FISCAL CONSTITUTIONS AND THE DETERMINACY OF INTERGENERATIONAL TRANSFERS

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Abstract

We study the impact of fiscal constitutions on intergenerational transfers by analyzing how political veto power influences social security. Transfers in this paper are outcomes of an infinite-horizon social security game among selfish agents whose lifecycles we embed in an overlapping generation model with a linear technology. Policies are decided one period at a time and may change later at zero cost. Simple majoritarian systems, which accord the current median voter maximum fiscal discretion and minimal influence over future policy, are known to sustain as subgame perfect equilibria all individually rational allocations. Among these are a continuum of stationary sequences (including dynamically inefficient ones) as well as a double continuum of non-stationary sequences (including cyclical or chaotic ones). We investigate how equilibrium is pinned down by constitutional "rules" that give minorities veto power over fiscal policy changes proposed by the majority. Veto power turns out to be equivalent to precommitment. Among subgame perfect equilibria, it eliminates fluctuating and dynamically inefficient transfers, reducing the equilibrium set to weakly increasing transfer sequences that converge to the golden rule. Veto power combined with Markov perfect equilibrium results in a unique, dynamic efficient allocation – the golden rule.

Key Words: Intergenerational Transfers, Veto Power, Constitutional Rules.
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1. INTRODUCTION

This paper studies how political constitutions influence intergenerational transfers, in particular, how veto power affects the determinacy and intertemporal efficiency of social security systems.

Indeterminacy is a property of economies in which finitely lived median voters are free to change the decisions of their predecessors at zero resource cost. Policymaking looks forward in environments of this type: today's decision depends on expectations of how tomorrow's median voter will react to the situation she expects to prevail the day after tomorrow, and so on ad infinitum. Policy choice is indeterminate because there is no way to pin down uniquely either the degree of coordination among successive generations of voters or the behavior of the "asymptotic" median voter.

As a counterpoint to median voter discretion, we propose an environment dominated by a fiscal constitution, that is, a framework which restricts the freedom to alter policies inherited from the past. In particular, a constitution that gives current voters or policymakers some veto power over changes in future policies brings to public choices an element of precommitment that helps pin down fiscal policy. Constitutional restrictions make future policies easier to predict when one knows past policies. They also deliver desirable properties of determinacy and optimality claimed for policy "rules" by Friedman (1948, 1968) and by Kydland and Prescott (1977).

The specific policy question we study is the evolution of pure intergenerational social security transfers among finitely-lived households in an infinite economy where individual preferences over fiscal transfers are single peaked, and policy conforms to the wishes of a well-defined "median voter" household. Analyzing social security naturally sheds light on a number of issues related to other intergenerational resource transfers, e.g., public debt, currency, and the generational distribution of the tax burden. As we
shall see in later sections, the reasons why societies maintain a social security system stem in part from a social compact and, hence, apply with equal force to issues like defaulting on public debt and preserving the purchasing power of currency.

Here we limit ourselves to social security in an overlapping generations model of production in which selfish individuals live two periods and consume one private good. Claims on this good are the only asset in the entire economy. We assume away altruistic preferences and the provision of public goods -- two key elements in the political economy of fiscal policy\(^2\) -- in order to bring out more clearly the impact of political institutions on intergenerational transfers.

Political institutions in this paper define the authority of the government, that is, of the median voter, to tax away income. We study two institutional environments that allow more and less of this power. The more discretionary of the two political systems we study is pure majority voting which permits the larger one of the two homogeneous population groups ("young" and "old") in our economy to reduce transfer payments to zero. The alternative is a constitution requiring the majority to obtain the approval of the minority to any changes in social security taxes and benefits.

Constitutional limits to fiscal transfers serve as an endogenous mechanism of partial policy commitment, one that encourages cooperation among successive median voters. Fiscal constitutions with veto power reduce the indeterminacy of allocations which occurs under majority voting. In particular, the set of subgame perfect equilibria shrinks to monotone, dynamically efficient transfer sequences converging to the golden rule. Even more impressively, veto power shrinks to a unique point the set of Markov perfect equilibria, and that point turns out to be the golden rule.

Section 2 describes a production economy whose subgame perfect equilibria we analyze in sections 3 and 4 under the majoritarian and constitutional systems. Section 5
studies Markov perfect equilibria. We discuss the literature relating to social security games and some extensions of our main results in sections 6 and 7.

2. A LINEAR PRODUCTION ECONOMY

To analyze the allocative effects of different fiscal policy under alternative political regimes, we start with a simple economic environment in which the government has a socially useful role. The economy is a standard dynamically inefficient overlapping generations model with a linear technology: it consists of an infinite number of two period lived cohorts. At any point in time only two generations are alive; we call them young and old. Individuals are identical within generations.

Agents in cohort \( t = 1, 2, \ldots \) evaluate consumption bundles \( (c'_t, c'_{t+1}) \) by the utility function

\[
u_t = U(c'_t) + \beta U(c'_{t+1})
\]

where \( c'_t \) represents the consumption at time \( t \) of the generation born at time \( t \) (the young), and \( c'_{t+1} \) is the consumption at time \( t+1 \) of the same generation (the old). The utility function is concave, twice differentiable and additively separable with \( \beta > 0 \).

Capital depreciates fully each period, and the production function is linear in labor and capital: \( F(K_t, L_t) = L_t + R \cdot K_t \) with \( R > 0 \). The marginal product of capital equals \( R \), and that of labor equals one. Agents supply labor inelastically. They are endowed with a non-negative vector \( (\varepsilon_1, \varepsilon_2) \) of efficiency labor units in youth and in old age. Population grows at a rate \( n > 0 \). There is no fiat money or public debt in this economy.

We assume that a competitive equilibrium without social security displays positive aggregate saving and dynamic inefficiency. Taken together, these two assumptions mean
The institution of social security in this economy allows the government to transfer resources into the future more efficiently than the accumulation of private capital. For instance, the associated "laissez-faire" lifecycle utility of a typical generation $t = 1, 2, \ldots$ is

$$v' = v\left(R, \varepsilon_1 + \frac{\varepsilon_2}{R}\right),$$

the indirect utility achieved when the interest rate is $R$ and the present value of lifecycle income is $\varepsilon_1 + \frac{\varepsilon_2}{R}$.

