

Working Paper 98-15
Statistics and Econometrics Series 09
February 1998

Departamento de Estadística y Econometría
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax 34 - 91- 624.9849

DETRENDING PROCEDURES AND COINTEGRATION TESTING: ECM TESTS UNDER
STRUCTURAL BREAKS

Miguel A. Arranz and Alvaro Escribano*

Abstract

It is well known that all the test for unit roots and cointegration depend on the deterministic elements that are in mean of the variables; constant, trend, breaks, outliers, segmented trends, etc. This is a serious inconvenient for empirical work. In this paper we analyze if those problems could be solved by forming the cointegration tests on extended models, on the components of the series obtained from trend cycle decompositions. We do that by Monte Carlo Simulations allowing for several structural breaks in the data generating process.

Key Words

Cointegration testing; ECM tests; structural breaks; trend-cycle decompositions; Hodrick-Prescott filter; Baxter-King filter.

*Arranz, Departamento de Fundamentos del Análisis Económico, Universidad de Alicante; e-mail: arranz@est-econ.uc3m.es; Escribano; Departamento de Estadística y Econometría, Universidad Carlos III de Madrid, e-mail: alvaroe@est-econ.uc3m.es.



DETRENDING PROCEDURES AND COINTEGRATION TESTING: ECM TESTS UNDER STRUCTURAL BREAKS

MIGUEL A. ARRANZ AND ALVARO ESCRIBANO

ABSTRACT. It is well known that all the tests for unit roots and cointegration depend on the deterministic elements that are in the mean of the variables: constant, breaks, outliers, segmented trends, etc. This is a serious inconvenient for empirical work. In this paper we analyze if those problems could be solved by forming the cointegration tests on extended models or on the components of the series obtained from trend-cycle decompositions. We do it by Monte Carlo simulations allowing for several structural breaks in the data generating process.

1. INTRODUCTION

The properties of the cointegration test based on the single equation error correction model (ECM test) are well known. The dependence of the critical values, and the power of the test on nuisance parameters are documented in Banerjee et al. (1986), Engle and Granger (1987), Kremers et al. (1992), Park and Phillips (1988, 1989), and Banerjee et al. (1993)¹.

From the univariate point of view, the effects of having breaks when applying unit root test, like Dickey and Fuller test, etc., are well documented. Perron (1989) is a good starting point to see those impacts. A structural break essentially corresponds to a shock with a lasting effect on the series (see Perron and Vogelsang, 1992). If this shock is not explicitly taken into account, standard unit root tests will mistake the structural break for a unit root. The results in Perron and Vogelsang (1992) indicate that a neglected shift in the mean also leads to spurious unit roots. Rappoport and Reichlin (1989) is probably the first reference to check if we want to know the impact of having segmented trends as an alternative to a unit root model. Other references about the topic are Hendry and Neale (1990), Banerjee et al. (1992), Zivot and Andrews (1992), Leybourne et al. (1998). Andrés et al. (1990) extended the analysis of Rappoport and Reichlin to more than one break point in the trend. Rappoport and Reichlin (1989) is probably the first reference to check if we want to know the impact of having segmented trends as an alternative to a unit root model.

Key words and phrases. Cointegration testing, ECM tests, structural breaks, trend-cycle decompositions, Hodrick-Prescott filter, Baxter-King filter.

This research has been partially financed with projects CICYT PB95-0298 and TMR ERB-4061 PL 97-0994.

¹See Appendix A for a brief list of the main results.

Once again, to implement the Dickey and Fuller test in that context requires the use of dummy variables and segmented trends. Andrés et al. (1990) extended the analysis of Rappoport and Reichlin to more than one break point in the trend. The main problem with this literature, that has expanded dramatically since then, is that we always have to add dummy variables to capture the structural breaks in order to correctly apply unit root tests and therefore the C.V. obtained depend on the size of the break, and on the timing of the break. Again, another vast literature emerged searching for unknown break points using recursive and segmented CUSUM tests (Andrews, 1993, Andrews et al., 1996, Bai, 1997, Bai and Perron, 1998, Vogelsang, 1997).

Another type of atypical event is the additive outlier. This is an event with a large, temporary effect on the series. In certain cases, this effect dominates the remaining information contained in the series and biases unit root inference towards rejection of the unit root hypothesis even if the null hypothesis of a unit root is correct, as reported in Lucas (1995*a,b*), Franses and Haldrup (1994).

For multivariate time series, however, the situation is even worse. Extending the comments of the paragraphs above may become in too many possibilities. We should decide on the type of models for generating the anomalous observations (breaking trends, additive outliers, . . .) and take into account that the irregularities need not occur simultaneously or to all the variables.

In empirical applications it is more the rule than the exception to include dummy variables in order to obtain parameter constant ECM models. The effects of including dummy variables to the capture structural breaks in ECM models have been analyzed in the econometric literature, see for example Kremers et al. (1992), and Campos et al. (1996). The fact that critical values (C.V.) depend on the particular type of dummy variable included is particularly a nuisance when doing applied work.

We can avoid using dummy variables with robust estimation techniques. This is the approach taken by Lucas (1995*a,b*) for the univariate case and Lucas (1997), Franses and Lucas (1997*a,b*) in the multivariate case.

In this paper we follow a different route. We want to find robust procedures to test for unit roots in the presence of structural breaks in an ECM context. Instead of including dummy variables in ECM models, we allow to approximate those breaks by having more dynamic terms, as determined by the SBIC criterion, or by including some lags of the error correction term (extended ECM model). By doing that we look at the critical values, study the size of the test under different MA(1) errors, and finally analyze the power of the ECM test with Monte Carlo simulations. We

analyze three different cases: simultaneous co-breaking, co-breaking in levels (not in differences) and co-breaking in differences (not in levels).

We also analyze if the robustness properties of the ECM test improve by following the same steps not on the observable variables, but on the trend components obtained from trend-cycle decompositions. In particular, we study two filters, the Hodrick and Prescott (1980, 1997) filter, and the Baxter and King (1995) filter, HP and BK from now on.

The structure of the paper is as follows. In section 2 we analyze the effects of having deterministic elements (constant terms, deterministic trends, dummy variables, segmented trends, etc.) on alternative specifications of the ECM models, and in particular on the cointegrating relationship. Three types of deterministic possibilities are studied in detail: simultaneous co-breaking, co-breaking in levels (not in differences) and co-breaking in differences (not in levels). Finally, everything is particularized for the case of having several structural breaks. The Monte Carlo experiments is explained with detail in Section 3. The extended ECM is studied in detail section 4. Section 5 discusses the alternative error correction models (ECM) for the trend components obtained from different filters (trend-cycle decompositions). Section 6 presents some conclusions.

2. ERROR CORRECTION MODELS WITH AND WITHOUT SIMULTANEOUS CO-BREAKING

Consider the following conditional error correction model (ECM)

$$\Delta(y_t - \mu_{y,t}) = a\Delta(z_t - \mu_{z,t}) + b[(y_{t-1} - \mu_{y,t-1}) - \alpha(z_{t-1} - \mu_{z,t-1})] + u_{1t} \quad (2.1a)$$

$$\Delta(z_t - \mu_{z,t}) = u_{2t} \quad (2.1b)$$

Assume that $\dots, y_{-1}, y_0 = 0$ and $\dots, z_{-1}, z_0 = 0$, let $\mu_{y,t} = E(y_t)$, $\mu_{z,t} = E(z_t)$ be the corresponding unconditional means which include all possible deterministic components like: constant terms, deterministic trends, dummy variables, segmented trends, outliers, etc. Define B as the back-shift operator, $B^k y_t = y_{t-k}$, $\Delta = (1 - B)$ is the first differencing operator, and let $(1, -\alpha)$ be the cointegrating vector. The stochastic errors u_{1t} and u_{2t} are jointly, and serially uncorrelated with zero mean, and constant variances $var(u_{1t}) = \sigma_1^2$ and $var(u_{2t}) = \sigma_2^2$.

Model (2.2a)–(2.2c) can be written in terms of the observable variables y_t and z_t as follows,

$$\Delta y_t = c_t + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \quad (2.2a)$$

$$\Delta z_t = \Delta \mu_{z,t} + u_{2t} \quad (2.2b)$$

$$c_t \equiv \Delta \mu_{y,t} - a\Delta \mu_{z,t} - b(\mu_{y,t-1} - \alpha \mu_{z,t-1}) \quad (2.2c)$$

In this paper we investigate the effects of having alternative models for the intercept c_t of (2.2a) on the ECM test for non-cointegration ($b = 0$).

The cointegrating relationship in terms of the observable variables can be obtained from (2.2a)–(2.2c)

$$y_t - \alpha z_t = \frac{1}{1 - (b+1)B} c_t + \frac{a - \alpha}{1 - (b+1)B} \Delta \mu_{z,t} + \frac{a - \alpha}{1 - (b+1)B} u_{2,t} + \frac{1}{1 - (b+1)B} u_{1,t} \quad (2.3)$$

and therefore we would need to include a lot of deterministic regressors through the first two dynamic blocks of (2.3) based on c_t and $\Delta \mu_{z,t}$. The cointegrating regression based on the observable variables requires the following minimum set of regressors

$$y_t = \alpha z_t + \frac{1}{1 - (b+1)B} c_t + \frac{a - \alpha}{1 - (b+1)B} \Delta \mu_{z,t} + v_t \quad (2.4)$$

Notice that the problem of having to add arbitrary and influential deterministic regressors is not solved by simply conditioning on $\Delta z_t, \Delta z_{t-1} \dots$, since

$$y_t = \alpha z_t + \frac{a - \alpha}{1 - (b+1)B} \Delta z_t + \frac{1}{1 - (b+1)B} c_t + w_t \quad (2.5)$$

where $w_t = \frac{1}{1 - (b+1)B} u_{1,t}$, and we still have to add the dynamic deterministic effects of c_t .

Definition 2.1. Let $E(y_t) = \mu_{y,t}$ and $E(z_t) = \mu_{z,t}$, we say that the time series y_t and z_t have co-breaks in levels if $\mu_{y,t} - \alpha \mu_{z,t} = c_l$, where c_l is a finite constant parameter.

Definition 2.2. Let $E(y_t) = \mu_{y,t}$ and $E(z_t) = \mu_{z,t}$, we say that the time series y_t and z_t have co-breaks in differences if $\Delta \mu_{y,t} - a \Delta \mu_{z,t} = c_d$, where c_d is a finite constant parameter.

Definition 2.3. Let $E(y_t) = \mu_{y,t}$ and $E(z_t) = \mu_{z,t}$, we say that the time series y_t and z_t have simultaneous co-breaks if $\Delta \mu_{y,t} - a \Delta \mu_{z,t} - b(\mu_{y,t} - \alpha \mu_{z,t}) = c_s$, where c_s is a finite constant parameter.

From definitions 2.1 and 2.2, it is clear that if y_t and z_t are co-breaks in levels and in differences (full co-break), we have a particular case of simultaneous co-breaking. In the case of simultaneous co-breaking, the intercept c_t from (2.2c) is constant, $c_t = c$ and the error correction model from (2.2a) becomes the standard conditional ECM model where the only deterministic regressor is the constant term, c .

$$\Delta y_t = c + a \Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \quad (2.6)$$

On the other hand, even if $c_t = c$, the cointegration regression (2.4) has a constant term, say $\tilde{c} = \frac{1}{1-(b+1)B}c$, and lags of $\Delta\mu_{z,t}$

$$y_t = \tilde{c} + az_t + \frac{a - \alpha}{1 - (b + 1)B} \Delta\mu_{z,t} + v_t. \quad (2.7)$$

Therefore, we would have to add a complicated dynamic expression of dummy variable when $\Delta\mu_{z,t}$ has structural breaks. This problem is solved now by conditioning on $\Delta z_t, \Delta z_{t-1}, \dots$, since (2.7) becomes

$$y_t = \tilde{c} + az_t + \frac{a - \alpha}{1 - (b + 1)B} \Delta z_t + w_t. \quad (2.8)$$

This regression is simplified if there is a common factor restriction, $a = \alpha$ (COMFAC restriction), since then $\Delta\mu_{z,t}$ has no effect on the cointegrating regression (2.7). From equation (2.4), it is clear that to have $a = \alpha$ (COMFAC restriction) is not a universal solution because the cointegration regression takes the form,

$$y_t = \alpha z_t + \frac{1}{1 - (b + 1)B} c_t + u_t \quad (2.9)$$

and we have a strange cointegrating regression with a complicated structure through the lagged deterministic elements of c_t

Several possible intermediate cases are of interest in empirical applications and will be considered in the the simulation experiments later on.

Case 2.1. Co-break in levels but not in differences.

Co-break in levels ($\mu_{y,t} - \alpha\mu_{z,t} = c_t$). Taking first differences, we have $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = 0$. But from equation (2.2c)

$$c_t = \Delta\mu_{y,t} - a\Delta\mu_{z,t} - bc_t = (\alpha - a)\Delta\mu_{z,t} - bc_t, \quad (2.10)$$

and equation (2.2a) becomes

$$\Delta y_t = -bc_t + (\alpha - a)\Delta\mu_{z,t} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \quad (2.11)$$

Remark 2.1. Co-break in levels \Rightarrow co-break in differences if $a = \alpha$ (COMFAC restriction).

Case 2.2. Co-break in differences but not in levels.

Co-break in differences: $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d$ implies that $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = (a - \alpha)\Delta\mu_{z,t} + c_d$. From recursive substitution $\mu_{y,t} - \alpha\mu_{z,t} = (\mu_{y,0} - \alpha\mu_{z,0}) + c_d t + (a - \alpha)\mu_{z,t}$, and c_t becomes

$$c_t = c_d - b(\mu_{y,0} - \alpha\mu_{z,0}) - bc_d(t - 1) - b(a - \alpha)\mu_{z,t-1} \quad (2.12)$$

and equation (2.2a) becomes

$$\Delta y_t = c_m + bc_d t - b(a - \alpha)\mu_{z,t-1} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t}, \quad (2.13)$$

where c_m is a constant equal to $c_m = c_d - b(\mu_{y,0} - \alpha\mu_{z,0}) + bc_d$.

Remark 2.2. Assuming that $\mu_{y,0} - \alpha\mu_{z,0} = \text{constant}$, co-break in differences \Rightarrow co-break in levels if $a = \alpha$ (COMFAC) and $c_d = 0$.

In general, without having any co-break in levels or in differences, the most parsimonious representation is the conditional ECM model (2.1a), and in terms of observable variables is representation (2.2a), because it only requires to add the deterministic regressors coming from the contemporaneous values of c_t . Clearly, if we are interested in estimating the parameters a, α and b , it is much easier and more parsimonious to estimate them by 1-step procedures (OLS or NLS) in ECM representations (2.1a) or (2.2a) than in any other of the representations discussed. However, to do that we need to know or to first estimate the unconditional means $\mu_{y,t}$ and $\mu_{z,t}$, and this can incorporate arbitrary assumptions about unknown events.

2.1. Error Correction Models with Simultaneous Cobreaking. From equations (2.2a)–(2.2c) and the analysis of Escribano (1987) and Andrés et al. (1990), it is clear that any error correction model in terms of the observable variables should account for the joint effects of the following elements: $\Delta\mu_{y,t}$, $\Delta\mu_{z,t}$, $\mu_{y,t-1}$ and $\mu_{z,t-1}$.

Previous error correction models with cobreaks have been treated in Campos et al. (1996) and Hendry (1996). In this section we consider deterministic segmented trends with y_t and z_t have *simultaneous co-breaks* if $\Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t} - \alpha\mu_{z,t}) = c_s$ (see Definition 2.3), where $c_s = 0$. In this case, (2.2a) and (2.2b) can be simplified to

$$\Delta y_t = a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \quad (2.14a)$$

$$\Delta z_t = \Delta\mu_{z,t} + u_{2,t} \quad (2.14b)$$

$$\Delta\mu_{z,t} = sD_{j,t} \quad (2.14c)$$

where (2.14a) has the form of the usual single equation error correction without a constant term since $c_t = 0$. s is a parameter that measures the size of the break, and $D_{j,t}$ is a dummy variable that takes the value 0 before the break and the value 1 at the break and after the break, see section 3.1 for details.

From equations (2.14a) and (2.14b) it is clear that $y_t \sim I(1)$, $z_t \sim I(1)$ with segmented trends, and that they are cointegrated with cointegration vector equal to $(1, -\alpha)$, in the sense of Engle

and Granger (1987). We are allowing for segmented trends in the 'exogenous' variable z_t that show simultaneous co-break ($c_s = 0$) with the endogenous variables y_t . The cointegrating relationship can explicitly be written as

$$y_t - \alpha z_t = \frac{a - \alpha}{1 - (b + 1)B} \Delta z_t + \frac{1}{1 - (b + 1)B} u_{1t}. \quad (2.15)$$

Therefore, from model (2.14a)–(2.14b), it is clear that if $-2 < b < 0^2$ the variables are cointegrated, $(y_t - \alpha z_t) \sim I(0)$, and if $b = 0$, $(y_t - \alpha z_t) \sim I(1)$, and therefore not cointegrated.

Substituting equations (2.14b) and (2.14c) in (2.15) we get an interesting explicit relationship,

$$y_t - \alpha z_t = \frac{a - \alpha}{1 - (b + 1)B} sD_{j,t} + \frac{a - \alpha}{1 - (b + 1)B} u_{2t} + \frac{1}{1 - (b + 1)B} u_{1t}. \quad (2.16)$$

It is clear that to estimate the cointegrating parameter α in (2.16) we need to include lags of the deterministic explanatory variable $D_{j,t}$ unless the common factor restriction, $a = \alpha$, holds. This conclusion affects most of the available static regression models using parametric or non-parametric procedures to estimate α in the regression

$$y_t = \alpha z_t + \epsilon_t \quad (2.17)$$

such as OLS (Engle and Granger, 1987), FM-OLS (Phillips and Hansen, 1990) or canonical cointegration (Park, 1992). The solution is simple if $\Delta \mu_{z,t}$ is a constant or a trend, but it is not that simple if it has level shifts, segmented trends or other types of unknown breaks that occur in practice. In this situation of simultaneous cobreaking, there is a clear advantage in the conditional dynamic model (2.14a) because the cointegrating parameter can be efficiently estimated by OLS without needing any deterministic explanatory variables. To summarize. Dynamic conditional error correction models based on variables that have simultaneous co-breaking do not require the use of dummy variables. However, for static cointegrating regressions we might have to use them if the contemporaneous short run parameters differ from the cointegrating parameter $a \neq \alpha$ (no COMFAC) and these results are maintained even when $a = 0$.

2.2. Error Correction Models without Simultaneous Co-breaking. In the previous section we have discussed the cases of cobreaking in levels and in differences. Our purpose now is to discuss several interesting intermediate cases

Case 2.1. Co-breaking in levels, but not co-breaking in differences.

²It is not uncommon to find the cointegration condition $-1 < b < 0$. See for example Kremers et al. (1992) and Campos et al. (1996).

From equation (2.14a) we have

$$\Delta y_t = -bc_l + (\alpha - a)sD_{j,t} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \quad (2.18)$$

Therefore the breaks in the marginal process of Δz_t affect the error correction model unless the COMFAC restriction is satisfied ($a = \alpha$).

Case 2.2. Co-breaking in differences, but not co-breaking in levels.

From equation (2.13)

$$\Delta y_t = c_m + bc_d t - b(a - \alpha)\mu_{z,t-1} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \quad (2.19a)$$

$$\mu_{z,t} = \mu_{z,0} + s \sum_{i=1}^t D_{j,i} \quad (2.19b)$$

Depending on the type of dummy variable that we define, $D_{j,t}$, we could have segmented trends with one or several breaking points, see section 3

The final possibility there is no cobreaking in levels nor in differences. This likely empirical situation is the result of joining equations (2.18) and (2.19a)–(2.19b) and it is a particular case of equation (2.2a). However, it is enough to show that for cases (2.18) and (2.19a) there are difficult situations one has to face in practice in the presence of structural breaks. In the next section we analyze the impact of having different structural breaks in terms of the empirical critical values (C.V.) and on the size and the power of the ECM test for $b = 0$.

3. MONTE CARLO SIMULATION EXPERIMENT

Our data generating process (DGP) will be based on the one used by Kremers et al. (1992) and Campos et al. (1996). It is a linear first-order vector autorregression with normal disturbances, Granger causality in only one direction ($z \rightarrow y$), and a possible structural break in the strongly exogenous variables (Δz_t) for the parameters a and α of interest.

$$\Delta y_t = c_t + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \quad (3.1a)$$

$$\Delta z_t = \Delta \mu_{z,t} + u_{2t} \quad (3.1b)$$

$$c_t = \Delta \mu_{y,t} - a\Delta \mu_{z,t} - b(\mu_{y,t-1} - \alpha \mu_{z,t-1}) \quad (3.1c)$$

$$\Delta \mu_{z,t} = g_z + s_z D_t \quad (3.1d)$$

where

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \sim IIN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \right). \quad (3.1e)$$

We allow three kinds of dummies in order to simulate either a *single break* in the deterministic trend (segmented trends), at two *different break points* ($T/4$ or $T/2$) where T is the sample size,

$$D_{1t} = \begin{cases} 1 & t \geq T/4 \\ 0 & \text{otherwise} \end{cases} \quad D_{2t} = \begin{cases} 1 & t \geq T/2 \\ 0 & \text{otherwise} \end{cases}$$

and a *double break* at points $T/4$ and $3T/4$.

$$D_{3t} = \begin{cases} 1 & T/4 \leq t \leq 3T/4 \\ 0 & \text{otherwise} \end{cases}$$

Without loss of generality, we take $\sigma_1^2 = 1$, $g_z = 0$ and $\alpha = 1$. Thus, the experimental design variables are the parameters a, b, s where $\sigma_2 = s$, and the sample size, T .

The experiment is a full factorial design with:

$a = 0.0, 0.5, 1$ (contemporaneous correlation in first differences)

$b = 0.0$ (no cointegration), $-0.05, -0.1, -0.25, -0.5, -0.75$ (cointegration)

$s = 1, 6, 16$ (size of the breaks)

$T = 25, 50, 100, 200, 500, 1000$ (sample size)³

and allowing the possibilities of no breaks (NO), one time breaks at $T/4$ or $T/2$ (D1 and D2) or a double-break (D3) where all of the breaks considered are jumps of size s . This represents 216 experiments for each value of b . Notice that when $a = 1$ there is a common (COMFAC) restriction in the error correction model ($a = \alpha = 1$).

To obtain the empirical critical values we simulate the y_t and z_t series following the DGP (3.1a)–(3.1e) with $b = 0$ and we estimate the equations

$$\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \quad (\text{Model 1})$$

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \quad (\text{Model 2})$$

where we have imposed $\alpha = 1$, and the orders of the polynomials $\phi(B)$ and $\theta(B)$ are chosen by means of the SBIC criterion. The lower 5% tail of the distribution of the $t(\hat{b})$ statistic is the empirical critical value. The empirical size of the test is analyzed by adding an MA(1) to the errors $u_{1,t}, u_{1,t} + \theta u_{1,t-1} = v_t$ with parameter values ($\theta = \pm 0.5$). The empirical power of the test is

³Full set of tables are available upon request.

calculated analogously by simulating the series with the other parameter values of b ($b \neq 0$), and computing the percentages of rejections obtained with the previous empirical critical values. We consider the cointegrating vector as known, $(1, -\alpha)$, $(1, -1)$.

We impose simultaneous co-breaking by making $c_t = \Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t} - \alpha\mu_{z,t}) = 0$. In order to get only cobreaks in differences, we impose that $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d = 0.5$. On the other hand, to simulate a set of series with only cobreaks in levels, we impose $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = 0$, see Appendix B for a detailed derivation.

Notice that if we had set $c_d = 0$, the critical values would be the ones obtained in the case of simultaneous co-breaking, since under $b = 0$ we would have simultaneous cobreaking with $c_s = 0$. Furthermore, in the case that $a = \alpha = 1$, co-breaks in differences would imply co-breaks in levels (full cobreaking). As we can see in Appendix B, under the COMFAC restriction, co-breaks in levels imply co-breaks in differences (full co-break).

