On the stability of recurrence relations for Kummer functions

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Abstract

We consider the three term recurrence relations

$$y_{n+1} + a_n y_n + b_n y_{n-1} = 0$$

satisfied simultaneously by confluent hypergeometric functions $M(a+kn; c+mn; x)$ and $U(a+kn; c+mn; x)$ (up to normalizations not depending on x). The parameters a, c, x are fixed and $k, m = 0, \pm 1$. The existence of minimal solutions when $n \to \infty$ is a crucial piece of information when we intend to use a recurrence relation for computation. However, in some cases the behavior of the solutions for moderate values of n can be opposite to the asymptotic behaviour. We provide numerical examples of this phenomenon, already noted by W. Gautschi in the case $(k, m) = (1, 1)$, both for the recurrence relations and for the associated continued fractions.