THE ROLE OF DEMAND DEPOSITS IN RISK SHARING

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Abstract

Based on the work of Hellwig [12], this paper compares the implementation of the second best allocation (non traded solution) by a financial intermediary to the one achieved in a walrasian market in which individuals hold the assets directly (traded solution). In this framework, in which individuals have smooth preferences, the traded and non traded solutions are no longer welfare equivalent; in fact, the non traded solution allows for greater risk sharing than the traded one. This result, and contrary to Hellwig’s work, shows that financial intermediaries do provide for a positive role in the economy.

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Banking, Deposit Contracts, Equity contracts, Risk Sharing
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The role of demand deposits in risk sharing

1.- Introduction

Modern financial intermediation literature has been concerned with explaining the nature of optimal bank liability contracts (deposits) intended to ensure preference shocks and coordination problems derived from the above contracts leading to phenomena such as bank runs and measures to cope with them. Examples of such work are Diamond and Dybvig [9], Bryant [7] and Chari and Jagannathan [8] among others. However, these papers are primarily concerned with modeling bank runs and therefore the role of demand deposits is not fully explored.

Another branch of literature has analyzed the different degrees of risk sharing provided by demand deposits and traded equity contracts. The purpose of these papers is to show whether financial intermediaries have a positive role in the economy, by ensuring liquidity needs or on the contrary, these same services can be provided by other non-financial intermediaries without the risk of bank runs. The papers of Jacklin [13], Jacklin [14], Hellwig [12] and Jacklin and Bhattacharya [4] are examples of such work. As Jacklin [13] has noted, and is confirmed also in the work of Hellwig, the Diamond Dybvig specification with no aggregate uncertainty about preferences, has the feature that the ex-ante optimal allocation is also implementable through trading, where shares of the investment portfolio (of short and long term assets) could be traded at date 1 as with a mutual fund. This rules out any specialness on the side of a financial intermediary. In a later paper, Jacklin [14] shows that unless there is both aggregate uncertainty about preferences and banks assets are risky, with depositors asymmetrically informed about asset quality, then traded equity contracts can provide the same services as demand deposit contracts, without the possibility of panics. The message of his paper is that liquidity transformation can and should be provided using equity contracts where the underlying assets may or not be risky, but where there is little or no potential for asymmetries of information about asset quality. The above papers considered models in which individuals have corner preferences, that are not considered a realistic
characterisation of individuals’ preferences. With smooth preferences and no aggregate uncertainty about preferences, Jacklin [13] has shown that non traded demand deposit contracts and traded equity are not welfare equivalent. In fact, demand deposits are shown to provide greater risk sharing than equity shares. Jacklin and Bhattacharya [4] also considered the relative degree of risk sharing provided by traded and non traded contracts, in a framework in which bank assets are risky and individuals (with smooth preferences) are informed concerning bank asset quality. The basic result is that deposit contracts tend to be better for financing low risk assets.

The motivation of this paper is to compare the traded and non traded solutions in this economy, with smooth preferences and in which there exists a reinvestment opportunity from date 1 to date 2. It will be shown, that in this framework, demand deposits and traded equity are no longer welfare equivalent.

The structure of the paper is as follows: The basic framework of the model is presented in section 2. The first-best and second-best allocations under complete and incomplete information, respectively, are derived in subsections 2.1 and 2.2. Section 3 compares the second-best allocation (non-traded solution) with the allocations that are achieved in a walrasian market in which individuals hold the assets directly. Section 4 concludes the paper.

2.- Description of the model

The hypothesis of the model are summarized as follows:

a.- Three period economy: $T = 0, 1, 2$

b.- One good per period

c.- There are three investment opportunities:

i.- A short-term asset at $T=0$ that yields a sure return $b_{o1}$ at $T=1$

ii.- A long-term asset at $T=0$ that yields a sure return $b_{o2}$ at $T=2$, premature liquidation of the asset is feasible but the rate of return is only $b_1 < b_{o1}$

iii.- A short-term asset at $T=1$ that yields a random return $\tilde{b}_{12}$ at $T=2$. The random variable is known at $T=1$ but not at $T=0$, at date 0, only the distribution function is known$^3$.

d.- On the household side of the economy, there is a continuum of unit mass of ex ante identical consumers that are uncertain at $T=0$ about their consumption needs. They are subject at $T=1$ to a privately observed
uninsurable risk of being of type-1 with probability \( t \) or of type-2 with probability \( 1 - t \).

Preferences will be represented by an additive utility function which is of the form:

\[
U'(C_t, C_{t-1}, \rho_i) = \frac{c_1^{1-\gamma} - \rho_i c_2^{1-\gamma}}{1-\gamma} + \rho_i c_2^{1-\gamma}
\]

where: \( 0 \leq \rho_i \leq 1 \), \( i = 1, 2 \) (type), \( \rho_1 < \rho_2 \) and \( \gamma \) is the CRRA coefficient.