The Monte Carlo experiments are based on 2000 replications of each experiment where the first 50 observations of the simulated series are dropped to consider random initial conditions.

		Model 1	Model 2
Simultaneous Cobreaking	C V	Unstable (a, s) for $a \neq 1$	Unstable (a, s) for $a \neq 1$ (shifted $T=25$)
	Power	High (High with $a = 1$)	High (Low $T=25, a=1$)
Not cobreak in levels (Cobreak in diffs.)	C V	Unstable (a, s) for $a \neq 1$	Unstable for $a \neq 1$
	Power	Low	Low
Not cobreak in diffs. (Cobreak in levels) ($a = 1$ FC)	C V	Unstable	Unstable
	Power	High	High (except $T=25$)

TABLE 1. Results of the ECM Test

3.1. Monte Carlo simulation experiment: ECM with Simultaneous Cobreaking .

Critical values of the ECM test with Simultaneous Cobreaking. The 5% critical values from the left tail of the empirical distribution of the t -ratio of \hat{b} are given in table 1a. Table 1a is generated by making σ_2^2 , the variance of $u_{2,t}$ (see equations (3.1b) and (3.1d)) equal to the jump size (s) for

$s = 1, 6, 16$. Alternatively, we fixed the variance of $u_{2,t}$ ($\sigma_2^2 = 1$) but varies $s = 1, 6$, and 16 to make the jump in Δz_t to be more pronounced, but the obtained C.V. are very similar⁴.

Several comments are worth making. When there is not COMFAC, the distribution of the t -ratio (t_b) is shifted to the right as the jump size (s) increases. The larger the jump in Δz_t , the more likely that we under-reject the null hypothesis of non-cointegration by using the usual C. V. for non-cointegration tests (too many unit roots in the cointegrating errors). However, those effects are not very pronounced and the critical values are similar for different types of jumps (D_1, D_2 , and D_3). For $T = 100$ the 5% C.V are between -1.6 and -2.2. For larger samples ($T = 1000$), the 5% C.V. are stable around -1.7.

The main effects on the empirical distribution are obtained while changing the short-run parameter a ($a=0, 0.5, 1$). This is not a surprise as it is explained in Appendix A where the limiting distribution is given for different situations. From equation (A.2) of the Appendix A it is clear that the asymptotic critical values depend on the short run parameter a . When $a = 1$ (COMFAC restriction) $q = -(a - 1)\sigma_{u1}/\sigma_{u2}$ is zero and the limit distribution coincides with the one obtained by Dickey-Fuller (see equation (A.4) of Appendix A). Therefore, one should expect that going from $a = 0$ to $a = 1$, the empirical distribution be shifted to the left, creating higher critical values in absolute values, see table 1a. ⁵ When there is a COMFAC ($a = 1$) restriction, the C.V. obtained are those of the DF distribution. Notice that the C.V. does not depend on T , or on the break size or on the type of the break either. The 5% C.V. is around -3.0 for $T = 25$ and for $T = 100$ and $T = 1000$ is around -2.9, which is good for empirical applications.

We also got the Critical Values allowing for the empirical uncertainty the dynamics of the model was unknown and thus lags of the variables had to be included. The number of lags were estimated by means of the SBIC criterion. Our our conclusions remain unchanged (see table 2a) for sample sizes 100 and 1000. However, for $T = 25$ the distribution is shifted towards the left with 5% C.V. between -3.2 and -4.9.

Empirical Size of the ECM test with Simultaneous Cobreaking. In order to assess the robustness of our test we analyzed the empirical by using the previous critical values obtained under the null but with longer dynamics generated by a MA(1) process on the errors. In particular, we simulated our data by $\Delta y_t = c_t + a\Delta z_t + u_{1t} + \theta u_{1,t-1}$, where θ was given values 0.5 and -0.5. We found a dramatic size distortion depending on the sign of θ if we do not include relevant dynamic terms.

⁴Results are available upon request.

⁵Notice that when $a = 0$ the critical values for NO, D1, D2, D3 do not numerically coincide since in the regression under H_1 we are including Δz_t , which depends on $D_{j,t}$ and that marginally affects the critical values obtained.

For $\theta = 0.5$ the size distortions are large when $a \neq 1$, and for $\theta = -0.5$ the largest distortions are for $a = 1$.

The problem is mitigated when we add dynamic terms as selected by SBIC criterion. Table 5.1 reports the values for $\theta = 0.5$ and Table 6.1 for $\theta = -0.5$. The size of the test given in Table 5.1 is near 5%, but the size given in Table 6.1 goes to 31% in some cases when $a = 1$ and drops to 1% when $a = 0.5$. Therefore, we need to look for a more reliable test when there is a risk of having a MA(1) with parameter θ equal to 0.5.

Power of the ECM test with Simultaneous Cobreaking. The power of the ECM-test (t_b) is analyzed by generating data from the DGP under H_1 for values of b that satisfy $-2 < b < 0$. Several parameter values for b are considered, $b = -0.05, -0.1, -0.25, -0.5,$ and -0.75 . Remember from equation (2.15) that the cointegrating error has an autoregressive representation that depends on the parameter b . If we call ρ_1 the first order autoregressive parameter, $\rho_1 = b + 1$, then $\rho_1 = 0.95, 0.9, 0.75, 0.5,$ and 0.25 , corresponding to the values of the parameter b . Therefore, for $b = -0.05$ we would expect to have low power against stationary alternatives, since the AR(1) parameter, 0.95, is close to the unit root. This intuition can be explained by using the results of appendix A, equation (A.6). When b increases is like increasing h relative to T . Therefore, from equation (A.7) we should obtain a reduction in power when the COMFAC restriction ($a = 1$) is satisfied.

In general, the power of the test given in Table 3.1 is high for all possible jump sizes ($s = 1, 6, 16$) and for all sample sizes. The lowest power of the ECM test occurs for values of $a = 1$ (COMFAC restriction) and especially for small sample sizes with small values of b . Remember that $a = 1$ corresponds to $q = 0$, see equation (A.7) from Appendix A and in that case the limiting distribution of the test gets close to the Dickey and Fuller limiting distribution. This fact motivated Kremers et al. (1992), Hansen (1995), and Banerjee et al. (1997) to suggest the addition of variables like Δz_t in the test regression equation to increase the power of the test for non-cointegration ($b = 0$), or in the univariate context for a unit root.

In summary, when we are in the presence of structural breaks that are cobreaking in levels and differences, the approach based on testing for non-cointegration (t_b) in an error correction model in terms of the observable variables is remarkably robust when there is no COMFAC ($a \neq 1$) and if σ_2 is large relative to σ_1 . When $a = 1$ the power is low for $T = 25$ and 100 and for values $-0.5 < b \leq 0$, but increases when $T = 1000$. Similar results are obtained for $a \neq 1$ and $s = 1$. Yet, the power increases with the sample size and the size of the breaks, $s = 6, 16$, see Table 7.1. The problem might appear when the variables are not cobreaking in levels and differences, and the analysis of this question is the main purpose of the following section.

3.2. Monte Carlo simulation experiment: ECM without Simultaneous Cobreaking .

Critical Values of the ECM test without Simultaneous Cobreaking. Critical values are obtained for different breaks. Suppose that we ignore that those breaks have occurred and we run the ECM-test on the usual error correction equation (2.2a) assuming that c_t is constant. Table 1c includes the 5% critical values of t_b when cobreaking occurs only in levels ($\mu_{y,t} - \alpha\mu_{z,t} = 0$) but not in differences ($\Delta z_t - a\Delta y_t \neq \text{constant}$). From table 1c it is clear that the break affects dramatically the empirical critical values. For example, with sample size 1000 and $a = 0.5$, the critical values span from -5.8 to -34.56 if s goes from 1 to 16. This large change is preserved for other sample sizes. These types of results complicates testing for cointegration from an empirical point of view since we must have to find the particular C.V. for every particular break and for every particular jump size (s) and error variance σ_2^2 , as well as for any value of a .

Table 1b considers the opposite departure from simultaneous co-breaking. Cobreaking occurs only in differences ($\Delta z_t - a\Delta y_t = 0.5$), but not in levels ($\mu_{y,t} - \alpha\mu_{z,t} \neq \text{constant}$). We can see that these critical values are not affected as much. They depend on s and T , not so much on a , even for $a = 1$.

Our comments on the C.V remain even when we try to approximate the misspecification of the break by adding dynamics to the model. These conclusions based on misspecified ECM models make the empirical analysis even more dependable on the use of the correct C.V, which depend on the particular type of level shift occurred.

Empirical Size of the ECM test without Simultaneous Cobreaking. We see that the ECM test is invalidated has wrong empirical size under misspecification, especially when there is not co-break in levels, and for not co-breaking in differences when the parameter of the MA(1) is $\theta = -0.5$. However, these negative conclusions are tempered when we add dynamics since our size distortion is not affected by the sign of the MA(1) parameter, θ , although it is far from the desired level of 5%. See Tables 5 and 6.

Power of the ECM test without Simultaneous Cobreaking. Tables 3.2 and 4.2 present the results of the power of the ECM-test (t_b) when there is only cobreaking in differences but not in levels and we ignore them and proceed as if no breaks occurred in the dynamic ECM model. Similar situations could be analyzed by introducing dummy variables for the breaks at the wrong unknown date.

The results of tables 7b and 8b indicate that the ECM-test based on an equation that misspecifies the deterministic breaks in levels has no power for any analyzed parameter value or any sample size.

Higher power of the ECM test is obtained for the alternative extreme case. Consider that there is only cobreaking in levels but not in differences⁶. Then, the power of the test is higher than before, but it is still low when there is a break for $a \neq 1$ and if $-0.5 < b < 0$, see Tables 3.3 and 4.3. In this case, the power of the test is highly affected by the jump size. The power is reasonably good for parameter values of b larger or equal to 0.5 in absolute value, but this can be influenced by the size distortion of the ECM test.

Those results are not satisfactory for applied work since we are never certain whether there is no cointegration or whether there is cointegration but without cobreaking in levels or in differences. Since the critical values and the power of the ECM-test depend on the type of structural break considered, there are many possible alternative combinations of breaks that could change completely the results on cointegration testing. In the following section we analyze whether this problem could be solved or reduced by smoothing the variables using the trend component and by expressing the ECM-test in terms of first differences and levels of the trend components.

4. EXTENDED ECM MODELS

In this case we are following Toda and Yamamoto (1995) and Dolado and Lutkepohl (1996). The idea is to add some lags of the error correction term. In the case of no lags in the model, we just include in the regression one extra lag of the error correction term, rendering

$$\Delta y_t = c + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + d(y_{t-2} - z_{t-2}) + u_t, \quad (4.1)$$

and when there are more dynamics in the model, it is

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + d(y_{t-k-2} - \alpha z_{t-k-2}) + u_t, \quad (4.2)$$

where

$$\begin{aligned} \phi(B) &= 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_{p^*} B^{p^*} \\ \theta(B) &= a - \theta_1 B - \theta_2 B^2 - \dots - \theta_{q^*} B^{q^*} \\ k &= \max\{p^*, q^*\} \end{aligned}$$

⁶Notice that if $a = 1$ co-breaking in levels implies co-breaking in differences, and that is why the power of the ECM test improves so much in Tables 3.3 and 4.3.

4.1. Monte Carlo Simulation Experiment: Extended ECM Models. In this section we analyze the behavior of the critical values and the power of the ECM test with and without simultaneous co-breaking but using the extended ECM models, as shown in equations (4.1) and (4.2).

$$\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + d(y_{t-2} - z_{t-2}) + u_t \quad (\text{Model 3})$$

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + d(y_{t-k-2} - z_{t-k-2}) + u_t \quad (\text{Model 4})$$

As we can see from table 2, when we estimate Model 3, we get very stable critical values in all cases. It is important to notice that the critical values are not close to those obtained by Dickey and Fuller, but close to -1.8, which would be the Gaussian distribution value. There is a minor instability in the case co-break in levels and not in differences, since critical values range between -1.67 to -2.5. It is remarkable that the power of the test is very good in all cases.

		Model 3	Model 4
Simultaneous Cobreaking	C V	Stable	Slightly unstable (<i>a</i>)
	Power	High (High with <i>a</i> = 1)	High (Low <i>T</i> =25, <i>a</i> =1)
Not cobreak in levels (Cobreak in diffs.)	C V	Stable	Stable
	Power	High	Low for <i>a</i> ≠ 1
Not cobreak in diffs. (Cobreak in levels) (<i>a</i> = 1 FC)	C V	Slightly unstable (<i>s</i>)	Unstable
	Power	High	High (except <i>T</i> =25)

TABLE 2. Results of the Extended ECM Model

The behavior of the test when include dynamics in the estimated model by means of the SBIC criterion is similar in terms of the critical values. However, we lose power, especially in the case of co-breaks in differences but not in levels. We tried some other especifications, such as

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + d(y_{t-2} - z_{t-2}) + u_t$$

$$\phi(B)\Delta y_t = c + a\Delta z_t + \theta(B)(y_{t-1} - z_{t-1}) + u_t$$

$$\Delta y_t = c + \theta(B)\Delta z_t + \phi(B)(y_{t-1} - z_{t-1}) + u_t$$

but we could not get better results. In particular, we got a trade-off between critical values stability and power of the test.

5. FILTERS, COINTEGRATION AND ERROR CORRECTION MODELS.

The usual aim of a filter in macroeconomic time series is to extract the components different than the cycle so that it can be analyzed. We are interested in splitting an observed time series in two components,

$$y_t = y_t^g + y_t^c \quad (5.1)$$

where y_t^g is the growth component, and y_t^c is the cyclical component.

Let $\bar{b}(B)$ be a general two-sided moving average filter where we impose some constraints in the \bar{b}_k coefficients so that it is a low-pass filter (see section 5.1 for details), and call $y_t^g = \bar{b}(B)y_t$ the corresponding trend component. Then, multiplying equation (3.1a) by $\bar{b}(B)$ we get

$$\Delta \bar{b}(B)y_t = \bar{b}(B)c_t + a\Delta \bar{b}(B)z_t + b[\bar{b}(B)y_{t-1} - \alpha \bar{b}(B)z_{t-1}] + \bar{b}(B)u_{1,t} \quad (5.2)$$

which is an ECM model for the trend component

$$\Delta y_t^g = c_t^g + a\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \bar{b}(B)u_{1,t} \quad (5.3)$$

Since $\bar{b}(B)u_{1,t}$ might have some autocorrelation, we can consider the more dynamic version of the ECM for the trend components given by

$$\phi_y(B)\Delta y_t^g = c_t^g + a\phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \eta_t \quad (5.4)$$

where η_t is considered white noise and the lags of $\phi_y(B)\Delta y_t^g$ and $\phi_z(B)\Delta z_t^g$ are determined by the SBIC criterion. We might expect that for significant smoothing, c_t^g can be approximated by a constant or a linear trend.

From equation (3.1a) we can write the ECM model for a general trend-cycle decomposition as,

$$\Delta(y_t^g + y_t^c) = c_t + a\Delta(z_t^g + z_t^c) + b[(y_{t-1}^g + y_{t-1}^c) - \alpha(z_{t-1}^g + z_{t-1}^c)] + u_{1,t} \quad (5.5)$$

and grouping terms, equations (5.5) and (3.1d) can be written as

$$\Delta y_t^g = c_t^* + a\Delta z_t^g + b(y_{t-1}^g - \alpha z_{t-1}^g) + u_{1,t} \quad (5.6a)$$

$$c_t^* = (\Delta \mu_{y,t} - \Delta y_t^c) - (\Delta \mu_{z,t} - \Delta z_t^c) - b[(\mu_{y,t-1} - \alpha \mu_{z,t-1}) - (y_{t-1}^c - \alpha z_{t-1}^c)] \quad (5.6b)$$

In practice, since we do not know where the breaks occur, we would like to approximate (5.6a)–(5.6b) by

$$\phi_y(B)\Delta y_t^g = c_0^* + \phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \epsilon_t \quad (5.7)$$

In the rest of the section we do a short overview of several filters that are proposed to be used with macroeconomic time series and that will be used in our simulation study.

5.1. **Baxter and King filter (BK)** . In general, decomposition (5.1) can be written as

$$y_t = y_t^* + (y_t - y_t^*) \tag{5.8}$$

Depending on the type of filter used, y_t^* could be either the trend or the business cycle component.

Most of the filters used in macroeconomic time series are two-sided infinity-order moving averages, see for example King and Rebelo (1993) and Baxter and King (1995), but in practice we approximate it with the two-sided symmetric finite MA(k)

$$y_t^* = a_0 + \sum_{h=1}^k a_h (B^h + B^{-h}) y_t$$

The implications of the filters are clearly seen in the frequency domain by looking at the *frequency response function* of the filters. The frequency response function of the two-sided MA(k) the frequency response function is $\alpha(\omega) = \sum_{h=-k}^k a_h e^{-i\omega h}$ Baxter and King showed that when $\sum_{h=-k}^k a_h = 0$, y_t^* has no trend if the growing component of y_t was generated by deterministic trends (linear or quadratic) or by I(1) or I(2) processes. Notice that the trend reduction condition, $\sum_{h=-k}^k a_h = 0$, implies that the frequency response function satisfies $\alpha(0) = 0$. Therefore, the spectrum of y_t^c is zero at the zero frequency and y_t^* is associated with the business cycle component (y_t^c) and $(y_t - y_t^c)$ with the trend (y_t^d). These trend reducing filters are called *high-pass filters* since they pass components of the data with frequency larger than a predetermined value $\underline{\omega}$ close to 0. and $\beta(\omega) = 1$ for $|\omega| \geq \underline{\omega}$.

On the other hand, *low-pass filters* are determined so that low frequencies, (long term movements) remain unchanged while others are canceled out. In terms of the finite symmetric MA(k) filter, this means that low-pass filters must satisfy $\sum_{h=-k}^k a_h = 1$. Baxter and King (1995) showed that an ‘ideal’ approximate *low-pass filters* can be obtained by choosing the coefficients of the two-sided MA(k) filter, equal to $a_0 = \frac{1}{\pi} \underline{\omega}$ and $a_h = \frac{1}{h\pi} \sin(hw)$ for $h = 1, 2, 3, \dots$. Therefore, the complementary *high-pass filter* has coefficients $(1 - a_0)$ at $h = 0$ and $-a_h$ for $h = 1, 2, 3, \dots$

When the filter passes frequencies between $\underline{\omega}$ and $\bar{\omega}$ of the spectrum where $0 < |\underline{\omega}| < |\bar{\omega}| < \pi$ it is called *band-pass filter* and can for example be obtained by subtracting two low-pass filters. This band-pass filter is what we are calling the BK filter in the simulations.

5.2. Hodrick and Prescott filter (HP). The Hodrick and Prescott (1980, 1997) filter is widely used in macroeconomics to detrend series in order to study of the stylized facts of an economy along the business cycle. The basis of this filter is the following: starting from (5.1) they define the trend component as the solution to the following optimization problem

$$\min_{\{y_t^g\}} \sum_{t=1}^T \left[(y_t - y_t^g)^2 + \lambda (\Delta^2 y_{t+1}^g)^2 \right] \quad (5.9)$$

The first term of (5.9) might be regarded as a measure of the goodness of fit of the trend component to the observed series, while the second one imposes a penalty in order to get a smooth trend component.

Let $Y = (y_1, y_2, \dots, y_T)'$, then in matrix notation, the objective function given in (5.9) can be written as

$$\min_{\{y_t^g\}} \left[(Y - Y^g)'(Y - Y^g) + \lambda ((AY^g)'(AY^g)) \right]$$

From the first order conditions, the decomposition (5.1) can be written in vector form as

$$Y = Y^g + Y^c \quad (5.10)$$

where the $T \times 1$ vector of growth components is $Y^g = (\lambda A'A + I)^{-1} Y = A^g Y$, and the $T \times 1$ of business cycle components is $Y^c = [I - (\lambda A'A + I)^{-1}] Y = A^c Y$. It is important to realize that the rows of A^g sum to 1 ($\sum_{i=-k}^k a_{ij}^g = 1, \forall j = 1, 2, \dots, T$), and the rows of A^c sum to 0 ($\sum_{i=-k}^k a_{ij}^c = 0, \forall j = 1, 2, \dots, T$)

5.3. Monte Carlo experiment with growth components ECM . From the ECM for the trend components, see equations (5.6a)–(5.6b), we want to investigate the effects of the types of structural breaks that were introduced in section 3.1 through equation (3.1b) and the dummy variable $D_{1,t}$, $D_{2,t}$, and $D_{3,t}$. In particular, we want to analyze the effects on testing for non-cointegration based on the t -ratio $t(\hat{b})$ in the presence of structural breaks.

The details of the data generating process were given in section 3.1. To the generated data we apply the Baxter and King (BK) and Hodrick and Prescott (HP) filters. For the BK filter we use the band-pass filter proposed by Baxter and King for annual series ($\underline{\omega} = \pi/4, \bar{\omega} = \pi, k = 3$) to obtain the cyclical component y_t^c , so that $y_t^g = y_t - y_t^c$ is the trend obtained by the BK filter ⁷. The frequency interval is associated with the NBER business cycle duration as defined by Burns and Mitchell (1946) where $\underline{\omega}$ corresponds to 32 quarters (8 years) and $\bar{\omega}$ to 6 quarters (1.5 or 2 years) Baxter and King (1995) conclude that their filter is very similar to HP for $\lambda = 10$. For that

⁷Notice that in this case we are adding to the trend component some noise from the very high frequencies.

reason, and to make comparisons about the two trend-cycle decompositions we carefully compare those results obtained, the size and the power of the test based on the C.V.

Growth components ECM. HP: $\lambda = 10$		ECM	Extended ECM
Simultaneous Cobreaking	C V	Slightly unstable (a) for $a \neq 1$	Stable
	Power	High for $T > 100$ (Lower with $a = 1$)	High for $T > 100$ (Lower with $a = 1$)
Not cobreak in levels (Cobreak in diffs.)	C V	Stable	Stable
	Power	Low	Low for $a \neq 1$
Not cobreak in diffs. (Cobreak in levels) ($a = 1$ FC)	C V	Slightly unstable (s)	Stable
	Power	High for $T > 100$ with small s	High for $T > 100$ with small s

TABLE 3. Results of Growth Components ECM Test

Once we simulate our data, we apply the filter and estimate the ECM model

$$\phi_y(B)\Delta y_t^g = c + \phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \epsilon_t$$

ot the extended ECM model

$$\phi_y(B)\Delta y_t^g = c + \phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g]d[y_{t-k-2}^g - \alpha z_{t-k-2}^g] + \epsilon_t.$$

Critical Values. We can see from Tables 11 and 12 that critical values are not affected very much by the type of detrending filter (HP or BK). Critical values are more robust to s when there if simultaneous cobreaking, although they are somehow affected by the value of a . They are affected by small sample sizes, $T = 25$.

When there is only co-break in differences and not in levels, critical values are extremely stable and independent from nuisance parameters. However, in the contrary case, co-break in levels and not in differences, critical values are somehow affected by the value of s . Given that it also happens when no breaks occur, we conclude that critical values are robust to the presence of structural breaks, the kind of break, its position, or its size, but are affected by the variance ratio, although it is not noticeable in moderate sample sizes ($T = 100$).

As it was expected, whe we use the extended ECM model, critical values become stable and centered around -1.8. (See Table 19 in Appendix C.)

Empirical size. Tables 15 and 16 show the empirical size of the test when the trends of the series are estimated using the HP filter and there is an MA(1) process with parameter $\theta = \pm 0.5$, and lags are determined by the SBIC criterion. We can see that it is very stable and close to the nominal level in all cases except for some cases when there is cbreak in levels but not in differences. These conclusions remain when BK filter is employed to estimate the growth components of the series. (See Tables 17 and 18 in Appendix C.)

