LONG-TERM AND SHORT-TERM LABOR CONTRACTS VERSUS LONG-TERM AND SHORT-TERM DEBT FINANCIAL CONTRACTS

Josep Tribo

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As a direct implication of our theoretical model, some empirical tests are proposed which are particularly relevant to describe some features of the current Spanish economy.

Key Words
Financial Contracts, Banks, Markets.

* This paper has benefitted from the comments made by Miguel Angel Garcia Cestona, Ines Macho, Yosif Spiegel, and the participants to the ASSET Meeting (1996), Las XII Jornadas de Economia Industrial (1996), and El XX Simposio de Analisis Economico (1995).

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1. INTRODUCTION

In this paper, we build up a model to show the existence of essential links between labor contracts (owner-worker) and financial contracts (entrepreneur-financial institution). These two contracts interact with each other, and this affects their length. The optimal length will depend on several environmental features. In particular, we consider uncertainty, the type of project, workers' skills, investment specificity, and the type of financial institution that provides the funds.

Traditionally, the economist at the time of analyzing the optimal length of labor contracts have focused on the existing trade-off between the commitment value of the long-term contracts and the flexibility of the short-term ones, Gray (1978), Dye (1985), Danzinger (1988), Anderson (1991). Thus, when the degree of uncertainty increases (for a given discount factor), the owner signs short-length contracts, in order to have enough flexibility to better match future situations. The cost he has to bear doing that, is a lower effort level implemented by workers. They have no security to be employed in the future and therefore they act in a short-sighted way.

On the other side, the analysis regarding the financial contracts Williamson (1988), Von Thadden (1994), shows that the credit structure is the result of two effects: A disciplinary effect of short-term credits (due to the fact that these contracts allow not to supply the required new funds in the future periods if several short-term conditions are not met); and a commitment effect that provides the firm with the possibility to undertake long-term actions and achieve higher expected returns.

What is new in our work is to consider, in an integrated manner, the optimal length of both contracts (labor-financial). Under our point of view, this is the "natural" way to reflect the vision of a firm as a nexus of contracts, using terminology introduced by Jensen & Meckling (1976), Grossman and Hart (1986). This approach facilitates a better relationship between the length of external firm's contracts (type of project) and the length of the internal contracts (type of labor-financial contract).
Furthermore, this treatment allows us to rationalize several stylized facts concerning the interaction between labor and financial contracts. In particular, we provide an explanation of the linkage between the short-term project's financing and the highly flexible labor markets that characterize the Anglo-Saxon markets. The complementary approach, observed in Japan and Germany, provides our second stylized fact: the financial and labor contracts are long-term, and the projects undertaken by firms are more of long-term type. Finally, a third stylized fact shows the type of financing observed in each paradigm: market financing in the first case and bank financing in the second.

Certainly there are some historical reasons behind the above correlations. The Glass–Steagal Act (1933) in US and the Bank Charter act (1844) in the UK have kept banks smaller in size than their corresponding counterparts in Germany and Japan. Allen, F. & Gale, D. (1995) measure the size of the extended banking system in Germany, through the total balance sheet as a percentage of GDP, and found it is more than double of the American figure. Berglöf, E. (1994), emphasizes that the non-existence of the above restrictions in Germany and Japan has affected not only banks' size but also firm's ownership to be less dispersed compared to US-UK. In the former countries, we find a situation where a small number of big banks have a large shares in the main enterprises. This fact has lead to a lower turnover in firm's ownership and also in financial-firm's relationships. Furthermore, with these longer relationships, firms have been able to develop long-term projects, and linked with these projects, to define longer relationships with their employees. In contrast, firm's ownership in the US is much dispersed among households, Prowse, S. (1990) reports that the percentage of outstanding corporate equity in households' hands is three times higher in the US than in Japan. This composition, as Porter, M. (1992) shows, favors the percentage of stocks held for long-term considerations be almost double in Germany than in the US. Consequent with this reality, American firms' managers are more willing to invest in short-term projects and to define a shorter labor relationships as Jacobs, M. T. (1991) points out.

To explain the above economic facts, we propose a dynamic model with two sequential bargaining stages: (lender-entrepreneur and entrepreneur-worker). The outcome of the first bargaining is a financial contract that can be short-term ($ST$) or of long-term ($LT$). This specific choice will be the result of balancing the disciplinary effect of the $ST$ contracts and the commitment effect of the $LT$ contracts which leads to an increase in the entrepreneur's bargaining power.
respect workers. This trade-off will induce, through the labor negotiation, the workers' incentives and at a last stage the expected project's returns. The second bargaining defines the labor contracts, and can also be of short-term type ($\equiv ST$) and long-term one ($\equiv LT$). The number of workers hired in each contract, depends on the bargaining power allocation, that is a consequence of the first negotiation process.

We show that the financial structure conditions labor contracting through the distribution of bargaining power between the negotiating parts. The labor relations which are the output of this process will condition project's return, and backwardly the initial credit structure. In this way we obtain the double financial-labor interaction.

There are a few other papers that deal with some aspects of labor and financial interaction in contracts definition. For instance, Farmer, R. (1988) presents a model in which money and bonds are both held as a result of legal restrictions on the banking system. This fact raises the cost of credit which ultimately causes firms to write labor contracts in which layoffs occur more frequently ($ST$ contracts in our terminology). This dynamics points in the same direction as ours, that is, to link financial constraints in the British case to $ST$ labor contracts.

Three types of results we find. Firstly, a direct linkage between the length of financial contracts and the length of labor contracts. Secondly, a positive correlation between the contract's temporal length and the projects' one, that can also be long-term ($\equiv LT$) or short-term ($\equiv ST$). Finally, we show that contracts' length in bank-financed firms is higher than in market-financed firms. All these results are in consistency with the stylized facts previously commented that defines the US-UK and German-Japanese paradigms.

The paper is organized as follows. In Section 2 we define the model. In Section 3 we characterize the Nash equilibrium and carry out a comparative static analysis. The comparison of the obtained results and the features of the US-UK and the German-Japanese paradigms is made in Section 4, where it is also proposed some empirical observation to test our model.

The paper finishes with some concluding remarks in section 5.

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1 The Bank of England, differently to the Bundesbank or the Japanese Central Bank, under the Bank Charter Act (1844) is tightly constrained to provide short-term liquidity needs. We have shown recently an expression of this policy in the Barings Bank collapse.
2. THE MODEL

2.1. Description of the Model

Let us consider separately the characteristics of the different components that define our two-period model:

Projects

We can characterize each project by three elements. The output a worker can generate in period 1 \((X_1)\) and period 2 \((X_2)\), being \(X_1 \leq X_2\). The liquidation value, \(L\), at the end of the first period. And thirdly, the first-period and the second-period probability of success, that is, \(p_1 = \theta p\) and \(p_2 = \theta (0 \leq \theta, p \leq 1)\), where we assume that \(p\) and \(\theta\) are exogenously given and of public knowledge. Parameter \(\theta\) represents the quality of the project, and \(p\) is linked to an environmental "noise" that hinders the action of the manager-entrepreneur to develop the project. Furthermore, we require \(\theta p\) to be bigger than \(\frac{1}{2}\). In case of failure, the project generates no returns. If it has been successful in period \(i\), generates an output \(Q_i\) (see the firm characterization).

We distinguish between LT and ST projects. There are two differences among them. Firstly, the \(L\) value is higher for the ST projects. Secondly, ST-project \(X_1^S\) \((X_2^S)\) is higher (lower) than LT-project \(X_1^L\) \((X_2^L)\).

Workers

We will use risk neutral workers. They will be hired using LT contracts (two-period contracts) or ST contracts (one-period contract). By convention, \(n^L\) is the number of workers contracted for both periods (denoted as LT workers); and \(n_i^S\) is the number of workers hired just for period \(i\) \((i = 1, 2)\) (denoted as ST workers). The workers have to implement a verifiable normative effort, \(\nu\), \(\mu\) in each period contracted. They also can implement a voluntary effort \(\equiv e\) in the first period, to acquire some expertise, and become more productive in the second period. This effort is assumed to be observable, but it is not verifiable

\[\text{\textsuperscript{2}}\text{ That is, in each period the probability of success is higher than the probability of failure. This is in connection with Proposition 2, as a necessary condition for firms to hire workers through long-term contracts.}\]

\[\text{\textsuperscript{3}}\text{ Whenever the project is not liquidated after the first period, this workers will still be employed in the second period.}\]

\[\text{\textsuperscript{4}}\text{ To consider this effort } e \text{ (training effort) a private action, we want to assign a commitment weight to this action. When workers decide to implement this effort } e \text{ to become more efficient in the next period, they are signaling an engagement with the firm's future.}\]
leading to a moral hazard problem, as $e$ cannot be contracted ex-ante. The reward for this effort is given in the second period, once the higher workers' productivity generates an increase in the second-period per worker output. Finally, workers' effort cost function is given by $c = C + \frac{1}{2} A e^2$.

**Lender**

We assume risk neutrality in the agents that provide funds to the firm. We distinguish between two types of lending, through a bank (bank financing) and through the market (market financing). In the former situation, the firm demands funds to a particular bank which extracts all the expected funds the project can generate. The opposite is true in the latter framework. The firm offers participation in the project financing to the best bid. Therefore, in this case, the entrepreneur can retain all the rents the project can generate.

