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The Role of Observability in Futures Markets

José Luis Ferreira*

*Universidad Carlos III de Madrid, jlferr@eco.uc3m.es

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Abstract

Allaz (1992) and Allaz and Vila (1993) show that in an oligopolistic industry the introduction of a futures market that operates prior to the spot market induces more competitive outcomes. Hughes and Kao (1997) show that this result presumes that firms' future positions are perfectly observed, and that when firms' positions are not observed the Cournot outcome arises. We study an alternative formulation of observability, where the behavior of participants in the futures market is explicitly analyzed, and show that this approach leads to different results. Imperfect observability induces more competitive outcomes than Allaz and Vila's model.

KEYWORDS: futures markets, observability, arbitrage, cournot competition

*Departamento de Economía, Universidad Carlos III de Madrid, C/ Madrid, 126, 28903 Getafe, Madrid, Spain. I wish to thank Diego Moreno for many helpful comments. Earlier versions of this paper were written while visiting the ITAM (México D.F.). Financial support from DGI grant SEJ2005-08633 (Ministerio de Educación y Ciencia) is gratefully acknowledged.

1 Introduction

The strategic impact of futures markets in oligopolistic contexts has attracted attention recently. In a Cournot duopoly, Allaz (1992) shows that, if firms can sell in a futures market previous to the spot market, the strategic interactions result in a more competitive outcome (in the sequel we will refer to this model as Allaz's basic model). In a later work, Allaz and Vila (1993) show that this pro-competitive effect increases as the futures markets open more often. However, Ferreira (2003) shows that if the futures market has infinitely many moments in which trade is allowed, the result is that not only the pro-competitive effect may disappear, but that the monopolistic outcome can be sustained in equilibrium. Taking these results together, the implications for the design of a futures market in this context are that the market maker must set specific (finitely many) moments to trade in the futures markets.

Hughes and Kao (1997) explore the role played by the observability of futures positions for the result in Allaz (1992) to hold. More specifically, they take Allaz's basic model and find that, in the spirit of Bagwell (1995), if firms do not observe each other's actions in the futures market, the pro-competitive effect does not arise, and the model yields the standard Cournot outcome. This result agrees with intuition; it is the knowledge that the other firm observes one's commitment in the futures markets that makes firms willing to use this market strategically. The implications for the design of futures markets, thus, seem to favor transparency.

More recently, Mahnec and Salani  (2004) show that, if firms compete *a la* Bertrand with differentiated products, the introduction of a forward market softens competition when this market opens once before the spot market. Hence, the design of futures markets ought to take into account the strategic variable of the oligopolistic firms. Liski and Montero (2005) consider an infinitely repeated oligopoly where, in each period, both a futures and a spot market are open. In the futures market firms may contract quantities for any future spot market. In this situation, they find that the presence of the futures market makes collusion easier in the sense that it requires a lower discount rate than in the case without futures market.

All previous works impose the no-arbitrage condition that futures and spot market prices must be the same. This condition is standard and can be easily justified. Allaz (1992) explicitly models a set of speculators who act competitively to drive the profits to zero, resulting in equal futures and spot prices. Hughes and Kao (1997) implicitly assume the existence of these

speculators to justify equal futures and spot market prices. However, the behavior of these speculators is not modeled and, thus, the model in Hughes and Kao (1997) must be understood as model in a reduced form.

In this paper we also take Allaz's basic model and study the role of observability of positions in the futures market, but we model explicitly the role of the speculators that trade in the futures markets. We propose a simple futures market game in which first firms decide how much to sell in this market, and then competitive speculators simultaneously set a price at which to buy those quantities. The speculator that offers the highest price buys the total quantity (up to a tie-breaking rule). In Allaz's basic model the speculators choose the level of forward purchases based on their risk attitude. However, in our model, since speculators are risk neutral, they will be willing to buy as much as they can if the futures price is lower than the expected spot price. In this setting we find that if speculators observe perfectly firms' futures positions then the result is the same as in Allaz's model when speculators' risk attitude tends to risk neutrality.

However, when trying to model the absence of observability, things are far from trivial. In our model, the knowledge of the speculators may be transmitted to the firms through the equilibrium price in the futures market. Hence we need to assume that speculators do not observe total futures market quantities in order to have imperfectly informed firms. With this new assumption we find that the conclusion in Hughes and Kao (1997) may not follow, *i.e.*, that the equilibrium does not necessarily imply a reversal to the Cournot outcome. In fact, both the Cournot and the competitive outcomes are sustained in equilibrium. However, only the competitive outcome looks reasonable after certain stability considerations are added. The reason is that now firms take advantage of the speculators' lack of knowledge and sell as much as they can in the futures market if the equilibrium price in this market is above the competitive equilibrium. Being anticipated, this behavior can only lead to the competitive price in equilibrium. However, we do not read this result as a case for opaque futures markets (at least not yet), as the assumption of total ignorance of positions in the futures market is clearly an extreme one. Rather, this result says that Hughes and Kao's reduced form may not be the only possibility to extend Allaz's basic model, and therefore that Hughes and Kao's result is not satisfied in general as the role of observability may depend drastically on the way the futures markets is organized.

Finally, we consider the case of partial observability that includes the

previous analysis as particular cases. In this general setting we characterize the set of prices that can be sustained in a Bayesian perfect equilibrium in pure strategies for some knowledge structure. If we rule out the extreme case in which positions are not observed (when we get the competitive price), we find that this set of prices is bounded away from the competitive price, and that it has the price in Allaz's basic model as a maximum.

In the absence of other reasons for using futures markets, the conclusion for designing futures markets thus seem to favor opacity, but only as long as it is possible to make speculators at least partially ignorant of total quantities in this market. If speculators know these quantities, futures prices will be informative regardless of the opacity of this market, and the decision on transparency versus opacity must be taken based on other grounds.

In Section 2 we present the basic model. In Section 3 we present the models for different assumptions on observability. Section 4 presents a discussion and Section 5 concludes. Figures are provided at the end.