Power. The comments on the power of the test and how it is affected by the parameter values are very similar to the ones made before for the ECM test based on observable variables when the lags were chosen by SBIC criterion. As expected, it is always small, especially for small sample sizes ($T = 25$). The main problem remaining is the low power obtained when there is not co-breaking in levels. (See Tables 13 and 14 in Appendix C.)

In the case that we use the extended ECM model, we improve the power of the test only in the case of co-break in differences but not in levels. We still get low power unless $a = 1$. (See Table 20 in Appendix C.)

6. CONCLUSIONS

We have analyzed the effect of different structural breaks on the ECM test for non-cointegration when no dummies were included in the model. It has been showed that the critical values (C.V.) depend on the type of break and other nuisance parameters. We have also analyzed the dependence on the different timing of the break, different sizes of the break and even allowed for the possibility of having two breaks on the first difference of the mean (segmented trends in levels). Another problem of the ECM test under structural breaks is that it can have large size distortions when there is a COMFAC restriction and when there is a MA(1) with negative coefficient equal to -0.5.

The fact that the C.V's depend on nuisance parameters and that the ECM test has very low power under misspecification of the co-breaks in the level of the series, opens the possibility of improving the robustness of the results by using extended ECM models. Our experiments show that critical values of the ECM test are very stable and, when no extra dynamic terms are included, its power is excellent under any co-breaking circumstances. Unfortunately, we might still get size distortions.

Another possible solution is using some smoothing procedures. Several trend-cycle decompositions, like the HP and BK filters, were studied. The critical values obtained from the error correction model of the trend component are more robust to structural breaks, although they differ in small samples ($T = 25$). The size of the ECM test is remarkably more robust and it does not

depend so much on the values of the coefficients of the MA(1) process. Its power is generally lower than the usual ECM test based on observable variables, and follows similar patterns. The worst power is obtained when we misspecify the break in levels by just adding a constant to the regression when in fact a segmented trend is required. Unfortunately, this conclusion remains in the case of using extended ECM models on growth components.

APPENDIX A. ASYMPTOTIC DISTRIBUTION OF COINTEGRATION TESTS.

Consider the DGP given by (2.14a) and (2.14b) with $\Delta\mu_{z,t} = 0$. The distribution of the t-ratio of the parameter b under the null hypothesis that $b = 0$ (no cointegration) was analyzed by Banerjee et al. (1986), Kremers et al. (1992), Park and Phillips (1988, 1989) and Banerjee et al. (1993).

Assuming that α is known and equal to 1, $\alpha = 1$ one could consistently estimate the parameters of equation (2.14a) by OLS,

$$\Delta y_t = \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{u}_{1,t} \quad (\text{A.1})$$

The asymptotic distribution of the t-ratio of \hat{b} under the null hypothesis ($b = 0$) is

$$t_{\hat{b}} \Rightarrow \frac{(a-1) \int B_{u_2} dB_{u_1} + r^{-1} \int B_{u_1} dB_{u_1}}{\sqrt{(a-1)^2 \int B_{u_2}^2 + 2(a-1)r^{-1} \int B_{u_2} B_{u_1} + r^{-2} \int B_{u_1}^2}} \quad (\text{A.2})$$

where ' \Rightarrow ' denotes weak convergence. B_{u_1} and B_{u_2} are independent Brownian motions and $r = \sigma_1/\sigma_2$. In terms of the 'signal to noise ratio', $q = -(a-1)r = \frac{-(a-1)\sigma_1}{\sigma_2}$

$$t_{\hat{b}} \Rightarrow \frac{\int B_{u_2} dB_{u_1} + \frac{1}{q} \int B_{u_1} dB_{u_1}}{\sqrt{\int B_{u_2}^2 + 2\left(\frac{1}{q}\right) \int B_{u_2} B_{u_1} + \left(\frac{1}{q}\right)^2 \int B_{u_1}^2}}. \quad (\text{A.3})$$

Notice that when $q = 0$ (or $a = 1$, COMFAC restriction) (A.2) is reduced to

$$t_{\hat{b}} \Rightarrow \frac{\int B_{u_1} dB_{u_1}}{\sqrt{\int B_{u_1}^2}} \equiv DF \quad (\text{A.4})$$

where DF is the Dickey–Fuller distribution, (see Dickey and Fuller, 1979), of the t -ratio of \hat{b} from the OLS regression (A.9), which is the non-cointegration D–F test of Engle and Granger (1987) when the cointegration vector is known. From (A.3) and for large q

$$t_{\hat{b}} \Rightarrow \frac{\int B_{u_2} dB_{u_1}}{\sqrt{\int B_{u_2}^2}} + O_p(q^{-1}) \quad (\text{A.5})$$

Since B_{u_1} and B_{u_2} are independent Brownian motions, the leading term in the right hand side follows a standard Normal distribution (Park and Phillips, 1988).

When $\Delta\mu_{z,t} = 0$, the distribution of the t-ratio of the parameter b in (A.1) under a local alternative hypothesis, $b = \frac{h}{T}, h < 0$ was derived by Kremers et al. (1992) following Phillips (1987).

$$t_{\hat{b}} \Rightarrow h(1+q^2)^{1/2} \left(\int K_e^2 \right)^{1/2} + \frac{(a-1) \int K_{u_2} dB_{u_1} + r^{-1} \int K_{u_1} dB_{u_1}}{\sqrt{(a-1)^2 \int K_{u_2}^2 + 2(a-1)r^{-1} \int K_{u_2} K_{u_1} + r^{-2} \int K_{u_1}^2}} \quad (\text{A.6})$$

where $e_t = (a-1)\Delta z_t + u_{1t}$ and K_e is a diffusion process. Notice that for $h = 0$, $K_i = B_i$ (a Brownian motion), and (A.6) coincides with (A.2). Therefore, the power of $t_{\hat{b}}$ should increase with

h for a given T , or increase with the value of b , since the distributions of $t_{\hat{b}}$ under H_1 is shifted to the left of $t_{\hat{b}}$ under H_0 .

For $a=1$

$$t_{\hat{b}} \Rightarrow c \left(\int K_e^2 \right)^{1/2} + \frac{\int K_{u_1} dB_{u_1}}{\sqrt{\int K_{u_1}^2}} \quad (\text{A.7})$$

which is the distribution of the DF statistic under the local alternative. For a large q , (A.6) is approximately Normal conditional on u_{2t}

$$t_{\hat{b}} \Rightarrow N \left[\left(c(1+q^2) \right)^{1/2} \left(\int K_{u_2}^2 \right)^{1/2}, 1 \right] + O_p(q^{-1}) \quad (\text{A.8})$$

The unconditional mean of $t_{\hat{b}}$ is approximately $E(t_{\hat{b}}) \approx c(1+q^2)^{1/2} \frac{1}{\sqrt{2}}$. Therefore, for large q , since c is negative, the distribution of $t_{\hat{b}}$ under the local alternative can arbitrarily be shifted towards the left and hence the power of the test can be made arbitrarily close to 100%.

When $a = \alpha$, and α is known, $\alpha = 1$, equation (2.14a) can be written as

$$\Delta(y_t - z_t) = b(y_{t-1} - z_{t-1}) + u_{1t} \quad (\text{A.9})$$

which is a standard Dickey-Fuller equation. Estimating the unrestricted equation (2.14a) with $\alpha = 1$, the $t_{\hat{b}} \rightarrow DF$ distribution. However, in small samples the power of $t_{\hat{b}}$ in (2.14a) with $\alpha = 1$ can be lower than the power of $t_{\hat{b}}$ in (A.9) since the estimation of the unrestricted model (2.14a) is less efficient, therefore generating smaller t-ratios than the asymptotic ones in absolute values.

APPENDIX B. ERROR CORRECTION MODELS WITH CO-BREAKS IN LEVELS AND DIFFERENCES

Consider the DGP given by (2.2a)–(2.2c). There are four cases of interest

1. $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d$ and $\mu_{y,t-1} - \alpha\mu_{z,t-1} \neq \text{constant}$, co-break in differences.
2. $\Delta\mu_{y,t} - a\Delta\mu_{z,t} \neq \text{constant}$ and $\mu_{y,t-1} - \alpha\mu_{z,t-1} = c_l$, co-break in levels.
3. $\Delta\mu_{y,t} - a\Delta\mu_{z,t} \neq \text{constant}$ and $\mu_{y,t-1} - \alpha\mu_{z,t-1} \neq \text{constant}$, no co-break.
4. $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d$ and $\mu_{y,t-1} - \alpha\mu_{z,t-1} = c_l$, co-break in levels and differences.

Remark B.1. When the deterministic rates of growth are constant, say $\Delta\mu_z = g_z$ and $\Delta\mu_y = g_y$, then

$$\mu_{y,t-1} - \alpha\mu_{z,t-1} = \mu_y^0 - \alpha\mu_z^0 + (g_y - \alpha g_z)(t-1) \quad (\text{B.1a})$$

$$c_t = (g_y - \alpha g_z) - b(\mu_y^0 - \alpha\mu_z^0) - b(g_y - \alpha g_z)(t-1) \quad (\text{B.1b})$$

Assuming co-break in levels, the necessary condition is $g_y - \alpha g_z = 0$, but that implies that $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = (\alpha - a)g_z$, which is constant, so that the co-break in differences condition is

met. So, in the case of constant growth rates (no breaks), co-break in levels implies co-break in differences.

On the other hand, in the case of co-break in differences, it must be $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = g_y - ag_z = c_d$ and hence

$$\mu_{y,t-1} - \alpha\mu_{z,t-1} = \mu_{y,0} - \alpha\mu_{z,0} + c_d(t-1) + g_z(a-\alpha)(t-1).$$

Thus, co-break in differences implies co-break in levels only when $c_d = 0$ and $a = \alpha$ (COMFAC restriction).

Remark B.2. Assume that there is a break, so that $\Delta\mu_{z,t} = g_z + s_z D_t$ and $\Delta\mu_y = g_y + s_y D_t$, then

$$\mu_{y,t-1} - \alpha\mu_{z,t-1} = \mu_y^0 - \alpha\mu_z^0 + (g_y - \alpha g_z)(t-1) + (s_y - \alpha s_z)(t-1-t_1)D_{t-1} \quad (\text{B.2a})$$

$$c_t = (g_y - ag_z) + (s_y - as_z)D_t - b(\mu_y^0 - \alpha\mu_z^0) - b(g_y - \alpha g_z)(t-1) - b(s_y - \alpha s_z)(t-1-t_1)D_{t-1} \quad (\text{B.2b})$$

The necessary conditions to have co-breaks in differences are $g_y - ag_z = c_d$ and $s_y - as_z = 0$. In that case

$$\mu_{y,t-1} - \alpha\mu_{z,t-1} = \mu_y^0 - \alpha\mu_z^0 + c_d(t-1) + (a-\alpha)g_z(t-1) + (a-\alpha)s_z(t-1-t_1)D_{t-1}$$

and co-breaks in differences implies co-break in levels only when $a = \alpha$ (COMFAC restriction) and $c_d = 0$.

Inversely, the necessary conditions to have co-breaks in levels, from equation (B.2a), are $g_y - ag_z = 0$ and $s_y - \alpha s_z = 0$. It is clear that if $a = \alpha$, these conditions are the ones required for co-break in differences taking $c_d = 0$. In effect, under co-breaks in levels

$$\Delta\mu_{y,t} - a\Delta\mu_{z,t} = (a-\alpha)g_z + (a-\alpha)s_z D_t,$$

and in the case that $a = \alpha$ (COMFAC restriction), there would be co-break in differences too.

- Park, J. Y. (1992), 'Canonical cointegrating regression', *Econometrica* **60**, 119–43.
- Perron, P. (1989), 'The great crash, the oil price shock and the unit root hypothesis', *Econometrica* **57**, 1361–1401.
- Perron, P. and T. Vogelsang (1992), 'Nonstationarity and level shifts with an application to purchasing power parity', *Journal of Business and Economic Statistics* **10**, 301–320.
- Phillips, P. C. B. (1987), 'Towards a unified asymptotic theory for autoregression', *Biometrika* **74**, 535–47.
- Phillips, P. C. B. and B. Hansen (1990), 'Statistical inference in instrumental variables regression with I(1) processes', *Review of Economic Studies* **57**, 99–125.
- Rappoport, P. and L. Reichlin (1989), 'Segmented trends and nonstationary time series', *Economic Journal* **99**, 168–177.
- Toda, H. and T. Yamamoto (1995), 'Statistical inference in vector autorregression with possibly integrated processes', *Journal of Econometrics* **66**, 225–250.
- Vogelsang, T. (1997), 'Wald-type tests for detecting breaks in the trend of a dynamic time series', *Econometric Theory* **13**, 818–849.
- Zivot, E. and D. Andrews (1992), 'Further evidence on the great crash, the oil price shock and the unit root hypothesis', *Journal of Business and Economic Statistics* **10**.

MIGUEL A. ARRANZ, DPTO. FUNDAMENTOS DEL ANÁLISIS ECONÓMICO, UNIVERSIDAD DE ALICANTE

E-mail address: mangel@merlin.fae.ua.es

ALVARO ESCRIBANO, DPTO. DE ESTADÍSTICA Y ECONOMETRÍA, UNIVERSIDAD CARLOS III DE MADRID

E-mail address: alvaroe@est-econ.uc3m.es



APPENDIX C. TABLES



T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.768	-1.912	-1.814	-2.831	-2.134	-1.877	-3.027	-2.994	-2.991
	D1	-2.118	-1.833	-1.716	-2.648	-1.828	-1.801	-3.026	-3.088	-2.985
	D2	-2.284	-1.828	-1.813	-2.723	-2.010	-1.817	-3.069	-2.950	-3.095
	D3	-2.213	-1.818	-1.760	-2.662	-1.835	-1.720	-2.976	-3.071	-3.012
100	NO	-2.590	-1.921	-1.795	-2.811	-2.156	-1.796	-2.854	-2.852	-2.898
	D1	-1.817	-1.649	-1.633	-2.048	-1.632	-1.649	-2.892	-2.996	-2.925
	D2	-1.942	-1.723	-1.700	-2.268	-1.772	-1.801	-2.907	-2.954	-2.956
	D3	-1.795	-1.738	-1.641	-2.252	-1.788	-1.730	-2.954	-2.937	-2.889
1000	NO	-2.632	-1.880	-1.745	-2.797	-2.150	-1.856	-2.794	-2.872	-2.857
	D1	-1.696	-1.730	-1.653	-1.759	-1.609	-1.700	-2.901	-2.903	-2.984
	D2	-1.741	-1.639	-1.577	-1.771	-1.651	-1.735	-2.887	-2.887	-2.908
	D3	-1.722	-1.740	-1.602	-1.854	-1.696	-1.654	-2.906	-2.983	-2.900

Table 1.1: Critical values. $\sigma_2 = s$. Estimated model $\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.
Simultaneous Cobreaking

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.447	-1.913	-1.825	-2.483	-2.046	-1.882	-2.560	-2.391	-2.480
	D1	-2.478	-1.837	-1.713	-2.836	-1.850	-1.804	-2.460	-2.430	-2.469
	D2	-2.551	-1.824	-1.807	-2.879	-2.051	-1.836	-2.581	-2.582	-2.493
	D3	-2.550	-1.848	-1.754	-2.802	-1.934	-1.714	-2.538	-2.488	-2.410
100	NO	-2.012	-1.900	-1.824	-2.060	-2.066	-1.851	-2.039	-2.034	-1.964
	D1	-2.141	-1.649	-1.632	-2.662	-1.668	-1.644	-2.023	-2.060	-1.994
	D2	-2.189	-1.742	-1.677	-2.369	-1.814	-1.779	-1.997	-2.040	-2.053
	D3	-2.292	-1.739	-1.639	-2.567	-1.833	-1.737	-1.954	-2.018	-2.022
1000	NO	-1.796	-1.681	-1.757	-1.685	-1.691	-1.793	-1.664	-1.711	-1.775
	D1	-1.777	-1.737	-1.659	-1.969	-1.637	-1.689	-1.661	-1.735	-1.747
	D2	-1.965	-1.652	-1.588	-1.862	-1.683	-1.744	-1.741	-1.769	-1.746
	D3	-1.833	-1.754	-1.598	-1.911	-1.678	-1.645	-1.802	-1.704	-1.736

Table 1.2: Critical values. $\sigma_2 = s$. Estimated model $\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.
Cobreaking in Differences, Not Cobreaking in Levels

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.768	-1.912	-1.814	-2.831	-2.134	-1.877	-3.027	-2.994	-2.991
	D1	-3.397	-5.771	-6.252	-3.163	-4.548	-6.466	-3.026	-3.088	-2.985
	D2	-3.694	-6.894	-7.923	-3.375	-5.662	-7.692	-3.069	-2.950	-3.095
	D3	-3.299	-4.424	-4.583	-3.063	-4.044	-4.575	-2.976	-3.071	-3.012
100	NO	-2.590	-1.921	-1.795	-2.811	-2.156	-1.796	-2.854	-2.852	-2.898
	D1	-4.704	-11.035	-12.270	-3.558	-8.148	-11.497	-2.892	-2.996	-2.925
	D2	-4.998	-13.150	-15.340	-3.725	-9.686	-13.783	-2.907	-2.954	-2.956
	D3	-4.096	-7.876	-8.296	-3.391	-6.668	-8.135	-2.954	-2.937	-2.889
1000	NO	-2.632	-1.880	-1.745	-2.797	-2.150	-1.856	-2.794	-2.872	-2.857
	D1	-10.546	-32.748	-37.437	-6.432	-23.787	-34.546	-2.901	-2.903	-2.984
	D2	-12.429	-38.894	-43.772	-7.134	-28.103	-42.559	-2.887	-2.887	-2.908
	D3	-9.038	-23.042	-23.910	-5.894	-18.265	-23.635	-2.906	-2.983	-2.900

Table 1.3: Critical values. $\sigma_2 = s$. Estimated model $\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.
Cobreaking in Levels, not Cobreaking in Differences

TABLE 1. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = var(u_{1t})$, and $\sigma_2^2 = var(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + u_t$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-4.177	-3.635	-3.108	-4.387	-3.870	-3.358	-4.738	-4.775	-4.443
	D1	-3.555	-3.226	-3.393	-4.388	-3.564	-3.511	-4.586	-4.941	-4.969
	D2	-4.076	-3.328	-3.435	-4.385	-3.611	-3.341	-4.779	-4.955	-5.176
	D3	-3.563	-3.336	-3.330	-4.421	-3.232	-3.151	-4.515	-4.680	-4.452
100	NO	-2.597	-1.936	-1.835	-2.820	-2.164	-1.825	-2.867	-2.859	-2.898
	D1	-1.838	-1.702	-1.646	-2.084	-1.681	-1.680	-2.970	-3.046	-2.980
	D2	-1.977	-1.746	-1.706	-2.327	-1.843	-1.827	-3.038	-3.014	-3.050
	D3	-1.800	-1.742	-1.676	-2.277	-1.804	-1.733	-2.989	-3.013	-2.922
1000	NO	-2.636	-1.882	-1.749	-2.797	-2.150	-1.856	-2.794	-2.872	-2.862
	D1	-1.698	-1.731	-1.653	-1.766	-1.635	-1.719	-2.929	-2.923	-3.005
	D2	-1.742	-1.639	-1.577	-1.774	-1.674	-1.735	-2.897	-2.920	-2.921
	D3	-1.722	-1.740	-1.602	-1.854	-1.696	-1.654	-2.911	-2.989	-2.919

Table 2.1: Critical values. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-4.139	-3.670	-3.061	-4.007	-3.722	-3.597	-4.041	-3.927	-3.961
	D1	-3.828	-3.225	-3.425	-4.256	-3.575	-3.546	-3.705	-4.230	-4.106
	D2	-4.004	-3.391	-3.413	-4.298	-3.624	-3.350	-4.277	-4.228	-4.261
	D3	-3.964	-3.467	-3.367	-4.620	-3.296	-3.238	-3.868	-4.064	-3.698
100	NO	-2.041	-1.936	-1.849	-2.056	-2.068	-1.890	-2.047	-2.048	-1.972
	D1	-2.170	-1.686	-1.643	-2.668	-1.689	-1.690	-2.098	-2.060	-1.994
	D2	-2.215	-1.778	-1.704	-2.369	-1.875	-1.805	-2.077	-2.139	-2.153
	D3	-2.300	-1.747	-1.678	-2.571	-1.850	-1.739	-1.957	-2.033	-2.035
1000	NO	-1.796	-1.676	-1.757	-1.685	-1.691	-1.794	-1.658	-1.711	-1.775
	D1	-1.777	-1.743	-1.659	-2.001	-1.651	-1.700	-1.680	-1.753	-1.779
	D2	-1.965	-1.652	-1.588	-1.867	-1.731	-1.743	-1.745	-1.771	-1.765
	D3	-1.833	-1.754	-1.598	-1.911	-1.678	-1.645	-1.802	-1.704	-1.732

Table 2.2: Critical values. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$.

$$\text{Estimated model } \hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-4.177	-3.635	-3.108	-4.387	-3.870	-3.358	-4.738	-4.775	-4.443
	D1	-4.412	-4.838	-4.846	-4.518	-4.394	-5.029	-4.586	-4.941	-4.969
	D2	-4.946	-5.781	-4.884	-4.977	-5.186	-5.611	-4.779	-4.955	-5.176
	D3	-4.779	-4.910	-4.162	-4.960	-4.685	-4.580	-4.515	-4.680	-4.452
100	NO	-2.597	-1.936	-1.835	-2.820	-2.164	-1.825	-2.867	-2.859	-2.898
	D1	-4.646	-2.918	-2.153	-3.552	-4.810	-2.649	-2.970	-3.046	-2.980
	D2	-4.951	-3.362	-2.683	-3.752	-6.739	-3.159	-3.038	-3.014	-3.050
	D3	-4.043	-2.797	-2.322	-3.411	-3.249	-2.535	-2.989	-3.013	-2.922
1000	NO	-2.636	-1.882	-1.749	-2.797	-2.150	-1.856	-2.794	-2.872	-2.862
	D1	-10.085	-2.965	-2.128	-6.415	-5.318	-2.888	-2.929	-2.923	-3.005
	D2	-11.962	-3.481	-2.576	-7.084	-5.916	-3.443	-2.897	-2.920	-2.921
	D3	-6.262	-2.786	-2.299	-5.779	-3.388	-2.363	-2.911	-2.989	-2.919

Table 2.3: Critical values. Cobreaking in Levels, not cobreaking in differences. $\sigma_2 = s$.

$$\text{Estimated model } \hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

TABLE 2. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t}$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	a	NO												D1												D2												D3											
		b=-0.05				b=-0.1				b=-0.25				b=-0.5				b=-0.75				b=-0.05				b=-0.1				b=-0.25				b=-0.5				b=-0.75											
		4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000																
25	0.0	4	7.40	14.75	46.30	90.70	89.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
		6	71.00	95.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
	0.5	4	27.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
		6	32.55	64.15	86.60	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
	100	0.0	4	4.90	8.00	7.70	13.80	46.35	86.05	83.00	86.85	84.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
			6	41.80	50.05	53.75	51.60	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90															
0.5		4	99.20	99.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
		6	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20																	
1000		0.0	4	61.15	58.70	52.50	44.20	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15																
			6	92.50	92.00	90.15	91.10	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00															
	0.5	4	11.15	37.55	33.85	34.55	23.15	49.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40																
		6	69.20	66.45	61.80	59.75	53.15	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40																

Table 3.1: Power of the test. $\sigma_2 = s$ Estimated model $\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$. Simultaneous Cobreaking