A social planner may easily achieve a higher level of lifecycle utility for all generations $t = 0, 1, \ldots$ by adopting the superior social security technology. One example is a stationary reallocation that levies on each person a lump sum tax $\tau \in [0, \tau_g]$, where $\tau_g$ is the golden-rule transfer, and distributes the proceeds equally among the older generation. The resulting utility,

$$v = \begin{cases} U(\varepsilon_1 - \tau) + \beta \cdot U(\varepsilon_2 + (1+n) \cdot \tau) & \text{if } t \geq 1 \\ U(\varepsilon_2 + (1+n) \cdot \tau) & \text{if } t = 0 \end{cases}$$

is higher than $v'$ because $1+n > R$. Recall that $\tau_g$ maximizes the RHS of the top line of equation 4 and provides the typical generation with the golden-rule utility $v_g$.

3. FISCAL POLICY UNDER MAJORITY VOTING

Many policy decisions in democracies require approval by a simple majority, that is, by 50% plus one vote in a chamber of deputies representing the electoral body. In practice, the will of an electoral majority may be thwarted by non-proportional representa-
tion, veto power from the executive or judiciary branches of government, or by voting blocks advancing the interests of particular groups. Still majoritarian systems are both descriptive and analytically tractable; for simple cases in which voters' preferences over policies outcomes are single peaked, the Median Voter Theorem enables us to aggregate individual tastes and obtain as an equilibrium of the voting process the outcome most preferred by a well defined agent -- the median voter.

Our analysis of majority voting over social security is in the spirit of Hammond (1975). We postulate a pay-as-you-go transfer system with no commitment technology: there is a vote every period which determines the social security tax $\tau_t$ levied on each young household, as well as the benefit $(1+n)\tau_t$ paid out to each old household. The current median voter in this arrangement cannot compel future voters to pay tax $\tau_t$ for any $s > t$.

To reflect electoral realities and provide some incentives toward intergenerational cooperation, we assume that the median voter belongs in the young generation $t$.

The set $Y$ of feasible fiscal policies contains all tax/transfer schemes that ensure consumption by the young is non-negative,

\begin{equation}
\tau_t \in Y = [0, \epsilon_t]
\end{equation}

and the median voter in generation $t$ maximizes

\begin{equation}
U(\epsilon_t - \tau_t) + \beta \cdot U(\epsilon_{t+1} (1 + n)\tau_t)
\end{equation}

subject to (5) and given the strategy followed by the median voter of the succeeding cohort $t+1$.

3.1 Open-Loop Equilibrium

A convenient benchmark to start with is open-loop strategies that depend purely on calendar time and not at all on history. These strategies are independent of the actions of
preceding players, both in and out of equilibrium, and hence provide no incentives for cooperation among generations. Hammond (1975) and Sjoblom (1985), in fact, recognized that the open loop outcome is zero social security; Loewy (1988) also found that the open loop equilibrium of a monetary economy shrinks to zero the purchasing power of currency. Therefore, in open loop equilibria the social security technology is not used, and agents obtain the "laissez-faire" lifecycle utility.

3.2 Subgame Perfect Equilibria

If selfish median voters are who behave in the apparently cooperative fashion that sustains a social security system, they do so from the vantage point of enlightened self-interest, that is, because each cohort is individually better off with a social security system in place than without one. Incentives to coordinate fiscal policies over cohorts of median voters may be thought of as social compacts or "norms" enforced by a system of rewards and punishments.

Here is an example of how reinforcement works. Cohorts that transfer to the old the resources specified by the norm expect to receive in their own old age a normal payment; cohorts that defect from the norm in their youth expect to receive a zero transfer in old age. Social norms in this example are enforced by a sequence of trigger strategies that connect the decisions of median voters with the behavior of their predecessors. As we know from Kandori (1992) and Salant (1991), these strategies make cooperation individually rational when it is unfeasible to commit to a future policy course.

Simple majoritarian systems turn out to sustain any individually rational allocation as a subgame perfect equilibrium by the use of an appropriate trigger strategy profile. This folk-like result, conjectured in Hammond (1975), has the following formal statement:

**Proposition 1** (Majoritarian Folk Theorem): For every feasible profile \((v, *)\) of lifecycle utilities bounded below by the "laissez faire", or open-loop, equilibrium
utility level $v'$, there exists a subgame perfect Nash equilibrium of the
majoritarian social security game that starts at $t$ and pays off $v_s^* \geq v'$ for all $s > t$.

To see this consider the following strategy profile $(\tau_s^*)^\infty$ for the median voter, consistent
with payoffs $(v_s^*)^\infty$:

\[
\tau_t^* = \begin{cases} 
\tau_t^* & \text{if } \tau_{t-1}^* = \tau_{t-1}^*, \\ 
0 & \text{otherwise}
\end{cases}, \quad \text{ for } i = 1, \ldots, t.
\]

We will show that there are no gains from deviating from the above strategies, that is, no
median voter will be the first to deviate from the optimal policy $\tau^*$; and it is incentive
compatible to punish all defectors.

The payoff from deviating is the "laissez faire" equilibrium utility level $v'$; while the
payoff from the strategy $\tau^*$ is $v'$ which exceeds $v'$ by construction. Hence, $\tau^*$ is
incentive compatible. Furthermore, the utility of punishing a defector is still the "laissez
faire" equilibrium level $v'$ which exceeds the utility from not punishing because

\[U(\varepsilon_1 - \tau^* - s) + \beta \cdot U(\varepsilon_2 + \beta \cdot s) > v' \quad \text{for } \tau^* > 0.\]

Fiscal policies$^5$ sustaining subgame perfect equilibria under majority voting are
ones that make each cohort prefer intergenerational cooperation to the open-loop outcome.

