	37	1.2313	0.985046	3.4132	14.3367
$JUMPNEG_t$	40	-1.00348	0.659994	-3.59903	16.5665

Table VI. Estimates of the Jump-Diffusion Models

This table contains the parameter estimates (with t-statistics in parentheses) of the two alternative jump-diffusion models for Spain Interbank interest rates. The sample period is from January 1988 to March 1994. The parameters are estimated by means of the Generalized Method of Moments and they are obtained from equations (25)-(26) and (27)-(28).

Model	a	b	σ^2	d	d^+	d^-	R^2
(25)-(26)	0.1490	-0.0082	0.0088	0.0113	----	----	0.0064
	(2.46)	(-1.77)	(-43.74)	(0.28)			
(27)-(28)	0.1248	-0.0070	0.0030	----	1.1533	-1.08	0.5441
	(3.17)	(-2.33)	(-46.27)		(31.19)	(29.84)	

Table VII. The Short Rate and the TI Rate

This table provides regression results on the relationship between the short rate and the TI rate. The sample period is from January 1988 to March 1994. The regression is corrected for first order autocorrelation (coefficient Φ) and is run on contemporaneous and one lagged values of the BS target rate TI. Robust standard errors are computed using the Newey-West covariance matrix estimator.

Variables	Constant	TI(t)	TI(t-1)	Φ	Adj. R^2	D-W
Estimate	-0.122	0.882	0.146	0.827	0.472	2.10
t-stat	-0.301	15.53	2.57	53.19		

Table VIII. Coincidence between Jumps and BS Actions. Jumps Size: Descriptive Statistics

This table provides summary statistics of the jumps that occur simultaneously with a change in the BS target rate. The sample period is from January 1988 through March 1994. We report these statistics for the following variables: $JUMPBS_t$, which denotes jumps size, $JUMPPSBS_t$, which includes the size of positive jumps, and $JUMPNGBS_t$, which shows the size of negative jumps. N denotes, for each type, the number of located jumps.

Variables	Number of Jumps	Mean	Standard Deviation	Skewness Coefficient	Excess Kurtosis
$JUMPBS_t$	22	0.01111	1.60083	-0.4879	1.6869
$JUMPPSBS_t$	10	1.3962	0.8150	1.4609	2.0053
$JUMPNGBS_t$	12	-1.1431	1.0729	-2.9163	9.0796

Table IX. Examples of Pricing Bonds and Options using the FIFD method

This table contains bond values for maturities of 3 months and 1 year at different values of the current overnight rate. Values are for zero-coupon bonds with face values of 1.0 and **r**=0.08, 0.09, 0.10, 0.11, 0.12. The arrival rate is **h**=12.3 jumps per year, and the jumps are distributed ~ N(0, 1.392). The other parameters are a=0.149, b=-0.008, σ=0.0088, λ=0.0. Option prices are computed for the three month option on the one year discount bond. The strike price is 0.95. The pricing results are identical for European and American call options.

Security	r=0.08	r=0.09	r=0.10	r=0.11	r=0.12
Diffusion					
Bond (3m)	0.9791	0.9779	0.9756	0.9748	0.9722
Bond (1y)	0.9295	0.9232	0.9176	0.9108	0.9042
Call	0.0087	0.0051	0.0022	0.0011	0.0003
Put(Euro)	0.0021	0.0051	0.0087	0.0116	0.0131
Put(Amer)	0.0097	0.0176	0.0192	0.0271	0.0342
Jump-Diff					
Bond (3m)	0.9794	0.9782	0.9758	0.9749	0.9725
Bond (1y)	0.9297	0.9234	0.9177	0.9110	0.9046
Call	0.0091	0.0054	0.0026	0.0015	0.0008
Put(Euro)	0.0021	0.0051	0.0087	0.0116	0.0131
Put(Amer)	0.0097	0.0176	0.0190	0.0271	0.0340

