A NON-WALRASIAN GENERAL EQUILIBRIUM MODEL
WITH MONOPOLISTIC COMPETITION AND BARGAINING

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Abstract

In a general equilibrium framework, this paper tries to reproduce an important stylized fact of real economies: firms set prices under demand uncertainty while consumption decisions are taken when prices are already known. Under these conditions, there is place for a quantity rationing equilibrium since preferences are revealed when prices are already set and market-clearing can not be attained through changes in prices. "Demand heterogeneity" is introduced in the model and related to "demand uncertainty": when firms set prices, their own market shares are not known with certainty, even if aggregate demand and the distribution of market shares are common knowledge. The main properties of the aggregate equilibrium are: (a) some markets are demand constrained while other markets are supply constrained, (b) aggregate production is smaller than aggregate demand and full-employment output, (c) there is (involuntary) unemployment, and (d) effective demand is greater than notional demand, implying a positive spill-over effect.

Key Words
Monopolistic Competition; Non-Walrasian General Equilibrium; Quantity Rationing Model; Unemployment; Wage Bargaining

* Universidad Carlos III de Madrid. I thank Jean-Pascal Benassy and Jacques Drèze for helpful conversations. I also had the opportunity to discuss a first draft of this paper at the Catholic University of Louvain and at the Universidad Carlos III de Madrid.
1 Introduction

During the seventies the so-called “disequilibrium theory” shifted the economic adjustments from prices to quantities. The fundamental criticism against Walrasian models is that prices in many real markets do not adjust instantaneously to clear the market. Transactions are thus made at non-Walrasian prices and the economic agents may face some type of quantity rationing. Malinvaud’s (1977) lectures, based on Barro and Grossman (1971), apply the notion of a fixed price equilibrium to macroeconomic problems, integrating in a unified framework both Keynesian and Classical macroeconomic policies. Disequilibrium models have yielded interesting empirical results, in particular under the “aggregation over micromarkets in disequilibrium” hypothesis. This literature sees the economy as the aggregation of a large number of small heterogeneous markets, each one being constrained by demand or supply. Under some plausible assumptions on the distribution of supplies and demands over micromarkets, aggregate production is a function of aggregate demand and aggregate supply. Lambert (1988) identifies sufficient conditions to approximate the aggregate production as a CES function, whose arguments are aggregate demand and aggregate capacity.

During the eighties a considerable effort was devoted to the search for an endogenous explanation of the price formation process in macroeconomic models, originating what is known as the “New Keynesian” approach. In this approach, endogenous price rigidities essentially depend on market imperfections and “menu costs” (the costs of changing prices).

Integrating both theoretical schools in order to look for the microfoundations of price rigidities in disequilibrium models seems to be a very promising task. An effort in this direction was made by Sneessens (1987), who assumes that firms operate in a monopolistically competitive economy and that they pre-set prices before knowing with certainty demand, capacities and labor supply constraints. Imposing the same conditions as Lambert on the joint

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1 Even though this literature was initially concerned with price formation in imperfectly competitive economies, “disequilibrium theory” is generally associated with “fix-price equilibrium models.” See Benassy (1976) in this respect.

2 This point was initially stressed by Muellbauer (1978) and Malinvaud (1980). Koornman (1984) and Lambert (1988), using somewhat different approaches, were the first to give empirically estimable forms to this idea.

3 Sneessens and Drèze (1986) build a macroeconometric model based on Lambert’s approach. Empirical applications to Europe indicate that demand and capacity constraints are both relevant in explaining economic fluctuations, implying that a mix of Keynesian and Classical policies is required. See Drèze and Bean (1990) in this respect.

4 Akerlof and Yellen (1985) and Mankiw (1985) are the main references. See Blanchard and Fischer (1989) for a survey of this literature.
distribution of the stochastic shocks, Sneessens shows that the markup rate charged by firms depends, in addition to the price elasticity of demand, on "demand pressures".

Building from this theoretical background, this paper tries to reproduce an important stylized fact of real economies: firms set prices under demand uncertainty while consumption decisions are taken when prices are already known. Under these conditions, there is place for a quantity rationing equilibrium since preferences are revealed when prices are already set and market-clearing cannot be attained through changes in prices. "Demand heterogeneity" is introduced in the model to reproduce another stylized fact, namely, that market conditions are different among firms. The CES utility function is a useful instrument to introduce demand heterogeneity in a very simple way: the firm market shares depend linearly on some parameter of the utility function. The model is closed by a direct relation between demand heterogeneity and "demand uncertainty": when firms set prices, their own market shares are not known with certainty, even if aggregate demand and the distribution of market shares are common knowledge.

