AN EXTENSION OF A MODEL OF FINANCIAL CONTRACTS AND LABOR CONTRACTS

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Keywords: Labor Contracts, Financial Contracts, Debt, Equity.

JEL Classification: D21, D23, G20, J41.

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Using an article by Garvey and Swan (GS) 1992 as a benchmark, we extend their model to deal with the issue of the optimal financial structure for a firm when the interaction between labor and financial contracts is considered. The GS article concludes that debt financing is Pareto superior to equity financing. We show that once we introduce a model, with more “complete” contracts, and some dynamic features, their results are no longer valid.

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1. INTRODUCTION

There are quite a few articles that deal with the issue of the optimal financial structure for a firm when considering the relationship between financial contracts and labor contracts. Among them, the pioneering paper by Garvey & Swan (GS) 1992 is a particularly remarkable article by its simplicity and the clarity of their results. The main objective of this note is to extend their model to account for more dynamic effects, in order to test the soundness of their conclusions. More specifically, in their original article GS show, using a one-period model where cooperation between workers plays an important role and where labor contracts are incomplete, that when firms are mainly debt financed (mentioned as the normal financial source in the Japanese corporations), workers become less selfish in comparison to the case where they work for equity-financed companies (which is the most common scenario in the US system). This fact allows the authors to conclude that under their one-shot model, debt-financed firms are Pareto superior with respect to equity-financed corporations.

Their argument makes use of a reward scheme for the workers that combines payoffs, assigned through a tournament process on an ex-ante basis, and a bonus, established by the manager after the workers have made their effort.
Under this scenario, and assuming a manager's utility function that weighs shareholders', bondholders' and workers' interest, the authors show that the manager's incentive to bonus helping behavior increases as debtholders' interests become prominent to shareholders' ones, whenever workers' interests is considered by the manager.

We show that GS's main result concerning debt financing superiority over equity financing to stimulate workers to cooperate cannot be maintained, once labor contracts are made more complete. More specifically, we prove that in situations where the workers' power is not negligible, the first-best solution can be achieved through a combination of financing that weighs equity more heavily than debt. This result, which is opposite to GS, is relevant because fits better a dynamic setting where possible longer manager-workers relationships tend to make contracts more complete. To model this fact, we extend the GS model to two periods, and we consider the workers are hired with a two-period labor contract. This longer relationship brings up implicit agreements between the agents to reduce uncertainty. In particular, we assume that workers know at the end of the first period how their second-period helping efforts will be paid. This measure will undoubtedly result in upgrading the completeness of the contract, leading the workers to reinforce their cooperative behavior, in spite of the existence
of some degree of contract incompleteness, à la GS, as the manager determines ex-post first-period bonus.

Beyond the GS article, in a framework with more complete contracts, the degree of workers' power becomes the critical parameter to define the superiority of one type of financing over the other. An exclusive equity financing scheme is the first-best solution when worker power is high. As the worker influence in the management of the firm diminishes, it is optimal for the firm to obtain an increasing share of the funds through debt, until debtholders' power becomes equal to shareholders'. Beyond this situation, for lower workers' power values, the optimal financial structure cannot lead to the first-best solution. Even, in the extreme situation, when the workers' interests are not considered by the manager, the classical Modigliani-Miller result is recovered, stating the financial irrelevance of firm's policy.

We have organized this note in four sections. The second section describes the GS model and their main results, while the third describes an alternative model. We finish with some concluding remarks.
2. GARVEY AND SWAN MODEL

Description of GS Model

1/ Two risk-neutral identical workers \((i = 1,2)\) with a null reservation utility, hired in a competitive labor market. These agents make two types of effort. An individual one, \(a_i\), and a helping one, \(h_i\), which is attributed to non-\(i\) worker.

2/ An opportunity cost of effort described by a function \(C(a_i, h_i)\) identical for both workers and that satisfies: \(\frac{\partial C}{\partial a_i} > 0, \frac{\partial C}{\partial h_i} > 0, \frac{\partial^2 C}{\partial a_i^2} = 0, \frac{\partial^2 C}{\partial a_i h_i} > 0, \frac{\partial^2 C}{\partial h_i^2} > 0\)

3/ Workers' performance measure is stochastically conditioned by a random variable \(\phi\) with a distribution function \(F\) and a density function \(f\) which is symmetrically and unimodal. Specifically; worker 1's performance = \(a_1 + h_2 - \frac{\phi}{2}\) and worker 2's = \(a_2 + h_1 + \frac{\phi}{2}\)

4/ Total worker output is given by \(Q = a_1 + a_2 + h_1 + h_2\).