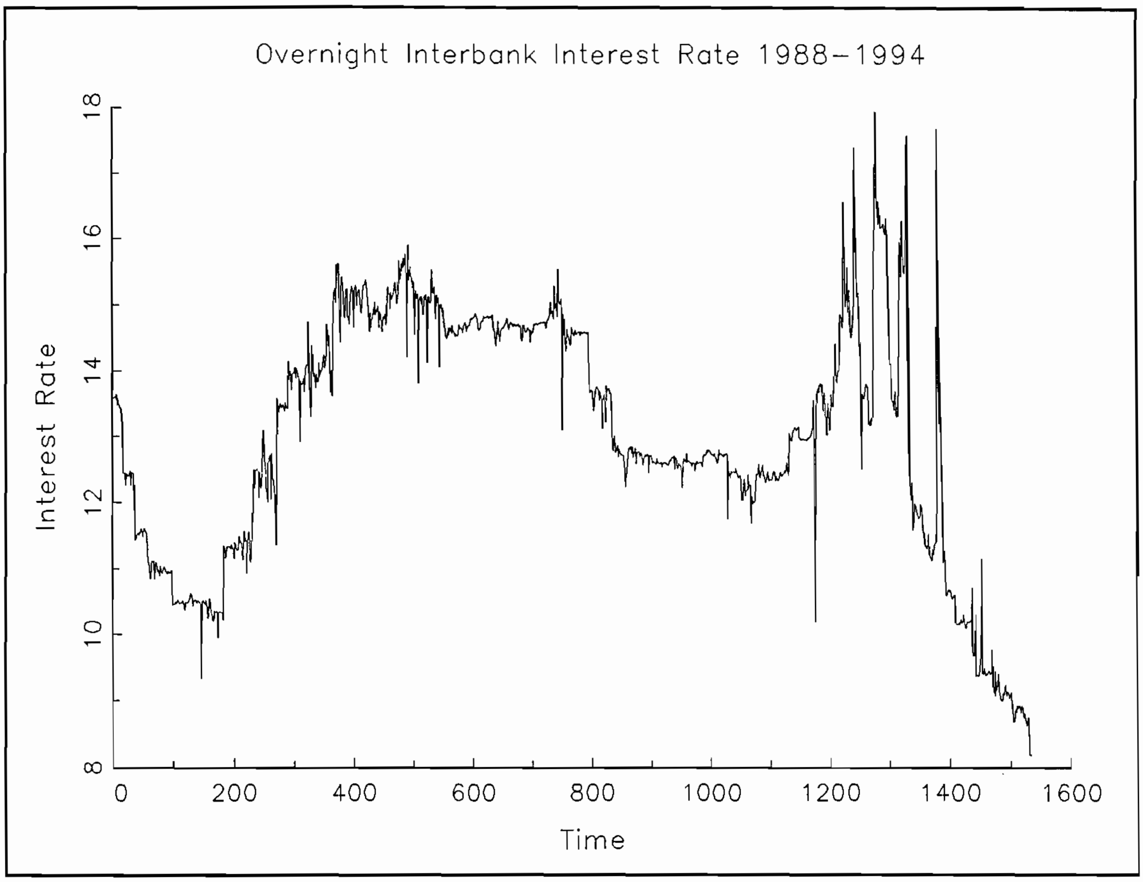
Overnight Interbank Interest Rate 1988-1994
Interest Rate
8
10
12
14
16
18
0
200
400
600
800
1000
1200
1400
1600
Time