T	a	NO												D1												D2												D3											
		b=-0.05				b=-0.1				b=-0.25				b=-0.5				b=-0.75				b=-0.05				b=-0.1				b=-0.25				b=-0.5				b=-0.75											
		4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000	4	6	16	1000																
25	0.0	4	5.95	6.30	7.85	7.55	9.15	7.10	8.40	10.40	15.45	17.20	26.10	36.25	39.55	42.15	45.75	47.90	50.05	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40	50.40																	
		6	66.25	92.40	99.50	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
	0.5	4	28.80	48.55	78.80	1.35	9.85	12.65	22.15	32.10	28.50	23.15	13.45	24.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10	5.10																	
		6	80.20	97.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
	100	0.0	4	4.10	2.88	1.20	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10																
			6	93.25	95.20	95.45	94.55	96.10	40.00	37.80	34.05	24.25	18.85	17.15	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00	44.00																
0.5		4	11.80	50.05	53.75	51.60	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90	48.90																
		6	99.20	99.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																
1000		0.0	4	61.15	58.70	52.50	44.20	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15	41.15																
			6	92.50	92.00	90.15	91.10	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00	93.00															
	0.5	4	11.15	37.55	33.85	34.55	23.15	49.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40	17.40																
		6	69.20	66.45	61.80	59.75	53.15	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40	49.40																

Table 3.2: Power of the test. $\sigma_2 = s$ Estimated model $\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$. Cobreak in differences, not in levels

T	a	s	NO															D1															D2															D3														
			b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75																														
1	0.0	1	7.50	14.75	46.30	90.70	99.75	6.95	9.85	24.10	60.60	91.05	7.75	10.20	20.95	50.80	83.30	7.50	11.10	26.60	62.40	91.20	7.50	11.10	26.60	62.40	91.20	7.50	11.10	26.60	62.40	91.20	7.50	11.10	26.60	62.40	91.20																									
		6	71.00	95.95	100.00	100.00	100.00	9.25	13.35	26.55	59.95	94.00	8.70	12.35	15.10	28.20	58.65	85.05	11.75	15.85	32.45	72.15	97.25	11.75	15.85	32.45	72.15	97.25	11.75	15.85	32.45	72.15	97.25	11.75	15.85	32.45	72.15	97.25																								
		16	99.25	100.00	100.00	100.00	100.00	6.70	13.60	24.00	56.20	93.70	8.55	9.15	12.20	18.40	46.00	77.80	7.80	10.15	20.30	55.90	89.25	10.15	16.25	35.65	72.80	96.65	10.15	16.25	35.65	72.80	96.65	10.15	16.25	35.65	72.80	96.65																								
		6	7.70	10.70	28.05	69.40	95.20	7.05	7.25	17.55	51.70	87.70	4.45	5.85	13.20	40.10	81.15	78.65	10.25	16.85	32.95	71.30	97.15	16.85	32.95	71.30	97.15	97.15	16.85	32.95	71.30	97.15	97.15	16.85	32.95	71.30	97.15	97.15																								
		16	32.55	64.55	98.60	100.00	100.00	10.50	16.60	31.00	72.00	96.10	10.75	11.85	17.75	36.10	78.55	88.35	6.70	8.00	15.80	50.20	86.00	8.00	8.00	15.80	50.20	86.00	8.00	8.00	15.80	50.20	86.00	8.00	8.00	15.80	50.20	86.00																								
		16	83.30	98.70	100.00	100.00	100.00	8.25	10.55	19.75	48.25	90.00	7.75	8.75	11.60	20.00	48.35	82.20	6.70	7.70	14.10	48.15	82.60	7.70	7.70	14.10	48.15	82.60	7.70	7.70	14.10	48.15	82.60	7.70	7.70	14.10	48.15	82.60																								
	0.5	1	4.65	7.45	14.85	48.05	83.00	4.65	6.75	7.35	15.50	49.30	85.00	5.50	7.50	13.60	47.05	83.20	5.30	7.70	14.10	48.15	82.60	5.30	7.70	14.10	48.15	82.60	5.30	7.70	14.10	48.15	82.60	5.30	7.70	14.10	48.15	82.60																								
		6	4.90	8.00	14.25	48.85	86.05	4.65	6.75	7.35	15.50	49.30	85.00	5.50	7.50	13.60	47.05	83.20	6.10	7.75	14.10	48.15	82.60	7.75	7.75	14.10	48.15	82.60	7.75	7.75	14.10	48.15	82.60																													
		16	4.75	7.70	15.80	48.55	84.75	6.95	8.00	15.90	51.60	86.10	10.25	6.15	15.20	46.25	86.65	84.10	15.85	18.85	46.50	98.85	100.00	15.85	18.85	46.50	98.85	100.00	15.85	18.85	46.50	98.85	100.00																													
		6	40.05	81.50	100.00	100.00	100.00	8.75	16.70	60.05	99.80	100.00	10.25	16.25	16.25	46.25	98.95	100.00	12.95	18.85	46.50	98.85	100.00	12.95	18.85	46.50	98.85	100.00	12.95	18.85	46.50	98.85	100.00																													
		16	100.00	100.00	100.00	100.00	100.00	7.20	8.00	18.25	86.20	100.00	6.55	3.90	5.20	28.90	92.55	13.50	13.50	13.50	45.30	98.80	100.00	13.50	13.50	45.30	98.80	100.00	13.50	13.50	45.30	98.80	100.00																													
		16	100.00	100.00	100.00	100.00	100.00	7.80	6.65	17.10	80.75	100.00	3.50	2.50	7.20	11.30	73.75	99.95	13.75	22.05	52.90	99.60	100.00	13.75	22.05	52.90	99.60	100.00	13.75	22.05	52.90	99.60	100.00																													
100	0.5	1	18.35	47.45	99.50	100.00	100.00	11.55	22.65	82.00	100.00	100.00	10.30	19.35	72.30	100.00	100.00	12.55	31.65	97.75	100.00	100.00	12.55	31.65	97.75	100.00	100.00	12.55	31.65	97.75	100.00	100.00	12.55	31.65	97.75	100.00	100.00																									
		6	95.25	100.00	100.00	100.00	100.00	6.70	7.85	19.35	85.60	100.00	5.45	3.50	14.75	71.75	99.95	13.40	14.85	45.30	98.80	100.00	14.85	45.30	98.80	100.00	100.00	14.85	45.30	98.80	100.00	100.00																														
		16	100.00	100.00	100.00	100.00	100.00	8.25	12.20	38.35	97.35	100.00	7.20	7.05	14.75	71.75	99.95	13.40	14.85	45.30	98.80	100.00	7.20	7.05	14.75	71.75	99.95	7.20	7.05	14.75	71.75	99.95																														
		6	13.90	35.85	97.65	100.00	100.00	13.30	35.30	96.85	100.00	100.00	13.05	35.00	97.00	100.00	100.00	12.40	34.15	96.25	100.00	100.00	12.40	34.15	96.25	100.00	100.00	12.40	34.15	96.25	100.00	100.00																														
		16	13.40	33.80	97.70	100.00	100.00	9.85	28.55	96.05	100.00	100.00	12.85	31.65	95.90	100.00	100.00	14.05	34.15	97.75	100.00	100.00	14.05	34.15	97.75	100.00	100.00	14.05	34.15	97.75	100.00	100.00																														
		16	11.65	31.20	97.45	100.00	100.00	11.90	33.25	96.40	100.00	100.00	12.05	30.25	97.15	100.00	100.00	16.05	39.10	100.00	100.00	100.00	16.05	39.10	100.00	100.00	100.00	16.05	39.10	100.00	100.00	100.00																														
	1.0	1	100.00	100.00	100.00	100.00	100.00	4.85	26.60	100.00	100.00	100.00	.70	5.25	99.15	100.00	100.00	1.25	4.20	82.30	100.00	100.00	1.25	4.20	82.30	100.00	100.00	1.25	4.20	82.30	100.00	100.00																														
		6	100.00	100.00	100.00	100.00	100.00	.00	.45	100.00	100.00	100.00	.00	.00	100.00	100.00	100.00	44.70	97.70	100.00	100.00	100.00	44.70	97.70	100.00	100.00	100.00	44.70	97.70	100.00	100.00	100.00																														
		16	100.00	100.00	100.00	100.00	100.00	29.35	94.25	100.00	100.00	100.00	14.40	79.80	100.00	100.00	100.00	3.45	9.05	97.35	100.00	100.00	3.45	9.05	97.35	100.00	100.00	3.45	9.05	97.35	100.00	100.00																														
		6	100.00	100.00	100.00	100.00	100.00	.20	.50	60.30	100.00	100.00	.05	.00	1.35	99.99	100.00	2.10	2.55	79.05	100.00	100.00	2.10	2.55	79.05	100.00	100.00	2.10	2.55	79.05	100.00	100.00																														
		16	100.00	100.00	100.00	100.00	100.00	.05	.05	3.05	100.00	100.00	.00	.00	100.00	100.00	100.00	106.00	100.00	100.00	100.00	100.00	106.00	100.00	100.00	100.00	100.00	106.00	100.00	100.00	100.00	100.00																														
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																													
1000	0.5	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
	1.0	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																									
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.																									

T	a	NO										D1										D2										D3									
		b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75															
25	0.0	1	7.38	8.80	14.70	36.20	54.75	12.70	13.25	35.55	61.55	69.10	8.45	14.31	27.38	49.45	65.80	13.80	19.30	35.65	62.30	70.00	13.80	19.30	35.65	62.30	70.00														
		6	27.55	56.85	74.55	74.20	75.50	70.20	79.05	79.70	77.90	79.35	65.10	75.25	78.88	77.75	81.05	67.35	76.15	79.55	78.50	79.05	67.35	76.15	79.55	78.50	79.05														
		16	73.55	80.85	81.25	80.95	78.70	77.95	79.30	79.20	79.00	78.60	77.85	77.88	77.40	78.75	77.70	79.70	79.70	79.55	81.35	80.45	79.70	79.55	81.35	80.45	80.45														
		1	6.00	6.00	10.80	16.20	33.50	6.30	7.60	10.70	18.25	35.80	7.25	9.05	12.60	21.35	39.30	6.45	8.15	11.45	20.30	33.50	6.45	8.15	11.45	20.30	33.50														
		6	12.05	20.35	50.50	65.05	71.05	69.30	64.75	66.05	68.90	72.80	27.40	48.05	61.40	69.05	74.10	37.60	59.35	70.40	77.75	77.75	37.60	59.35	70.40	77.75	77.75														
		16	38.35	60.10	70.70	74.95	76.90	69.20	72.55	72.85	79.20	71.32	66.40	71.85	74.60	75.50	78.35	71.00	74.75	75.45	77.25	79.20	71.00	74.75	75.45	77.25	79.20														
100	0.0	1	5.05	5.20	5.55	5.75	5.85	5.30	5.45	5.40	5.45	5.35	5.45	5.70	5.75	5.80	6.10	5.90	6.10	6.55	6.65	6.10	5.90	6.10	6.55	6.65															
		6	4.75	5.20	6.40	7.40	8.45	24.20	6.35	6.45	6.95	8.45	4.80	5.70	6.15	7.05	7.80	7.10	7.05	7.95	8.75	7.10	7.05	7.95	8.75	8.75															
		16	40.35	80.20	99.30	100.00	100.00	99.85	99.85	99.85	99.85	99.85	98.35	98.35	98.85	99.85	100.00	99.70	99.85	100.00	100.00	100.00	99.70	99.85	100.00	100.00	100.00														
		1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
1000	0.0	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00															
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00															
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
		1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00														

Table 4.1: Power of the test. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model $\hat{\varphi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.

T	a	NO										D1										D2										D3									
		b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75															
25	0.0	1	4.75	5.00	4.30	3.00	2.75	6.70	6.20	9.10	5.30	3.70	1.45	1.15	13.55	12.20	8.40	7.15	8.40	17.10	8.50	7.15	8.75	9.40	12.10	15.95															
		6	24.95	46.50	54.30	27.95	6.15	12.25	13.65	2.10	2.10	1.45	11.15	13.55	8.25	3.40	3.40	21.80	15.70	20.30	24.80	18.75	21.80	22.20	20.30	24.80	24.80														
		16	73.45	78.50	65.60	27.70	9.05	13.35	19.85	2.75	2.75	1.25	17.10	12.20	8.25	3.30	4.25	4.15	4.75	5.70	8.70	4.80	4.15	4.75	5.70	8.70	8.70														
		1	10.15	13.80	18.25	16.15	5.05	8.25	5.85	5.90	13.75	13.75	6.85	7.10	6.85	3.05	3.05	7.60	12.35	13.60	14.30	15.85	7.60	12.35	13.60	14.30	15.85														
		6	31.90	49.90	47.55	15.20	5.65	9.45	8.95	2.15	2.00	1.30	11.50	6.52	3.45	3.30	2.15	14.25	18.75	20.65	23.00	24.55	14.25	18.75	20.65	23.00	24.55														
		16	4.40	3.85	3.60	3.05	2.05	4.10	4.15	3.85	3.85	3.85	4.25	4.40	4.30	4.80	4.70	4.70	4.35	3.90	3.90	2.25	4.35	3.90	3.90	3.90	2.25														
100	0.0	1	5.60	5.75	7.90	4.5	.03	6.75	5.85	5.15	.55	.20	28.45	10.55	7.65	10.70	3.75	8.25	19.65	5.40	4.10	8.25	19.65	5.40	4.10	9.80															
		6	88.40	54.95	65.55	1.65	.50	8.88	8.38	5.15	.00	.00	3.45	1.30	1.70	7.20	19.30	4.05	2.80	11.75	2.10	4.05	2.80	11.75	2.10	31.15															
		16	1.80	1.20	8.30	1.0	.28	25.60	16.15	7.5	.00	.00	26.95	19.40	2.70	.85	.00	33.15	18.90	8.55	14.75	16.05	26.95	19.40	2.70	.85	16.05														
		1	39.95	33.15	7.10	.45	.15	20.30	3.35	.05	.20	.10	19.00	2.70	.85	.15	.30	1.35	1.50	.05	.00	.00	19.00	2.70	.85	.15	.00														
		6	1.25	.05	.05	.00	.00	1.55	.45	.35	.15	.15	1.80	.65	.18	.25	.30	1.50	.05	.00	.00	.00	1.80	.65	.18	.25	.00														
		16	5.50	2.40	.00	.00	.00	6.10	1.80	.45	.15	.15	3.52	3.65	.80	.30	.45	7.85	8.75	1.55	1.55	.30	7.85	8.75	1.55	1.55	.30														
1000	0.0	1	1.70	.80	.00	.00	.00	1.85	.00	.00	.00	5.85	.00	.00	.00	.00	5.90	1.35	.00	.00	.00	5.90	1.35	.00	.00	.00															
		6	16.35	2.05	.00	.00	.00	1.85	.00	.00	.00	2.35	1.85	.00	.00	.00	20.40	12.30	4.20	2.15	.90	20.40	12.30	4.20	2.15	.90															
		16	18.60	3.75	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	11.20	1.25	.00	.30	.70	11.20	1.25	.00	.30	.70															
		1	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00														
		6	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00														
		16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00														

Table 4.2: Power of the test. Cobreak in differences, not in levels. $\sigma_2 = s$. Estimated model $\hat{\varphi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	6.200	5.900	7.600	5.900	7.800	7.650	4.350	4.400	6.500
	D1	6.800	7.150	6.250	5.700	6.350	5.550	5.650	5.200	4.700
	D2	5.550	7.500	6.150	6.550	7.950	8.450	6.250	5.650	5.100
	D3	7.800	6.850	7.000	5.100	8.000	8.500	7.200	5.900	7.450
100	NO	9.000	6.950	5.250	7.100	8.150	6.250	7.850	8.250	7.650
	D1	7.050	6.050	6.500	9.500	7.450	6.300	10.050	7.650	7.950
	D2	8.050	6.600	6.900	9.850	7.300	4.900	9.000	8.350	7.700
	D3	9.050	5.700	6.000	7.550	6.350	6.000	10.250	8.950	9.550
1000	NO	7.200	6.400	5.550	6.050	5.500	6.750	4.950	5.650	4.700
	D1	5.850	4.600	4.700	7.150	5.050	4.900	5.350	5.050	4.150
	D2	5.850	4.850	5.600	10.550	6.300	4.200	6.900	6.900	7.100
	D3	5.050	5.050	5.600	5.600	5.100	5.150	5.700	4.650	6.300

Table 5.1: Empirical size of the test. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	5.400	5.200	7.850	5.050	8.150	6.300	5.850	6.800	7.850
	D1	7.300	7.200	6.100	7.400	7.000	5.800	8.750	6.700	6.450
	D2	7.500	7.400	6.300	6.600	7.750	8.450	6.800	6.500	7.250
	D3	7.400	6.300	6.800	4.800	8.400	8.200	7.600	6.000	9.200
100	NO	7.450	6.100	5.050	8.700	7.600	6.450	8.200	6.650	8.050
	D1	8.550	6.500	6.450	9.550	7.550	6.050	9.500	9.750	9.700
	D2	8.100	6.150	6.750	10.400	7.350	5.150	13.300	8.150	7.150
	D3	7.400	5.850	6.050	8.350	5.900	6.000	9.450	8.350	6.900
1000	NO	4.550	6.950	5.100	6.900	7.000	6.250	7.450	6.550	6.150
	D1	6.000	4.600	4.650	11.150	5.100	5.100	9.000	6.900	6.000
	D2	5.250	4.700	5.400	7.700	5.650	4.200	8.550	5.800	6.200
	D3	6.400	5.000	5.550	7.150	5.350	5.300	5.650	6.400	6.550

Table 5.2: Empirical size of the test. Cobreaking in differences, not cobreaking in levels.

$$\sigma_2 = s. \text{ Estimated model } \hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	6.200	5.900	7.600	5.900	7.800	7.650	4.350	4.400	6.500
	D1	6.000	5.200	4.300	5.450	5.950	4.650	5.650	5.200	4.700
	D2	5.200	4.100	4.250	5.200	6.150	3.800	6.250	5.650	5.100
	D3	6.250	4.200	7.600	4.700	5.350	5.250	7.200	5.900	7.450
100	NO	9.000	6.950	5.250	7.100	8.150	6.250	7.850	8.250	7.650
	D1	.800	8.200	6.250	2.700	.850	6.350	10.050	7.650	7.950
	D2	.800	6.600	5.100	2.750	.000	5.750	9.000	8.350	7.700
	D3	2.550	4.500	6.600	3.750	4.500	4.000	10.250	8.950	9.550
1000	NO	7.200	6.400	5.550	6.050	5.500	6.750	4.950	5.650	4.700
	D1	.000	12.400	6.450	.000	9.150	11.250	5.350	5.050	4.150
	D2	.000	11.550	6.250	.100	9.450	9.400	6.900	6.900	7.100
	D3	3.250	8.350	5.000	.050	7.750	8.050	5.700	4.650	6.300

Table 5.3: Empirical size of the test. Cobreaking in Levels, not cobreaking in differences.

$$\sigma_2 = s. \text{ Estimated model } \hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

TABLE 5. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} + 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t}$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.400	3.350	4.900	5.600	3.950	3.850	6.550	7.500	10.900
	D1	3.600	4.200	3.750	3.000	2.800	3.200	8.600	6.000	6.050
	D2	3.250	5.700	4.150	3.650	3.400	4.800	9.250	9.550	7.000
	D3	3.850	4.100	3.700	2.650	4.000	4.250	10.000	9.800	11.800
100	NO	2.900	2.450	2.000	14.500	1.450	1.400	32.250	29.800	29.850
	D1	2.800	2.850	2.850	4.900	2.450	2.350	34.600	29.100	30.950
	D2	2.700	3.000	3.150	6.150	2.050	1.600	32.650	31.200	30.250
	D3	3.100	2.000	2.700	1.150	1.400	1.750	34.950	31.800	31.900
1000	NO	1.650	3.300	2.950	5.050	1.300	2.350	10.800	12.050	10.700
	D1	3.550	3.100	2.950	5.550	2.850	2.650	12.650	11.500	10.000
	D2	3.800	3.650	4.100	8.900	4.000	2.400	15.500	12.750	15.400
	D3	2.750	3.250	4.000	2.300	2.600	2.200	13.250	10.600	11.400

Table 6.1: Empirical size of the test. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	2.450	3.850	5.650	2.850	3.800	3.550	2.700	3.300	3.350
	D1	3.300	4.250	3.600	6.350	2.850	3.150	2.550	2.700	2.350
	D2	3.900	5.850	4.450	4.000	3.100	5.050	2.750	2.800	3.050
	D3	4.050	3.600	3.600	3.450	4.100	4.100	3.000	3.000	4.000
100	NO	1.600	2.350	1.900	1.300	1.450	1.200	1.300	.850	1.200
	D1	2.200	3.150	2.800	3.950	2.600	2.350	1.100	1.650	2.150
	D2	2.150	2.750	3.200	1.000	2.100	1.600	1.650	1.250	.800
	D3	1.600	2.050	2.600	1.400	1.200	1.600	1.700	1.150	.950
1000	NO	2.250	3.750	3.050	3.050	2.900	2.450	4.050	3.000	2.350
	D1	2.750	3.050	2.850	2.250	2.500	2.900	3.350	2.700	3.100
	D2	2.400	3.250	4.050	2.000	3.050	2.250	3.350	2.950	3.050
	D3	2.800	3.150	4.000	2.250	2.700	2.300	2.300	2.300	2.750

Table 6.2: Empirical size of the test. Cobreaking in differences, not cobreaking in

levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t$.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.400	3.350	4.900	5.600	3.950	3.850	6.550	7.500	10.900
	D1	4.450	2.950	3.150	6.150	2.800	2.950	8.600	6.000	6.050
	D2	4.150	5.400	5.250	5.700	4.600	5.650	9.250	9.550	7.000
	D3	5.700	4.400	7.300	4.950	5.400	6.050	10.000	9.800	11.800
100	NO	2.900	2.450	2.000	14.500	1.450	1.400	32.250	29.800	29.850
	D1	7.900	2.700	5.200	19.900	12.250	3.250	34.600	29.100	30.950
	D2	13.350	2.500	5.350	21.850	10.400	2.700	32.650	31.200	30.250
	D3	8.950	3.300	7.850	16.050	4.550	4.200	34.950	31.800	31.900
1000	NO	1.650	3.300	2.950	5.050	1.300	2.350	10.800	12.050	10.700
	D1	3.750	1.650	4.300	13.450	3.400	3.650	12.650	11.500	10.000
	D2	3.050	1.800	4.300	15.000	2.950	1.850	15.500	12.750	15.400
	D3	2.550	3.000	4.800	6.800	2.900	3.900	13.250	10.600	11.400

Table 6.3: Empirical size of the test. Cobreaking in Levels, not cobreaking in differences.

$$\sigma_2 = s. \text{ Estimated model } \hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + u_t.$$

TABLE 6. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} - 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t}$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.500	-1.869	-1.806	-2.425	-2.076	-1.787	-2.614	-2.661	-2.635
	D1	-1.953	-1.731	-1.703	-2.372	-1.814	-1.764	-2.658	-2.698	-2.607
	D2	-1.948	-1.792	-1.761	-2.187	-1.796	-1.782	-2.594	-2.599	-2.517
	D3	-2.000	-1.877	-1.735	-2.190	-1.797	-1.734	-2.660	-2.566	-2.719
100	NO	-2.006	-1.737	-1.582	-2.094	-1.800	-1.729	-2.133	-2.057	-2.107
	D1	-1.796	-1.722	-1.727	-1.783	-1.623	-1.603	-2.105	-2.114	-2.150
	D2	-1.737	-1.620	-1.704	-1.757	-1.735	-1.730	-2.069	-2.100	-2.069
	D3	-1.704	-1.722	-1.762	-1.823	-1.701	-1.697	-2.162	-2.063	-2.046
1000	NO	-1.691	-1.672	-1.636	-1.871	-1.704	-1.757	-1.733	-1.673	-1.838
	D1	-1.696	-1.664	-1.720	-1.701	-1.574	-1.672	-1.813	-1.749	-1.777
	D2	-1.650	-1.624	-1.557	-1.735	-1.619	-1.675	-1.809	-1.822	-1.739
	D3	-1.796	-1.608	-1.634	-1.727	-1.673	-1.637	-1.834	-1.810	-1.827

