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P-VALUES FOR NON-STANDARD DISTRIBUTIONS WITH AN APPLICATION TO THE DF TEST

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Abstract

The literature of unit roots and structural breaks has produced numerous tests that follow non-standard asymptotic distributions. This paper by fitting a seminonparametric model to them proposes a new simple way of calculating the p-values.

Key Words
DF Tests; non-standard distributions; p-values; SNP Approximations; Unit roots.

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1 Introduction

Most of the tests for structural breaks and for unit roots follow non-standard asymptotic distributions. These distributions are summarized by reporting a set of critical values (CV) (1 % , 5 % and 10 %). With this information applied researchers can not calculate the p-values and therefore can not set their own level of significance.

This paper presents a new way of approximating the p-values ($\hat{p}(x)$) by using the semi-nonparametric (SNP) techniques introduced by Gallant and Nychka (1987) and Fenton and Gallant (1994). Instead of reporting a set of CV's, we propose to report a set of parameters that describe a polynomial approximation to the non-standard empirical density. Simple integration of that polynomial produces the approximate p-values. This can be a very efficient way for econometrics software packages to store the empirical distributions of most of the unit root tests and structural break tests.

Previous attempts to approximate p-value functions for non-standard test statistics in econometrics were made by Hansen (1992, 1995[1]) and Mackinnon (1994). These works basically approximate the p-value functions by $H(f(x,\theta))$ were $f(x)$ is a simple polynomial in x with parameters θ and $H(.)$ is the Normal distribution function in Mackinnon (1994), a transformation of the Chi-square distribution in Hansen (1995) and the identity function in Hansen (1992). In Hansen (1992) and Mackinnon (1994) the parameters θ are estimated by a least squares regression of $H^{-1}(p)$ on a set of quantiles (x). The approach in Hansen (1995) differs in the sense of using an L^r norm, for r large, to measure the distance $|\hat{p}(x)-p|$.

This paper follows an approach that can be seen as more general than the one in the previous literature, in the sense of not being problem specific. We do not need to assume any ad-hoc $H(.)$ function depending on whether the test is a unit root test or a structural break test. We can also fit the whole p value function with one model, instead of doing the fitting by regions like the previous literature.

This paper is organized as follows. Section 2 presents the general framework and methodology. Section 3 shows with the particular example of the DF test for unit roots how to approximate in practice the p-values of a non-standard test. Section 4 concludes.

[1] We received a copy of Hansen (1995) after this research was done.

2 The General Framework and Methodology

Our goal is to approximate a function p-value $= p(T_n)$ which maps an observed test statistic T_n into the appropriate value in the range $[0,1]$. This test statistic has an asymptotic distribution $F(x)$ with the corresponding density function $f(x)$. We approximate the p-value function $p(x) = F(x)$, by integrating a polynomial approximation of the density function. In order to do that, we simply put the density equal to a Hermite series following the SNP approach introduced by Gallant and Nychka (1987) and Fenton and Gallant (1994). This approach consists on approximating the density function of T_n by

$$f_n(x,\theta) = \left[\sum_{i=0}^{k} \theta_i w_i((x-m)/\sigma)\right]^2 / \sum_{i=0}^{k} \theta_i^2 \qquad (1)$$

where

$$\begin{aligned} w_0(x) &= (\sqrt{2\pi})^{-1/2} e^{-x^2/4} \\ w_1(x) &= (\sqrt{2\pi})^{-1/2} x e^{-x^2/4} \\ w_i(x) &= [x w_{i-1}(x) - \sqrt{(i-1)}\, w_{i-2}(x)]/\sqrt{i}. \end{aligned} \qquad (2)$$

The parameters $\theta = (m, \sigma, \theta_i)$ can be estimated by maximum likelihood like in Fenton and Gallant (1994). Instead of doing that, this paper proposes to estimate θ by minimizing the following loss function

$$d_k(\theta) = \sum_{j=1}^{q} ((1/\alpha_j)(\alpha_j - \int_{-\infty}^{cv_{\alpha_j}} f_n(x,\theta)dx))^2 \qquad (3)$$

where the first element of θ, θ_0 is set to one and cv_{α_j} is the critical value at the α_j % significance level. According to our experience the minimization problem (3) produces better approximations for the p-values and converges faster than the equivalent maximum likelihood problem.