There is also another difference between these two complementary financial frameworks. We assume that a bank has a superior commitment than “the market” in the management of the firm. We can argue in terms of historical relationships, or the possibility the bank can coordinate more efficiently the actions to take in extreme situations (like financial distress). Consistently with this, if the lender decides to refinance an initially unsuccessful project, the firm will be allowed to retain a higher share of second-period funds under a bank financing scheme than under a market financing one. Noting as $F$ this share, then, bank’s $F_B$, is higher than market’s $F_M$.

Finally, the lender has the possibility to liquidate (not refinance) an unsuccessful project. In this case it receives a $L$ amount.

**The Financial Contracts**

It is a vector $\{I_1, I_2, R_1, R_2, \Phi, F\}$ defined in the first period; where $I_1, I_2$ are the firm's investments requirements in periods 1 and 2 to develop the project. We note with $R_1, R_2$ the returns the lender fixes for its credit of $I_1$ in the first period and $I_2$ in the second period. These quantities are determined balancing the

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5 This can be seen as a local approximation to a more general cost function that satisfies: $\frac{\partial c}{\partial e} > 0$, $\frac{\partial^2 c}{\partial e^2} > 0$

6 We have argued that this $F$ is an expression of the commitment of the lender towards the firm. It would be wrong to consider this parameter a measure of the entrepreneur's bargaining power, because a high $F$ could be related with high values of $R_1$ and $R_2$, leading at the end to low entrepreneurs profits. In fact, we have assumed a null (absolute) entrepreneur's bargaining power under a bank-financing (market-financing) scheme.

7 This amount is linked to some factors such as the project's quality $\theta$, the labor costs, and other development costs.
power between the lender and the entrepreneur. Under a bank-financing scheme, a zero-entrepreneur-profit conditions works. By contrast, the lender of a market-financed firm, fixes $R_1$ and $R_2$ through a zero-lender-profit condition.

We focus on $ST$ redeployable debt contracts. Therefore, whenever the entrepreneur pays $R_1$ to the lender at the end of the first period, he wins the right to receive the new funds $I_2$, otherwise the lender would receive a high punishment. On the other hand, in the case of a project failure, firm’s control rights are transferred to the lender. In that situation, it will decide with a probability $\Phi$ to renegotiate or not the contract. If not, a liquidation is triggered and it receives an $L$ amount. If so, the project will be refinanced with an $I_2$ amount. The “cost” for the entrepreneur, besides the initial $R_2$ payment, is the lost of a share $1 - F$ of the second-period profits, that goes directly to the lender.

The probability $\Phi$ of a contract renegotiation can be determined ex-ante, because we deal with a symmetric information game, without informational acquisition along the time. Therefore, the ex-ante probability coincides with the ex-post probability. Obviously as $\Phi$ tends to one, these redeployable $ST$ contracts, become simply $LT$ contract. In this sense, we denote as $1 + \Phi$, the “length” of the financial contract.

**The Labor Contract**

It is a vector $\{w, \lambda, T\}$ also defined ex-ante. Parameter $w$ is the fix part of the workers wage. This amount rewards the normative effort $\nu$, to assure the workers participation constraint, and is paid every period. There is also a variable component in the workers wage. This is paid in the second period to the workers that have increase their productivity through the implementation of the already mentioned effort $c$ in the first period. Specifically, this output-related component of workers wage is given by $\lambda w \Delta Q_2$, where $\Delta Q_2$ is the increase in the verifiable second-period per-worker productivity generated by these more productive workers, (see the firm characterization). As only the $LT$ workers Therefore in the second period, as only the $LT$ workers receive a wage of $w(1 + \lambda \Delta Q_2)$. Finally $T$ is the length of the contract through which the workers are hired. $T = 1 \ (2)$ represents a $ST \ (LT)$ contract.

---

8 This variable component in workers' wages results to be particularly important in countries such as Japan where the long-term schemes in the labor relationships are more common. Kanemoto, Y. et al (1992) reports that the average workers' payments in a non-fix basis is up to 40% of the total wage.
Firm

We assume a risk neutral entrepreneur-manager, (there is no separation between control rights and ownership rights). The firm has limited liability with regards to the financial claims. Regarding to the workers claims, the firm burdens an unlimited liability concerning the fix part of the workers wage, \( w \). Furthermore, we assume that the LT workers have the legal right to receive \( w \) in the second period even if the project is liquidated. We can consider that to obtain a legal permission to develop a project, the entrepreneur has to assure the reservation utility of the workers, \( w \). Therefore, at the beginning of period one, the firm to fulfill its labor liabilities has to place in a public fund \(^9\) an amount \( w(2n^L + n^S) \). In this expression, there are two factors: \( w2n^L \) to face the payment of the first-period and the second-period LT normative efforts. \( wn^S \) to reward first-period ST normative efforts. With these assumptions, we want to emphasize the cost to contract 2-period (LT) workers. As the firm has initially no internal funds, this labor costs are paid directly from the \( I_1 \) amount lent by the financial institution in the initial period. Similarly, in the second period, an amount \( wn^S \) is paid in advance to the ST workers hired.

The firm is a monopoly in project's output market. This is to avoid strategic considerations in the contract length analysis. Specifically, the demand side is modeled by a linear function of the type \( p_i = a_i - bQ_i \). With regard to the supply side, the production is given in each period by \( \bar{Q}_1 = (1 + \bar{c}_1)(n^L + n^S)X_1 \) and \( \bar{Q}_2 = (1 + \bar{c}_2)[n^S + n^L(1 + re)]X_2 \) where \( X_i \) is the per-worker period-i output that results from the workers' normative effort \( \nu \). The \( r \) parameter is exogenous, and defines the influence of the workers' voluntary effort \( e \) in the second-period output. We are going to use this \( r \) as a measure of the project's quality. High-quality projects require some specific workers' training to be completed. To distinguish the real output from the verifiable output, we introduce in period \( i \), the stochastic variable \( \bar{c}_i \). The existence of this variable is essential, otherwise the reward for the voluntary effort \( e \), would be \( w\lambda \Delta Q^S_2 \equiv w\lambda \Delta Q_2 = w\lambda \Delta e \), therefore, dependent on \( e \). This fact would have dropped the moral hazard problem. This \( \epsilon \) variable is assumed to be white noise, and can be linked to factors like manager's actions to hide conveniently the real increase in productivity, or simply environmental uncertainty.

From the previous hypothesis we can distinguish between two different frameworks to be studied separately: A/ LT Projects' framework B/ ST Projects' framework

\(^9\) In Spain the name is FOGASA.
2.2. Timing of the game

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<th>1st period</th>
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1/ The entrepreneur initially has a project (LT or ST) and bargains with a financial institution (a bank or the market) to seek for funds 10.
2/ After signing the financial contract, the entrepreneur hires a number $n^L$ of LT workers and $n^S$ of ST workers.
3/ Workers implement their efforts (normative $\nu$ and voluntary $e$).
4/ First-period results are made. If successful, there is a payment $R_1$ to the lender who provides new funds. If not successful, the lender decides to renegotiate or liquidate the contract with a probability $\Phi$.
5/ If the contract is renegotiated, new funds are provided.
6/ The entrepreneur has the choice of whether hire ST workers or not.
7/ Workers implement second-period effort, which is only the normative $\nu$ 11.
8/ Second-period payments are made.

2.3. Dynamics of the Game

The driving force in our model comes from the fact that hiring LT workers generates two opposite effects. Firstly, there is an increase in the productivity of the firm in the second period. Secondly, there is also a raise in firm’s fixed labor costs. It is more costly to hire one two-period (LT) worker than to hire two one-period (ST) workers. This is so, due to the stated legal requirement to pay second-period reservation wage $w$ to the LT workers, even, if the project is liquidated. Therefore, the marginal fix cost to hire one LT worker is $w(1 - p_e)$, where $p_e$ is the project continuation probability.

As the loan becomes of LT type ($\Phi$ high, and $p_e$ high), the first effect rises, as LT workers are more willing to implement higher voluntary efforts $e$. On the other hand, the second effect diminishes because of the lower marginal saving to contract ST workers, that is, $w(1 - p_e)$ diminishes. Both effects go in the same direction to link LT financial contracting, with LT labor contracting.

10 In order to first consider the financial arrangement, we are assuming that the firms have more financial constraints than labor ones. This is for example the case when we deal with low-quality projects.
11 In LT worker’s case, due to the training effort $e$, this $\nu$ is much more productive.
3. SOLVING THE MODEL

3.1. LT PROJECTS

3.1.1/ Second-Period Analysis

In order to compute the perfect Nash equilibrium in this symmetric information model, we have to use the Kuhn algorithm and solve the game backwards. The first step, would be to determine the number of ST workers the entrepreneur will hire in the second period \(n^S_2\). The second step will be to discover if the lender will decide, ex-post, whether to continue the project or not after the first-period results.

Second-Period ST Workers

The entrepreneur has to solve the maximization problem \(^{12}\): 

\[
Max_{n^S_2} E_2 U_2^{En} = E_2 \{\tilde{P}_2 \tilde{Q}_2\} + I_2 - n^S_2 w - \theta [R_2 + n^L w \lambda \epsilon] \quad ^{13}
\]

The FOC leads to the following result:

\[
\frac{\partial}{\partial n^S_2} U_2^{En} = 0 \implies n^S_2 = Max\{N^*_2 - n^L (1 + \epsilon)\_0\} \quad (1)
\]

With  \(N^*_2 \equiv \frac{a - \tilde{w}_2}{2bX_2(1+\sigma^2)}\)

Where \(E_2\) are the expectations at the beginning of the second period, \(\sigma^2\) is the shock variance and \(\tilde{w}_2 \equiv \frac{w}{\partial X_2}\) is the second-period per-worker relative wage with regard to output.