5/ Total output of the two-worker team \((Q)\) is observed by the manager, but in court \(Q + \gamma\) is only verifiable. \((\gamma\) is a random variable defined over an interval \((\gamma, \bar{\gamma})\) with a density function \(g\) and a distribution \(G\)).

6/ Workers' reward scheme consists of two parts: The first one is set prior to workers' participation, based on a tournament scheme to provide incentives to individual efforts. It pays \(p\) to the best-performance worker and \(w\) to the worst.
An additional compensation may be offered after workers have made their efforts. This enables the manager to reward each worker with an amount $\delta$ in order to motivate helping efforts $h_i^1$.

The Time-Line of the Game:

1/ Founder sets corporate governance and workers’ reward scheme.

2/ Investors and workers choose whether or not to participate.

3/ Workers can choose direct effort and/or helping effort.

4/ Manager observes performance and chooses $\delta$.

5/ Workers receive payments $\{p, w\}$

3. GS RESULTS

The authors consider a general situation where the firm can be financed through debt and/or equity, and workers can have a voice on the board. They model this framework, making use of a manager’s utility function that weighs shareholders’, debtholders’ and workers’ interests.
Specifically, the problem to be solved by the manager is:

\[
\text{Max}_{\{\delta\}} U^{\text{Mger}} \equiv \alpha U^{\text{Sholders}} + \beta U^{\text{Dholder}} + 2(1 - \alpha - \beta)U^{\text{Worker}}
\]

\[
U^{\text{Sholder}} = \int_{\gamma}^{\gamma'} (Q - D - w - p - 2\delta + \gamma)dG(\gamma)
\]

\[
U^{\text{Dholder}} = \int_{\gamma}^{\gamma'} (Q - w - p - 2\delta + \gamma)dG(\gamma) + D(1 - G(\gamma))
\]  

(1)

\[
U^{\text{Worker}} = pF(\epsilon_{1,2}) + w(1 - F(\epsilon_{1,2})) - C(a_1, h_1) + \delta
\]

s.t. \(\epsilon_{1,2} = a_1 + h_2 - a_2 - h_1\) \(\tilde{\gamma} = D - Q + w + p + 2\delta\)

Where \(\tilde{\gamma}\) is the critical level of \(\gamma\) that defines the default zone \(\gamma < \tilde{\gamma}\). \(U^{\text{Worker}}\) is the utility function for worker 1, that can be taken as the representative worker by the symmetry of the problem. Its expression is obtained taking into account that worker 1 wins the tournament premium, \(p\), if \(a_1 + h_2 - \frac{\phi}{2} > a_2 + h_1 + \frac{\phi}{2}\) \(\Leftrightarrow \phi < a_1 + h_2 - a_2 - h_1\).

Bonus \(\delta\) value is determined by making use of the following derivative:

\[
\frac{\partial U^{\text{Mger}}}{\partial \delta} = -2(\alpha(1 - G(\tilde{\gamma})) + \beta G(\tilde{\gamma})) + 2(1 - \alpha - \beta)
\]  

(2)

To sign this expression, the authors distinguish different financing situations:

- **Equity financing**

In this case \(\{\beta, \tilde{\gamma}\} = 0\), and (2) becomes \(\frac{\partial U^{\text{Mger}}}{\partial \delta} = 2(1 - 2\alpha)\), which is a negative expression under the normal assumption of giving workers no absolute control over the executive board \((\alpha > \frac{1}{2})\). This fact leads to an optimal \(\delta^* = 0\), and
moving backwards to the workers' maximization problem, to a null helping effort. It is important to note that this result relies critically on the fact that this is a one-period problem and workers have no bargaining power over the manager, because they cannot punish low manager's reward, $\delta$, through the implementation of low effort levels in future periods. Thus, workers do not cooperate among themselves. Furthermore, this behavior is maintained independently of their presence on the board ($1 - \alpha$). This is rather remarkable because one of the justifications for allowing workers to have a voice on the executive board is precisely to try to stimulate their specific investments (cooperative effort). The more they cooperate the higher their bargaining power, leading eventually to higher rewards. This intuitive result is recovered in the two-period model of the next section. In that case, workers have a real voice on the board $^2$, because of their ability to retaliate in future periods with measures such as strikes (low effort implementation).
Debt and Equity Financing

This situation is modeled considering a $\beta > \alpha$ and $\alpha < \frac{1}{2}$. In that case an interior solution $\delta^* > 0$ follows, and this generates a positive helping effort $h^* > 0$. Although in a one-period framework, workers now enjoy a certain bargaining power because they can punish the manager through changes in the threshold level, $\gamma$, that defines the default zone. Once they made a low effort $h$, there will be an increase of $\gamma$ which generates a higher default probability that will end up penalizing the debtholders who are the most important individuals for the manager (remember, $\beta > \alpha$). Therefore, the manager will try to diminish these perverse effects, linking the discretionary reward $\delta$ to the production level $Q^3$. This fact will ultimately lead the workers to implement a positive helping effort.

