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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 98 75

# TERM PREMIUM AND EQUITY PREMIUM IN ECONOMIES WITH HABIT FORMATION*

Santiago Budría [a], Antonia Díaz[b†],

[a] Universidade da Madeira and CEEAplA
[b]Department of Economics, Universidad Carlos III, 28093 Getafe Madrid, SPAIN

***Abstract***

In this paper we investigate the size of the risk premium and the term premium in an representative agent exchange model economy where households preferences are subject to habit formation. As a novel feature, we develop theoretical measures for risk premium and term premium that can be used even when the consumption growth process is serially autocorrelated. We find that habit formation increases risk aversion significantly but increases much more the aversion to variations of consumption across dates. This induces a substantial increase in the precautionary demand of short term assets and a significant fall in the precautionary demand of long term assets. As a result, the term premium increases substantially with habit formation. Next we calibrate our model economy and examine the quantitative predictions of our theoretical measures of equity premium, risk premium and term premium. In line with previous literature, we show that it is possible to find a reasonable calibration for which the equity premium is that observed in the data. However, we find that around 70 percent of the equity premium is just term premium. That is, a very large fraction of the increase in the equity premium is due to the asymmetric effect that habit formation has on the precautionary demand of an asset depending on its maturity.

**Keywords: Term premium, equity premium, habit formation, consumption autocorrelation.**

**JEL** Classification: **E43, G12, G13.**

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† Corresponding Author: Professor Antonia Díaz, Department of Economics Universidad Carlos III, Calle Madrid, 126, 28903 Getafe, Madrid, Spain; Telephone: +34-91-624-5733; Fax: +34-91-624-9875; E-mail: andiaz@eco.uc3m.es

# 1 Introduction

Models whit habit forming preferences have been widely used to in the asset pricing literature to understand the equity premium puzzle. For instance, Abel (1990) and Constantinides (1990) show that adding habit formation to an otherwise standard exchange model economy, the equity premium puzzle, as stated by Mehra and Prescott (1985), disappears. The same result is obtained by Heaton (1995), Boldrin, Christiano, and Fisher (1997) and Campbell and Cochrane (1999). Since then, the properties of habit formation preferences have been tested in a variety of issues ranging from effects of the monetary policy (see Fuhrer 2000, Amato and Laubach 2004), behavior of the aggregate saving rate in a growth economy, (see Carroll, Overland, and Weil 2000) to movements of the current account (see Gruber 2004). In all these studies habit formation helps to bring the response of aggregate consumption closer to its observed behavior, mainly because habit formation makes consumption responses to any innovation more sluggish.

Notwithstanding its success in those literatures, it seems that in production economies habit formation fails to account for the observed equity premium, and for the very same reason that makes it so successful in those mentioned literatures: habit forming agents save so much for precautionary reasons that they can shield their consumption very well against fluctuations. Due to this behavior, Jermann (1998) has to introduce high adjustment cost of capital in a stochastic growth model without labor-leisure choice to obtain an equity premium close to the data. Boldrin, Christiano, and Fisher (2001) resort to limited reallocation of labor in a two sector business cycle model to match the observed equity premium, whereas Pijoan-Mas (2006) finds in a general equilibrium model with uninsurable idiosyncratic risk and liquidity constraints that the Sharpe ratio is much smaller than that implied by the data. Moreover, Lettau and Uhlig (2000) show that introducing habit forming preferences in a standard business cycle model further reduces the already small consumption volatility and can lead to contracyclical fluctuations in hours worked.

The failure of habit formation preferences to account for the equity premium in production economies led us to take a step back and inspect closely the pricing mechanism implied by this type of preferences. We use a exchange economy with a representative agent. In this way we isolate any possible effect of saving or wealth heterogeneity from affecting prices. Thus, prices should reflect solely changes in curvature of the utility function and in the valuation of consumption at different states of nature and dates. For simplicity we assume that all assets available are discount securities of various maturities and, as in Abel ( 1999, 2005), we allow for leverage.

First of all, we develop theoretical measures for the degree of risk aversion, which reflects how an individual values consumption across states of nature, and the Intertemporal Elasticity of Substitution ($IES$ hereafter), which reflects the valuation of consumption across dates. Our measures are sufficiently general to accommodate the two dominant ways in which habit formation has been modeled in the literature: as a ratio or as a difference. Next, we follow Abel (1999) and construct a log-normal approximation of assets returns. Using this approximation we can distinguish analytically the three driving forces that shape the return of any asset (conversely, its demand): the effect of consumption growth, the precautionary demand of the asset and the effect of return uncertainty. The particular advantage of our theoretical approach is that it gives us a precise description of how the level of risk aversion and the $IES$ determine the size of each of the three forces mentioned above. We find that habit formation changes the precautionary demand of any asset more drastically than its demand due to the return uncertainty. This is so because the effect of habit formation on the aversion to intertemporal fluctuations in consumption is quantitatively much larger than its effect on the level of risk aversion.

Next, we decompose the equity premium in a term (the spread between risk free assets of different maturities) and a risk premium (the excess return of a risky and a risk free asset of the same maturity). We find that the existence of habits increases both the risk premium and the term premium. Habit formation increases the risk premium because agents fear variations of consumption across states of nature more than agents with standard preferences. However, habit formation increases much more the term premium. This is so because habit formation has an asymmetric effect on the precautionary demand of assets depending on their maturity. The reason of this asymmetric effect is that agents fear fluctuations of consumption more when their habits stock is given, that is, in the short term, than in the long run, where the habit stock varies along with consumption. That is, agents would like to save in the form of short term assets and borrow in the form of long term assets. As a consequence, the net demand of precautionary savings brings a positive and large term premium that pushes up the equity premium.

Next we turn to calibrate our model economy and examine the quantitative predictions of our measures of equity premium, risk premium and term premium. We show that it is possible to find a plausible calibration for which the equity premium is that observed in the data. Using our previous theoretical measures, we decompose the equity premium in risk and term premium and we find that around 70 percent of the equity premium is just term premium, which is more than twice the magnitude that is observed. This result is found for any level of consumption

autocorrelation considered and for plausible levels of leverage. The explanation for this finding is already outlined in the previous paragraph. The level of term premium depends on the size of the net precautionary demand of savings. This net demand increases substantially with habit formation. In an exchange economy this augmented precautionary demand pushes up the term premium. This result is consistent with what is found in production economies: habit formation brings a substantial increase in precautionary savings so that agents can shield their consumption very well against fluctuations.

Our paper is very close to Abel (2005), who extends the analysis to keeping/catching up with the Joneses type of preferences but only considers consumption processes that are i.i.d. over time. Jermann (1998) uses a production economy and finds that about 90 percent of the equity premium is term premium in habit formation economies. Thus, the novelty of our paper is to study the determinants of the term premium. Our paper is also close in spirit to Boldrin, Christiano, and Fisher (1997). They use a different decomposition for the equity premium: the fraction due to changes in curvature in the utility function imposed by habit formation and what they call the capital gains channel, which includes the effect of the precautionary demand of the asset. They find that over 90 percent of the increase in the mean equity premium resulting from a switch from power utility to habit formation is due to the operation of the capital gains channel. Lettau and Uhlig (2002) exploit the log-linear approximation to obtain closed form solutions for the equity premium under different types of habit forming preferences. Their theoretical measures can be directly compared to ours. They only focus on the equity premium, disregarding the effects of habit formation on risk and term premium. They do not consider consumption processes that have serial autocorrelation, as we do.

This paper is related to the extensive literature on the term structure of interest rates. Backus, Gregory, and Zin (1989) already showed that a exchange model economy with standard preferences cannot reproduce the observed term structure of interest rates in terms of its means and volatility. More recently there is a host of papers trying to account for these facts. See, for instance, Seppala (2004), Ravenna and Seppala (2005), Seppala and Xie (2005) or Watcher (2006). While the focus of these papers are different from ours, we think that our approach is complementary to theirs since we assess the ability of habit formation models in accounting for the observed term structure of real interest rates.

The rest of the paper is organized as follows: in section 2 we develop our theoretical measures for risk aversion and the $IES$. Section 3 presents an endowment economy and use the log-normal

approximation to obtain closed form solutions for the expected return of assets of various maturities. In section 4 we calibrate our model economy and assess the ability of the habit formation model to account for the observed equity and term premium jointly. Section 5 concludes.

# 2 Measures of risk aversion

In an representative agent exchange economy prices are determined by the individual's attitude towards risk and intertemporal fluctuations in consumption. That is, prices depends on how individuals value consumption at different dates and states of nature. Individuals with standard preferences do not distinguish between dates and states of nature, whereas individuals with habit formation do. This has been already pointed out, for instance, by Constantinides (1990), and Boldrin, Christiano, and Fisher (1997). Here we review the concepts of aversion to intertemporal fluctuations and risk aversion and derive theoretical measures for the $IES$ and the coefficient of risk aversion. To gain intuition about how these measures differ under habit forming preferences we present their definitions in a very simple economy.

## 2.1 A simple economy

Assume that there is no aggregate uncertainty and that the economy is populated by a large number of infinitely lived households. Assume further that the interest rate is given and there are perfect credit markets. In this economy the problem solved by a household is

$$\begin{aligned} V(w_t, h_t) = \quad & \max_{\{c_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, h_{t+i}) \\ \text{s. t.} \quad & c_{t+i} + s_{t+i+1} = (1+r)^i w_t + (1+r) s_{t+i}, \text{ for all } i, \\ & h_{t+1} = f(c_t, h_t), \text{ for all } t \geq 1, \end{aligned} \tag{2.1}$$

where $w_t$ denotes household's net worth at the beginning of period $t$ and $r$ denotes the net interest rate. The solution to this problem is a sequence of functions of the state $(w_t, h_t)$ that we denote as $\{g_{t+i}(w_t, h_t)\}_{i=0}^{\infty}$. We also introduce some notation and call

$$U_t = \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, h_{t+i}). \tag{2.2}$$

That is, $U_t$ denotes the intertemporal level of utility starting at time $t$ for a given sequence of consumption. We denote as $\Lambda_t$ the first partial derivative of $U_t$ with respect to $c_t$, where the derivative takes into account the impact of the change in $c_t$ in all future values of the habit stock $h_t$. $\Lambda_{t,\,s}$ is the first partial derivative of $\Lambda_t$ with respect to $c_s$.

Finally, to obtain closed-form solutions of the IES and the risk aversion measure we need to specify the type of preferences we are focusing on. There are two competing ways in which habits have been introduced in the literature. On the one side, there is a *survival consumption* branch. Past consumption piles up into a habit stock that determines a minimal consumption for today, below which utility is not defined. This way of modeling habits was pioneered by Ryder, Jr., and Heal (1973) and followed for instance by Constantinides (1990), Heaton (1995), Boldrin, Christiano, and Fisher (1997) or Dynan (2000). On the other side, there is a *relative consumption* branch. Past consumption piles up into a habit stock that enters utility dividing today's consumption, capturing the notion that, under habit formation, it is not the absolute level but consumption relative to the stock what matters. This notion has been used, for instance, by Abel (1990), Carroll, Overland, and Weil (2000) or Fuhrer (2000). Therefore, the two different approaches differ in two dimensions. First, the *survival consumption* household cares about the absolute difference between consumption and habit stock whereas the *relative consumption* consumer cares about the relative difference. Second, for the *survival consumption* household, consuming below the minimal level given by the habit stock is not defined (death) whereas it is well defined for the *relative consumption* consumer. The functional forms used are for relative and survival habits, respectively,

$$u(c_t, h_t) = \frac{[c_t\, h_t^{-\gamma}]^{1-\tau}}{1-\tau}, \tag{2.3}$$

$$u(c_t, h_t) = \frac{[c_t - \gamma\, h_t]^{1-\tau}}{1-\tau}. \tag{2.4}$$

The literature assumes that the stock of habits evolves according to the law

$$h_{t+1} = (1-\lambda)\, h_t + \lambda\, c_t. \tag{2.5}$$

The parameter $\gamma$ measures the intensity of habits. If $\gamma = 1$, households only care about the consumption to habits ratio, in the case of relative habits, and about the difference in the case of survival habits. The parameter $\lambda$ measures the persistence of habits. The higher the level of $\lambda$, the higher its fluctuation with consumption. For the purpose of this paper we are going to assume

that $\lambda = 1$; that is, the current level of habits is just consumption in the previous period.

## 2.2 The Intertemporal Elasticity of Substitution

The measure that captures how an individual values consumption at different dates is the inverse of the $IES$. Here we provide a closed form solution for the inverse of the $IES$ and study how it is affected by the presence of habits. The Intertemporal Elasticity of Substitution at the steady state is given by the inverse of the Arrow-Pratt coefficient,

$$\frac{1}{IES_t} = AP_{t+1} = -\frac{\Lambda_{t+1,\,t+1}}{\Lambda_{t+1}}. \tag{2.6}$$

(See Appendix A). In a steady state allocation the consumption path satisfies $c_t = \eta\, c_{t+1}$, for all $t$, where $\eta$ denotes the steady state growth factor. Under relative habits the expression shown above becomes

$$\frac{1}{IES^r} = AP^r = \tau\,\frac{1+\gamma^2\xi}{1-\gamma\,\xi} - \frac{\gamma\,\xi\,(1+\gamma)}{1-\gamma\,\xi}, \quad \xi = \beta\,\eta^{(1-\gamma)(1-\tau)}. \tag{2.7}$$

For survival habits expression (2.6) becomes

$$\frac{1}{IES^s} = AP^s = \frac{\tau}{1-\varphi}\,\frac{1+\varphi^2\zeta}{1-\varphi\,\zeta}, \quad \varphi = \frac{\gamma}{\eta}, \quad \zeta = \beta\,\eta^{1-\tau}. \tag{2.8}$$

In both cases the $AP$ collapses to $\tau$, the risk aversion parameter, when $\gamma = 0$, that is, when there are no habits. For relative habits the $AP$ is larger than $\tau$ only if $\tau > 1$. This is not the case for survival habits, where the $AP$ is always greater than $\tau$. To see more clearly how the intensity of habits affects the curvature of the utility function we have plotted expressions (2.7 ) and (2.8) in figure 1 for several values of the intensity of habits, $\gamma$. Notice that the coefficient increases with $\gamma$ and is always larger under survival habits. That is, under habit forming preferences, households are less willing to intertemporally substitute consumption than without habits. The reason is the following: the $AP$ measures the elasticity of the variation in the valuation of future consumption in terms of current consumption with respect to a change in the consumption growth rate. Under standard preferences an individual is willing to take an increase in the consumption growth rate if the price of future consumption falls. Under habit forming preferences the fall in the price must be larger (larger $AP$) because habits induce a complementarity between current and future consumption. In other words, under habit forming preferences, households want to smooth not only the level of

consumption but also its growth rate. To see this more clearly, let us rewrite the instantaneous utility function as

$$u\left(c_t, X_t\right) = \frac{\left(c_t^{1-\gamma} X_t^{\gamma}\right)^{1-\tau}}{1-\tau}, \quad u\left(c_t, X_t\right) = \frac{\left(c_t\left(1-\frac{\gamma}{X_t}\right)\right)^{1-\tau}}{1-\tau},$$

where $X_t = c_t/c_{t-1}$. Under relative habits households not only want to smooth the level of consumption over time, but also its growth rate. This is also the case under survival habits but, additionally, the growth rate cannot fall below $\gamma$. Thus, households with survival habits fear more a decrease in consumption. This is why the $AP$, the inverse of the $IES$, is always higher for survival than for relative habits.