If we check the expression (1), it can be a corner solution (if \(n^L\) or \(\tilde{w}\) are high). To avoid this situation, we consider an upper bound to \(r\) and \(\tilde{w}_2\) \(^{14}\).

\(^{12}\) This is considering a discount factor equals to one.

\(^{13}\) We account in first-period profits the cost of the fix part of LT workers wage, \(w\). This ex-ante accounting is a consequence of the legal requirement to pay LT workers their reservation wage \(w\), in both periods, even if the project is liquidated.

\(^{14}\) Without loss of generality, if we consider that in the LT project, \(X_1^L = X_2^L \equiv X\) and in the ST project \(X_1^S > X\) and \(X_2^S < X\), then, the \(r\) upper bound value to assure \(E_1\{n^S_2\} > 0\) for both type of projects is given by \(r^2 \leq \tilde{v} \equiv \frac{2A(1-p)}{\lambda \phi X_1(a - \tilde{w})}\); where we have used expressions (2) and (5) and that \(X_1\) to be high enough to fulfill \(\tilde{w}_1 \equiv \frac{X_1^S}{\phi X_1} < \frac{a}{2}\). This last assumption is to make certain that \(\tilde{v}\) is also valid for the ST projects.
It is straightforward to show that increases in second-period per-worker production ($X_2$) produce two different effects in $n^S$. Firstly, there is a decrease in the relative wage $\tilde{w}_2$ that leads to an increase in the number of labor contracts because it is cheaper to hire workers. The second effect, which is linked to the increase in the per-worker production, leads the firm to demand a lower number of workers. In this trade-off, the first effect is bigger (lower) than the second one if relative wages are high (low) enough.

With regard to the influence of the environmental uncertainty in the labor contracts, we can show that increases in $\sigma^2$, generates two effects: The first one (substitution effect) leads to a lower $n^L$ and therefore to an increase in $n^S$. The second one (absolute effect) motivates an overall decrease in all labor contracts. If first-period labor contracts are basically $ST$, then, the second effect will offset the first. This logic is reversed when first-period contracts are basically $LT$.

Following the timing of the game, we have to analyze the lender problem with regard to his decision of whether to continue the project or not.

**Lender Decision Over Project Continuation**

We deal with $ST$ redeployable debt financial contracts. Therefore, first-period results will determine two different situations:

- If the project has been successful in the first period, then, the entrepreneur pays $R_1$ to the lender, and he wins the right to obtain a refinancing. The entrepreneur is interested to continue the project, because he has limited liability concerning the financial claims. As we have assumed that the lender would receive a high punishment not providing the new funds, the refinancing is assured.

- If there is no first-period returns, there is a control rights transference to the lender. In that situation, the liquidation is not immediately triggered, and a renegotiation starts with a probability $\Phi'$. The outcome of this process is a refinancing. The cost for the entrepreneur, besides the payment of $R_2$, is that he has to give up a share $1 - F$ of second-period profits.

To make consistent $\Phi'$ with the ex-ante probability to refinance a failed project, $\Phi$, the financial contract conditions settled by the lender have to make it ex-post indifferent whether to continue the project or not. That is, $L = (1 - F)E_2\{U^{EN}_T\} +$

15 Specifically, when $\tilde{w}_2 < \frac{W_1}{2}$ (which is assumed in the last footnote as $\tilde{w}_2 < \frac{\tilde{w}_1}{2}$), then, increases in second-period per-worker output $X_2$ leads to a decrease in the number of second-period labor contracts. The opposite is true when $\tilde{w}_2 > \frac{W_2}{2}$.

16 $ST$ contracts in this uncertain framework are more attractive for the entrepreneur than $LT$ ones, due to their higher flexibility.
LT-ST Labor and Debt Financial Contracts

\( \theta R_2 - I_2 \). Consequently, the lender will randomize his decision. In equilibrium, to achieve his decision with the ex-ante expectations made by all the agents in this symmetric information game, it will choose \( \Phi' = \Phi \).

3.1.2/ First-Period Analysis

In the first period all agents make decisions:

a/ \( LT \) workers decide their voluntary effort which generates the second-period increases in productivity.

b/ Using workers' policy, the entrepreneur will decide ex-ante the conditions that will characterize the labor contracts and the number of \( LT \) and \( ST \) workers that he is going to contract in the first period.

c/ In the first stage, the lender will characterize the financial contract.

Workers Effort in the First Period

The problem to be solved by the \( LT \) worker in this first period is the following:

\[
Max_e U^{LW} = w[2 + p_e(\theta \lambda e)] - C - \frac{1}{2} \theta e^2
\]

Where \( p_e \equiv \theta p + (1 - \theta) \Phi \).

The FOC leads to:

\[
\frac{\partial}{\partial e} U^{LW} = 0 \implies e = \frac{1}{A}(\theta r \lambda w_p)
\] (2)

The previous expression tells us that workers' effort increases with \( \Phi \). As project continuation probability increases, \( LT \) workers implement higher efforts since they think that they are more likely to be rewarded accordingly in the second period. This is the commitment effect of \( LT \) financial contracts.

Making use of this fact, and expression (1) of second-period \( ST \) workers; it is

17 If this relationship would have been satisfied with inequality, that is, \( \theta R_2 - I_2 + (1 - F)E_2(U^E_{2n}) \leq L \), then \( \Phi' = 0 \), and the financial contract had been a pure \( ST \) contract or a pure \( LT \) contract.

18 Where the continuation probability, \( p_c \), is composed of two terms. The first-period probability of success \( \theta p \), and the ex-ante probability of continuing an unsuccessful project \((1 - \theta p) \Phi \).
It is straightforward to obtain a negative correlation between \( ST \) workers and the \( \Phi \) parameter. This is stated in the following lemma:

**LEMMA 1**

*If there is an increase in project continuation probability (credit is more of \( LT \) type), then, there is an increase in the proportion of \( LT \) workers in the second period \( (\equiv \delta_2) \). Analytically:

\[
\frac{\partial \delta_2}{\partial \Phi} > 0 \quad (3)
\]

**Proof:**

Directly by the reduction of \( n_2^S \) with \( \Phi \), and the fact, proved in Proposition 1, that \( n^L \) is an increasing step function on \( \Phi \).

**First-Period Entrepreneur Problem**

The entrepreneur’s decision set in the first period \( (\equiv D^{En}) \), circumscribes to compute the \( LT \) workers \( (n^L) \), the first-period \( ST \) workers \( (n_1^S) \) and the share of the second-period increases in the verifiable output that goes to the \( LT \) workers \( (\lambda) \). In order to do so, we have to maximize the ex-ante entrepreneur utility \( (\equiv U^{En}) \).

\( \max_{\{D^{En}\}} E_1 \{\tilde{P}_1 \tilde{Q}_1\} + I_1 - (2n^L + n_1^S)w - \theta p R_1 + p_{\phi} U_2^{En} \quad (3) \)

Where \( p_{\phi} \equiv \theta p + (1 - \theta p) \Phi F \)

The *FOC* is shown in the first point of the Appendix 19. The result for the share \( \lambda \) is the following:

\[
\frac{\partial}{\partial \lambda} U_1^{En} = 0 \implies \lambda = \frac{1}{2\theta} \quad (4)
\]

19 To solve the maximization procedure we have used the hypothesis made in footnote 15 to assure that \( E_1 \{n_2^S\} > 0 \).
Expression (4) shows us that a high-quality entrepreneur ($\theta$ high), tries to minimize the free-rider problem with regard to $LT$ workers, reducing the share of second-period rents that go to workers (low $\lambda$). Reversing the above logic when $\theta$ is low; then, $\lambda$ is high to give incentives to workers’ effort.

With regard to the entrepreneur’s worker hiring policy:

$$\frac{\partial}{\partial n^1} U_1^{En} = 0 \implies n^L + n^S = N_1^* \equiv \frac{a - \hat{w}_1}{2bX(1 + \sigma^2)}$$  \hspace{1cm} (5)

$$\frac{\partial}{\partial n^L} U_1^{En} = \frac{\partial}{\partial n^S} U_1^{En} + \Delta[\Phi]$$ \hspace{1cm} (6)

With $\Delta[\Phi] = p_o p_r \eta - w(1 - p_c)$ \hspace{1cm} [\eta \equiv \frac{\sigma w^2}{\lambda^2}] \hspace{1cm} (7)$

The polynomial expression $\Delta[\Phi]$ represents the difference between the entrepreneur’s marginal utility to contract a $LT$ worker and to contract a $ST$ worker. Expression (7) shows that $\Delta[\Phi] \neq 0$ for a given $\Phi$. This fact shows us that in equilibrium all first-period workers are $LT$ or $ST$ with the $\Phi$-parameter defining the different cases according to the sign of $\Delta[\Phi]$.