GS show that the higher workers’ willingness to implement helping efforts makes this financing situation Pareto superior to the previous one.
4. ALTERNATIVE GAME

Now we extend the GS model to study the soundness of their results. Specifically, we review the following points:

1/ The linkage between financial structure and workers' cooperative behavior.

2/ The irrelevance of the workers' presence on the board for the exclusive equity financing case unless they enjoy absolute control.

3/ The Pareto superior condition of the debt-financing approach with regard to the equity-financing approach.

To develop our analysis, we consider a game that intends to be the natural extension of the GS game for a two-period case. In particular, the first period is identical to the GS model, the second period is different in two points. First, investors do not receive returns at the end of the first period, just when the game finishes (long-term financing). Secondly, workers' bonuses $\delta + \delta'$, are paid at the end of the game, but, their structure is fixed at the end of the initial period. Furthermore, we also assume that both bonus have the same structure, that is, if the manager decides a first-period bonus $\delta = \lambda Q$, then, $\delta' = \lambda Q'$ being $Q$ ($Q'$) first-period (second-period) output. This assumption, that makes labor
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can contract more complete, is in some sense a natural complement to the increase in the number of periods of the game. To motivate this, we have to consider each workers' helping effort as an individual commitment action favoring team results. This is because there is a misattribution between team members of their particular helping effort. Within this scheme, the entrepreneur will be interested in signaling the commitment of the firm with regard to their workers in order to reduce the uncertainty of a longer (two-period) relationship. A natural way to do so, is defining the reward scheme of the labor contract as ex-ante as possible.

Another question is to settle \( \lambda' = \lambda \). This is made for simplicity in the computation. Beyond this, we may also justify this assumption if we think that defining a certain structure of contract is complex enough that it would be optimal to maintain the same pattern for at least two periods. In fact, this is what is seen in the real world, where firms do not change the percentage of revenues devoted to bonus the workforce very frequently.

Finally for tractability, we particularize, to a second-order polynomial effort cost function \( C(a_i, h_i) = \frac{1}{2}(a_i^2 + h_i^2) \) which satisfies the required derivative conditions.
Timing of the Game:

1/ 2/ 3/ 4/ 5/ 6/ 7/

The first five stages are as in game 1.

6/ Workers after observing λ and the previously defined \( p \) and \( w \), choose second-period individual and helping efforts \((a'_i, h'_i)\).

7/ Stochastic variable \( \gamma \) is realized and workers and investors receive returns contingent on the overall output \( Q + Q' \).

5. SOLVING THE GAME

5.1. Equity Financing with Workers' Voice on the Board

We model this situation as GS, that is, we assign a weight \( \alpha \) and \( 1 - \alpha \) in the manager's utility function to account for shareholder's and workers' power on the board. The equilibrium is computed straightforwardly using backward induction. The result is given in the following proposition:
PROPOSITION 1

The first-best solution when firms are financed exclusively by equity can be obtained when the workers’ power is equal to one-half. For lower values of the workers’ power \(1 - \alpha\), the equilibrium of the game, is given by:

\[
a_1^* = a_2^* = h_1^* = h_2^* = \frac{\alpha(3\alpha-1)}{89\alpha^2 - 78\alpha + 17} \quad \lambda^* = \frac{\alpha(11\alpha-5)}{89\alpha^2 - 78\alpha + 17}
\]

\[
p^* = w^* < 0 \quad a_1'' = a_2'' = h_1'' = h_2'' = \lambda^*
\]

Proof

See point one in the Appendix.

We have characterized an equilibrium result with positive helping effort, under an equity-financing scheme, which is in contrast to the GS model. Moreover, the symmetry of the effort cost function considered leads the helping effort to be linked with the individual one in a way which makes it nonsensical to talk about selfish behavior in effort implementation. Consequently, the tournament scheme looses its meaning \((p^* - w^* = 0)\), and the rewards are shared symmetrically by both workers as a team. Note the importance of considering a two-period scheme, workers in this situation enjoy bargaining power to punish a possible manager’s low reward through their low second-period efforts. Under this dynamic it makes sense to obtain workers who are bonded with the firm \(\{p^*, w^*\} < 0\). In fact,
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this is a common feature in models where workers have to invest in firm-specific human capital (helping effort in our terminology)\(^6\). Moreover, this payment is a necessary condition to set high workers' reward (high \(\lambda\)) which, in the end, will induce workers to implement high efforts, even as high as the social optimum \((a = h = 1)\). This is the case, whenever the workers' power is equal to the shareholders'. This is in contrast to GS's, where the incompleteness of labor contracts leads not only to a superiority of debt financing, but makes the first-best solution unreachable.

