ON NATURAL SELECTION IN OLIGOPOLISTIC MARKETS[1]

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ABSTRACT. *In this paper I analyze the kind of behavior which can be considered evolutively stable in an oligopolistic market*

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1. INTRODUCTION

The theory of oligopoly studies how the behavior of interdependent firms yields a market equilibrium. This approach usually consists of a description of the basic economy -cost and demand functions-, some assumptions on how firms react to each other -Cournot, Bertrand, etc-, a proof of the existence, and in certain cases uniqueness and stability, of equilibrium and an analysis of the properties of this equilibrium.

This research program has produced fundamental insights on the understanding of oligopoly but is not free from trouble: the structure of the game -the class of admissible strategies (prices vs. quantities) and the timing (one-shot,etc)- are taken as given, and there are several classes of behavior and objectives of firms which are equally plausible. In other words equilibrium, if it exists, is indeterminate (this is the so-called folk theorem of game theory, see Kalai-Fershtman-Judd (1987) p.2).

A possible way to reduce the multiplicity of equilibria is to impose a rationality criterion (perfectness or the like). However this approach is not always successful since " the subgame perfectness concept imposes no real restriction in repeated games" (van Damme (1987) p. 165). A more subtle difficulty is the following: suppose that in a symmetric game a player makes crazy choices (and looks crazy too !) but she does at least as good in terms of payoffs as any other player. Can she be called irrational? (certainly she is successful !). Or more generally, why rationality -i.e. informed maximization of utility- should be central to economic modeling?. Up to my knowledge there are two possible answers. On the one hand it may be argued that rational behavior, as opposed to the chaos of irrationality, yields clear-cut conclusions. On the other hand there is the presumption that irrational players will be wiped out by the rational ones. However we have seen that rational behavior indeed produces a very large set of equilibria. Also the assertion that only rational players survive has not been proved. Furthermore as Koopmans (1957) points out " if this is the basis of our belief in profit maximization then we should postulate that basis itself and not the profit maximization which it implies in certain circumstances" (p. 140).

In this paper, we try to overcome the excessive multiplicity of equilibria over imposing on the usual equilibrium story a natural selection mechanism of the following sort. Suppose that firms can select their behavior from a set of reaction functions. Each of these functions might be rationalized as arising from some maximization program (i.e. maximization of profits, sales, etc for some given conjectures). A behavior may be understood as a type. We will say that a type is a <u>survivor</u> if no matter how other firms behave (i.e. the type they choose) this firm obtains at least as much profits as any competitor (see Definitions 1 and 2). In other words we would expect that a type survives in a market if the profits generated by this behavior overcome the profits obtained by competitors with any possible behavior. The idea behind that is that profits can be used as a buffer against bad times or to expand the firm. Also the possibility of survival in the event of

a price war is positively related to the quantity of profits. This suggests that to choose a survivor type may be rational in some sense. In the Sections 3 and 5 we will discuss the relationship between rationality and surviving behavior.

We will show that the introduction of evolutive considerations may help to solve the multiplicity problem mentioned before. In Proposition 1 we prove that the Walrasian type (i.e.the reaction function which arises from the maximization of profits taken the market price as given) is a survivor type. Conversely, under smoothness, symmetry and concavity assumptions the market equilibrium arising from firms which select survivor types is a Walrasian Equilibrium. This is our Proposition 2. We also show that under increasing returns and more than two firms there are no survivors (Proposition 3). Finally Proposition 4 applies a weaker notion -namely that of a successful type- to markets in which average costs are decreasing. It is shown there that optimistic firms (those choosing higher outputs) are successful types.

The rest of the paper goes as follows. The next Section explains the basic economy we are working with. In Section 3 we define the basic evolutive concepts and their relationship with "Rational" behavior. Section 4 gathers our main results. Finally Section 5 offers some comments on the significance of results.

2. THE MODEL

There is a homogeneous market served by $n > 1$ firms. Let N be the set of firms. Let us denote by A the set of states of nature with finite cardinality m. Let $q_1, \ldots, q_m$ be the true probability distribution of the states of nature. The price of the product is denoted by p. Let $p = p(x,a)$ be the inverse demand function where $a \in A$ and $x = \sum_{i \in N} x_i$, being x_i the output of firm i. The range of variation of x is taken to be compact and convex.

