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WP–AD 98–06

Editor: Insituto Valenciano de Investigaciones Económicas
First Edition; April 1998
ISBN: 84–482–1767–5
Depósito Legal: V–1170–1998

IVIE working–papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

*We thank J. Alcalde, P. Amorós, S. Chattopadhyay, L. Corchón, E. Dierker, J. Massó, F. Mertens, J. Naeve, J. Peris, J. Silvestre, M. Socorro and W. Trockel for their comments. Financial support from the Instituto Valenciano de Investigaciones Económicas and from the Ministry of Education under project no. PB 94–1504 is gratefully acknowledged.

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ABSTRACT

In economies with public goods, and agents with quasi–linear preferences, we give a characterization of the welfare egalitarian correspondence in terms of three axioms: Pareto optimality, symmetry and solidarity. This last property requires that an improvement in the ability to exploit the public goods of some of the agents should not decrease the welfare of any of them.

KEYWORDS: Public Goods; Welfare Egalitarian.

1 Introduction

The welfarist approach to Social Choice Theory proposes to allocate resources in such a way that all the information not contained in the the utility possibility set is ignored. This approach is exemplified in Bargaining Theory in which all the relevant information about the agents is summarized in the utility possibility set (the outcomes available to the agents when they cooperate) and a threat point (representing the payoff in case of disagreement). In this context, the set of utility profiles is associated, in a way not determined by the theory, with the set of alternatives available to the agents.

Bargaining theory studies utility allocation mechanisms defined on this abstract setting. Characterizing those mechanisms by a minimal set of properties (or axioms) is at the core of the theory. These axioms summarize the properties of fairness of the mechanism determined by them and provide the foundation for proposing it.

This line of research has been questioned by J. Roemer ([11, 12, 13]). J. Roemer's critique to Axiomatic Bargaining Theory is based on the observation that much of the relevant economic information is lost when the problem is presented as one of dividing up utility. As a result he points out some potentially severe shortcomings of Axiomatic Bargaining Theory.

Firstly, he provides some examples of genuinely different economic models giving rise to the same utility possibility set and the same threat point. Thus, it is hard to justify the usage of mechanisms which depend only on the information contained in the utility possibility set. Secondly, Roemer argues that some of the axioms commit the mechanism to much more than the intuition motivating them suggests. Furthermore, the intuition provided by some economic environments may not be valid for others. And yet they could correspond to the same situation in the bargaining setting. Finally, empirical results ([19]) suggest that the notions of fairness observed in people are based on more information than just that contained in the attainable utilities.

In [12], Roemer characterizes five classical solutions of Axiomatic Bargaining Theory: the Nash solution, the Kalai-Smorodinsky solution, the egalitarian solution, the family of monotone utility path solutions and the family of proportional solutions. He argues that to reconstruct the standard axiomatic characterizations of Bargaining Theory, one has to consider commodity spaces of unbounded dimension. And it is no longer clear that the economic analogues of the axioms of Bargaining Theory still characterize a solution in more realistic and smaller domains. Thus, he views his work as "...demonstrating the lengths to which one must go to preserve the axiomatic characterization of the standard bargaining mechanisms on economic environments" ([12, p. 32]). He concludes that classical Bargaining Theory is unacceptable as a positive model of the bargaining process as well as a normative model of resource allocation.

The present paper is an attempt to reconcile some of the principles contained in Bargaining Theory with the work of Roemer by showing that E. Kalai's ([3]) characterization of the egalitarian solution can be transplanted from classical Bargaining Theory into (at least) some economically meaningful environments.

We consider a set of agents endowed with preferences on vectors of public goods and a single private good (money), which can be represented by quasi–linear utility functions with constant marginal utility in the private good. There is a common technology which can produce bundles of public goods using the private good as an input. The agents differ in their valuations of the public goods and the issue is to implement a production plan (a bundle of public goods to produce) and a financing scheme for it (a vector of inputs to be provided by the agents).

We adopt the point of view of social choice theory in that we seek a solution determined by some equitable properties. We focus on the three key properties considered by Kalai in his characterization of the egalitarian solution, Pareto efficiency, symmetry and monotonicity. The axiom of Pareto optimality needs no modification in our context. However, the other two properties have to be reinterpreted within the economic situation at hand.