We should note that the elasticity of the intertemporal rate of substitution with respect to an increase in the consumption growth rate is different if we assume a permanent increase in the consumption growth rate. In a case of a permanent increase in the consumption growth rate, at the steady state, it can be shown that the inverse of the $IES$ is given by

$$\frac{1}{IES^r} = APS^r = \tau + \gamma\left(1-\tau\right), \; \frac{1}{IES^s} = APS^s = \tau. \tag{2.9}$$

(see Appendix A). Figure 1 shows the differences between this measure (labeled $APS$) and the standard $AP$. We could think of the measure $APS$ as the inverse of the Intertemporal Elasticity of Substitution across steady states. Notice that the across steady state $APS$ is smaller than the $AP$. The reason is that, across steady states, the habit stock and consumption move together and the effect of the intertemporal complementarity in consumption is eliminated. In the words of Carroll, Overland, and Weil (2000), "the gain or loss in utility associated with a given increase or decrease in consumption over a long horizon will be diminished by the associated movement in the habit stock". For survival habits the inverse of the across steady state $IES$ is just $\tau$, thus, the curvature of the utility function is the same that without habits. For relative habits, however, preferences exhibit less curvature and the across steady state $IES$ decreases with the intensity of habits $\gamma$. In other words, households desire less consumption smoothing since the habits stock moves to accommodate changes in consumption.

## 2.3 Risk aversion

To understand how preferences towards consumption at different states of nature are affected by the presence of habits we need to give a measure of risk aversion. We follow Boldrin, Christiano, and Fisher (1997) and define risk aversion in consumption, which measures how much an individual is willing to pay to avoid a fair gamble in consumption holding next period's wealth constant. Thus, the measure of risk aversion is

$$RRA_c = -\frac{u_{cc_t} + \beta V_{h_{t+1},h_{t+1}} \left(\frac{\partial h_{t+1}}{\partial c_t}\right)^2}{u_{c_t} + \beta V_{h_{t+1}} \frac{\partial h_{t+1}}{\partial c_t}} \, c_t, \tag{2.10}$$

where $V_{h_{t+1}}$ denotes the partial derivative of $V\left(w_{t+1}, h_{t+1}\right)$ with respect to the stock of habits and $V_{h_{t+1},h_{t+1}}$ is its second derivative. The function $V\left(w_{t+1}, h_{t+1}\right)$ solves the problem shown in (2.1) at period $t+1$. The expressions $u_{c_t}$ and $u_{cc_t}$ denote, respectively, the first and second derivative of the instantaneous utility function with respect to consumption, that is, without taking into account the effect of the change in current consumption on future habits. It is shown in Appendix A that we can express the coefficient of risk aversion in consumption as

$$RRA_c = -\frac{\Lambda_{t,\,t}}{\Lambda_t} \, c_t - \frac{\Lambda_{t,\,t+1}}{\Lambda_t} \xi_{t+1} \, c_{t+1}, \tag{2.11}$$

where $\xi_s$ denotes the elasticity of $g_s\left(w_{t+1}, h_{t+1}\right)$ with respect to $h_{t+1}$, for any $s \geq t+1$. Let us assume the economy is at the steady state and that the elasticity $\xi_{t+1}$ is around one. Then, risk aversion in consumption is the sum of two terms: the Arrow-Pratt coefficient plus a term that comprises changes in future utility due to changes solely in the stock of habits,

$$RRA_c \simeq -\frac{\Lambda_{t,\,t}}{\Lambda_t} \, c_t - \frac{\Lambda_{t,\,t+1}}{\Lambda_t} \, c_{t+1}. \tag{2.12}$$

In a steady state allocation the expression shown above becomes

$$RRA_c^r = AP^r - \frac{\gamma \, \xi \, (\tau - 1)}{1 - \gamma \, \xi}, \tag{2.13}$$

for relative habits, whereas for survival habits the coefficient is equal to

$$RRA_c^s = AP^s - \frac{\tau}{1 - \varphi} \, \frac{\varphi \, \zeta}{1 - \varphi \, \zeta}. \tag{2.14}$$

Expression (2.12) shows that risk aversion in consumption is lower than the $AP$ coefficient. A

fall in current consumption comes together with an increase in its price. This is measured by the first component, $-\Lambda_{t,t}\, c_t/\Lambda_t$. But a decrease in current consumption induces a fall in future habits that forces a fall in future consumption which, due to the complementarity of current and future consumption, decreases the price of current consumption. This is captured by the second term $-\Lambda_{t,t+1}\, c_{t+1}/\Lambda_t$. Thus, the level of risk aversion in consumption is lower than the inverse of the $IES$.

Figure 2 shows that as the intensity of habits rises both risk aversion and the $AP$ coefficient rise, but the increase in the $AP$ coefficient is larger. That is, habits intensity increases risk aversion but decreases, in a larger proportion, the $IES$. In other words, households with habit forming preferences fear variations of consumption across states of nature more than households with standard preferences, but they fear intertemporal variations in consumption even more. This effect will be key when we decompose the premium of a risky asset in the sum of a risk premium and a term premium.

# 3 Risk premium and term premium in theory

In this section we set our benchmark economy and obtain closed form solutions for the returns of risk free and risky assets, as well as for the equity, risk and term premium.

## 3.1 An exchange economy

The utility function of the representative household is

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right). \tag{3.1}$$

The stock of habits at time $t$ is just the level of consumption at period $t-1$, $h_t = c_{t-1}$. The instantaneous utility function is the one specified in expressions (2.4) and (2.3). There is a production unit that produces commodity $c_t$. The growth rate in $c_t$ is denoted as $x_{t+1} = \ln\left(c_{t+1}/c_t\right)$ and it follows an AR(1) process,

$$x_{t+1} = (1-\rho)\overline{x} + \rho\, x_t + \varepsilon_{t+1}. \tag{3.2}$$

The random component $\varepsilon_{t+1}$ is normal and i.i.d. with mean zero and variance $\sigma_\varepsilon^2$. The parameter $\rho$ denotes the autocorrelation coefficient. We denote by $\sigma^2$ the variance of consumption growth, which is equal to $\frac{\sigma_\varepsilon^2}{1-\rho^2}$.

There is a discount security with maturity $n$ that is competitively traded; it is a claim to a fraction of the output of the production unit. We denote as $y_t(\nu,\rho)$ the fraction of the output accrued as the payoff of the discount security. Its growth rate is $z_{t+1}(\nu,\rho) = \ln\left(y_{t+1}(\nu,\rho)/y_t(\nu,\rho)\right)$ and it follows the process

$$z_{t+1}(\nu,\rho) = (1-\rho)\overline{x} + \rho\, z_t(\nu,\rho) + \nu\,\varepsilon_{t+1}, \quad 0 < \theta \leq 1, \quad \nu \geq 0. \tag{3.3}$$

Notice that if $\nu = 1$ the payoff of the security is the entire output of the production unit. If $\nu = 0$ the payoff is constant and if $\nu > 1$ the volatility of the security payoff is larger than the volatility of the output. We model the payoff of this security in this way to introduce leverage in a simple way (see Abel 1999). In Appendix C we show that the covariance between the consumption and the dividend process is $Cov(x_{t+j}, z_t) = \rho^{|j|}\nu\sigma^2$. Additionally to the discount security, households can trade a risk free asset of maturity one period. Thus, the household's problem can be written as

$$\begin{aligned}
\max_{c_t, a_{t+1}, b_{t+1}, d_{t+1}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t\, u\left(c_t, h_t\right) \\
\text{s. t.} \quad & c_t + \sum_{i=0}^{n-1} p_t\left(n-i,\nu,\rho\right) a_t\left(n-i,\nu,\rho\right) + p_t\left(1,0,\rho\right) a_t\left(1,0,\rho\right) \leq \\
& a_{t-1}\left(0,\nu,\rho\right) y_t\left(\nu,\rho\right) + \sum_{i=1}^{n-1} p_t\left(n-i,\nu,\rho\right) a_{t-1}\left(n-i+1,\nu,\rho\right) + a_{t-1}(0,0,\rho), \\
& h_{t+1} = c_t, \text{ for all } t,
\end{aligned} \tag{3.4}$$

where $p_t(n-i,\nu,\rho)$ is the price at period $t$ of a discount security that pays off the dividend $y_{t+n-i}\left(\nu,\rho\right)$ and expires at period $t+n-i$, for $i = 0,\ldots,n-1$. $a_t(n-i,\nu,\rho)$ denotes the beginning of period $t+1$ holdings of a discount security that pays at period $t+n-i$ before expiration, for $i = 0, ..., n-1$. Thus, $a_{t-1}\left(0,\nu,\rho\right)$ denotes the beginning of period $t$ holdings of a security that pays off today and, therefore, was issued at period $t-n$ . $p_t(1,0,\rho)$ denotes the price at period $t$ of the one period risk free asset ($\nu = 0$) that will pay off at $t+1$. Thus, $a_{t-1}(0,0,\rho)$ is the beginning period $t$ holdings of the risk free asset that expires today. In the next section we turn to analyze asset pricing in this economy.

### 3.2 Asset pricing

**The expected return of the asset**

Solving the household's problem we find that the price of the security must satisfy

$$p_t(n-i,\nu,\rho) = E_t\left[\frac{\Lambda_{t+n-i}}{\Lambda_t}\, y_{t+n-i}\,(\nu,\rho)\right],\ i=0,...,n-1. \tag{3.5}$$

Using the convention

$$p_t(0,\nu,\rho) = y_t(\nu,\rho), \tag{3.6}$$

we can write the gross return of the security at time $t$ $i$ periods before its expiration as

$$R_{t+1}(n-i,\nu,\rho) = \frac{p_{t+1}(n-i-1,\nu,\rho)}{p_t(n-i,\nu,\rho)}. \tag{3.7}$$

For our study we are going to use a log-normal approximation to the equilibrium expression of prices. The method follows the procedure used by Abel (1999) and it is described in Appendix B.

In Appendix C we show that the first and second moments of the return on a one period security are approximated by the expressions

$$\ln E[R_{t+1}(1,\nu,\rho)] = -\ln\left(\frac{\Lambda^{ss}_{t+1}}{\Lambda^{ss}_t}\right) - \Psi_1\,\frac{\sigma^2}{2} + \Psi_2\,\nu\,\sigma^2, \tag{3.8}$$

$$Var[R_{t+1}(1,\nu,\rho)] = \left(\frac{1}{\phi}\,(AP - RRA_c)\right)^2\sigma^2 + \left[\nu - 2\rho\left(\frac{1}{\phi}\,(AP-RRA_c)\right)\right]\nu\,\sigma^2, \tag{3.9}$$

$$\Psi_1 = AP^2 + (AP-RRA_c)\left(\left(\frac{\phi^2-1}{\phi}\right)(AP-RRA_c) - 2\rho AP\right), \tag{3.10}$$

$$\Psi_2 = AP - \rho\,(AP-RRA_c)\left(\frac{\phi+1}{\phi}\right), \tag{3.11}$$

where the parameter $\phi$ is the effective discount factor and is equal to $\beta\, e^{\overline{x}(1-\gamma)(1-\tau)}$ for relative habits and $\beta\, e^{\overline{x}(1-\tau)}$ for survival habits. Expression (3.8) shows that the return to a one-period asset is the sum of three terms. The first one is a composition of the effect of the discount factor and the effect of growth. This term is equal to $-\ln(\beta) + (\tau + \gamma\,(1-\tau))\ \overline{x}$ for relative habits and $-\ln(\beta) + \tau\,\overline{x}$ in the case of survival habits. It shows that the return of any asset is lower the larger the discount factor is and the second part is just the inverse of the Intertemporal Elasticty

of Substitution with respect to a permanent change in the consumption growth rate. It implies that, in the presence of consumption growth, households want to borrow against future income to smooth their consumption path so that the return of the asset must rise to prevent them from doing so. We will refer to this term as the consumption growth effect for simplicity.

The second term, $\Psi_1 \frac{\sigma^2}{2}$, captures the effect of the demand for precautionary savings and, as in the standard case without habits, is always positive. This term arises because, in a world of uncertainty, agents would like to hedge against future unfavorable consumption realizations by building "buffer stocks" of the consumption good. Hence, in equilibrium, the interest rate falls to counter this enhanced demand of savings. The third term, $\Psi_2 \nu \sigma^2$, is always positive and measures the effect of uncertainty on the return of the asset. Notice that both terms depend on the difference between the $AP$ coefficient and the $RRA_c$ coefficient. That is, the precautionary demand of the asset and the uncertainty effect both depend on how the individual values consumption at different states of nature and dates. Nevertheless, the precautionary demand of savings depends more strongly on the aversion to intertemporal variations in consumption than the uncertainty effect.