2 The basic model and the extreme cases

2.1 Case 1. Allaz's basic model

Consider a duopoly where firms produce a homogeneous good at zero costs and compete *a la* Cournot in a market where demand is given by $p = A - q$ if $q \leq A$, and $p = 0$ if $q > A$. Suppose that before producing and selling in the spot market, firms may sell in advance part of their production in a futures market. Denote by s_i and f_i the quantities sold by firm i in the spot and futures market, respectively, and write $q_i = s_i + f_i$, with $q = q_1 + q_2$. Assume that (f_1, f_2) are observed when firms sell in the spot market. A strategy for firm i is a pair $(f_i, s_i(f_1, f_2))$. Finally, let us introduce a no-arbitrage condition that requires that both futures and spot prices be the same. The game tree is depicted in Figure 1.

Firms' payoffs are $\Pi_i = p(f_i + s_i)$. This game corresponds to the model in Allaz and Vila (1993) when there is only one period to sell in the futures market. It also corresponds to the model in Allaz (1992) when firms are risk neutral, and where a no-arbitrage condition substitutes the speculators.

The unique subgame perfect equilibrium (*SPE*) in non-weakly dominated strategies is readily calculated backwards. If positions in the futures market

are observable, given f_1 and f_2 , in the spot market firm i solves:

$$\begin{aligned} & \max_{s_i} p s_i \\ \text{s.t } p &= A - f_1 - f_2 - s_1 - s_2 \end{aligned}$$

The solution gives the reaction function $s_i = \frac{A-f_1-f_2-s_j}{2}$ or, in terms of total quantities, $q_i = \frac{A+f_i-q_j}{2}$. Solving for s_1 and s_2 , one finds $s_i = \frac{A-f_1-f_2}{3}$, $q_i = \frac{A+2f_i-f_j}{3}$, and $p = \frac{A-f_1-f_2}{3}$.

Knowing this reaction, firms' position in the futures market is calculated as follows:

$$\begin{aligned} & \max_{f_i} p (f_i + s_i) \\ \text{s.t } s_i &= p = \frac{A - f_1 - f_2}{3} \end{aligned} \tag{1}$$

The solution to this problem is $f_i = \frac{A-f_j}{4}$. Solving for f_1 and f_2 , and substituting in the expressions for the other variables, the result is $f_i = p = s_i = \frac{A}{5}$, $q_i = \frac{2}{5}A$, with profits given by $\Pi_i = \frac{2}{25}A^2$. The equilibrium is thus showing a pro-competitive effect of the futures market as the Cournot equilibrium without it ($f_1 = f_2 = 0$) is $q_i = p = \frac{A}{3}$, with $\Pi_i = \frac{A^2}{9}$. To understand why this occurs, notice that if, for whatever reason, it is known that Firm 2 does not use the futures market, then Firm 1 chooses $f_2 = \frac{A}{4}$ and then it follows that $p = s_i = \frac{A}{4}$, $q_1 = \frac{1}{2}A$, $q_2 = \frac{1}{4}A$. The pro-competitive effect is caused as firms strategically commit some production level in order to change the Cournot game they play on the spot market.

The strategy profiles in which $q_1, q_2 \geq A$ are also *SPE*, but with firms playing dominated strategies. Instead of ruling out dominated strategies, we could have introduced a production cost (as in Allaz and Vila, 93), but this option has the consequence of making the analysis in Section 3 far more complicated. Likewise, we have assumed that firms can only take a short position in the futures markets. A long position by Firm i is implied by a negative value of f_i . In this section, as well as in the next, all equilibria that come after interior solutions to the firms' maximization problems include non negative forward positions. Nevertheless, to simplify matters in equilibria after corner solutions, we will keep this assumption in the sequel. If contracted quantities in the futures markets are physical deliveries that take place at the spot market opening, then it is natural that producers cannot take long positions.

2.2 Case 2. The result in Hughes and Kao

Hughes and Kao (1997) consider Allaz's basic model and show that when firms' futures positions are not observed, the introduction of a futures market has no impact. To see this, consider the same model as before, and assume that quantity f_i is not observed by firm $j \neq i$, (in particular, the futures market price is not observed). A strategy for firm i is now a pair $(f_i, s_i(f_i))$, and payoffs are $\Pi_i = p(f_i + s_i)$. As before, and following Hughes and Kao (1997), the no-arbitrage condition is imposed. The game tree is depicted in Figure 2.

The next proposition shows the only Bayesian perfect equilibrium (*BPE*) of this game in non-dominated strategies. The presentation of the model and the proof are somewhat different from those in Hughes and Kao (1997), but the results and the arguments are essentially the same.

Proposition 1 *Consider the game in Figure 2, with payoffs as described before. Then, the only BPE in which players use non-dominated strategies is the following: Firm i chooses $f_i = 0$, and $s_i = \frac{A-f_i}{3}$, and Firm i believes with probability one that it is in the node after $f_j = 0$.*

Proof. First show that the strategy is indeed a *BPE*. In the equilibrium firms are playing the Cournot outcome, with profits given by $\Pi_i = \frac{A^2}{9}$. Standard Cournot analysis shows that there is no profitable deviation from s_i . To check that there is no profitable deviation from $f_i = 0$ consider $f'_i > 0$, while $f_j = 0$. After this deviation s_i changes to $s'_i = \frac{A-2f'_i}{3}$, p to $p' = \frac{A-f'_i}{3}$, and Π_i to $\Pi'_i = \frac{A-f'_i}{3} \left(\frac{A-2f'_i}{3} + f'_i \right) = \frac{A^2 - (f'_i)^2}{9} < \frac{A^2}{9}$.