The main characteristics of this economy are described in section 2. In Section 3 consumer behavior is formulated as an extension of the Dixit and Stiglitz (1977) model. Since markets do not necessarily clear, optimal consumption decisions are derived under quantity constraints. As is standard in disequilibrium theory, "effective demand" is distinguished from "notional demand", which yields "spill-over effects": if some quantity constraints are binding the consumers spill their unsatisfied demands over to other markets.

The supply side of the economy is developed in Section 4. Monopolistic competition and efficient bargaining allow for endogenous prices and wages. Demand uncertainty modifies the price behavior of firms allowing for expected demand pressures to increase the monopoly power of the firm. The bargaining outcome determines the labor share as a function of union power, and expected employment as a function of utility parameters. This result can be interpreted as, following the literature on wage bargaining, the "battle of the markups."

The equilibrium of the aggregate economy is studied in Section 5. Since firms and unions are ex-ante (before the revelation of preferences) identical, prices and wages are the same for each firm and the aggregation problem over prices and wages is avoided. However, employment and production are de-
cided ex-post (when preferences are already revealed) and both differ among firms. To allow for aggregation, some particular assumptions about preferences are imposed such that the “aggregation over micro markets in disequilibrium” hypothesis holds. The main properties of the aggregate equilibrium are analyzed: (a) some markets are demand constrained while other markets are supply constrained, (b) aggregate production is smaller than aggregate demand and full-employment output, (c) there is (involuntary) unemployment, and (d) effective demand is greater than notional demand, implying a positive spill-over effect.

2 The Economy

There are three types of economic agents: households, unions and firms. Each household supplies a given quantity of labor to a particular firm, demands goods and holds money. Money plays the role of numeraire and is the only asset in the economy. Households are represented by unions, which are organized at the firm level. Firms hire labor from households and produce differentiated goods. Firms and unions bargain at the firm level.

A particular information structure is assumed: there are two periods in the model, ex-ante (before the revelation of individual preferences over goods) and ex-post (when all relevant information is public). Households supply labor and firms and unions decide (at the firm level) wages and prices without knowing with certainty the demand for the good produced by the firm. When prices and wages are public information, households demand goods and firms hire workers and produce.

3 The Demand Side

This section develops the demand side of the economy. As stated in the previous section, consumption decisions are taken ex-post when all relevant information is common knowledge. Households behave as in Dixit and Stiglitz (1977) but, as it will be showed later, markets do not necessary clear, implying that they must take into account quantity rationing constraints.

To better understand household behavior in the goods market in this paper, I use the distinction, proposed by Clower (1965), between “notional demand” and “effective demand”. “Notional demand” for a particular good is defined as the demand function when all rationing constraints for other goods are not binding. In the same sense, “effective demand” is the demand
function when at least some rationing constraints are binding. This distinction allows us to introduce the idea of "spill-over effect," i.e., the amount by which the unsatisfied demands are transferred from the constrained goods to the unconstrained ones.

All of the households have the same utility function and offer the same given quantity of labor. However, in the labor market workers must be treated asymmetrically: some may find a job while others may be unemployed. Revenues of households are different, even if all have the same non-human wealth. In order to analyze the behavior of a representative consumer facing rationing constraints, it is convenient to assume that differences in revenues do not affect optimal consumption rules. To this end, let us assume that "rationing schemes" are proportional, i.e., all households are rationed in each market proportionally to their revenues.

This section is mainly concerned with the derivation of "effective demand" functions for a set of differentiated goods. It will be shown that the distribution of the aggregate demand among goods depends, in addition to good's prices, on the parameters of the utility function. This property allows us to introduce in an endogenous way the "aggregation over micromarkets" assumptions proposed by Lambert (1988).