As commented in the introduction, it is assumed a more general preference structure with respect to Hellwig as individuals derive utility from consumption in both periods, with type-1 agents deriving relatively more utility from consumption in the first period with respect to type-2 agents.

Consumers are endowed with \( k_0 \) units of the good at \( T=0 \) to be divided between short-term and long-term investments.

It is assumed no aggregate uncertainty, so that with probability one a fraction \( t \) of consumers are of type-1 and a fraction \( 1 - t \) of type-2.

The economy must deal with the following allocation problem:

- At \( T=0 \) the initial endowment must be divided between short and long term investments \( (k_0 = k_{o1} + k_{o2}) \)
- At \( T=1 \) the fraction \( (0 \leq \mu \leq 1) \) of the long-term investment that is liquidated must be determined, this may depend on the observed value of \( b_{12} \).
- At \( T=1 \) the returns from short-term assets and possibly liquidated long-term investments must be divided between consumption and new short-term investments, this may also depend on the observed realization of \( b_{12} \).

As already mentioned in the introduction, the main objective of this work is to compare the second best allocation or non-traded solution (social optimum constrained by incentive compatibility) with the allocations achieved in a walrasian market or traded solution.

As a benchmark case, the complete information situation is studied first.

### 2.1. First best allocations under complete information

In the complete information case, it is assumed the type of the consumer is publicly observable and in this
situation, the efficient allocation will be the solution to the following problem:

$$\max_{c_{10}, c_{11}, c_{20}, c_{21}} E[U(c_{11}, c_{21}, \rho_1) + (1-t)U(c_{12}, c_{22}, \rho_2)]$$

s.t. 

$$k_{10} + k_{20} = k_0$$

$$t\tilde{c}_{11} + (1-t)\tilde{c}_{12} \leq b_{10}k_{10} + b_{11}\tilde{k}_{20}$$

$$t\tilde{c}_{21} + (1-t)\tilde{c}_{22} = b_{20}(1-\tilde{\mu})k_{20} + b_{21}\tilde{k}_{10} \mu k_{20} - t\tilde{c}_{11} - (1-t)\tilde{c}_{12}$$

$$b_1\tilde{b}_{12} < b_{20}$$

$$\tilde{\mu} \leq 1$$

$$\tilde{c}_{ij} \geq 0$$

$$\mu \geq 0$$

The utility function is the one described above in Point d of Page 3. $c_{11}$, $c_{21}$ represents the prior plan indicating the consumption bundle allocated to type-1 consumers and $c_{12}$, $c_{22}$ the plan allocated to type-2 consumers. The feasibility constraints are the second and third constraints respectively. The second one requires that aggregate consumption at $T=1$ should be less or equal to aggregate resources per capita available from short-term investments and possibly liquidated long-term ones. Similarly, the third constraint requires that aggregate consumption at $T=2$ should be covered by non liquidated long-term investments plus short-term reinvestments of unused resources at $T=1$. The fourth constraint states that at date 1, it is never desirable to liquidate long-term investments in order to make room for new short-term ones.

This maximization problem is solved as a three-step problem:

a.- In a first step, the initial investment choices, $(k_{10}, k_{20})$ are considered as exogenous parameters and the optimal consumption levels and liquidation policy are determined.

b.- In a second step the indirect utility function derived in the first step is maximized on $k_{10}$ and $k_{20}$, and so the optimal levels of the initial investments are obtained.

c.- Finally, the optimal levels of $k_{10}$ and $k_{20}$ are substituted back into the first step problem, and the final solution is reached. Although this last step is obvious, it has just been added to clarify how the numerical solutions presented in the figures have been derived.

The solution to the first best problem gives the main result of the section, expressed by the following proposition:
Proposition 1. Let \((k_{o1}, k_{o2}, c_{11}^*, c_{12}^*, c_{21}^*, c_{22}^*, \mu^*)\) be a solution to the first best problem and define:

\[
\begin{align*}
\beta_{\lim} &= \left( \frac{b_{o1}k_{o1} \left\{ \frac{\rho_1}{\rho_2} \right\}^{-1/\gamma}}{b_{o2}k_{o2}} \right)^{-1} \\
\end{align*}
\]

Then:

- if \(\tilde{\beta}_{12} < \beta_{\lim}\) CASE A \((\lambda_1 > 0)\)

\[
\begin{align*}
c_{12}^* &= b_{o1}k_{o1} \\
c_{21}^* &= \frac{b_{o2}k_{o2}}{t^* (1-t)} \left[ \frac{\rho_1}{\rho_2} \right]^{-1/\gamma} \\
\end{align*}
\]

\[
\begin{align*}
c_{11}^* &= b_{o1}k_{o1} \\
c_{22}^* &= \left[ \frac{\rho_1}{\rho_2} \right]^{-1/\gamma} \\
\mu^* &= 0
\end{align*}
\]