To show that the *BPE* is unique consider any other strategy (f_1, f_2) in which one firm, say i , chooses $f_i > 0$. Again, Cournot analysis in the second stage indicates that, in equilibrium, $s_1 = s_2 = \frac{A-f_1-f_2}{3}$. Suppose now that, instead of f_i , firm i plays f'_i . After positions (f'_i, f_j) , the other variables take values $s'_i = \frac{A-2f'_i+f_i-f_j}{3}$, $p = A - s'_i - s_j - f'_i - f_j = \frac{A-f'_i-f_j}{3}$, and $\Pi'_i = \frac{A-f'_i-f_j}{3} \frac{A+f'_i+f_i-f_j}{3}$. In the last expression observe that, for every initial f_i , profits achieve a unique maximum at $f'_i = \frac{f_i}{2}$. This means that the only case with no profitable deviations is $f_i = 0$. The equilibria in dominated strategies are similar to those in Case 1. ■

This result shows that firms are unable to commit unless their futures positions are observable. Observability is therefore a necessary condition to

obtain the pro-competitive effect of futures markets. As we shall see, when the futures market is modeled in more detail, this conclusion does not hold in general.

3 A model with speculators

In this section we study the role of observability in more detail. Hughes and Kao (1997) suggest that the pro-competitive effect of a futures market found by Allaz (1992) arises only if positions in the futures market are observed, and that if they are not, the equilibrium outcome reverts to Cournot. However, Hughes and Kao's result is based on two implicit assumptions. The first one is a strong version of the no-arbitrage condition: the price in the futures market must be the same as in the spot market *in every contingency*. This means that the two prices have to coincide, not only along the equilibrium path, but also along deviations from the equilibrium. The second assumption is that firms do not observe prices in the futures market. Firms detect deviations only when they produce and sell in the spot market. Let us discuss these assumptions in more detail.

(i) *The strong no-arbitrage condition.* It is most natural to require that, in equilibrium, futures and spot prices must be the same. If not, some speculator can profit from buying in one market and selling in the other. Arbitrage opportunities exist unless prices are identical. Take, then, an equilibrium situation in which the no-arbitrage condition is satisfied, and consider the consequences of a firm deviating by selling a larger quantity in the futures market. This implies an increase in the total quantity sold in the aggregation of both markets, and a lower price in the spot market. Should this imply also a lower (and identical) price in the futures market? If agents have perfect information of what is going on in the futures market, this seems the only reasonable consequence. On the other hand, if agents in the futures market do not have perfect information, they may not detect deviations (or, at least, the exact size of them), and the price in the futures market may not reflect the price that will prevail in the spot market, where Cournot competition in the residual demand (after discounting futures sales) reveals the spot price. The existence of an agent that can foresee this difference in prices may not be the only natural assumption. In order for the futures price to reflect the new quantity, the deviation must be anticipated. However, there are two reasons why this may not be the case. First, once a strategy profile is considered

as an equilibrium candidate, there may be more than one possible deviation to anticipate. How do agents know which one is actually taking place to adjust the price? Second, the profitability of a deviation may depend on the reaction of other agents, and, then, on the price induced by the deviation. These considerations call for a detailed model linking deviations with prices.

(ii) *Observability of prices in the futures market.* In the case where agents have perfect information about quantities sold in the futures market it is hard to justify that firms may not know about them. After all, by selling a small quantity in the futures market, any firm may enter the market and know the price, and *a fortiori*, the quantities sold. Therefore, if one wants a model in which firms are not fully aware of quantities sold in the futures market (out of the equilibrium path) then one also needs to have that speculators are not fully informed either. Understanding these issues requires modeling how agents and firms gather information.

From the discussion above, at least three possibilities arise:

Case 1. Future and spot market prices coincide *in every contingency*, and firms observe quantities (or prices) in the futures market (Allaz, 92).

Case 2. Future and spot market prices coincide *in every contingency*, but firms do not observe prices or quantities in the futures market (Hughes and Kao, 97), and

Case 3. Future and spot market prices coincide *along the equilibrium path* and firms observe prices, but not quantities in the futures market.

In the previous section we presented game forms for cases 1 and 2. Next we present explicit game forms representing subcases of Case 3, which deals with a weaker version of the no-arbitrage condition and opens up new possibilities, depending of how deviations from equilibrium are observed and how they affect the price in the futures market. The differences in the game form highlights the merits of each one of the cases. In particular, cases 1 and 2 do not explicitly model the demand side of the futures market, while Case 3 does. Furthermore, in Case 3, the arbitrage condition is a consequence of the equilibrium itself, not an assumption, as in cases 1 and 2. In Subcase 3.1 below (Proposition 2) in which firms do not observe quantities in the futures market our result is equivalent to that in Case 1 as the price in the futures market is fully informative. However it is remarkable that Subcase 3.2 (Proposition 3) in which firms observe futures prices that are not informative is very different to Case 2, and yields a reversal to perfect competition rather than to Cournot. Subsection 3.3 analyzes the general case of imperfect observability.

3.1 Weak version of the no-arbitrage condition: $f_1 + f_2$ observed by speculators.

In the previous cases prices in the futures market were automatically set equal to spot market prices. To introduce prices in the futures market as an independent variable, we add new players that select these prices. The new game is as follows. In a first stage firms simultaneously decide positions in the futures market (f_1 and f_2). In the second stage, and after observing the actions in the first stage, n speculators simultaneously offer a price at which to buy the total quantity $f_1 + f_2$. If Speculator j offers the highest price (say p_j) she buys all future quantities and makes profits given by $\Pi^j = (p_s - p_f)(f_1 + f_2)$, where p_s is the price in the spot market. In this case, the futures price is $p_f = p_j$. If the highest price (and, then, futures price) p_f is offered by m speculators, each one of them buys $\frac{1}{m}$ of the futures quantities and has profits given by $\Pi^j = (p_s - p_f) \frac{f_1 + f_2}{m}$. Note that, under these conditions, speculators behave competitively (like in a standard Bertrand model). In the third stage, and after observing prices in the futures market, firms sell in the spot market. Since we are interested in the role of observability in the futures market, we keep things simple in the spot market in the sense that, as in the previous models, firms compete for the residual demand, which is obtained after subtracting the forward positions. In other words, we assume that speculators play no role (behave competitively) in the spot market. The game is depicted in Figure 3 where, for simplicity, only two speculators are shown. Nodes a^1, a^2, a^3 and a^4 belong to different information sets of Speculator 1. Similarly nodes $\{b^k\}_k$ (alt. $\{c^k\}_k, \{d^k\}_k$) belong to different information sets of Speculator 2 (alt. Firm 1, Firm 2). A strategy for Speculator j is a function $p_j(f_1 + f_2)$. A strategy for Firm i is a pair $(f_i, s_i(f_i, p_f))$.