Observing the equilibrium, we also note that first-period efforts are lower than the second-period ones. The logic behind this result is the following: in the first period workers choose low efforts, otherwise it might be optimal for the manager to define a take-the-money-and-run-strategy by setting \(\lambda = 0\) and giving up second-period production \(Q'\). The manager, in so doing, reacts by establishing a high \(\lambda\) to stimulate high second-period efforts, (in the Appendix we show that \(\frac{\partial \lambda}{\partial (a)} = \frac{\partial \lambda}{\partial (h)} < 0\), and that \(\frac{\partial a'}{\partial \lambda} = \frac{\partial h'}{\partial \lambda} = 1\).

Regarding the impact of workers' power \((1 - \alpha)\) in the equilibrium, we find, a negative dependence between \(\lambda\) and \(\alpha\). This fits with the intuitive linkage between the workers' bonus settled by the manager (proportional to \(\lambda\)) and the workers' weight in the manager compensation scheme \((1 - \alpha)\). What is relevant is that this
negative correlation does not require the workers to have absolute control of the board. Consistent with this fact, we observe an increase in the effort implemented by the workers in both periods as $1 - \alpha$ increases. Finally, it is also remarkable that the participation of the workers in firm's management tends to equilibrate the efforts they provide in each period $(\frac{d}{da} (d' + h') > 0)$, and with this, each period production ($Q \to Q'$). This is quite important for the firm, to maintain a balanced production pattern.

We can conclude by saying that, in this equity-financing scheme, workers’ presence in the management of the firm (given the reward scheme proposed), generates benefits in terms of the firm's value. In the extreme situation where shareholders and workers share equal responsibility ($\alpha = \frac{1}{2}$), the social optimum value for the firm can be obtained.
5.2. Debt and Equity Financing

In this framework, we are interested to characterize the optimal combination of debt and equity through which a firm can reach its first-best value. We show that this combination critically depends on the workers' power, \( \omega = 1 - \alpha - \beta \). The specific result is given in the following proposition.

**PROPOSITION 2**

When firms can be financed with debt and equity, the first-best solution can be reached once the workers' power, \( \omega \), is between \( \frac{1}{3} \) and \( \frac{1}{2} \). Within this dominion, shareholders' power has to be superior to debtholders', in particular, \( \alpha = \frac{1}{2} - \frac{3}{4} \omega + \left( \frac{1}{2} - \frac{3}{4} \omega \right)^2 - \frac{1}{2} (1 - 3 \omega + \omega^2) \)^{1/2} \geq \beta \equiv 1 - \alpha - \omega \), being \( \beta = 0 \) for \( \omega = \frac{1}{2} \), and \( \beta = \frac{1}{3} \) for \( \omega = \frac{1}{3} \). For \( \omega \) values lower than \( \frac{1}{3} \), it is not possible to implement the first-best solution by varying the financial structure of the firm, and even this structure becomes irrelevant under the extreme situation of null worker power.

**Proof**

See point 2 in the Appendix.

We show that once labor contracts become more complete, the workers' power turns out to be the key parameter that determines the type of financing providing
a higher firm value. This parameter is so nodal within this alternative model that only within a range of positive \( \omega \) values, the first-best solution can be defined. If we draw a graphical representation of the relationship between \( \alpha, \beta \) and \( \omega \), that lead to the first-best solution, we get the following picture:

We see that in the first-best dominium, any decrease in the workers' power has to be balanced with a reduction in shareholders' power and a simultaneous increase in debtholders'. Using this strategy, the managerial's disincentives to reward workers with decreasing power are exactly offset by the incentives linked to a rise in debtholders' power with regard to shareholders' \(^7\). The result is a maintenance of workers' social optimum efforts \((a = h = 1)\). This dynamic works up to the point \( \omega = \frac{1}{3} \). For lower values of \( \omega \), the formal condition that lead to
the first-best outcome implies a lower shareholders' power with regard to workers', but, we neglect this scenario as we do not consider worker-managed firms.

This exposition clarifies the importance of allowing workers to participate in firm's decisions. In so doing, they increase their power and the financial structure of the firm turns out to be relevant in generating profitability.

6. DISCUSSION AND CONCLUSIONS

The main goal of this note has been to extend some of the results obtained by Garvey and Swan (1992) in a model where there is an interaction between financial and labor contracts. We have shown that their conclusions critically depend on the particular framework they consider, specifically the degree of labor contract incompleteness.

The basic result of their article is that debt financing is Pareto-superior to equity financing, because they induce, under a certain type of contracts, a more cooperative behavior (in fact a non-null cooperative behavior). This occurs independently of the presence of workers on the executive board.