- if \(\tilde{\beta}_{12} \geq \beta_{\lim}\) CASE B \((\lambda_1 = 0)\)

\[
\begin{align*}
\tilde{c}_{12}^* &= \tilde{c}_{11}^* \\
\tilde{c}_{21}^* &= \left[ \frac{\rho_1}{\rho_2} \right]^{-1/\gamma} + \tilde{c}_{22}^* \\
\mu^* &= 0
\end{align*}
\]

Where \(\lambda_1\) is the Lagrange multiplier associated with the second resource constraint.

**Proof:** See Appendix A.

Lemma 1. The optimal \(k_{o1}^*\) satisfies: \(k_{o1, crit} \leq k_{o1}^* \leq 1\)

where:

\[
\begin{align*}
k_{o1, crit} &= \frac{\frac{b_{o2}^{-1/\gamma}}{b_{o1} b_{o1}^{-1/\gamma}} \left[ \frac{\rho_1}{\rho_2} \right]^{-1/\gamma} + b_{o2}^{-1/\gamma} \frac{\rho_1^{-1/\gamma}}{\rho_1^{-1/\gamma}}}{t^* (1-t) \left[ \frac{\rho_1}{\rho_2} \right]^{-1/\gamma}}
\end{align*}
\]

**Proof:** See Appendix A.

Given that \(k_{o1}\) is an endogenous variable, this characterization may seem awkward, but it is understood in terms of dynamic programming considerations. As mentioned before, the maximization problem [2] has been solved as a three step problem: In the first step, \(k_{o1}\) and \(k_{o2}\) were considered as exogenous parameters and the
optimal consumption levels were obtained, in the second step, the optimal levels of the initial investments were
derived, maximizing on \( k_{e1} \) and \( k_{e2} \) the indirect utility function of the first step problem:

- Given that this \( k_{e1} \) is always above the critical value, \( k_{e1, \text{crit}} \), the optimal solution involves no liquidation
  of the long-term asset. The consumption levels are the ones specified in Proposition 1. It is observed that
  for low values of the random return, and up to a limit value, consumption is independent of \( \delta_{12} \), but once
  this limit value is achieved, first period consumption decreases, second period consumption increases with
  the random return. The explanation is that given the high value of the random return, it becomes
  advantageous to reinvest some of the return from the short-term asset available at \( T=1 \), in the new short-
  term technology. As mentioned in Hellwig, from an ex-ante point of view, the uncertainty about the
  random return is seen as a source of opportunities rather than a threat. While long-term investments are
  earmarked for consumption at date 2, short-term investments are not necessarily earmarked for
  consumption at date 1. The choice between consumption and investment depends on the rate of return \( \delta_{12} \)
on the new short-term investments.

2.1.1.- Numerical simulations

In order to provide a graphical plot of the optimal solution, some numerical simulations have been developed
for the input data given in Table I. The optimal consumption levels corresponding to the first best allocation
are shown in Figure 1.

2.2.- Second best allocations under incomplete information

In this case it is assumed that the realization of the timing of the consumption needs is private information of
the consumer. Given this information asymmetry, an allocation can only be implemented if it is incentive
compatible, that is, if it gives no consumer an incentive to lie or deviate about what he actually wants to
consume.

In the case of a type-2 agent, incentive compatibility requires that the consumption bundle he receives if
he is honest \((c_{12}, c_{22})\), should be at least as large as what he gets by lying and behaving like a type-1 agent
\((c_{11}, c_{21})\) and then reinvesting in the backyard technology in the optimal way for him. If he reinvested part of
his first period allocation \((c_{11})\) in the new short term asset, his optimal consumption levels in both periods

c\(^*_1\), c\(^*_2\) are the solution to the following problem:

\[
\max_{c_1, c_2} \left[ \frac{c_1^{1-\gamma} c_2^{1-\gamma}}{1-\gamma} \right] \\
\text{s.t.} \quad c_1 \leq c_{11} \\
\quad c_2 = (c_{11} - c_1) b_{12} c_{21}
\]  

which yields:

\[
c_1^* = \left( b_{21} \right)^{-\frac{1-\gamma}{\gamma}} \frac{c_{21} + b_{12} c_{11}}{1 + b_{12}^{(1-\gamma)/\gamma} - 1/\gamma} \leq c_{11} \]
\[c_2^* = \frac{c_{21} + c_{11} b_{12}}{1 + b_{12}^{(1-\gamma)/\gamma} - 1/\gamma}
\]

The incentive compatibility constraint for a type-2 agent is then:

\[
\frac{c_1^{1-\gamma}}{1-\gamma} + \frac{c_2^{1-\gamma}}{1-\gamma} \leq \frac{c_1^{1-\gamma}}{1-\gamma} + \frac{c_2^{1-\gamma}}{1-\gamma}
\]

The incentive constraints for a type-1 agent would be obtained in a similar way.