### 3.1 The Representative Consumer

The representative consumer optimization problem is

\[
\begin{align*}
\text{Max } U &= \left( \frac{C}{\gamma} \right)^\gamma \left( \frac{M/P}{1 - \gamma} \right)^{1-\gamma} \\
\text{subject to } & C = \sum_{i=1}^{n} \left( \frac{u_i}{n} \right)^\theta \left( \frac{s_i}{P} \right)^{1-\theta} \\
& 1 > \gamma > 0, \quad \theta > 1 \quad \text{and } \sum_{i=1}^{n} u_i = n; \\
& \sum_{i=1}^{n} p_i c_i + M = I \\
& c_i \leq s_i \quad \forall i.
\end{align*}
\]

6 "Rationing schemes" in disequilibrium markets are analyzed by Benassy (1982).
C represents the consumption of a composite good, M represents money holdings, P the aggregate price index, c_i and p_i are, respectively, the consumption and the price of the good i, n is the number of differentiated goods, s_i represents the quantity rationing constraint associated to the good i and I represents total wealth and revenues of the representative consumer. The quantity constraints s_i are taken as given by the consumer, but they are endogenous variables of the model which will be determined later. As in monopolistically competitive models, n is assumed large enough to prevent changes in p_i affecting the price index P^7. The parameters γ, θ and u_i, ∀i ∈ \{1, 2, ..., n\} are given and their meaning will become clear in the next sections.

As in the Dixit-Stiglitz framework, the utility function of the representative consumer depends on the consumption of a composite good and the final stock of real balances. The main differences between this paper and the model of Dixit and Stiglitz are: (a) consumers are not necessarily able to buy all they want at the given prices and (b) the weights u_i are not necessary unity, even if they add up to n.

Optimality conditions for M/P and c_i are^8

\[ P\lambda = \left( \frac{(1 - \gamma)}{\gamma} \frac{PC}{M} \right) ^\gamma \]

\[ c_i = \begin{cases} s_i & \text{if } \lambda_i > 0 \\ \left( \frac{p_i}{P} \right) ^{-\theta} \left( \frac{(1 - \gamma)PC}{\gamma M} \right) ^{-\theta} C \frac{u_i}{n} & \text{if } \lambda_i = 0 \end{cases} \]

and

\[ \sum_{i=1}^{n} p_i c_i + M = I \]

where λ is the marginal value of money and λ_i is the marginal value of the supply constraint s_i.

3.1.1 Notional Demands

If the consumer does not face any rationing constraint (i.e., when λ_i = 0 ∀i) the solution of the optimality conditions is equivalent to that of Dixit-Stiglitz.

^7In this model, the assumption of "large enough" is extended to the relation among quantities: changes in quantities in a particular market have not effects on aggregate quantities. This point will be stressed later.

^8Since the utility function is concave in its arguments and the budget and the quantity constraints are linear, the Kuhn-Tucker conditions are necessary and sufficient for a maximum.
Demands for the composite good and real balances, which are qualified as "notional demands" and denoted, respectively, by $C_n$ and $M_n/P_n$, are

$$C_n = \frac{1}{P_n}$$

and

$$M_n = (1 - \gamma)I$$

where

$$P_n = \left( \sum_{i=1}^{n} \left( \frac{u_i}{n} \right) p_i^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

$P_n$ is the "notional price index".

From equation (3), since $\lambda_i = 0 \forall i$, the "notional demand" for good $i$, denoted by $d^n_i$, is equivalent to the Dixit-Stiglitz solution and can be written as

$$d^n_i = \left( \frac{p_i}{P_n} \right)^{-\theta} \gamma I \frac{u_i}{P_n}$$

(4)

The demand for the composite good, $\frac{\gamma I}{P_n}$, is distributed among the differentiated goods depending on the relative prices and the $\frac{u_i}{n}$ weights.

### 3.1.2 Effective Demands

As in Benassy (1982), let us define "effective demand" for good $i$ in the solution to problem (1) when the $i$ constraint is not taken into account. Since $n$ is "large enough," eliminating only one constraint must not affect the aggregates, allowing us to use equation (3) to define the effective demand for all goods $i$,

$$d_i = \Lambda d^n_i$$

(5)

where

$$\Lambda = \left( \frac{(1 - \gamma)PC}{\gamma M} \right)^{-\theta} \frac{PC}{\gamma I} \left( \frac{P_n}{P} \right)^{1-\theta}$$

(6)

$\Lambda$ represents the "spill-over effect". Since $n$ is large, changes in $d_i$ or $p_i$ do not have significant effects on $\Lambda$, nor on $P$. When none of the quantity constraints is binding, or equivalently when $\lambda_i = 0 \forall i$, the spill-over effect is equal to one, implying that effective demand is identical to notional demand. In general $\Lambda \geq 1$.

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9Note that $P_n$ is the "true" price index associated with the representative household utility function when the quantity constraints are not binding.
As stated in equation (6), the spill-over effect is not indexed by \( i \), i.e., it is the same for all goods. This property depends on the particular form of the utility function and is very useful since we are interested in the behavior of the aggregates more than in the explanation of the market shares for each particular product.