In particular, we have the following:

**Proposition 2 (Characterization Theorem):** There is a continuous, increasing, convex
function $\phi : \mathbb{R} \rightarrow \mathbb{R}$ and a maximum sustainable tax $\tau_{\text{max}}$ such that the set of
feasible, individual rational fiscal policy $(\tau_s^*)^\infty$ satisfies

\[
\begin{aligned}
(i) & \quad \tau_{s+1}^* \geq \tau_s^* \frac{R}{1 + n} \quad \text{if } R \geq MRS(\varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s + 1 + n) \\
& \text{or } R < MRS(\varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s + 1 + n)
\end{aligned}
\]
Proof of Proposition 2: Define the function $\phi : Y \rightarrow Y$ from

$$U(e_1 - \tau_s^*) + \beta \cdot U(e_2 + (1 + n) \cdot \tau_{s+1}^*) = v'.$$

Part (i) of this proposition is derived from individual rationality. Moreover, it implies that $\tau_{s+1}^*$ is an increasing and quasi-concave function of $\tau_s^*$ with $\phi(e_1) \geq e_1$. Thus, it exists another fixed point of the function defined in part (i), $\tau = \tau_{max}$.

Q.E.D.

As shown in Figure 1, the map defined by part (i) of proposition 2, which connects today's social security tax with the lowest incentive compatible tax for tomorrow, has two fixed points. These are the zero transfer, and a higher value, $\tau_{max}$, above which individually rational transfers explode and youthful consumption becomes negative in finite time. Figure 2 displays the corresponding equilibrium allocations.

3.3 Indeterminacy of Majoritarian Equilibria

Any feasible social security sequence $(\tau_s)_t$ that satisfies the inequalities (i) and (ii) listed in Proposition 2 is a subgame perfect equilibrium of the majority voting system. Figures 1 and 2 display all these sequences both directly and also in terms of the old-age consumption that corresponds to each one. Specifically, the set of equilibria contains:

(i) a continuum of constant sequences $\tau_t = \tau \in Y = [0, \tau_{max}] \ \forall t$;

(ii) dynamically inefficient sequences bounded above by the golden rule, e.g., sequences that satisfy $\tau_s \leq \tau_s - \gamma$ for some $\gamma > 0$ and $\forall t$;

(iii) volatile, cyclical and chaotic sequences generated by the tentlike map drawn in Figure 3.

Note also in Figures 1 and 2 that subgame perfect equilibria exist that pay off every cohort, except the initial old, more than the golden rule utility level. These are associated
with the superior social security technology, at \( \tau_r = \tau_g \), and with zero capital accumulation. This bonanza is made possible by "inventing" social security in some finite period with an initial benefit below the golden rule value \( \tau_g \). The resulting surplus may then be spread among all subsequent cohorts by a rising sequence of benefits which converges to \( \tau_g(1 + n) \).

The large amount of indeterminacy present in Figures 1 and 2 stems directly from the inability of voters to commit their successors to a particular course of fiscal policy. Section 4 and 5 explore how refinements in political institutions bring about drastic changes in both the size and the volatility of fiscal policies.

4. CONSTITUTIONAL RULES

Large policy adjustments in a democratic society often require wider approval than that of a simple legislative majority. This observation applies particularly when: (i) there is uncertainty over the identity or preferences of future policymakers; and also when (ii) the policy change under consideration contains the seeds of its own reversal because it affects adversely the interests of, and will likely draw loud objection from, a politically significant group.

Case (i) is similar to the one examined in Tabellini and Alesina (1990) where today's median voter does not know for sure the tastes of his successor over public goods. To perpetuate his own favorite bundle of public goods, today's median voter runs a large deficit which reduces the resources available to his successor.

In what follows we ignore all future uncertainty and focus on case (ii). Specifically, we consider a political arrangement that partly precommits fiscal policy by awarding the current median voter veto power over future policy changes. Veto power is exercised through a Constitution, assumed to be fixed and immutable for the time being; we discuss in Section 6 how this arrangement may come about. The Constitution
empowers the younger cohort at time $t$ to set up a binary fiscal policy agenda $T_t$, and entrusts the old with choosing from the agenda the actual policy $\tau_t$ to be implemented this period.

Formally, we have

$$\tau_t \in T_1 = \{\tau_{t-1}, p_t\}$$

where $\tau_{t-1}$ is last period's actual policy and $p_t \in [0, c_1]$ is the new social security tax level proposed by the young. The status quo fiscal policy $\tau_{t-1}$ plays the role of a state variable here and makes all the difference between the constitutional political structure and the majoritarian one.

Constitutionalism in this setting encourages commitment. The old can guarantee themselves the same social security as their immediate predecessors by vetoing any change $p_t \neq \tau_{t-1}$. The young, too, can ensure a constant fiscal policy sequence by choosing $p_t = \tau_{t-1}$, i.e., with an offer to maintain the status quo ante.

We assume that the economy starts off in autarky, without a social security system, and the initial agenda in period one is $T_1 = \{0, p_1\}, p_1 \geq 0$.

Old generations have a simple decision: they pick the largest item on the agenda because their utility is monotone in the size of the transfer. The old would clearly choose to exercise their power to veto any reduction in social security; hence transfer sequences will be non-decreasing.

Agenda setting by the younger cohort guarantees them the golden rule payoff $v_g$. This is easiest to see when the social security system is invented; starting from autarky at $t = 1$, any young cohort can get a unanimous vote to raise the social security transfer level from zero to the golden rule value $(1+n)\tau_g$, and veto any fiscal changes in the subsequent period $t = 2$. In the Appendix we show that any young generation that wishes
to alter inherited fiscal policy can guarantee itself the golden rule payoff which is simply the utility of defecting from the equilibrium path.

Given the defection payoff, it is now straightforward to prove the following constitutional analogs of Propositions 1 and 2. The proofs are in the Appendix.

**Proposition 3** (Constitution Folk Theorem): For every feasible profile $\left( v_s^* \right)_t$ of lifecycle utilities whose lower bound is the golden rule level $v^g$, there exists a subgame perfect Nash equilibrium of the constitutional social security game that starts at $t$ with zero social security and pays off $v_s^* \geq v^g$ for all $s > t$.