Firms have identical technologies[2] which are represented by a common cost function $c(x_i,a)$ such that $c(0,a) = 0 \quad \forall a \in A$. Therefore for given $a \in A$ profits for firm i can be written as $\pi_i = p(x,a).x_i - c(x_i,a) \equiv \pi(x_i,x,a)$. Notice that this profit function is identical for all firms. True expected profits for firm i are therefore $\sum_{a \in A} q_a \pi(x_i,x,a) \equiv E(x_i,x)$.

As we remarked in the Introduction, the behavior of any two firms may be different. Essentially, behavior of a firm, is determined by the following items.

a) The objectives to be maximized (i.e. sales, profits, etc.). If the firm follows a rule of thumb such as a fixed output or price equals average cost plus a given markup, this can be interpreted as the minimization of the distance between these

[2] The reason why we assume identical firms is that we want to focuss in differences in behavior, i.e. we do not want a firm to be a survivor just because it is more efficient.

targets (i.e. the output or the markup in the above examples) and the actual policies, assumed to be feasible.

b) Subjective probabilities about the occurrence of the elements of A plus a Von Neumann-Morgestern utility function reflecting the firm's attitude toward risk and defined over the objectives described in part a) above.

c) Conjectures held by a firm about how x reacts to x_i (i.e. Cournot, Bertrand, Walrasian or Perfectly Competitive, etc.)

We formalize this by saying that every firm, say i, is of some type t_i which belong to the set of possible types for this firm T_i. Let $T \equiv \prod_{i \in N} T_i$ and $T_{-i} \equiv \prod_{j \neq i} T_j$ with typical elements t and t_{-i} respectively. In general T will be a functional space i.e. a space of reaction functions. An interesting special case arises when a type specifies an output. The interpretation of this case is that the firm is committed to some particular output. If the set T_i consist of all possible outputs for firm i we will say that T_i(or T) is a direct space.

Given the set of firms and a profile of types, i.e. a behavior for each firm, let us denote by $e: T \longrightarrow R_+^{n+1}$ the mapping which yields equilibrium outputs given a profile of types $t \in T$. This mapping will be called the equilibrium mapping. The first n components are outputs for firms 1, ..., n. and the last component is total output, i.e. $e(t) = (x_1, \ldots, x_n, x)$.

In general let types $t_1,\ldots,t_n$ be represented by reaction correspondences $f_1(x_{-1}),\ldots,f_n(x_{-n})$ where x_{-i} represents a list of all outputs except x_i. The values in the range of each $f_i(\ )$

represent the output set by the corresponding firm as a function of outputs of other firms. Then, $e(t_1,\ldots,t_n)$ is the fixed point of these correspondences[3].

Assumption 1. $\forall i \in N$, T_i *contains only all upper-hemicontinuous and convex-valued correspondences.*

The role of this assumption is twofold. On the one hand it implies -via Kakutani fixed point theorem- the existence of e( ) (see Roberts-Sonnenschein (1977) for examples in which an equilibrium fails to exist). On the other hand it guarantees that type spaces are "rich" enough so most oligopoly theories are covered. We will also assume that $e(t)$ is single valued $\forall t \in T$. We do not justify this assumption here since it will be shown to hold under not unreasonable conditions (see Assumptions 3-6 and the second step in Proposition 2 below). Moreover our definitions can be adapted to a multi-valued e( ) at cost of some complications.

In the next Section we over impose on the equilibrium story, i.e. the mapping $e(\ )$, a Natural Selection mechanism in order to see which type will survive

3 Notice that given an arbitrary function f (x_{-i}) there is a utility function $U_i = x_i . f_i(x_{-i}) - x_i^2/2$ such that if $x_i = f_i(x_{-i})$, x_i maximizes this utility function for given x_{-i}, i.e. any reaction function can be rationalized.

3. THE NATURAL SELECTION MECHANISM

Let us think of firms as having an exogenous behavior which may be subject to (possibly random) changes, i.e. mutations. Alternatively we may think of firms imitating the behavior of the others. Intuitively a firm will survive if its type matched against any other possible type will yield an expected profit for this firm greater or equal than profits of any other firm. In other words a firm survives in a market if it sticks to some behavior and no matter what competitors do, it does as least as good as any of them.

It must be remarked that the above idea does not necessarily implies that agent are single minded. On the contrary there may be very sophisticated players (belonging to some type say, t_i some $i \in N$). The purpose of our analysis is to identify which kind of behavior (sophisticated or not) will survive in the long run.