Now we turn to the longer term assets. Let us denote as $E\left[R_{t+1}(\infty, \nu, \rho)\right]$ the expected return of a discount security when its maturity period is arbitrarily large, $E\left[R_{t+1}(\infty, \nu, \rho)\right] \equiv \lim_{n\to\infty} E\left[R_{t+1}(n, \nu, \rho)\right]$. We can characterize its first and second moments in the following way:

$$\ln E[R_{t+1}(\infty, \nu, \rho)] = -\ln\left(\frac{\Lambda_{t+1}^{ss}}{\Lambda_t^{ss}}\right) - \Upsilon_1 \frac{\sigma^2}{2} + \Upsilon_2 \nu \sigma^2, \tag{3.12}$$

$$Var[R_{t+1}(\infty, \nu, \rho)] = \left[2\left(\frac{1}{\phi}\left(AP - RRA_c\right)\right)^2 + \left(\nu + \frac{2(1-\rho)}{\phi}\left(AP - RRA_c\right)\right)\right]\nu\sigma^2, \tag{3.13}$$

$$\Upsilon_1 = \frac{4\rho}{\phi^2(1-\rho)}\left[AP - (1+\phi)RRA_c\right]^2 - \left(\frac{1-2\rho}{\phi^2} - \frac{2+\phi}{\phi}\right)\left(AP - RRA_c\right)^2 + AP\left(AP - \frac{2(1+\phi)}{\phi}\left(AP - RRA_c\right)\right), \tag{3.14}$$

$$\Upsilon_2 = \frac{2\rho}{\phi(1-\rho)}\left[AP - (1+\phi)RRA_c\right] + \left[\frac{\rho}{\phi}\left(AP - RRA_c\right) + RRA_c\right]. \tag{3.15}$$

(See Appendix C). The first term measures the growth effect, the second one, $\Upsilon_1 \sigma^2/2$, comprises the effect due to the precautionary demand of savings and the third term, $\Upsilon_2 \nu \sigma^2$, is due to uncertainty.

**Risk premium and term premium**

Here we provide a measure of equity premium, which is defined as the excess return on equity over a one period risk free asset. We decompose the equity premium as the sum of two components: one entirely due to risk, the risk premium, whereas the other is due to the differences in asset maturity and is labeled the term premium. In terms of our notation:

$$EP(\nu,\rho) = \ln\, E[R_{t+1}(\infty,\nu,\rho)] - \ln\, E[R_{t+1}(1,0,\rho)]. \tag{3.16}$$

We define the risk premium as the excess return of a long term risky asset over a long term risk free asset,

$$RP(\nu,\rho) = \ln\, E[R_{t+1}(\infty,\nu,\rho)] - \ln\, E[R_{t+1}(\infty,0,\rho)]. \tag{3.17}$$

Term premium is defined as the excess return of a risk free asset over its one period counterpart

$$TP(\nu,\rho) = \ln\, E[R_{t+1}(\infty,0,\rho)] - \ln\, E[R_{t+1}(1,0,\rho)]. \tag{3.18}$$

## 3.3 The effect of habits

In this section we want to discuss the effect of habits on asset expected returns and the premia defined above. For simplicity we will talk of one period assets, whose return is shown in (3.8), and long term assets, shown in (3.12). The moments of the risk free assets are obtained setting $\nu = 0$ in (3.8) and (3.12), respectively. Figure 3 depicts the expected return of the asset as a function of the habits intensity, $\gamma$. Figure 4 shows the level of equity premium, risk premium and term premium for any habits intensity. For the clarity of exposition we study here the case in which the consumption growth process is i.i.d., $\rho = 0$. The effect of non zero serial autocorrelation will be studied in the next section.

**The one period risk free asset**

Let us examine first the return of the one period risk free assets under survival habits (panel a of figure 3, second column). In this case the consumption growth effect does not depend on the habits intensity, $\gamma$, so that the fall in the expected return of the asset is due solely to the enhanced demand of precautionary savings. The larger $\gamma$ is, the higher the demand of savings is to hedge

against bad times. Thus, to prevent households from increasing their savings the return of the asset must fall. Under relative habits, however, the fall in the return of the asset is not only due to the precautionary demand of the asset but also is due to the dependence of the growth effect on the habits intensity. As $\gamma$ increases, the $IES$ with respect to a permanent increase in the consumption growth rate increases too (the growth effect). As a result, households are willing to take more intertemporal variations in consumption and are willing to save more today. Thus, the return of the asset must fall to prevent them from doing so.

If $\gamma = 0$ we are back in the standard case without habits. Thus, introducing habits helps to obtain a lower return on the one period risk free asset. That is, as Kocherlakota (1996) argues, habit formation helps to resolve the "risk free rate puzzle" stated by Weil (1989). Nevertheless, the presence of habits increases the standard deviation of the asset. At $\gamma = 0$ the standard deviation of the risk free asset is zero, whereas it is positive for a positive $\gamma$ (see expression 3.9). This is so because habits introduce a dependence of the return of the asset on the future consumption growth. The larger $\gamma$ is, the stronger the habits level and the complementarity in consumption. Thus, the reduction in the return of the risk free asset comes at the cost of a higher variance.

**The long term risk free asset and the term premium**

Now we turn to analyze the behavior of the long term risk free asset under survival habits (panel c of figure 3, second column). The behavior of this asset is solely governed by changes in the precautionary demand of the asset, as that of its one period counterpart. Its expected return, however, increases with the habits intensity. That is, households are willing to save for precautionary reasons using a long term risk free asset only if its premium is positive. In other words, habit formation affects the term structure of interest rates. To see this in a simple example consider the case of a two period risk free asset and its one period counterpart. It is easily checked that we can write

$$\ln E[R_{t+1}(2,0,\rho)] = \ln E[R_{t+1}(1,0,\rho)] - cov\left(\frac{\Lambda_{t+2}}{\Lambda_{t+1}}, \frac{\Lambda_{t+1}}{\Lambda_t}\right), \tag{3.19}$$

where the last term denotes the covariance between the marginal rate of substitution at time $t$ with its counterpart at period $t+1$. The last term is the term premium of the two period asset over the one period asset. Under standard preferences the marginal rate of substitution, $\Lambda_{s+1}/\Lambda_s$, only depends on the consumption growth rate. If consumption autocorrelation is zero, the covariance is zero and there is no term premium. This is exactly the case shown in Figure 3 (panel c, column 2) for

$\gamma = 0$. That is, under standard preferences households are indifferent between one period and long term risk free assets if the consumption growth autocorrelation is zero. This was already pointed out by Backus, Gregory, and Zin (1989). The presence of habits, however, induces a negative serial correlation in the intertemporal marginal rate of substitution even if the consumption process is not serially autocorrelated. This implies that habits have an asymmetric effect on the precautionary demand of an asset depending on its maturity, which is exactly what we see in Figure 3: agents would want to save more in the form of the one period risk free asset (so that its return must fall) whereas they would like to borrow in the form on the long term asset (and its return must rise). Another way of understanding the term premium is the following: the habit stock is fixed at the short run whereas it moves accordingly with consumption at the long run. Thus, households fear much more short term than long term fluctuations. Therefore, they would like to borrow using long term assets and save in the form of one period assets. In a representative agent exchange economy this behavior brings a fall in the return of the one period asset and a rise in the return of the long term asset and, therefore, a positive term premium.

Let us turn now to the relative habits specification (see panel 3, column 1 of figure 3). The expected return of the long term risk free asset is a non monotonic function of the habits intensity. That is, it initially decreases, as its one period counterpart, but increases afterwards. This is due to the composition of two effects. On the one hand, as in the case of survival habits, households need to receive a positive premium to hold the long term risk free asset instead its one period counterpart. On the other hand, the growth effect implies that the return of the asset decreases with $\gamma$. For values of $\gamma$ sufficiently high the first effect dominates and the asset expected return augments with the level of habits intensity.

**The risk premium**

Now we turn to analyze the effect of habits on the risky assets. Comparing the return of the risk free asset with its risky counterpart, both under relative and survival habits (see panel b of figure 3), we obtain the effect of uncertainty. Since we have assumed that the consumption process is i.i.d. the effect of uncertainty is given by the level of risk aversion in consumption. As we have seen in Figure 2, the $RRA_c$ coefficient increases with $\gamma$, therefore, the risk premium increases with the habits intensity. That is, as $\gamma$ increases individuals are less willing to save in the form of the risky asset and, hence, its premium must increase.

**The equity premium and the term premium**

Figure 4 shows that, as we already know, the size of the equity premium is larger for larger levels of habits intensity. This figure also suggests that habits produce a modest augment in the risk premium and a substantial increase in the term premium. These assertions will be made more forcefully in the section where we quantify the size of the risk premium and the term premium. Nevertheless, before turning to the quantitative exercise we want to discuss the connection between precautionary savings, term premium and equity premium. In our notation, the size of the equity premium is given by

$$EP(\nu, \rho) = (\Psi_1 - \Upsilon_1) \frac{\sigma^2}{2} + \Upsilon_2 \nu \sigma^2. \tag{3.20}$$

The first term is the term premium and the second term is the risk premium. Under standard preferences and zero consumption growth autocorrelation (we will discuss later the case of serial autocorrelation) the size of the precautionary demand of savings of a particular asset is invariant with respect to its maturity, that is, $\Psi_1 = \Upsilon_1$. In other words, there is no term premium. This implies that the size of the precautionary demand of savings does not affect the equity premium. This is no longer the case under habit forming preferences. Habits have an asymmetric effect on the precautionary demand of the asset depending on its maturity; that is, $\Psi_1$ is no longer equal to $\Upsilon_1$. As a matter of fact, households would like to borrow in the form long term assets and save using one period assets (recall figure 3). In a representative agent exchange economy this behavior implies a rise in the term premium since agents cannot go short in any asset. Moreover, the term premium increases with the difference $\Psi_1 - \Upsilon_1$. This difference, which can be viewed as the size of the net precautionary demand of savings, increases with the habits intensity $\gamma$ (see figure 3). Therefore, under habit forming preferences the size of the net precautionary demand of savings determines the term premium.

## 3.4 Changes in the consumption growth process autocorrelation

In the previous subsection we have analyzed the effect of habits on asset prices. Our assertion were made using an i.i.d. consumption growth process. Here we want to investigate the effects of habit formation when the consumption process has a non zero autocorrelation. We proceed as Otrok, Ravikumar, and Whiteman (2002) and conduct the following exercise: we vary $\rho$, the parameter that measures the persistence of the consumption growth process, and the variance of

the consumption innovations, $\sigma_\varepsilon$, so that the consumption growth variance remains unchanged. In this way, changing $\rho$ amounts to changing only the frequency at which consumption fluctuations occur but not the overall volatility of the process. Figure 5 depicts the expected return of assets as a function of $\rho$ and figure 6 shows the equity, risk and the term premium. To clarify how habit formation and consumption autocorrelation interact to determine asset returns we focus first on the case of standard preferences.

**The standard preferences case**

The first column of figure 5 shows the case of standard preferences. Panel (a) shows that, under standard preferences, the expected return of one period assets is not affected by the level of consumption autocorrelation. The return of long term assets, though, decreases with $\rho$. This is due to a combination of the change in the precautionary demand of the asset and the uncertainty effect. We analyze each in turn. Let us focus first in the behavior of the risk free long term asset compared with its one period counterpart. By looking at panel (c), column 1 of figure 5 we observe that the long term risk free asset commands a positive premium with respect to its one period counterpart if $\rho$ is negative and a negative premium otherwise. In the case of zero autocorrelation both assets command the same expected return. Thus, the consumption growth persistence affects the term structure of interest rates in a similar manner to habit formation. This is so because a negative consumption growth autocorrelation induces a negative serial autocorrelation in the intertemporal marginal of substitution (recall expression 3.19). As a consequence, households expect higher intertemporal fluctuations in the short run than in the long run when $\rho$ is negative than when it is positive. This implies a positive premium for long term assets when $\rho$ is negative (this can be seen in figure 6, first panel). Reversely, if persistence is large and positive the serial autocorrelation in the intertemporal marginal rate of substitution is positive and the premium to long term assets is negative. This was already pointed out by Backus, Gregory, and Zin (1989). Thus, habit formation has the same qualitative effect that a negative autocorrelation in consumption.

Now we turn to analyze the behavior of the long term risky asset with respect to its risk free counterpart (panel b, column 1). Notice that the return of the risky asset falls more sharply than the return of the risk free asset so that the the difference (the risk premium) becomes negative for sufficiently high levels of $\rho$. Remember that the difference in the return of both assets is given by the uncertainty component shown in (3.12). This component decreases with $\rho$ and, eventually, becomes negative. The reason of this behavior is the following: for negative autocorrelation large

persistence of the process means that high growth today is followed by low expected future growth and vice versa. That, is, consumption growth fluctuates around its unconditional mean. Since the household would like to smooth its consumption path, the premium needed to hold the risky asset must be positive. If $\rho$ is positive and sufficiently large, persistence means that high growth today implies high expected future growth tomorrow and vice versa. Holding the risk free asset, which yields the unconditional mean of the consumption process, may imply, in expected terms, a larger fluctuation in consumption than holding the risky asset. Thus, the premium may become negative for sufficiently large $\rho$. This can be seen in figure 6.

Summarizing, the persistence of the consumption process affects the size of the term and the risk premium. By looking at figure 6 we can see that the equity premium falls for large and positive levels of consumption growth autocorrelation.

**Habit forming preferences**

Now we can analyze the interaction between habit formation and the level of consumption growth autocorrelation. Let us look first to the return to a one period risk free asset (panel a, columns 2 and 3 of figure 5). Notice that the expected return augments as the consumption growth process becomes more persistent. That is, as the consumption process becomes more persistent the precautionary demand of the asset falls so that its return must increase. This is so because larger persistence implies less frequent consumption fluctuations (and smaller size of innovations). Therefore, households do not need to keep so much precautionary savings and the return of the asset goes up.

