UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 07-07
Statistic and Econometric Series 02
February 2007

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249849

THE SIGN OF ASYMMETRY AND THE TAYLOR EFFECT IN STOCHASTIC VOLATILITY MODELS *

Helena Veiga [1]

Abstract

According to the *Taylor-Effect* the autocorrelations of absolute financial returns are higher than the ones of squared returns. In this work, we analyze this empirical property for three different asymmetric stochastic volatility models, with short and/or long memory. Specially, we investigate how the Taylor-Effect relates to the most important model characteristics: its asymmetry and its capacity to generate volatility persistence and kurtosis. Finally, we realize Monte Carlo experiments to infer about possible biases of the sample Taylor-Effect and fit the models to the return series of the Dow Jones.

Keywords: Asymmetry, Kurtosis, Long and Short Memory, Taylor-Effect.
JEL Classification: C22

*

1 I am very grateful to Tim Bollerslev for his suggestions. The author also acknowledges financial support from the Spanish Ministry of Education and Science, research project SEJ2006-03919. All remaining mistakes are mine. Address: Department of Statistics, Universidad Carlos III Madrid, Calle Madrid 126, 28903 Getafe, Spain. E-mail: mhveiga@est-econ.uc3m.es

The Sign of Asymmetry and the Taylor Effect in Stochastic Volatility Models

Helena Veiga*

February 20, 2007

Abstract

According to the *Taylor-Effect* the autocorrelations of absolute financial returns are higher than the ones of squared returns. In this work, we analyze this empirical property for three different asymmetric stochastic volatility models, with short and/or long memory. Specially, we investigate how the Taylor-Effect relates to the most important model characteristics: its asymmetry and its capacity to generate volatility persistence and kurtosis. Finally, we realize Monte Carlo experiments to infer about possible biases of the sample Taylor-Effect and fit the models to the return series of the Dow Jones.

Keywords: Asymmetry, Kurtosis, Long and Short Memory, Taylor-Effect.

JEL-Classifications: C22.

1 Introduction

Taylor (1986), Granger et al. (1999), and Dacorogna et al. (2001) have shown, among others, that the absolute autocorrelations of financial return series are usually higher than the ones of squared observations. This phenomena is known as *Taylor-Effect*, first defined by Granger and Ding (1995). Recently, He and Teäsvirta (1999), Malmsten and Teräsvirta (2004) and Mora-Gálan et al. (2004) concluded that the GARCH, EGARCH, and the symmetric autoregressive stochastic volatility model (ARSV(1)) have difficulties in generating the Taylor-Effect, specially if the implied kurtosis is not big enough.

The aim of this paper is threefold: First, we relate the sign of asymmetry to the Taylor-Effect in the context of stochastic volatility. Second, we analyze the influence of volatility persistence and kurtosis on the Taylor-Effect and, finally, we perform Monte Carlo experiments in order to see if this empirical property is a sampling phenomena caused by the existence of biases in the sample autocorrelations.

The paper is organized as follows: In the next section, we present the models and derive their autocorrelation structure. We run Monte Carlo experiments in Section 3. In Section 4, we report the estimation results and, in Section 5, we conclude.

*I am very grateful to Tim Bollerslev for his suggestions. The author also acknowledges financial support from the Spanish Ministry of Education and Science, research project SEJ2006-03919. All remaining mistakes are mine. Address: Department of Statistics, Universidad Carlos III Madrid, Calle Madrid 126, 28903 Getafe, Spain. E-mail: mhveiga@est-econ.uc3m.es

2 Stochastic Volatility Models and the Taylor-Effect

In this section, we review first the two factor long memory stochastic volatility model (2FLMSV) of Veiga (2006). The objective of the first factor is to capture persistence in volatility and is similar in its spirit to the volatility process of Breidt et al. (1998). The second factor accommodates the short run dynamics and helps generating extra kurtosis. Formally,

$$\begin{aligned} y_t &= \varepsilon_t \sigma \exp\left(\frac{h_{1t}+h_{2t}}{2}\right) && (1) \\ h_{1t} &= \phi h_{1t-1} + \eta_t && (2) \\ h_{2t} &= (1-L)^{-d}\zeta_t. && (3) \end{aligned}$$