Table 7.1: Critical values. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model

$$\Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.230	-1.843	-1.817	-2.215	-1.993	-1.775	-2.113	-2.187	-2.119
	D1	-2.173	-1.726	-1.705	-2.670	-1.829	-1.762	-2.217	-2.121	-2.121
	D2	-2.135	-1.790	-1.762	-2.347	-1.799	-1.806	-2.176	-2.112	-2.043
	D3	-2.135	-1.881	-1.735	-2.386	-1.787	-1.735	-2.203	-2.205	-2.342
100	NO	-1.848	-1.752	-1.584	-1.855	-1.787	-1.731	-1.848	-1.776	-1.830
	D1	-1.810	-1.726	-1.723	-2.064	-1.628	-1.598	-1.831	-1.857	-1.808
	D2	-1.787	-1.620	-1.700	-1.936	-1.732	-1.725	-1.830	-1.826	-1.799
	D3	-1.715	-1.726	-1.764	-1.873	-1.711	-1.700	-1.873	-1.810	-1.780
1000	NO	-1.639	-1.669	-1.625	-1.785	-1.707	-1.728	-1.670	-1.659	-1.742
	D1	-1.682	-1.662	-1.722	-1.774	-1.570	-1.672	-1.764	-1.664	-1.686
	D2	-1.673	-1.635	-1.558	-1.784	-1.609	-1.670	-1.715	-1.737	-1.677
	D3	-1.778	-1.608	-1.634	-1.734	-1.672	-1.635	-1.756	-1.665	-1.748

Table 7.2: Critical values. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$.

$$\text{Estimated model } \Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-2.500	-1.869	-1.806	-2.425	-2.076	-1.787	-2.614	-2.661	-2.635
	D1	-2.474	-2.716	-2.852	-2.695	-2.470	-2.784	-2.658	-2.698	-2.607
	D2	-2.535	-3.054	-3.380	-2.582	-2.770	-3.080	-2.594	-2.599	-2.517
	D3	-2.555	-2.801	-2.873	-2.537	-2.602	-2.737	-2.660	-2.566	-2.719
100	NO	-2.006	-1.737	-1.582	-2.094	-1.800	-1.729	-2.133	-2.057	-2.107
	D1	-2.165	-2.397	-2.519	-2.112	-2.181	-2.453	-2.105	-2.114	-2.150
	D2	-2.081	-2.625	-2.849	-2.198	-2.373	-2.602	-2.069	-2.100	-2.069
	D3	-2.045	-2.402	-2.403	-2.081	-2.250	-2.371	-2.162	-2.063	-2.046
1000	NO	-1.691	-1.672	-1.636	-1.871	-1.704	-1.757	-1.733	-1.673	-1.838
	D1	-1.826	-2.225	-2.412	-1.792	-2.133	-2.363	-1.813	-1.749	-1.777
	D2	-1.907	-2.388	-2.510	-1.912	-2.169	-2.591	-1.809	-1.822	-1.739
	D3	-1.819	-2.112	-2.297	-1.789	-2.059	-2.088	-1.834	-1.810	-1.827

Table 7.3: Critical values. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$.

$$\text{Estimated model } \Delta y_t = \hat{c} + \hat{a}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t.$$

TABLE 7. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a= 0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-4.234	-3.490	-3.067	-4.531	-3.552	-3.180	-5.224	-5.287	-4.748
	D1	-2.921	-2.671	-2.663	-3.877	-2.820	-2.325	-5.297	-4.875	-5.016
	D2	-3.461	-3.172	-3.166	-4.754	-3.341	-2.803	-5.119	-5.876	-5.616
	D3	-3.274	-2.915	-3.181	-4.256	-2.901	-2.269	-4.882	-5.233	-5.217
100	NO	-2.638	-1.926	-1.770	-2.888	-2.152	-1.792	-2.918	-2.989	-2.997
	D1	-1.795	-1.693	-1.593	-2.098	-1.635	-1.708	-2.964	-3.027	-3.004
	D2	-1.938	-1.710	-1.734	-2.306	-1.854	-1.756	-3.060	-3.044	-3.083
	D3	-1.844	-1.746	-1.705	-2.318	-1.738	-1.690	-3.048	-3.021	-2.914
1000	NO	-2.657	-1.886	-1.735	-2.843	-2.163	-1.863	-2.896	-2.893	-2.932
	D1	-1.688	-1.751	-1.662	-1.763	-1.616	-1.695	-2.971	-2.940	-3.033
	D2	-1.761	-1.655	-1.603	-1.762	-1.674	-1.719	-2.969	-2.968	-2.964
	D3	-1.749	-1.720	-1.627	-1.830	-1.691	-1.629	-2.901	-2.985	-2.936

Table 9.1: Critical values. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-3.776	-3.249	-3.093	-3.637	-3.762	-3.232	-3.545	-3.514	-3.585
	D1	-3.610	-2.721	-2.741	-4.338	-2.885	-2.327	-3.380	-4.266	-3.786
	D2	-4.198	-3.257	-3.311	-4.334	-3.357	-2.884	-4.054	-4.573	-4.413
	D3	-3.990	-3.074	-3.225	-4.870	-3.192	-2.358	-3.599	-4.604	-3.719
100	NO	-2.043	-1.938	-1.748	-2.045	-2.072	-1.814	-2.080	-2.033	-1.995
	D1	-2.170	-1.705	-1.615	-2.739	-1.666	-1.712	-2.056	-2.098	-2.034
	D2	-2.294	-1.759	-1.757	-2.371	-1.869	-1.771	-2.078	-2.089	-2.058
	D3	-2.333	-1.736	-1.698	-2.613	-1.786	-1.695	-1.977	-2.058	-2.045
1000	NO	-1.796	-1.694	-1.741	-1.696	-1.694	-1.799	-1.668	-1.734	-1.751
	D1	-1.779	-1.731	-1.654	-2.008	-1.617	-1.677	-1.671	-1.718	-1.793
	D2	-1.960	-1.663	-1.615	-1.854	-1.707	-1.719	-1.752	-1.756	-1.741
	D3	-1.852	-1.717	-1.613	-1.893	-1.709	-1.638	-1.813	-1.705	-1.729

Table 9.2: Critical values. Cobreak in differences, not in levels. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_t.$$

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-4.234	-3.490	-3.067	-4.531	-3.552	-3.180	-5.224	-5.287	-4.748
	D1	-4.615	-4.891	-5.055	-4.949	-4.465	-5.539	-5.297	-4.875	-5.016
	D2	-5.125	-5.951	-4.929	-5.061	-5.215	-5.839	-5.119	-5.876	-5.616
	D3	-5.088	-5.127	-4.571	-4.982	-4.567	-4.942	-4.882	-5.233	-5.217
100	NO	-2.638	-1.926	-1.770	-2.888	-2.152	-1.792	-2.918	-2.989	-2.997
	D1	-4.490	-2.952	-2.275	-3.491	-3.560	-1.574	-2.964	-3.027	-3.004
	D2	-4.759	-3.425	-2.659	-3.763	-5.542	-1.560	-3.060	-3.044	-3.083
	D3	-4.028	-2.867	-2.427	-3.420	-1.711	-1.467	-3.048	-3.021	-2.914
1000	NO	-2.657	-1.886	-1.735	-2.830	-2.169	-1.863	-2.893	-2.893	-2.932
	D1	-9.623	-3.096	-2.397	-6.318	.227	-1.128	-2.971	-2.940	-3.033
	D2	-11.496	-3.543	-2.673	-6.863	.237	-1.175	-2.969	-2.968	-2.964
	D3	-5.627	-3.074	-2.645	-5.608	.702	-1.202	-2.901	-2.985	-2.936

Table 9.3: Critical values. Cobreak in levels, not in differences. $\sigma_2 = s$. Estimated model

$$\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_t.$$

TABLE 9. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_{1t}$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a= 0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	α	NO													D1			D2			D3		
		$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$		
25	0.0	1	6.25	8.65	27.25	79.90	98.75	10.10	16.35	46.55	93.25	99.75	10.25	13.90	30.20	92.55	98.70	9.55	14.75	46.55	91.50	99.65	
		6	35.15	78.90	100.00	100.00	100.00	43.15	86.55	100.00	100.00	100.00	40.25	85.60	100.00	100.00	100.00	37.65	86.50	99.95	100.00	100.00	
		16	92.65	100.00	100.00	100.00	100.00	96.85	100.00	100.00	100.00	100.00	97.35	100.00	100.00	100.00	100.00	96.80	100.00	100.00	100.00	100.00	
	0.5	1	7.25	9.95	22.15	62.50	92.75	6.30	8.75	24.00	65.85	94.70	18.55	22.25	29.35	73.50	96.05	9.50	12.05	30.95	73.45	97.15	
		6	12.60	28.90	91.60	100.00	100.00	19.40	42.05	95.80	100.00	100.00	56.55	64.05	97.15	100.00	100.00	20.70	44.25	96.50	100.00	100.00	
		16	50.85	93.35	100.00	100.00	100.00	58.40	96.55	100.00	100.00	100.00	95.55	96.40	100.00	100.00	100.00	62.10	97.15	100.00	100.00	100.00	
100	0.0	1	5.80	7.80	15.85	48.75	81.30	5.30	7.05	14.85	46.40	82.80	5.90	8.15	15.80	50.50	84.10	6.00	8.20	16.60	52.40	84.60	
		6	4.05	6.65	13.35	45.60	82.20	4.50	6.45	13.55	44.50	85.50	6.10	8.45	15.45	48.50	83.45	6.40	8.50	16.60	52.40	84.60	
		16	5.05	6.55	14.35	45.60	82.20	6.45	7.85	15.45	48.95	85.50	7.30	8.45	16.30	53.55	87.15	8.40	9.20	16.25	42.95	81.50	
	0.5	1	13.70	31.10	93.65	100.00	100.00	17.15	38.25	96.35	100.00	100.00	17.60	41.45	96.35	100.00	100.00	13.05	42.65	97.05	100.00	100.00	
		6	88.95	100.00	100.00	100.00	100.00	92.25	100.00	100.00	100.00	100.00	93.15	100.00	100.00	100.00	100.00	83.10	100.00	100.00	100.00	100.00	
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
1000	0.0	1	48.25	85.25	100.00	100.00	100.00	54.25	92.80	100.00	100.00	52.60	96.55	100.00	100.00	100.00	100.00	54.75	97.40	100.00	100.00	100.00	
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
	0.5	1	43.50	81.70	100.00	100.00	100.00	49.10	94.35	100.00	100.00	48.40	99.90	100.00	49.60	100.00	100.00	49.50	100.00	100.00	100.00	100.00	
		6	99.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
1000	1.0	1	48.30	91.50	100.00	100.00	100.00	41.25	92.65	100.00	100.00	42.40	91.70	100.00	100.00	100.00	44.15	92.70	100.00	100.00	100.00		
		6	50.40	93.75	100.00	100.00	100.00	46.20	92.35	100.00	100.00	46.75	91.95	100.00	47.50	91.55	100.00	47.40	91.55	100.00	100.00	100.00	
		16	42.40	91.60	100.00	100.00	100.00	41.60	92.65	100.00	100.00	44.45	92.40	100.00	44.45	91.95	100.00	42.20	91.95	100.00	100.00	100.00	

Table 8.1: Power of the test. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model $\Delta y_t = \hat{c} + \hat{\alpha}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t$.

T	α	NO													D1			D2			D3		
		$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$	$b=-0.05$	$b=-0.1$	$b=-0.25$	$b=-0.5$	$b=-0.75$		
25	0.0	1	5.90	6.70	10.30	28.45	58.65	5.20	6.50	12.35	29.45	60.00	6.50	8.70	22.30	48.55	74.90	7.45	10.30	20.35	48.40	72.25	
		6	33.40	74.40	100.00	100.00	100.00	15.50	28.70	62.35	89.60	96.10	14.40	29.00	66.30	79.50	92.80	14.10	31.00	60.55	81.00	94.40	
		16	91.45	99.95	100.00	100.00	100.00	37.05	56.15	74.55	87.95	97.35	35.95	48.55	65.85	85.85	92.40	43.25	61.00	73.20	87.30	96.05	
	0.5	1	4.75	5.20	6.95	19.10	44.05	4.60	6.25	14.50	44.35	78.00	6.75	7.40	10.25	26.55	53.85	6.30	7.45	12.30	31.80	59.50	
		6	12.80	24.40	67.70	93.25	99.05	9.05	13.45	35.75	71.30	91.75	8.85	14.65	38.70	68.30	90.20	10.25	17.45	42.20	74.25	90.90	
		16	49.50	89.70	100.00	100.00	100.00	18.25	36.95	66.20	86.85	96.75	17.45	35.50	61.75	80.95	94.75	23.80	43.80	67.15	86.75	95.30	
100	0.0	1	5.30	6.25	7.30	17.85	39.25	4.70	4.90	5.95	16.90	35.65	5.15	5.55	7.35	17.85	38.60	5.00	4.80	6.40	15.55	34.15	
		6	4.65	4.75	6.55	16.30	34.80	5.55	6.65	7.05	15.20	37.45	6.70	6.30	7.20	17.15	38.65	4.90	4.90	6.00	15.55	35.25	
		16	6.20	5.40	7.35	17.95	37.25	6.15	6.35	7.75	17.20	39.00	6.70	6.50	7.65	17.80	42.60	10.15	9.00	12.60	30.80	39.30	
	0.5	1	7.50	12.25	42.55	94.70	100.00	8.25	11.35	42.50	92.30	100.00	8.25	16.05	46.40	94.55	98.95	10.15	17.60	48.05	94.70	99.80	
		6	61.70	94.15	100.00	100.00	100.00	31.95	60.80	94.55	100.00	100.00	33.45	56.35	94.85	100.00	100.00	38.25	59.35	94.15	99.90	100.00	
		16	100.00	100.00	100.00	100.00	100.00	60.15	74.55	97.35	100.00	100.00	54.75	69.40	96.20	100.00	100.00	69.10	77.90	94.15	99.95	100.00	
1000	0.0	1	6.45	9.65	28.95	84.20	99.65	17.20	15.80	41.45	91.10	99.85	7.00	10.70	32.90	84.65	98.80	8.05	11.60	34.00	84.25	99.80	
		6	19.45	40.15	94.90	100.00	100.00	18.10	37.60	87.45	99.95	100.00	14.85	29.80	82.05	100.00	100.00	29.00	38.05	84.70	99.75	100.00	
		16	88.00	99.70	100.00	100.00	100.00	44.80	70.50	97.45	100.00	100.00	37.30	61.40	92.30	100.00	100.00	48.45	67.85	93.75	99.95	100.00	
	0.5	1	5.70	8.20	26.30	74.55	99.10	5.75	8.90	26.40	76.10	98.75	7.15	9.85	26.70	78.00	98.60	7.45	9.70	28.50	75.00	98.65	
		6	6.45	9.65	28.50	78.95	99.05	6.70	7.70	26.95	75.75	98.60	7.20	8.55	28.30	78.80	99.15	7.40	8.20	26.65	76.55	98.65	
		16	5.70	8.60	24.80	75.60	98.85	6.00	11.20	29.35	77.80	99.25	6.80	8.80	28.95	77.95	98.70	7.30	10.20	29.40	79.40	99.45	
1000	0.0	1	26.90	71.10	100.00	100.00	100.00	25.50	66.95	100.00	100.00	100.00	28.60	68.80	100.00	100.00	100.00	22.05	65.65	100.00	100.00	100.00	
		6	99.20	100.00	100.00	100.00	100.00	89.95	99.85	100.00	100.00	100.00	88.55	99.80	100.00	100.00	100.00	85.20	99.55	100.00	100.00	100.00	
		16	100.00	100.00	100.00	100.00	100.00	95.60	100.00	100.00	100.00	100.00	96.65	100.00	100.00	100.00	100.00	91.65	99.95	100.00	100.00	100.00	
	0.5	1	17.40	49.95	99.65	100.00	100.00	21.30	51.15	93.95	100.00	100.00	18.05	50.75	99.90	100.00	100.00	19.65	52.55	99.80	100.00	100.00	
		6	75.75	99.95	100.00	100.00	100.00	70.95	98.80	100.00	100.00	100.00	66.70	98.55	100.00	100.00	100.00	64.65	97.90	100.00	100.00	100.00	
		16	99.95	100.00	100.00	100.00	100.00	92.05	100.00	100.00	100.00	100.00	90.40	99.95	100.00	100.00	100.00	87.30	99.45	100.00	100.00	100.00	
1000	1.0	1	20.15	46.30	99.40	100.00	100.00	15.75	42.75	93.10	100.00	100.00	16.05	43.05	98.95	100.00	100.00	17.25	44.55	98.80	100.00	100.00	
		6	19.75	46.35	99.40	100.00	100.00	18.10	46.15	99.30	100.00	100.00	16.05	42.70	99.10	100.00	100.00	19.00	44.25	99.35	100.00	100.00	
		16	15.50	44.10	99.00	100.00	100.00	16.15	46.50	93.35	100.00	100.00	19.05	46.40	99.10	100.00	100.00	15.90	43.55	99.20	100.00	100.00	

Table 8.2: Power of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\Delta y_t = \hat{c} + \hat{\alpha}\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-2} - z_{t-2}) + u_t$.

T	α	f	NO												D1												D2												D3											
			b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.95	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.95	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.95	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.95	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.95																		
1000	0.0	1	6.28	8.65	27.28	79.90	98.75	7.58	10.40	57.80	75.20	60.55	8.70	9.80	27.70	71.55	95.50	5.68	8.55	34.10	69.20	93.30	9.20	24.60	69.20	93.30	9.20	24.60	69.20	93.30	9.20	24.60	69.20	93.30																
		6	35.15	78.90	100.00	100.00	100.00	1.170	18.55	58.80	98.55	100.00	5.50	18.45	34.70	88.25	99.55	9.50	14.50	37.10	86.25	99.55	8.55	15.10	43.10	93.60	99.75	17.10	44.80	93.70	99.95	15.10	44.80	93.70	99.95															
		16	92.65	100.00	100.00	100.00	100.00	1.152	18.50	58.75	98.15	100.00	5.55	18.50	34.70	88.15	99.75	9.55	14.50	37.10	86.25	99.75	8.55	15.10	43.10	93.60	99.75	17.10	44.80	93.70	99.95	15.10	44.80	93.70	99.95															
		6	7.28	9.95	32.18	62.50	92.75	4.55	6.85	16.15	51.25	88.10	9.05	6.00	17.85	35.55	88.30	99.35	5.05	16.05	40.45	82.05	99.35	9.25	15.70	42.65	90.30	99.65	15.70	44.70	94.50	100.00	15.70	44.70	94.50	100.00														
		16	50.85	93.35	100.00	100.00	100.00	1.050	17.50	50.15	93.80	99.55	11.35	11.35	16.05	40.45	89.20	99.30	11.35	16.05	40.45	89.20	99.30	9.25	15.70	42.65	90.30	99.65	15.70	44.70	94.50	100.00	15.70	44.70	94.50	100.00														
		16	5.80	7.80	15.88	48.75	81.30	3.50	7.05	14.85	46.40	82.60	5.30	6.15	15.85	38.00	84.10	97.00	5.30	6.15	15.85	38.00	84.10	97.00	6.00	15.70	42.65	90.30	99.65	15.70	44.70	94.50	100.00	15.70	44.70	94.50	100.00													
100	0.0	1	5.05	6.65	13.35	45.60	82.35	4.50	6.45	13.45	44.00	81.90	5.10	8.45	18.45	35.55	87.15	4.40	8.45	18.45	35.55	87.15	4.40	8.45	18.45	35.55	87.15	4.40	8.45	18.45	35.55	87.15	4.40	8.45	18.45	35.55														
		6	13.70	31.10	53.85	100.00	100.00	10.50	24.00	86.70	100.00	100.00	12.25	26.90	66.45	100.00	100.00	18.40	36.20	69.85	100.00	100.00	20.40	43.05	99.75	100.00	100.00	20.40	43.05	99.75	100.00	20.40	43.05	99.75	100.00															
		16	88.95	100.00	100.00	100.00	100.00	23.00	50.60	100.00	100.00	100.00	16.40	36.45	72.65	100.00	100.00	18.40	36.45	72.65	100.00	100.00	17.70	41.10	98.95	100.00	100.00	17.70	41.10	98.95	100.00	17.70	41.10	98.95	100.00															
		6	9.35	21.65	39.35	78.75	100.00	10.50	22.55	50.60	100.00	100.00	9.05	18.65	36.45	72.65	100.00	100.00	18.40	36.45	72.65	100.00	100.00	17.55	41.10	98.95	100.00	100.00	17.55	41.10	98.95	100.00	17.55	41.10	98.95	100.00														
		16	43.50	90.70	100.00	100.00	100.00	20.50	48.45	99.60	100.00	100.00	18.00	36.10	71.80	99.80	100.00	100.00	19.75	36.10	71.80	99.80	100.00	19.75	41.10	98.95	100.00	100.00	19.75	41.10	98.95	100.00	19.75	41.10	98.95	100.00														
		16	98.25	100.00	100.00	100.00	100.00	23.00	50.60	100.00	100.00	100.00	16.40	36.45	72.65	100.00	100.00	18.40	36.45	72.65	100.00	100.00	17.70	41.10	98.95	100.00	100.00	17.70	41.10	98.95	100.00	17.70	41.10	98.95	100.00															
100	0.5	1	8.80	15.90	27.38	59.85	100.00	10.50	18.75	39.55	89.30	100.00	10.50	20.20	40.50	89.90	100.00	18.40	36.20	69.85	100.00	100.00	17.55	41.10	98.95	100.00	100.00	17.55	41.10	98.95	100.00	17.55	41.10	98.95	100.00															
		6	9.95	19.10	30.20	68.45	100.00	12.00	16.70	38.25	89.30	100.00	10.50	14.25	38.45	71.80	99.90	10.50	14.25	38.45	71.80	99.90	10.50	17.70	41.10	98.95	100.00	10.50	17.70	41.10	98.95	100.00	10.50	17.70	41.10	98.95														
		16	8.45	17.30	28.45	68.45	100.00	12.00	16.70	38.25	89.30	100.00	10.50	14.25	38.45	71.80	99.90	10.50	14.25	38.45	71.80	99.90	10.50	17.70	41.10	98.95	100.00	10.50	17.70	41.10	98.95	100.00	10.50	17.70	41.10	98.95														
		6	70.85	99.65	100.00	100.00	100.00	63.45	39.35	100.00	100.00	100.00	58.50	38.45	100.00	100.00	100.00	61.55	38.45	100.00	100.00	100.00	61.55	38.45	100.00	100.00	100.00	61.55	38.45	100.00	100.00	61.55	38.45	100.00	100.00															
		16	100.00	100.00	100.00	100.00	100.00	93.40	100.00	100.00	100.00	100.00	82.25	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	88.55	100.00	100.00	100.00															
		16	100.00	100.00	100.00	100.00	100.00	93.95	100.00	100.00	100.00	100.00	82.25	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	100.00	88.55	100.00	100.00	100.00	88.55	100.00	100.00	100.00															
1000	0.5	1	48.25	95.25	100.00	100.00	100.00	43.95	100.00	100.00	100.00	43.95	100.00	100.00	100.00	100.00	43.95	100.00	100.00	100.00	100.00	43.95	100.00	100.00	100.00	100.00	43.95	100.00	100.00	100.00	43.95	100.00	100.00	100.00																
		6	99.95	100.00	100.00	100.00	100.00	86.80	100.00	100.00	100.00	86.80	100.00	100.00	100.00	100.00	86.80	100.00	100.00	100.00	100.00	86.80	100.00	100.00	100.00	100.00	86.80	100.00	100.00	100.00	86.80	100.00	100.00	100.00																
		16	100.00	100.00	100.00	100.00	100.00	93.55	100.00	100.00	100.00	93.55	100.00	100.00	100.00	100.00	93.55	100.00	100.00	100.00	100.00	93.55	100.00	100.00	100.00	100.00	93.55	100.00	100.00	100.00	93.55	100.00	100.00	100.00																
		6	48.30	91.50	100.00	100.00	100.00	41.25	92.85	100.00	100.00	41.25	92.85	100.00	100.00	41.25	92.85	100.00	41.25	92.85	100.00	100.00	41.25	92.85	100.00	100.00	41.25	92.85	100.00	100.00	41.25	92.85	100.00	100.00																
		16	50.40	93.75	100.00	100.00	100.00	46.20	92.85	100.00	100.00	46.20	92.85	100.00	100.00	46.20	92.85	100.00	46.20	92.85	100.00	100.00	46.20	92.85	100.00	100.00	46.20	92.85	100.00	100.00	46.20	92.85	100.00	100.00																
		16	42.40	91.60	100.00	100.00	100.00	41.60	92.65	100.00	100.00	41.60	92.65	100.00	100.00	41.60	92.65	100.00	41.60	92.65	100.00	100.00	41.60	92.65	100.00	100.00	41.60	92.65	100.00	100.00	41.60	92.65	100.00	100.00																

Table 8.3: Power of the test. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\Delta y_{it} = \hat{c} + \hat{a}\Delta z_{it} + \hat{b}(y_{it-1} - \alpha z_{t-1}) + \hat{d}(y_{it-2} - \alpha z_{t-2}) + \alpha_1$.