The tuning parameters in (3) are

- (i) the q p-values to match $(p_1\%, p_2\%, ..., p_q\%)$
- (ii) the number k of parameters θ_i.

The next section shows how to choose those tuning parameters in a particular example.

3 An Example: The DF Test

The methodology introduced in section 2 is valid for obtaining approximate p-values of any non-standard distribution. In this section we apply our method to the DF distribution.

Suppose we wish to test the null hypothesis that the variable y_t has a unit root. The simplest and most commonly used unit root tests are the DF tests. These tests are based on ordinary least squares estimates of any of the following regressions:

$$
\begin{aligned}
R(1): &\quad \Delta y_t = \lambda y_{t-1} + \textstyle\sum_{i=1}^{p} \delta_i D y_{t-i} + e_t \\
R(2): &\quad \Delta y_t = \mu_0 + \lambda y_{t-1} + \textstyle\sum_{i=1}^{p} \delta_i D y_{t-i} + e_t \\
R(3): &\quad \Delta y_t = \mu_0 + \mu_1 t + \lambda y_{t-1} + \textstyle\sum_{i=1}^{p} \delta_i D y_{t-i} + e_t
\end{aligned}
$$

where $\Delta y_t \equiv y_t - y_{t-1}$, t is a linear deterministic time trend, e_t is an error term, and λ is a parameter that equals zero under the null hypothesis of a unit root. The DF(i) test considered in this paper is the t statistic for λ to equal 0 in the regression R(i). This test has a well known non-standard asymptotic distribution (see Dickey and Fuller (1979)).

In order to calculate the approximated p-values of DF(i) (i=1, 2, 3) we follow five simple steps:

- (i) Calculate the quantiles (x) from the empirical distribution function of DF(i) test constructed from 50,000 independent draws from their (approximate) asymptotic distributions. In each draw the sample size is set to n=2000.

- (ii) Standardize x by estimating the mean (m) and the standard deviation (σ) by their sample counterparts. This does not change the results but increases the speed of convergence of the minimization problem (3).

- (iii) Decide which q p-values to match in (3). In this application we have chosen only sixteen p-values (.01, .02, .03, .05, .06, .08, .10, .12, .15, .17, .20, .25, .30, .40, .50, .60). There is a higher concentration in the left tail because of the nature of the DF test.

- (iv) Minimize the function $d_k(\theta)$ for different values of k. Plot $d_k(\theta_i)$ on k, like Figure 1, and choose k accordingly. From Figure 1, k has been set equal to six.

[Figure 1 about here]

- (v) Calculate the approximate p-values by simple integration of $f_n(x, \theta)$ from $-\infty$ to cv_{α_i}. Once the parameters θ_i are estimated, the integration can be done by any standard software (Gauss, Mathematica, Matlab, ...).

Table 1 shows the estimated parameters of the polynomial approximation $f_n(x, \theta)$ and Table 2 the approximate p-values and the absolute error of the approximation. Notice that although only sixteen p-values are chosen to be matched in (3), the approximation works uniformly very well for all the p-values from .01 to .60. In fact for the case we focus in this application (DF(3)) the $\hat{p}(x)$ are very close to the true ones in the whole range [0, 1].

[Tables 1 and 2 about here]

Similar results are available for different sample sizes. They are not reported here to make the note short.

The calculations have been done using the Optimum library in GAUSS. The programs are available upon request.

4 Conclusion

This note shows a simple and a very general way of approximating the p-values of non-standard distributions.