In equation (1), σ denotes a scale parameter, σ_t^2 is the conditional variance of y_t, ε_t is $NID(0,1)$, and η_t and ζ_t are $NID(0,\sigma_\eta^2)$ and $NID(0,\sigma_\zeta^2)$, respectively. Veiga (2006) assumed additionally that $(\varepsilon_t, \zeta_{t+1})'$ follows the bivariate normal distribution

$$\begin{pmatrix} \varepsilon_t \\ \zeta_{t+1} \end{pmatrix} \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta\sigma_\zeta \\ \delta\sigma_\zeta & \sigma_\zeta^2 \end{pmatrix}\right), \qquad (4)$$

where δ, the correlation between ε_t and ζ_{t+1}, induces correlation between returns and changes in volatility, (see Taylor, 1994; Harvey and Shephard, 1996). We relate the asymmetry to the long memory volatility factor due to the results found in Durham (2006) and Bollerslev et al. (2006). They evidenced for daily and high frequency data, respectively, that the correlation with the persistent volatility factor was large and negative and the correlation with the short-run volatility factor was small and positive.

As suggested by Ruiz and Veiga (2006), equations (1) and (4) together with

$$(1-\phi_1)(1-L)^d h_t = \zeta_t \qquad (5)$$

define the asymmetric extension of the LMSV specification of Breidt et al. (1998). We call it the ARLMSV(1) model. On the other hand, the equations (1) and (2) together with the hypothesis that $(\varepsilon_t, \eta_{t+1})'$ follows a bivariate normal distribution similar to equation (4) specifies the asymmetric ARSV(1) model.

Although the series of returns is a martingale difference and, consequently, an uncorrelated sequence, it is not independent. Next, we provide the expressions of the first order autocorrelations of the absolute ($c=1$) and squared returns ($c=2$) for the 2FLMSV and the ARLMSV(1) models. We simplify the analysis by considering first order autocorrelations throughout. Observe that the analogous for the ARSV(1) model can be obtained by restricting the following expressions accordingly. For the 2FLMSV model (equations (1)-(4)) we obtain that

$$corr(|y_t|^c, |y_{t+1}|^c) = \frac{\exp\left(\frac{c^2}{4}\left(\sigma_{h_1}^2\rho_{h_1}(1)+\sigma_{h_2}^2\phi\right)\right)\left(1+\delta^c\sigma_\zeta^c\left(\frac{4}{k_c}\right)^{\frac{c-2}{2}}\right)-1}{k_c\exp\left(\frac{c^2}{4}\sigma_{h1}^2+\sigma_{h_2}^2\right)-1}, \qquad (6)$$

where $k_c = \frac{\Gamma(c+0.5)\Gamma(0.5)}{[\Gamma(0.5(c+1))]^2}$, $\rho_{h_1}(1) = \frac{\Gamma(1-d)\Gamma(1+d)}{\Gamma(d)\Gamma(2-d)}$, and $\sigma_\zeta^2 = \frac{\Gamma(1-2d)}{\Gamma(1-d)}$ ($\Gamma(.)$ denotes the gamma function). Moreover, the excess kurtosis of y_t is given by $EK = 3[\exp(\sigma_{h_1}^2 + \sigma_{h_2}^2) - 1]$, (see Veiga, 2006).