Fig. 1

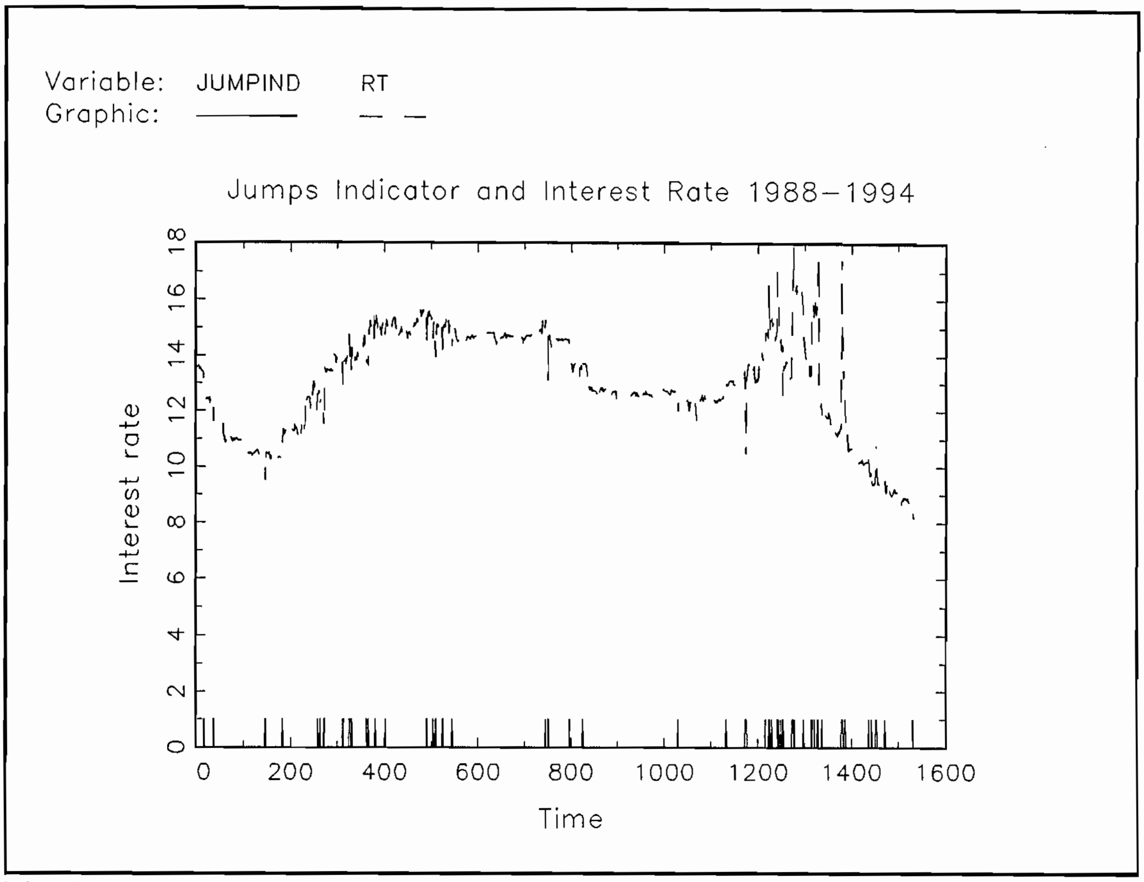
Variable: JUMPIND RT
Graphic:
Jumps Indicator and Interest Rate 1988–1994
Interest rate
0
2
4
6
8
10
12
14
16
18
0
200
400
600
800
1000
1200
1400
1600
Time