The idea behind the axiom of symmetry is that agents which cannot be distinguished with the information available in the model should be treated equally. In Bargaining theory this means that if the utility possibility set is symmetric, then all the agents should end up with the same utility level. However, here, as in Roemer's example, it will be the case that different economic situations with genuinely different agents correspond to the same utility possibility set. Thus, in the context of our modeling, it seems more appropriate to postulate the following axiom: whenever all the agents have the same preferences, they should all pay the same amount of private good (of course, the level of the public goods enjoyed by the agents is, by definition, the same for all of them).

The monotonicity axiom requires that enlarging the set of alternatives available to the agents should benefit all of them. The well known intuition supporting this principle is that if the pie gets larger, then everybody should benefit (perhaps differently) from it. In the quasi–linear world a bigger pie corresponds to having a larger surplus to share. However, all the agents contribute to the surplus. And it is possible that it becomes larger, because of the greater contributions of some of the agents, even though some others reduce their participation in the common project. Is it fair then to demand axiomatically that all of them benefit? To make monotonicity more palatable we consider the following modification. If some agent raises his valuation of the public goods but the rest do not modify their valuations (so total surplus is now higher and nobody contributes less than before to it), then the payoff of every agent should also increase.

This idea is not entirely new in the literature. It has been used before in [16, 14, 15] in a slightly different form. The reasoning therein is that whenever there is a change in the preferences of some agents, the ones whose preferences remain the same should be affected in the same direction. In this literature, this principle is called solidarity and that is the name we have adopted here as well. Necessary and sufficient conditions for some notion of solidarity are provided in [5], where solidarity means now that if the exogenous parameter changes, then the welfare of every agent moves in the same direction. These authors show that a solution satisfies solidarity if and only if it coincides with some monotone welfare path. This concept is also related with the notions of population solidarity ([18]) and skill solidarity ([2]).

The main result of our work is to show that on the set of economies with quasi–linear preferences, the three axioms we have just discussed determine the same rule as in classical Bargaining Theory. That is the welfare egalitarian correspondence which splits the surplus equally among the agents. In contrast with Roemer's findings, we restrict ourselves to standard economic domains with, for example, a fixed commodity space and make use of a reduced number of axioms. Thus, our findings provide (as in [11, 12, 13]) an alternative foundation of welfarism based on economic principles.

One should make clear at this point that our model makes use of interpersonal comparison of utilities. The extent to which this is legitimate or meaningful is debatable. Nevertheless, some comparison of the welfare of the agents seems hard to avoid in a theory which is about equity and fairness. Especially, a notion of equality of welfare presumes this type of comparisons. Our result holds for domains of preferences which are much more general than the quasi–linear ones. But, since it is not within the aim of this paper to add to the interpersonal comparison of utilities controversy, we have restricted our results to the case of quasi–linear preferences. In this world, individual utilities are measured in terms of the private good in an objective way and it is meaningful to compare them.

There is a rather extensive literature studying the egalitarian solution in addition to Kalai's characterization. R. B. Myerson ([9]) uses a condition on decomposability with respect to sequences of bargaining problems and enough invariance under ordinal utility transformations to determine a solution which equalizes the gains of the agents in some ordinal utility space. W. Thomson ([18]) provides another characterization in terms of four axioms, monotonicity with respect to changes in the number of agents, weak Pareto optimality, independence of irrelevant alternatives and continuity. Moulin ([6]) considers social choice functions which share equally the surplus above a reference utility level. In this model, there is no cost function so the private good is only used to make monetary compensations among the agents. He proposes two axioms to study these egalitarian social choice functions, no advantageous reallocation and no disposal of utilities.

In the context of ordinal preferences, Y. Sprumont ([15]) has axiomatized the welfare egalitarian solution by means of solidarity with respect to changes in the feasibility constraints and preferences. The key axiom is a generalized form of solidarity. Whenever a change occurs in the feasibility correspondence and/or some of the agent's preferences, all the agents whose preferences have not changed are similarly affected. This author shows that a choice rule satisfies his axiom of solidarity if and only if it equalizes welfare with respect to some complete preordering on preferences and indifference classes of outcomes. In contrast, the axiom of solidarity as presented in the present work only applies to a restricted class of changes in the preferences of the agents. In addition, we define welfare egalitarianism in terms of the preferences of the agents themselves and not with respect to some abstract preordering on the space of preferences and indifference classes of allocations as in [15]. Furthermore, we also insist that the modeling and the axioms we consider stem directly from the fundamentals of the economy.