Similarly, we obtain for the ARLMSV(1) model that

$$corr(|y_t|^c, |y_{t+1}|^c) = \frac{\exp\left(\frac{c^2}{4}\sigma_h^2\rho_h(1)\right)\left(1 + \delta^c\sigma_\zeta^c\left(\frac{4}{K_c}\right)^{\frac{c-2}{2}}\right) - 1}{K_c\exp(\frac{c^2}{4}\sigma_h^2) - 1}, \qquad (7)$$

where $\sigma_h^2 = \sigma_\zeta^2\frac{\Gamma(1-2d)}{[\Gamma(1-d)]^2} \cdot \frac{F(1,1+d;1-d;\ \phi_1)}{(1+\phi_1)}$ ($F(.,.;.;.)$ denotes the hypergeometric function) and $\rho_h(1)$, the autocorrelation of order 1 of h_t, is equal to $\frac{d}{1-d} \cdot \frac{F(1,d+1;1-d+1;\phi_1)+F(1,d-1;1-d-1;\phi_1)-1}{(1-\phi_1)F(1,1+d;1-d;\phi_1)}$. Finally, the excess kurtosis of y_t was shown to be equal to $3\left[\exp(\sigma_h^2) - 1\right]$, (see Ruiz and Veiga, 2006).

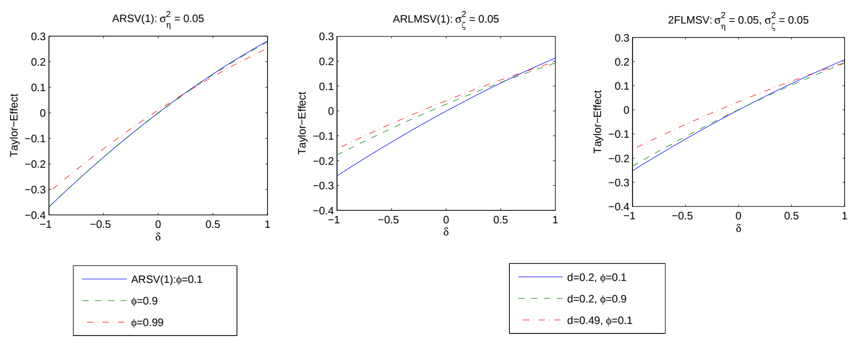

Figure 1: Relationship between the Taylor-Effect and δ, the parameter of asymmetry.

Figure 1 shows the relationship between the Taylor-Effect and δ, the parameter that captures the correlation between the volatility factors and the return process. We see that the models are only able to generate the Taylor-Effect when the correlation is positive. Moreover, it seems to exist a positive relationship between the parameters that induce volatility persistence (d, ϕ, and ϕ_1) and the Taylor-Effect. In particular for $\delta < 0$, the higher the values of these parameters the less negative is the difference between $corr(|y_t|, |y_{t+1}|)$ and $corr(y_t^2, y_{t+1}^2)$. This is more evident in the ARLMSV(1) and 2FLMSV models.

Malmsten and Teräsvirta (2004) and Mora-Gálan et al. (2004) showed that if the kurtosis of the returns is relatively small, neither the EGARCH nor the ARSV(1) are able to reproduce the Taylor-Effect. Table 1 reports the kurtosis for all specifications. The kurtosis of y_t ranges from 3.16 to 6.71 for M1 and M2 and it increases substantially for the third specification of the ARSV(1) model. Since we observe that an increase of ϕ leads to a less negative difference between the autocorrelations of the absolute and squared observations for the ARLMSV(1) model, specially when $\delta < 0$, we have considered additionally the values $\{\phi, d, \sigma_\zeta^2, \delta\} = \{0.5, 0.49, 0.05, -0.8\}$. This specification produces a Taylor-Effect of 0.0274 and a kurtosis of 71.14. This allows us to highlight that if K_y is "unrealistic" high, it is possible to annulate the effect of negative δ's and reproduce the Taylor-Effect. The same happens in the 2FLMSV model.

Models	ARSV(1)	ARLMSV(1)	2FLMSV
M1	3.16	3.18	3.33
M2	3.90	6.71	4.12
M3	37.0	8.17	7.15

Table 1: Kurtosis generated by the models.

3 Finite Sample Properties

So far we have seen that the three stochastic volatility models do not always generate the Taylor Effect. Harvey and Streibel (1998), Pérez and Ruiz (2003), and Mora-Gálan et al. (2004) showed for symmetric stochastic volatility models that the sample autocorrelations are negative biased and that the biases of the sample autocorrelations of squared returns are higher than the ones of absolute returns. Hence, it is possible to observe the Taylor-Effect empirically even if it does not exist in the population and *viceversa*.