Fig. 2

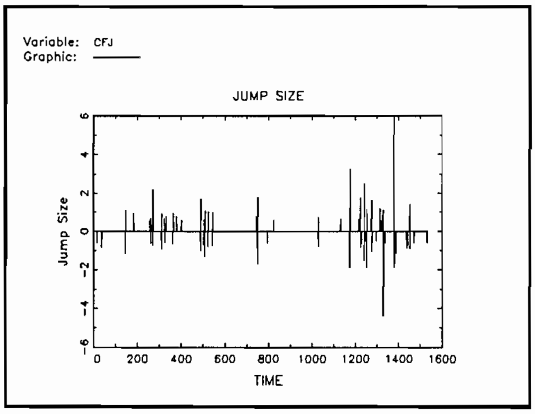

Fig. 3a

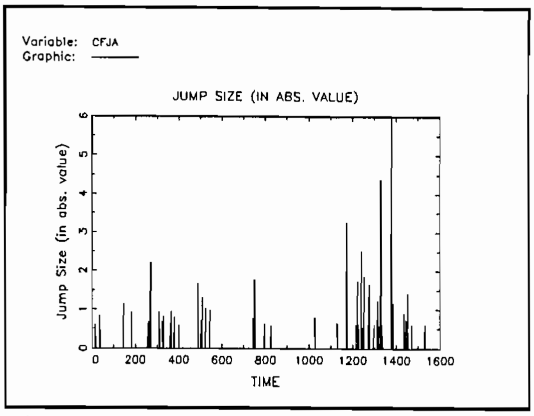

Fig. 3b

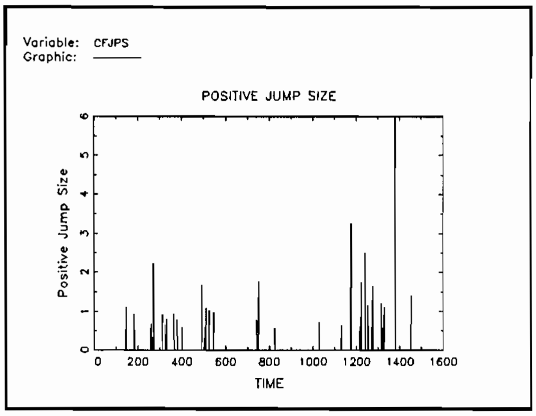

Fig. 3c

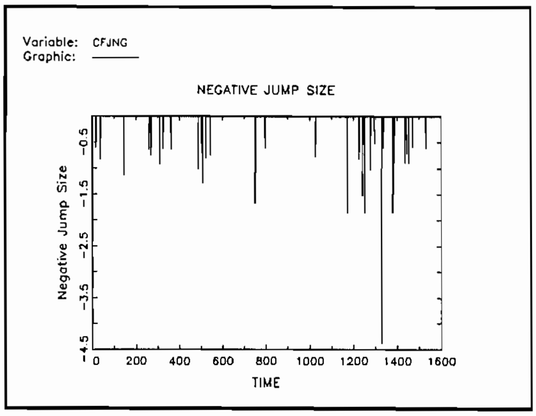

Fig. 3d

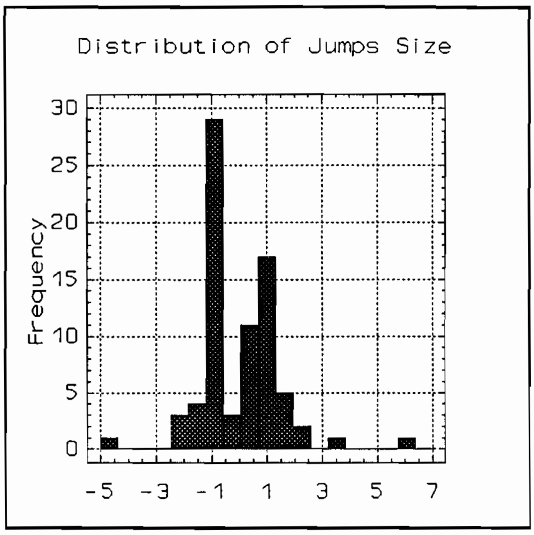

Fig. 4a