Fiscal policies that support these payoffs are described in

**Proposition 4** (Characterization Theorem): There is a continuous, increasing, convex function $\psi : Y \to Y$ and a maximum sustainable tax $\tau^*$ such that the set of feasible, individual rational fiscal policy $\left( \tau_s^* \right)_t$ satisfies

$$\begin{cases} 
\tau^*_{s+1} \geq \frac{\tau^*_s}{1+n} + \frac{R}{1+n} & \text{if } R \geq MRS\left[ \varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s, (1+n) \right] \\
\tau^*_{s+1} \geq \psi(\tau^*_s) & \text{if } R < MRS\left[ \varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s, (1+n) \right]
\end{cases}$$

where $x$ is uniquely defined from $v^* R, \varepsilon_1 + \frac{\varepsilon_2 + x}{R} = v^*$

(i) $\tau^*_{s+1} \geq \frac{\tau^*_s}{1+n} + \frac{R}{1+n}$ if $R \geq MRS\left[ \varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s, (1+n) \right]$

(ii) $\tau^*_{s+1} \geq \psi(\tau^*_s)$ if $R < MRS\left[ \varepsilon_1 - \tau^*_s, \varepsilon_2 + \tau^*_s, (1+n) \right]$

(iii) $\tau^*_s \in [0, \tau^*_s]$

Recall that $v^* R, \varepsilon_1 + \frac{\varepsilon_2 + x}{R}$ is the indirect utility as a function of the interest rate and lifecycle income. Once more, part (i) in Proposition 4 solves the median voter’s individual rationality constraint.
with the equal sign holding. The function $\psi$ -- interpreted again as a map between today's actual tax and tomorrow's minimum incentive compatible tax -- has only one fixed point this time, just $\tau_g$, as Figures 4 and 5 show.

Since no stationary allocation can pay more than the golden rule, one consequence of the individual rationality constraint in eq.(9), is the following

**Corollary:** The golden rule is the unique stationary subgame perfect equilibrium.

Of course, it is possible for non-stationary equilibria to pay off more than the golden rule utility because an initial old generation received less. However, the distance of any non-stationary equilibrium from the golden rule must asymptotically shrink to zero because it would take an explosive sequence of transfers to keep welfare bounded away from the golden rule payoff. In fact, one easily demonstrates the following result.

**Proposition 5:** All constitutional equilibria support allocations that converge to the golden rule.

The proof is straightforward: the tax sequence $(\tau^*_i)$ is bounded and weakly increasing by definition. It converges to the golden rule value $\tau_g$ because it would be individually irrational for $\tau^*_i$ to remain bounded above by a number less than $\tau_g$, and infeasible to remain bounded below by a number bigger than $\tau_g$. The irrationality is simple to show; if $\tau^*_i < \tau_g$, then equation (9) tells us that a proposal to raise the social security tax to $\tau_g$ immediately, and to veto subsequent changes would receive the unanimous approval of all currently existing generations. The infeasibility of maintaining $\tau^*_i$ some distance above $\tau_g$ forever comes again from Part (i) of Proposition 4 which requires transfer payments to
increase faster than the rate of growth \( n \) if \( \tau_i^* > \tau_g \).

The main insight from this section is to look at Propositions 3 and 4 jointly and conclude that a constitutional grant of veto power to the minority is sufficient to eliminate all volatility and all dynamic inefficiency from majoritarian subgame perfect equilibria. Figure 4 shows how much the policy commitment emanating from this power shrinks the set of equilibrium allocations. Only one steady state survives; all cyclical, chaotic and dynamically inefficient equilibria disappear.

5. MARKOVIAN FISCAL POLICIES

One way to restrict fiscal policy is to regard it as a stable function of some state variable and not as a sequence which depends on calendar time. Krusell, Quadrini and Ríos-Rull (1997), Grossman and Helpman (1995) and other authors study Markovian policies for which the stock of physical capital is the relevant state variable. These policies restrict economies with different histories but identical structure and capital stock to adopt the same fiscal policy.

In this section we impose this restriction on intergenerational transfer decisions and show how it reduces the indeterminacy of equilibrium allocations by one degree. However, not all of the resulting equilibrium transfer sequences need be efficient.

Young individuals take as given the tax sequence and choose savings to solve the following economic optimization problem:

\[
\max_{\{c_t, c_{t+1}\}} U(c_t) + \beta U(c_{t+1})
\]

s.t.

\[
c_t \leq \varepsilon_1 - \tau_t - s_t
\]

\[
c_{t+1} \leq \varepsilon_2 + (1 + n)\tau_{t+1} + Rs_t
\]

Moreover, the median voter in each young cohort has to determine the level of
intergenerational transfer to be awarded to the old generation. We restrict the choice of the median voter to stationary, or Markovian, policy functions of the following type:

\[ \tau_i = \begin{cases} \theta_i(k_i) & \text{for } k_i > 0 \\ A_i & \text{for } k_i = 0 \end{cases} \]

Thus, the median voter’s optimal decision can be obtained by maximizing her lifecycle utility with respect to the function \( \theta_i(k_i) \), subject to the accumulation relation \( s_i = (1 + n)k_{i+1} \) which equates net private wealth to the aggregate capital stock.

The first order condition for the median voter’s maximization problem is

\[ -R + (1 + n) \frac{\partial \theta_{i+1}}{\partial k_{i+1}} \frac{\partial k_{i+1}}{\partial \theta_i} = 0 \]

The first term, \( R \), is the marginal cost of increasing today’s tax rate, whereas the second term represents the marginal benefit, which comes from the increase in \( \theta_i \) on tomorrow’s capital stock. The saving function can be written in terms of lifecycle after-tax endowments \( s_i = z[\varepsilon_1 - \theta_i(k_i), \varepsilon_2 + (1 + n) \cdot \theta_{i+1}(k_{i+1})] \).