When $\Delta[\Phi] < 0$, we have $\frac{\partial}{\partial n^L} U_1^{En} < 0$, and the equilibrium situation is such that all first-period labor contracts are of $ST$ type. As $\Delta[\Phi]$ is increasing with $\Phi$, only for low $\Phi$ values can this situation be possible. Note also that if $\eta$ is high enough ($\eta \geq \eta^{*} \equiv \frac{w(\frac{1}{(1 + \sigma^2)})}{r}$), then, $\Delta[\Phi = 0] > 0$ and it will not be first-period $ST$ labor contracting. This is reasonable, because $\eta$ is proportional to workers’ quality ($\frac{1}{A}$) to project’s quality ($r$), and to the wages that reward the non-voluntary effort ($w$), which are factors that give incentives to $LT$ labor contracting (see Lemma 2). It is also interesting to emphasize that $\eta^{*}$ is independent of the type of lender, that is, for high $\eta$ values, both market-financed and bank-financed firms will only contract $LT$ workers in the first period. In Proposition 3, we are going to show that for low $\eta$ values, this symmetric outcome is broken, and there is a bias towards $ST$ labor contracting by market-financed firms.

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20 $\hat{w}_1 = \frac{w}{p^X}$ is the period-one per-worker relative wage.

21 Except for a set of values of zero dimension.

22 This is a consequence of the symmetric information problem we are dealing with, as well as the use of linear functions, which make this analysis a first-order one.

23 As $A$ increases, workers feel that it is more costly to implement the voluntary effort. In this sense $\frac{1}{A}$ is a measure of workers’ quality.
If $\Delta[\Phi] > 0$, expression (6) shows that $\frac{\partial}{\partial n^L} U^E n > 0$, then, all first-period labor contracts are $LT$ ($n^L = N_1^*)$. It is concluded that as the financial structure is of $LT$ type, then, first-period labor contracts are also of $LT$ type.

As a final comment, it is interesting to point out that the expression $\Delta[\Phi]$, is composed of two terms. The first $(p_c p_s \eta)$, which is positive and is weighted by the ex-ante probability to continue the project ($p_c$), accounts for the benefits of $LT$ labor contracting. This first term is also composed of two factors that give incentives to contract on a long-term basis; that is, the ex-ante proportion of second-period profits $p_s$ belonging to the entrepreneur, and the $\eta$-factor, already mentioned, which is a mixture of the structural parameters that are correlated with the benefits of hiring $LT$ workers. This positive term is the expression of the initially referred commitment effect of $LT$ contracting.

The second term $((1 - p_c) w)$, which is negative, and is weighted by the ex-ante probability to not continue the project, is the expression of the marginal cost to contract a $LT$ worker in stead of a $ST$ one. This cost, comes from the entrepreneur's legal responsibility to satisfy the payment $w$ in the second period to all $LT$ workers, independently of the project's interruption. Under this view, we obtain that this term shapes the rigidity effect of $LT$ contracting.

From this trade-off we see that as $\Phi$ increases, the first (second) term becomes bigger (lower) as the project continuation probability, $p_c$, increases. Therefore the equilibrium in these circumstances is based on $LT$ workers.

If we represent the polynomial $\Delta[\Phi] = 0$ we obtain:

24 Note that this is the case when $\Phi \to 1$, then $p_c = p_s = 1$ and this implies that the value $\Delta[\Phi = 1] = \eta > 0$
From this figure, we can see that for $\Phi = \Phi^*$, there is a balancing of the previously mentioned two effects. If the financial contract, which is defined through $\Phi$, results to be lower than $\Phi^*$, then, all first-period workers are hired on a short-term basis. On the other hand, if $\Phi$ is higher than the threshold value $\Phi^*$, then, the result is reversed and only $LT$ labor contracts are signed. These results are outlined in the following proposition:
PROPOSITION 1

To define the entrepreneur's worker hiring policy in the first period, we have to consider two scenarios:

1/ If \( \eta \geq \eta^* = \frac{1-w}{(\Phi^*)^2} \) then, the threshold \( \Phi^* \) that satisfies \( \Delta[\Phi^*] = 0 \) is equal to zero and consequently there are only LT labor contracts in the first period.

2/ If \( \eta \leq \eta^* = \frac{1-w}{(\Phi^*)^2} \) in this case \( \Phi^* \) will be interior to the interval \((0,1)\). This fact leads to define two different scenarios:

When \( \Phi < \Phi^* \), the first-period contracts are only of ST type, and the equilibrium of the game is:

\[
n^L = 0 \quad n^S_1 = N^*_1 \quad n^S_2 = N^*_2 \quad (9)
\]

When \( \Phi > \Phi^* \) there is only LT labor contracting, and the equilibrium of the game is given by:

\[
n^S_1 = 0 \quad n^L = N^*_1 \quad n^S_2 = N^*_2 - (1 + re)N^*_1 \quad e = \frac{1}{2A}(rwpc) \quad \lambda = \frac{1}{2\rho} \quad (10)
\]

We can assure, as a consequence of the above points, that the proportion of LT workers in the first period increases when credits are more of LT nature (\( \Phi \) high). This is put formally as follows:

\[
\frac{\partial \delta_1}{\partial \Phi} > 0 \quad (11)
\]

Proof

By construction.

Using the expression \( \Delta[\Phi] \) we can make an easy comparative static analysis with the different structural parameters of the model.

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25 Assuming as a hypothesis the upper bound to \( r^2 \) given in footnote 14
**LEMMA 2**

An entrepreneur in a bank-financed firm is more willing to contract LT workers than an entrepreneur in a market-financed firm.

On the other hand, the likelihood the entrepreneur to contract LT workers is positively correlated with the η parameter, that is, with project's quality (τ), entrepreneur's quality (θ) and workers' quality (∏).

**Proofs:**
For the first part of the Lemma, see Point 2 in the Appendix. For the second part, looking at (7), it is straightforward to compute the variations of Δ with regard to different parameters. As Δ[Φ] is a monotone increasing function in the interval [0,1], then, increases (decreases) in Δ lead to a lower (higher) Φ*. These movements, will determine a higher (lower) probability Φ to be bigger than Φ* which is the way to note that the LT (ST) contracts are more probable.

The reason why an entrepreneur is more willing to hire LT workers in a bank-financed firm than in a market-financed firm, lies on the higher commitment of the bank with regard to the market as a fund provider. This is shown in the outcome of the renegotiation process that might follow when the project becomes unsuccessful in the initial period. In that situation, the firm can retain a higher share of second-period profits if the lender is a bank than if it is the market, that is, \( F_B > F_M \). This is relevant because the positive effect to contract LT workers is only present in the second period, when they become more productive. As the entrepreneur of a bank-financed firm can enjoy a higher share of this benefits, he is more willing to offer LT contracts to the workers.

**Lender Problem**

In this first period, the lender has to define ex-ante the variables \( \{R_1, R_2, \Phi\} \) of the financial contract. The problem to solve is different under a bank financing than under a market financing, therefore we spit the analysis accordingly.

---

\( ^{26} \) That is \( \frac{\partial \Phi^*}{\partial F} < 0 \), and other things equal, the LT labor contracting zone (\( \Phi \geq \Phi^* \)) is widened under the high \( F \) scenario (bank financing).
Bank Financing

Under this financial scheme, the maximization problem faced by the bank is:

\[
Max_{DFin} I_1 + \theta p(R_1 + \theta R_2 - I_2) + (1 - \theta p)L \quad (12)
\]

S.t. \[ I_1 + \pi_1 - \theta pR_1 + \Phi E_2[U_{2}^{En}] = 0 \quad (13)
\]

S.t. \[ \theta R_2 - I_2 + (1 - F)E_2[U_{2}^{En}] = L \quad (14)
\]

In this case the lender has all the bargaining power, therefore it will define a financial contract, maximizing his profit function, and extracting ex-ante all the entrepreneur's rents (eq. 13). We use equality (14) as a selection criterion among different possible financial contract that satisfy equation (13). The idea, which has been commented previously \(^{27}\), is to focus on contracts that induce the lender to adequate the ex-post probability to renegotiate a failed project to the ex-ante one, \(\Phi\). This will only happen, when the lender of a failed project is ex-post indifferent whether to continue the project or not. This is precisely what states equation 14. In equilibrium, the indifferent lender adequates his decision to renegotiate an unsuccessful project with the ex-ante expectations made by all the agents in this symmetric information game.

Let us now see, the lender problem under the alternative financing mechanism.

Market Financing

We model this situation, considering that the entrepreneur offers to finance the project to the best bid. Therefore, the lender's objective will be to maximize the entrepreneur's profit function, subject to his own participation constraint (lender zero-profit condition). This is shown in the following expressions:

\[
Max_{DFin} I_1 + \pi_1 - \theta pR_1 + \Phi E_2[U_{2}^{En}] \quad (12')
\]

S.t. \[ - I_1 + \theta p(R_1 + \theta R_2 - I_2) + (1 - \theta p)L = 0 \quad (13')
\]

S.t. \[ (14)
\]

The important point is that the lender determines \(\Phi\) maximizing the same formal function under both financing scenarios. That is:

\[
Max_{c} \pi_1 + p_c \pi_2 + (1 - p_c)L \quad (15)
\]

Where \(\pi_1\) and \(\pi_2\) are the operative profits in period one and two respectively.

\(^{27}\) See footnote 17.
In point 3 of the Appendix are made explicit their expressions:

\[ \pi_1 = (\theta p - \frac{1}{2})X_1N_1^*(a - \hat{w}_1) \quad \text{and} \quad \pi_2 = (\theta - \frac{1}{2})X_2N_2^*(a - \hat{w}_2) + 2p_n\eta N_1^* \] (15')

Both problems are formally identical. The only distinction between them is the value of \( F \ (F_B > F_M) \). Obviously the determination of \( R_1 \) and \( R_2 \) in each scenario, is made using different constraints. Therefore the values are different.