		T=500			T=1000			T=5000		
$\{\phi_1, \sigma_\eta^2, \delta\}$	T.E.	MC.T.E.	R.B.	S.D.	MC.T.E	R.B.	S.D.	MC.T.E	R.B.	S.D.
{0.1,0.05,-0.8}	-0.21	-0.006	-0.97	0.02	-0.007	-0.97	0.02	-0.008	-0.96	0.01
$\{0.1, 0.05, -0.2\}$	-0.05	0.0001	-1.00	0.02	-0.0007	-0.99	0.02	-0.0005	-0.99	0.01
$\{0.1, 0.05, 0.0\}$	-0.0002	0.0006	-4.00	0.02	-0.0002	0.00	0.02	-0.00002	0.90	0.01
$\{0.1, 0.05, 0.2\}$	0.05	0.0003	-0.99	0.02	-0.0008	-1.02	0.02	-0.0004	-1.01	0.01
$\{0.1, 0.05, 0.8\}$	0.17	-0.006	-1.04	0.02	-0.007	-1.04	0.02	-0.007	-1.04	0.01
$\{0.9, 0.05, -0.8\}$	-0.19	0.004	-1.02	0.04	0.002	-1.01	0.03	-0.0002	-1.00	0.02
$\{0.9, 0.05, -0.2\}$	-0.05	-0.0001	-1.00	0.03	-0.001	-0.98	0.03	-0.001	-0.98	0.01
$\{0.9, 0.05, 0.0\}$	-0.002	-0.0001	-0.95	0.03	-0.001	-0.50	0.03	-0.001	-0.50	0.01
$\{0.9, 0.05, 0.2\}$	0.04	0.0001	-1.00	0.03	-0.001	-1.03	0.03	-0.002	-1.05	0.01
$\{0.9, 0.05, 0.8\}$	0.16	0.005	-0.97	0.04	0.001	-0.99	0.03	-0.001	-1.01	0.02
$\{0.99, 0.05, -0.8\}$	-0.02	0.12	-7.00	0.07	0.15	-8.50	0.07	0.18	-10.0	0.07
$\{0.99, 0.05, -0.2\}$	0.11	0.09	-0.18	0.07	0.11	0.00	0.07	0.13	0.18	0.06
$\{0.99, 0.05, 0.0\}$	0.14	0.08	-0.43	0.07	0.10	-0.29	0.06	0.13	-0.07	0.06
$\{0.99, 0.05, 0.2\}$	0.16	0.09	-0.44	0.07	0.11	-0.31	0.06	0.14	-0.13	0.06
$\{0.99, 0.05, 0.8\}$	0.23	0.13	-0.43	0.08	0.15	-0.35	0.07	0.18	-0.22	0.06

Table 2: Monte Carlo finite sample Taylor effect (MC.T.E), relative biases (R.B.), Monte Carlo standard deviations (S.D.), Taylor-Effect (T.E.) in ARSV(1) models. T is the sample size.

In order to investigate if this also occurs in the context of asymmetry, we run several Monte Carlo experiments. All results are based on 1000 replicates of the models. We have selected fourteen cases for each model and in all cases we have imposed a scale parameter, σ, of one. The results are presented in Tables 2-4. The first conclusion is that the biases exist and are of big magnitude. Second, the models have difficulties to generate the Taylor-Effect even when it is observed in the population. This happens for the parametrizations $\{0.1, 0.05, 0.2\}$ and $\{0.1, 0.05, 0.8\}$ in the ARSV(1) model, for the parametrizations $\{0.1, 0.2, 0.05, 0.2\}$ and $\{0.1, 0.49, 0.05, -0.2\}$ in the ARLMSV(1) model, and for $\{0.1, 0.2, 0.05, 0.05, 0.2\}$ and $\{0.1, 0.49, 0.05, 0.05, 0.2\}$ in the 2FLMSV model. Third and less frequent, the Taylor-Effect seems to be a sampling consequence of the estimation biases of the sample autocorrelations. This occurs for $\{0.99, 0.05, -0.8\}$ in the ARSV(1) model and for $\{0.9, 0.2, 0.05, -0.8\}$ in the ARLMSV(1) model. Note that these parametrizations have a very negative asymmetry in common that has not been reported so far in the literature. In particular, Sandmann and Koopman (1998) and Yu (2005), for the asymmetric ARSV(1) model, estimated values of δ that ranged between -0.32 till -0.48. Considering the 2FLMSV model, the same phenomena is only observed for the relatively small sample size T=1000.