It follows that

\[ (1 + n) \frac{\partial k_{i+1}}{\partial \theta_i} = -\frac{z_1}{1 - z_2} \cdot \frac{\partial \theta_{i+1}}{\partial k_{i+1}} \]

where, for \( i=1,2,\ldots \), \( z_i \) represents the derivative of the saving function with respect to the period-i net endowment. Substituting (13) into (12) and rearranging terms we obtain the differential equation

\[ \frac{\partial \theta_{i+1}}{\partial k_{i+1}} = -\frac{R}{z_1 - Rz_2} \]

Integrating we obtain,

\[ \theta_{i+1}(k_{i+1}) = A - \frac{R}{z_1 - Rz_2} k_{i+1} \]
The constant of integration, $A$, is a free parameter which is pinned down by the first median voter’s expectations of future policies. Notice also that, for the period-$t$ median voter, the political optimization problem turns out not to depend on the function $\theta$, as long as the next median voter also chooses the fiscal policy according to equation (15).^7

In order to simplify the analysis, we assume that the utility function displays constant elasticity of substitution: $U(c_i) = c_i^{1-\lambda} / (1 - \lambda)$ for $\lambda \neq 1$. It is easy to show that equation (15) then simplifies to:

\begin{equation}
\theta_{t+1}(k_{t+1}) = A - Rk_{t+1}.
\end{equation}

Combining this policy function with the saving function implied by a CES utility function and the accumulation relation, we derive the following law of motion for the stock of capital:

\begin{equation}
k_{t+1} = \max \left\{ 0, \frac{\varepsilon_1 - A}{1 + n} - \frac{\varepsilon_2 + A(1 + n)}{(1 + n)(\beta R)^{1/\lambda}} + \frac{R}{1 + n} k_t \right\}
\end{equation}

subject to $k_{t+1} \leq \frac{A}{R} \forall t$.

The capital accumulation dynamics depend on the median voter’s expectations through the free parameter $A$. The capital stock converges to its steady state value

\begin{equation}
k_{SS} = \max \left\{ 0, \frac{\varepsilon_1 - A}{1 + n - R} - \frac{\varepsilon_2 + A(1 + n)}{(1 + n - R)(\beta R)^{1/\lambda}} \right\}.
\end{equation}

In particular, there are two possible cases. Let us define by $A^* = (\beta R)^{1/\lambda} \varepsilon_1 - \varepsilon_2$ the value of the free parameter that sets to zero the intercept of the law of motion from equation (17). If $A < A^*$, then the economy converges to a steady state associated with a positive capital stock, as figure 6a shows. If $A \geq A^*$, on the other hand, the economy converges to a zero capital stock steady state along the path shown in figure 6b or 6c. The restriction
in (17) guarantees that the social security tax rate is non-negative.

The evolution of capital has its mirror image in the dynamics of equilibrium tax rates. In fact, using the Markovian policy function in equation (11), and the law of motion for the stock of capital in equation (17), we can write the law of motion of the equilibrium tax rate as follows:

\[
\theta_{t+1} = \min \left\{ A, \frac{(\beta R)^{\frac{1}{2}} + R}{(\beta R)^{\frac{1}{2}}} A - \frac{R}{1+n} \frac{(\beta R)^{\frac{1}{2}} \varepsilon_1 - \varepsilon_2}{(\beta R)^{\frac{1}{2}}} + \frac{R}{1+n} \theta_t \right\}.
\]

Here again, depending on the median voter’s expectations through the parameter \(A\), we have three possible cases. We define \(A' = \frac{R}{1+n} \frac{(\beta R)^{\frac{1}{2}} \varepsilon_1 - \varepsilon_2}{(\beta R)^{\frac{1}{2}} + R} = \frac{R}{1+n} \frac{(\beta R)^{\frac{1}{2}} + 1+n A^*}{(\beta R)^{\frac{1}{2}} + R}\) to be the value that sets to zero the intercept of equation (19). If \(A > A^* > A'\), then the equilibrium social security tax rate sequence is monotonically increasing to its maximum value, \(\theta = A\), as figure 7a shows. If \(A > A'\) and \(A < A^*\), then the sequence converges to its steady state value

\[
\theta_{SS} = \frac{1+n}{1+n-R} \frac{(\beta R)^{\frac{1}{2}} + R}{(\beta R)^{\frac{1}{2}}} A - \frac{R}{1+n-R} \frac{(\beta R)^{\frac{1}{2}} \varepsilon_1 - \varepsilon_2}{(\beta R)^{\frac{1}{2}}} < A,
\]

as in figure 7b. The last case corresponds to \(A < A' < A^*\); here the tax rate monotonically decreases to zero, as shown in figure 7c, and resources are transferred into the future exclusively by accumulating physical capital.

The parallel between capital accumulation and equilibrium tax dynamics is straightforward. Equilibria with a positive level of capital are clearly inefficient as agents do not fully exploit the superior social security technology. Indeed, even zero capital equilibria might be inefficient as long as the social security tax rate is lower than the golden rule level. In fact, efficient allocations are obtained for zero capital and \(\theta = A \geq \tau_g\).
To summarize, for a given initial level of capital stock, the set of equilibrium social security tax rates associated with a stationary or Markovian policy function contains:

- a continuum of decreasing sequences (indexed by the parameter $A$) converging to a steady state with zero social security and positive capital;
- a continuum of monotonically increasing or decreasing sequences (again indexed by $A$) converging to the steady state level $\theta_{\text{SS}}$ defined in equation (20), and to a positive capital stock;
- a continuum of monotonically increasing sequences converging to a steady state tax rate level $A$, and a zero capital stock; and
- an increasing sequence converging to the golden rule transfer

\[ \theta = A = \tau_{\rho} = \frac{(\beta R)^{1/2} e_1 - e_2}{(\beta R)^{1/2} + 1 + n}. \]

The adoption of stationary or Markovian policy functions decreases the intrinsic indeterminacy of these intergenerational transfer schemes to a one dimensional indeterminacy. The long run dynamics of the system are indexed on the median voters’ expectations through the parameter $A$, which also determines the efficiency properties of the equilibrium allocations.