In a related work, H. Moulin and J. Roemer ([7]) study the justification for the existence of inequalities in a model with publicly owned technology and two identical agents whose skills are privately owned. They propose three properties, in addition to efficiency, which reflect the public and private property rights of the agents. They also find that the produced goods must be distributed in a way which equalizes the welfare of the agents. Our model can also be regarded along similar lines. The role played by the "private ownership of skill" in [7] is played in our model by the private contribution of each player to the overall surplus which is to be distributed among the players. They also present their work as one in which the outcome is not based solely on how it treats utility. Similarly, we consider axioms reflecting ethical concerns akin to those in Bargaining Theory. But, as in [7], our assumptions and modeling are on the basic data of the economy and have an interpretation within the economic scenario considered.

Perhaps the reason why Roemer's observations apply to Bargaining Theory is because the latter is a rather ambitious theory with a scope which is unrealistically universal. After all, its proposals apply, in principle, to every conceivable conflict. Yet, at least some of its ideas and intuitions are applicable if one is willing to work at a smaller scale and incorporate the relevant economic considerations into the model. Of course, the price one has to pay is a loss in the universality of the solution, which might now depend on the family of economic models of interest (see section 3).

The rest of the paper is organized as follows. In section 2, the formal model with public goods and the main results are discussed. In section 3, we modify the previous setting and consider the case of private goods to exemplify how the justification for the axioms and the solution obtained might depend on the concrete economic features of the model under study. We conclude in section 4 by clarifying the relationship between the model proposed here and the ones studied in Bargaining Theory.

2 The model

Given two vectors x, z in some Euclidean space $\mathbf{R}^p_+$, the notation $x \geq z$ (resp. $x >> z$) means that $x_i \geq z_i$ (resp. $x_i > z_i$) for every $i = 1, \ldots, p$. We write $x \not> z$ to indicate that $x_i \leq z_i$ for some $i = 1, \ldots, p$. Finally, $x > z$ means that $x \geq z$ and $x \neq z$.

The space of public goods is $Y = \mathbf{R}^m_+ = \{y \in \mathbf{R}^m : y \geq 0\}$. The technology to produce them is jointly owned by the agents and it is described by a function $c : Y \rightarrow \mathbf{R}_+$ measuring the cost of producing each bundle of public goods in terms of the single private good of the economy. Throughout this paper we will consider a fixed cost function c satisfying the following hypotheses.

Assumption 2.1 *The mapping c is lower semicontinuous, non decreasing and satisfies $c(0) = 0$ and $\lim_{||y|| \rightarrow \infty} c(y) = +\infty$.*

Recall that a mapping $c : Y \rightarrow \mathbf{R}$ is lower semicontinuous if for each $z \in Y$ we have $f(z) \leq \liminf_{y \rightarrow z} f(y)$. A lower semicontinuous function is bounded below on every compact set and it attains its minimum value. Assumption 2.1 allows for technologies with jumps, so initial fixed costs are not ruled out in the model.

We let $N = \{1, 2 \ldots, n\}$ denote the set of agents. The set $X_i = \mathbf{R}$ represents the possible payments, in terms of the private good, made by agent $i \in N$. We let $X = X_1 \times \cdots \times X_n$. An allocation $(z; t) = (z; t_1, \ldots, t_n) \in Y \times X$ is *feasible* whenever $c(z) \leq \sum_{i=1}^n t_i$. The preference relation of agent $i = 1, \ldots, n$ is represented by a quasi–linear utility function $u_{\pi_i}(y; t) = \pi_i(y) - t_i$, with $(y; t) = (y; t_1, \ldots, t_n) \in Y \times X$ and $\pi_i : Y \rightarrow \mathbf{R}$, which represents the utility obtained by agent $i \in N$ when the bundle $y \in Y$ of public goods is implemented and he has to contribute the amount t_i towards its financing. For convenience, we write $u_{\pi_i}(y; t)$ as depending on $t = (t_1, \ldots, t_n) \in X$ even though agent $i \in N$ is only interested in the consumption of the public goods and of his private good so $u_{\pi_i}(y; t)$ depends only on $y \in Y$ and $t_i \in X_i$. The following assumption is made on the preferences of the agents.