Table 8: $H_0 : \Delta y_{it} = c_t + a\Delta z_{it} + u_{1t}$, $\Delta z_{it} = sD_{it} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under $\Delta y_{it} = c_t + a\Delta z_{it} + b(y_{it-1} - \alpha z_{t-1}) + u_{1t}$ and $\sigma_1^2=1$. The estimated model is $\Delta y_{it} = c + a\Delta z_{it} + b(y_{it-1} - \alpha z_{t-1}) + d(y_{it-2} - \alpha z_{t-2}) + u_{1t}$. Critical values of the test are provided at table 1 for different sample sizes (T=25, 100, 1000), different short run parameter values ($\alpha=0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

REFERENCES

- Andrés, J., A. Escribano, C. Molinas and D. Taguas (1990), *La inversión en España: Econometría con restricciones de equilibrio*, A. Bosch.
- Andrews, D. (1993), 'Tests of parameter instability and structural change with unknown break point', *Econometrica* **61**, 821–856.
- Andrews, D., W. Lee and W. Ploberger (1996), 'Optimal changepoints tests for normal linear regression', *Journal of Econometrics* **70**, 9–38.
- Bai, J. (1997), 'Estimating multiple breaks one at a time', *Econometric Theory* **13**, 315–352.
- Bai, J. and P. Perron (1998), 'Estimating and testing linear models with multiple structural changes', *Econometrica* **66**, 47–78.
- Banerjee, A., J. Dolado, D. Hendry and G. Smith (1986), 'Exploring equilibrium relationships in econometrics through static models: Some Monte Carlo evidence', *Oxford Bulletin of Economics and Statistics* **48**, 853–877.
- Banerjee, A., J. Dolado and F. Mestre (1997), 'ECM test for cointegration in a single equation framework', *Journal of Time Series Analysis* (Forthcoming).
- Banerjee, A., J. Dolado, J.W. Galbraith and D.F. Hendry (1993), *Cointegration, error-correction, and the econometric analysis of non-stationary data*, Oxford University Press.
- Banerjee, A., R. Lumsdaine and J. Stock (1992), 'Recursive and sequential tests of unit root and trend break hypothesis: Theory and international evidence', *Journal of Business and Economic Statistics* **10**, 271–287.
- Baxter, M. and R. King (1995), Measuring business cycles: Approximate band-pass filters for economic time series, Working Paper 5022, NBER.
- Burns, A. and W. Mitchell (1946), *Measuring Business Cycles*, NBER.
- Campos, J., N. Ericsson and D. Hendry (1996), 'Cointegration tests in the presence of structural breaks', *Journal of Econometrics* **70**, 187–220.
- Dickey, D. and W. Fuller (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association* **74**, 427–431.
- Dolado, J. and H. Lutkepohl (1996), 'Making Wald tests work for cointegrated VAR systems', *Econometric Reviews* **15**, 369–386.
- Engle, R. F. and C. W. J. Granger (1987), 'Cointegration and error-correction: Representation, estimation, and testing', *Econometrica* **55**, 251–276.
- Franses, P. and A. Lucas (1997a), Outlier robust cointegration analysis of Dutch interest rates, in 'Proceedings of the 1996 Meeting of the American Statistical Association', ASA, pp. 106–109.

- Franses, P. and A. Lucas (1997b), Outlier robust cointegration tests, VU Research Memorandum 97-45, Vrije Universiteit Amsterdam.
- Franses, P. and N. Haldrup (1994), 'The effects of additive outliers on unit roots and cointegration', *Journal of Business and Economic Statistics* **12**, 471-478.
- Guay, A. and P. St-Amant (1997), Do the Hodrick-Prescott and Baxter-King filters provide a good approximation of business cycles?, *Cahiers de Recherche* 53, CREFE.
- Hansen, B. (1995), 'Rethinking the univariate approach to unit root testing: Using covariates to increase power', *Econometric Theory* **11**, 1148-1172.
- Hendry, D. (1996), A theory of co-breaking, in 'Time Series Seminar Proceedings', NBER-NSF.
- Hendry, D. and A. Neale (1990), The impact of structural breaks on unit root tests, in P. Hackl and A. Westlung, eds, 'Economic Structural Change: Analysis and Forecasting', Berlin: IIASA and Springer-Verlag, pp. 95-119.
- Hodrick, R. and E. Prescott (1980), Post-war U.S. business cycle: An empirical investigation, Technical report, Carnegie-Mellon University.
- Hodrick, R. and E. Prescott (1997), 'Post-war U.S. business cycle: An empirical investigation', *Journal of Money, Credit and Banking* **29**, 1-16.
- King, R. and S. Rebelo (1993), 'Low frequency filtering and real business cycles', *Journal of Economic Dynamics and Control* pp. 207-31.
- Kremers, J., N. Ericsson and J. Dolado (1992), 'The power of cointegration tests', *Oxford Bulletin of Economics and Statistics* **54**, 349-67.
- Kydland, F. and E. C. Prescott (1990), 'Business cycles: real facts and a monetary myth', *Federal Reserve Bank of Minneapolis Quarterly Review* **14**, 3-18.
- Leybourne, S., T. Mills and P. Newbold (1998), 'Spurious rejections by Dickey-Fuller tests in the presence of a break under the null', *Journal of Econometrics* **87**, 191-203.
- Lucas, A. (1995a), 'An outlier robust unit root tests with an application to the extended Nelson-Plosser data', *Journal of Econometrics* **66**, 153-173.
- Lucas, A. (1995b), 'Unit roots tests based on M estimators', *Econometric Theory* **11**, 331-346.
- Lucas, A. (1997), 'Cointegration testing using pseudo likelihood-ratio tests', *Econometric Theory* **13**, 149-169.
- Park, J. and P. Phillips (1988), 'Statistical inference in regressions with integrated processes: Part I', *Econometric Theory* **4**, 458-497.
- Park, J. and P. Phillips (1989), 'Statistical inference in regressions with integrated processes: Part II', *Econometric Theory* **5**, 95-131.

T	α	NO										D1										D2										D3									
		b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75										
25	0.0	1	6.35	7.83	13.80	29.10	49.40	13.85	23.60	44.50	68.30	73.45	10.55	16.45	37.45	57.55	65.00	10.50	17.05	38.75	57.35	65.00	10.50	17.05	38.75	57.35	65.00	10.50	17.05	38.75	57.35	65.00									
		6	23.40	49.30	70.55	71.65	70.95	73.05	78.85	78.85	78.85	79.15	72.15	65.60	74.35	74.35	77.85	75.85	69.60	69.45	77.85	77.85	75.85	75.85	77.85	77.85	75.85	75.85	77.85	77.85	75.85	75.85									
		16	65.15	74.70	78.35	76.90	73.60	76.70	78.75	80.30	79.00	79.00	79.15	71.25	72.95	73.45	76.30	74.25	72.35	73.35	76.30	74.25	74.25	73.35	76.30	74.25	74.25	73.35	76.30	74.25	74.25	73.35	76.30								
		1	6.35	7.05	9.75	13.30	26.20	46.50	8.60	10.10	13.85	26.85	44.80	5.05	6.55	11.10	17.45	22.45	4.55	6.05	11.10	17.45	22.45	4.55	6.05	11.10	17.45	22.45	4.55	6.05	11.10	17.45	22.45								
		6	12.85	21.45	51.65	65.15	71.20	36.05	53.55	68.00	68.00	74.45	76.65	22.45	43.10	63.25	68.20	71.95	39.60	48.65	63.25	68.20	71.95	71.95	48.65	63.25	68.20	71.95	71.95	48.65	63.25	68.20	71.95								
		16	31.90	56.40	68.35	72.25	74.00	65.85	74.25	77.40	78.50	78.50	80.45	57.40	68.15	73.30	75.25	75.25	64.45	75.15	73.30	75.25	75.25	73.30	75.15	73.30	75.25	75.25	73.30	75.15	73.30	75.25	75.25								
100	0.5	1	4.35	5.20	5.70	3.25	10.85	3.80	5.70	6.10	4.70	5.05	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70									
		6	4.55	5.20	5.70	7.05	7.05	9.70	5.85	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10	6.10								
		16	6.05	6.20	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35	6.35								
		1	38.35	78.55	99.30	100.00	100.00	92.55	98.30	98.30	98.30	98.30	98.30	94.15	97.50	99.25	100.00	100.00	95.20	97.50	99.25	100.00	100.00	100.00	99.25	97.50	99.25	100.00	100.00	100.00	100.00	100.00	100.00								
		6	99.10	100.00	100.00	100.00	100.00	99.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00								
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00								
1000	1.0	1	98.40	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00									
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00								
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00							
		1	98.55	100.00	100.00	100.00	100.00	99.35	99.35	99.35	99.35	99.35	99.35	99.20	99.35	99.35	99.35	99.35	99.20	99.35	99.35	99.35	99.35	99.35	99.20	99.35	99.35	99.35	99.35	99.35	99.35	99.35	99.35								
		6	99.60	100.00	100.00	100.00	100.00	99.75	99.75	99.75	99.75	99.75	99.75	99.55	99.75	99.75	99.75	99.75	99.55	99.75	99.75	99.75	99.75	99.75	99.55	99.75	99.75	99.75	99.75	99.75	99.75	99.75	99.75								
		16	99.50	100.00	100.00	100.00	100.00	99.70	99.85	99.85	99.85	99.85	99.85	99.80	99.85	99.85	99.85	99.85	99.80	99.85	99.85	99.85	99.85	99.85	99.80	99.85	99.85	99.85	99.85	99.85	99.85	99.85	99.85								

Table 10.1: Power of the test. Simultaneous Cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_t$.

T	α	NO										D1										D2										D3									
		b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75										
25	0.5	1	4.35	4.65	4.80	4.15	4.95	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70									
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		16	4.45	4.75	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85								
		1	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		16	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
100	0.0	1	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80									
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80									
		16	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		1	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		16	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
1000	1.0	1	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80									
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80									
		16	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		1	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		6	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								
		16	4.30	4.65	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80								

Table 10.2: Power of the test. Cobreak in differences, not in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z$

T	α	k	NO																							
			b=-0.035	b=-0.1	b=-0.225	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75				
25	0.0	1	6.55	7.85	13.80	29.10	49.40	5.90	6.80	9.65	19.60	36.95	5.55	6.30	8.45	13.95	26.00	5.95	6.65	8.15	14.25	25.60	5.95	6.65	8.15	
		6	23.40	49.30	70.55	71.65	70.95	10.45	14.30	26.10	47.40	56.95	5.50	5.95	8.80	14.55	15.55	6.25	6.85	8.00	14.60	16.95	14.60	14.60	14.60	14.60
		16	65.15	74.70	76.35	76.90	73.60	13.75	21.75	32.80	53.70	58.40	5.65	5.10	11.45	11.90	11.20	7.25	8.00	8.00	13.00	12.95	10.05	20.05	20.05	20.05
	0.5	1	6.35	7.05	9.75	13.30	26.20	5.70	5.75	7.55	9.50	18.05	4.95	6.85	7.45	9.65	17.65	5.80	6.90	8.10	11.25	11.25	20.05	20.05	20.05	
		6	12.85	21.45	51.65	65.15	71.20	8.70	12.60	18.40	41.00	56.90	7.60	7.80	9.70	13.85	21.10	7.40	9.75	11.85	13.05	14.65	14.65	14.65	14.65	
		16	31.90	56.40	68.35	72.25	74.00	9.00	10.25	18.95	36.10	52.40	5.05	4.95	5.90	4.10	8.05	5.90	6.35	6.55	8.55	6.55	11.55	11.55	11.55	
100	0.0	1	4.35	5.20	5.20	7.05	10.85	3.80	5.10	4.90	6.75	10.05	5.30	5.55	6.70	7.55	13.80	6.15	6.35	6.35	9.30	10.15	10.15	10.15		
		6	4.55	6.20	5.20	7.05	9.70	5.85	6.10	6.70	9.70	15.80	3.75	4.50	6.05	6.35	8.90	5.90	6.50	6.50	8.25	9.80	10.45	10.45		
		16	6.05	6.20	6.35	9.70	17.90	5.85	5.90	7.15	8.70	14.35	10.20	4.90	4.90	5.90	7.75	10.20	5.35	5.35	5.35	8.25	8.25	8.25		
	0.5	1	36.35	76.55	99.50	100.00	100.00	9.65	17.55	58.80	87.30	99.10	10.20	15.95	43.15	68.85	95.25	14.45	14.45	14.45	27.45	35.35	35.35	35.35		
		6	99.10	100.00	100.00	100.00	100.00	13.30	14.15	15.10	10.40	17.60	8.40	8.40	7.20	4.75	1.90	13.15	13.15	13.15	12.80	13.45	13.45	13.45		
		16	100.00	100.00	100.00	100.00	100.00	7.85	6.85	14.15	20.70	22.75	4.35	2.55	3.40	3.20	2.50	10.15	10.15	10.15	3.15	3.15	3.15	3.15		
1000	0.0	1	16.75	41.80	97.85	99.90	100.00	11.90	22.40	81.35	99.80	99.95	9.50	18.05	68.35	99.30	99.80	12.70	20.75	26.25	81.15	91.95	91.95	91.95		
		6	91.95	99.45	100.00	100.00	100.00	2.75	2.35	12.50	32.60	57.05	1.45	1.40	0.85	4.85	1.55	11.70	11.70	11.70	7.75	8.15	8.15	8.15		
		16	99.90	100.00	100.00	100.00	100.00	9.85	26.40	92.95	99.90	100.00	11.35	27.05	92.70	99.95	100.00	14.00	14.00	14.00	32.90	32.90	32.90	32.90		
	0.5	1	12.60	31.95	93.85	100.00	100.00	11.55	30.30	93.85	100.00	100.00	10.30	27.45	92.45	99.95	100.00	11.25	27.05	32.90	81.15	91.95	91.95	91.95		
		6	11.50	27.10	94.40	99.95	100.00	9.85	26.40	92.95	99.90	100.00	11.35	27.05	92.70	99.95	100.00	14.00	14.00	14.00	32.90	32.90	32.90	32.90		
		16	9.90	26.30	93.80	99.90	100.00	10.65	27.30	93.30	100.00	100.00	9.30	24.85	90.95	99.80	100.00	14.00	14.00	14.00	32.90	32.90	32.90	32.90		
1000	0.0	1	99.40	100.00	100.00	100.00	100.00	1.85	20.25	17.20	99.55	100.00	39.95	59.00	44.90	32.05	36.50	34.30	42.15	42.15	42.15	42.15	42.15			
		6	100.00	100.00	100.00	100.00	100.00	63.55	80.75	75.20	98.80	88.80	0.05	3.10	10.45	89.75	93.50	34.30	34.30	34.30	34.30	34.30	34.30			
		16	100.00	100.00	100.00	100.00	100.00	15.35	10.05	12.30	23.90	27.70	7.95	4.30	3.15	5.05	5.65	7.70	7.70	7.70	7.70	7.70	7.70			
	0.5	1	99.60	99.95	100.00	100.00	100.00	25.00	64.50	99.95	100.00	100.00	13.65	46.15	99.90	100.00	100.00	36.30	47.90	47.90	47.90	47.90	47.90			
		6	100.00	100.00	100.00	100.00	100.00	48.35	99.00	100.00	100.00	100.00	9.00	13.45	100.00	100.00	100.00	41.65	41.65	41.65	41.65	41.65	41.65			
		16	100.00	100.00	100.00	100.00	100.00	12.35	21.15	67.70	98.60	99.95	9.00	13.45	47.45	95.30	99.10	14.75	14.75	14.75	14.75	14.75	14.75			
1000	1.0	1	99.55	100.00	100.00	100.00	100.00	99.35	99.95	100.00	100.00	100.00	99.20	99.90	100.00	100.00	100.00	99.30	99.30	99.30	99.30	99.30	99.30			
		6	99.60	100.00	100.00	100.00	100.00	99.75	99.85	100.00	100.00	100.00	99.55	99.90	100.00	100.00	100.00	99.60	99.60	99.60	99.60	99.60	99.60			
		16	99.50	100.00	100.00	100.00	100.00	99.70	99.85	100.00	100.00	100.00	99.80	100.00	100.00	100.00	100.00	99.65	99.65	99.65	99.65	99.65	99.65			

Table 10.3: Power of the test. Cobreak in levels, not in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_t$.

Table 10: $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2 = 1$. The estimated model is $\hat{\phi}(B)\Delta y_t = \hat{c} + \hat{\theta}(B)\Delta z_t + \hat{b}(y_{t-1} - z_{t-1}) + \hat{d}(y_{t-k-2} - z_{t-k-2}) + u_{1t}$. 5% critical values are provided at table 2 for different sample sizes (T=25, 100, 1000), different short run parameter values ($a=0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-7.053	-5.897	-5.922	-7.179	-6.448	-6.181	-8.042	-8.092	-8.020
	D1	-6.161	-5.639	-5.725	-7.253	-6.509	-6.050	-7.736	-8.128	-8.286
	D2	-6.459	-5.925	-5.720	-7.345	-6.396	-6.036	-7.782	-8.460	-8.357
	D3	-6.279	-5.698	-5.626	-6.604	-6.292	-5.969	-7.574	-7.963	-7.899
100	NO	-2.845	-2.243	-2.173	-3.277	-2.487	-2.094	-3.403	-3.330	-3.429
	D1	-2.233	-2.149	-2.136	-2.608	-2.074	-2.106	-3.600	-3.654	-3.619
	D2	-2.429	-2.249	-2.209	-2.982	-2.348	-2.237	-3.772	-3.812	-3.752
	D3	-2.215	-2.154	-2.112	-2.584	-2.159	-2.215	-3.605	-3.666	-3.674
1000	NO	-2.608	-1.946	-1.804	-2.828	-2.164	-1.886	-2.842	-2.895	-2.826
	D1	-1.771	-1.704	-1.610	-1.931	-1.632	-1.692	-3.005	-3.091	-3.080
	D2	-1.779	-1.731	-1.659	-2.063	-1.794	-1.682	-3.144	-3.172	-3.106
	D3	-1.741	-1.800	-1.694	-1.920	-1.768	-1.704	-3.067	-3.129	-3.064

Table 11.1: Critical values. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-6.490	-5.947	-5.748	-7.238	-6.564	-6.392	-6.646	-7.160	-6.938
	D1	-6.414	-5.750	-5.742	-7.479	-6.561	-6.052	-6.720	-7.043	-6.855
	D2	-6.730	-5.969	-5.655	-6.582	-5.934	-5.923	-6.466	-6.887	-7.319
	D3	-6.392	-5.670	-5.701	-6.566	-6.198	-6.076	-6.734	-7.103	-7.155
100	NO	-2.277	-2.327	-2.148	-2.440	-2.440	-2.166	-2.328	-2.266	-2.322
	D1	-2.468	-2.167	-2.139	-3.159	-2.110	-2.099	-2.453	-2.471	-2.448
	D2	-2.597	-2.239	-2.192	-2.758	-2.397	-2.225	-2.370	-2.605	-2.604
	D3	-2.529	-2.107	-2.128	-3.016	-2.181	-2.196	-2.357	-2.267	-2.373
1000	NO	-1.815	-1.738	-1.821	-1.712	-1.728	-1.837	-1.731	-1.745	-1.799
	D1	-1.833	-1.704	-1.620	-2.113	-1.650	-1.706	-1.863	-1.837	-1.882
	D2	-1.999	-1.742	-1.654	-1.934	-1.788	-1.695	-1.881	-1.890	-1.850
	D3	-1.873	-1.795	-1.694	-1.996	-1.750	-1.706	-1.872	-1.731	-1.799

Table 11.2: Critical values. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-7.053	-5.897	-5.922	-7.179	-6.448	-6.181	-8.042	-8.092	-8.020
	D1	-6.946	-5.287	-5.230	-7.184	-6.420	-5.946	-7.736	-8.128	-8.286
	D2	-6.957	-6.321	-6.028	-7.247	-7.046	-6.451	-7.782	-8.460	-8.357
	D3	-6.953	-6.714	-6.335	-7.387	-7.445	-7.694	-7.574	-7.963	-7.899
100	NO	-2.845	-2.243	-2.173	-3.277	-2.487	-2.094	-3.403	-3.330	-3.429
	D1	-3.448	-2.226	-1.819	-3.574	-2.698	-2.232	-3.600	-3.654	-3.619
	D2	-3.686	-3.026	-2.824	-3.725	-3.269	-3.098	-3.772	-3.812	-3.752
	D3	-3.438	-2.687	-2.519	-3.563	-3.402	-2.861	-3.605	-3.666	-3.674
1000	NO	-2.608	-1.946	-1.804	-2.828	-2.164	-1.886	-2.842	-2.895	-2.826
	D1	-5.117	-2.268	-1.804	-4.176	-3.270	-2.073	-3.005	-3.091	-3.080
	D2	-5.579	-2.822	-2.372	-4.383	-3.618	-2.567	-3.144	-3.172	-3.106
	D3	-4.008	-2.371	-2.205	-3.707	-2.579	-2.300	-3.067	-3.129	-3.064

Table 11.3: Critical values. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

TABLE 11. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_{1t}$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a= 0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-6.949	-5.912	-5.796	-7.872	-7.582	-7.268	-8.262	-11.377	-11.806
	D1	-6.034	-5.737	-5.707	-7.630	-7.447	-7.892	-9.487	-12.809	-13.005
	D2	-6.306	-5.893	-6.100	-7.938	-7.708	-6.974	-9.298	-12.428	-13.442
	D3	-6.253	-5.857	-6.153	-7.267	-6.651	-6.449	-9.826	-11.819	-12.150
100	NO	-2.885	-2.308	-2.285	-3.314	-2.495	-2.108	-3.415	-3.494	-3.453
	D1	-2.289	-2.145	-2.145	-2.663	-2.020	-2.031	-3.685	-3.726	-3.698
	D2	-2.375	-2.195	-2.195	-3.124	-2.361	-2.262	-3.871	-3.905	-3.879
	D3	-2.329	-2.205	-2.248	-2.630	-2.262	-2.184	-3.703	-3.839	-3.713
1000	NO	-2.652	-2.049	-1.972	-2.809	-2.182	-1.873	-2.768	-2.890	-2.798
	D1	-1.904	-1.912	-1.748	-2.201	-1.747	-1.688	-2.935	-3.136	-3.088
	D2	-1.862	-1.867	-1.798	-2.361	-1.799	-1.693	-3.137	-3.126	-3.072
	D3	-1.934	-1.991	-1.866	-2.070	-1.774	-1.731	-2.985	-3.105	-3.082