One way to allow the median voter to form her expectations about future policy is to introduce some degree of commitment, along the lines suggested in the previous section. Specifically, if we restrict the decision space of the voters by introducing veto power, the free parameter $A$ is pinned down. In this setting, the expectation parameter $A$ equals the extreme right-hand side of equation (21), and the economy converges to golden rule with zero capital and a social security tax rate equal to $A$. 
6. RELATED LITERATURE

Social security has a similar role to public debt in reallocating consumption among successive population cohorts. Like public debt and fiat money, social security is a "social contrivance" whose value as a transfer payment mechanism depends on mutual trust among cohorts and on some degree of intergenerational cooperation. In plain language, social security is like a bubble, and it would be useful to relate the social security equilibria we studied in Sections 2 and 3 with the dynamics of public debt and fiat money we have learned from Wallace (1980), Tirole (1985) and others. The connection is easiest to establish in situations of zero primary budget deficits. Consider, for example, an "actuarially fair" tax sequence \((\tau, \ldots, \tau, \ldots)\) such that

\[
-t + \frac{1+n}{R_{t+1}} \tau_{t+1} = 0
\]

where

\[
R_{t+1} = \frac{U'(e - \tau)}{\beta U'(e + (1+n)\tau_{t+1})}
\]

This sequence adds zero present value to each generation's lifecycle income computed at interest rates that correspond to marginal rates of substitution at the consumption vector \(c' = (e_1 - \tau, e_2 + (1+n)\tau_{t+1})\) implied by the sequence \((\tau, \ldots, \tau, \ldots)\). Each element of this sequence represents excess supply by a typical member of generation \(t\) as well as \(1/(1+n)\) times the excess demand by each member of generation \(t-1\). Equations (22) and (23), taken together, describe the reflected offer curve of a generation-\(t\) household.

Figure 8 reminds us that this curve coincides with the phase diagram of equilibrium in pure-exchange economies with a given stock of fiat money or public debt. All we need to reinterpret actuarially fair social security as public debt or currency is to
think of $\tau_r$ as the real per capita value of the government liability, and of eq. (22) as the government budget constraint in an economy with zero public consumption and zero primary budget deficit. Then it is easy to see that the golden rule outcome is the only stationary actuarially fair equilibrium, and one that is likely to prevail under a credible constitutional arrangement which commits to maintaining the purchasing power of social contrivances -- or bubbles -- like currency, public debt or social security.

By the same token, the indeterminacy of equilibrium we encounter in economies with bubbles is directly related to the absence of a credible promise from the Treasury or the Central Bank to preserve the future value of the bubble. Another source of indeterminacy creeps in if, in addition, we permit governments or median voters to deviate from the "fairness" of the present value relation (22) by running a primary budget deficit of their choosing. Then majoritarian equilibria will display the two degrees of indeterminacy exhibited by the subgame perfect allocations of Figures 1 or 2.

Reducing the large set of subgame perfect equilibria has been a priority in the fiscal policy literature ever since Hammond (1975); it is typically achieved by ruling out trigger strategies. Kotlikoff, Persson and Svensson (1988) and Esteban and Sakovics (1993) restrict the strategy sets of the median voter to costly Markovian strategies of the form $\tau_{i+1} = \phi(\tau_i)$ which assign a fixed resource cost $k > 0$ to any change in the social security tax. The resource cost is a form of partial commitment. For economies starting with zero transfers, the non-cooperative pure-strategy outcome in the case of "small" $k$ is to reach a transfer somewhat short of the golden rule and to remain there forever.

More eclectic equilibrium refinements include Boldrin and Rustichini (1995), who assign the entire surplus from inventing social security to the generation that actually invents the system. Cooley and Soares (1995) study majoritarian stationary equilibria in a calibrated pure exchange economy with four-period lived households, focusing on the tax
rate preferred by the second youngest generation; it is this generation which closely corresponds to the age profile of the median U.S. voter.

7. HETEROGENEOUS VOTERS AND OTHER EXTENSIONS

We discuss here the robustness of the conclusion reached earlier about the operating characteristics and the social desirability of constitutional rules for fiscal policy.

First of all, it is fairly straightforward to see under what circumstances our basic results extend to economies which may wish to redistribute income from richer to poorer individuals. How will intragenerational transfers to the poor interfere with, or influence, intergenerational transfers to the old? The answer is typically "not at all" if society possesses two independent instruments for these two redistributive targets; but social security allocations may well respond to changes in income distribution if the social security structure is progressive in taxes or transfers.

To see this in a particular simple case, we emulate Meltzer and Richard (1981) indexing households by the amount \( e \in [0,e'] \) of efficiency labor units supplied in youth. Suppose that each individual type is public knowledge and that types are distributed according to a cumulative function \( H \) such that

\[
H(0) = 0, \quad H(e) = 1 \quad \forall e \geq e'
\]

Aggregate supply of efficiency labor per unit population continues to equal one, e.g.,

\[
\int edH = 1
\]

A fiscal policy is now a triple \((\tau_t, p_t, s_t)\) for each period, consisting of a proportional tax rate \( \tau_t \in [0,1] \), a per capita subsidy to the young \( p_t \geq 0 \), and a proportional social security transfer \( e\sigma_t \) to retirees of type \( e \). The parameter \( p_t \) controls the distribution of income within generation \( t \) while \( s_t \) regulates how resources are shared between
generations \( t-1 \) and \( t \).

Among individuals alive at time \( t \) only the poorer within generation \( t \) would favor a positive lump-sum intragenerational transfer payment \( p \). Richer members of the same generation will be against this transfer scheme because they are net contributors to it. Older folk, who care only about social security pensions, may also be opposed to the extent that intragenerational transfers subtract tax revenue away from pensions. In fact, voters will reject welfare payments to the poor whenever the fraction of people with below-median income is not too large. In the Appendix we prove the following result:

**Proposition 6:** If \( H(1) < \frac{2+n}{2+2n} \), any fiscal policy \((\tau, p, s)\) with \( p \geq 0 \) is defeated in a pairwise majority vote by another \((\tau, 0, s + \delta)\) for some small \( \delta > 0 \).

This result implies that, unless the distribution of income is extremely skewed toward the left tail, dynamically inefficient economies will vote for purely intergenerational transfers.