**LEMMA 3**

Under both financing scenarios, the lender defines the length of the financial contracts, \( 1 + \hat{\phi} \), maximizing the same formal objective function.

**Proof:**

See Point 3 of the Appendix.

Making use of Lemma 2 and Lemma 3, we can fully characterize the optimal length of the financial contracts defined by the financier. This is shown in the following proposition:

**PROPOSITION 2**

Lender’s definition of the optimal financial contract’s length \( (1 + \hat{\phi}) \) is given by the liquidation value \( L \) of the project:

- If \( L \leq L_F \), it is optimal to define a LT financial contracts, that is, \( \hat{\phi} = 1 \)
- When \( L_F < L \leq \bar{L}_F \), the lender will define the minimum possible credit’s length to induce the entrepreneur to contract LT, that is, \( \hat{\phi} = \Phi^* \)
- For \( \bar{L}_F < L \), a pure short-term financial contract, \( \hat{\phi} = 0 \), is the optimum.

In case of \( \eta \geq \eta^* \), we know from Proposition 1 that \( \Phi^* = 0 \). The lender will only define exclusive LT credits or exclusive ST credits. Furthermore, the threshold value that differentiates both contract situations, results to coincide under both type of lenders, that is \( L_F = \bar{L}_F = \hat{L} \)

**Proof:**

See Point 4 in the Appendix, where we compute the values of \( L_F \) and \( \bar{L}_F \) for each financing scenario.

We have obtained a negative correlation between the liquidation value \( L \) and the length of the credit. The higher is \( L \), the higher is the willingness of the lender to liquidate an unsuccessful project in period one, and obtain a sure return \( L \). Note that, if it provides new funds \( I_2 \), it risks to obtain nothing because the liquidation value in the second period is zero.
We now consider the influence of the lender's type in the credit's length definition \(1 + \Phi\). We find two effects. Firstly, Lemma 2 links the commitment of the lender inside the firm (\(F\)), with the willingness of the entrepreneur to hire workers through \(LT\) contracts. This fact shows that a rise in \(F\) generates a marginal benefit to increase the length of the credit. This is so because in that situation it is easier to induce the entrepreneur to hire \(LT\) workers which are valuable for the lender because they are more productive and generate more profits in the second period. We call this positive effect \textit{labor incentive effect}. Secondly, there is a negative effect linked to \(F\). As \(F\) raises, whenever a failed project is renegotiated, the share of second period profits obtained by the lender \((1 - F)\) is lower. This fact gives disincentive to refinance an unsuccessful project, (decrease \(\Phi\)). The lender will prefer to liquidate a first-period unsuccessful project, instead of risking new capital for a lower premium. We call this second effect \textit{lender's premium effect}. The \textit{labor incentive effect} points in the direction of inducing a higher credit's length under a bank-financing scheme rather than under a market financing one. On the other hand, the \textit{lender's premium effect} goes in the opposite direction to give incentives to reduce the financial contract's length in bank-financed firms. The result of this trade-off is stated in the following proposition.

\textbf{PROPOSITION 3}

\textit{Credit's length dependence on lender's type is given by the following facts:}

If \(L \leq L_B\), under both financial alternatives there are exclusive \(LT\) credits, that is, \(\Phi_B = \Phi_M = 1\). As a consequence the firm only offers \(LT\) contracts in the first period, independently of the lender considered.

When \(L_B < L \leq L_M\), the average credit's length is lower under the bank-financing scheme. Specifically \(\Phi_B = \Phi_M^* < \Phi_M = 1\). Furthermore, the firm's labor policy is based on exclusive \(LT\) hirings under both scenarios.

For \(L_M < L \leq L_B\), the firm only obtains \(ST\) credits with a market financing, \(\Phi_M = 0\), but the bank defines credits with a positive length, \(\Phi_B = \Phi_B^*\). Under these conditions market-financed firms only offer \(ST\) contracts to first-period workers. By contrast, bank-financed firms hire only \(LT\) workers.

For \(L > L_B\), both types of lenders provides financing only by means of \(ST\) credits. Consequently, in the first period the firm only hires \(ST\) workers.

Considering a uniform distribution over project liquidation values \(L\) in an interval \([0, L_H \geq L_B]\), the overall length average of the credits defined under both financing alternatives is the same. By contrast, the overall length average of the labor contracts is higher under the bank-financing scheme.
**Proof:**

See Point 5 in the Appendix.

It is interesting to point out that although \( L_M < L_B \); for \( L_F \), the logic is reversed. In other words, a bank reduces the length of their credits for a lower liquidation values than the market. Moreover, the amount of the length reduction is stronger, that is from \( \hat{\Phi} = 1 \) to \( \hat{\Phi} = \Phi^*_B < \Phi^*_M \). Therefore, for \( L < L_M \) we can assure that the credit length is lower under bank financing than under market financing \( (\hat{\Phi}_B \leq \hat{\Phi}_M) \). This result, for low-\( L \), values shows the superiority of the previously stated lender's premium effect with regard to the labor incentive effect. On the other hand, in the high-\( L \) zone, the length of the credit is superior under the bank-financing scheme. This is so, because a lower credit’s length is required to incentive the entrepreneur to hire \( LT \) workers \( (\Phi^*_B < \Phi^*_M) \). This effect, previously called labor incentive effect, results in being higher than the lender’s premium effect for high liquidation values. By this, we can see a rise in credit’s length under the bank-financing scenario.

A graphical representation of optimal credit’s length \( \hat{\Phi} \) versus liquidation value \( L \)
Although we cannot talk in absolute terms of a higher or lower length average in the credits defined by a bank or the market, we have proved in Point 5 that in a set of projects with enough diversity\textsuperscript{28} the overall credit length average coincides under both types of lenders. Credit's length differences in the low-$L$ zone are exactly offset with the opposite differences in the high-$L$ zone. Furthermore, Proposition 3 states, labor contract’s length results in being higher under a bank funding scheme than under a market funding scheme. In this sense, we can talk of a higher length in the overall firm’s contract structure under bank funding rather than under market funding.

Analyzing the threshold liquidation values $L_F$, we can make a comparative static study relating the length of the financial contract with different structural parameters. The results are stated in the following lemma:

**LEMMA 4**

Financial contract’s length is positively correlated with the $\eta$-parameter, that is, with project’s quality ($r$), entrepreneur’s quality ($\theta$) and workers’ quality ($\theta$). In all cases, by Lemma 2, the higher is the credit’s length, the higher is the labor contract’s length.

**Proof:**

See Point 6 in the Appendix, where the derivatives of the threshold liquidation value $L_F$ with regard to $\eta$ are computed.

In this symmetric information game, the lender has the same information as the entrepreneur. Therefore both agents can coordinate their actions to obtain a higher profits\textsuperscript{29}. In this way, we can understand how the length of the labor contracts interact with the length of the financial contracts. Specifically, we have shown that an increase in the $\eta$-parameter leads to a lower $\Phi^*$ in the labor contract definition, and therefore, to an extension of the $\Phi$-region where $LT$ labor contracts are defined. On the other hand, Lemma 4 shows that the lender also increases the credit’s length, $1 + \Phi$. Therefore, both effects are reinforced among themselves in giving incentives to the $LT$ contractual relationships.

As a final note, we can say that a more (less) uncertain framework, modeled with a lower (higher) $p$\textsuperscript{30}, leads to a larger (lower) $ST$ contract proportion.

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\textsuperscript{28} We have modeled this project’s diversity with a uniform liquidation value distribution covering a wide spectrum (a null lower bound and a higher-than-$L_M$ upper bound).

\textsuperscript{29} It is in his interest to do so, because a share $1 - F$ of the second-period returns are enjoyed by the lender when he renegotiates a failed project.

\textsuperscript{30} A high “noise” in the environment leads to a low $p$. See the project’s characterization.
3.2. **ST PROJECT**

The way we have modeled the difference between the LT and ST projects, allows us to solve the ST project's problem in a symmetric way than the LT one, just considering a higher liquidation value \( L \), and a lower (higher) second-period (first-period) per-worker output \( X_1(X_2) \). Taking into account the former changes, we just have to modify the expressions in a convenient way. The results we get, compared with the LT results, will permit us to analyze the impact of the project's type in the proposed equilibrium contracts.

To begin with, we study the proportion of the first-period and the second-period LT workers' contract (\( \delta_1 \) and \( \delta_2 \)).

**Analysis of Second-Period and First-Period Labor Hirings**

As in the LT project's case, we have to distinguish two frameworks, depending if \( \Phi \) is higher or lower than the threshold value \( \Phi^* \). This value does not change, when we consider ST projects, because the polynomial \( \Delta[\Phi] = 0 \) is invariable to \( S \). In case of \( \Phi < \Phi^* \) we obtain an exclusive first-period ST labor contracting, and for \( \Phi > \Phi^* \), there is only LT labor contracting in the first period. The difference is that \( N_1^* \) is lower and \( N_2^* \) is higher in ST projects. Making use of this result we can state the following lemma:

**LEMMA 5**

In the ST project we obtain that in both periods the number of ST workers is higher, and the proportion of LT workers (\( \delta \)) is positively correlated with the financial contract's length, that is, \( \frac{\partial \delta_1}{\partial \Phi} > 0 \) \( \frac{\partial \delta_2}{\partial \Phi} > 0 \).

