FIXED PRICE AND QUALITY SIGNALS*

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ABSTRACT

In this paper I study the existence and efficiency of equilibrium for a fixed price economy in which the quality of goods is variable.

1.- INTRODUCTION

One of the most notable achievements of Fixed Price Theory (see Barro-Grossman (1976), Benassy (1975), Drèze (1975), Malinvaud (1977) and Younès (1975)) has been to show the existence of **quantity** signals under whose guidance agents can attain compatible decisions. The message is that prices are not the unique relevant signal generated by the economic environment. In this paper we will use a signal which differs from prices or quantity constraints, in order to achieve market balance.

The main idea of our paper is that in markets in which prices do not reflect relative scarcity, qualities will take their role (see Sheshinski (1976) for the case of a regulated monopoly which can manipulate the quality of the product at regulated prices, and Tirole (1988) p.287 for an example of oligopolistic firms competing in qualities with fixed prices). How important this adjustment is remains to be shown empirically, but casual empiricism suggests that it may be significantly so big, especially in planned economies in which the demands of the Central Authority are sometimes fulfilled, altering the size, weight, etc. of goods. Apparently this is one of the reasons for the introduction of "market-oriented" incentives. As we will see, our results imply that the market does not necessarily solve these problems.

In this paper it is shown, that in a fixed-price economy in which there is no rationing[1], if the **quality** of goods can be changed, a market equilibrium exists. As we said above, the idea behind our notion of equilibrium is that under fixed-prices a (notional) excess demand of a good creates a negative pressure on the quality of this good. Similarly excess supply gives incentives to enhance quality. Examples such as hospitals, food, universities, Public Transport, restaurants, furniture, coffee, etc. come easily into one's mind.

In this paper firms and consumers are assumed to be "quality-takers", i.e. they maximize their respective objective functions subject to the usual constraints, taking qualities as given. This means that we do not model how qualities are set by agents. Instead we stick to a much more elusive procedure. We simply assume that when there is some discrepancy between supply and demand, "the market" changes the quality of goods in order to match them[2].In this way, quality standards in our model act very much the same as prices (which we take as exogenously given) in the usual approach. Therefore our model is successful in determining quality standards at the cost of leaving price levels completely unexplained.

2) Bohm, Maskin, Polemarchakis and Postlewaite (1983) proved that in some cases to establish quantity constraints is not in the interest of a monopoly.

3) Whether or not such an assumption is compatible with some kind of quality-making firms must be subject of further research

We show that under standard continuity and convexity conditions plus an additional assumption which ensures that utility functions and production sets are actually affected by the quality levels, there is an allocation in which all agents maximize and market balances. We call this allocation a quality-taker equilibrium. The formal proof of the existence of such an allocation is recorded as Proposition 1.

Two remarks are in order. Firstly, we assume that quality varies continuously, with a range contained in a compact subset of the real line. This is not only a natural approach in many cases but it is also convenient since the continuity of the signal is an almost necessary requirement in order for quality to achieve market balance. Secondly, we assume no uncertainty and therefore many interesting problems associated with asymmetrical information (see, for example, the pioneering paper by Akerloff (1970)) are entirely forgotten.

The following step is to study the efficiency of this kind of equilibrium. It is very easy to show that even though first order conditions of Pareto efficiency and market equilibrium will generally differ, the allocation of goods is Pareto efficient given quality standards (see Proposition 2). This follows from the fact that, for given quality level, our equilibrium is an Arrow-Debreu-McKenzie (perfectly competitive) equilibrium in which all agents maximize their objective functions taking prices as given and all markets clear. Moreover, under additional assumptions we show that

1.- A quality-taker equilibrium can dominate -in the Pareto sense- other quality-taker equilibria even if there is a strong agreement of preferences on the quality level. We recall that the fact that some equilibria may, in the Pareto sense, dominate other equilibria occurs in other second-best problems (for instance see Hart (1975) in the context of an economy with missing markets).