In the absence of any other backyard technology (for converting date 2 consumption into date 1) there is

no other incentive constraint to be considered.

Taking the incentive constraints into account, the second-best problem is a solution to the following one:

\[
\max_{c_{11}, c_{12}, \ldots} E[U(c_{11}, \bar{c}_{21}, \bar{c}_{12}) + (1-\bar{t})U(\bar{c}_{12}, \bar{c}_{22}, \bar{c}_{21})]
\]

s.t

\[
k_{o1} + k_{o2} = k_o \\
t \bar{c}_{11} + (1-t) \bar{c}_{12} \leq b_{o1} k_{o1} + b_{1} \bar{c}_{k2} \\
t \bar{c}_{21} + (1-t) \bar{c}_{22} = b_{o2} (1-\bar{\mu}) k_{o2} + b_{12} \bar{c}_{o1} k_{o1} + b_{1} \bar{c}_{k2} - t \bar{c}_{11} (1-t) \bar{c}_{12} \\
\]

\[
b_{1} \bar{c}_{k2} < b_{o2} \\
\bar{\mu} \leq 1 \\
\bar{c}_{o2} \geq 0 \\
\bar{\mu} \geq 0
\]  

IC constraints
2.2.1.- Numerical simulations

The analytical treatment of the second-best solution is quite a tedious one, therefore numerical solutions have been computed. The working procedure is the same as for the first-best case, i.e., the problem is solved in three steps.

There are some remarks to be pointed out:

a.- In the second-best allocation the incentive constraint for type-1 agents is binding, whereas that of type-2 agents is never binding.

b.- The second-best allocation does not involve liquidation of the long-term asset. This result differs from Hellwig as in his case the second best allocation may involve liquidation of the long-term technology. Although this result is based on numerical analysis, it seems that similarly to the first best allocation, the utility function is always a continuous and increasing function in \( k_{o1} \) in Case C', and therefore, the optimal level of the initial investment will be at least \( k_{o1}^{\text{crit}} \). On the contrary, in Hellwig's case, the utility function (in the liquidation solution) is increasing in \( k_{o1} \) but it is not continuous in the limit case \( k_{o1}^{\text{crit}} \), that distinguishes the liquidation and non-liquidation solutions, and therefore, the optimal \( k_{o1}^{*} \) may occur in the liquidation case, for values of \( k_{o1} \) sufficiently close but below \( k_{o1}^{\text{crit}} \).

c.- The optimal solution has been derived for the input data of Table I. A graphical plot of the optimal solution is given by Figure 2.

3.- Comparison with a walrasian market.

This section will compare the second best allocation (non-traded solution) to the competitive equilibrium in an equity economy (traded solution). Suppose that at \( T=1 \), there was a walrasian market for date 1 and date 2 consumption goods, in which consumers participate with endowments consisting of \( b_{o1}k_{o1} \) units of the date 1 good and \( b_{o2}k_{o2} \) units of the date 2 good. Let \( R_{2} = 1 + r \) be some equilibrium interest rate at which individuals are willing to trade good 1 in exchange for good 2, and so that for any agent j:
Comparison with a walrasian market

\begin{equation}
\begin{aligned}
c_{ij} &= b_{oi}k_{oi} + B_j \\
c_{ij} &= b_{oi}k_{oi} - R_2 B_j \quad j = 1, 2
\end{aligned}
\end{equation}

where \( B_j \) is the quantity demanded (or supplied) of good 1 in exchange for good 2 and with \( \sum_j B_j = 0 \) across agents determining \( R_2 \), subject to the caveat \( R_2 \geq b_{12} \), the short term realized (storage) rate from \( T=1 \) to \( T=2 \).

If storage (with \( R_2 = b_{12} \)) is done then \( 0 \geq \sum_j B_j \geq -b_{oi}k_{oi} \) is the constraint overall.