The behavior of the long term assets, as in the standard preferences case, is affected by the precautionary savings demand effect and the uncertainty effect. We discuss each in turn. By comparing the return of the one period risk free asset to its long term counterpart we see that the long period asset commands a positive premium that decreases with $\rho$ (see panel c, columns 2 and 3). That is, compared to the case of standard preferences, the premium commanded by the long term asset, although decreasing, is positive for positive $\rho$. The reason is that habit formation induces negative autocorrelation in the intertemporal marginal rate of substitution which partially counteracts the positive autocorrelation induced by the positive consumption growth autocorrelation. As a consequence, the premium is positive for $\rho = 0$.

Now we turn to analyze the behavior of the long term risky asset with respect to its risk free

counterpart (panel b, columns 2 and 3). Notice that the premium commanded by the risky asset decreases with $\rho$. Again, the mechanism operating is the same that under standard preferences but partially counteracted because of the negative autocorrelation in the intertemporal marginal rate of substitution implied by habits. As a result, the premium becomes negative for a much larger level of persistence than under standard preferences.

Summarizing, the higher the persistence of the consumption process the lower the size of the equity premium, the term and the risk premium. A visual inspection of Figure 6 suggests that the risk premium is less responsive to changes in the consumption growth autocorrelation than the term premium. Thus, persistence in the consumption process partially offsets the strong effect of habits on the term premium. In the following section we give a measure of the quantitative importance of each effect.

# 4 A quantitative exercise

In this section we turn to calibrate our model economy to asses quantitatively the size of the risk premium and the term premium.

## 4.1 The benchmark calibration

Our model period is a quarter. Boldrin, Christiano, and Fisher (1997) use quarterly consumption data from 1959 to 1989 and obtain an average consumption growth rate, $\overline{x}$, equal to 0.45 percent. Lettau (2003) uses quarterly data from 1948 to 1996 and finds $\overline{x} = 0.5$ percent. Since Lettau covers a longer time span, we chose $\overline{x} = 0.005$. The volatility of consumption growth, $\sigma = 0.0053$, is taken from Boldrin, Christiano, and Fisher (1997) since Lettau (2003) does not report it. As for the autocorrelation factor, $\rho$, Boldrin, Christiano, and Fisher (1997) set $\rho = 0.34$, whereas Campbell and Cochrane (1999) use an i.i.d process. We have chosen an intermediate value, $\rho = 0.15$. In our model, $\nu$ is the proportion between the standard deviation of dividend growth and consumption growth. Depending on the data source, the sample period, the time aggregation, and the definition of dividends, estimates of $\nu$ range from about 3 to 11. Abel (1999) uses $\nu = 2.74$. In Campbell and Cochrane (1999) the quarterly standard deviation of dividend growth is 5.6 percent, which implies that dividends are 11 times more volatile than consumption. With these numbers in mind, we have chosen an intermediate value of $\nu = 7$.

Estimates of the quarterly equity premium range from 1.61 ( Campbell and Cochrane) to 2.00 (Lettau). We target a value of 1.80. The composition of the equity premium is sensitive to the sample period considered. Lettau (2003) uses the postwar sample period and finds that only a 7 percent of the equity premium can be accounted for by a term premium. It differs from that reported in Jermann (1998) and Abel (1999). They consider the 1923-1996 sample period, and report that one third of the total premium is a term premium. Here we assume that the term premium comprises 11 percent of the equity premium.

Finally, we have to choose values for the preferences parameters. We have set the discount factor $\beta = 1$. In this way, we give the model the highest possible chance of reproducing a large risk premium. For the relative habits setting we set $\tau$, the risk aversion parameter, equal to 5 and the habits parameter, $\gamma$, is chosen so that the model reproduces the desired level of equity premium, 1.80 percent. This implies a value of $\gamma = 0.7799$ and a value for the across steady state $IES$ equal to 1.88. For the survival habits case we set $\tau = 1.88$ to keep constant the across steady state $IES$. The needed value of $\gamma$ to match the observed equity premium is 0.6986. It is very interesting to note that our calibration is very close to the estimates found by Fuhrer (2000). He estimates the utility function parameters of a representative agent that has relative habits so that the optimal consumption path matches the properties of aggregate quarterly data. Using quarterly data from 1966 to 1995 Fuhrer (2000) estimates a value for $\gamma = 0.8$ and $\tau = 6.1$. Thus, we think that our calibration is very reasonable.

### 4.2 The size of the equity premium and the term premium

Table 1 shows the size of the equity premium and its decomposition in risk and term premium for the standard preference case and the case with habits. It also reports the standard deviations of the three types of assets. The first thing we need to note is that both habits economies (the one with relative habits and the other with survival habits) deliver the same statistics. That is, assuming the same across steady state $IES$, the asset pricing implications of both specifications are the same. This is why we no longer distinguish between both types of habits.[1] Notice that the habits model economy matches the equity premium by construction whereas under standard preferences is almost one order of magnitude lower. This is so because we have set the same across steady state $IES$ for both the habits economies and the standard preferences case. As a consequence, $\tau$ is 1.88 under

[1]This statement only means that assuming either type of habits in aggregate consumption has the same asset price implications. In economies with heterogeneous agents this might not be the case, see Díaz, Pijoan-Mas, and Ríos-Rull (2003).

standard preferences which implies a very low equity premium. Let us turn to the decomposition of the equity premium in risk and term premium in the habits case. The risk premium accounts for less than 20 percent of the equity premium in the model whereas is close to 90 percent in the data. That is, most of the equity premium implied by the presence of habits is term premium. The reason was already outlined in section 3.2. The presence of habits amounts to imposing a stronger intertemporal complementarity of consumption than under standard preferences. This enhanced intertemporal complementarity of consumption induces strong changes in the demand of precautionary savings because agents fear short term intertemporal changes in consumption much more than in the case of standard preferences. This increased net demand of precautionary savings drives up the size of the term premium to a magnitude much higher than what is observed in the data.

It could be argued that these quantitative assessments are conditional on the margins that we have shut in our model economy: production and the possibility of household's borrowing. Both of them affect asset prices and the size of the equity premium. In a production economy where agents cannot borrow, the household behavior just described would imply a substantial increase in the size of household's wealth due to precautionary reasons. That is, households would reduce the fluctuations of their consumption path through self insurance which would affect negatively the size of the risk premium. This is exactly the main finding obtained by Jermann (1998), Boldrin, Christiano, and Fisher (2001). Allowing for borrowing would reduce the price of risk, as it is found by Pijoan-Mas (2006).

### 4.3 Consumption growth autocorrelation and the size of the term premium

Otrok, Ravikumar, and Whiteman (2002) and Chapman (2002) document that the autocorrelation of the consumption growth process was negative in the first third of the XXth century. Otrok, Ravikumar, and Whiteman (2002) report a $-0.26$ percent autocorrelation for annual data for the period 1890-1930 and Chapman (2002) reports $-0.16$ for the period 1890-1948. As Boldrin, Christiano, and Fisher (1997) point out, the consumption process used by Mehra and Prescott (1985) has an autocorrelation of $-0.14$. This is why we also report results assuming $\rho = -0.15$ in Table 1. We have recalibrated the habits parameter so that the equity premium for the habits economies is 1.80. As we can see, the main result still holds: the size of the risk premium is much smaller than that observed in the data.

We further investigate the responsiveness of the term premium to changes in the level of con-

sumption growth autocorrelation. This is shown in Table 2. Here we have recalibrated the habits model for every level of autocorrelation so that the equity premium is 1.80. Notice that the larger $\rho$ the larger the habits intensity needed, $\gamma$, so that that the equity premium is 1.80. Notice that the fraction of the equity premium that is term premium goes from 78.7 percent when $\rho = -0.6$ to 94.0 percent when $\rho = 0.6$. Thus, we can conclude that, although the level of consumption growth persistence affects the size of the term premium, it is not responsible of the term premium being so large in our habits economy.

### 4.4 Leverage and the size of the term premium

Jermann (1998) suggests that introducing leverage may decrease the importance of the term premium. Table 3 shows that the existence of leverage reduces the fraction of the equity premium accounted for by the term premium. However, given reasonable values for leverage, it is not enough for the model to match the data. As we can see we need a value for $\nu = 100$, which implies that stocks are 100 times more volatile than consumption, in order for the term premium to account for a fraction of the equity premium as observed in the data.

### 4.5 A robustness check

It could be argued that our analysis, based on discount securities, cannot tell us much about standard securities. Using standard securities and assuming non zero autocorrelation in the consumption process we cannot resort to our log normal approximation and we need to use simulations. Table 4 shows the standard securities case. Asset returns are calculated using the parameterized expectations approach described in Marcet and Lorenzoni (1998). We use a third degree polynomial and 10,00 quarters of artificial data to compute asset moments. As we can see, we can match the equity premium but the term premium, as a fraction of the equity premium, is within the bounds found for discount securities. It is always larger than 75 percent of the equity premium. Thus, we think that our analysis goes through with standard securities.

## 5 Final comments

In this paper we have investigated the asset pricing mechanism implied by habit formation. A calibrated exchange representative model economy can reproduce the observed equity premium.

Nevertheless, when we decompose the equity in risk and term premium we find that the model predicts a size of the term premium seven times larger as that observed in the data. This is so because habit formation has an asymmetric effect on the precautionary demand of assets depending on their maturity. In particular, agents would like to save in the form of short term assets and borrow in the form of long term assets. This is so because agents fear more fluctuations of consumption when their habits stock is given, that is, in the short run than in the long run. In other words, habit formation affects very much how agents price consumption at different dates. This asymmetric effect opens a wedge in the precautionary demand of assets depending on their maturity. We argue that this wedge is given by the net precautionary demand of savings and that it determines the size of the term premium. This result relies heavily on the margins we have shut: production and the possibility of borrowing. Nevertheless, we think that this result points out why production models economies with habit formation fail to deliver an equity premium close to that observed in the data. The large increase in the net precautionary demand of savings is responsible of a large term premium in exchange representative agent model economies whereas it would induce either a large volume of precautionary savings or a substantial amount of borrowing. Both effects drive down the equity premium.

We have considered a particular type of habits where the persistence in the habit stock is very small. If we had assumed larger persistence (as in Díaz, Pijoan-Mas, and Ríos-Rull 2003 or Pijoan-Mas 2006) the result would be enhanced. Larger persistence in the habit stock would imply larger negative correlation in the intertemporal marginal rate of substitution which, in its turn, would increase the term premium.

## References


Abel, A. (1990). Assets prices under habit formation and catching up with the joneses. *American Economic Review 89*(2), 38–42.

Abel, A. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics 43*, 3–33.

Abel, A. (2005). Equity premia with benchmark levels of consumption, leverage, imperfect correlation of consumption and dividends, and distorted beliefs: Closed-form results. Unpublished Manuscript, University of Pennsylvania.

Amato, J. D. and T. Laubach (2004). Implications of habit formation for optimal monetary policy. *Journal of Monetary Economics 51*(2), 305–325.

Backus, D. K., A. W. Gregory, and S. E. Zin (1989). Risk premiums in the term structure: Evidence from artificial economies. *Journal of Monetary Economics 24*, 371–399.

Boldrin, M., L. Christiano, and J. Fisher (1997). Habit persistence and asset returns in an exhange economy. *Macroeconomic Dynamics 1*(2), 312–332.

Boldrin, M., L. Christiano, and J. Fisher (2001). Habit persistence and asset returns and the business cycle. *American Economic Review 91*(1), 149–166.

Campbell, J. and J. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate shock market behavior. *Journal of Political Economy 107*(2), 205–251.

Carroll, C., J. Overland, and D. Weil (2000). Saving and growth with habit formation. *American Economic Review 90*(3), 341–355.

Chapman, D. A. (2002). Does intrinsic habit formation actually resolve the equity premium puzzle? *Review of Economic Dynamics 5*, 618–645.

Constantinides, G. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy 98*(3), 519–543.

Díaz, A., J. Pijoan-Mas, and J.-V. Ríos-Rull (2003). Precautionary savings and wealth distribution under habit formation. *Journal of Monetary Economics 50*(6), 1257.

Dynan, K. E. (2000). Habit formation in consumer preferences: Evidence form panel data. *American Economic Review 90*(3), 391–406.

Fuhrer, J. C. (2000). Habit formation in consumption and its implications formonetary policy models. *American Economic Review 90*(3), 367–90.

Gruber, J. W. (2004). A present value test of habits and the current account. *Journal of Monetary Economics 51*(7), 1495–1507.

Heaton, J. (1995). An empirical investigation of asset pricing with temporally dependent preference specification. *Econometrica 63*(3), 681–717.

Jermann, U. (1998). Asset pricing in production economies. *Journal of Monetary Economics 41*, 257–275.

Kocherlakota, N. R. (1996, March). The equity premium: It's still a puzzle. *Journal of Economic Literature 34*(1), 42–71.

Lettau, M. (2003). Inspecting the mechanism: The determination of asset prices in the RBC model. *Economic Journal 113*, 550575.

Lettau, M. and H. Uhlig (2000). Can habit formation be reconciled with business cycle facts. *Review of Economic Dynamics 3*, 79–99.

Lettau, M. and H. Uhlig (2002). The sharpe ratio and preferences: A parametric approach. *Macroeconomic Dynamics 6*(2), 242–265.

Marcet, A. and G. Lorenzoni (1998). Parameterized expectations approach: Some practical issues. In R. Marimon and A. Scott (Eds.), *Computational Methods for the Study of Dynamic Economics*, Chapter 7, pp. 143–171. Oxford University Press.

Mehra, R. and E. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*, 145–61.

Otrok, C., B. Ravikumar, and C. Whiteman (2002). Habit formation: A resolution of the equity premium puzzle? *Journal of Monetary Economics 49*, 1261–1288.

Pijoan-Mas, J. (2006). Pricing risk in economies with heterogeneous agents and incomplete markets. Mimeo, CEMFI.