A possible motivation for the concept of survivor is that expected profits can be interpreted as measure of the future growth of the firm (i.e. its reproductive power[4]). Therefore a survivor is a firms whose growth possibilities are not over taken by any competitor. Alternatively, relative expected profits can be thought as indicating the relative probability of survival of a firm subject to large random shocks and/or a price war.

[4] It must be remarked that under uncertainty, more complex measures of the expected reproductive power -involving for instance the mean and the variance of profits- are possible.

Definition 1.- *Type t' is a survivor if $\exists i$ such that $t' \in T_i$ and $\forall\, t_{-i} \in T_{-i}$ we have that if $e(t',t_{-i}) = (x'_1, \ldots, x'_n, x')$*

$$E(x'_i,x') \geq E(x'_j,x') \quad \forall j \in N.$$

In order to see clearly what is involved in Definition 1 we may rewrite it in a different form. Let $V_i(t_i,t_{-i})$ be the indirect expected profit function of firm i, i.e. $V_i(t_i,t_{-i}) \equiv E(e_i(t),e_{n+1}(t))$ where $e_j(\ )$ is the j^{th} component of $e(\ )$, $j = 1, \ldots, n+1$. Then, we have

Definition 2.- *t' is a survivor if $\exists i$ such that $t' \in T_i$ and*

$$V_i(t',t_{-i}) \geq V_j(t',t_{-i}), \quad \forall j \in N,\ \forall t_{-i} \in T_{-i}.$$

At it is clear, this definition bears some similarity with the concept of an Evolutionary Stable Strategy (see J. Maynard Smith (1982)). Main differences are that we require that survival is a global property (i.e. $\forall t_{-i}$) instead of a local one, that survivors do not mutate and that we do not impose any symmetry on the types of firms. These features reflect that in economics, mutations are not random but they are conciously made by agents in the hope of obtaining better results. Therefore if a firm behaves in such a way that it obtains more profits than their competitors a mutation of this firm is very unlikely. Conversely those firms fearing badly are good candidates to change their behavior.

EXAMPLE 1. *In Table 1 below we present an abstract situation with $n = 2$ and three types for each agent. As usual entries represent expected profits for firms 1 and 2. It is easily seen that no type* *is a survivor. If for example firm 1 takes* $t_1 = 1$*, firm 2 can obtain more profits choosing* $t_2 = 3$*, so on and so forth.*

(INSERT TABLE 1 ABOUT HERE)

The fact that there are more than two strategies is essential for the above example. In fact it is easy to prove that if the game is symmetric and we have two players and two strategies for each of them, a survivor type exists. For instance in a Prisoners-Dilemma situation if types are identified with strategies it can be proved that the strategy "To Confess" (i.e. to defect unilaterally from the cooperative agreement) is the unique survivor type. However since there are important cases in which there are no survivor types (e.g. when there are increasing returns) it will be useful to have a weaker concept. Therefore we consider the following definition which requires only "locally successful" behavior. In order to do that, let us assume that T_i, $i = 1, \ldots, n$ are metric spaces and let us denote by $B(c,d)$ the intersection of a ball with center $c \in T_{-i}$ and radius d with T_{-i}.

Definition 3. *Let* $t = (t_i, t_{-i})$ *be a given profile.* t_i *is a successful type in the profile t if* $\exists \delta > 0$ *such that* $\forall t'_{-i} \in B(t_{-i}, r)$, $\forall r < \delta$ *we have that*

$$V_i(t_i, t'_{-i}) \geq V_j(t_i, t'_{-i}) \quad \forall j \in N.$$

In words a type is successful in a given profile, if it does as good as any other possible type for small mutations of competitors.

We now discuss the relationship between surviving and rational behavior. A natural definition of rational behavior is that firms choose their types in order to maximize expected utility.

Definition 4. $(t^*_1, \ldots, t^*_n)$ *is Rational if* $\forall i \in N$ *we have that*

$$V_i(t^*_1, \ldots, t^*_n) \geq V_i(t_i, t^*_{-i}) \quad \forall t_i \in T_i$$

In words, a profile is Rational if it is a Nash Equilibrium in which types are the strategies of the game. Special cases of this equilibrium are Reasonable Conjectural Equilibrium (see Hahn (1978), Grossman (1981), Hart (1985) and Klemperer and Meyer (1989)), Incentive Equilibrium (see Fershtman-Judd (1987), Sklivas (1987) , Vickers (1985), and Corchón-Silva (1989)) and the Manipulative Nash Equilibrium studied in the theory of mechanisms for resource allocation (for general surveys see Hurwicz (1985) and Thomson (1985). For an application to the Oligopoly case see Alkan-Sertel (1981)). This concept is appealing but, in general, it produces too many equilibria[5].