Another issue is to endogenize the voting structure, permitting the electorate to choose a majoritarian or constitutional fiscal structure, and to switch from one to the other according to well-specified rules. Fiscal structures may be thought of as the outcomes of an enlarged game in which the young propose both the choice of institutional arrangement (majoritarian or constitutional) and the level of the transfer. The rules of this game require that, if society is organized along constitutional lines, the consent of the old must be obtained prior to any change, e.g., before either a change in the transfer level or a switch from the constitutional to a majoritarian system. A simple majority suffices for all other decisions. Proposition 4 is likely to generalize in this setting, that is, weakly increasing transfer sequences converging to the gold rule continue to be equilibria of the "grand" constitutional game. Starting with a majoritarian system and zero transfers, the median voter can guarantee herself the golden rule utility by switching to a constitution and
opposing all future changes; the rest of the argument proceeds exactly like Proposition 4.

A third problem is the inflexibility of constitutional fiscal structures. Imagine, in particular, that voters set the tax at a numerical level independent of any realizations of the income and other parameters of the economy. An instructive example of this phenomenon is to endow the young with a binary stochastic income stream.

\[ s_i = \begin{cases} \alpha & \text{with probability } p \\ \beta > \alpha & \text{with probability } 1 - p \end{cases} \]

We keep old-age labor endowment constant at \( \varepsilon_2 \) as before, and assume that voting takes place each period after endowment realizations. Hence the first young generation that experiences the high income realization, \( \beta \), will obtain constitutional approval for raising the transfer level to the golden rule of the economy with deterministic income vector \( (\beta, \varepsilon_2) \). In later periods, old voters in this economy will resist any proposal to lower the transfer to the level implied by the golden rule of the economy with the smaller deterministic income vector \( (\alpha, \varepsilon_2) \), even if \( \alpha \) happens to be the income of the actual tax-paying generation. It is, in this sense that constitutions or veto power may lead to "excessive" social security. Similar arguments can be made about random changes in population parameters.

We conclude with a comment on the robustness of Proposition 4. Are constitutionally set social security transfers likely to go up over time if changes in the system require an elevated majority of voters rather than complete unanimity? This situation may be explored in a model with more than two coexisting generations like Cooley and Soares (1995). In particular, would a coalition of say, 1/4 or more of all voters attempt to block a proposal to reduce permanently social security taxes or benefits
by $\delta > 0$? The answer depends on the demographic and income structure of the economy and on several other things, but it appears to us to be in the affirmative. This proposal will surely be opposed by all retirees and all persons sufficiently near retirement because it reduces the present value of their remaining lifetime incomes. The cutoff age depends on the prevailing rate of interest, but assuming this age to be about 53 years, opposition against a reduction in social security will unite 20 or more cohorts in the 56-75 age group, with 30 cohorts in the 21-50 age group (including the median voting cohort) being in favor of reform, and the 51 to 55 year olds standing on the margin. If this opposition group has veto power, Proposition 4 will extend to economies with richer demographies than the one we have studied here.
REFERENCES


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TECHNICAL APPENDIX

Proof of Proposition 1: Consider the following strategy for the median voter at \( t+j, \forall j > 0 \):

\[
\tau_{t+j} = \begin{cases} 
\tau_{t+j}^* & \text{if } \text{either } \tau_{t+i,j-1} = \tau_{t+i,j-1}^* \quad \forall i = 1, \ldots, 2n \text{ and } n = 1,2,\ldots \\
0 & \text{otherwise}
\end{cases}
\]

We will show that there are no gains from deviating from the above strategy, that is, no median voter will be the first to deviate from the optimal policy \( \tau^* \); and it is incentive compatible to punish all defectors.

The payoff from deviating is the "laissez faire" equilibrium utility level \( v' \), while the payoff from the strategy \( \tau^* \) is \( v^* \) which exceeds \( v' \) by construction. Hence, \( \tau^* \) is incentive compatible. In addition, the utility of punishing a defector,

\[
v(R, \varepsilon_1 + [\varepsilon_2 + (1 + n) \cdot \tau_{t+i,j-1}^*] / R)
\]

clearly exceeds that of not punishing,

\[
v(R, \varepsilon_1 - \tau_{t+i,j}^* + \varepsilon_2 / R).
\]

Proof of Proposition 2: Define the function \( \phi: Y \rightarrow Y \) from

\[
U(\varepsilon_1 - \tau_i^*) + \beta \cdot U(\varepsilon_2 + (1 + n) \cdot \tau_{t+i,j}^*) = v^*
\]

and note that individual rationality implies part (i) of this Proposition. The definition of \( \phi \) implies that it is increasing and concave with \( \phi(\varepsilon_1) > \varepsilon_1 \), which implies that \( \phi \) has another fixed point, \( \tau = \tau_{\text{max}} \), in the interval \( (\tau_g, \varepsilon_1) \).

Proofs of Propositions 3 and 4: We show first that the utility of defecting here equals the golden rule payoff \( v_g \). Payoffs are

\[
v^* = v(R, \varepsilon_1 - \tau_{t+i,j}^* + [\varepsilon_2 + (1 + n) \cdot \tau_{t+i,j}^*] / R)
\]

if a generation-t voter maintains the transfer policy \( (\tau_{t+i,j}^*, \tau_{t+i,j}^*) \), and
\[ v_i^D = \max_{\tau_{r_i-1}} v(R, \epsilon_1 - \tau + \left[ \epsilon_2 + (1 + n) \cdot \tau \right] / R) \] if she defects by changing upward the sequence and blocking all subsequent attempts to alter the new sequence. The utility of defection is clearly

\[
\begin{align*}
(A-3) \\
v_i^D = \begin{cases} \\
v_g & \text{if } \tau_{r_i-1}^* \leq \tau_g \\
U(\epsilon_1 - \tau_{r_i-1}) + \beta U(\epsilon_2 + (1 + n) \cdot \tau_{r_i-1}) & \text{if } \tau_{r_i-1}^* > \tau_g .
\end{cases}
\end{align*}
\]

But setting \( \tau_{r_i-1}^* > \tau_g \) cannot be part of any individually rational sequence \( \{\tau_i^*\} \) starting from \( \tau^* = 0 \) for that sequence would have to cross the golden rule transfer \( \tau_g \); this means the existence of a subsequence \( \{\tau_i^*\} \) such that \( \tau_{k-1}^* < \tau_g, \tau_k^* > \tau_g \). To maintain individual rationality, this subsequence would have to pay off at least \( v_g \) to any median voter in it, which requires its elements to increase rapidly past \( \tau_g \) and diverge.