Ravenna, F. and J. Seppala (2005). Monetary policy and the term structure of interest rates. Mimeo, University of Illinois.

Ryder, H. E. Jr., and G. M. Heal (1973). Optimal growth with intertemporally dependent preferences. *Review of Economic Studies 40*(1), 1–31.

Seppala, J. (2004). The term structure of real interest rates: Theory and evidence from U.K. index-linked bonds. *Journal of Monetary Economics 51*, 1509–1549.

Seppala, J. and L. Xie (2005). The cyclical properties of the term structure of interest rates. Mimeo, University of Illinois.

Watcher, J. (2006). A consumption based model of the term structure of interest rates. *Journal of Financial Economics 79*, 365–399.

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics 24*, 401–421.

# Appendices

## A Measures of risk aversion

**Proposition 1.** *The Intertemporal Elasticity of Substitution at the steady state is given by the inverse of the Arrow-Pratt coefficient,*

$$\frac{1}{IES_t} = AP_{t+1} = -\frac{\Lambda_{t+1,\,t+1}}{\Lambda_{t+1}}. \tag{A.1}$$

*Proof.* We define the intertemporal elasticity of substitution, $IES$ as the percentage change in consumption from time $t$ to time $t+1$ induced by a 1% change in the interest rate at time $t$, other things equal. Conversely, the inverse of the $IES$ is the elasticity of the marginal rate of substitution, denoted as $MRS_t$, with respect to the consumption growth rate. Thus, if we define $X_{t+1} = c_{t+1}/c_t$, we can write,

$$\frac{1}{IES_t} = -\frac{d\,\ln\,MRS_t}{d\,X_{t+1}}\,X_{t+1}. \tag{A.2}$$

Let us write $c_{t+1}$ as $X_{t+1}\,c_t$, and $c_{t+2}$ as $X_{t+2}\,X_{t+1}\,c_t$ in $\Lambda_t$ and $\Lambda_{t+1}$. We take the ln of the $MRS_t$ and we make a first order linear approximation around the steady state,

$$\begin{aligned}\ln(MRS_t) = \quad & \ln\left(\Lambda_{t+1}\right) - \ln\left(\Lambda_t\right) \simeq \\ & \ln\left(\Lambda^{ss}_{t+1}\right) - \ln\left(\Lambda^{ss}_{t}\right) + \frac{\Lambda^{ss}_{t+1,\,t}}{\Lambda^{ss}_{t+1}}(c_t - c^{ss}_t) + \frac{\Lambda^{ss}_{t+1,\,t+1}}{\Lambda^{ss}_{t+1}}\,(X_{t+1}\,c_t - c^{ss}_t) + \\ & + \frac{\Lambda^{ss}_{t+1,\,t+2}}{\Lambda^{ss}_{t+1}}\,(X_{t+2}\,X_{t+1}\,c_t - c^{ss}_t) - \frac{\Lambda^{ss}_{t,\,t+1}}{\Lambda^{ss}_{t}}\,(X_{t+1}\,c_t - c^{ss}_t) - \frac{\Lambda^{ss}_{t,\,t}}{\Lambda^{ss}_{t}}(c_t - c^{ss}_t).\end{aligned} \tag{A.3}$$

Differentiating $\ln(MRS_t)$ with respect to $X_{t+1}$ we obtain

$$\frac{d\,\ln\,MRS_t}{d\,X_{t+1}} = \frac{\Lambda^{ss}_{t+1,\,t+1}}{\Lambda^{ss}_{t+1}}\,c_t + \frac{\Lambda^{ss}_{t+1,\,t+2}}{\Lambda^{ss}_{t+1}}\,X_{t+2}\,c_t - \frac{\Lambda^{ss}_{t,\,t+1}}{\Lambda^{ss}_{t}}\,c_{t+1}. \tag{A.4}$$

At the steady state we know that $\left(\Lambda^{ss}_{t+1,\,t+2}/\Lambda^{ss}_{t+1}\right)c^{ss}_{t+2} = \left(\Lambda^{ss}_{t,\,t+1}/\Lambda^{ss}_{t}\right)c^{ss}_{t+1}$. Thus,

$$\frac{d\,\ln\,MRS_t}{d\,X_{t+1}}\,X_{t+1} = \frac{\Lambda^{ss}_{t+1,t+1}}{\Lambda^{ss}_{t+1}}\,c_{t+1}, \tag{A.5}$$

which, by definition, is the Arrow-Pratt coefficient. □

**Proposition 2.** *The elasticity of the Intertemporal Elasticity of Substitution with respect a permanent increase in the consumption growth rate is* $\frac{d\ln MRS_t}{d\eta}\eta = \frac{\Lambda_{t,\,t-1}}{\Lambda_t}c_{t-1} + \frac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}c_{t+1}$.

*Proof.*

$$\frac{d\,\ln MRS_t}{d\,\eta} = t\,\frac{\Lambda_{t+1,\,t}}{\Lambda_{t+1}}\,\eta^{t-1}\,c + (t+1)\,\frac{\Lambda_{t+1,\,t+1}}{\Lambda_{t+1}}\,\eta^{t}\,c + (t+2)\,\frac{\Lambda_{t+1,\,t+2}}{\Lambda_{t+1}}\,\eta^{t+1}\,c \\ - (t-1)\frac{\Lambda_{t,\,t-1}}{\Lambda_t}\,\eta^{t-2}\,c - t\,\frac{\Lambda_{t,\,t}}{\Lambda_t}\,\eta^{t-1}\,c - (t+1)\,\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,\eta^{t}\,c. \quad \text{(A.6)}$$

$$\frac{d\,\ln MRS_t}{d\,\eta}\,\eta = t\,\frac{\Lambda_{t+1,t}}{\Lambda_{t+1}}\,c_t + (t+1)\,\frac{\Lambda_{t+1,\,t+1}}{\Lambda_{t+1}}\,c_{t+1} + (t+2)\,\frac{\Lambda_{t+1,\,t+2}}{\Lambda_{t+1}}\,c_{t+2} \\ - (t-1)\frac{\Lambda_{t,\,t-1}}{\Lambda_t}\,c_{t-1} - t\,\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t - (t+1)\,\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}. \quad \text{(A.7)}$$

At the steady state

$$\frac{\Lambda_{t+1,\,t}}{\Lambda_{t+1}}\,c_t = \frac{\Lambda_{t,\,t-1}}{\Lambda_t}\,c_{t-1},\ \frac{\Lambda_{t+1,\,t+1}}{\Lambda_{t+1}}\,c_{t+1} = t\,\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t,\ (t+2)\,\frac{\Lambda_{t+1,\,t+2}}{\Lambda_{t+1}}\,c_{t+2} = (t+1)\,\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}. \quad \text{(A.8)}$$

Therefore,

$$\frac{d\ln MRS_t}{d\,\eta}\,\eta = \frac{\Lambda_{t,\,t-1}}{\Lambda_t}\,c_{t-1} + \frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}. \quad \text{(A.9)}$$

Particularizing for each type of habits we can find the expressions shown in (2.9). □

**Proposition 3.** *Risk aversion in consumption is*

$$RRA_c = -\,\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t - \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,\xi_{t+1}\,c_{t+1}, \quad \text{(A.10)}$$

*Proof.* This proof draws heavily from Díaz, Pijoan-Mas, and Ríos-Rull (2003). It can be shown

that

$$u_{c_t} + \beta V_{h_{t+1}} \frac{\partial h_{t+1}}{\partial c_t} = \Lambda_t + \beta \frac{d\, h_{t+1}}{d\, c_t} \Lambda_{t+1} \left[ \sum_{i=0}^{\infty} \beta^i \frac{\Lambda_{t+1+i}}{\Lambda_{t+1}} \frac{\partial\, g_{t+1+i}\,(w_{t+1}, h_{t+1})}{\partial\, h_{t+1}} \right]. \tag{A.11}$$

Recall that $h_{t+1} = c_t$ and that $[\beta\,(1+r)]^i\,\Lambda_{t+1+i} = \Lambda_{t+1}$, we obtain

$$u_{c_t} + \beta V_{h_{t+1}} \frac{\partial h_{t+1}}{\partial c_t} = \Lambda_t + \beta\,\Lambda_{t+1} \left[ \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \frac{\partial\, g_{t+1+i}\,(w_{t+1}, h_{t+1})}{\partial\, h_{t+1}} \right]. \tag{A.12}$$

The expression inside the brackets is the derivative of the household's budget constraint with respect to $h_{t+1}$ and it is equal to zero, hence

$$u_{c_t} + \beta V_{h_{t+1}} \frac{\partial h_{t+1}}{\partial c_t} = \Lambda_t. \tag{A.13}$$

Differentiating again,

$$u_{cc_t} + \beta V_{h_{t+1},h_{t+1}} \left( \frac{\partial h_{t+1}}{\partial c_t} \right)^2 = \Lambda_{t,\,t} + \sum_{i=1}^{\infty} \Lambda_{t,\,t+i} \frac{\partial\, g_{t+i}\,(w_{t+1}, h_{t+1})}{\partial\, h_{t+1}}. \tag{A.14}$$

Notice that $\Lambda_{t,\,t+i} = 0$ for all $i > 2$. Then, dividing equation (A.14) by (A.13) we obtain

$$-\frac{u_{cc_t} + \beta V_{h_{t+1},h_{t+1}} \left( \frac{\partial h_{t+1}}{\partial c_t} \right)^2}{u_{c_t} + \beta V_{h_{t+1}} \frac{\partial h_{t+1}}{\partial c_t}}\, c_t = -\frac{\Lambda_{t\,t}}{\Lambda_t}\, c_t - \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, \xi_{t+i}\, c_{t+i}, \tag{A.15}$$

where

$$\xi_{t+i} = \frac{\partial\, g_{t+i}\,(w_{t+1}, h_{t+1})}{\partial\, h_{t+1}} \frac{h_{t+1}}{c_{t+i}}. \tag{A.16}$$

and the result follows. □

# B The log-normal approximation

The expression for the prices shown in (3.5) can be written as follows

$$p_t(n-i, \nu, \rho) = \, y_t(\nu, \rho)\, E_t \left[ \frac{\Lambda_{t+n-i}}{\Lambda_t} \prod_{j=1}^{n-i} Z_{t+j}(\nu, \rho) \right], \tag{B.1}$$

where

$$Z_{t+j}(\nu,\rho)=\frac{y_{t+j}(\nu,\rho)}{y_{t+j-1}(\nu,\rho)}. \tag{B.2}$$

Let us assume that the economy is at the steady state at time $t-1$. Then we can express consumption in terms of deviations with respect its steady state level as

$$c_{t+j}=\exp\left(\widetilde{x}_{t+j}+.....+\widetilde{x}_{t-1}\right)c_{t+j}^{ss}. \tag{B.3}$$

Applying a Taylor expansion of degree one to $\frac{\Lambda_{t+n-i}}{\Lambda_t}$ around the steady state we find

$$\ln\left(\frac{\Lambda_{t+n-i}}{\Lambda_t}\right)\simeq\ln\left(\frac{\Lambda_{t+n-i}^{ss}}{\Lambda_t^{ss}}\right)+\sum_{j=-1}^{1}\frac{\Lambda_{t+n-i,\,t+n-i+j}^{ss}}{\Lambda_{t+n-i}^{ss}}\,c_{t+n-i+j}^{ss}\left(\exp\left(\sum_{l=-1}^{n-i+j}\widetilde{x}_{t+l}\right)-1\right)$$
$$-\sum_{j=-1}^{1}\frac{\Lambda_{t,\,t+j}^{ss}}{\Lambda_t^{ss}}\,c_{t+j}^{ss}\left(\exp\left(\sum_{l=-1}^{j}\widetilde{x}_{t+l}\right)-1\right) \tag{B.4}$$

Since $\exp(a)-1\approx a$ we have,

$$\ln\left(\frac{\Lambda_{t+n-i}}{\Lambda_t}\right)\simeq\ln\left(\frac{\Lambda_{t+n-i}^{ss}}{\Lambda_t^{ss}}\right)+\sum_{j=-1}^{1}\frac{\Lambda_{t+n-i,\,t+n-i+j}^{ss}}{\Lambda_{t+n-i}^{ss}}\,c_{t+n-i+j}^{ss}\left(\sum_{l=-1}^{n-i+j}\widetilde{x}_{t+l}\right)$$
$$-\sum_{j=-1}^{1}\frac{\Lambda_{t,\,t+j}^{ss}}{\Lambda_t^{ss}}\,c_{t+j}^{ss}\left(\sum_{l=-1}^{j}\widetilde{x}_{t+l}\right). \tag{B.5}$$

Taking into account that

$$\frac{\Lambda_{t+n-i,\,t+n-i+j}^{ss}}{\Lambda_{t+n-i}^{ss}}\,c_{t+n-i+j}^{ss}=\sum_{j=-1}^{1}\frac{\Lambda_{t,\,t+j}^{ss}}{\Lambda_t^{ss}}\,c_{t+j}^{ss} \tag{B.6}$$

and that

$$\frac{\Lambda_{t,\,t-1}^{ss}}{\Lambda_t^{ss}}\,c_{t-1}^{ss}=\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}^{ss}}{\Lambda_t^{ss}}\,c_{t+1}^{ss}, \tag{B.7}$$

where $\phi$ is the effective discount factor, which is equal to $\beta\, e^{\overline{x}\,(1-\gamma)(1-\tau)}$ for relative habits and $\beta\, e^{\overline{x}\,(1-\tau)}$ for survival habits we find

$$\ln\left(\frac{\Lambda_{t+n-i}}{\Lambda_t}\right) \simeq \ln\left(\frac{\Lambda^{ss}_{t+n-i}}{\Lambda^{ss}_t}\right) + \frac{\Lambda^{ss}_{t,\,t}}{\Lambda^{ss}_t}\, c^{ss}_t \left(\sum_{l=-1}^{n-i} \widetilde{x}_{t+l} - \sum_{l=-1}^{0} \widetilde{x}_{t+l}\right) + \frac{\Lambda^{ss}_{t,\,t+1}}{\Lambda^{ss}_t}\, c^{ss}_{t+1} \left[\left(\sum_{l=-1}^{n-i+1} \widetilde{x}_{t+l} - \sum_{l=-1}^{1} \widetilde{x}_{t+l}\right) + \frac{1}{\phi}\left(\sum_{l=-1}^{n-i-1} \widetilde{x}_{t+l} - \widetilde{x}_{t-1}\right)\right]. \quad \text{(B.8)}$$

Thus, the asset pricing equation can be written as

$$p_t(n-i,\nu,\rho) \simeq\, y_t(\nu,\rho)\, E_t\Bigg[\exp\Bigg(\ln\left(\frac{\Lambda^{ss}_{t+n-i}}{\Lambda^{ss}_t}\right) + \frac{\Lambda^{ss}_{t,\,t}}{\Lambda^{ss}_t}\, c^{ss}_t \sum_{l=1}^{n-i} \widetilde{x}_{t+l} + \frac{\Lambda^{ss}_{t,\,t+1}}{\Lambda^{ss}_t}\, c^{ss}_{t+1} \left(\sum_{l=2}^{n-i+1} \widetilde{x}_{t+l} + \frac{1}{\phi} \sum_{l=0}^{n-i-1} \widetilde{x}_{t+l}\right) + \sum_{l=1}^{n-i} z_{t+l}(\nu,\rho)\Bigg)\Bigg] \quad \text{(B.9)}$$

where $z_{t+j}(\nu,\rho) = \ln(Z_{t+j}(\nu,\rho))$.