The individuals' maximization problems are shown below:

Type-1 problem at \( T=1 \)

\begin{equation}
\begin{aligned}
\text{max}_{e_1} & \quad \left\{ \frac{c_{11}^{1-\gamma} + c_{21}^{1-\gamma}}{1 \quad \text{or} \quad 1-\gamma} \right\} \\
\text{s.t} & \quad c_{11} = b_{oi}k_{oi} + B_1 \\
& \quad c_{21} = b_{oi}k_{oi} - R_2 B_1
\end{aligned}
\end{equation}

with solution:

\begin{equation}
B_1 = \frac{(\rho_1 R_2)^{-1/\gamma} b_{oi}k_{oi} - b_{oi}k_{oi}}{1 + (\rho_1 R_2)^{-1/\gamma} R_2}
\end{equation}

Type-2 problem at \( T=1 \)

\begin{equation}
\begin{aligned}
\text{max}_{e_2} & \quad \left\{ \frac{c_{12}^{1-\gamma} + c_{22}^{1-\gamma}}{1 \quad \text{or} \quad 1-\gamma} \right\} \\
\text{s.t} & \quad c_{12} = b_{oi}k_{oi} + B_2 \\
& \quad c_{22} = b_{oi}k_{oi} - R_2 B_2
\end{aligned}
\end{equation}

with solution:

\begin{equation}
B_2 = \frac{(\rho_2 R_2)^{-1/\gamma} b_{oi}k_{oi} - b_{oi}k_{oi}}{1 + (\rho_2 R_2)^{-1/\gamma} R_2}
\end{equation}

A). If \( R_2 \geq b_{12} \)

From the equilibrium condition \( \sum_j B_j = 0 \), the following non-linear equation in \( R_2 \) is obtained, that is:
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\[
\frac{1}{1 + (p_1 R_2^{-1/2})} \frac{(p_2 R_2)^{1/2} k_{o1} - b_{o1} k_{o1}}{1 - (1 - t) (p_2 R_2)^{1/2} k_{o2} - b_{o2} k_{o2}} = 0
\]

The value of \( R^*_2 \) is obtained as a solution to the above equation, and from it the values of \( B^*_1 \) and \( B^*_2 \) are derived. These values are substituted in the expressions for \( c_{11}, c_{21}, c_{12}, c_{22} \), to calculate ex-ante expected utility in this economy.

B). If \( b_{o2} > R_2 \)

In this case the equilibrium interest rate is \( R^*_2 = b_{o2} \), the realized short term return.

The optimal levels of \( B^*_1 \) and \( B^*_2 \) are:

\[
B^*_1 = \frac{(p_1 b_{o2})^{1/2} k_{o2} - b_{o1} k_{o1}}{1 + (p_1 b_{o2})^{1/2} b_{o2}} ; \quad B^*_2 = \frac{(p_2 b_{o2})^{1/2} k_{o2} - b_{o1} k_{o1}}{1 + (p_2 b_{o2})^{1/2} b_{o2}}
\]

and from them, the optimal consumption levels and the value of the expected utility are obtained.

Table II shows the numerical computations of the traded solution for the input data of Table I.

In order to compare the expected utility obtained in the non traded solution with respect to the traded one, some numerical examples have been computed. The input data are those corresponding to Table I, where the variations in the exogenous parameters and the results of these comparisons are shown in Figure 3 to Figure 5.

It is observed that in all the examples the non-traded solution is always welfare superior, also, as the value of the random return increases, the difference in utility diminishes. This result differs from Hellwig, as in his case, the traded and non-traded solutions coincide.

It may be concluded that, if preferences are represented by an additive utility function, the allocations obtained in the non-traded solution are welfare superior with respect to the ones achieved in the traded one. The intuition for this result can be viewed in terms of individual versus coalition incentive compatibility. (See Jacklin [13]). Since demand deposits cannot be traded, they can be used to achieve any allocation that is individually incentive compatible. On the other hand, the allocation achieved in the traded solution is a competitive equilibrium and thus represents an element in the core of the economy in which individuals start trading with identical initial endowments. By definition all elements in the core are not only individually incentive compatible but also coalitionically incentive compatible (that is, there does not exist a coalition of individuals each of whom can be made better off by following a strategy specified by the coalition and then redistributing the coalition’s total allocation).
In both the demand deposit and equity economies, the same objective function (i.e. ex ante expected utility) is being maximized, but in the non traded solution it is maximized over a less constrained set and therefore demand deposits can generally achieve greater risk sharing than equity contracts.

4.- Concluding remarks

The main purpose of the paper has been to analyze the role of financial intermediaries versus Walrasian markets, in ensuring preference shocks, in a framework in which there is no aggregate uncertainty, individuals have smooth preferences and there exists a short term investment opportunity between dates 1 and 2.

It has been shown that demand deposit contracts provide greater risk sharing than equity contracts. This result (and contrary to Hellwig's model) shows that financial intermediaries do provide a positive role in the economy.