Table 12.1: Critical values. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-6.323	-5.933	-5.749	-6.759	-7.657	-7.678	-7.382	-8.005	-8.491
	D1	-6.578	-5.736	-5.678	-7.575	-7.735	-7.920	-7.205	-9.049	-8.789
	D2	-6.724	-5.799	-6.037	-7.252	-7.681	-6.822	-7.332	-8.888	-9.486
	D3	-6.775	-5.877	-6.080	-7.693	-6.834	-6.447	-7.502	-8.115	-8.019
100	NO	-2.424	-2.307	-2.235	-2.391	-2.394	-2.160	-2.285	-2.322	-2.366
	D1	-2.525	-2.154	-2.151	-3.183	-2.066	-2.026	-2.385	-2.433	-2.478
	D2	-2.638	-2.194	-2.207	-2.798	-2.333	-2.228	-2.429	-2.590	-2.641
	D3	-2.681	-2.226	-2.220	-3.058	-2.244	-2.184	-2.382	-2.320	-2.331
1000	NO	-1.960	-1.861	-1.942	-1.856	-1.785	-1.859	-1.787	-1.762	-1.804
	D1	-1.954	-1.890	-1.752	-2.014	-1.757	-1.696	-1.687	-1.870	-1.887
	D2	-2.162	-1.880	-1.799	-1.814	-1.799	-1.714	-1.669	-1.902	-1.852
	D3	-1.999	-1.975	-1.874	-2.067	-1.766	-1.724	-1.905	-1.733	-1.804

Table 12.2: Critical values. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-6.949	-5.912	-5.796	-7.872	-7.582	-7.268	-8.262	-11.377	-11.806
	D1	-7.169	-5.808	-4.831	-8.055	-8.494	-7.411	-9.487	-12.809	-13.005
	D2	-8.368	-8.786	-8.563	-8.689	-10.843	-13.059	-9.298	-12.428	-13.442
	D3	-7.336	-7.612	-7.322	-8.290	-13.711	-17.138	-9.826	-11.819	-12.150
100	NO	-2.885	-2.308	-2.285	-3.314	-2.495	-2.108	-3.415	-3.494	-3.453
	D1	-3.489	-2.198	-1.862	-3.686	-2.649	-2.063	-3.685	-3.726	-3.698
	D2	-3.784	-2.977	-2.883	-3.860	-3.371	-3.070	-3.871	-3.905	-3.879
	D3	-3.441	-2.751	-2.581	-3.607	-3.425	-2.735	-3.703	-3.839	-3.713
1000	NO	-2.652	-2.049	-1.972	-2.809	-2.182	-1.873	-2.768	-2.890	-2.798
	D1	-5.465	-2.340	-1.859	-4.331	-3.319	-2.107	-2.935	-3.136	-3.088
	D2	-5.886	-2.849	-2.432	-4.544	-3.726	-2.563	-3.137	-3.126	-3.072
	D3	-4.231	-2.418	-2.225	-3.755	-2.582	-2.287	-2.985	-3.105	-3.082

Table 12.3: Critical values. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

TABLE 12. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = var(u_{1t})$, and $\sigma_2^2 = var(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	a	NO															D1															D2															D3														
		b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75																														
25	0.0	1	3.25	3.70	6.43	6.35	5.25	7.85	9.60	9.95	10.40	8.95	7.40	8.15	8.70	8.20	8.25	8.15	8.70	8.60	7.70	8.25	8.15	8.70	8.60	7.70	8.25	8.15	8.70	8.60	7.70																														
		6	10.10	11.95	7.85	6.43	6.20	14.35	12.50	8.60	9.50	6.55	7.25	10.60	11.95	9.05	6.15	5.35	14.20	10.80	8.45	6.60	6.35	14.20	10.80	8.45	6.60	6.35	14.20	10.80	8.45																														
	16	9.20	7.35	5.05	4.90	5.60	8.60	9.30	5.60	4.95	5.25	5.25	8.65	8.50	6.05	4.10	4.25	6.90	6.40	4.25	4.25	6.90	6.40	4.25	4.25	6.90	6.40	4.25	4.25	6.90																															
	6	4.35	4.90	5.85	5.90	6.40	5.55	6.05	5.95	5.70	5.70	5.70	4.45	5.15	6.85	6.15	6.15	6.85	6.80	6.45	6.20	6.35	6.80	6.45	6.20	6.35	6.80	6.45	6.20	6.35																															
100	0.5	1	7.55	8.95	8.95	7.00	7.55	8.20	10.05	8.75	9.95	6.60	4.70	8.75	9.60	9.00	7.30	5.30	10.95	11.20	5.30	5.30	10.95	11.20	5.30	5.30	10.95	11.20	5.30	5.30																															
		6	10.40	9.40	6.70	5.10	3.40	11.35	10.05	6.95	5.20	3.55	4.80	8.60	9.10	6.70	4.80	4.80	6.70	4.45	4.45	6.70	4.45	4.45	6.70	4.45	4.45	6.70	4.45	4.45																															
	16	4.20	4.30	4.05	4.25	4.90	5.15	5.45	5.65	5.10	5.20	4.60	5.00	5.30	5.30	5.30	5.30	5.30	4.70	4.70	5.30	5.30	5.30	4.70	4.70	5.30	5.30	5.30	5.30																																
	6	6.35	5.70	6.05	6.55	4.90	4.95	5.70	6.60	6.35	5.15	4.95	4.95	3.95	4.80	6.30	6.30	6.30	5.35	5.35	6.30	6.30	6.30	5.35	5.35	6.30	6.30	6.30	6.30																																
1000	1.0	1	4.35	4.10	5.95	6.40	5.75	7.20	8.25	8.75	8.15	6.55	6.55	7.65	8.15	8.15	8.15	8.15	7.65	7.65	8.15	8.15	8.15	7.65	7.65	8.15	8.15	8.15	8.15																																
		6	83.10	81.05	56.85	46.60	39.80	91.15	83.75	62.55	48.15	41.70	90.95	83.60	61.55	36.70	26.55	21.65	77.35	80.20	65.80	50.10	43.30	80.20	65.80	50.10	43.30	80.20	65.80	50.10																															
	16	75.00	76.85	36.35	26.00	21.95	79.60	62.65	38.25	28.95	22.50	79.90	79.90	61.55	36.70	26.55	21.65	77.35	79.90	61.55	47.40	39.90	77.35	79.90	61.55	47.40	39.90	77.35	79.90																																
	6	11.35	17.10	34.40	49.45	62.05	41.75	50.80	64.70	75.80	84.80	70.80	70.80	77.55	76.90	73.00	61.65	63.65	86.40	83.70	63.65	63.65	86.40	83.70	63.65	63.65	86.40	83.70	63.65	63.65																															
1000	0.0	1	90.00	14.45	28.75	34.05	31.15	9.60	15.40	25.35	31.35	28.10	8.35	13.15	23.05	25.30	25.30	25.30	25.30	14.70	14.70	25.30	25.30	25.30	14.70	14.70	25.30	25.30	25.30																																
		6	12.40	17.50	33.55	40.55	34.85	8.45	14.05	24.55	30.00	26.75	8.30	12.40	22.45	25.75	25.75	25.75	25.75	24.30	24.30	25.75	25.75	25.75	24.30	24.30	25.75	25.75	25.75																																
	16	8.90	16.80	28.60	37.20	33.65	9.10	14.40	26.85	30.75	28.30	8.30	14.45	25.65	29.30	29.30	29.30	29.30	24.25	24.25	29.30	29.30	29.30	24.25	24.25	29.30	29.30	29.30																																	
	6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																
1000	0.5	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																
	16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																	
	6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																	
1000	1.0	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																
		6	99.90	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	99.95	99.80	99.95	100.00	100.00	100.00	99.85	99.85	100.00	100.00	99.85	99.85	100.00	100.00	99.85	99.85	100.00																																
	16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																	
	6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00																																	

Table 13.1: Power of the test. Simultaneous cointegration. $\sigma_2 = \epsilon$. Estimated model $\hat{\phi}(B)\Delta z_t^y = \hat{c} + \hat{\theta}(B)\Delta z_t^y + \hat{b}(y_t^y - z_{t-1}^y) + u_t$. Growth components obtained with HP10 filter.

T	a	NO															D1															D2															D3														
		b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75																														
25	0.0	1	5.40	4.35	4.90	4.65	3.50	5.00	4.55	4.25	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50																															
		6	8.80	9.00	7.15	6.70	5.50	6.25	5.25	5.85	6.70	7.35	4.00	4.60	5.65	6.80	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35	7.35																															
	16	10.10	7.60	5.15	6.15	8.15	5.15	4.80	7.50	8.25	8.25	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70	4.70																																
	6	4.00	4.15	2.45	2.30	3.05	4.70	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75																																
100	0.5	1	7.20	6.80	4.55	4.55	4.10	4.50	5.10	4.35	3.20	3.65	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75																																
		6	9.55	7.45	5.60	4.75	5.45	5.85	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65	4.65																																
	16	5.15	4.90	3.30	3.65	2.55	2.85	3.80	4.35	4.35	3.53	3.45	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35	4.35																																	
	6	4.65	5.15	4.50	3.70	3.35	4.20	4.60	4.95	4.30	4.40	3.95	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20																																	
1000	1.0	1	4.55	3.35	1.25	4.0	4.0	2.60	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85	2.85																																
		6	14.70	5.25	2.85	4.70	5.50	4.10	1.60	1.00	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50																																	
	16	22.65	9.35	10.75	10.60	10.85	8.15	3.00	3.85	5.05	6.40	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15	6.15																																	
	6	3.05	1.80	0.70	4.0	4.0	3.15	1.155	1.55	3.30	2.35	2.85	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75																																		
1000	0.5	1	6.05	2.60	1.10	1.05	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50																																	
		6	14.85	7.35	5.85	8.35	8.00	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35	2.35																																	
	16	2.40	1.60	0.55	0.90	0.45	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25																																		
	6	3.25	1.60	0.55	0.90	0.45	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15	3.15																																		
1000	1.0	1	1.90	0.45	0.00	0.00	0.00																																																						

T	a	s	NO															D1			D2			D3		
			b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-1.0	b=-1.25	b=-1.5	b=-1.75	b=-2.0	b=-2.25	b=-2.5	b=-2.75	b=-3.0	b=-3.25	b=-3.5	b=-3.75							
25	0.0	1	5.26	5.70	6.48	6.85	5.25	5.15	5.95	6.20	7.65	6.45	5.05	3.90	5.45	6.55	4.95	5.80	5.70	6.10	6.35	5.50				
		6	10.10	11.95	7.85	6.45	6.20	8.15	10.85	14.25	14.70	14.55	4.95	4.85	4.85	5.00	4.70	5.00	5.15	5.70	5.60	6.35	6.20			
		16	9.20	7.55	5.05	4.90	5.60	9.85	12.85	15.30	13.80	12.30	4.15	3.85	3.85	5.40	5.10	6.20	6.55	6.50	5.70	6.10	6.00			
		1	4.35	4.90	5.85	5.90	6.40	5.00	5.70	5.60	6.80	4.75	5.35	6.65	5.75	5.75	6.55	4.60	5.30	5.30	5.80	5.80	6.05			
		6	7.55	8.95	8.95	7.00	5.50	6.60	7.85	8.65	8.65	8.10	4.65	5.25	5.25	5.15	4.45	3.30	6.85	6.15	6.35	6.35	6.45			
		16	10.40	9.40	6.70	5.10	3.40	8.60	11.75	12.75	11.75	8.35	7.00	5.05	5.05	4.30	5.25	5.35	4.75	5.05	5.35	4.55	5.25			
100	0.0	1	4.20	4.30	4.05	4.20	3.70	5.15	5.45	5.65	5.10	5.20	4.95	5.00	5.30	6.30	4.65	5.35	4.70	5.45	5.35	5.75				
		6	6.35	5.70	6.05	5.55	4.90	4.95	5.70	6.60	6.35	5.15	4.95	4.95	4.80	5.30	4.70	5.35	5.35	6.95	6.55	6.90				
		16	4.35	5.55	6.30	6.55	4.65	5.80	5.75	5.30	5.30	4.75	4.90	5.90	5.50	4.25	4.25	4.60	6.15	6.15	6.20	6.55				
		1	24.05	41.40	59.65	64.80	57.95	7.60	11.60	23.10	29.30	26.65	7.05	10.20	13.80	22.35	18.20	18.20	9.30	14.25	14.25	31.75	30.25			
		6	83.10	61.05	56.85	46.60	39.80	10.70	19.00	19.20	20.95	20.65	6.45	6.45	7.30	5.20	4.35	6.50	11.35	13.40	13.40	10.20	10.70			
		16	75.00	57.65	36.35	29.00	21.95	7.80	12.85	22.35	24.30	22.05	5.85	5.85	4.25	3.30	2.65	3.05	11.35	11.00	10.40	8.55	8.55			
1000	0.0	1	11.35	17.10	34.40	49.45	62.05	12.35	21.20	33.40	47.75	8.25	11.45	18.30	28.40	37.95	37.95	8.70	13.60	13.60	22.85	32.50	42.55			
		6	50.55	59.95	63.75	53.35	56.05	8.35	13.45	17.65	18.25	17.25	6.20	7.35	6.75	4.65	5.60	6.70	8.20	8.20	25.95	25.95				
		16	80.30	76.00	60.00	40.30	36.55	9.35	13.40	17.65	17.10	17.40	5.20	4.95	4.95	3.05	3.25	6.00	7.30	7.30	6.70	6.70				
		1	9.00	14.45	28.75	34.05	31.15	9.60	15.40	25.35	31.35	28.10	8.35	13.15	23.05	25.30	22.60	22.60	9.50	14.70	14.70	26.80	32.10			
		6	12.40	17.50	33.55	40.55	34.85	8.45	14.05	24.55	30.00	26.75	8.30	12.40	22.45	25.75	24.30	24.30	9.35	13.80	13.80	25.25	29.40			
		16	8.90	16.80	28.60	37.20	33.65	9.10	14.40	26.85	30.75	28.30	9.75	14.45	25.65	29.30	24.25	24.25	8.30	13.00	13.00	23.50	28.95			
1000	0.5	1	100.00	100.00	100.00	100.00	100.00	28.40	79.80	96.15	99.20	10.05	52.20	79.25	93.75	97.45	97.45	70.70	97.60	97.60	100.00	100.00				
		6	100.00	100.00	100.00	100.00	100.00	13.05	21.35	39.00	43.90	1.95	3.80	8.25	11.95	13.10	13.10	12.40	15.00	15.00	92.85	92.85				
		16	100.00	100.00	100.00	100.00	100.00	18.30	22.30	27.80	27.80	4.05	4.75	5.35	5.45	5.95	8.30	8.30	8.95	10.85	10.85	99.95				
		1	100.00	100.00	100.00	100.00	100.00	72.85	98.95	100.00	100.00	58.85	97.70	100.00	100.00	100.00	100.00	100.00	87.15	99.75	99.75	100.00				
		6	100.00	100.00	100.00	100.00	100.00	4.95	18.30	48.85	73.60	1.35	4.40	22.65	43.75	44.50	44.50	44.50	24.80	43.55	43.55	84.15	84.15			
		16	100.00	100.00	100.00	100.00	100.00	14.95	24.05	35.10	40.05	3.50	8.75	11.05	11.05	11.80	11.80	11.80	10.85	13.50	13.50	23.85	24.40			
1000	1.0	1	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00				
		6	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00				
		16	100.00	100.00	100.00	100.00	100.00	99.90	100.00	100.00	100.00	99.85	100.00	100.00	100.00	100.00	100.00	99.70	100.00	100.00	100.00	100.00				

Table 13.3: Power of the test. Cobreak in levels, not cobreak in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = c + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

Table 13: $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{it} + u_{2t}$, where $\sigma_1^2 = var(u_{1t})$, and $\sigma_2^2 = var(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = c + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. 5% critical values are provided at table 9 for different sample sizes ($T=25, 100, 1000$), different short run parameter values ($a=0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	α	NO															
		b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	
25	0.0	1	5.85	7.75	9.50	11.35	11.85	10.90	14.40	16.90	17.85	17.45	9.55	12.80	15.05	17.90	20.80
		6	16.00	22.15	24.20	25.10	21.70	23.20	24.35	28.30	28.40	29.10	23.50	26.60	28.65	30.35	30.50
		16	24.20	26.80	30.10	30.40	27.30	27.15	33.30	34.25	30.20	29.50	23.40	27.05	29.45	30.30	27.60
	0.5	1	9.75	6.65	1.05	1.70	6.30	6.80	6.80	6.30	7.45	9.15	6.90	6.45	7.30	7.30	8.70
		6	9.70	12.60	13.10	10.90	9.05	14.20	17.45	16.30	13.15	11.25	10.45	14.25	17.10	12.60	16.85
		16	13.75	14.30	20.10	14.30	13.30	17.60	17.20	17.20	10.10	10.60	11.85	22.95	23.20	17.00	15.30
100	0.0	1	6.95	7.50	5.00	1.75	6.05	5.65	5.25	5.40	5.10	4.00	6.25	6.70	7.15	6.15	5.35
		6	6.90	5.70	7.05	5.95	6.05	3.55	5.45	4.60	4.00	2.55	5.25	4.55	7.80	6.50	4.20
		16	5.45	5.95	6.10	5.30	4.15	8.010	4.40	5.10	5.10	2.65	4.95	5.15	6.50	5.75	2.85
	0.5	1	23.60	42.75	69.80	77.15	74.65	80.10	89.60	95.00	95.55	91.45	72.60	86.00	94.90	90.65	81.50
		6	90.80	91.25	82.60	79.70	81.15	97.10	96.00	89.60	88.35	86.60	97.55	95.60	89.75	88.45	86.90
		16	90.15	83.00	81.05	80.00	81.30	94.50	90.60	89.20	87.95	88.50	94.55	90.55	88.45	87.10	84.50
1000	0.0	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	0.5	1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
		16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Table 14.1: Power of the test. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^c = \hat{c} + \hat{\theta}(B)\Delta z_t^c + \hat{b}(y_{t-1}^c - z_{t-1}^c) + u_t$. Growth components obtained with BK filter.

T	α	NO															
		b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	b=0.05	b=0.1	b=0.25	b=0.5	b=0.75	
25	0.0	1	5.90	5.85	5.20	3.90	3.50	5.50	6.10	4.65	3.55	2.50	6.65	5.95	7.35	5.25	6.65
		6	14.80	15.85	12.80	9.10	7.45	7.45	6.25	5.45	4.20	4.40	7.70	7.85	7.30	7.80	14.15
		16	24.45	22.95	17.10	9.80	7.90	6.30	5.75	4.15	4.35	5.05	7.20	7.50	11.60	14.40	18.85
	0.5	1	5.45	5.55	4.15	3.10	2.10	7.25	6.85	7.40	7.50	5.75	5.90	5.20	6.85	4.35	5.05
		6	8.75	9.20	6.65	2.95	2.30	4.25	5.00	3.05	1.65	1.05	6.45	5.65	4.40	3.80	3.70
		16	11.80	12.05	8.60	3.30	3.05	4.15	3.20	2.00	.95	.65	6.55	6.40	6.80	8.45	9.15
100	0.0	1	4.90	4.40	3.25	1.50	1.80	4.00	4.50	3.35	2.10	1.60	4.40	4.55	3.65	3.25	1.15
		6	4.15	2.80	.55	.25	.10	4.60	3.20	.75	.80	.05	5.40	5.60	2.85	2.05	1.70
		16	21.50	4.65	.70	.20	.20	2.55	1.75	.80	.70	1.45	2.30	3.85	.85	1.45	1.65
	0.5	1	3.35	1.95	.75	.45	.15	8.55	12.35	23.60	42.15	52.90	5.50	5.45	6.00	14.85	21.90
		6	6.95	2.45	.40	.10	.15	4.20	1.80	.60	1.20	.95	4.10	1.40	1.40	.75	.85
		16	16.60	3.55	.45	.25	.15	2.10	1.15	1.95	1.10	1.65	1.25	1.45	1.70	1.60	1.10
1000	0.0	1	3.05	1.40	.70	.10	.15	3.05	1.70	.75	.70	.75	4.85	3.95	1.90	.30	.80
		6	2.30	1.20	.35	.35	.25	2.85	2.25	.60	.30	.35	3.75	3.50	1.45	.45	.80
		16	2.10	.65	.60	.25	.15	2.35	2.05	.65	.35	.25	4.00	2.60	1.55	1.30	1.00
	0.5	1	.25	.10	.00	.00	.00	21.25	58.60	99.25	100.00	100.00	11.35	36.35	94.50	99.20	100.00
		6	.90	.30	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
		16	.90	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00

Table 14.2: Power of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^d = \hat{c} + \hat{\theta}(B)\Delta z_t^d + \hat{b}(y_{t-1}^d - z_{t-1}^d) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	5.950	6.150	4.800	4.800	7.600	5.950	4.350	5.100	5.050
	D1	4.900	7.300	5.300	4.750	5.600	7.000	5.000	5.000	4.050
	D2	4.950	5.950	5.400	5.100	5.750	5.150	4.850	3.650	4.450
	D3	5.400	6.600	6.200	4.500	5.300	5.500	5.500	4.500	4.250
100	NO	7.300	6.850	6.600	5.300	7.400	8.300	2.950	2.250	2.100
	D1	8.200	7.400	7.150	9.500	9.250	9.100	3.750	1.850	1.800
	D2	7.850	6.400	7.200	7.400	7.150	6.850	2.750	2.050	1.750
	D3	9.650	7.950	6.950	8.500	8.600	8.200	4.100	3.150	2.100
1000	NO	7.550	6.600	5.550	4.500	6.550	8.400	4.600	3.350	3.000
	D1	5.700	5.650	5.050	2.850	6.600	8.150	7.000	2.200	2.550
	D2	5.950	5.300	5.750	2.550	6.250	7.150	4.900	2.050	3.000
	D3	5.700	5.100	5.650	4.600	6.800	7.700	4.950	3.200	3.400

Table 15.1: Empirical size of the test. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	5.900	5.950	5.050	4.400	7.150	5.250	5.900	5.450	6.200
	D1	5.500	6.750	5.150	4.950	5.500	6.950	6.250	6.450	7.500
	D2	5.000	6.150	5.550	5.050	6.800	5.200	6.900	5.650	4.650
	D3	6.450	6.650	6.100	6.100	5.600	5.600	6.400	6.100	5.700
100	NO	9.050	5.750	7.150	7.950	7.300	8.750	8.300	9.300	8.500
	D1	8.750	7.400	7.150	6.050	9.150	9.200	8.800	8.750	9.150
	D2	8.600	6.800	7.500	9.150	7.300	7.300	11.550	6.700	6.050
	D3	8.000	8.700	6.800	6.800	8.400	8.400	10.300	10.000	9.650
1000	NO	5.100	6.750	4.900	7.300	8.100	7.950	7.450	8.700	8.150
	D1	6.350	5.600	4.800	15.000	6.700	8.050	14.800	8.150	7.400
	D2	5.050	5.500	5.800	11.250	6.150	6.900	15.800	6.850	7.450
	D3	6.450	5.250	5.600	7.550	7.000	7.700	6.150	9.150	8.500

Table 15.2: Empirical size of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	5.950	6.150	4.800	4.800	7.600	5.950	4.350	5.100	5.050
	D1	5.400	5.950	5.300	5.700	5.500	5.300	5.000	5.000	4.050
	D2	5.700	4.850	4.350	5.450	5.200	6.250	4.850	3.650	4.450
	D3	5.100	5.400	6.600	4.900	5.500	4.100	5.500	4.500	4.250
100	NO	7.300	6.850	6.600	5.300	7.400	8.300	2.950	2.250	2.100
	D1	4.100	5.300	5.850	5.150	6.200	5.100	3.750	1.850	1.800
	D2	5.050	4.600	5.550	4.800	4.450	3.350	2.750	2.050	1.750
	D3	4.900	7.150	7.000	4.500	1.800	2.450	4.100	3.150	2.100
1000	NO	7.550	6.600	5.550	4.500	6.550	8.400	4.600	3.350	3.000
	D1	3.250	8.100	6.700	.400	9.500	9.100	7.000	2.200	2.550
	D2	1.800	7.450	5.850	.450	10.200	7.950	4.900	2.050	3.000
	D3	5.100	6.600	4.750	1.450	6.750	5.200	4.950	3.200	3.400