# C Risk premium and term premium in theory

**Proposition 4.** *The covariance of consumption growth and dividends growth satisfies* $cov\,(x_{t+j}, z_t) = \rho^{|j|}\nu\,\sigma$.

*Proof.* To obtain the covariance formula, write the $AR(1)$ processes in its $MA(\infty)$ version:

$$x_{t+1} = \frac{\overline{x}}{1-\rho} + \sum_{i=0}^{\infty} \rho^i \varepsilon_{t+1-i} + \rho^{t+1} x_0, \quad \text{(C.1)}$$

and

$$z_{t+1}(\nu,\rho) = \frac{\overline{x}}{1-\rho} + \nu \sum_{i=0}^{\infty} \rho^i \varepsilon_{t+1-i} + \rho^{t+1} z_0(\nu,\rho), \quad \text{(C.2)}$$

where the last term in both equations can be neglected for a sufficiently large $t$. Then

$$E[x_{t+j}\, z_t] = \frac{\overline{x}^2}{(1-\rho)^2} + \rho^{|j|}\nu \sum_{i=0}^{\infty} (\rho^2)^i E[\varepsilon^2_{t-i}] \quad \text{(C.3)}$$

$$= \frac{\overline{x}^2}{(1-\rho)^2} + \rho^{|j|}\, \nu\, \frac{\sigma^2_\varepsilon}{1-\rho^2}. \quad \text{(C.4)}$$

Finally, taking into account that $cov(x_{t+j}, z_t) = E[x_{t+j}\, z_t] - E[x_{t+j}]\, E[z_t]$ with $E[x_{t+j}] = \frac{\overline{x}}{1-\rho}$, $E[z_t] = \frac{\overline{x}}{1-\rho}$, and $\widetilde{x}_t = x_t - \overline{x}$, we get

$$Cov(x_{t+j}, z_t) = Cov(\widetilde{x}_{t+j}, z_t) = \rho^{|j|}\nu\frac{\sigma_\varepsilon^2}{1-\rho^2} = \rho^{|j|}\nu\sigma^2. \tag{C.5}$$

□

**Proposition 5.** *The price of a discount security can be written as*

$$p_t(1,\nu,\rho) \simeq y_t(\nu,\rho)\, \exp\left[\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] \exp\left[\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, \widetilde{x}_t\right]\, E_t\left[\exp\left(q_t(1,\nu,\rho)\right)\right], \tag{C.6}$$

*where*

$$q_t(1,\nu,\rho) = \frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t\, \widetilde{x}_{t+1} + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, \widetilde{x}_{t+2} + z_{t+1}(\nu,\rho), \tag{C.7}$$

$$p_t(2,\nu,\rho) \simeq y_t(\nu,\rho)\, \exp\left[\ln\left(\frac{\Lambda_{t+2}}{\Lambda_t}\right)\right] \exp\left[\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, \widetilde{x}_t\right]\, E_t\left[\exp\left\{q_t(2,\nu,\rho)\right\}\right], \tag{C.8}$$

$$q_t(2,\nu,\rho) = \left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t + \frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\right)\widetilde{x}_{t+1} + \left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\right)\widetilde{x}_{t+2} + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, \widetilde{x}_{t+3} + z_{t+1}(\nu,\rho) + z_{t+2}(\nu,\rho). \tag{C.9}$$

*For any* $n \geq 3$,

$$p_t(n-i,\nu,\rho) \simeq y_t(\nu,\rho)\, \exp\left[\ln\left(\frac{\Lambda_{t+n-i}}{\Lambda_t}\right)\right] \exp\left[\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, \widetilde{x}_t\right]\, E_t\left[\exp\left\{q_t(n-i,\nu,\rho)\right\}\right], \tag{C.10}$$

*where*

$$q_t(n-i,\nu,\rho) = \left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t + \frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\right)\widetilde{x}_{t+1} + \left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\left(1+\frac{1}{\phi}\right)\right)\left(\sum_{l=2}^{n-i-1} x_{t+l}\right) + \left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\, c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\right) x_{t+n-i} + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\, c_{t+1}\, x_{t+n-i+1} + \sum_{l=1}^{n-i} z_{t+l}(\nu,\rho). \tag{C.11}$$

*Proof.* It follows from the log-linear approximation described in Appendix B. □

### The one period assets

Using Proposition App. 5 we can write the return of a one period asset as

$$R_{t+1}(1,\nu,\rho) = \frac{y_{t+1}(\nu,\rho)}{p_t(1,\nu,\rho)} = \frac{\exp\left[z_{t+1}(\nu,\rho) - \frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\,\widetilde{x}_t\right]}{E_t\left[\exp(q_t(1,\nu,\rho))\right]\exp\left(\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right)} \tag{C.12}$$

where

$$E_t[\exp(q_t(1,\nu,\rho))] = \exp\left[\overline{x} + \left(\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t\right)^2 + \left(\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2 + \nu\left(\nu + 2\,\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t\right) + 2\,\rho\left(\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \nu\right)\right)\frac{\sigma^2}{2}\right] \tag{C.13}$$

Taking the unconditional expectation,

$$E\left[R_{t+1}(1,\nu,\rho)\right] \simeq \exp\left\{-\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \left(\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t\right)^2 + \left(\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2\left(\frac{\phi^2-1}{\phi^2}\right)\right)\frac{\sigma^2}{2}\right\} \times \exp\left\{-\left[\nu\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t\right) + \rho\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t\right)\left(\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right) + \nu\rho\left(\frac{\Lambda_{t,\,t+1}}{\Lambda_t}, c_{t+1}\left(\frac{\phi+1}{\phi}\right)\right)\right]\sigma^2\right\}. \tag{C.14}$$

Finally, rearranging terms, and using (2.12) and (2.6) expression (3.8) follows. To calculate the second moment, note that $Var\left[R_{t+1}(1,\nu,\rho)\right] = E\left[R_{t+1}(1,\nu,\rho)^2\right] - E\left[R_{t+1}(1,\nu,\rho)\right]^2$. Some algebra gives

$$Var[R_{t+1}(1,\nu,\rho)] \simeq \left[\left(\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2 + \nu\left(\nu - 2\rho\left(\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\right)\right]\sigma^2. \tag{C.15}$$

Finally, using (2.12) and (2.6) expression (3.9) follows.

### The $n$-period assets

Using the definition (3.7), and (C.10) we have

$$R_{t+1}(n-i,\nu,\rho) = \exp\left[-\ln\left(\frac{\Lambda_{t+n-i}}{\Lambda_t}\right) + z_{t+1}(\nu,\rho) + \frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}(x_{t+1}-x_t)\right]\frac{E_{t+1}[\exp(q_{t+1}(n-i-1,\nu,\rho))]}{E_t[\exp(q_t(n-i,\nu,\rho))]} \tag{C.16}$$

It can be checked that for $n \geq 3$,

$$E_t\left[\exp(q_t(n,\nu,\rho))\right] = \frac{\exp(b)}{\exp(a)^{n-2}}\,\Gamma(n,\nu,\rho) \tag{C.17}$$

where

$$\begin{aligned} b = 2\overline{x} + \left[\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2 + \left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2\right]\tfrac{\sigma^2}{2} + \\ \left[\left(\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)^2 + 2\nu\left(\nu + 2\tfrac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)\right]\tfrac{\sigma^2}{2} \end{aligned} \tag{C.18}$$

$$a = -\left[\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\frac{1}{\phi}\right)\right)^2 + \nu\left[\nu + 2\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}\,c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\frac{1}{\phi}\right)\right)\right]\right]\frac{\sigma^2}{2} \tag{C.19}$$

and $\Gamma(n,\nu,\rho)$ is a complicate function of cross-correlation terms,

$$\begin{aligned} &\Gamma(n,\nu,\rho) = \exp\left\{\left[\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)\sum_{i=1}^{n-2}\rho^i + \right.\right. \\ &\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\rho^{n-1} + 2\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)^2\sum_{j=0}^{n-3}\textstyle\sum_{i=1}^{n-j-2}\rho^i + \\ &\left(\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\rho^n + \left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)\sum_{i=1}^{n-2}\rho^i \\ &\left.\left.\left(\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)\rho\sum_{i=1}^{n-2}\rho^i + \left(\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\rho\right]\sigma^2\right\}\times \\ &\exp\left\{\left[\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\textstyle\sum_{i=1}^{n-1}\rho^i + 2\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\left(1+\tfrac{1}{\phi}\right)\right)\textstyle\sum_{j=0}^{n-3}\sum_{i=1}^{n-j-2}\rho^i\right. \\ &\left.\left(\tfrac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\textstyle\sum_{i=1}^{n-1}\rho^i + \left(\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}\,c_{t+1}\right)\sum_{i=1}^{n}\rho^i\right]\nu\,\sigma^2\right\}. \end{aligned} \tag{C.20}$$

We define,

$$\Omega(\infty,\nu,\rho) = \lim_{n\to\infty}\frac{\Gamma(n-1,\nu,\rho)}{\Gamma(n,\nu,\rho)} \tag{C.21}$$

and, after some algebra, we obtain

$$\Omega(\infty,\nu,\rho) = \exp\left[-2\frac{\rho}{1-\rho}\left(\frac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}c_{t+1}\left(1+\frac{1}{\phi}\right)\right)\left[\frac{\Lambda_{t,\,t}}{\Lambda_t}c_t + \frac{\Lambda_{t,\,t+1}}{\Lambda_t}c_{t+1}\left(1+\frac{1}{\phi}\right) - \nu\right]\sigma^2\right] \tag{C.22}$$

Then, taking the limit when $n \to \infty$,

$$\lim_{n\to\infty} \frac{E_{t+1}[\exp\{q_{t+1}(n-i-1,\nu,\rho)\}]}{E_t[\exp\{q_t(n-i,\nu,\rho)\}]} = \exp(a)\,\Omega(\infty,\nu,\rho), \tag{C.23}$$

we can write the interest rate on a infinite period security as,

$$R_{t+1}(\infty,\nu,\rho) = \exp\left[-\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) + z_{t+1}(\nu,\rho) + \frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}(\widetilde{x}_{t+1} - \widetilde{x}_t)\right]\exp(a)\,\Omega(\infty,\nu,\rho). \tag{C.24}$$

Taking the unconditional expectation, and using (A.10) and (2.6) we can write,

$$\begin{aligned} &E\left[\exp\left[z_{t+1}(\nu,\rho) + \tfrac{1}{\phi}\tfrac{\Lambda_{t,\,t+1}}{\Lambda_t}(\widetilde{x}_{t+1} - \widetilde{x}_t)\right]\right] \simeq \\ &\exp\left[\overline{x} + \left[\tfrac{\nu^2}{2} + \tfrac{1}{\phi^2}\left(AP - RRA_c\right)^2 + \tfrac{\nu}{\phi}(AP - RRA_c) - \right.\right. \\ &\left.\left.\tfrac{\rho}{\phi}\left(AP - RRA_c\right)\left(\tfrac{1}{\phi}\left(AP - RRA_c\right) + \nu\right)\right]\sigma^2\right], \end{aligned} \tag{C.25}$$

$$\begin{aligned} E\left[\exp(a)\right] \simeq \exp\Bigg[ -\overline{x} - \Bigg[ &\left(\left(AP - RRA_c\right)\left(1 + \frac{1}{\phi}\right) - AP\right)^2 \\ &+ \nu\left(\nu + 2\left(\left(AP - RRA_c\right)\left(1 + \frac{1}{\phi}\right) - AP\right)\right)\Bigg]\frac{\sigma^2}{2}\Bigg]. \end{aligned} \tag{C.26}$$

The formula for $E[R_{t+1}(\infty,\nu,\rho)]$ follows after rearranging terms. To obtain the second moment, note that $Var\left[R_{t+1}(\infty,\nu,\rho)\right] = E\left[R_{t+1}(\infty,\nu,\rho)^2\right] - E\left[R_{t+1}(\infty,\nu,\rho)\right]^2$. After some algebra, we get

$$Var[R_{t+1}(\infty,\nu,\rho)] \simeq \left[2\left(\frac{1}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}c_{t+1}\right)^2 + \nu\left(\nu + \frac{2(1-\rho)}{\phi}\frac{\Lambda_{t,\,t+1}}{\Lambda_t}c_{t+1}\right)\right]\sigma^2. \tag{C.27}$$

Using (A.10) and (2.6) again, the formula for $Var[R_{t+1}(\infty,\nu,\rho)]$ follows.