The payoffs for the firms are calculated as $\Pi_i = p_f f_i + p_s s_i$. A speculator is to be understood as an agent that can make a potential profit from buying in one market and selling in another one at a higher price. The demand in the futures market is thus created if the spot price is expected to be higher than the price in the futures market. Proposition 2 shows that this case is equivalent to Case 1. As mentioned before, the reason is that futures prices are fully informative of the futures positions (because speculators are fully informed of the quantities sold in this market).

Proposition 2 *Consider the game in Figure 3 with payoffs as described*

above. Then, in all Bayesian perfect equilibria firms sell $f_1 = f_2 = \frac{A}{5}$ in their first move, speculators set prices $p_1 = \dots = p_n = \frac{A-f_1-f_2}{3}$, and firms sell $s_1 = s_2 = \max\{p_1, \dots, p_n\}$ in the spot market.

Proof. In the third stage firms are playing the Cournot outcome, $s_1 = s_2 = \frac{A-f_1-f_2}{3}$, which implies $p_s = \frac{A-f_1-f_2}{3}$, and therefore, will not deviate from it. Knowing this, in the second stage, speculators will choose $p_j = p_s = \frac{A-f_1-f_2}{3}$ as the only possibility in equilibrium. In the first stage, firms must solve (1), as in Case 1, to find their best reply at this stage. The solution gives $f_1 = f_2 = \frac{A}{5}$. To see that the equilibrium is unique notice that, in the third stage firms are playing the unique equilibrium strategy, speculators will deviate from strategies that do not imply $p_1 = p_2 = \frac{A-f_1-f_2}{3}$, and that the solution to firms' maximization problem in the first stage is unique. ■

3.2 Weak version of the no-arbitrage condition: $f_1 + f_2$ not observed by speculators.

We have just seen that the introduction of informed speculators does not make any difference with respect to the situation without speculators, but with informed firms, as the price perfectly reflects the information. Next we show that the introduction of uninformed speculators is not equivalent to the model without speculators and with uninformed firms. We start by looking at the extreme case, opposite to Case 3.1 above where $f_1 + f_2$ is observed, in which buyers in the futures market do not observe total positions in this market, *i.e.*, we have a game as before, except that speculators cannot condition their actions (p_1, \dots, p_n) on the quantity $f_1 + f_2$. In the next subsection, we will consider a general case that includes 3.1 and 3.2 as subcases. We chose to present the extreme cases first to facilitate the discussion. The game form is the same as in Figure 3, except that now nodes $\{a^k\}_k$ ($\{b^k\}_k$) belong to the same information set of Speculator 1 (2). Finally, $\{c^1, c^2\} \in u_1$, $\{c^3, c^4\} \in v_1$, $\{d^1, d^3\} \in u_2$ and $\{d^2, d^4\} \in v_2$, where u_i and v_i are different information sets of Firm i .

The key feature of this game is that the demand in the futures market does not react to firms' deviations. This opens the possibility for different prices in the two markets as a result of deviations. The lack of information in the model does not allow arbitrage between them. However, in equilibrium, both prices have to coincide since the equilibrium must be anticipated by all players in the game.

Proposition 3 shows that, in this setting, both the competitive and Cournot outcomes can be obtained. However we will see that only the competitive outcome satisfies certain equilibrium selection criteria.

Proposition 3 *In the game defined above there are two types of Bayesian perfect equilibria: (i) firms choose $f_1 \geq A$, $f_2 \geq A$, and $s_1 = s_2 = 0$, and (ii) firms choose $f_1 = f_2 = 0$, and $s_1 = s_2 = \frac{A}{3}$.*

Proof. (i) First find an equilibrium with $p_s = p_f$. Cournot behavior in the spot market implies $s_1 = s_2 = p_s$. Now see that in equilibrium $p_s = p_f = 0$. Suppose, to the contrary, that $p_f = p_s = A - s_1 - s_2 - f_1 - f_2 > 0$. In this situation, if Firm i increases its future positions by Δf_i its profits increase by $\Delta f_i \times p_f$. Even if Δf_i implies that p_s decreases, $\Delta f_i \times p_f$ can be made arbitrarily large and, thus, the deviation is profitable.

To show that $f_1 \geq A$, $f_2 \geq A$ consider first the case where $f_j < A$, but still $f_1 + f_2 \geq A$ (implying $p_f = 0$), and consider, further, that Firm i deviates to $f'_i = 0$. This deviation does not change the futures price ($p_f = 0$), as it is not observed by speculators. Now firm i can sell $s_i = \frac{A-f_j}{2}$ with the consequence of $p_s = \frac{A-f_j}{2}$, and profits $\Pi_i = \left(\frac{A-f_j}{2}\right)^2 > 0$. Finally, if $f_1 + f_2 < A$, standard Cournot analysis implies $s_1 = s_2 = p_s > 0$ in equilibrium, in contradiction with the already established result that $s_1 = s_2 = p_s = 0$. Hence, the equilibrium requires $f_1 \geq A$, $f_2 \geq A$, $p_s = p_f = s_i = 0$. It is straightforward to check that this is indeed an equilibrium as profits are zero regardless of unilateral deviations.

(ii) The only possibility for an equilibrium with $p_s \neq p_f$ requires that $p_f = 0$. As in (i), $p_f > 0$ makes any firm willing to sell as much as possible in the futures market. Also, $p_f = 0$ is only possible if speculators cannot profit by buying in the futures market to sell in the spot market, and this can only happen if $F = f_1 + f_2 = 0$ (no one is selling in the futures market.) Once $F = 0$ is established, the standard Cournot equilibrium must follow in the spot market. Firms do not want to deviate from $f_i = 0$ to $f'_i > 0$ as this implies no profits in the futures market and will decrease profits to $\frac{A-f'_i}{3}$ in the spot market. ■

Notice that only equilibria of type (i) satisfy the weak version of no-arbitrage. The no-arbitrage condition is not a behavioral imposition but it is rather a consequence of the equilibrium. Equilibrium (ii) does not satisfy the no-arbitrage condition because nothing is sold in the futures market.