2.- Even if, for some economy, the quality-taker allocation is Pareto efficient, this economy is "exceptional" in the set of economies, i.e. the economy with a Pareto efficient quality taker equilibria is "surrounded" by economies with non Pareto efficient quality taker equilibria.

Points 1 and 2 are formally recorded as Propositions 3 and 4. We point out that this kind of inefficiency is of a different nature from that studied by Nayak (1980) and Silvestre (1985) in the framework of a fixed-price economy with rationing, but it does bear some similarity with problems considered by Hart (1980), and Makowsky (1980) in large, purely competitive economies.

Finally, the rest of the paper goes as follows. In the next Section the main concepts, including that of a quality-taker equilibrium, are defined. In Section 3 we prove the existence of such an equilibrium. Welfare is studied in Section 4. The final Section is devoted to some comments on the implications of our approach and some suggestions for future research.

2.- THE MODEL

There are n goods, h consumers and f firms (n, h and f are assumed to be natural numbers). Let H = {1 ... h}, N = {1 ... n} and F = {1 ... f} be the sets of consumers, goods and firms respectively . Let j, i and g be typical elements of H, N and F respectively. Consumer j's initial endowments and shares are denoted respectively by non-negative vectors $\bar{x}_j = (\bar{x}_{j1} \ldots \bar{x}_{jn})$ and $\Theta_j = (\Theta_{j1} \ldots \Theta_{jf})$ such that shares of any firm add up to one and $\sum_{j \in H} \bar{x}_j >> 0$ (i.e. there is a positive initial quantity of each good). Let k_i be the quality of good i where $k_i \in [0, \bar{k}_i] \equiv K_i$, $\bar{k}_i \neq 0$ $\forall i \in N$.

Let $K = \mathop{X}_{i=1}^{n} K_i$ be the quality space. $\forall j \in H$, $\exists\, U_j: X_j \times K \rightarrow R$ where X_j is a non empty subset of R^n_+ and $U_j(x_j, k)$ is the utility function of j. $\forall g \in F$, $\exists \bar{Y}_g : K \rightarrow R^n$ where $\bar{Y}_g(k)$ is a non-empty subset of R^n which can be interpreted as the possible input (taken to be negative) -output (which are positive) combinations if signal $k = (k_1 \ldots k_n) \in K$ is received. Prices are strictly positive $(\bar{p}_1 \ldots \bar{p}_n) > 0$. An economy e is a tuple

$$e = (\bar{p}_i, X_j, U_j, \bar{x}_j, \Theta_j, \bar{Y}_g, K_i)_{\substack{i = 1\ldots n \\ j = 1\ldots h \\ g = 1\ldots f}}$$

Notice that the main differences with the standard definition of an economy are that: a) utility functions and production sets depend on quality standards. In particular, notice that there is a production set for every vector in K. b) Prices are part of the environment since they are exogenously given.

Also notice that implicit in our definitions is the fact that $\bar{x}_j$ can be freely converted into initial endowments of any quality. If any input is needed for such a conversion, we will say that this agent is a producer, i.e., he has access to a production set which transforms a part of his initial endowments into a good with the required quality. A different approach to the same problem would consist in taking $\bar{x}_j$ as being dependent on k. This can be accomplished in our model with straightforward modifications.

DEFINITION 1.- *A (quality-taker) equilibrium for the above economy is a tuple* $(x^*_1 \ldots x^*_h)\ (y^*_1 \ldots y^*_f)\ (k^*_1 \ldots k^*_n)$ *such that* $\forall i \in N,\ \forall j \in H,\ \forall g \in F$

a) $k^*_i \in K_i$

b) $Z^*_i \equiv \sum_{j \in H} x^*_{ji} - \sum_{g \in F} y^*_{gi} - \sum_{j \in H} \bar{x}_{ji} = 0$

c) x^*_j *maximizes* $U_j(x_j, k^*)$ *over* $B_j = \left\{ x_j / \ \bar{p} \circ x_j \leq \bar{p} \circ \bar{x}_j + \sum_{g \in F} \theta_{jf}\ \bar{p} \circ y^*_g \right\}$

d) y^*_g *maximizes* $\bar{p} \circ y_g$ *over* $Y_g \in \bar{Y}_g(k^*)$.