Table 15.3: Empirical size of the test. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

TABLE 15. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = var(u_{1t})$, and $\sigma_2^2 = var(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} + 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\phi(B)\Delta y_t^g = c + \theta(B)\Delta z_t^g + b(y_{t-1}^g - z_{t-1}^g) + u_{1t}$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.900	3.750	4.150	3.750	4.500	4.750	3.800	4.550	4.550
	D1	3.450	4.500	5.050	2.950	3.650	4.750	3.600	4.400	3.300
	D2	3.950	4.900	5.950	3.350	4.650	4.450	4.000	3.950	4.250
	D3	3.850	5.050	4.500	3.450	3.550	4.450	4.900	4.350	4.500
100	NO	3.850	4.700	4.650	3.100	5.000	5.950	2.100	2.150	2.750
	D1	5.600	5.700	5.150	4.100	8.550	7.150	2.950	2.200	1.650
	D2	5.250	5.100	5.300	3.200	6.100	5.700	2.050	2.100	2.250
	D3	6.250	5.000	5.600	3.250	7.000	6.500	3.550	2.650	2.450
1000	NO	2.200	3.600	3.400	3.250	2.250	3.950	5.950	6.950	6.300
	D1	4.350	5.000	3.950	2.700	4.550	4.900	6.750	6.100	5.900
	D2	4.150	4.350	5.350	3.100	4.200	4.400	5.550	5.150	7.650
	D3	3.750	4.300	4.650	2.850	3.450	4.050	6.050	5.900	6.850

Table 16.1: Empirical size of the test. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	4.000	4.100	4.150	2.450	3.400	4.400	3.300	3.900	3.400
	D1	2.700	4.550	5.150	2.900	3.700	4.850	3.000	3.550	4.800
	D2	3.150	5.000	5.750	3.950	5.900	4.950	3.600	4.000	3.500
	D3	3.350	5.100	4.350	5.200	3.900	4.000	3.600	2.950	3.600
100	NO	4.600	3.600	5.300	4.950	5.500	5.350	5.750	5.850	5.350
	D1	4.550	5.650	5.200	2.700	8.500	7.100	5.400	4.600	5.200
	D2	4.100	5.250	5.400	2.900	5.550	5.850	6.350	3.950	3.150
	D3	4.100	5.400	5.400	2.850	6.500	6.550	7.000	7.000	5.900
1000	NO	3.250	4.500	3.750	4.750	4.900	3.550	4.950	4.450	4.550
	D1	3.550	5.000	3.800	3.850	4.650	4.850	3.700	3.050	3.550
	D2	2.850	4.250	5.300	4.600	4.450	3.950	4.350	3.150	4.150
	D3	3.350	4.200	4.750	3.450	3.750	4.100	3.000	3.850	3.850

Table 16.2: Empirical size of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.900	3.750	4.150	3.750	4.500	4.750	3.800	4.550	4.550
	D1	2.500	3.750	2.950	3.600	3.350	4.250	3.600	4.400	3.300
	D2	5.000	5.950	4.450	5.250	6.650	5.650	4.000	3.950	4.250
	D3	4.950	5.300	6.800	4.700	5.000	4.100	4.900	4.350	4.500
100	NO	3.850	4.700	4.650	3.100	5.000	5.950	2.100	2.150	2.750
	D1	2.650	3.850	5.550	3.800	3.000	5.200	2.950	2.200	1.650
	D2	2.850	2.900	4.700	3.200	3.750	4.550	2.050	2.100	2.250
	D3	2.950	8.250	6.250	2.550	1.900	8.450	3.550	2.650	2.450
1000	NO	2.200	3.600	3.400	3.250	2.250	3.950	5.950	6.950	6.300
	D1	3.650	2.700	5.450	10.350	1.050	4.250	6.750	6.100	5.900
	D2	3.250	2.350	5.000	11.800	1.100	3.650	5.550	5.150	7.650
	D3	2.800	4.000	4.600	5.800	2.650	6.400	6.050	5.900	6.850

Table 16.3: Empirical size of the test. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with HP10 filter.

TABLE 16. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{j_t} + u_{2t}$, where $\sigma_1^2 = var(u_{1t})$, and $\sigma_2^2 = var(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} - 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	6.600	7.950	6.250	5.600	7.900	6.250	6.650	6.050	6.400
	D1	7.100	8.500	7.650	6.350	7.350	4.700	4.450	5.750	5.850
	D2	7.650	6.550	6.150	5.650	7.450	7.300	6.650	6.250	4.950
	D3	9.000	7.400	5.600	6.300	6.750	7.150	4.850	6.500	7.400
100	NO	6.600	7.050	5.000	4.050	6.400	7.600	2.750	2.100	2.300
	D1	7.900	7.250	7.650	9.300	9.050	8.750	2.950	1.900	2.100
	D2	8.050	7.000	6.900	6.750	6.200	5.300	2.900	2.600	2.450
	D3	9.150	7.200	6.850	7.650	6.850	7.150	3.550	2.300	2.750
1000	NO	6.900	4.550	3.450	4.350	5.450	7.450	3.950	3.450	3.400
	D1	4.000	3.150	3.300	2.400	5.700	7.500	4.450	2.200	2.600
	D2	4.550	3.800	4.050	1.600	6.300	5.550	3.150	3.400	3.850
	D3	3.550	3.000	3.800	3.750	6.400	6.400	4.500	3.800	3.600

Table 17.1: Empirical size of the test. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	8.550	7.500	6.600	7.100	8.900	5.700	6.450	7.800	8.200
	D1	7.450	8.350	7.650	7.200	6.750	4.250	7.300	7.350	7.800
	D2	6.650	7.000	6.450	7.600	7.550	7.800	7.500	7.600	6.600
	D3	6.600	7.550	5.650	5.800	6.650	7.050	8.450	8.500	9.650
100	NO	8.300	6.200	5.900	8.650	7.950	7.400	8.800	8.000	7.200
	D1	8.800	7.200	7.500	5.500	9.000	8.800	8.900	8.350	8.100
	D2	8.450	7.250	6.750	8.650	6.800	5.500	9.250	7.050	5.600
	D3	7.300	7.400	7.100	5.500	7.800	7.250	9.550	8.350	9.300
1000	NO	3.350	5.100	3.500	5.450	7.400	6.950	7.200	7.650	7.250
	D1	3.950	3.450	3.300	14.500	5.800	7.550	11.300	7.600	6.450
	D2	3.050	3.900	4.150	11.200	6.350	5.250	11.950	6.750	7.500
	D3	4.400	3.100	3.700	6.000	6.250	6.450	6.300	8.100	7.550

Table 17.2: Empirical size of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	6.600	7.950	6.250	5.600	7.900	6.250	6.650	6.050	6.400
	D1	6.750	8.650	7.950	7.050	7.500	6.950	4.450	5.750	5.850
	D2	4.450	5.250	4.450	5.450	6.550	5.550	6.650	6.250	4.950
	D3	6.450	6.050	4.900	4.750	3.600	4.300	4.850	6.500	7.400
100	NO	6.600	7.050	5.000	4.050	6.400	7.600	2.750	2.100	2.300
	D1	3.850	5.400	4.600	4.500	6.300	5.600	2.950	1.900	2.100
	D2	4.050	5.000	5.900	4.200	4.750	4.150	2.900	2.600	2.450
	D3	5.400	6.000	7.000	4.300	1.600	3.700	3.550	2.300	2.750
1000	NO	6.900	4.550	3.450	4.350	5.450	7.450	3.950	3.450	3.400
	D1	2.000	7.200	5.900	.400	9.850	8.450	4.450	2.200	2.600
	D2	1.150	7.350	5.450	.350	9.750	8.500	3.150	3.400	3.850
	D3	3.850	6.600	5.400	1.450	7.150	5.100	4.500	3.800	3.600

Table 17.3: Empirical size of the test. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

TABLE 17. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} + 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = c + \theta(B)\Delta z_t^g + b(y_{t-1}^g - z_{t-1}^g) + u_t$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values (a= 0.0, 0.5, 1.0), and different jump sizes (s=1, 6, 16).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.300	4.200	3.250	2.900	3.200	2.850	4.500	2.200	2.250
	D1	3.650	4.050	3.650	2.300	2.650	2.450	2.500	1.850	1.750
	D2	3.050	4.550	3.200	2.850	3.350	4.400	3.650	2.550	1.900
	D3	3.150	3.500	2.600	2.750	3.250	3.700	2.450	2.900	3.200
100	NO	3.500	3.950	3.300	2.350	4.900	5.250	2.650	1.950	2.300
	D1	3.900	4.700	4.600	4.300	7.200	6.800	2.700	2.000	1.600
	D2	3.800	4.300	4.200	2.350	4.450	3.700	2.550	1.600	1.400
	D3	4.050	3.950	4.200	2.900	5.500	5.750	3.550	1.850	2.250
1000	NO	2.300	3.950	3.500	3.900	2.300	4.050	6.450	6.900	6.400
	D1	3.900	3.900	4.300	3.450	4.150	4.900	8.550	5.200	5.300
	D2	4.250	4.300	4.950	3.350	4.800	4.050	5.600	6.150	8.650
	D3	3.900	3.850	4.450	2.950	3.800	4.000	7.550	6.350	6.500

Table 18.1: Empirical size of the test. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.350	3.800	3.200	3.200	3.100	2.700	2.850	3.000	3.050
	D1	3.250	4.150	3.600	3.800	2.550	2.500	2.450	2.300	3.050
	D2	3.550	4.850	3.400	4.150	3.400	4.600	1.950	2.550	2.450
	D3	2.950	3.700	2.700	3.250	3.300	3.700	2.000	1.700	3.300
100	NO	3.350	3.100	3.950	3.550	5.200	5.200	5.050	4.250	3.650
	D1	3.700	4.550	4.550	2.850	6.950	6.850	5.200	4.550	4.750
	D2	3.100	4.400	4.150	3.050	5.400	4.000	4.750	3.200	2.450
	D3	3.150	3.900	4.450	2.050	5.450	5.750	5.500	4.150	4.900
1000	NO	3.500	5.100	3.750	4.100	4.150	3.450	4.500	4.450	4.450
	D1	3.650	4.250	4.250	3.100	4.200	4.900	4.450	3.600	3.850
	D2	2.400	4.200	5.000	3.600	4.550	3.900	4.750	3.250	4.400
	D3	3.550	4.050	4.450	2.850	3.700	4.000	2.750	4.300	4.250

Table 18.2: Empirical size of the test. Cobreaking in differences, not cobreaking in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	3.300	4.200	3.250	2.900	3.200	2.850	4.500	2.200	2.250
	D1	3.250	3.050	2.600	3.050	2.200	2.100	2.500	1.850	1.750
	D2	3.750	3.500	3.750	3.850	5.400	4.300	3.650	2.550	1.900
	D3	5.850	5.200	5.100	4.050	3.850	3.950	2.450	2.900	3.200
100	NO	3.500	3.950	3.300	2.350	4.900	5.250	2.650	1.950	2.300
	D1	2.050	3.450	4.200	3.450	2.950	6.350	2.700	2.000	1.600
	D2	2.550	4.300	4.600	3.150	3.800	4.850	2.550	1.600	1.400
	D3	3.300	6.000	6.100	3.000	2.900	7.800	3.550	1.850	2.250
1000	NO	2.300	3.950	3.500	3.900	2.300	4.050	6.450	6.900	6.400
	D1	1.650	2.350	4.950	7.650	.950	4.300	8.550	5.200	5.300
	D2	1.050	2.600	4.500	8.050	.750	3.700	5.600	6.150	8.650
	D3	1.750	3.400	3.450	4.700	2.400	6.000	7.550	6.350	6.500

Table 18.3: Empirical size of the test. Cobreaking in levels, not cobreaking in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + u_t$. Growth components obtained with BK filter.

TABLE 18. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under $\Delta y_t = c_t + a\Delta z_t + u_{1t} - 0.5u_{1,t-1}$ and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = c + \theta(B)\Delta z_t^g + b(y_{t-1}^g - z_{t-1}^g) + u_{1t}$. 5% Empirical size of the test are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a= 0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-5.765	-5.934	-5.076	-7.558	-6.136	-5.736	-8.289	-8.500	-8.876
	D1	-4.659	-4.650	-4.068	-6.112	-5.232	-4.043	-7.610	-8.196	-7.962
	D2	-5.315	-4.761	-5.701	-7.186	-5.537	-4.191	-8.098	-8.660	-8.797
	D3	-4.820	-4.837	-4.882	-6.576	-5.053	-3.856	-8.196	-8.517	-8.176
100	NO	-1.639	-1.694	-1.715	-2.158	-1.857	-1.592	-2.477	-2.614	-2.607
	D1	-1.645	-1.656	-1.669	-2.160	-1.848	-1.676	-2.507	-2.638	-2.572
	D2	-1.505	-1.595	-1.605	-2.102	-1.829	-1.609	-2.481	-2.613	-2.587
	D3	-1.605	-1.681	-1.681	-2.214	-1.901	-1.545	-2.497	-2.645	-2.609
1000	NO	-1.654	-1.701	-1.622	-2.487	-1.672	-1.673	-1.931	-1.667	-1.632
	D1	-1.622	-1.560	-1.622	-2.539	-1.604	-1.693	-1.833	-1.638	-1.603
	D2	-1.547	-1.663	-1.669	-2.517	-1.658	-1.556	-1.923	-1.657	-1.612
	D3	-1.666	-1.679	-1.575	-2.548	-1.611	-1.540	-1.882	-1.631	-1.738

Table 19.1: Critical values. Simultaneous cobreaking. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-5.013	-5.592	-5.091	-5.101	-5.404	-5.577	-6.332	-7.705	-6.166
	D1	-5.713	-4.660	-4.122	-7.209	-5.207	-3.956	-6.785	-7.197	-6.944
	D2	-6.673	-4.817	-5.647	-7.652	-5.214	-4.039	-6.818	-6.891	-7.727
	D3	-6.247	-5.110	-4.873	-7.145	-5.157	-3.879	-6.995	-6.960	-7.113
100	NO	-1.654	-1.685	-1.720	-2.172	-1.897	-1.611	-2.498	-2.698	-2.665
	D1	-1.608	-1.656	-1.672	-2.220	-1.854	-1.677	-2.538	-2.666	-2.613
	D2	-1.581	-1.591	-1.606	-2.228	-1.832	-1.608	-2.518	-2.657	-2.641
	D3	-1.589	-1.684	-1.676	-2.214	-1.901	-1.549	-2.589	-2.663	-2.636
1000	NO	-1.658	-1.717	-1.650	-2.473	-1.695	-1.681	-1.921	-1.761	-1.726
	D1	-1.625	-1.561	-1.624	-2.555	-1.602	-1.693	-1.870	-1.741	-1.677
	D2	-1.592	-1.681	-1.669	-2.500	-1.667	-1.550	-1.935	-1.741	-1.762
	D3	-1.637	-1.680	-1.574	-2.542	-1.610	-1.540	-1.893	-1.711	-1.842

Table 19.2: Critical values. Cobreak in differences, not in levels. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_t$. Growth components obtained with HP10 filter.

T	DUM	a=0			a=0.5			a=1		
		s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
25	NO	-5.765	-5.934	-5.076	-7.558	-6.136	-5.736	-8.289	-8.500	-8.876
	D1	-7.019	-5.229	-5.624	-7.430	-6.580	-6.100	-7.610	-8.196	-7.962
	D2	-7.464	-6.474	-5.962	-7.020	-6.755	-6.278	-8.098	-8.660	-8.797
	D3	-6.634	-6.590	-6.768	-7.542	-8.366	-6.600	-8.196	-8.517	-8.176
100	NO	-1.639	-1.694	-1.715	-2.158	-1.857	-1.592	-2.477	-2.614	-2.607
	D1	-2.040	-1.791	-1.746	-2.308	-1.922	-1.504	-2.507	-2.638	-2.572
	D2	-2.060	-1.872	-1.791	-2.264	-1.923	-1.581	-2.481	-2.613	-2.587
	D3	-2.002	-1.821	-1.743	-2.269	-1.904	-1.545	-2.497	-2.645	-2.609
1000	NO	-1.654	-1.701	-1.622	-2.487	-1.672	-1.673	-1.931	-1.667	-1.632
	D1	-2.985	-1.805	-1.388	-2.480	-1.612	-1.624	-1.833	-1.638	-1.603
	D2	-3.119	-1.860	-1.442	-2.491	-1.630	-1.510	-1.923	-1.657	-1.612
	D3	-3.122	-1.527	-1.369	-2.480	-1.503	-1.525	-1.882	-1.631	-1.738

Table 19.3: Critical values. Cobreak in levels, not in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_t$. Growth components obtained with HP10 filter.

TABLE 19. $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}$, $\Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2=1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = \hat{c} + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-1}^g - z_{t-1}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_{1t}$. 5% critical values are provided for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a = 0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).



T	a	NO															D1			D2			D3		
		b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75	b=-0.05	b=-0.1	b=-0.25	b=-0.5	b=-0.75				
25	0.0	1	7.50	8.70	9.15	9.55	10.05	4.50	5.30	5.60	8.10	6.55	5.05	5.70	5.60	6.30	4.90	6.55	6.00	7.70	8.15	7.05	6.40		
		6	7.40	5.75	7.45	7.30	5.75	6.30	8.60	11.35	11.00	10.05	4.15	6.00	6.00	5.55	5.85	5.40	5.15	6.60	6.15	6.15	6.30		
		16	10.45	8.80	6.20	5.55	8.15	7.10	9.35	10.35	9.75	8.20	4.70	4.00	4.70	4.70	5.40	6.35	4.90	4.45	4.80	4.60	6.30		
	0.5	1	4.75	5.40	6.10	6.55	5.85	5.15	5.75	7.15	6.25	4.50	6.45	6.70	6.05	6.70	5.20	5.20	5.20	5.20	5.20	5.20	4.40		
		6	6.85	7.50	7.80	7.70	6.85	5.75	6.35	7.30	7.90	7.65	5.70	6.05	5.95	6.10	5.65	5.65	4.15	4.55	4.15	4.35	5.35		
		16	6.85	6.45	7.25	6.65	5.40	7.05	8.05	8.20	8.25	7.40	6.20	6.65	5.30	6.70	5.10	5.10	6.05	6.25	7.15	7.25	5.35		
100	1.0	1	3.75	4.85	4.95	5.10	5.25	6.45	5.40	6.30	6.05	5.75	4.30	4.75	5.85	6.10	5.25	5.30	5.30	5.30	5.30	5.30			
		6	4.90	4.85	4.95	5.60	5.00	5.45	5.20	6.20	6.00	6.45	5.15	5.15	5.70	5.95	5.95	4.60	5.65	5.65	6.35	6.35			
		16	5.15	4.75	5.00	5.65	5.40	6.25	6.70	6.70	5.80	6.75	4.90	4.50	4.50	5.40	5.25	5.05	5.35	6.40	6.70	6.70	7.25		
	0.0	1	11.25	20.50	37.85	50.50	43.40	8.85	11.70	21.70	29.40	23.05	11.85	12.70	20.10	25.45	22.10	22.10	7.45	10.45	10.45	12.55	12.55		
		6	52.85	53.00	11.05	19.05	28.85	11.60	19.45	2.35	4.20	12.85	11.85	16.15	1.95	6.40	2.20	9.20	11.60	23.55	23.55	4.05	4.05		
		16	34.35	10.05	19.30	23.15	24.25	13.80	6.70	4.85	5.50	4.35	14.10	5.40	4.30	4.30	4.15	4.15	12.55	13.05	13.05	13.05	13.05		
1000	0.5	1	4.35	5.60	15.30	37.10	59.35	4.75	4.55	10.30	29.75	54.30	4.75	5.05	9.15	28.95	52.95	4.60	4.80	4.80	11.20	11.20			
		6	7.10	9.40	21.00	32.30	47.85	4.90	6.30	7.95	5.25	13.50	5.45	5.85	6.40	3.35	9.80	5.35	5.35	8.05	8.05	1.95	9.40		
		16	7.95	10.65	24.05	41.45	34.85	6.20	7.50	13.45	13.25	14.40	5.10	5.75	10.35	7.80	10.25	4.00	4.35	4.35	4.35	4.35	2.75		
	1.0	1	4.00	4.30	5.15	3.35	4.00	5.10	4.15	4.05	4.25	3.60	5.40	4.45	4.45	5.30	3.60	3.70	4.35	4.35	4.35	4.35	4.35		
		6	5.20	4.40	4.75	3.80	3.25	4.30	3.95	5.45	3.55	1.85	4.45	4.50	4.50	5.30	3.90	2.05	4.70	4.70	4.70	4.70	4.70		
		16	4.30	4.25	5.15	4.10	3.05	5.00	4.90	5.15	4.95	2.60	4.90	4.75	4.75	5.60	4.30	3.80	5.45	5.45	5.45	5.45	5.45		
10000	0.0	1	37.45	85.00	57.75	22.25	76.65	33.35	58.80	24.95	5.75	40.20	30.45	51.15	22.10	4.00	33.75	26.40	48.20	48.20	22.62	22.62			
		6	32.05	17.55	64.20	97.20	98.55	6.30	1.75	4.85	5.80	11.75	6.30	1.80	4.45	5.10	8.90	17.15	4.00	6.85	6.85	9.45	9.45		
		16	32.45	12.00	95.95	91.25	85.20	8.45	9.25	9.00	10.15	1.75	8.40	8.40	8.40	8.70	8.65	8.90	8.15	4.20	4.20	4.20	4.20		
	0.5	1	13.10	18.05	36.10	36.95	96.35	10.40	33.95	39.85	39.65	41.60	10.10	17.85	42.55	38.15	43.20	43.20	4.45	6.20	6.20	17.45	17.45		
		6	7.30	10.85	35.25	99.90	96.35	5.05	7.70	20.60	41.80	48.65	5.05	6.15	19.20	36.45	43.35	43.35	4.85	5.70	5.70	14.85	14.85		
		16	12.60	28.95	83.80	99.70	97.90	4.05	3.75	4.20	18.75	78.35	6.20	7.30	29.15	18.70	78.70	78.70	7.80	7.80	7.80	32.25	32.25		
1.0	1.0	1	6.95	11.35	32.35	78.05	74.10	7.90	12.85	32.60	78.75	78.35	6.20	10.35	29.15	18.70	83.25	78.35	11.05	11.05	11.05	23.65	23.65		
		6	5.70	7.85	21.50	72.45	83.30	6.20	8.85	21.70	73.80	73.80	6.35	7.30	22.95	17.80	83.25	83.25	8.40	8.40	8.40	23.25	23.25		
		16	5.30	8.40	21.25	72.90	83.95	8.55	8.20	23.80	73.85	85.05	8.00	8.85	23.35	17.15	84.60	84.60	5.00	5.00	5.00	23.25	23.25		

Table 20.3: Power of the test. Cobreak in levels, not in differences. $\sigma_2 = s$. Estimated model $\hat{\phi}(B)\Delta y_t^g = c + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-k-2}^g - z_{t-k-2}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_t$. Growth components obtained with HP10 filter.

Table 20: $H_0 : \Delta y_t = c_t + a\Delta z_t + u_{1t}, \Delta z_t = sD_{jt} + u_{2t}$, where $\sigma_1^2 = \text{var}(u_{1t})$, and $\sigma_2^2 = \text{var}(u_{2t})$. The DGP is generated under H_0 and $\sigma_1^2 = 1$. The estimated model is $\hat{\phi}(B)\Delta y_t^g = c + \hat{\theta}(B)\Delta z_t^g + \hat{b}(y_{t-k-2}^g - z_{t-k-2}^g) + \hat{d}(y_{t-k-2}^g - z_{t-k-2}^g) + u_{1t}$. 5% critical values are provided at table 9 for different sample sizes (T= 25, 100, 1000), different short run parameter values ($a=0.0, 0.5, 1.0$), and different jump sizes ($s=1, 6, 16$).