			T=500			T=1000			T=5000	
$\{\phi_1, d, \sigma^2_\zeta, \delta\}$	T.E.	MC.T.E.	R.B.	S.D.	MC.T.E	R.B.	S.D.	MC.T.E	R.B.	S.D.
$\{0.1, 0.2, 0.05, -0.8\}$	-0.21	-0.0009	-1.00	0.02	-0.0009	-1.00	0.02	-0.0005	-1.00	0.01
$\{0.1, 0.2, 0.05, -0.2\}$	-0.05	-0.0008	-0.98	0.02	-0.0008	-0.98	0.02	-0.0003	-0.99	0.01
$\{0.1, 0.2, 0.05, 0.0\}$	-0.01	-0.0007	-0.93	0.02	-0.0008	-0.92	0.02	-0.0003	-0.97	0.01
$\{0.1, 0.2, 0.05, 0.2\}$	0.05	-0.0007	-1.01	0.02	-0.0008	-1.02	0.02	-0.0003	-1.01	0.01
$\{0.1, 0.2, 0.05, 0.8\}$	0.17	-0.0008	-1.00	0.02	-0.0009	-1.01	0.02	-0.0004	-1.00	0.01
$\{0.9, 0.2, 0.05, -0.8\}$	-0.13	0.002	-1.02	0.03	0.003	-1.02	0.03	-0.003	-0.98	0.01
$\{0.9, 0.2, 0.05, -0.2\}$	-0.01	-0.002	-0.80	0.03	-0.003	-0.70	0.02	-0.003	-0.70	0.01
$\{0.9, 0.2, 0.05, 0.0\}$	0.03	-0.002	-1.07	0.03	-0.003	-1.10	0.02	-0.003	-1.10	0.01
$\{0.9, 0.2, 0.05, 0.2\}$	0.06	-0.002	-1.03	0.03	-0.003	-1.05	0.02	-0.003	-1.05	0.01
$\{0.9, 0.2, 0.05, 0.8\}$	0.16	0.002	-0.99	0.03	0.001	-0.99	0.03	0.001	-0.99	0.02
$\{0.1, 0.49, 0.05, -0.8\}$	-0.11	-0.002	-0.98	0.03	-0.002	-0.98	0.02	-0.004	-0.96	0.01
$\{0.1, 0.49, 0.05, -0.2\}$	0.01	-0.002	-1.20	0.03	-0.003	-1.30	0.02	-0.003	-1.30	0.01
$\{0.1, 0.49, 0.05, 0.0\}$	0.04	-0.002	-1.05	0.03	-0.003	-1.08	0.02	-0.003	-1.08	0.01
$\{0.1, 0.49, 0.05, 0.2\}$	0.08	-0.002	-1.03	0.03	-0.003	-1.04	0.02	-0.003	-1.04	0.01
$\{0.1, 0.49, 0.05, 0.8\}$	0.17	-0.002	-1.12	0.03	-0.003	-1.02	0.02	-0.003	-1.02	0.01

Table 3: Monte Carlo finite sample Taylor effect (MC.T.E), relative biases (R.B.), Monte Carlo standard deviations (S.D.), Taylor effect (T.E.) in ARLMSV(1) models. T is the sample size.