In an alternative game, which extends author's one-shot model to allow for a longer workers-manager relationship, we reach the conclusion that workers' behavior in both financing scenarios it is a non-null cooperative. We have argued this
result is connected with the rise in the completeness of the labor contracts. This is so because a more complete contract signals a commitment from the entrepreneur to the workers who transfer their commitment to the team by implementing a high cooperative effort. Additionally, under a long-term (two-period) relationship workers enjoy real power, and even accept to pay an amount in the first period to enter the firm, as they can retaliate in future periods against arbitrary actions implemented by the manager.

Our study shows that the financial structure is an ingredient of the corporate governance of the firm as a balance of power among its agents. We have described the social optimum firm’s value as the outcome to combine a minimum workers’ power and a financial structure that weighs equity more heavily than debt. Shareholders’ power is a mechanism to counterweigh high workers’ power. On the contrary, debtholders’ power leans workers’ interests against shareholders’ profits. Finally, with no agent to counterbalance (no workers’ power), the financial structure of the firm does not generate value on its own.

To conclude we can say that the in general, it is not a question of Pareto superiority of one type of financial contract over the other. The issue becomes one of jointly determining the optimal combination of labor and financial contracts as a unified definition of the corporate governance of the firm. In this sense, the
message of this paper is to emphasize the problem of talking about Pareto superiority of debt financial contracts over equity contracts when this comparison is absolutely conditioned by the labor contract considered. In the spirit of this analysis, if we see the firm as a nexus of contracts, it would seem natural to study at a deeper level the relationship between these contracts. On the basis of this, we can make a fair comparison between the so-called US paradigm (equity-financed firms with short-term labor relationships and payoffs fixed in a tournament basis) and the Japanese paradigm, based in debt-financed enterprises that define long-term labor relationships with their workers in order to stimulate cooperative behavior. This is left for future research.
FOOTNOTES

1 Note that worker 1’s performance also depends on the helping effort made by worker 2. This extreme assumption, later relaxed in the model, would lead workers to implement a null helping effort if the reward scheme were only based on a tournament proposal. To stimulate a positive $h$, part of the workers’ wage must be linked to the overall output $Q$; which justifies $\delta$

2 Even if they are not in majority.

3 Which is the standard reward scheme to minimize the agency costs.

4 See point 3/ of the GS model description.

5 Although to account for the feature of the contract incompleteness, we follow GS by not allowing the overall labor reward scheme to be defined ex-ante.


7 See the explanation given in the debt and equity financing part of GS model.
APPENDIX

We are going to compute the equilibrium of the game for both financing scenarios using the same formal framework. To solve the game, we use a backward induction approach:

**Fifth Stage (Workers Problem in the Second Period)**

At this stage the maximization problem is identical to GS, and for worker 1, as the representative worker, it is given by:

\[
Max \{a'_1, h'_1\} U^\text{worker}_I = pF(\epsilon_{1,2}) + w(1 - F(\epsilon_{1,2})) + \lambda Q' - c(a'_1, h'_1) \quad (A1.1)
\]

Where \(\epsilon_{1,2} = a'_1 + h'_2 - a'_2 - h'_1\) and \(Q' = a'_1 + h'_2 + a'_2 + h'_1\)

Particularizing for \(c(a'_1, h'_1) = \frac{1}{2}[(a'_1)^2 + (h'_1)^2]\)

The FOC over \(a'_1\) and \(h'_1\) lead respectively to:

\[
a'_1 = \lambda + \Delta f(0) \quad (A1.2)
\]
\[
h'_1 = \lambda - \Delta f(0)
\]

By the symmetry of the equilibrium \(a'_1 = a'_2 \equiv a' \quad h'_1 = h'_2 \equiv h' \quad \epsilon'_{1,2} = \epsilon'_{2,1} = 0\)

It is clear to see that the SOC are also satisfied.
Fourth Stage (The Manager Problem)

As we have commented in the text, the manager's objective function is given by:

$$\max_{\{\lambda\}} U^M_{\text{G}er} = \alpha U_{\text{Sh}older} + \beta U^D_{\text{holder}} + (1 - \alpha - \beta)(2U_{\text{W}or\_ker}^1 + 2U_{\text{W}or\_ker}^2)$$

$$U_{\text{Sh}older} = \int_{\gamma} \left[ (Q + Q')(1 - 2\lambda) - 2(w + p) - D + \gamma \right] dG(\gamma)$$

$$U^D_{\text{holder}} = D(1 - G(\hat{\gamma})) + \int_{\gamma} \left[ (Q + Q')(1 - 2\lambda) - 2(w + p) + \gamma \right] dG(\gamma)$$

$$U_{\text{W}or\_ker}^1 = pF(\epsilon_{1,2}) + w(1 - F(\epsilon_{1,2})) + \lambda Q - c(a_1, h_1)$$

$$U_{\text{W}or\_ker}^2 = pF'(\epsilon'^{1,2}) + w(1 - F'(\epsilon'^{1,2})) + \lambda Q' - c(a'^1, h'^1)$$

$$\hat{\gamma} = D + 2(w + p) - (1 - 2\lambda)(Q + Q')$$

Where $U_{\text{W}or\_ker}^1, U_{\text{W}or\_ker}^2$ denotes first (second)-period worker 1 utility function.