The analysis of this paper may be of interest in the design of financial systems in emerging markets. An example is Eastern Europe, in which a new financial system is being established almost from scratch, and poses the question, whether it should be a stock-market based, American type system or a bank-based German type system.
Appendix A: Additive Utility Function

A.- First Best allocation

The first best allocation is obtained as a solution to the following problem:

$$\max_{q_1, p_1} \left\{ \frac{c_{11}^{1-\gamma} + p_1 c_{21}^{1-\gamma}}{1-\gamma} + (1-t) \frac{c_{12}^{1-\gamma} + p_2 c_{22}^{1-\gamma}}{1-\gamma} \right\}$$

s.t.

$$tc_{11} + (1-t)c_{12} \leq b_{o1} k_{o1} + \mu k_{o2} b_1$$
$$tc_{21} + (1-t)c_{22} = (1-\mu) b_{o2} k_{o2} + [\mu k_{o2} b_1 + b_{o1} k_{o1} - tc_{11} - (1-t)c_{12}] b_{12}$$
$$c_{ij} \geq 0$$
$$\mu \geq 0$$

The Lagragian is formed by using the lagrangian multipliers $\lambda_1$ and $\lambda_2$ of the two first resource constraints.

The Kuhn-Tucker conditions are:

$$tc_{11} - \lambda_1 t + tb_{12} \lambda_2 = 0$$
if $c_{11} > 0$ [a]
$$t \rho_1 c_{21} + t \lambda_2 = 0$$
if $c_{12} > 0$ [b]
$$(1-t) c_{12} - \lambda_1 (1-t) + (1-t) b_{12} \lambda_2 = 0$$
if $c_{22} > 0$ [c]
$$(1-t) \rho_2 c_{22} + (1-t) \lambda_2 = 0$$
if $\mu > 0$ [d]
$$k_{o2} b_1 \lambda_1 - k_{o2} (b_{o1} b_{12} - b_{o2}) \lambda_2 = 0$$
if $\lambda_1 > 0$ [e]
$$tc_{11} + (1-t)c_{12} - b_{o2} k_{o2} - \mu k_{o1} b_1 = 0$$
if $\lambda_1 > 0$ [f]
$$tc_{21} + (1-t)c_{22} - (1-\mu) b_{o2} k_{o2} - [\mu k_{o2} b_1 + b_{o1} k_{o1} - tc_{11} - (1-t)c_{12}] b_{12} = 0$$
if $\lambda_2 > 0$ [g]

A.1.- First-step solution:

The following cases may be considered:

A.1.1.- CASE A: ($\lambda_1 > 0$)

The equations to be solved are:

$$c_{11}^\gamma - \lambda_1 + b_{12} \lambda_2 = 0$$
if $c_{11} > 0$ [4]
$$\rho_1 c_{21}^\gamma + \lambda_2 = 0$$
if $c_{12} > 0$ [5]
$$c_{12}^\gamma - \lambda_1 + b_{12} \lambda_2 = 0$$
if $c_{22} > 0$ [6]
$$\rho_2 c_{22}^\gamma + \lambda_2 = 0$$
if $\mu > 0$ [7]
$$tc_{11} + (1-t)c_{12} - b_{o2} k_{o2} = 0$$
if $\lambda_1 > 0$ [8]
$$tc_{21} + (1-t)c_{22} - b_{o2} k_{o2} = 0$$
if $\lambda_2 > 0$ [9]
Appendix A: Additive Utility Function. 1st Best

The optimal solution to this problem yields:

\[
\begin{align*}
    &c_{11}^* = c_{12}^* = b_{o2}k_{o1} \\
    &c_{21}^* = \frac{b_{o2}k_{o2}}{t + (1 - \theta) \left[ \frac{\rho_1}{\rho_2} \right]^{-\frac{1}{\gamma}}} \\
    &\mu^* = 0
\end{align*}
\]

In Case A it is assumed \( \lambda_1 > 0 \), from [4] and [5]:

\[
\lambda_1 = c_{11}^\gamma - b_{12} \rho_1 c_{21}^\gamma > 0
\]

Substituting \( c_{11}^* \) and \( c_{21}^* \) in the expression for \( \lambda_1 \), the following condition on \( b_{12} \) for this case to hold is obtained:

\[
b_{12} < \frac{1}{\rho_1} \left( \frac{b_{o2}k_{o1}}{b_{o2}k_{o2}} \right) T
\]

Similarly it is assumed \( \mu^* = 0 \), that means \( \frac{\partial L}{\partial \mu} \leq 0 \)

\[
\frac{\partial L}{\partial \mu} = k_{o2} b_1 \lambda_1 - k_{o2} (b_1 b_{12} - b_{o2}) \lambda_2 \leq 0
\]

Substituting \( \lambda_1 = c_{11}^\gamma - b_{12} \rho_1 c_{21}^\gamma \) and \( \lambda_2 = -\rho_1 c_{21}^\gamma \) in the above expression, the condition on \( k_{o1} \), for this case to hold is obtained:

\[
k_{o1} = \left( \frac{b_{o1}}{b_{o2} \rho_1^{\frac{1}{\gamma}}} \right)^{\frac{\gamma - 1}{\gamma}} \left( t + (1 - \theta) \left[ \frac{\rho_1}{\rho_2} \right]^{-\frac{1}{\gamma}} \right)^{-\frac{\gamma}{\gamma - 1}} b_{o2}^{\frac{1}{\gamma}} \rho_1^{\frac{1}{\gamma}}
\]

If the optimal level of the initial investment is above this limit value \( (k_{o1}^{crit}) \) there is no liquidation in the optimal solution.