It turns out that the fact that equilibrium (ii) does not satisfy the weak version of no-arbitrage makes it a less likely equilibrium. First we construct an intuitive argument to support this claim. Then we connect it to the literature of equilibrium selection. In Section 4.4, when discussing risk aversion, we provide one more reason to rule out the equilibrium in (ii)¹.

One of the first things to highlight in this case is not only that speculators observe nothing about quantities offered in the futures market, but that the winner is forced to buy any quantity at the quoted price. With these premises it should not come as a surprise that firms will sell a high quantity in the futures market at any positive price. Being aware of this behavior, speculators choose $p_j = 0$ in both types of equilibria.

To understand why equilibrium (ii) is not reasonable consider again the strategy consisting of not selling in the futures market and selling the Cournot quantity in the spot market. Speculators' best reply correspondence contains all possible prices (profits are zero in all cases), in particular, it contains $p_j = 0$, which, together with $f_1 + f_2 = 0$, drives the equilibrium. If there is a small possibility that speculators choose $p_j = \varepsilon > 0$ (e.g., by mistake), firms will anticipate profits by entering the futures market and offer a positive quantity in this market. A large enough quantity will even compensate for any cost to enter in this market.

The concept of stable set of equilibria in Kohlberg and Mertens (1986) actually rules out equilibrium (ii) on these grounds. More technically, this particular perturbation of speculators' choices has no equilibrium that is close to equilibrium (ii), as $f_i = 0$ cannot be a best reply for firms. Equilibria in (i), on the other hand, are stable as any perturbation on the way players choose have equilibria close to them.

Nevertheless, as we will see next, this is a very extreme and peculiar case of a more general and interesting one, where the issues discussed here are not relevant.

3.3 A general case: $f_1 + f_2$ observed imperfectly by speculators.

In Case 3.1, speculators observed perfectly firms' futures positions, whereas in Case 3.2 they did not observe them. In this section we model a situa-

¹I thank an anonymous referee for pointing out the existence of equilibrium (ii) and for suggesting the argument in Section 4.4.

tion of partial observability, in which speculators only observe whether total quantities in the futures market belong to a certain information set.

In the first stage firms choose simultaneously quantities f_1 and f_2 within the interval $[0, \frac{B}{2}]$, where $\frac{B}{2} > A$. Setting an upper bound to the quantities firms may sell only prevents us from considering subgames in which futures positions are infinite. Consider the set of partitions on the interval $[0, B]$ in which sets are either intervals of length of at least a pre-fixed $\delta > 0$ or real numbers. When the set is an interval, it will be called an *interval of uncertainty*. Denote this set of partitions by \mathfrak{U} . Given $F = f_1 + f_2$, in the second stage Speculator j 's information partition is a set $U_j \subset \mathfrak{U}$. These assumptions imply that speculators observe F or, at least, are able to determine that F is in a certain interval of uncertainty. All speculators have the same information partition on futures quantities; *i.e.*, for all j , $U_j = U$. Speculators choose prices contingent on information sets. In the third stage firms observe prices set by speculators and decide s_1 and s_2 . Profits are calculated as in the previous cases.

The game form is the same as in Figure 3 except that now nodes belong to the same information set of a given player according to the stated condition (recall that a firm always knows its own past actions). Case 3.1 is the limit of this general case when speculators' information partition gets finer, and Case 3.2 corresponds to the situation in which speculators' information partition has only one information set containing all nodes such that $f_1 + f_2 \in [0, B]$.

Denote by p the price in the spot and futures markets when both coincide. Proposition 4 shows that the range of prices that can be obtained with different levels of partial observability is $\frac{A}{6} \leq p \leq \frac{A}{5}$ (in addition to $p = 0$ and $p_s = \frac{A}{3}$). Recall that $p = \frac{A}{5}$ is the price obtained with perfect observability. The intuition is the same as in Proposition 3: speculators know that the imperfect observability of futures positions induce firms to increase their sales in the futures markets to take advantage of a high price. The size of this increase depends on the precise information structure. In any case, speculators know that firms will produce too much and anticipate lower prices with imperfect observability. For very low prices (lower than $\frac{A}{6}$), however, deviations are no longer beneficial for the firms because the gains in the futures market do not compensate for the losses in the spot market.

The case of $p = 0$ is explained because of the discontinuity of the demand function at this point, which allows for arbitrary deviations when the lack of observability is absolute. Finally, the case of $p_s = \frac{A}{3}$ occurs because of the discontinuity of speculators' best reply (they may set $p_f = 0$ at $F = 0$, but

$p_f = \frac{A-F}{3}$ if F is very small), and only in a non-stable equilibrium.

Proposition 4 *In the game described above the only prices that can be sustained in a BPE in pure strategies with an information partition $U \subset \mathfrak{U}$ are $p = 0$, $p_s = \frac{A}{3}$ and $\frac{A}{6} \leq p \leq \frac{A}{5}$.*

Proof. Propositions 2 and 3 already show that $p = 0$, $p = \frac{A}{3}$ and $p = \frac{A}{5}$ can be sustained in a BPE, only notice that speculators' information sets are in \mathfrak{U} . The rest of the proof is dedicated to showing that the only other prices that can be sustained in equilibrium are $\frac{A}{6} \leq p < \frac{A}{5}$. Recall that to sustain a price $p > 0$ in a BPE, the required total quantity must be $q = A - p$, and also that the spot quantities must satisfy $s_1 = s_2 = p$. Total futures positions compatible with these conditions require $f_1 + f_2 = F = q - s_1 - s_2 = A - 3p$ or $p = \frac{A-F}{3}$. To sustain p take $f_1 > 0$ and $f_2 > 0$ such that $F = f_1 + f_2 = A - 3p$. In a BPE speculators must assign probability 1 to the total quantity F , and offer prices $p_j = p$. If firms choose f_1 and f_2 , Firm one's profits are $\Pi_1 = p(f_1 + s_1) = \frac{A-f_1-f_2}{3} \frac{A+2f_1-f_2}{3}$. To consider possible deviations from the proposed scenario we need to distinguish several cases. Given the structure of information sets, it is straightforward to see that there are only three possibilities for F :

- (i) $F \in u$ for some interval $u \in U$. (F cannot be known with certainty.)
- (ii) $F \in U$ and all points in a neighborhood of T are also elements of U . (F and a neighborhood around it can be known with certainty.)
- (iii) $F \in U$ and is the frontier of an interval $u \in U$ such that $F \notin u$. (F can be known with certainty, but is in the frontier of an interval of uncertainty.)