Making use of (A1.2), the FOC becomes:

$$\alpha [1 - G(\gamma)]\left[ \frac{\partial Q'}{\partial \lambda}(1 - 2\lambda) - 2(Q' + Q) \right] + \beta [G(\gamma)]\left[ \frac{\partial Q'}{\partial \lambda}(1 - 2\lambda) - 2(Q' + Q) \right] +$$

$$+ 2(1 - \alpha - \beta)(Q + Q' + \lambda \frac{\partial Q'}{\partial \lambda} - 2\lambda) = 0$$

Applying $Q' = 4\lambda$ the previous expression adopts a more compact form:

$$0 = [\alpha(1 - G(\gamma)) + \beta G(\gamma)(1 - \alpha - \beta)]4(1 - 4\lambda) - 2Q + 4(1 - \alpha - \beta)(1 - \lambda) \quad (A1.4)$$

At this stage, as we consider an exclusive equity-financing scenario, we have to fix $\beta = 0 \Rightarrow \hat{\gamma} = \gamma$ and (A1.4) transforms to:
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\[ 0 = (2\alpha - 1)[4(1 - 4\lambda) - 2Q] + 4(1 - \alpha)(1 - \lambda) \Rightarrow \lambda = \frac{\alpha}{7\alpha - 3} - \frac{Q(2\alpha - 1)}{2(7\alpha - 3)} \quad (A1.4') \]

Regarding the SOC, it is straightforward to see from (A1.2) and (A1.4) that for \( \alpha \geq \frac{1}{2} \Rightarrow \frac{\partial^2 U}{\partial \lambda^2} < 0 \)

It is also direct to see \( \frac{\partial \lambda}{\partial a_1} = \frac{\partial \lambda}{\partial h_1} = -\frac{Q(2\alpha - 1)}{2(7\alpha - 3)} \quad (A1.4'') \) which implies precisely that low workers’ first-period efforts leads the manager to increase the reward in order to encourage second-period efforts.

**Third Stage (Workers Problem in the First Period)**

Using symmetry we can focus on worker’s 1 maximization problem:

\[
\begin{align*}
\max_{(a_1, h_1)} & U_{1t}^{Worker} + U_{1t}^{Worker} \\
\text{s.t.} & (A1.4)
\end{align*}
\]

**FOC** over \( a_1 \) and \( h_1 \) leads to:

\[
\begin{align*}
\frac{\partial}{\partial a_1} \left\{ U_{1t}^{Worker} + U_{1t}^{Worker} \right\} = 0 \Rightarrow a &= \lambda + \Delta f(0) + (Q + Q') \frac{\partial \lambda}{\partial a_1} \quad (A1.5') \\
\frac{\partial}{\partial h_1} \left\{ U_{1t}^{Worker} + U_{1t}^{Worker} \right\} = 0 \Rightarrow h &= \lambda - \Delta f(0) + (Q + Q') \frac{\partial \lambda}{\partial h_1} \quad (A1.5'')
\end{align*}
\]

Where we have applied symmetry \( a_1 = a_2 \equiv a \quad h_1 = h_2 \equiv h \quad \epsilon_{1,2} = \epsilon_{2,1} = 0 \)

Adding the above expressions and using (A1.4''), we get

\[
a + h = 2\lambda \frac{3\alpha - 1}{11\alpha - 5} \]

which combined with (A1.4) defines the following values:

\[
\begin{align*}
\lambda &= \frac{\alpha(11\alpha - 5)}{89\alpha^2 - 78\alpha + 17} \\
a + h &= \frac{2\alpha(3\alpha - 1)}{89\alpha^2 - 78\alpha + 17} \quad (A1.6)
\end{align*}
\]
Making use of expressions (A1.4''), (A1.6) and anticipating the result of the next stage \( \Delta = 0 \), we can ensure the SOC are satisfied:

\[
\frac{\partial^3 U}{\partial (a,h)^3} = \frac{\partial \lambda}{\partial (a,h)} (-2 + 4 \frac{\partial \lambda}{\partial (a,h)}) + \frac{\partial^2 \lambda}{\partial (a,h)^2} (Q + Q') - 1 < 0 \text{ for } \alpha \geq \frac{1}{2}.
\]