Let us define the "nominal aggregate effective demand" as the sum of money that the representative household want to spend on each good to realize its effective demands, i.e.,

\[
\sum_{i=1}^{n} d_{i} p_{i} = \Lambda \gamma I. \tag{7}
\]

Dividing both sides by the aggregate price index \( P \), the aggregate effective demand, denoted by \( D \), is

\[
D = \Lambda \left( \frac{\gamma I}{P} \right). \tag{8}
\]

The aggregate effective demand is the product of the spill-over effect \( \Lambda \) times a fraction \( \gamma \) of real income \( I/P \).

From previous equations the "effective demand" for good \( i \) can be rewritten as

\[
d_{i} = \left( \frac{p_{i}}{P} \right)^{-\theta} \left( \frac{P}{P_{n}} \right)^{1-\theta} D \frac{u_{i}}{n}. \tag{9}
\]

This equation says that aggregate demand \( D \), corrected by the ratio of the effective price index to the notional price index, is distributed among goods following the distribution of the \( u_{i} \) parameters and another factor which depends on individual prices relative to the aggregate price index.

This property can also be analyzed by looking at the effectively demanded market shares. From equations (8) and (9), good \( i \)'s market share is given by

\[
\frac{p_{i} d_{i}}{PD} = \left( \frac{p_{i}}{P_{n}} \right)^{1-\theta} \frac{u_{i}}{n}. \tag{10}
\]

The distribution of the effectively demanded market shares among goods depends on the distribution of the individual prices and the distribution of the \( u_{i} \) parameters. When all prices are equal, the distribution of the effectively demanded market shares depend only on the distribution of the \( u_{i} \) parameters in the utility function.

This definition of "effective demand" allows us to write the optimality condition (3) in an equivalent way

\[
c_{i} = \min \{ s_{i}, d_{i} \}. \tag{10}
\]
Optimal consumption \( c_i \) is equal to the quantity constraint \( s_i \) when \( \lambda_i > 0 \) and equal to the effective demand \( d_i \) when \( \lambda_i = 0 \).

4 The Supply Side

As stated in the Introduction, the aim of this paper is to integrate "quantity rationing" and "monopolistic competitive" models. For this purpose, we impose two essential assumptions that avoid market clearing. The first assumption is the existence of "nominal rigidities": prices and wages are decided under demand uncertainty, implying that when the stochastic demand takes place, prices and wages are already set and no price adjustment allows market-clearing. Second, some assumptions are introduced to allow for "real rigidities", in the particular sense that each firm faces some full-employment constraint as an upper-bound on production.

Labor supply, prices and wages are decided ex-ante (i.e., before the realization of the stochastic demand), but labor demand and production are decided when the demand for goods is public information.

In the supply side of this economy there are \( n \) monopolistically competitive firms, each of them producing a variety of the unique single good. Workers are assumed to offer \( l_s \) units (hours) of labor to one specific firm. There are also \( n \) unions, which represent the workers offering their labor to each firm. Let us assume the following institutional arrangement: at the firm level, firms and unions bargain over wages and prices under demand uncertainty. Assuming that the bargaining process is at the same time over wages and prices is equivalent to assuming that firms and unions bargain over wages and expected employment (efficient bargaining). This assumption is not essential, but the results are easier to obtain than under the alternative assumption that both parties bargain only over wages and prices are set by the firm alone\(^{10}\).

Workers are uniformly distributed among firms and \( l_s \) represents the hours offered by the representative worker\(^ {11}\). Another important assumption is added to produce real rigidities: labor markets are segmented. Each worker is offering his labor to a specific firm and, if a firm decides not to hire a

\(^{10}\)Arnsperger and de la Croix (1990) compare both "bargaining regimes" in a very similar framework.

\(^{11}\)The representative household solve the problem (1) in section 3.1. Because the marginal desutility of labor is zero, he is optimally working the maximum feasible time \( l_s \). Additionally, workers do not know the labor demand for each firm and they are interested in to be distributed uniformly among them.
worker, this worker is unable to offer his labor to another firm. Under these assumptions each firm faces an upper-bound on production given by the "full-employment output," defined as

\[ y_f = A l_s, \]

where \( A \) is the technical coefficient for labor, which is assumed given for simplicity.

Uncertainty comes into the model because firms, unions and households do not know with certainty the specific \( u_i \) assigned to a particular good. This uncertainty disappears if all households reveal their preferences. But, if each particular household bears some small cost of revelation (for example, the cost of phoning all firms to announce his preferences), it would be interested in revealing its preferences only if all other households do the same, and that is not guaranteed in a decentralised economy. Thus, uncertainty could be interpreted as a "coordination failure."