The difference with the LT project is that the second derivative is lower than the equivalent in the previous project's framework, that is \( \frac{d}{dS} \{ \frac{\partial \delta_2}{\partial \Phi} \} < 0 \).

---

31 In fact to simplify things, we can consider, as we did in footnote 14, that in the LT project \( X_1 = X_2 = X \), and in the ST project \( X_1 > X \) and \( X_2 < X \).

32 N.B. All the economic magnitudes are indexed with \( L(S) \) to account for project's type.

33 We make the convention to note by \( S \) the changes that differentiate both type of projects, that is, an increase in \( X_1 \), a decrease in \( X_2 \) and an increase in the liquidation value \( L \).

34 Where we have assumed as it is stated in footnote 15 that \( \{ \tilde{w}_2, \tilde{w}_1 \} \leq \frac{X}{2} \), which makes \( N_2^* \) decreasing in \( X_2 \) and \( N_1^* \) decreasing in \( X_1 \). See expressions (1) and (5).
Proof:
See Point 7 in the Appendix, where we compute $\frac{dN^*_1}{dS} < 0$ and $\frac{dN^*_2}{dS} > 0$

To determine the optimal credit's length, $\hat{\Phi}$, chosen by the lender, we make use of Proposition 2 and the following two facts. Firstly, a $ST$ project has a higher liquidation value than a $LT$ one, and by this, the lender is more willing to define low credit's length. Secondly, the threshold value $\bar{L}_F$ and $\bar{L}_F$ are decreasing with $S$. With this two facts we can state the following proposition:

PROPOSITION 4

When the projects have high (low) second-period returns and low (high) first-period, the financial institution is more willing to provide funds through $LT$ ($ST$) credits, that is $\hat{\Phi}_L \geq \hat{\Phi}_S$. This fact leads, at a second stage, to induce the entrepreneur to contract workers in a $LT$ ($ST$) basis.

Proof:
See point 7 in the Appendix.

This is an intuitive result to link the temporal nature of the project with the temporal nature of the financial contracts and ultimately of the labor contracts.

4. DISCUSSION AND EMPIRICAL TESTS

If we keep in mind Propositions 3 and 4 we can provide a theoretical explanation of some stylized facts with economic interest.

The first one refers to the correlation between labor and financial elements with the project's type that characterizes the US-UK and German-Japanese paradigms. In the German-Japanese framework, the projects are basically of $LT$ nature, similar to the labor and financial contracting. On the other hand, the US-UK paradigm basically shows that these relationships are made in a $ST$ basis: projects, labor and financial contracts are based in a short-term approach.

The second stylized fact, which is also connected with the first, refers to the impact of the lender's type in the definition of the contract structure of the firm. We consider two types of fund providers: a bank and the financial markets. In the first type, characteristic of the German-Japanese paradigm, the credits are long-term in nature and banks play an active role. In the second case, more common
in the US-UK paradigm, the funds borrowed by the firms have to be returned following a short-term pattern.

Regarding to the results we have obtained; we model the first stylized fact when we endogenize the bargaining lender-entrepreneur and entrepreneur-workers, and obtain the stated correlation that defines both paradigms. The German-Japanese approach, with \( LT \) projects \( \leftrightarrow \) \( LT \) financial contracts \( \leftrightarrow \) \( LT \) labor contracts; and the US-UK, with \( ST \) projects \( \leftrightarrow \) \( ST \) financial contracts \( \leftrightarrow \) \( ST \) labor contracts.

With regard to the second stylized fact; we characterize the type of financing (bank-market) by proportion of rents the entrepreneur can keep in the second period, \( F \), as a result to renegotiate a failed project \(^{35} \). Using this model we find that the length of the credits the entrepreneur obtains from a bank, is higher than the corresponding to the credits he can obtain through the market. This result is consistent with the previous description of economic paradigms.

Making use of this results, it is possible to define a set of empirical tests that particularly fits the Spanish Economy. In particular:

1/ To determine if the Spanish labor reforms in the 80's, have biased the \( ST \) labor contracting towards the \( ST \)-financed firms. In the same line, if the test has produced positive results, we can forecast the effects of the 1997 Spanish labor reform that try to give incentives to the firms to hire workers by means of \( LT \) contracts. Our model predicts (Proposition 1) that this measure will be most effective in firms with a \( LT \) financial structure.

2/ It can also be interesting to implement a test by sectors, in order to check for a higher labor contract's length in those mature industries (manufacture sector), where firm's financial structure is more \( LT \). By contrast, in more young sectors (services), we should find a shorter labor contract's length.

3/ A last test would be to analyze a possible change in the labor contracting policy followed by IPO's firms and by firms that have increased its market capitalization. Our model (Proposition 3) forecasts a rise in the proportion of workers hired through a \( ST \) contract.

\(^{35} \) We consider that bank's \( F \) is higher than market's \( F \).
5. FINAL REMARKS

Our main objective of this paper has been to study the optimal length of the contracts that define a firm (labor and financial). Moreover, we have approached this problem in an integrated way with other external features that strongly condition firm's policy. In particular project's type, workers' skills and the characteristics of the financial institution which provides funds.

We have followed the modern view of what is known as corporate governance of the firm. This idea applied to our framework leads to a unified treatment of the problem of determining the length of the labor and financial contracts. This method of analysis is radically different to the classical one, where both problems were dealt separately, and always considering the other type of contracts as exogenously given. Specifically, in the determination of the optimal length of the labor contracts, the financial structure of the firm has been traditionally ignored. Simply a balance was made between the commitment value of long-term contracts and the bigger flexibility provided by the short-term ones. Apart from that, when financial contracts have been studied, the institution that provides the funds, designed the credit ignoring the effects of its decision on firms' labor contracting. In fact it balances the disciplinary effect of \( ST \) credits (due to the possibility of cutting the provision of funds in future periods) and the commitment effect of \( LT \) ones that leads to higher returns.

In our work, we have found the mentioned trade-off in the labor contracting. Furthermore, we have shown the importance to introduce the length of the credits to properly analyze how the entrepreneur defines the length of the labor contracts. In this sense, the key factor in our study has been the ex-ante probability to renegotiate an initially unsuccessful project (\( \equiv \Phi \)). Parameter directly linked to the average length of the financial contract (\( \equiv 1 + \Phi \)). We have proved that when the credit's length is higher (lower) than a given threshold level, \( 1 + \Phi^* \), then, all labor contracts signed initially are of \( LT \) (\( ST \)) type.

Regarding the internal consequence of project's type, we have stated another result. The lender supplies credits with a higher length when the financing is over a \( LT \) projects than over a \( ST \) project. This fact under the claim made in Proposition 1, determines a higher probability the entrepreneur will sign \( LT \) labor contracts when the firm undertakes a \( LT \) project instead of a \( ST \) project. If we put all together, we can state a positive correlation between (projects's type -
financial contract's type - labor contract's type) in a direction that is consistent with the so called US-UK and German-Japanese paradigms.

A second type of results, complementary to the above ones, is stated in Proposition 3, where we find a relationship between "bank financing", "market financing" and contract's length. Banks favor LT and markets favor ST.

Concerning possible limitations and shortcuts of the model, we can mention the rude characterization of the lender, with an exogenously given commitment parameter $F$, and with an extreme assumption over its bargaining power (zero-market, one-bank). Another clear avenue of further research is the informational structure. The current symmetric nature is difficult to conciliate with the proper existence of financial intermediaries, that are not neutral funds provider but are involved in firms' decisions taking through monitoring and incentive activities. Finally we consider only two periods, and this does not allow us to study reputation problems that are so important in the analysis of contract's characterization.

An extension of this model, proposed by Fama (1990), and partially made by John, K. & John, T. (1993) consists in the analysis of the interaction between both types of contracts (labor-financial), not under a temporal perspective, but considering the fix and variable nature they present. In this sense, financial contracts "fixed - debt-type" ("variable - equity-type") are correlated with labor contracts that are of fixed (variable) rewards.

Further research should address all these questions in order to improve our knowledge of this important link between financial and labor issues.

$^{36}$ Payment related performance
APPENDIX

[1] In the entrepreneur’s maximization problem, the variables are $n^L, \lambda,$ and $n^s$. We first compute the derivative with respect to $n^s$, that is:

$$\frac{\partial}{\partial n^s} E_1\{U^{En}\} = \frac{\partial}{\partial n^s} E_1\{U^{En}\} = \theta p X(a_1 - 2bX(1 + \sigma^2)N_1) - w =$$

$$= -2b\theta p X^2(1 + \sigma^2)(N_1 - N^*_1) \quad [N^*_1 = N^*_1 \equiv \frac{\frac{\partial w}{\partial X}}{2\theta p X(1 + \sigma^2)}] \quad (A1.1)$$

With regard to the $\lambda$ variable:

$$\frac{\partial}{\partial \lambda} E_1\{U^{En}\} = \frac{\partial}{\partial \lambda} E_1\{U^{En}\} = \frac{\partial}{\partial \lambda} \text{ wnr}^L - p^2(\theta e + \frac{\partial e}{\partial \lambda}(1 - \theta \lambda)) \quad (A1.2)$$

If we make use of expression (2) of first-period workers' effort $e = \frac{1}{\lambda}(\theta r \lambda w p_c)$, (A1.2) assures that $\lambda = \frac{1}{2\theta r \lambda w p_c}$. Finally, making use (A1.1) and the fact that in the optimal contract it is true that $\frac{\partial}{\partial n^s} E_1\{U^{En}\} = 0$, we get in the following expression:

$$\frac{\partial}{\partial n^L} E_1\{U^{En}\} = \frac{\partial}{\partial n^L} E_1\{U^{En}\} = \frac{\partial}{\partial n^L} E_1\{U^{En}\} - w(1 - p_c) + p_c \theta e(1 - \theta \lambda) \quad \text{as } \lambda = \frac{1}{2\theta r \lambda w p_c}, \quad (A1.3)$$

Direct computations assures that $SOC$ are also satisfied.