**First and Second Stage (The Entrepreneur Problem)**

The Entrepreneur's maximization problem is given by:

\[
\text{Max}_{\{\Delta, p\}} U^{\text{Foun}}
\]

\[
U^{\text{Foun}} = \int_{\gamma}((Q + Q')(1 - 2\lambda) - 2(w + p) + \gamma)dG(\gamma) \quad (A1.7)
\]

s.t. \( p + w + \lambda(Q + Q') = c(a, h) + c(a', h') \)

Besides, workers' participation constraint is obtained making use of a zero reservation utility, and the fact that \( F(\varepsilon) \) is unimodal and symmetric around zero \((F(\varepsilon_{1,2}) = F(\varepsilon'_{1,2}) = F(0) = \frac{1}{2})\).

If we introduce the restriction into the objective function we can obtain:

\[
U^{\text{Foun}} = \int_{\gamma} \left\{ (Q + Q') - 2[c(a, h) + c(a', h')] + \gamma \right\}dG(\gamma) \quad (A1.7')
\]

The necessary conditions over \( \Delta \) lead to: (\( p \) will be computed using the participation constraint of the workers)

\[
\frac{\partial U^{\text{Foun}}}{\partial \Delta} \equiv 0 \Rightarrow 0 = \frac{\partial a}{\partial \Delta}(1 - a) + \frac{\partial h}{\partial \Delta}(1 - h) + \frac{\partial a'}{\partial \Delta}(1 - a') + \frac{\partial h'}{\partial \Delta}(1 - h') \quad (A1.7'')
\]
Using (A1.2) and (A1.5') we see \( \frac{\partial \kappa}{\partial \Delta} = \frac{\partial \kappa'}{\partial \Delta} = f(0) \) and \( \frac{\partial \phi}{\partial \Delta} = \frac{\partial \phi'}{\partial \Delta} = -f(0) \)

which transforms expression (A1.7") to: \(-4(\Delta f(0)^2) = 0 \Rightarrow \Delta = 0\) (A1.8)

The SOC are trivially satisfied

Using (A1.8) and (A1.2) we get \( a' = h' = \lambda \)

On the other hand by (A1.8), expressions (A1.5') and (A1.5") ensures that \( a = h \)

Finally if we introduce (A1.8) in the participation constraint of the workers (A1.7), we obtain \( p = -\frac{1}{2}\{3(a')^2 + a(4a' - a)\} < 0. \)

If we put it all together we recover the equilibrium given in Proposition 1

\[ \lambda^* = \frac{a(11a-5)}{8a^2-78a+17}, \quad a_1^* = a_2^* = h_1^* = h_2^* = \frac{a(3a-1)}{8a^2-78a+17} \quad (A1.9) \]

\[ p^* = w^* < 0 \quad a_1'' = a_2'' = h_1'' = h_2'' = \lambda^* \]

First, we are going to show when the workers' power is null, then, financial structure becomes irrelevant. To do so, we focus on the expression (A1.4), and fix \( 1 - \alpha - \beta = 0. \) In that case, we get the following FOC for \( \lambda: \)

\[ 0 = (\alpha + G(\gamma)(1 - 2\alpha))(\frac{\partial Q'}{\partial \Delta})(1 - 2\lambda) - 2(Q' + Q) \]

As the first factor \( \alpha + G(\gamma)(1 - 2\alpha) \neq 0 ; \) when \( 0 \leq \alpha \leq 1, \) then:

\[ (\frac{\partial Q'}{\partial \Delta})(1 - 2\lambda) - 2(Q' + Q) = 0 \Rightarrow \lambda = \frac{1}{4} - \frac{1}{8}Q \quad (A2.1) \]

The point is that this \( \lambda \) expression can be obtained from (A1.4'), when \( \alpha = 1. \)
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Using (A1.2) and (A1.5') we see \( \frac{\partial g}{\partial \Delta} = \frac{\partial a}{\partial \Delta} = f(0) \) and \( \frac{\partial h}{\partial \Delta} = \frac{\partial h'}{\partial \Delta} = -f(0) \)

which transforms expression (A1.7') to:

\[ -4(\Delta (f(0)^2) = 0 \implies \Delta = 0 \] (A1.8)

The SOC are trivially satisfied

Using (A1.8) and (A1.2) we get \( a' = h' = \lambda \)

On the other hand by (A1.8), expressions (A1.5') and (A1.5'') ensures that \( a = h \)

Finally if we introduce (A1.8) in the participation constraint of the workers (A1.7), we obtain:

\[ p = -\frac{1}{2} \{3(a')^2 + a(4a' - a)\} < 0. \]