(i) Consider firm one's deviation to (f'_1, s'_1) with $f'_1 + f_2$ belonging to the same information set as F , and with s'_1 being derived from the reaction function $s'_1 = \frac{A-f'_1-f_2-s_2}{2}$. Because speculators do not observe the deviation, the futures price does not change along the equilibrium path: *i.e.*, for all $j \in N$, $p_j = \frac{A-f_1-f_2}{3}$. Because the price in the futures market does not change, Firm 2 does not change its spot quantity along the equilibrium path, thus $s_2 = \frac{A-f_1-f_2}{3}$, and then $s'_1 = \frac{2A-3f'_1-2f_2+f_1}{6}$. The spot price, however, changes to $p' = A - f'_1 - f_2 - s'_1 - s_2 = \frac{2A-3f'_1-2f_2+f_1}{6} = s'_1$ as Firm 1 changed its quantities in both the forward and spot markets. Then Firm 1's profits are given by $\Pi'_1 = \frac{A-f_1-f_2}{3} f'_1 + \left(\frac{2A-3f'_1-2f_2+f_1}{6} \right)^2$. The difference in profits is $\Pi'_1 - \Pi_1 = \frac{1}{4} (f'_1 - f_1)^2$. This difference is positive for any $f'_1 \neq f_1$ such that $f'_1 + f_2$ belong to the same information set as F for speculators, which means that no equilibrium exists in this case.

(ii) A deviation to f'_1 within a neighborhood of f_1 gives profits $\Pi'_1 = \frac{A-f'_1-f_2}{3} \frac{A+2f'_1-f_2}{3}$. The difference in profits is now given by the expression $\Pi'_1 - \Pi_1 = \frac{1}{9} (f'_1 - f_1) (A - 2f'_1 - 2f_1 - f_2)$. If $A - 4f_1 - f_2 > 0$, take $f'_1 = f_1 + \varepsilon$, with $\varepsilon > 0$ small enough, to get $\Pi'_1 - \Pi_1 > 0$. If $A - 4f_1 - f_2 < 0$, take $f'_1 = f_1 - \varepsilon$.

(iii) There are four relevant cases. (iii.a) $A - 4f_1 - f_2 < 0$ and $f_1 + f_2$ is located at the left end of an interval of uncertainty (open to the left), (iii.b) $A - 4f_1 - f_2 > 0$ and $f_1 + f_2$ is located at the right end of an interval of uncertainty (open to the right), (iii.c) $A - 4f_1 - f_2 > 0$ and $f_1 + f_2$ is located at the left end of an interval of uncertainty, (iii.d) $A - 4f_1 - f_2 < 0$ and $f_1 + f_2$ is located at the right end of an interval of uncertainty. The cases when $A - 4f_1 - f_2 = 0$ and $A - 4f_2 - f_1 = 0$ are solved like in Proposition 2 and give $p = \frac{A}{5}$ in equilibrium. In the first two cases, the deviation is perfectly observed, in Case (iii.a) repeat Case (ii) with $f'_1 = f_1 + \varepsilon$, and in (iii.b) repeat (ii) with $f'_1 = f_1 - \varepsilon$ to conclude that there are no equilibria.

For the other two cases, (iii.c) and (iii.d), notice that if futures positions are not perfectly observed after the deviation, speculators will offer a price p' , their contingent price for the information set induced by Firm 1. The new profits for Firm 1 are given by $\Pi' = p'f'_1 + \left(\frac{A-f'_1-f_2-p'}{2}\right)^2$. In Case (iii.c) $p' < \frac{A-f_1-f_2}{3}$ and the maximum for this expression restricted to $f'_1 \geq f_1$ corresponds to $f'_1 = f_1$. This implies that if Firm 1 deviates from f_1 to induce p' , it had better do it with a very small deviation. *I.e.*, $f'_1 = f_1 + \varepsilon$ for a small $\varepsilon > 0$. The gain in profits are (except for terms in ε) $\Pi' - \Pi = \frac{1}{36} (5A - 17f_1 - 5f_2 - 3p') (A - f_1 - f_2 - 3p')$. The expression in the second parenthesis is positive. The expression in the first parenthesis attains its infimum with respect to p' , and restricted to $p' < \frac{A-f_1-f_2}{3}$ at $p' = \frac{A-f_1-f_2}{3}$. This infimum is $4(A - 3f_1 - f_2) > 0$ if $A - 4f_1 - f_2 > 0$, as required in this case. In Case (iii.d) $p' > \frac{A-f_1-f_2}{3}$, and the maximum of Π' also corresponds to $f'_1 = f_1$, which implies that firm one's deviation should be $f'_1 = f_1 - \varepsilon$. The expression for $\Pi' - \Pi$ is the same as in (iii.c). The second parenthesis is negative, and the first parenthesis is always non negative if $A - 3f_1 - f_2 \geq 0$. By symmetry, to get that no profitable deviations exist for the other firm we have that $A - 4f_2 - f_1 \geq 0$ and $A - 3f_2 - f_1 \geq 0$. *I.e.*, there are equilibria as long as $2A - 5f_1 - 5f_2 \geq 0$ and $2A - 4f_1 - 4f_2 \geq 0$, which implies $\frac{4}{5}A \leq f_1 + f_2 \leq A$ and $\frac{A}{6} \leq p \leq \frac{A}{5}$. ■

It is interesting to notice that, according to the proof of Proposition 4, the futures quantity $f_1 + f_2$, and the information partition $U \subset \mathcal{U}$ that support

prices $p \in [\frac{A}{6}, \frac{A}{5})$ in a *BPE* in pure strategies satisfy one of two following conditions:

- (a) (Case iii.c in the proof) $A - 4f_1 - f_2 > 0$ and $f_1 + f_2$ is located at the left end of an interval of uncertainty,
- (b) (Case iii.d in the proof) $A - 4f_1 - f_2 < 0$ and $f_1 + f_2$ is located at the right end of an interval of uncertainty.