[2] To get in the first result of Lemma 2, we have to prove that the threshold level for bank-financed firms, $\Phi_B^*$, is lower than for market-financed firms, $\Phi_M^*$. To do so, we make use of the definition of $\Phi_B^*$ and $\Phi_M^*$, that is $\Delta^B[\Phi_B^*] = 0$ and $\Delta^M[\Phi_M^*] = 0$, where $\Delta[\Phi]$ is given by (7). As $\Delta[\Phi]$ is increasing in $\Phi$, we only have to prove that $\Delta^B[\Phi_M^*] > 0$.

From (7), $\Delta^B[\Phi_M^*] = \eta \Phi_M^* \text{pc}[\Phi_M^*] - w(1 - p_c[\Phi_M^*]) \Rightarrow \text{by the definition of } \Phi_M^*$, $\Delta^B[\Phi_M^*] = \eta \Phi_M^* \{p^r_M[\Phi_M^*] - p^R_M[\Phi_M^*]\} = \eta \Phi_M^*(1 - \theta p)(F_B - F_M) > 0$

The previous inequality assures that $\Phi_B^* < \Phi_M^*$ (A2.1)

[3] To get in the required expression, which is only $\Phi$—dependent, we have to use the constraints (13) and (14) to express $R_1 + \theta R_2$ in terms of $\Phi$. We subtract (13) from $\theta p$ times (14), we find:

$$\theta p(R_1 + \theta R_2) - \theta p I_2 - I_1 - \pi_1 + [\theta p(1 - F) - p_c]U^{En}_2 = \theta p L \quad (A3.1)$$

With (A3.1), lender's utility function transforms to:

37 Where we have used $\frac{\partial w}{\partial X} = \frac{\theta r \lambda w p_c}{A}$ and the fact that $E_1\{n^s\} > 0$, that leads to $\frac{\partial w}{\partial X} = 0$

38 Remember that we have assumed that $F_B > F_M$
\[ U^{\text{Fin}} = -I_1 + \theta p R_1 + \theta R_2 - I_2 + (1 - \theta p) L = \pi_1 + L + [p_\Phi - \theta p(1 - F)] E_2 \{ U_2^{\text{En}} \} \]

To arrange this, we make use of \( p_\Phi - \theta p(1 - F) = F p_c \) (A3.3)

\[ E_2 \{ U_2^{\text{En}} \} = E_2 \{ \tilde{P}_2 Q_2 \} + I_2 - \theta R_2 - n_2^L w - n_2^L w (1 + \lambda \theta re) \] (A3.4)

By (14), we know that \( \theta R_2 - I_2 = L - (1 - F) E_2 \{ U_2^{\text{En}} \} \), then (A3.4) becomes:

\[ F E_2 \{ U_2^{\text{En}} \} = -L + E_2 \{ \tilde{P}_2 Q_2 \} - n_2^L w - n_2^L w (1 + \lambda \theta re) \] (A3.5)

If we plug (A3.5) and (A3.3) in (A3.2), we obtain:

\[ U^{\text{Fin}} = \pi_1 + L + p_c (\pi_2 - L) \] which is the expression (15).

As a matter of completeness, we compute \( \pi_1 \) and \( \pi_2 \):

\[ \pi_1 \equiv E_1 \{ \tilde{P}_1 \tilde{Q}_1 \} - (n_1^S + n^L) w = \theta p N_1^* X_1 (a - b(1 + \sigma_z) N_1^*) - N_1^* w =
\]

\[ = (\theta - \frac{1}{2}) X_1 N_1^* (a - \tilde{w}_1) \] (A3.6)

Where we have used (5) to show \( n_1^S + n^L = N_1^* \).

To compute \( \pi_2 \) we have to distinguish two situations, depending if \( \Phi \leq \Phi^* \), which generates that \( n_2^L = 0 \) or \( n_2^L = N_2^* \) respectively.

For \( \Phi < \Phi^* \):

\[ \pi_2 = E_2 \{ \tilde{P}_2 \tilde{Q}_2 \} - (n_2^S + n_2^L) w = \theta N_2^* X_2 (a - b(1 + \sigma_z) N_2^*) - N_2^* w =
\]

\[ = (\theta - \frac{1}{2}) X_2 N_2^* (a - \tilde{w}_2) \] (A3.7)

On the other hand, for \( \Phi \geq \Phi^* \):

\[ \pi_2 = E_2 \{ \tilde{P}_2 \tilde{Q}_2 \} - w (n_2^S + n_2^L [1 + \lambda \theta re]) = \theta N_2^* X_2 (a - b(1 + \sigma_z) N_2^*) -
\]

\[ -N_2^* w + w n_2^L re = (\theta - \frac{1}{2}) X_2 N_2^* (a - \tilde{w}_2) + 2 p_c \eta N_2^* \] \( \eta \equiv \frac{r^2 w^2}{4 A} \) (A3.8)

**Market Financing**

Let us now prove what Lemma 3 states, that is that the market-financing problem, also leads to expression (15).

\[ \max_{\{U^{\text{Ent}}\}} E_1 \{ U^{\text{Ent}} \} = I_1 + \pi_1 - \theta p R_1 + p_\Phi E_2 \{ U_2^{\text{En}} \} \] (12'')

\[ \text{s.t.} \quad -I_1 + \theta p (R_1 + \theta R_2 - I_2) + (1 - \theta p) L = 0 \] (13'')

\[ \text{s.t.} \quad \theta R_2 - I_2 + (1 - F) E_2 \{ U_2^{\text{En}} \} = L \] (14)

To arrange (12''), we make use of \( I_1 - \theta p R_1 \) from (13'') and the definition of \( p_\Phi = \theta p + (1 - \theta p) \Phi F \):

\[ E_1 \{ U^{\text{Ent}} \} = \pi_1 + (1 - \theta p) L + \theta p (\theta R_2 - I_2 + E_2 \{ U_2^{\text{En}} \}) + (1 - \theta p) \Phi F E_2 \{ U_2^{\text{En}} \} \]

From (14) we make explicit \( FE_2 \{ U_2^{\text{En}} \} \), to obtain:

\[ E_1 \{ U^{\text{Ent}} \} = \pi_1 + p_c (\theta R_2 - I_2 + E_2 \{ U_2^{\text{En}} \}) + (1 - p_c) L \]

The definitions of \( E_2 \{ U_2^{\text{En}} \} \) and \( \pi_2 \) given in (A3.4) and (A3.7), leads to:

\[ E_1 \{ U^{\text{Ent}} \} = \pi_1 + p_c \pi_2 + (1 - p_c) L \]

This is the same objective function of the bank-financing scenario, therefore we have completed the proof of Lemma 3

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\[39\] It is important to point out that to take expectations over period one is the same as to take expectations over period 2, due to our IID assumption over shocks.
To prove Proposition 2, we make use of expressions (15) and (A3.8).

\[ U = \pi_1 + p_c \pi_2 + (1 - p_c) L, \]

where \( \pi_1 \) and \( \pi_2 \) are given by (A3.7) and (A3.8) respectively. We make the convention to note by \( L \equiv \theta - \frac{1}{2} X_2 N_2 (a - \bar{a}_2) \) and \( D \equiv 2p_c \eta N_1^* \) which is \( \Phi \)-dependent through \( p_c \). We also define \( \bar{D} \equiv D[\Phi = 1] \) and \( D[\Phi = 0] \). The \( \Phi \)-derivative with regard to \( U \) leads to:

\[
\frac{\partial}{\partial \Phi} U = \begin{cases} 
(1 - \theta p)[L - L] & \text{if } \Phi < \Phi^* \\
(1 - \theta p)[L + 2D - L] & \text{if } \Phi \geq \Phi^*
\end{cases}
\]

(A4.1)

From (15) we show that for \( \Phi = \Phi^* \) there is a shift in the lender utility function of an amount equal to \( p_c[\Phi^* ]D[\Phi^* ] > 0 \). This fact will assure that for \( \Phi = \Phi^* \) the lender considers optimal to increase \( \Phi \) towards the LT-labor-contracting zone.

We now study (A4.1) for different \( L \)-values.

1/ Trivially, for \( L \leq L \), \( \frac{\partial}{\partial \Phi} U > 0 \) for all \( \Phi \), therefore it is optimal for the lender to sign an exclusive LT contract, that is \( \Phi = 1 \).

2/ For \( L < L \leq L + 2D \), \( \frac{\partial}{\partial \Phi} U > 0 \) for \( \Phi < \Phi^* \) but \( \frac{\partial}{\partial \Phi} U > 0 \) for \( \Phi \geq \Phi^* \).