It is easy to find examples in which no rational type is a survivor and viceversa. For instance in Table 1 a Rational profile exists (namely $t_1 = t_2 = 3$) but there are no survivor types. This implies that the connexion between rational and surviving behavior must be found elsewhere (see p. 21 and Section 5 for additional comments on this issue).

[5] Klemperer and Meyer obtain uniqueness of the Rational profile under strong conditions.

4. RESULTS

In this Section we present our main findings. First we define a specific type of firm which will play an important role in Proposition 1 below. We will say that a type is Walrasian, if its objectives are profits, it is risk neutral with Rational Expectations (i.e. it has the correct probability distribution over A) and it conjecture that x does not depend on x_i, i.e. it is a price-taker. It must be noticed that the allocation resulting from all firms being Walrasian is not a Perfectly Competitive Equilibrium in the sense of General Equilibrium (see e.g. Arrow-Hahn (1971) cap. 5), since we are assuming that there are no contingent markets. Rather, it is an equilibrium in the sense used in the Rational Expectations literature. In order to simplify notation let $b \equiv x_{-i}$.

Definition 5. *A reaction correspondence f_w is called the Walrasian type if* $f_w(b) = \{ x_i \in R_+ / E(x_i, x_i+b) \geq E(x'_i, x_i+b) \ \forall \ x'_i \in R_+ \}$.

It must be noticed that f_w <u>is</u> <u>not</u> the usual supply function, so we need to show that f_w belongs to the class of admissible types according to assumption 1. We first assume

Assumption 2. $\forall a \in A$, *cost and inverse demand functions satisfy:*

a) *$c(\ )$ is continuously differentiable.*

b) *$dc(x_i,a) / dx_i$ (denoted as c') is non decreasing on x_i.*

c) *$\exists\ y$ such that $p(y',a) < c'(y',a)\ \forall y' \geq y$.*

d) *$p(\ )$ is strictly decreasing and continuous on x.*

Lemma 1. *Under assumption 2, f_w exists, it is single-valued and continuous.*

Proof: *Existence of f_w is equivalent to find, for a given b, an x_i' such that if $\partial E(0,0+b)/\partial x_i > 0$ then $\partial E(x_i', x_i'+b)/\partial x_i = 0$, since if the first inequality is reversed $0 \in f_w(b)$ (notice that in order to compute $\partial E(\ )/\partial x_i$ we assume that x is held fixed). By assumption 2 c) $\exists\ y$ such that $\partial E(y, b+y)/\partial x_i < 0$. Then, the mean value theorem yields the result.*

Now, let us prove single-valuedness. Suppose it is not and let $u \in f_w(b)$ and $v \in f_w(b)$ with $u > v$. Then $E(u,u+b,) \geq E(v,u+b,)$ and $E(v,v+b,) \geq E(u,v+b,)$. Combining these inequalities we get

$$(u-v).(\sum_{a \in A} q_a.(p(u+b,a)-p(v+b,a)) \geq 0$$

and this contradicts that $p(\ )$ is strictly decreasing.

Finally continuity follows trivially from the continuity of $p(\ )$ and $c(\ )$.■

Now we are ready to prove our first result.

Proposition 1. *Under assumptions 1 and 2 f_w is a survivor.*

Proof: *Suppose f_w is not a survivor. Therefore $\exists j$ and a $t'_{-1} \in T_{-1}$ such that if $(x'_1, \ldots, x'_n, x') = e(f_w, t_{-1})$ then $E(x'_j, x') > E(x'_1, x')$*

However from the definition of a Walrasian type we have that

$$E(x'_1, x') \ \geq \ E(x'_j, x') \quad \forall x'_i \in R,$$

Therefore we have reached a contradiction and the Proposition is proved.■

Figure 1 shows how Proposition 1 works. For any given $t \in T$ we have an equilibrium total output, say x^*. Therefore the values of $E(x_i,x^*) \equiv E_i$ represent expected profits for firms 1, ..., n given their outputs, which are measured in the horizontal axis. The maximum of this function corresponds precisely with the output set by the Walrasian type. In other words, the Walrasian type is a survivor since it takes the price -which is common for all firms- as given and maximizes accordingly.