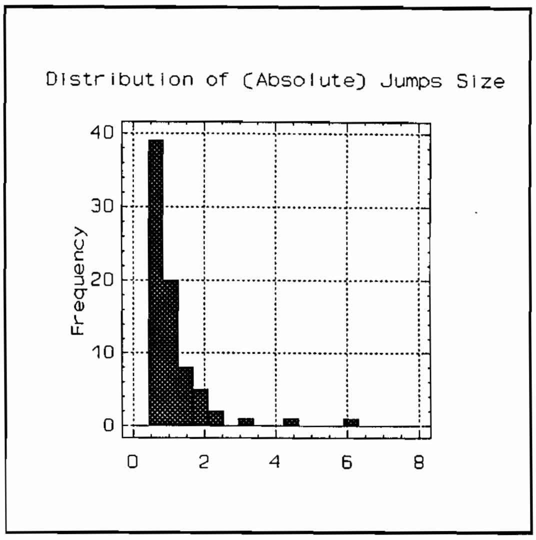

Fig. 4b

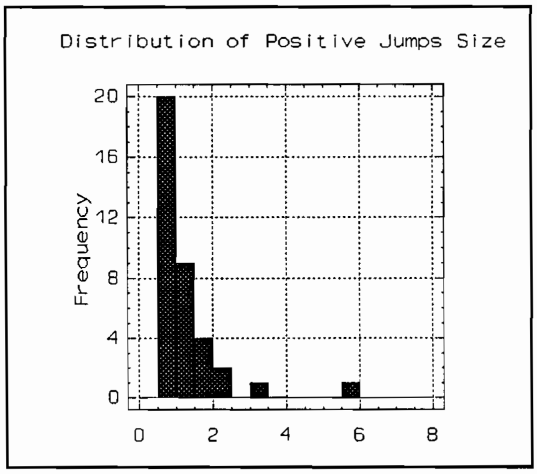

Fig. 4c

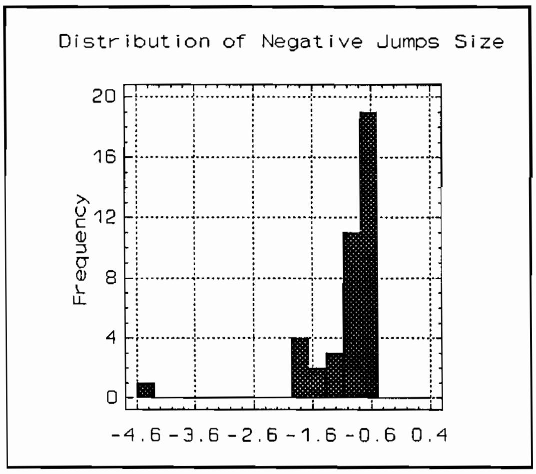

Fig. 4d

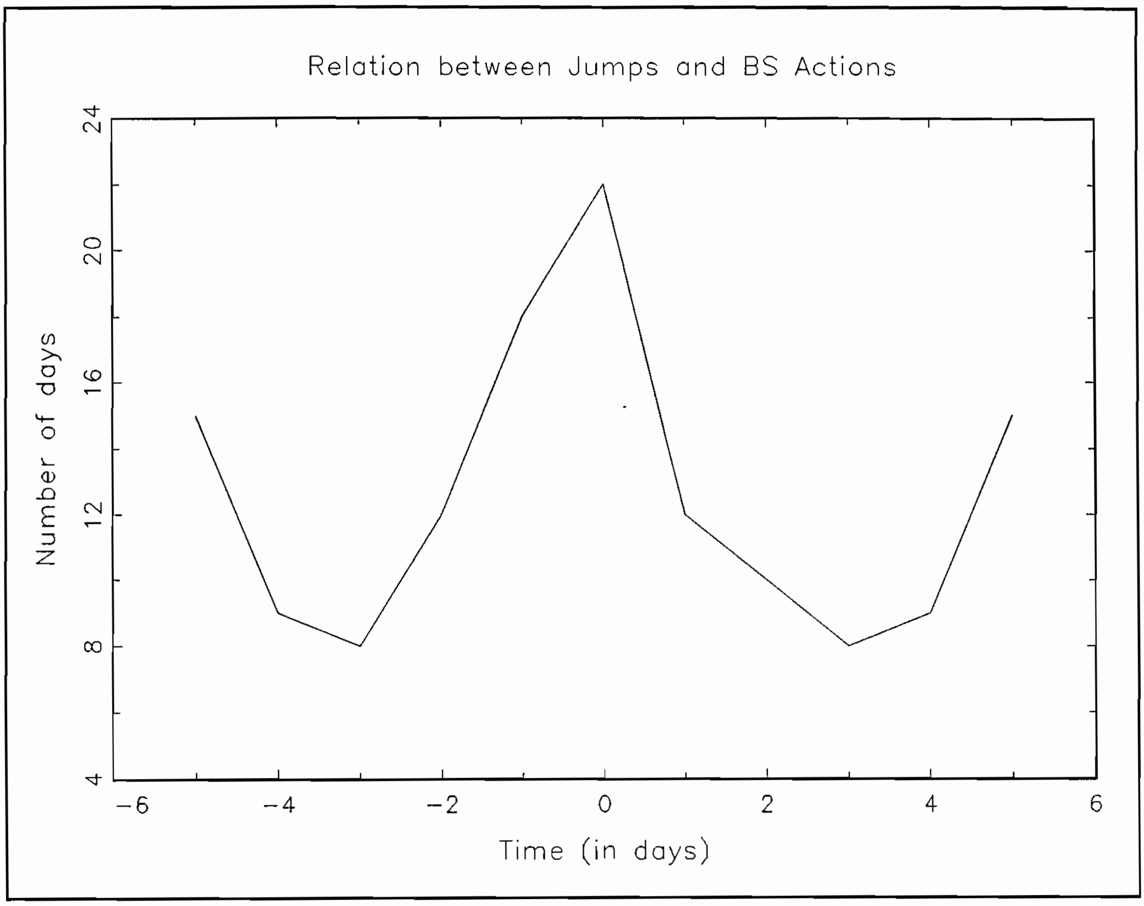
Relation between Jumps and BS Actions
Number of days
24
20
16
12
8
4
−6
−4
−2
0
2
4
6
Time (in days)

Fig. 5