Notice that our definition is similar to that of a competitive equilibrium. In particular c) and d) require the maximization of utility and profits at given prices and qualities. Meanwhile, b) is just the standard feasibility requirement which is assumed to hold with equality.

3.- THE EXISTENCE OF A QUALITY-TAKER EQUILIBRIUM.

The strategy of the proof is the following. We first prove the existence of a Nash equilibrium (in the quality space). In this equilibrium, consumers and producers maximize according to parts c) and d) in Definition 1 above, and we introduce a new, fictitious agent -the auctioneer- in order to set up qualities. We will see that a Nash equilibrium is not necessarily feasible -part b) of Definition 1- which means that we have to introduce an additional assumption in order to prove feasibility (i.e. that $Z_i = 0$).

DEFINITION 2.-*A tuple* $(x_j, y_g, k_i)_{\substack{i=1...n \\ g=1...f \\ j=1...h}}$ *is said to be a Nash Equilibrium (in the quality space) if conditions a), c) and d) in Definition 1 are satisfied and in addition*

e) $\sum_{i=1}^{n} k_i \circ Z_i$ *is a minimum over* K *for given* Z_i $\forall i \in N$.

Notice that e) attempts to capture the role of quality standards as balancing supply and demand, i.e. goods in excess demand are given zero quality standards and goods in excess supply are given the maximum possible quality. In other words, e) reflects the behavior of the auctioneer as equilibrating markets.

We will first give a list of the sufficient assumptions for the existence of a Nash equilibrium.

A.1.- $X_j = \mathbb{R}^n_+ \quad \forall j \in H.$

A.2.- $U_j(x_j, k)$ *is continuous in* x_j, k *and quasi concave in* x_j *for given* k, $\forall j \in H$.

A.3.- $\exists \tilde{x}_j \in X_j,\ \bar{p}\circ\tilde{x}_j < \bar{p}\circ\bar{x}_j \quad \forall j \in H.$

A.4.- If $k_i = \bar{k}_i$ *some* $i \in N$, *then* $U_j(\ )$ *displays no local satiation* $\forall j \in H$.

A.5.- The correspondence $\bar{Y}_g(k)$ *is continuous with convex and compact image sets and* $0 \in \bar{Y}_g(k)$ $\forall k \in K$, $\forall g \in F$.

All these assumptions are standard except A.4. An interpretation of this assumption is that if good i is supplied at the highest quality, no consumer is satiated by it. Then we have

Lemma 1.- *Under A.1-5 a Nash equilibrium exists.*

Proof: *Let* $x_j = x^*_j(k)$ *and* $y_g = y(k)_g$ *be the correspondences whose values maximize* $U_j(\ )$ *and* $\bar{p}y_g$ *respectively over the sets described under c) and d) in Definition 1. Then by the maximum theorem (see Debreu (1982) p. 701 Lemma 1),* $x^*_j(k)$ *and* $y^*_g(k)$ *are closed correspondences. Also, they are convex-valued. Then the usual argument (see Debreu (1982) p. 702 Theorem 3) proves the Lemma.*■

A Nash equilibrium is not necessarily feasible (part b) in Definition 1). For instance if utility functions and production sets do not depend on k, equilibrium will exist only in exceptional cases. In order to avoid such a problem we assume the following

A.6.- a) $\forall i \in N$ *if* $k_i = 0$, $U_j(\ )$ *is strictly decreasing on* x_{ji}, $\forall j \in H$.

b) $\forall i \in N$ *if* $k_i = 0$, *and* $y_g \in \bar{Y}_g(k)$, *with* $y_{gi} < 0$, *then,* y'_g *which is identical to* y_g *except that* $y'_{gi} = 0$ *is such that* $y'_g \in \bar{Y}_g(k)$ $\forall g \in F$.