			T=500			T=1000			T=5000	
$\{\phi_1, d, \sigma^2_\zeta, \sigma^2_\eta, \delta\}$	T.E.	MC.T.E.	R.B.	S.D.	MC.T.E	R.B.	S.D.	MC.T.E	R.B.	S.D.
$\{0.1, 0.2, 0.05, 0.05, -0.8\}$	-0.20	-0.002	-0.99	0.02	0.0005	-1.00	0.02	-0.001	-1.00	0.01
$\{0.1, 0.2, 0.05, 0.05, -0.2\}$	-0.05	-0.002	-0.96	0.02	0.001	-1.02	0.02	-0.001	-0.98	0.01
$\{0.1, 0.2, 0.05, 0.05, 0.0\}$	-0.0004	-0.002	-4.00	0.02	0.001	-3.50	0.02	-0.0005	0.25	0.01
$\{0.1, 0.2, 0.05, 0.05, 0.2\}$	0.04	-0.002	-1.05	0.02	0.001	-0.98	0.02	-0.0005	-1.01	0.01
$\{0.1, 0.2, 0.05, 0.05, 0.8\}$	0.17	-0.002	-1.01	0.02	0.0006	-1.00	0.02	-0.0006	-1.00	0.01
$\{0.9, 0.2, 0.05, 0.05, -0.8\}$	-0.17	-0.002	-0.99	0.03	0.0001	-1.00	0.03	-0.001	-0.99	0.01
$\{0.9, 0.2, 0.05, 0.05, -0.2\}$	-0.04	-0.002	-0.95	0.03	0.0001	-1.00	0.03	-0.001	-0.98	0.01
$\{0.9, 0.2, 0.05, 0.05, 0.0\}$	0.002	-0.002	-2.00	0.03	0.0001	-0.95	0.03	-0.001	-1.50	0.01
$\{0.9, 0.2, 0.05, 0.05, 0.2\}$	0.04	-0.002	-1.05	0.03	0.0002	-1.00	0.03	-0.001	-1.03	0.01
$\{0.9, 0.2, 0.05, 0.05, 0.8\}$	0.15	-0.002	-1.01	0.03	0.0003	-1.00	0.01	-0.001	-1.01	0.01
$\{0.1, 0.49, 0.05, 0.05, -0.8\}$	-0.12	-0.002	-0.98	0.03	-0.001	-0.99	0.02	-0.002	-0.98	0.01
$\{0.1, 0.49, 0.05, 0.05, -0.2\}$	-0.003	-0.002	-0.33	0.03	-0.001	-0.67	0.02	-0.002	-0.33	0.01
$\{0.1, 0.49, 0.05, 0.05, 0.0\}$	0.03	-0.002	-1.07	0.03	-0.001	-1.03	0.02	-0.002	-1.07	0.01
$\{0.1, 0.49, 0.05, 0.05, 0.2\}$	0.07	-0.002	-1.03	0.03	-0.001	-1.01	0.02	-0.002	-1.03	0.01
$\{0.1, 0.49, 0.05, 0.05, 0.8\}$	0.17	-0.001	-1.01	0.03	-0.0005	-1.00	0.02	-0.002	-1.01	0.01

Table 4: Monte Carlo finite sample Taylor effect (MC.T.E), relative biases (R.B.), Monte Carlo standard deviations (S.D.) and Taylor effect (T.E.) in 2FLMSV models. T is the sample size.

4 An Empirical Example

In this section, we take real data from the Dow Jones Industrial Index in order to determine whether the models are able to reproduce the empirical properties. The daily returns of the Dow Jones span the period 3/01/90 to 11/01/07 including a total of 4293 observations. The kurtosis of this series is 7.71 and the first order autocorrelations of the absolute and squared observations are 0.15968 and 0.15965, respectively. This implies a very small Taylor-Effect of 0.00003.

We have estimated the models using the Efficient Method of Moments (EMM) of Gallant and Tauchen (1996). The estimated parameters together with the implied Taylor-Effects are presented in Table 5. The results show that both the ARSV(1) and the 2FLMSV models are not able to reproduce the sample Taylor-Effect while the ARLMSV(1) model overestimates its magnitude.