4 Discussion

4.1 Market design

Our results may be sensitive to the particular way competition is modeled in the futures market. In our model firms first decide their futures positions, and then speculators bid to buy these quantities. This is a standard model of a first-price, sealed-envelope auction or a Bertrand price-competition. Notice that speculators know the equilibrium quantities, but may not know precisely the quantities after a deviation occurs. Speculators are, of course, aware of their ignorance and react to it. The equilibrium is the result of these reactions.

Other models are possible (and even more realistic) to describe the futures market. For instance, if firms are allowed to sell in the futures market with a positive reservation price (meaning that if the price is zero no trade takes place), then, no matter how close to zero is this reservation price, the strategy of $f_i = 0$ is dominated. But then, speculators may also have the possibility of buying only a limited quantity in the futures market. These alternatives may rule out some of the extreme cases, but open the possibility for more complicated supply and demand schedules as strategy choices, and will greatly complicate the model.

The present work makes the point that it is precisely the study of these alternatives that, in the end, will tell us about the role of observability in the futures markets. In this work we did not attempt to provide a detailed study of different institutional designs of futures markets. Rather we provide the first model that, to our knowledge, considers partial observability in the futures market, and explicitly takes into account the behavior of all agents in this market. Furthermore, we have shown that, by doing this, our findings are different from the ones found in the reduced form models studied in the literature.

Having said this, and admitting it is too early to derive recommendations for the design of futures markets in oligopolistic industries without further studies, it may still be worthwhile to provide a sketch of the design of a market that conforms with the assumptions in this paper.

First, the transparency of the futures markets is not required to promote competition. In fact, transparency may make collusion easier to implement as deviations can be monitored. Opacity, on the other hand, can promote competition if speculators are not informed of total quantities (if they are informed, the result is the same as with a transparent market). The question is, then, how to design a market that invites the participation of speculators without giving them any information. A literal reading of the model would require that the futures markets opens only once, and that the market takes the form of an auction where the auctioneer collected bids, in the form of price per unit, for an unknown quantity. The speculator that submitted the highest bid would be forced to buy the whole quantity.

Here we suggest a more realistic possibility. Given that there is an upper bound to what firms can sell in the futures market, say A , the auctioneer divides A in k lots of size $a = A/k$. Of course, if the total quantity is q , with $l \leq q/a \leq l + 1$, there will be l lots with a quantity of a , one lot with quantity $q - la$, and $k - l$ lots of quantity zero. All this is common knowledge. Then a first-price, sealed-envelope auction takes place for each one these lots. Participants in the auctions (speculators) decide in which of the auctions to participate, but there will be no information about q or l , or about the size of a particular lot. To spread speculators over all lots, a limited number of bidders for each lot could be set. All auctions take place simultaneously, and the actual quantity of each lot ($q_j \in \{a, q - la, 0\}$, $j = 1, \dots, l$) is revealed after the winner is proclaimed. The winner of lot j buys the total quantity q_j . Given that the demand is completely elastic at the anticipated futures price, p_f , each of the auctions will give p_f as the equilibrium price. (This is not true in models where the demand is not perfectly elastic.)

This type of auction has some resemblances with Treasury auctions, where it has been shown that first price auctions may be more efficient than second-price auctions.

4.2 Market microstructure

There is a large literature on Market Microstructure Theory (see O'Hara (1995) and references within) studying the exchange of assets under explicit

trading rules. One of the approaches in this literature takes the form of information-based models, where the trading process is viewed as a game involving traders with asymmetric information. However, we are not aware of any work in which intermediaries have an information structure comparable to the one we develop here. Furthermore, the source of asymmetric information in the Theory of Market Microstructure is usually the asset's true value. The present work suggests that the value of the asset itself may be affected by the traders decisions, as the value of a futures position depends on the quantities sold by any of the firms in the futures market. The reason lies, of course, in the oligopolistic structure of the underlying spot market. Thus we can see the present work as a motivation for a possible extension of the models in Market Microstructure Theory.

Even if the sources for asymmetric information are different, it is interesting to note that the literature on market microstructure has found that more transparency is not always better in terms of efficiency. See Madhavan (2000).

4.3 Speculators' information

In the general model, all speculators have the same information about futures quantities. However, we can extend the results to the case where speculators may have different information partitions (e.g., because they observed different private signals). To see this, consider Speculator j 's information partition, U_j , and define \widehat{U} as the union of all partitions U_j ($j \in \{1, \dots, n\}$). The possibilities for total future quantities F are now:

(i') $F \in u$ for some interval $u \in \widehat{U}$. (F cannot be known with certainty by any speculator.)

(ii') $F \in \widehat{U}$ and all points in a neighborhood of T are also elements of \widehat{U} . (F and a neighborhood around it can be known with certainty by some speculator.) And

(iii') $F \in \widehat{U}$ and is the frontier of an interval $u \in \widehat{U}$ such that $F \notin u$. (F can be known with certainty by some speculator, but is in the frontier of an interval of uncertainty of all speculators.)

One can now rewrite Proposition 4. The existence of no equilibrium in Case (i') is shown as in Case (i). In Case (ii'), the deviation f' is observed by some speculators, which may or may not change future prices (because the new price offered by these speculators may or may not be lower than the

price set by the others). If the futures market price changes, the argument is the same as in Case (ii) in the proposition, if it does not change, we argue as in Case (i). The same argument works for Case (iii').