### 4.1 Firm Behavior

To produce the single good has a given marginal productivity of labor: the inverse of the technical coefficient for labor \( A \). Production of a variety of the single good entails fixed costs, implying that each variety is produced by one firm only. The fixed costs are assumed equal for each variety and denoted by \( z \). The zero expected profits condition determines the number \( n \) of differentiated goods and firms. Since \( n \) is assumed to be large, each firm is small so that its decisions have no significant effects on the aggregate outcome.

Additionally, let us assume that the "representative firm" forms its expectations rationally. It knows the "effective demand function" (equation (10)) assigned to its particular variety. Aggregate effective demand and the distribution of the \( u_i ' s \) are common knowledge, but the representative firm does not know its own \( u_i \) with certainty.

Nominal and real rigidities imply that ex-post optimal production is given by the minimum of the two constraints, those of effective demand and full-employment output:

\[ y_i = \min\{d_i, y_f\}. \tag{11} \]

Expected profits are therefore

\[ E(\pi_i) = (p_i - w_i A^{-1}) E(y_i) - z, \tag{12} \]
where $z$ represents the fixed costs of differentiation. The monopolistic competitive assumption imposes that $E(\pi_i) = 0$, and allows us to determine the number of firms.

The next important point is the determination of $E(y_i)$. The full-employment constraint is given and the effective demand function $d_i$ comes from equation (10). Since $n$ is large, let us assume that the histogram of the parameters $u_i$ can be approximated by a lognormal density function. This additional restriction on preferences allows us to employ some useful results from "quantity rationing theory". Under the assumption that $u_i$ follows a lognormal distribution, expected production can be approximated by a CES function of expected demand and the full-employment constraint\footnote{See Lambert for a proof.}

\[
E(y_i) = \left( E(d_i)^{-p} + y_f^{-p} \right)^{-\frac{1}{p}},
\]

(13)

where $\rho$ depends on the variance of the distribution of the $u_i$ parameters and is positive for plausible values of this variance\footnote{More precisely, $\rho = -1 + \frac{2 f(-\sigma/2)}{\sigma F(-\sigma/2)}$ where $F$ is the standard normal distribution function, $f$ is the standard normal density function and $\sigma$ is the standard deviation of the distribution of the $u_i$ parameters. The standard deviation of demand must have $\sigma \geq 1.2$ which allows $\rho$ to be smaller than one.}. In particular, when there is no uncertainty, i.e., when the variance goes to zero, the parameter $\rho \rightarrow \infty$, implying that expected production is equal to the minimum of expected demand and full-employment output. This is equivalent to the standard monopolistic competitive model, where $u_i = 1 \forall i$.

From equation (14) we can calculate the elasticity of expected production to expected demand, denoted by $\Phi_d$, as

\[
\Phi_d = \left( \frac{E(y_i)}{E(d_i)} \right)^\rho \leq 1.
\]

(14)

This elasticity is smaller than one and decreases when expected demand increases. $\Phi_d$ is also a measure of the probability of excess supply (or demand constraint).

Before solving the bargaining outcome, let us analyze the behavior of the firm when the labor market is competitive and prices are set under demand uncertainty\footnote{See Nuesseens (1987).}. The firm maximizes expected profits, given by equation (12), subject to expected production, given by equation (13). The optimal price
rule is

\[ p_i = \left( \frac{1}{1 - \frac{1}{\theta \Phi_d}} \right) A^{-1} w_i. \]

(15)

Prices are a markup over marginal labor cost, with market power equal to the demand elasticity \( \theta \) times the elasticity of expected production to expected demand \( \Phi_d \). The main difference from standard theory comes from the existence of demand uncertainty: the market power of the firm is increased by the non-zero probability of an excess supply.

Ex-post, when demand is revealed, production is given by equation (11). If the demand shock is “good” the firm produces at the full-employment level and if the demand shock is “bad” the firm is not able to hire all its labor supply and some workers are unemployed in this particular market.

Finally, let us assume that the firm has a fall-back level equal to minus the fixed costs \( z \). If there is not agreement, the firm loses its investment in the diversification technology, i.e., the fixed cost \( z \).