The remainder of the proofs is directly analogous to that of Propositions 1 and 2, except for the veto power awarded to the old which simply implies that any subgame perfect equilibrium sequence \( \{\tau_i\} \) must be weakly increasing.

**Proof of Proposition 6:**

The fiscal policy \( \pi_i = (r_i, p_i, s_i) \) satisfies the public sector budget constraint

\[
(A-4) \\
r_i = p_i + s_i / 1 + n
\]

and implies a lifecycle net income vector \( ((1 - r)e + p_r, e_s, v) \) for an individual of type \( e \).

The corresponding lifecycle consumption vector is

\[
(A-5) \\
((1 - r)e + p_r - z(e), Rz(e) + e_s, v)
\]

where \( z(e) \geq 0 \) is saving by type \( e \) household.
Payoffs from the policy $\pi_t = (\tau_t, p_t, s_t)$ are

\begin{align}

v^o_t(e, \pi_t) &= RZ_{t-1}(e) + es_t & \text{for old people} \\
v^y_t(e, \pi_t) &= v\left(R, (1-\tau_t)e + p_t + \frac{s_{t+1}}{R}\right) & \text{for young people}
\end{align}

Here $v$ is an indirect utility function which depends on the interest rate and on the present value of lifecycle income.

Older people evaluate $\pi_t$ by its social security component alone, while young people care about the present value of net transfers,

\begin{equation}

b_t = -\tau_t e + p_t + es_{t+1} / R
\end{equation}

Suppose also that young people expect that $s_{t+1}$ does not depend in any way on the transfer payment $p_t$ but may depend in a weakly positive manner on the variable $s_t$.

For any policy $\pi_t = (\tau_t, p_t, s_t)$ satisfying (A-4) consider the alternative policy

$\pi_t' = \left(\frac{s_t + \delta}{1+n}, 0, s_t + \delta\right)$ for some $\delta > 0$. This one satisfies (A-4), raises the social security rate by $\delta > 0$ and abolishes altogether the intragenerational lump-sum subsidy $p_t$. Clearly all old people prefer $\pi_t'$ to $\pi_t$ for any $\delta > 0$; and young people also do provided that

$v^y_t(e, \pi_t) - v^y_t(e, \pi_t') = (1-e)p_t + e\delta \cdot \frac{1}{1+n} - \frac{1}{R} \leq 0$ which occurs for all $e > 1$ and $\delta > 0$. The coalition of retirees and young people with above median income constitute a majority if

$H(1) < \frac{2+n}{2+2n}$

Q.E.D.
FOOTNOTES

1 Ferejohn (1986) analyzes some of the pitfalls in the median voter equilibrium concept.

2 This issues are investigated in Tabellini and Alesina (1990) and Tabellini (1991).

3 The median voter in postwar U.S. presidential elections is a net saber whose age varies between 43 and 46 years. This description corresponds to a member of the “young” generation in a two-cohort economy, and of some intermediate generation in a multi-cohort framework like Cooley and Soares (1995). The alternative assumption of an old-generation median voter underlies Loewy’s (1989) analysis of open-loop government fiscal policies.

4 Fudenberg and Tirole (1991), pp. 130-34, discuss open-loop strategies.

5 The trigger strategy proposed above does not support renegotiation proof equilibria. In fact, if the social security system has been dismantled to punish a deviator, all future generations will prefer to reinstitute the system. A renegotiation proof strategy that supports the same set of equilibria is presented in the appendix. It requires the punishment of odd-numbered (first, third, etc.) successive defectors, in order to deter unprovoked deviations. The proof is completed by showing that no median voter will be the first to defect from the equilibrium policy \( \tau \), and that the best response to defection is immediate punishment by the next median voter.

6 If the old generation were to set the agenda, the game would have no equilibrium in pure strategy as the young would refuse to transfer resources to the old, preferring instead to start a social security system when they become old.

7 Therefore we assume that every median voter decides according to equation 15, lagged one period, in order to validate the previous median voter expectations.

8 In this case the veto power should be interpreted as contingent on the capital level, and therefore it could only be applied to the parameter \( A \). In other words, the veto power
could be used only if the reduction in the social security tax rate takes place for a given capital stock.

* See Azariadis (1993), Chs. 19 and 24, for a modern treatment of bubble dynamics in pure exchange economies.
Figure 1: Majoritarian Tax Equilibria

\[ \tau_{t+1} = \phi(\tau_t) \]

\[ \tau_{t+1} = R \frac{\tau_t}{1+n} \]
Figure 2: Majoritarian Allocations

- Golden Rule
- Open Loop
- Initial Endowment
- $c_{t+1}$
- $c_t$
- $\varepsilon_2$
- $\varepsilon_1$
- $\varepsilon$
- $1+n$
- $R$
Figure 3: Cyclical and Chaotic Allocations

\[ \tau_{t+1} = R \frac{\tau_t}{1+n} \]

\[ \tau_{t+1} = \phi(\tau_t) \]
Figure 4: Costitutional Tax Rate

\[ \tau_{t+1} = \psi(\tau_t) \]

\[ \tau_{t+1} = x + R \frac{\tau_t}{1+n} \]
Figure 5: Constitutional Allocations

ab: MRS > R
bd: MRS < R

Initial Endowment

Open Loop

Initial Endowment
Figure 5: Constitutional Allocations

ab: MRS > R
bd: MRS < R
Figure 6: Capital Accumulation

Fig. 6a
$A < A^*$

Fig. 6b
$A > A^*$

Fig. 6c
$A = A^*$
Figure 7: Markovian Equilibrium Tax Rate Dynamics

Fig. 7a
A>A* >A'

Fig. 7b
A<A*
A>A'

Fig. 7c
A<A' <A*
Figure 8: Equivalence of Social Security with Public Debt

\[ \tau_{t+1} = x + R \frac{\tau_t}{1+n} \]

\[ \tau_{t+1} = \psi(\tau_t) \]

\[ \tau_{t+1} = \phi(\tau_t) \]