	ϕ	ϕ_1	d	σ^2_η	σ^2_ζ	δ	σ	Estimated T.E.
ARSV(1)	0.98			0.02		-0.35	0.93	-0.04
ARLMSV(1)		0.93	0.40		0.01	-0.27	0.76	0.09
2FLMSV	0.99		0.41	0.01	0.06	-0.66	0.84	-0.11

Table 5: EMM estimates of the parameters. T.E. denotes Taylor-Effect. All parameters are statistical significant.

5 Conclusion

We have shown that not only the sign of asymmetry affects the Taylor-Effect but also the volatility persistence. In particular, a higher persistence and kurtosis lead to a more positive Taylor-Effect. These results are consistent with the ones found in the literature for the symmetric ARSV(1) model. Our Monte Carlo results reenforce the evidence that the models have difficulties in generating the Taylor-Effect even when it is present in the population. Finally, only in very special situations (high persistence and kurtosis) it happens that the Taylor-Effect is a sampling result due to the biases in the sample autocorrelations.

References

Bollerslev, T., J. Litvinova, and G. Tauchen (2006). Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics 4(3)*, 353–384.

Breidt, F., N. Crato, and P. de Lima (1998). On the detection and estimation of long memory in stochastic volatility. *Journal of Econometrics 83*, 325–348.

Dacorogna, M., R. Gencay, U. Muller, R. Olsen, and O. Pictet (2001). *An Introduction to High-Frequency Finance*. Academic Press, San Diego.

Durham, G. (2006). Monte carlo methods for estimating, smoothing, and filtering one and two-factor stochastic volatility models. *Journal of Econometrics 133*, 273–305.

Gallant, A. and G. Tauchen (1996). Which moments to match. *Econometric Theory 12*, 657–681.

Granger, C. and Z. Ding (1995). Some properties of absolute returns. An alternative measure of risk. *Annales d'Economie et de Statistique 10*, 67–91.

Granger, C., S. Spear, and Z. Ding (1999). Stylized facts on the temporal and distributional properties of absolute returns: an update. *Proceedings of the Hong Kong International Workshop on Statistics and Finance: and Interface*, 97–120.

Harvey, A. and N. Shephard (1996). Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics 4*, 429–434.

Harvey, A. and M. Streibel (1998). Testing for a a slowly changing level with special reference to stochastic volatility. *Journal of Econometrics 87*, 167–189.

He, C. and T. Teäsvirta (1999). Properties of moments of a family of GARCH processes. *Journal of Econometrics 92*, 173–192.

Malmsten, H. and T. Teräsvirta (2004). Stylized facts of financial time series and three popular models of volatility. *SSE/EFI Working Paper Series in Economics and Finance, Stockholm School of Economics*.

Mora-Gálan, A., A. Pérez, and E. Ruiz (2004). Stochastic volatility model and the taylor effect. *Working Paper, Universidad Carlos III de Madrid*.

Pérez, A. and E. Ruiz (2003). Properties of the sample autocorrelations of non-linear transformations in long memory stochastic volatility models. *Journal of Financial Econometrics 10*, 420–444.

Ruiz, E. and H. Veiga (2006). Modelling long-memory volatilities with leverage effect: A-lmsv versus fiegarch. *Working Paper, Universidad Carlos III de Madrid*.

Sandmann, G. and S. Koopman (1998). Estimation of stochastic volatility models via Monte Carlo maximum likelihood. *Journal of Econometrics 87*, 271–301.

Taylor, S. (1986). *Modelling Financial Time Series*. Wiley, New York.

Taylor, S. (1994). Modelling stochastic volatility: A review and comparative study. *Mathematical Finance 4*, 183–204.

Veiga, H. (2006). A two factor long memory stochastic volatility model. *Working Paper, Universidad Carlos III de Madrid*.

Yu, J. (2005). On leverage in a stochastic volatility model. *Journal of Econometrics 127*, 165–178.