(INSERT FIGURE 1 ABOUT HERE).

Notice that an implication of Proposition 1 is that if a firm, say i, takes the Walrasian type, all firms selecting survivor types must be producing the Walrasian output (but they are not necessarily of Walrasian type). Therefore the question is whether Walrasian equilibrium results from any survivor type. In order to prove that let us assume the following

Assumption 3. *$E(\ )$ is a continuously differentiable function.*

Assumption 4. $\forall\ i,j \in N,\ T_i = T_j \subseteq R^k$

Assumption 5. *$e(\ )$ is generated by a Nash Equilibrium in quantities in which each firm maximizes a continuously differentiable function $U_i: R_+^2 \times T_i \longrightarrow R$, $U_i = U_i(x_i, x, t_i)$ for given outputs of its competitors. Also $\forall t \in T$, $e(t) >> 0$.*

In order to interpret Assumptions 4 and 5 we may think of a firm as having some weighted sum of profits, sales, etc. as its objective. A type specifies a) these weights, b) a probability distribution on A and c) the risk aversion measured by some parameters. At cost of some additional complications, conjectures may also be considered. Alternatively we may think of T_i as direct spaces and $U_i(\ )$ as the Euclidean distance between the type (output) of the firm and any other possible output. It is easy to show that assumption 5 also holds in this case.

A necessary condition for a Nash Equilibrium is

$$\partial U_i(x_i,x,t_i)/\partial x_i + \partial U_i(x_i,x,t_i)/\partial x = 0$$

In order to save notation let us denote the left hand side of the above equation as $R_i(x_i,x,t_i)$.

Assumption 6. *$R_i(x_i,x,t_i)$ is a) strictly decreasing on x_i for given x and decreasing on x for given x_i and b) never constant (not even locally) on t_i.*

The second part of Assumption 6 means that different types will generate different behavior. An immediate consequence of the first part is that $U_i(\ )$ is concave on x_i. In order to relate this assumption to well-known cases we may assume that firms maximize profits and have point expectations on A (however their expectations may differ). The space T_i corresponds here to A. Then, Assumption 6 a) implies that $\forall a \in A$.

$$x_i.d^2p(x,a)/d^2x_i + dp(x,a)/dx \leq 0$$

and

$$dp(x,a)/dx + d^2c(x_i,a)/dx_i^2 < 0$$

These are the usual assumption in the Cournot model in order to prove existence and uniqueness of equilibrium (see Friedman (1982) p. 496). Another case in which Assumption 6 is satisfied is when T_i are direct spaces and the behavior of the firm is summarized by the minimization of the distance between any output and the corresponding t_i.

Finally, let us define the following

Definition 6. *z is a (Symmetrical) Walrasian Equilibrium if*

$$E(z,nz) \geq E(x,nz) \quad \forall x \in R_+.$$

Now we are ready to prove a partial converse to Proposition 1.

Proposition 2. *Let $t* \in$ interior T_i, $\forall i \in N$ be a survivor. Let $(y, ..., y, ny) = e(t*, ..., t*)$. Then under Assumptions 2b), 3-6, y is a (Symmetrical) Walrasian Equilibrium.*

Proof: *First we notice that under Assumptions 5, and 6 a Nash Equilibrium exists for any given $t \in T$ since $U_i(\ )$ is continuous and concave on x_i $\forall i \in N$ and the strategy space for each firm can be taken to be compact and convex.*

Second we show that this Nash Equilibrium is unique for any given $t \in T$. Suppose it is not. Let us denote by the superindex 1 and 2 two arbitrary equilibria. Notice that $x^1 = x^2$ is impossible since $R(\ )$ strictly decreasing on x_i would imply that $x_i^1 = x_i^2$ $\forall i \in N$. Without loss of generality let us assume that $x^1 > x^2$. Then, the first part of Assumption 6 implies that $x_i^1 \leq x_i^2$, $\forall i \in N$ which is impossible since $x = \sum_{i \in N} x_i$.