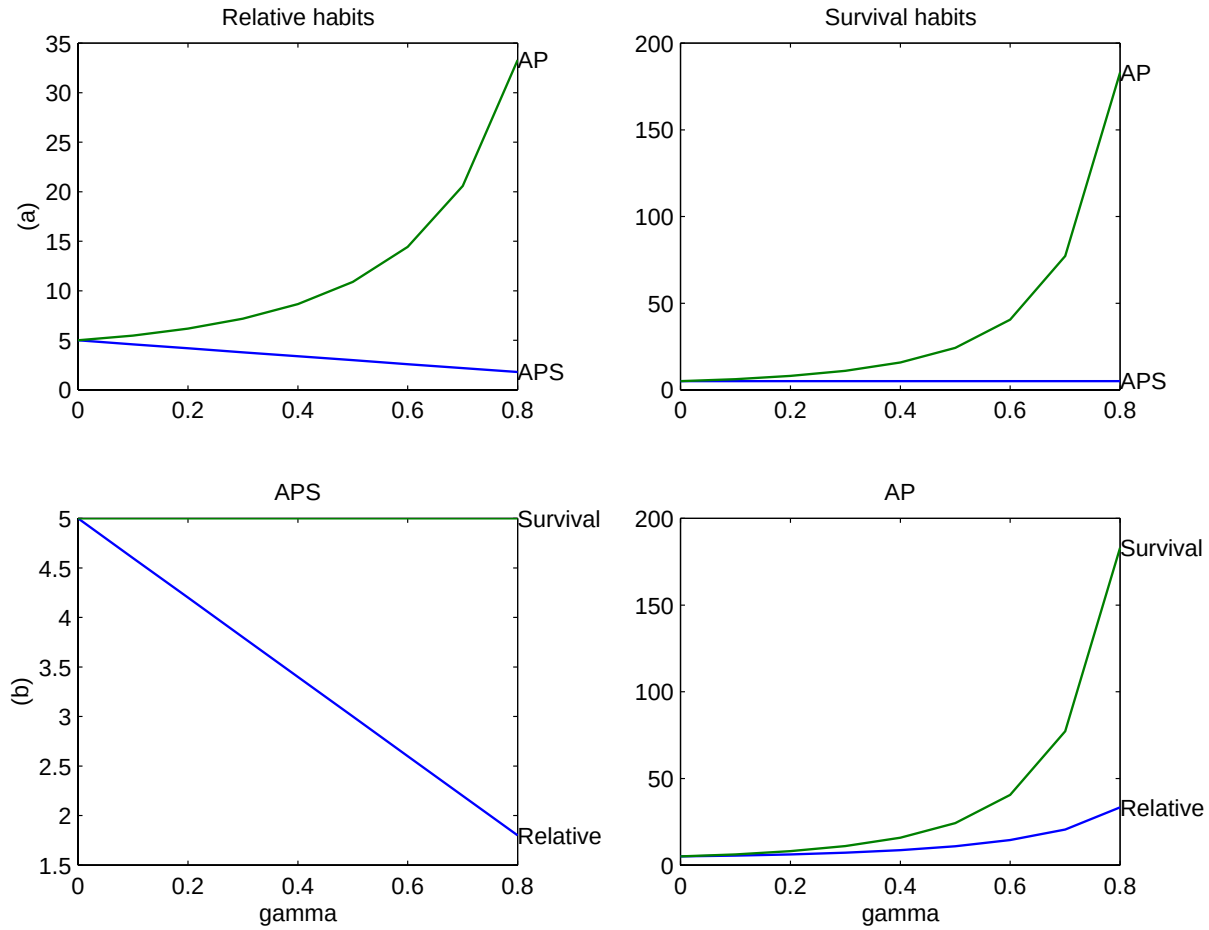


Figure 1: $AP$: inverse of the $IES$. $APS$: Inverse of the $IES$ for a permanent change in the consumption growth rate. $\beta = 1$, $\tau = 5$, $\eta = 1.0045$.

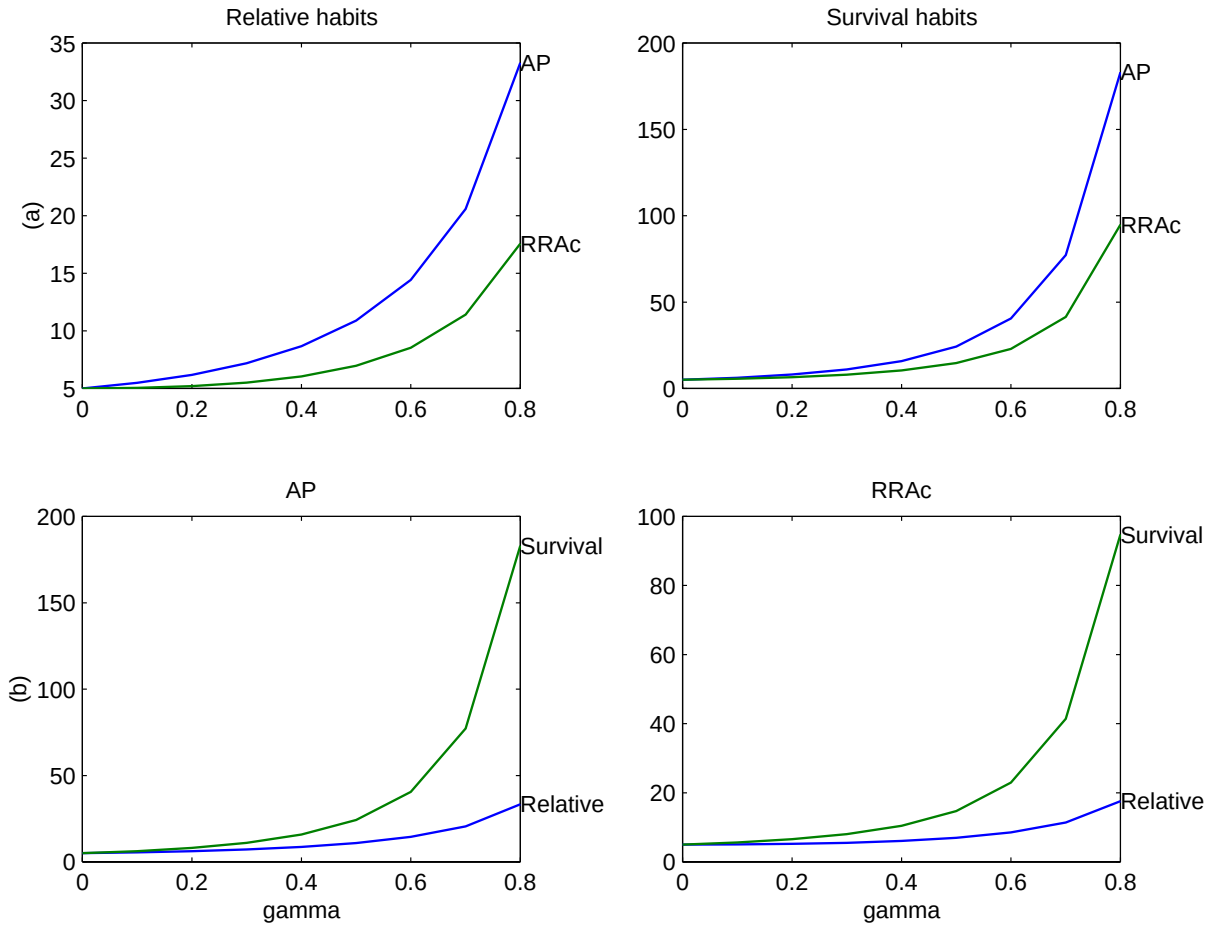


Figure 2: $AP$: Arrow-Pratt coefficient. $RRA_c$: risk aversion in consumption. $\beta = 1$, $\tau = 5$, $\eta = 1.0045$.

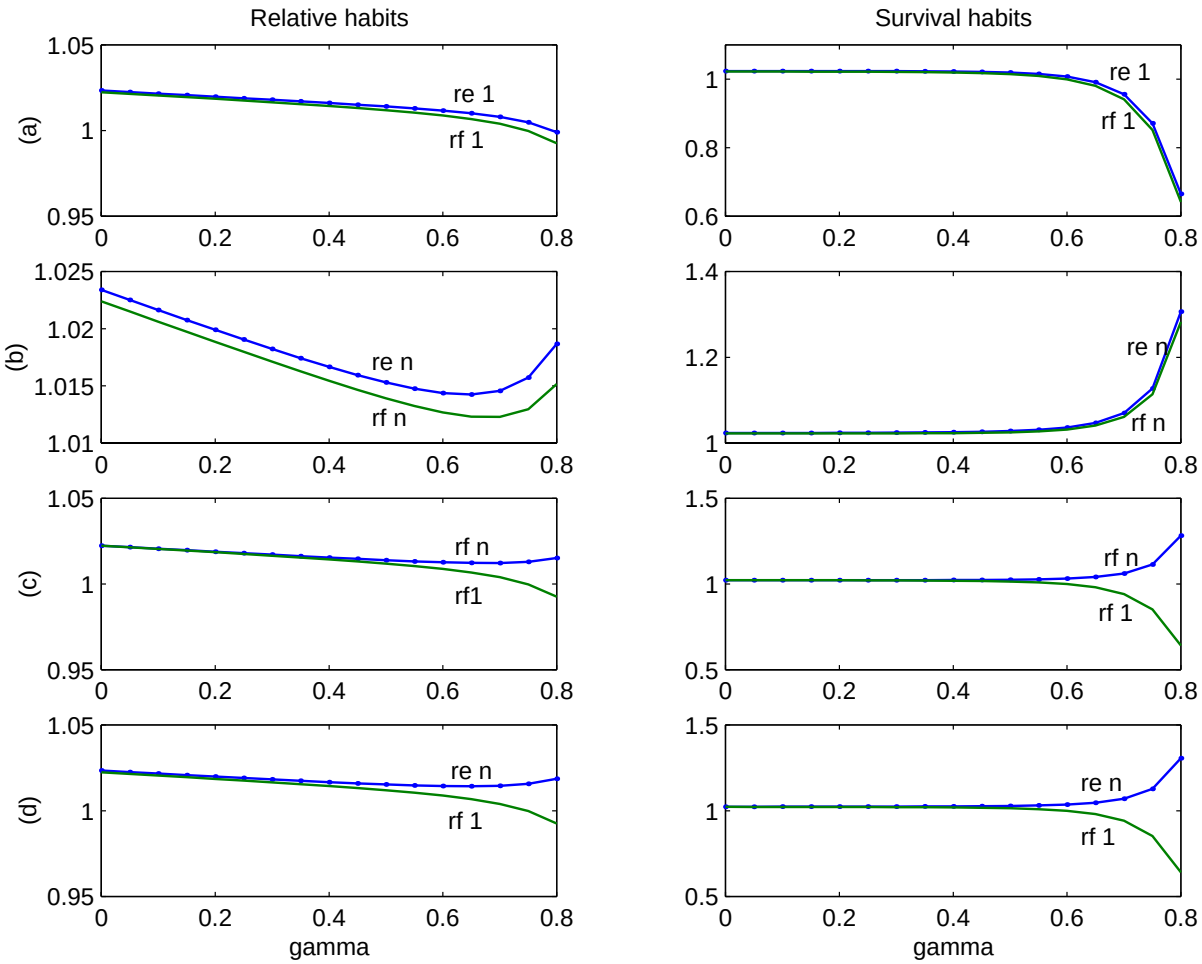


Figure 3: Expected return for various values of $\gamma$. $rf_1$: one period risk free asset, $re_1$: one period risky asset, $rf_n$: long term risk free asset, $re_n$: long term risky asset. $\beta = 1$, $\tau = 5$, $\overline{x} = 0.005$, $\rho = 0$, $\sigma = 0.0053$, $\nu = 7$.

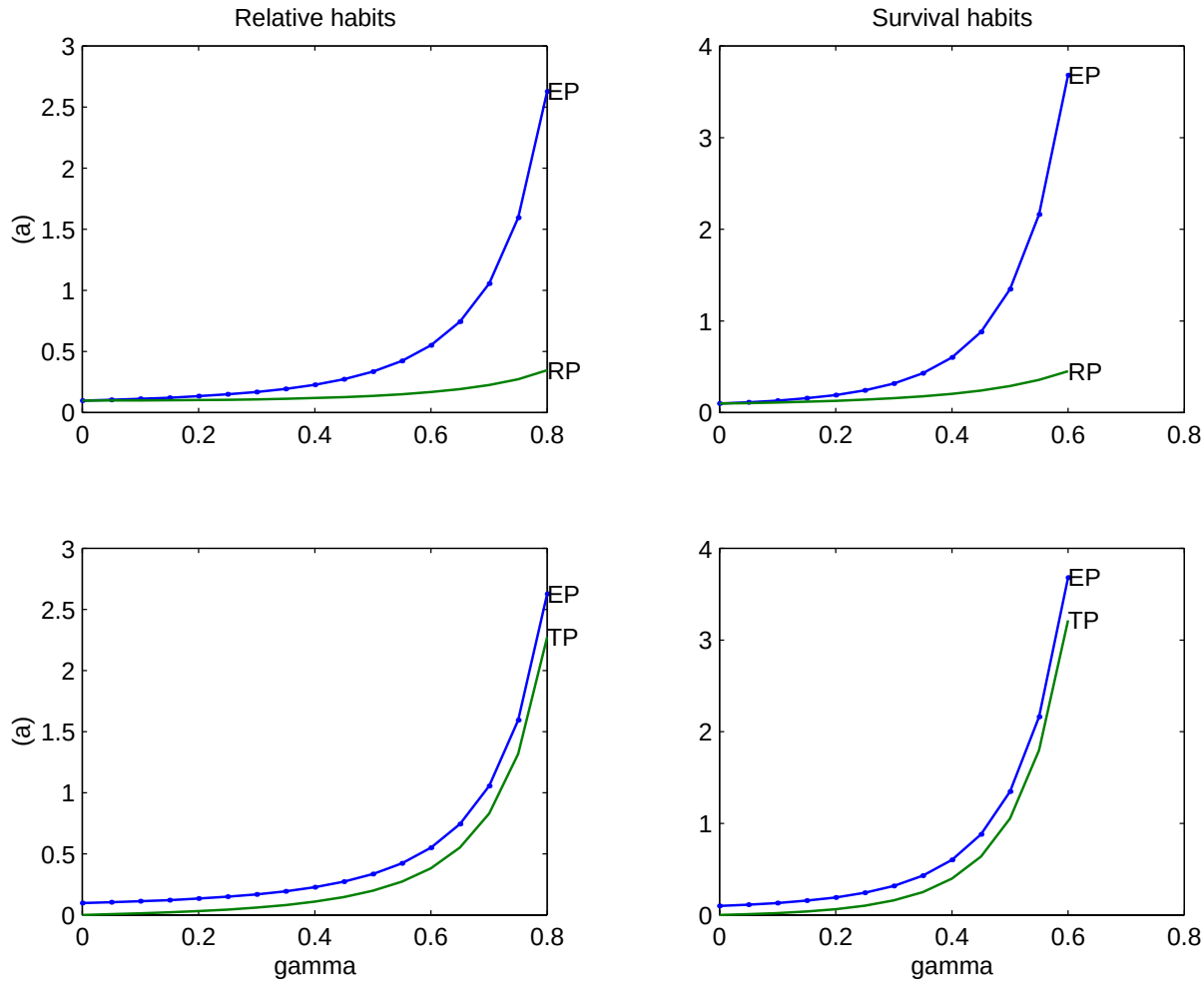


Figure 4: Equity premium, risk premium and term premium for various values of $\gamma$, $\beta = 1$, $\tau = 5$, $\overline{x} = 0.005$, $\rho = 0$, $\sigma = 0.0053$, $\nu = 7$.