Therefore to determine the optimal length, we have to compare the values of \( U \) for \( \Phi = 0 \) and \( \Phi = 1 \).

\[ U[\Phi = 0] = \overline{D} - (1 - \theta p)(L - L), \quad \text{if } L \leq L + \frac{D}{1 - \theta p} \]

(A4.2)

But for \( \theta p > \frac{1}{2} \) (which is assumed), it is true that

\[ L + 2D < L + \frac{D}{1 - \theta p} \Rightarrow \Phi = 1 \quad \text{for } L \leq L + 2D \]

1/ For \( L + 2D < L \leq L + 2D \)

In this situation, \( U \) is convex for \( \Phi > \Phi^* \), with the \( \Phi \) that leads to a minimum in \( U \) being increasing in \( L \), in such a way that for \( L = L + 2D \) the \( U \) is decreasing for all \( \Phi \geq \Phi^* \). Therefore, to determine the \( \Phi \) optimal in this zone, we have to compare \( \Phi = 1 \) with \( \Phi^* \).

\[ U[\Phi = 1] - U[\Phi^*] = 40 \overline{D} + L(1 - p_c^*) - (1 - p_c^*)(L_f) \]

(A4.3)

Finally, note that

\[ L + 2D < L + \overline{D}(1 + p_c^*) < L + 2D < L + \frac{D}{1 - \theta p} \]

(A4.4)

The last inequality, assures that \( \Phi \) will always be higher than \( \Phi^* \).

Then, in the interval of liquidation considered, making use of (A4.5) and (A4.2), we can assert that:

\[ \text{For } L + 2D < L \leq L + \overline{D}(1 + p_c^*) \Rightarrow \Phi = 1 \]

(A4.6)

And \[ L + \overline{D}(1 + p_c^*) < L \leq L + 2D \Rightarrow \Phi = \Phi^* \]

(A4.7)

40 Where we have used the convention \( p_c^* = p_c[\Phi^*] \) and the fact \( D[\Phi^*] = 2p_c^* \eta N_1^* = p_c^* \overline{D} \)

41 Simply by \( \theta p > \frac{1}{2} \)
In this zone, we know that $U^{\text{Fin}}$ is decreasing for all $\Phi$. Therefore the optimal credit's length will be $\hat{\Phi} = 0$ or $\hat{\Phi} = \Phi^*_+$. To choose the optimal value, we compute $U^{\text{Fin}}$ for both $\Phi$ values:

$$U^{\text{Fin}}[\Phi^*_+] - U^{\text{Fin}}[0] = p_c^*(L + p_c^*D) - \theta p L + (1 - p_c^*) (L_f) - (1 - \theta p) L \quad (A4.8)$$

(A4.8) assures that $U^{\text{Fin}}[\Phi^*_+] - U^{\text{Fin}}[0] \leq 0$ if $L \leq L + \frac{D}{1 - \theta p} (p_c^*)^2 \quad (A4.9)$

From (A4.9), we can assure that:

For $L + 2D < L \leq L + \frac{D}{1 - \theta p} (p_c^*)^2 \Rightarrow \hat{\Phi} = \Phi^*_+$ \quad (A4.10)

And $L + \frac{D}{1 - \theta p} (p_c^*)^2 < L \Rightarrow \hat{\Phi} = 0$ \quad (A4.11)

As a matter of completeness we put all together 42:

For $L_F < L \leq L_F \equiv L + \overline{D}(1 + p_c^*) \Rightarrow \hat{\Phi} = \Phi^*_+$ \quad (A4.13)

For $L_F < L \Rightarrow \hat{\Phi} = 0$ \quad (A4.14)

To finish the proof of Proposition 2, we have to show that $L_F = \overline{L}_F$ when $\eta \geq \eta^*$, which is the situation that leads to $\Phi^* = 0$ (pure LT labor contracting under both lender's type). To do so, we have to note that only threshold value $L_F$ is compatible with a situation where there is only LT labor contracts.

But $L_F[\Phi^* = 0] = L + \overline{D}(1 + \theta p) = L_F[\Phi^* = 0] \equiv \hat{L}$

To prove first part of proposition 3, we only have to show if $L_M \leq \overline{L}_B$, and $L_B \leq L_M$:

To prove $L + \frac{D}{1 - \theta p} (p_{c,B}^*)^2 \geq 43 \frac{D}{1 - \theta p} (p_{c,M}^*)^2$, we only have to note:

$$\frac{\partial}{\partial \Phi} \left[ (p_{c,B}^*)^2 \right] = -(\theta p^2) + (1 - \theta p)^2 < 0 \text{ for } \theta p \geq \frac{1}{2} \quad (A5.1)$$

But by (A2.1) $\Phi_B^* \leq \Phi_M^*$; therefore $\frac{(p_{c,B}^*)^2}{\Phi_B^*} \geq \frac{(p_{c,M}^*)^2}{\Phi_M^*} \Rightarrow L_B \leq L_M$.

Finally, note that $L_B = L + \overline{D}(1 + p_{c,B}^*) < L_M = L + \overline{D}(1 + p_{c,M}^*)$ by $(\Phi_B^* \leq \Phi_M^*)$.

With regard to the second part, if we see the graphic, we have to prove:

If $\Phi_B^*[L_B - L_B] - \Phi_M^*[L_M - L_M] - [L_M - L_B] = 0$

By (A4.12) and (A4.13), we can assure:

$\Phi_B^*[L_F - L_F] = \frac{D}{1 - \theta p} [(p_{c,B}^*)_2 - \theta (1 - \theta p)] \Phi_B^* = \frac{D}{1 - \theta p} [(p_{c,B}^*)_2 - (1 + p_{c,B}^*)(1 - \theta p)] \Phi_B^* = \frac{D}{1 - \theta p} [(1 + p_{c,B}^*)(p_{c,B}^* - \theta p)] = \frac{D}{1 - \theta p} [(\theta p + (1 - \theta p)] (A5.2)$

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42 Note that the dependence with the type of financier, is made through the value of $\Phi^*$, which is lower in the bank-financing's case than under the market-financing one ($\Phi_B^* < \Phi_M^*$).

43 With $p_{c,B}^* \equiv p_c[\Phi_B^*]$ and $p_{c,M}^* \equiv p_c[\Phi_M^*]$. 


Therefore by (A5.2) and (A4.12), we obtain:
\[ \Phi_B^*[\bar{L}_B - L_B] - \Phi_M^*[\bar{L}_M - L_M] - [L_M - L_B] = \overline{D}[p_{c,M}^* - p_{c,B}^* - (p_{c,M}^* - p_{c,B}^*)] = 0 \]

We have to prove that \( \bar{L}_F = L + \frac{\overline{D}}{1-\theta_p} \left( \frac{(\tilde{\phi})^2}{\tilde{\phi}_0^*} \right) \), which is the threshold value that defines the switching from LT scenario to ST, is increasing in \( \eta \).

To prove \( \frac{\partial \bar{L}_M}{\partial \eta} > 0 \), we have to note:
1. \( \overline{D} \equiv 2\eta N_1^* \) is positively correlated with \( \eta \)
2. By (A5.1), \( \frac{\partial}{\partial \eta} \{\frac{(\tilde{\phi})^2}{\tilde{\phi}_0^*} \} < 0 \)
3. By Lemma 2, \( \frac{\partial \phi^*}{\partial \eta} < 0 \Rightarrow \frac{\partial}{\partial \eta} \{\frac{(\tilde{\phi})^2}{\tilde{\phi}_0^*} \} > 0 \) (A6.1)

To prove Proposition 4, we have to check if \( \frac{\partial \bar{L}_M}{\partial S} < 0 \)

We first prove \( \frac{\partial N_1}{\partial S} < 0 \) and \( \frac{\partial N_2}{\partial S} > 0 \)

\[ \frac{\partial N_1}{\partial S} = \frac{\partial N_1}{\partial X_1} = \frac{1}{2\theta_p(1+\sigma^2)} \frac{\partial}{\partial X_1} \left( \frac{\tilde{w}_1}{\tilde{X}_1} \right) < 0 \text{ if } \tilde{w}_1 \equiv \frac{w}{\theta_p X_1} < \frac{a}{2} \]

Similarly:

\[ \frac{\partial N_2}{\partial S} = -\frac{\partial N_2}{\partial X_2} = -\frac{1}{2\theta(1+\sigma^2)} \frac{\partial}{\partial X_2} \left( \frac{\tilde{w}_2}{\tilde{X}_2} \right) > 0 \text{ if } \tilde{w}_2 \equiv \frac{w}{\theta X_2} < \frac{a}{2} \]

At this stage, to prove \( \frac{\partial \bar{L}_M}{\partial S} = \frac{\partial}{\partial S} \{L + \frac{\overline{D}}{1-\theta_p} \left( \frac{(\tilde{\phi})^2}{\tilde{\phi}_0^*} \right) \} < 0 \), is rather straightforward:
1. \( \frac{\partial \Phi^*}{\partial S} = 0 \), because \( \Delta[\tilde{\phi}] \) is independent on \( S \) (see expression 7).
2. \( \frac{\partial \bar{L}_M}{\partial S} < 0 \) by \( \frac{\partial N_1}{\partial S} < 0 \)
3. \( \frac{\partial \bar{L}_M}{\partial S} = \frac{\partial}{\partial S} \left( \theta - \frac{1}{2} \right) X_2 N_2^* \tilde{w}_2 = (\theta - \frac{1}{2}) \frac{\partial}{\partial S} \{X_2 N_2^* (a - \tilde{w}_2)\} < 0 \)

Similar argument, also assures \( \frac{\partial \bar{L}_M}{\partial S} < 0 \)

\[ ^{44} \text{See footnote 15 for this assumption, which is equivalent to consider that } X_i \text{ is high enough.} \]