Third we note that the determinant of the matrix with typical element $a_{ii} = \partial R_i(\ )/\partial x_i + \partial R_i(\ )/\partial x$ and $a_{ij} = \partial R_i(\ )/\partial x$ $\forall j \neq i$ is non vanishing, since all rows and columns are linearly independent (because Assumption 6 a)). Therefore e(t) is a continuously differentiable function in a vicinity or $(t, ..., t*)$ (notice that $t*$ is not an n-dimensional tuple but an element of T_i).*

Fourth we will prove that $\forall i \in N$, $\forall r = 1,...,k$ we have that $\partial e_i(t*)/\partial t_{ir} \neq \partial e_j(t*)/\partial t_{ir}$, some j. Consider the necessary condition for a Nash Equilibrium $R_i(y,ny,t*) = 0$. If the $r^t h$ component of $t*$ changes and the value of this function, says, increase, an increase of x_i must imply a decrease of x (because Assumption 6 a) again) and vice versa. Therefore if x decrease, some $j \in N$, $(j \neq i)$ must decrease as well.

Finally notice that if $t*$ is a survivor it must be that $\forall i \in N$ it maximizes $V_i(t,t*_{-i}) - V_f(t,t*_{-i})$ $\forall f \neq i$ where $t*_i$ must be understood as an n - 1 dimensional tuple with identical components $t*$. Then, first order condition of the above maximization plus symmetry imply that

$$\partial E(y,ny)/\partial x_i.(\partial e_i(t*)/\partial t_{ir} - \partial e_f(t*)/\partial t_{ir}) = 0$$

From the result obtained in the fourth step above setting $j = f$ *we get*

$$\partial E(y,ny)/\partial x_i = 0$$

And Assumption 2b) implies that y *is a Walrasian equilibrium.*■

It is easy to see that some Assumptions can be relaxed (i.e. interiority of all components of t^*, etc.) without changing our conclusion. Notice too that these assumptions are not very strong.

Two Remarks are in order. First, Proposition 2 does not assert that if we assume survivor types only, then each of these types are Walrasian. It just says that if a type is a survivor and we consider a profile which consists only of this type, if the resultant allocation is symmetrical, then it is Walrasian. Second, it would be tempting to interpret this Proposition saying that it shows that the Symmetrical Walrasian output is a survivor. However, this is only true when n=2 (see our comments on Shaffer's paper in the final Section).

Next we investigate the case in which we have economies of scale. In order to do that let us assume the following.

Assumption 7. *(Possibility of inaction)* $\forall j \in N$, $\exists t^\circ_j \in T_j$ *such that* $e_j(t^\circ_j, t_{-j}) = 0$, $\forall t_{-j} \in T_{-j}$.

Assumption 8. *Average costs are decreasing on output.*

Assumption 9. *(Feasibility of Duopoly). If* t_s *is a survivor type and* t *is a profile in which* $t_r = t_s$, $r = i,j$ *and* $t_u = t^\circ_u$, $\forall u \neq i,j$ $e_i(t)$ *and* $e_j(t)$ *are both positive.*

Then we have

Proposition 3. *Under Assumptions 3-9 if* $n > 2$ *there are no survivors.*

Proof: *Let us assume that contrarily to the Proposition we had a survivor, say* t_s. *Let us consider the allocation generated by a profile in which firms* i *and* j *are of type* t_s *and the rest of firms are inactive according to Assumption 7. Because of Assumption 9, Proposition 2 applies (with trivial modifications) to an economy consisting of two firms and therefore.*

$$\partial E(x_r, x)/\partial x_r = 0, \quad r = i,j.$$

But then Assumption 8 implies that $E(x_r,x) < 0$ *and* t_s *is not a survivor since for inactive firms* $E(0,x) = 0$.■

The interpretation of Proposition 3 is clear. Active firms will engage in cut-throat price equals marginal cost competition and therefore they make losses. Then the optimal strategy for any firm from the point of view of survival is to be inactive.

Proposition 3 suggests under which conditions a survivor may exists if increasing returns to scale are postulated: either we have a duopolistic market or reaction functions must be discontinuous, i.e. assumptions 3-6 are violated. The first alternative does not look very promising since survivor firms must be choosing an output for which price equals to marginal cost and therefore they are making losses (this follows from the first part of the proof of Proposition 3). The second alternative seems risky since the existence of the mapping e( ) is not guaranteed. Therefore we turn our attention to a weaker notion, namely that of a successful type. First, let us define the following

Definition 7. *Firm j is said to be optimistic at the profile t if*

$$e_j(t) \geq e_i(t), \quad \forall i \neq j.$$

Definition 8. *A profile t is said to be regular if*

a) There is a unique optimistic firm and

b) There is a firm, say i, such that $V_i(t) \geq 0$.