**A.1.2.- CASE B: (\( \lambda_1 = 0 \))**

The F.O.C. in this case are:
The role of demand deposits in risk sharing

\[ 0c_{11}^{\tau} + b_{12} \lambda_{2} = 0 \]  \hspace{1cm} [15] \hspace{1cm} c_{12}^{\tau} + b_{12} \lambda_{2} = 0 \] \hspace{1cm} [17]

\[ \rho_{1} c_{22}^{\tau} + \lambda_{2} = 0 \] \hspace{1cm} [16]

\[ c_{21}^{\tau} + (1 - \tau) c_{22} - b_{a2} k_{o2} - \left[ b_{a1} k_{a1} - t c_{11} - (1 - \tau) c_{12} \right] b_{12} = 0 \] \hspace{1cm} [19]

and the optimal solution is:

\[ c_{11}^{*} = c_{12}^{*} = \frac{1}{b_{11} \rho_{1}} \left[ b_{a2} k_{a2} \right] ^{-1} c_{21}^{*} \]

\[ c_{22}^{*} = \left[ \frac{\rho_{1}}{\rho_{2}} \right] ^{-1} c_{21}^{*} \]

\[ c_{21}^{*} = \frac{b_{a2} k_{a2} b_{12} - b_{a2} k_{o2}}{t + (1 - \tau) \left[ \frac{\rho_{1}}{\rho_{2}} \right] ^{-1} + \frac{\left[ \rho_{1} / \rho_{2} \right] ^{-1} \tau^{-1} \left[ \frac{\rho_{1}}{\rho_{2}} \right] ^{-1} \tau^{-1}}{b_{12} k_{o2}} \} \mu^{*} = 0 \] \hspace{1cm} [20]

In Case B it is assumed \( \lambda_{1} = 0 \), or equivalently:

\[ t c_{11}^{*} + (1 - \tau) c_{12}^{*} \leq b_{a1} k_{a1} \] \hspace{1cm} [21]

Substituting the optimal consumption levels, the following expression for the random return is obtained:

\[ b_{12} = \frac{1}{\rho_{1}} \left[ b_{a2} k_{o2} \right] ^{-1} \mu^{*} b_{12} \] \hspace{1cm} [22]

A.1.3.- CASE C: \( \lambda_{1} > 0, \mu^{*} > 0 \)

The equations to be solved are the [3] and the optimal solution is:

\[ c_{11}^{*} = c_{12}^{*} = \left[ \frac{b_{a2} \rho_{1}}{b_{11}} \right] ^{-1} c_{21}^{*} \]

\[ c_{21}^{*} = \frac{b_{11} b_{a2} (b_{a2} k_{a2} - b_{a2} k_{o2})}{\left[ \frac{\rho_{1}}{\rho_{2}} \right] ^{-1} b_{12} k_{o2} + \frac{\left[ \rho_{1} / \rho_{2} \right] ^{-1} \tau^{-1}}{b_{12} k_{o2}} \} \]

\[ c_{22}^{*} = \left[ \frac{\rho_{1}}{\rho_{2}} \right] ^{-1} c_{21}^{*} \]

\[ 0 < \mu^{*} = \frac{c_{11}^{*} + (1 - \tau) c_{12}^{*} - b_{a1} k_{a1}}{b_{11} k_{a1}} \leq 1 \] \hspace{1cm} [23]

In this case, it is assumed a value for \( \mu^{*} > 0 \), that is, substituting \( c_{11}^{*} \) and \( c_{12}^{*} \) in the expression for \( \mu^{*} \), the condition on \( k_{a1} \) for this case to be satisfied is obtained:
Appendix A: Additive Utility Function. 1st Best

\[ k_{ol} < \frac{\frac{r-1}{b_{ol}} \rho_1^{\gamma} y_t}{b_{ol} \left( t + (1 - \frac{\rho_1}{\rho_2}) \right) + b_{ol}^{\gamma} \rho_1^{\gamma - 1} y_t} = k_{ol}^{\text{crit}} \]  

\[ b_{ol}^{1 - \frac{\gamma}{\gamma + 1}} \left[ t \left( t + (1 - \frac{\rho_1}{\rho_2}) \right) + \frac{b_{ol} - b_{l}}{\rho_1^{\gamma} \left( b_{l} - b_{ol} \right)} \right] = k_{ol}^{\text{crit}} \]