Condition a) says that if the quality of good i is zero, the consumer dislikes it. In other words, if a commodity is supplied at the minimum quality, it is a "bad". Together with A.4 it implies monotonic preferences on goods with the highest quality. Condition b) says that if firm f uses input i at quality $k_i = 0$, this input is irrelevant for production purposes (presumably because the firm -freely- disposes of any quantity of such input). Therefore A.6 implies that if $k_i = 0$, $Z_i(k) < 0$. Under this assumption $k_i = 0$ might be interpreted as the worst possible quality for good i. Then we have

PROPOSITION 1.- *Under A.1-6 a quality taker equilibrium exists.*

PROOF: *In view of Lemma 1 we only have to show that a Nash Equilibrium is a quality taker equilibrium, i. e. that the feasibility condition holds. Suppose that in a Nash Equilibrium* $Z_i^+ > 0$ *some* $i \in N$. *Then, the minimization of* $k \circ Z^+$ *yields* $k_i^+ = 0$, *but this implies (by A.6) that* $Z_i^+ < 0$, *which is a contradiction. Therefore* $Z_i^+ \leq 0$ $\forall i \in N$. *If* $Z_i^+ < 0$ *some i,* $k_i^+ = \bar{k}_i$ *and then A.4 implies that* $\bar{p} \circ Z^+ = 0$. *But this contradicts* $Z_i^+ < 0$, *since* $\bar{p}$ *is assumed to be strictly positive. Hence* $Z^+ = 0$. ■

4.- SOME WELFARE IMPLICATIONS.

In this Section we will study the efficiency of our equilibrium concept. First notice that a quality-taker equilibrium is an Arrow-Debreu-McKenzie perfectly competitive equilibrium for given quality standards. Therefore, inefficiencies must be due to the (wrong) level of quality standards, since the distribution of goods given k^* is efficient. This is related to the work of Hart (1980) and Makowsky (1980) on the efficiency of Walrasian equilibria with inactive markets. The model studied here can be interpreted as a model in which many markets are not open and the number of potential goods is infinite.

PROPOSITION 2.- *Given any vector of equilibrium qualities, there is no feasible allocation of goods such that all consumers are better off.*

PROOF: *Identical to the first fundamental theorem in Welfare Economics.*■

Let us now study the full efficiency of the quality-taker equilibrium. Our results here must be understood as extended examples, i.e. as an illustration of certain possibilities. In particular we show that under some assumptions

a) There is a continuum of quality-taker equilibria and all consumers are better off in one of them (i.e. equilibria can be Pareto-ranked).

b) If an economy *e* has a Pareto-efficient quality-taker equilibrium there is a continuum of economies around *e* such that the equilibrium is inefficient.

In order to study a) above let us introduce two new assumptions.

A.7.- $\bar{Y}_g(k*) = \bar{Y}_g(\lambda k*), \quad \forall g \in F, \lambda \simeq 1.$

A.8.- $U_j(x_j, k)$ *in a vicinity of the quality-taker equilibrium is such that*

a) $U_j(x_j, k) > U_j(x_j, \lambda k), \forall \lambda < 1, \forall j \in H$ *and*

b) Preferences over any two bundles are not affected by a change from k to $\lambda k, \lambda \simeq 1 \; \forall j \in H.$

Notice that both assumptions are assumed to hold locally. A.7 says that production sets are invariant if all quality standards change slightly in the same proportion. Thus if a firm produces corn from labor and corn, and if the quality of both input and output changes slightly, the production possibility set remains the same. Sufficient conditions for A.8 to hold are that utility functions are homogeneous of degree one on quality standards and that they are additively separable on quality standards and quantities. Then a small proportional change in all quality levels transforms monotonically the utility function holding the indifference map among goods constant. Hence both assumptions imply that excess demand functions are homogeneous of degree zero on quality standards in a vicinity of the quality taker equilibrium. Then