Then, we have the following

Proposition 4. *Let us assume 3-6 and 8. Then if j is an optimistic firm at a regular profile* $t = (t_1,..t_j,..t_n)$, t_j *is a successful type.*

Proof: *It is easy to see that if average costs are decreasing on output it must be that* $V_j(t) > V_i(t) \quad \forall i \neq j$. *Also from Assumptions 3-6 e( ) is locally continuous at t. Then, there is a ball with radius d and center* t_{-j} *such that* $\forall t'_{-j} \in B(t_j, r) \cap T_{-j}$ *we have that* $V_j(t_j, t'_{-j}) \geq V_i(t_j, t'_{-j}) \quad \forall i \neq j, \ \forall r < d.$■

The interpretation of this Proposition is that in the short-run (i.e. in a situation in which mutations are small), those firms with optimistic expectations will do better than those with rational or pessimistic expectations.

This proposition bears some similarity with a result obtained by Vickers (1985), Fershtman-Judd (1987) and Sklivas (1987) (see Corchón-Silva (1989) for a generalization of this result to conditions comparables to assumptions 3-6 above). They show that non profit-maximizer managers can earn greater profits than profit-maximizers managers i.e. the profit-maximizing type is not Rational according with Definition 4 above. The explanation of this is that a more optimistic behavior causes a firm's reaction function to shift outwards, and up to a point, this increases profits relative to the "Rational Expectations" point. However this similarity is only apparent since a successful type is not necessarily rational according to Definition 4. Moreover in the case considered in Proposition 4 optimism is always good, which is not always the case in the rational approach.

5. FINAL COMMENTS

There are three basic conclusion of our paper.

First, the link between to survive and "maximizing and informed", i.e. rational, behavior is more subtle than it was thought. For instance Walrasian behavior (see Definition 6) implies Rational Expectations but it does not take advantage of all profitable opportunities (except in large economies) and therefore can not be fully rational. Also, under economies of scale firms with Rational Expectations will be wiped out by firms with optimistic expectations. Finally it was shown that a rational profile is not necessarily composed of surviving types and viceversa.

Second, if Walrasian Equilibrium exists, it is the unique possible outcome of the evolutive process irrespectively of the number of firms and the kind of postulated behavior. In this sense our theory differs from standard models in which a perfectly competitive outcome can only be guaranteed under strong assumptions about the behavior of firms (i.e. Bertrand models) or the number of competitors in the market (i.e. Cournot models)[6].

Third, the performance of the market is linked with the technological basis of the society. In other words, natural selection favors a behavior which is socially acceptable, i.e. efficient, only if economies of scale are small. In this sense our paper supports just partially the view that natural selection is a good screening mechanism in market economies. In particular industrial markets with strong economies of scale do not posses equilibrium in an evolutive sense and show tendency towards overproduction. Of course the right framework to analyze the last question is a full-fledged dynamic model. Elsewhere (see Corchón-Vega (1990)) we have begun to build up such a model

6 However it can be argued that we replace the assumption of a large number of competitors by the assumption that we are in the long-run.

Summing up, the evolutive model developed in this paper produces quite definitive results on the kind of behavior which is more likely to persist in the long run, and provides a fresh angle to discuss questions like the foundation of rational behavior and the social merits of free competition. It must be remarked though that our approach is not free from shortcomings. Thus payoffs must be comparable among players for our definition of a survivor to be meaningful. Also this concept makes sense only in the long-run. Therefore many questions concerning the impact of exogenous variables on price and output can not be answered in our framework.

Previous contributions to the theory of evolutionary process in economics include Shubik (1954)[7] (who studied a three person duel in which the fittest does not necessarily survive), Nelson and Winter (1982) and Shaffer (1989), who takes the closer approach to ours. In particular, he assumes no uncertainty and constant returns to scale and define a Symmetric Evolutionary

7 The idea behind Shubik's paper is similar to an old chinese story: two warriors fight for a treasure and kill each other. A pacific fisherman observes that, and finally collects the prize (the story is called "the fisherman advantage"). However, it is disputable if truly adverse selection occurs in his model since as it is suggested by the chinese story the pacific player is in some sense the fittest. I owe this reference to Tomoichi Shinotsuka.