4.4 Risk aversion

Except for equilibrium (ii) in Case 3.2, all of the results are robust to small departures from the specification of the model (linear and certain demand, and risk neutral firms). If firms are risk averse and demand is uncertain, equilibrium (ii) can no longer exist because risk averse firms have a strong reason to enter the futures market (to reduce risk). This means that $p_j = 0$ cannot be an equilibrium in this market unless the price in the spot market is also zero. Thus, risk aversion provides another argument against the Cournot outcome if positions in the futures market are not observed (not even by speculators).

It is interesting to note the similarity between this situation and the resolution of the famous Grossman-Stiglitz paradox (Grossman and Stiglitz (1980)) also in the context of futures contracts (with asymmetric information about the real value of the asset): if price reveals all private information, and can be observed costlessly, then there will be no incentive to gather costly private information. But then no private information will be revealed, and there will be an incentive to gather it. Grossman himself had argued that the addition of a suitable source of noise would avoid the paradox, but as Bray (1992) observes, all one requires is that there exists some hedging motive for trading the asset. In our context, the Cournot outcome occurs if no one enters the futures market, which allows for a zero price in this market (or any other). At a zero price there is no incentive to enter the market. If mistakes are added the zero price disappears, but this addition is not necessary as the existence of uncertainty and of risk averse firms may be enough to rule out the zero price.

4.5 Opacity versus transparency

According to Hughes and Kao (1997) firms perform better in a situation in which the futures market is opaque (they get the Cournot profits) than when it is transparent (they get the outcome found in Allaz, 1992). This means that, given the choice, they prefer an opaque market. However, if both types of markets are present, then firms face a prisoners' dilemma, as they have an

incentive to use the transparent market to get a higher market share. Our model, however, indicates that, if the demand side of the futures market is sensitive enough in the sense of setting prices that reflect actual quantities, the observability of futures quantities by firms does not make any difference. Therefore, firms will be indifferent if given the choice between the two types of markets. On the other hand, if prices in the futures market are not sensitive to changes in quantities, a more competitive outcome may result. In this case firms prefer the transparent market and to face no prisoners' dilemma.

The liberalization of the power market in England and Wales provides an interesting case. In this industry there were two futures markets: Contracts for Differences (CfD) and Electricity Forward Agreements (EFA), where the CfD is much more opaque. According to estimates in Power UK (1997 and 1998), around 1998 the coverture of the CfD's was nearly 90% of the market, while the EFA's accounted for less than 30%. This contradicts the conclusions of Hughes and Kao (1997), but not ours if prices are informative. Of course, our model does not explain the fact that firms prefer the opaque market, but is compatible with these preferences if there are reasons that justify them. For example, firms may show a collusive behavior that may be better implemented in real life in the more opaque market. This possibility has been suggested in Powel *et al.* (1994).

5 Conclusion

Allaz (1992) and Allaz and Vila (1993) show that, if a futures market is added to the spot market in an oligopolistic industry, firms show a more competitive behavior. Hughs and Kao (1997) argue that firms must have perfect information about futures quantities for this result to hold, and that without this information firms behave as in Cournot. We show that while perfect information leads to the result in Allaz and Vila (1993), the imperfect information may not lead to Cournot, but to a more competitive outcome. If the demand side in the futures market is informed, the fact that firms are not informed is irrelevant, as prices reveal the information. This later case provides a theoretical model for the observed behavior in the futures markets of the UK power industry, which contradicts the results in Hughs and Kao (1997). Our model suggests that the microstructure of the futures markets needs to be studied with more care, and that the conclusions obtained in a reduced form of the market may not be compelling.

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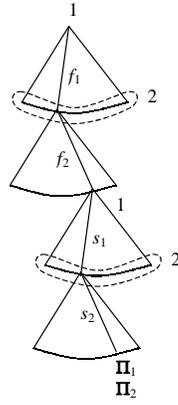


Figure 1

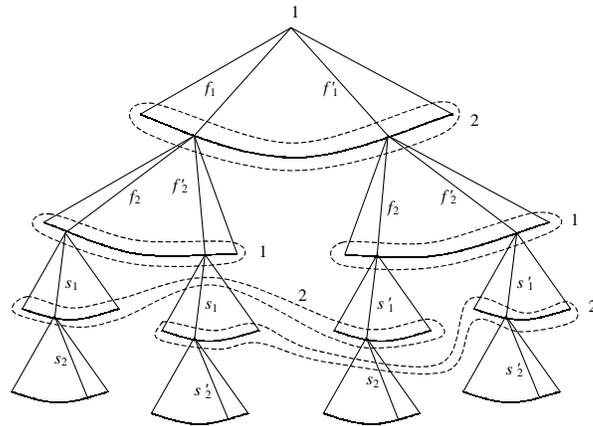


Figure 2

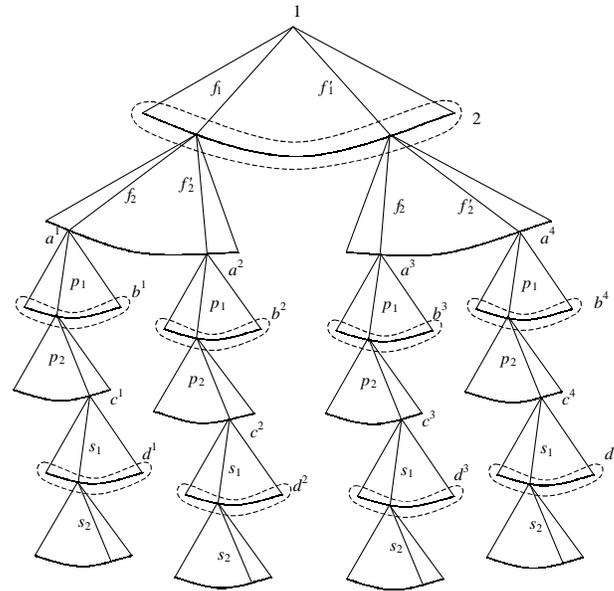


Figure 3