If we put it all together we recover the equilibrium given in Proposition 1

\[ \lambda^* = \frac{\alpha(11\alpha - 5)}{89\alpha^2 - 78\alpha + 17} \quad a^*_1 = a^*_2 = h^*_1 = h^*_2 = \frac{\alpha(3\alpha - 1)}{89\alpha^2 - 78\alpha + 17} \] (A1.9)

First, we are going to show when the workers' power is null, then, financial structure becomes irrelevant. To do so, we focus on the expression (A1.4), and fix \( 1 - \alpha - \beta = 0 \). In that case, we get the following FOC for \( \lambda \):

\[ 0 = (\alpha + G(\gamma)(1 - 2\alpha))(\frac{\partial Q'}{\partial \lambda})(1 - 2\lambda) - 2(Q' + Q)) \]

As the first factor \( \alpha + G(\gamma)(1 - 2\alpha) \neq 0 \); when \( 0 \leq \alpha \leq 1 \), then:

\[ (\frac{\partial Q'}{\partial \lambda})(1 - 2\lambda) - 2(Q' + Q) = 0 \implies \lambda = \frac{1}{4} - \frac{1}{8}Q \] (A2.1)

The point is that this \( \lambda \) expression can be obtained from (A1.4'), when \( \alpha = 1 \).
As all the stages of the game to be solved are formally identical to the previous exclusive equity financing scenario, we obtain an equilibrium which is given by (A1.9), with $\alpha = 1$, that is

$$
\begin{align*}
\lambda^* &= \frac{3}{14} \quad a_1^* = a_2^* = h_1^* = h_2^* = \frac{1}{14} \\
p^* &= w^* < 0 \quad a_1^{**} = a_2^{**} = h_1^{**} = h_2^{**} = \lambda^*
\end{align*}
$$

(A2.2)

In this way, we prove the irrelevance of the financial structure (given by $\{\alpha, \beta\}$), whenever the workers’ power $1 - \alpha - \beta = 0$.

Second, to define the combination of debt and equity that lead to the social optimum solution, we use again expression (A1.4), particularizing for $a = a' = h = h' = \lambda = 1$. This is so because under the first-best solution, the marginal cost of effort in each period has to be equal to the marginal revenue, that is: $(a = 1, a' = 1, h = 1, h' = 1)$. Regarding to $\lambda = 1$, this is obtained combining (A1.2), the previous equality $a' = h' = 1$ and the fact that $\Delta = 0$ also works for the founder problem in the first-best framework.

If we arrange in the adequate way the expression (A1.4), we obtain:

$$
\alpha[\omega] = -\frac{(1-\omega)\gamma(\gamma)}{1-2\gamma(\gamma)} \quad (A2.3)
$$

Making use of the workers’ participation constraint we characterize $\hat{\gamma}$:

$$
\hat{\gamma} = D + 2[C(a, h) + C(a', h')] - (Q + Q') \quad (A2.4)
$$
This expression allows to study the SOC for the manager problem that leads to expression (A2.3) in the first-best framework.

We compute \( \frac{\partial^2}{\partial x^2} = 0 \Rightarrow \frac{\partial^2 U_M}{\partial x^2} = -4\omega < 0 \)

Besides, (A2.4) allows also to characterize \( \gamma \) for the first-best solution. We obtain \( \gamma = D - 4 \), therefore, as \( D \) is exogenously given, we can connect in a natural way \( G(\gamma) \) with the debtholders’ power \( \beta \). In that case, expression (A2.3) becomes a second-order polynomial expression: \( \alpha^2 + \alpha(3\omega - 1) + \frac{1}{2}(\omega^2 - 3\omega + 1) = 0 \), where the positive root is:

\[
\alpha^*[\omega] = \frac{1}{2} - \frac{3}{2}\omega + \left[\left(\frac{1}{2} - \frac{3}{2}\omega\right)^2 - \frac{1}{2}(1 - 3\omega + \omega^2)\right]^{1/2} \quad (A2.5)
\]

Therefore, only for \( \omega \geq \omega^* = 2\sqrt{10} - 6 < \frac{1}{3} \), \( \alpha^*[\omega] \) is a real expression.

Finally, as \( \alpha^*[\omega = \left\{\frac{1}{3}, \frac{1}{2}\right\}] = \left\{\frac{1}{3}, \frac{1}{2}\right\} \) and \( \frac{d\alpha^*[\omega]}{d\omega} > 0 \), if we neglect worker-managed firms, that is \( \alpha > \omega \), then, only in the interval \( \frac{1}{3} \leq \omega \leq \frac{1}{2} \) is possible to reach the social optimum solution, where \( \alpha = \alpha^*[\omega] \) and \( \beta = 1 - \alpha^*[\omega] \).
BIBLIOGRAPHY