Equilibrium (SEE) (it is easy to show that a profile of surviving types is a SEE and that the converse is not necessarily true). Then, he proves that the symmetric Walrasian output is a SEE (actually, Jones (1980) obtained, in a very different framework, a similar result assuming that firms maximize an objective function which depends on the profits of all firms). It must be remarked, though, that Shaffer's results are different than our's since the asymmetrical Walrasian output is neither a SEE, nor a survivor type. Moreover, in a duopoly the symmetrical Walrasian output is a survivor (see Corchón-Vega (1990)) but with more than two firms it is not necessarily so. For instance if n=3, cost function is $c.x_1$ and demand function reads p= b-x the symmetrical Walrasian output is (b-c)/3 but if firm 1 produces b, firm 2 produces a small quantity ε, and firm 3 produces (b-c)/3 this output is not a survivor type since firm 2 has inferior losses. Also if firm 1 produces b-c-ε (which is an asymmetrical Walrasian output) and firms 2 and 3 produce ε each, for small values of ε firm 1 is not choosing a survivor type. Therefore it is not generally true that the Walrasian output is a survivor. Actually, what is proved in our paper is that the Walrasian <u>behavior</u> is a survivor. This illustrates that, in general, it is not possible to translate results from the type space to a direct space.

Finally we remark that some interesting questions -as the consideration of discontinuous reaction functions, heterogeneous products or technological change- were not addressed in this paper. We leave all this to future research.

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FIGURE 1

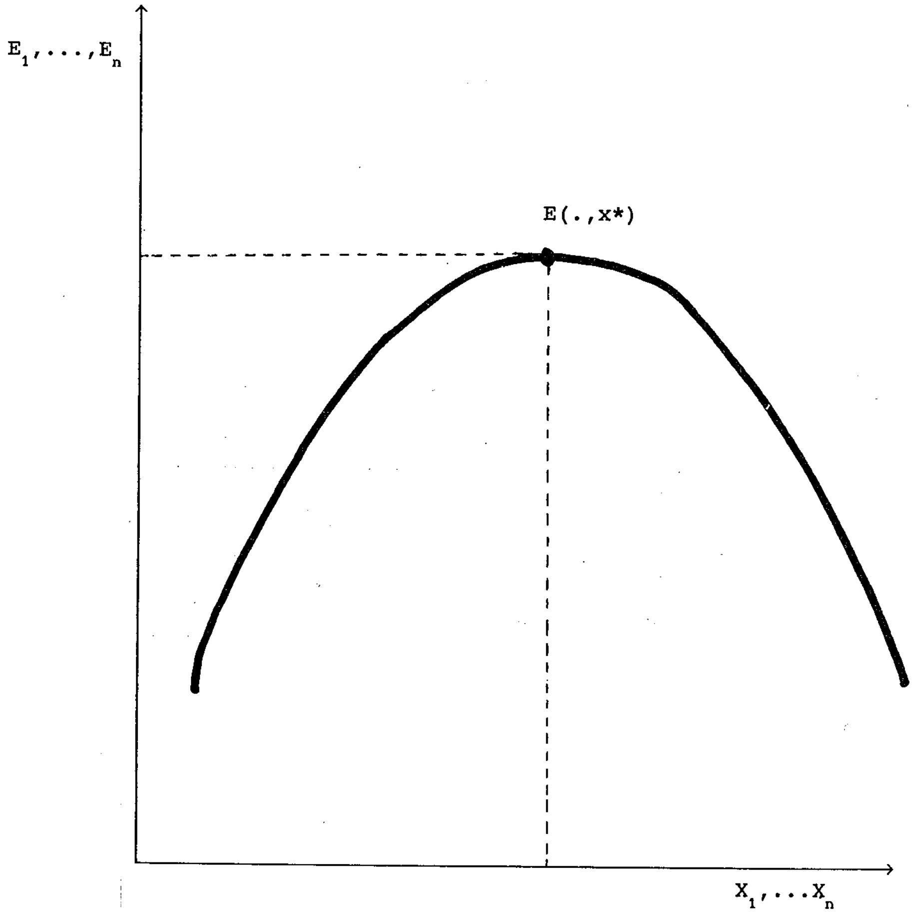

TABLE 1

Firm 1 \ Firm 2	t_1	t_2	t_3
t_1	3,3	6,4	4,6
t_2	4,6	10,10	10,8
t_3	6,4	8,10	20,20

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