PROPOSITION 3.- *Let us assume A. 1-8. Then, there is a continuum of Pareto-inefficient quality-taker equilibria which can be Pareto-ranked.*

PROOF: *Let* $(x^*_j, y^*_g, k^*_i)_{\substack{j\in H\\ i\in N\\ g\in F}}$ *be a quality-taker equilibria which exists because of A.1-6. Notice that since in equilibrium excess demands are zero, A.6 implies that* $k^*_i \neq 0 \ \forall\ i \in N$. *Consider the allocation* $(x^*_j, y^*_g, \lambda k^*_i)_{\substack{j\in H\\ i\in N\\ g\in F}}$ *with* $\lambda < 1$. *If* λ *is sufficiently close to 1, A.7 and A.8 b) imply that this allocation is also a quality-taker equilibrium. Because A.8 a) is Pareto-dominated by the old allocation. Finally notice that we can generate an infinite number of such equilibria in a vicinity of any equilibrium.*■

Proposition 3 has been obtained under rather strong assumptions, but the economic sense behind it is clear. If all quality standards change in a proportionally downwards sense, all consumer's welfare decreases, but quantities do not change, since for any good the quality of substitutes will have varied in the same proportion. In other words, Proposition 3 shows the inability of the market to find the right quality standards in situations in which excess demand depends on relative quality standards, but utility depends on absolute quality standards. Notice how under such circumstances, quality control would seem to besensible. The next Proposition will show that even if the Pareto-efficientlevel of quality standards is unique, generic inefficiency is obtained. First, let us define Pareto Efficiency in our framework.

DEFINITION 3.- *An allocation* $(x_j, y_g, k_i)_{\substack{j\in H\\ i\in N\\ g\in F}}$ *is Pareto-Efficient if*

a) $k_i \in K_i \ \forall i \in N$.

b) $\sum_{j\in H} x_{ji} - \sum_{g\in F} y_{gi} - \sum_{j\in H} \bar{x}_{ji} = 0 \quad \forall i \in N$.

c) $x_j \in X_j,\ y_g \in \bar{Y}_g(k), \quad \forall j \in H,\ \forall g \in F$.

d) There is no $(x'_j, y'_g, k'_i)_{g\in F}^{j\in H, i\in N}$ *such that it fulfills a), b) and c) above and such that* $U_j(x'_{hj}, k') > U_j(x_j, k)$ $\forall j \in H$.

Notice that Definition 3 is standard except in the fact that quality standards enter into the picture, i.e. if quality standards are fixed, Definition 3 is reduced to the usual definition of a Pareto efficiency. Also notice that we only allow **one** quality for each good to be produced in order to make a fair comparison between what can be achieved on the one hand by the market and on the other by a well informed and benevolent planner (it is clear that the planner can improve market allocations in many instances if two different quality standards of the same good could be produced). Now let us introduce some more assumptions.

A.9.- There is a unique consumer with a strictly quasi-concave and continuously differentiable utility function such that if $a \in$ *interior* X_j *and* $b \in$ *boundary* X_j, *then* $U_j(a,k) > U_j(b,k)$ *for k strictly positive.*

A.10.- The vector of Pareto-Efficient qualities for an economy e, written k(e) is strictly positive and unique.

A sufficient condition for A.9 to hold, is that preferences over goods can be represented by a Cobb-Douglas utility function. Notice that under A.9, A.10 is a weak assumption, i.e. if for some economy there are two vectors of Pareto-Efficient quality standards, a small perturbation of the utility function of the consumer will make A.10 true.