A.2.- Second-step solution:

The second step is the solution to the expression:

\[
\max_{k_{ol},b_{12}} \begin{cases} 
\int_{b_{\text{min}}}^{b_{12}} U^{(a)}(b_{12}) \, db_{12} + \int_{b_{\text{min}}}^{b_{12}} U^{(b)}(b_{12}) \, db_{12} & \text{if } k_{ol} \geq k_{ol}^{\text{crit}} \\
\int_{b_{\text{min}}}^{b_{12}} U^{(c)}(b_{12}) \, db_{12} & \text{if } k_{ol} < k_{ol}^{\text{crit}}
\end{cases}
\]

Then, if \( 0 < k_{ol} < k_{ol}^{\text{crit}} \):

\[
\max_{k_{ol}} \int_{b_{\text{min}}}^{b_{12}} U^{(c)}(b_{12}) \, db_{12} - \int_{b_{\text{min}}}^{k_{ol}^{\text{crit}}} \frac{\partial U^{(c)}}{\partial k_{ol}}(b_{12}) \, db_{12} = 0
\]

That is, if \( \frac{\partial U^{(c)}}{\partial k_{ol}} > 0 \) in the interval \( [b_{\text{min}}, b_{\text{max}}] \), the maximum is reached in \( k_{ol}^{\text{crit}} \). The proof is given by:

\[
\frac{\partial U^{(c)}}{\partial k_{ol}} = \begin{cases} 
\frac{c_1^{\gamma} - c_1^{\gamma} c_2^{\gamma} \frac{\partial c_2}{\partial k_{ol}}}{c_1^{\gamma} + \frac{\partial c_2}{\partial k_{ol}}} & \text{if } c_2 > 0 \\
(1 - t) \left[ \frac{c_1^{\gamma} - c_2^{\gamma} \frac{\partial c_2}{\partial k_{ol}}}{c_1^{\gamma} + \frac{\partial c_2}{\partial k_{ol}}} + \frac{\partial c_2}{\partial k_{ol}} \right]
\end{cases}
\]

where:

\[
\frac{\partial c_2}{\partial k_{ol}} = \frac{b_1 - b_{ol} (b_{ol} - b_1)}{t + (1 - t) \left( \frac{\rho_1^{\gamma}}{\rho_2} \right) + b_{ol}^{\gamma} \rho_1^{\gamma - 1} \frac{1}{\rho_1^{\gamma}}}
\]

\[
\frac{\partial c_1}{\partial k_{ol}} = \left[ \frac{b_{ol}^{\gamma} \rho_1^{\gamma}}{b_1} \right] \frac{\partial c_2}{\partial k_{ol}}
\]

By assumption \( b_1 < b_{ol} \) and therefore, \( \frac{\partial U^{(c)}}{\partial k_{ol}} > 0 \) which implies \( k_{ol}^{\text{crit}} < k_{ol}^{\text{crit}} \), this mans that, the optimal solution falls always in Cases A and B, with no liquidation of the long-term asset.
References


5.- Bhattacharya, S. 1994, The Economics of Bank Regulation, Université Catholique de Louvain.


List of Tables

Table I

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List of Figures

Figure 1.- Optimal consumption levels in the first-best allocation.
Figure 2.- Optimal consumption levels in the second-best allocation
Figure 3 to Figure 5.- Expected utility of non-traded minus traded solution
1. Traded equity contracts refer to the allocations achieved in a Walrasian market in which individuals hold the assets directly.

2. These models of intertemporal liquidity risk have been recently extended to compare banks versus markets, in a dynamic framework (see Fulghieri and Rovelli [11], Dutta and Kapur [10], Bhattacharya and Padilla [2] and Bhattacharya, Fulghieri and Rovelli [3] as examples of such work).

3. For simplicity, a triangular distribution for the random return is assumed. The use of this distribution, defined by its mean and standard deviation, does not affect the qualitative nature of the results with respect to Hellwig.

4. As shown in Jacklin [13] and is commented also in the work of Hellwig, the demand deposit contract can be used to achieve the constrained social optimum.

5. Given the observed realization of the short term return, \( \beta_{12} \), the decision between consumption and reinvestment in this new short term asset takes place. If there is no reinvestment, the second resource constraint would be satisfied as an equality.

6. The system of non-linear equations was solved by the Newton Raphson technique, with the use of a computer program, that was written in MSdos Qbasic.

7. See explanation of Case C in Appendix A