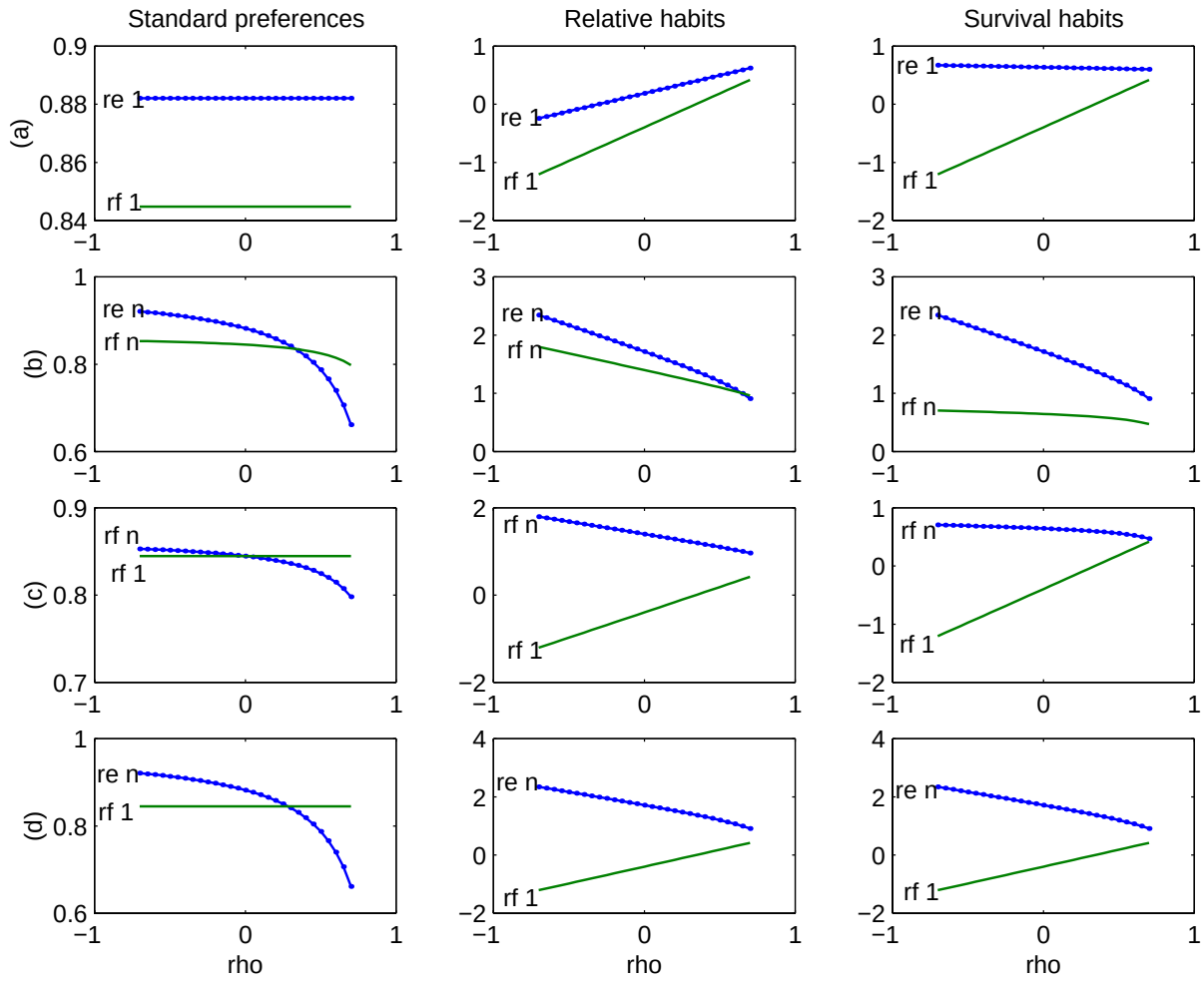


Figure 5: Expected return for various values of $\rho$. $rf_1$: one period risk free asset, $re_1$: one period risky asset, $rf_n$: long term risk free asset, $re_n$: long term risky asset. $\beta = 1$, $\tau = 5$, $\overline{x} = 0.005$, $\sigma = 0.0053$, $\nu = 7$. We assume $\gamma = 0.777$ for relative habits and $\gamma = 0.542$ for survival habits.

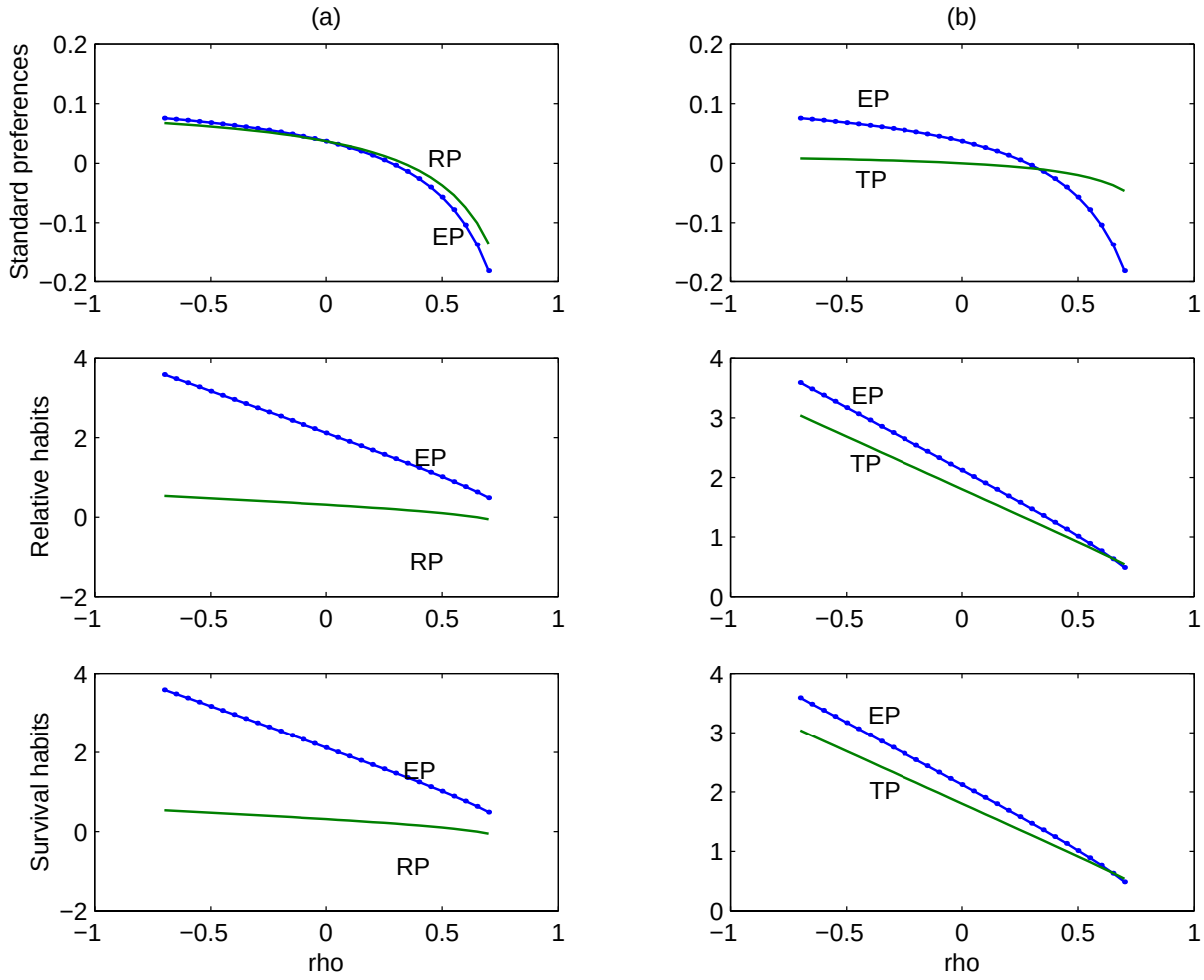


Figure 6: Equity premium, risk premium and term premium for various values of $\rho$, $\beta = 1$, $\tau = 5$, $\overline{x} = 0.005$, $\sigma = 0.0053$, $\nu = 7$. We assume $\gamma = 0.777$ for relative habits and $\gamma = 0.542$ for survival habits.

Table 1: EP, TP and standard deviations (%)

| | $EP$ | $TP/EP$ | $\sigma_{re_n}$ | $\sigma_{rf_n}$ | $\sigma_{rf_1}$ |
|---|---|---|---|---|---|
| | Standard preferences | | | | |
| $\rho$=0.15 | 0.020 | -17.167 | 3.710 | 0.000 | 0.000 |
| $\rho$=-0.15 | 0.053 | 5.601 | 3.710 | 0.000 | 0.000 |
| | Habits | | | | |
| $\rho$=0.15 | 1.800 | 85.455 | 13.047 | 10.476 | 7.407 |
| $\rho$=-0.15 | 1.800 | 82.225 | 11.994 | 8.781 | 6.209 |
| Data | 1.800 | 11.100 | 7.500 | 4.800 | 0.700 |

Asset returns are in percentage and quarterly terms. We have assumed $\sigma = 0.0053$, $\beta = 1$, $\overline{x} = 0.005$, $\nu = 7$. For relative habits $\tau = 5$ and $\gamma = 0.7799$ when $\rho = 0.15$ and $\gamma = 0.7480$ when $\rho = -0.15$. For survival habits $\tau = 1.88$ and $\gamma = 0.6986$ when $\rho = 0.15$ and $\tau = 2.01$ and $\gamma = 0.6671$ when $\rho = -0.15$.

Table 2: Term premium as percentage of the equity premium

| $\rho$ | Relative | Survival | | $TP/EP$ |
|---|---|---|---|---|
| | $\gamma$ | $\tau$ | $\gamma$ | |
| -0.600 | 0.709 | 2.163 | 0.629 | 78.689 |
| -0.450 | 0.721 | 2.115 | 0.641 | 79.749 |
| -0.300 | 0.734 | 2.064 | 0.654 | 80.919 |
| -0.150 | 0.748 | 2.008 | 0.667 | 82.228 |
| 0.000 | 0.763 | 1.947 | 0.682 | 83.717 |
| 0.150 | 0.780 | 1.880 | 0.699 | 85.455 |
| 0.300 | 0.799 | 1.806 | 0.717 | 87.555 |
| 0.450 | 0.820 | 1.720 | 0.739 | 90.240 |
| 0.600 | 0.846 | 1.618 | 0.765 | 94.010 |

Asset returns are in percentage and quarterly terms. We have assumed $\sigma = 0.0053$, $\beta = 1$, $\overline{x} = 0.005$, $\nu = 7$. $\tau = 5$ for relative habits.

Table 3: Term premium as percentage of the equity premium

| $\nu$ | Relative | Survival | | $TP/EP$ |
|---|---|---|---|---|
| | $\gamma$ | $\tau$ | $\gamma$ | |
| 1.000 | 0.792 | 1.833 | 0.711 | 97.787 |
| 7.000 | 0.780 | 1.880 | 0.699 | 85.455 |
| 11.000 | 0.772 | 1.914 | 0.690 | 78.083 |
| 50.000 | 0.678 | 2.290 | 0.599 | 32.567 |
| 100.000 | 0.529 | 2.885 | 0.459 | 11.034 |

Asset returns are in percentage and quarterly terms. We have assumed $\sigma = 0.0053$, $\beta = 1$, $\overline{x} = 0.005$, $\rho = 0.15$. $\tau = 5$ for relative habits.

Table 4: EP, RP and standard deviations (%)

| | $EP$ | $TP/EP$ | $\sigma_{RE}$ | $\sigma_{RLB}$ | $\sigma_{RF}$ |
|---|---|---|---|---|---|
| | Standard securities | | | | |
| Relative habits | 1.80 | 77.22 | 18.5 | 15.7 | 10.0 |
| Survival habits | 1.80 | 76.67 | 18.4 | 15.5 | 10.0 |

Asset returns are in percentage and quarterly terms. We have assumed $\rho = 0.15$, $\sigma = 0.0053$, $\beta = 1$, $\overline{x} = 0.005$, $\nu = 7$. For relative habits $\gamma = 0.868$ and $\tau = 5$. For survival habits $\gamma = 0.632$ and $\tau = 1.528$.