4.2 Union Behavior

In each market, a trade union represents the workers offering their labor to the firm producing the corresponding variety. Let us assume that the “representative union” has the same information as the “representative firm” and forms its expectations rationally. It knows the “effective demand function” (equation (10)) assigned to the variety \( i \). Aggregate effective demand and the distribution of the \( U_j \)'s are common knowledge, but the representative union does not know with certainty the \( u_i \) faced by the \( i \)th firm.

Total wealth and revenues of the representative member of the union \( i \) are

\[ I_i = \tilde{M}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij} \pi_j, \]

where \( \tilde{M}_i = \tilde{M} \) represents initial money holdings, \( l_i \) is employment and \( \theta_{ij} = \theta_j \) represents the shares of firm \( j \) held by the \( i \)th worker. Since the number of households is large, \( \theta_j \) is small, implying that the \( i \)th union do not care about the profits of the \( i \)th firm. The objective function of the union is

\[ E(V) = \left( \frac{w_i A^{-1} E(y_i)}{P l_s} \right), \]

(16)

where \( V \) is the indirect utility function of the risk-neutral representative
member after the deduction of the fall-back level. The fall-back level is

\[ \frac{M_i}{p} + \sum_{j=1}^{n} \theta_{ij} \frac{\pi_j}{p}, \]

i.e., the non-human wealth.

### 4.3 The Bargaining Outcome

The institutional arrangement is that the representative firm and the representative union bargain, at the level of the firm, over both prices and wages, or equivalently over expected employment and wages. The outcome of this bargaining process is the solution of the Nash product

\[ \max_{\{p_i, w_i\}} N = \left( \frac{w_i A^{-1} E(y_i)}{P l_s} \right)^\beta \left( \frac{(p_i - w_i A^{-1}) E(y_i)}{P} \right)^{1-\beta}, \]

where \(1 > \beta > 0\) represents the “union power” and \(1 - \beta\) the “firm power” in the labor market. \(E(y_i)\) is given by equation (13).

The optimality conditions for prices and wages are therefore

\[ p_i = \left( \frac{1}{1 - \frac{(1-\beta)}{\Phi_d}} \right) A^{-1} w_i \]

and

\[ w_i = \beta A p_i. \]

Equation (18) states that the firm (in agreement with the union) sets a markup over marginal costs. The monopoly power in the goods market is given by the inverse of the demand elasticity \(\theta\) weighted by the elasticity of expected production to expected demand \(\Phi_d\). Since the latter elasticity is smaller than one, it reinforces the power of the firm in the market: as stated by equation (15), \(\Phi_d\) could be interpreted as a measure of “demand pressures.” However, since the union cares about expected employment, the monopoly power of the firm is weighted by its power in the labor market. When the union has no power at all \((\beta = 0)\), the “Lerner index” is given by the monopoly power only.

Equation (19) could be interpreted as the union and the firm bargaining over the labor share, i.e., \(\frac{w_i}{\rho_i v_i}\). At the optimum, this share is equal to union power, implying that the greater this power is, the greater the share of total revenues appropriated by workers will be.
Equations (18) and (19) are linearly dependent on prices and wages, implying that the labor share must be equal in both equations. This is possible because the monopoly power in equation (18) adjusts until the labor share becomes equal to $\beta$. Equalizing the labor share from both equations implies

$$\Phi_d = \frac{1}{\theta^*},$$

where $\Phi_d$ is defined in equation (14). From (14) and (20) the expected unemployment rate, denoted by $E(v_1)$, can be determined as

$$E(v_1) = 1 - \left(\frac{\theta - 1}{\theta}\right)^{1/\rho}.$$  

(21)

This implies that in the negotiation process both parties are mainly interested in their own share of total revenues and in expected (un)employment.

From equations (13), (14), (19) and (20) the negotiated price and wage can be solved as functions of the aggregate effective demand $D$, the aggregate price index $P$ and the aggregate full-employment output $Y_f = n y_f$. They depend also on the parameters $\theta$ and $\rho$.

5 The Aggregate Economy and Equilibrium Conditions

The assumptions on a representative consumer, a representative firm and a representative union allow us to define some aggregates in a very straightforward manner. The only difference among agents comes from the distribution of demand among goods: that is, the distribution of the $u_i$ parameters.