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Table 1: Approximate empirical densitity of the DF t test for $p \leq .60$

$$f_n(x,\theta) = \left[\sum_{i=0}^{k} \theta_i w_i((x-m)/\sigma)\right]^2 / \sum_{i=0}^{k} \theta_i^2$$

$$w_0(x) = (\sqrt{2\pi})^{-1/2} e^{-x^2/4}$$

$$w_1(x) = (\sqrt{2\pi})^{-1/2} x e^{-x^2/4}$$

$$w_i(x) = [x w_{i-1}(x) - \sqrt{i-1}\, w_{i-2}(x)]/\sqrt{i}$$

parameter	DF(1)	DF(2)	DF(3)
m	-0.42483	-1.53206	-2.17745
σ	0.97505	0.83702	0.75122
θ_1	-0.30044	-0.65441	-0.04769
θ_2	0.02413	-0.61767	-0.09269
θ_3	0.50388	-0.23093	-0.10166
θ_4	0.51716	0.12300	-0.09983
θ_5	0.28817	0.17911	-0.06281
θ_6	0.02567	0.12666	-0.06952

Table 2: Approximate *p*-values for the DF t test

	Approximate p-values			Absolute Error		
p-values	DF(1)	DF(2)	DF(3)	DF(1)	DF(2)	DF(3)
1 %	1.00319	0.99984	1.00320	.00003	.00000	.00003
2 %	1.98983	2.00171	1.99539	.00010	.00002	.00005
3 %	2.98137	2.99925	2.95448	.00019	.00001	.00046
4 %	4.04239	3.99781	4.07692	.00042	.00002	.00077
5 %	5.04151	4.96833	5.09336	.00042	.00032	.00093
6 %	6.02473	6.05821	6.05094	.00025	.00058	.00051
7 %	7.01476	6.99862	7.06335	.00015	.00001	.00063
8 %	7.99055	7.96124	7.96878	.00009	.00039	.00031
9 %	9.01392	8.94637	8.93215	.00014	.00054	.00068
10 %	10.02747	9.97407	9.94182	.00027	.00026	.00058
11 %	11.00684	11.08123	10.92839	.00007	.00081	.00072
12 %	11.97744	12.05579	11.96005	.00023	.00056	.00040
13 %	12.98543	13.05882	12.99713	.00015	.00059	.00003
14 %	13.99103	14.04091	13.99807	.00009	.00041	.00002
15 %	14.98372	15.05265	14.93359	.00016	.00053	.00066
16 %	15.97357	15.93928	15.95400	.00026	.00061	.00046
17 %	16.94230	16.90459	16.93799	.00058	.00095	.00062
18 %	17.96399	17.94639	17.98578	.00036	.00054	.00014
19 %	18.96067	18.96677	19.02162	.00039	.00033	.00022
20 %	19.97044	20.00598	20.07539	.00030	.00006	.00075
21 %	21.01644	21.01417	21.05782	.00016	.00014	.00058
22 %	22.06442	21.99547	22.07475	.00064	.00005	.00075
23 %	23.03593	22.97540	23.03562	.00036	.00025	.00036
24 %	24.00016	24.00336	24.06584	.00000	.00003	.00066
25 %	24.98652	25.02584	25.05570	.00013	.00026	.00056
30 %	29.98828	29.97709	30.03994	.00012	.00023	.00040
35 %	35.02969	34.99348	34.96467	.00030	.00007	.00035
40 %	40.03144	39.99644	40.01450	.00031	.00004	.00015
45 %	45.03378	44.88271	45.08477	.00034	.00117	.00085
50 %	50.23323	50.03430	50.11525	.00233	.00034	.00115
55 %	55.25007	55.08260	54.96449	.00250	.00083	.00036
60 %	59.83218	59.98085	59.83991	.00168	.00019	.00160
65 %	63.89082	64.84299	64.91459	.01109	.00157	.00085
70 %	67.10778	69.25286	69.65490	.02892	.00747	.00345

Figure 1: Log of Objective Function ($d_k(\vartheta)$)
for the DF(3) test