We now specify our space of economies which will be denoted by $\mathcal{E}$. Let prices vary in a n-1 dimensional simplex and keep the rest of the characteristics constant. Therefore an economy is simply a price vector. We assume that A.1-6 and A.9-10 hold. Thus $\mathcal{E}$ contains an uncountable number of elements since under A.1-6 a quality-taker equilibrium exists for any given vector of positive prices. Then, we have

PROPOSITION 4.- *Under A.1-6, 9-10, there is only one economy in $\mathcal{E}$ such that the quality-taker equilibrium is Pareto-Efficient.*

PROOF: *First notice that for every $e \in \mathcal{E}$ the Pareto efficient allocation of goods to the consumer (denoted by $x(e)$) is the same. Moreover this allocation is unique because Pareto-efficient qualities are unique and A.5 and A.9. Also, it follows from A.9-10 that $x(e) \gg 0$. Therefore there is a single element in $\mathcal{E}$ (i.e. a unique price vector in the n-1 simplex) for which*

$$\frac{\partial U_j(x(e),\ k(e))/\partial\ x_i}{\partial U_j(x(e),\ k(e))/\partial\ x_r} = \frac{p_i}{p_r},\ \forall\ i,\ r \in N,$$

which is the necessary and sufficient condition of the consumer's maximization program in a quality-taker equilibrium. Thus the Proposition is proved.■

Again rather strong assumptions are used to justify an intuitive proposition: If quality standards are used to clear up markets, only in exceptional cases will these qualities coincide with the optimal ones.

5.- CONCLUSIONS

In this paper we have studied the existence and the efficiency of a market equilibrium in which prices are fixed and the quality of goods is used in order to clear up markets. Our main conclusions are that

1) If excess-demand functions are responsive enough to quality standards and, in particular, if for the lowest level of quality of any good, there is an excess supply of this good (assumption 6), under standard convexity and continuity assumptions a quality-taker equilibrium exists (Proposition 1).

2) Given equilibrium quality standards, the allocation of goods is such that no other feasible allocation of goods will improve the welfare of all consumers (Proposition 2).

3) Under additional assumptions we prove the existence of a continuum of equilibria which are Pareto-ranked (Proposition 3), and the generic inefficiency of equilibria (Proposition 4). Therefore, in this case, the wrong set of quality standards is chosen by the market.

A consequence of our approach is that market failure under fixed-prices may be of a different sort than that which arises from quantity rationing. In order to illustrate this, the following example is presented below.

Take the usual fixed-price model (e.g. see Malinvaud (1977)) and suppose that at some prices, there is unemployment (i.e. we are either in the Classical or the Keynesian unemployment region). Now consider that the quality of labor may change and workers may work harder or work extra hours. Then, there is a quality level of labor such that at the initial price level, supply equals demand in the labor market.

This allocation is Pareto-efficient given quality standards, and involves no involuntary unemployment. However, such an outcome may be very negative from the welfare point of view. The point is, that to distinguish between that which is voluntary (or notional) and that which is involuntary, may not be enough to enable us to understand all the possible hidden inefficiencies of an economy. In particular, characteristics which may be attributed to preferences (i.e. laziness of workers) may be due to the wrong levels of quality since any rational agent will be lazy under certain circumstances.

The above discussion suggests that what we call "quality" is open to a wide interpretation. It may be waiting time, so that up to some extent excess demand -or supply- can be interpreted as quality; or it may be any signal which consumers use in order to make up their minds about the characteristics of the product, i.e. advertising. A more appealing interpretation is that quality is, in fact, a public good. The meaning of a quality-taker equilibrium under this interpretation is that the planning authority fixes prices in advance (by, say, equity reasons or group's bargaining power) and it uses public goods to clear markets. This interpretation fits very well in some instances (i.e. the price of medical services may be rigid and Social Security services freely provided by a National Health Service are used to clear markets), but it relies on the existence of an equal number of both private and public goods which are substitutes up to some extent. Therefore the adequate framework would be a model in which the range of a priori variation of both prices and public goods is given, such that either the range of prices or quality standards is large enough to achieve market balance (the model presented here is a special case of this model). An interesting question would be to study the optimal policy under these circumstances.

Other possible applications of the basic idea of the paper are 1) the introduction of quality-making firms, 2) the study of the relative inefficiency of quantity-rationing equilibria versus quality-taker equilibria, 3) the consideration of price vs. quality cuts in a supergame framework and 4) the study of how quality -technical knowledge- changes with time. We leave all this to future research.

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