Since prices and wages are set before the firm (and the union) knows its own position in the effective demand distribution, all firms and unions agree on the same prices and wages. Formally, $p_i = P^* \forall i$ and $w_i = W^* \forall i$. From equations (18) and (19), the aggregate price $P^*$ and the aggregate wage $W$ are, at equilibrium, given by

$$P^* = \left(1 - \frac{1 - \beta}{\theta \Phi_d^*}\right)^{-1} W^* A^{-1},$$

(22)

and

$$W^* = \beta A P^*.$$  

(23)

\footnote{It is assumed that the aggregate price is a weighted sum of individual prices, which is in accord with the standard Paasche and Laspeyres index. However, since the representative household is rationed in some markets the “true price index” should include the “shadow prices” for the constrained goods. See Deaton and Muellbauer (1980).}
Taking into account that at equilibrium the aggregate price index has the property that \( P = P^* = P_n^* \), the equilibrium values of aggregate production and aggregate effective demand are, respectively, \( Y^* = n E(y_i) \) and \( D^* = n E(d_i) \). The elasticity of aggregate production to aggregate effective demand is, from equation (15),

\[
\Phi^*_d = \left( \frac{Y^*}{D^*} \right)^\rho.
\]  

As stated before, each firm produces the minimum of effective demand and full-employment output. This implies that the appropriate supply constraints for the representative consumer are \( s_i = y_f \forall i \). The equilibrium in the market for variety \( i \) is defined by the following condition: \( c_i = y_i \), where \( y_i = \min\{d_i, y_f\} \). Following Lambert, at the equilibrium, the aggregates are

\[
C^* = Y^* = \left( \frac{D_{-\rho} + Y_f^{-\rho}}{\rho} \right)^\frac{1}{\rho},
\]  

where \( C^* \) is aggregate consumption, \( Y^* \) is aggregate production, \( D^* \) is aggregate effective demand and \( Y_f \) is the aggregate full-employment output. The parameter \( \rho \) is the same as in equation (14). Aggregate production is a CES function of aggregate effective demand and full-employment output. From equations (6) and (8), the equilibrium value of aggregate effective demand \( D^* \) is given by

\[
D^* = \left( \Lambda \gamma \right) \frac{1}{P^*} = \left( \frac{(1 - \gamma)P^* P^* Y^*}{\gamma M} \right)^{\frac{1}{\rho}} Y,
\]  

since \( P_n^* = P^* \). Aggregate effective demand is a proportion \( \Lambda \gamma \geq \gamma \) of total real incomes. Since it is assumed that there is no money growth, the equilibrium in the money market implies \( M^* = M \).

The equilibrium for this economy, summarized in equations (22) to (26), shows some interesting properties.

**Property 1** The unemployment rate is generally positive and equal to the expected unemployment rate given by equation (21).

The "battle of the mark-ups" implicit in equations (22) and (23) determines the equilibrium value for \( \Phi_d \),

\[
\Phi^*_d = \frac{1}{\theta},
\]  

and from equations (24) and (25) the unemployment rate at the equilibrium is

\[
\nu^* = 1 - \left( \frac{\theta - 1}{\theta} \right)^\frac{1}{\rho}.
\]
which in general is positive. Note that aggregate unemployment is equal to expected unemployment, given by equation (21).

There is full employment only in two extreme cases: if there is perfect competition, i.e., if $\theta \to \infty$; or if there is no uncertainty, i.e., if $\rho \to \infty$.

**Property 2** Because households are unable to coordinate and reveal their preferences, the economy faces demand uncertainty and is unable to attain full-employment equilibrium.

From equations (24), (25) and (27), the equilibrium values of aggregate production and aggregate effective demand can be derived as

$$Y^* = \left(\frac{\theta - 1}{\theta}\right)^\frac{1}{\theta} Y_f \leq Y_f, \quad \text{and} \quad D^* = (\theta - 1)^\frac{1}{\theta} Y_f. \quad (28)$$

Aggregate production is generally smaller than full-employment output, which is consistent with a positive unemployment rate. The relation between aggregate effective demand and full-employment output depends crucially on the parameter $\theta$. In the particular case when $\theta = 2$, aggregate effective demand is equal to the full-employment output, but both are greater than production. This has an interesting interpretation: since some markets are in excess supply and other markets are in excess demand, aggregate production is always smaller than full-employment output, even if the latter is equal to aggregate effective demand.

Note that when there is no uncertainty, and $\rho \to \infty$, the economy is at full-employment equilibrium. Uncertainty is essential for our result: since households are unable to coordinate and reveal their preferences, the economy faces demand uncertainty and is unable to attain their full-employment equilibrium. When uncertainty is removed, i.e., when households reveal their preferences, "nominal rigidities" disappear and "real rigidities" are irrelevant.

**Property 3** A quantity theory of money is implicit in the model, with a velocity of money depending on the preferences parameters $\gamma$ and $\theta$ and on the uncertainty parameter $\rho$.

From equations (26) and (27) It can be shown that

$$P^*Y^* = \frac{\gamma}{\theta^2 r (1 - \gamma)} \bar{M}. $$

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This is a simple version of the quantity theory of money, where the velocity of money depends on the preference parameters \( \gamma \) and \( \theta \) and on the uncertainty parameter \( \rho \). When there is no uncertainty, i.e., as \( \rho \) goes to infinity, the firms satisfy demand, implying that our result is the same as in the standard "monopolistic competition model."\(^{16}\)

Since \( Y^* \) is defined in equation (28), the equilibrium solution to the price index \( P^* \) is

\[
P^* = \frac{\gamma}{(1-\gamma)} \left( \frac{\theta^{(1-\frac{1}{\theta})}}{\theta-1} \right)^{\frac{1}{\theta}} \frac{\bar{M}}{Y_f}.
\]

**Property 4** Effective consumption at equilibrium is smaller than notional consumption.

Equilibrium aggregate spending is

\[
P^* C^* = \left( \frac{\gamma}{\theta^{\frac{1}{\theta}}(1-\gamma)+\gamma} \right) I^* \leq \gamma I^* \leq \gamma I_n,
\]

where \( I_n \) represents notional wealth and revenues and it follows: \( I_n = \bar{M} + P_n Y_f \).

Because households are constrained in some markets they are consuming less than in the Dixit-Stiglitz equilibrium, even if they spill unsatisfied consumptions over to markets with excess supply. This arise from the combination of two convergent reasons: (1) people spend less than a proportion \( \gamma \) of total wealth and revenues on the composite good, and (2) revenues are smaller than notional revenues because production is smaller than full-employment output. These differences disappear when there is no uncertainty or no product differentiation.

**Property 5** At equilibrium the representative household saves more than a proportion \( (1-\gamma) \) of total wealth and revenues, and the marginal value of money is smaller than the inverse of the price index.

This is the reverse of Property 4. Equilibrium money holdings are

\[
M^* = \left( \frac{\theta^{\frac{1}{\theta}}(1-\gamma)}{\theta^{\frac{1}{\theta}}(1-\gamma)+\gamma} \right) I^* > (1-\gamma)I^*.
\]

\(^{16}\)See Blanchard and Fischer (1989).
This result is consistent with the result that, at the equilibrium, the marginal value of money $\lambda$ is lower than the inverse of the price index $P^*$,

$$\lambda^* = \left( \frac{1}{\theta} \right)^{\frac{1}{\theta}} \frac{1}{P^*}. $$

**Property 6** The representative consumer “spills-over” from the constrained demands to the non-constrained ones.

The equilibrium value for the spill-over effect is

$$\Lambda^* = \frac{\theta^\frac{1}{\theta}}{\theta^\frac{1}{\theta}(1 - \gamma) + \gamma} > 1.$$ 

6 Conclusions and Remarks

The behavior of households under monopolistic competition in the Dixit-Stiglitz framework is extended to a situation in which quantity constraints could be binding. Under these conditions, the distinction between “notional demand” and “effective demand” is shown to be relevant to the analyses of consumer behavior and market equilibrium. “Demand heterogeneity” is introduced through weights in the CES utility function, and is related to “demand uncertainty.” Since households do not reveal their preferences before prices are public information, from the firm point of view demand uncertainty is the consequence of demand heterogeneity. In an imperfectly competitive economy, with nominal and real rigidities, demand uncertainty is the main explanation of quantity rationing in the goods market and unemployment.

Given the difficulties of working with quantity constraints, the model presented in this paper introduced some important simplifying assumptions. First, since the model is static, firms are inhibited to learn about their own demand. To build a dynamic model of this type is not a very easy task, since households must take into account all future quantity constraints. However, it seems possible to reproduce the main results of this paper in an Overlapping Generation Model, where agents live for two periods. This will yield a better understanding of how households transfer constrained demands over time and how firms learn about their own demand. Second, the results in this paper are closely related to the assumption of “real rigidities,” mainly caused by “labor market segmentation.” Modelling the obstacles to labor mobility seems to be an important improvement. A third point is related to welfare considerations: the price index is defined as a weighted sum of
the observed prices, but it is not necessarily the “true price index.” Finally, an interesting extension of the model would be to study “open economies.” Domestic quantity rationing could be spilled over to foreign goods and affect both the current account and the exchange rate.
References