Assumption 2.2 *For each $i = 1, \ldots, n$, $\pi_i : Y \rightarrow \mathbf{R}_+$ is a continuous, non decreasing function satisfying $\pi_i(0) = 0$ and*

$$\limsup_{||y|| \rightarrow \infty} \frac{\pi_i(y)}{c(y)} = 0.$$

There are several interpretations for the mappings $\pi_1, \ldots \pi_n$ and we do not stick necessarily to any of them. On the one hand, the amount $\pi_i(y)$ represents the valuation that agent $i \in N$ has of the public goods y. One can also think of it as representing his private technology to exploit those public goods or the benefit (in terms of the private good) he would obtain if he could enjoy those public goods for free. If the status quo is no consumption of any of the public goods, then $\pi_i(y)$ is also the maximum amount of his private good that he is willing to relinquish for the consumption of the bundle y.

In addition, $\pi_i(y) - t_i$ is the net benefit agent $i \in N$ obtains when he has to contribute t_i units of his private good in order to enjoy the bundle y of public goods. Thus, $\pi_i(y) - t_i$ is also the net contribution that agent $i \in N$ makes towards the net surplus $\pi_1(y) + \cdots \pi_n(y) - c(y)$ that the society obtains from the consumption of the bundle $y \in Y$ of public goods.

We refer to a vector of utility functions $\pi = (\pi_1, \ldots, \pi_n)$ as a profile of utilities and we use the following notation $u_\pi(y;t) = (u_{\pi_1}(y;t), \ldots, u_{\pi_n}(y;t)) = (\pi_1(y) - t_1, \ldots, \pi_n(y) - t_n)$. The vector of utilities resulting from $\pi = (\pi_1, \ldots, \pi_n) \in E$ when π_i is replaced by the new utility function ν_i is denoted by $(\pi_{-i}, \nu_i) = (\pi_1, \ldots, \pi_{i-1}, \nu_i, \pi_{i+1}, \ldots, \pi_n)$. Given two utility profiles π and ν defined on Y we say that $\pi \geq \nu$ whenever $\pi(y) \geq \nu(y)$ for every $y \in Y$.

The technology is fixed and an economy is defined by a vector of utility profiles $\pi = (\pi_1, \ldots, \pi_n)$ satisfying 2.2. We let E denote the set of such economies. A *mechanism* is a function $R : E \to Y \times X$ which assigns to every economy $\pi \in E$ a feasible allocation $R(\pi)$. We denote by $\mathrm{P}(\pi)$ (resp. $\mathrm{P}^*(\pi)$) the set of *Pareto optimal* (resp. weakly Pareto optimal) allocations consisting of those feasible allocations $(y;t) \in Y \times X$ for which if $u_\pi(y;t) < u_\pi(z;r)$ (resp. $u_\pi(y;t) << u_\pi(z;r)$) for some other allocation $(z;r) \in Y \times X$, then $(z;r)$ is not feasible.

The problem faced by the agents is to find "the optimal" bundle of public goods and a fair share of its cost. According to the normative approach a mechanism which is "acceptable" should satisfy certain equitable requirements. The normative principles which we study in this work are described by the following three properties.

Axioms 2.3 *For every $\pi \in E$,*

(i) $R(\pi) \in \mathrm{P}(\pi)$.

(ii) If $\pi_1 = \cdots = \pi_n$ *then,* $u_{\pi_1}(R(\pi)) = \cdots = u_{\pi_n}(R(\pi))$.

(iii) If $\pi \geq \nu$, *then* $u_\pi(R(\pi)) \geq u_\nu(R(\nu))$.

Properties (i) and (ii) reflect, respectively, the notions of Pareto efficiency and symmetry. They are standard in the literature, so we will make no further comment about them. The novelty here lies on axiom 2.3 (iii). It is equivalent to the statement that if $\pi_{i_0} \geq \nu_{i_0}$ for some $i_0 \in N$ then $u_\pi(R(\pi)) \geq u_\nu(R(\nu))$ for the vectors of mappings $\pi = (\pi_1, \ldots, \pi_n)$ and $\nu = (\pi_{-i_0}, \nu_{i_0})$. Thus, it reduces to comparisons involving only changes of preferences in one agent.

One possible interpretation of this axiom is that if, after reaching an agreement, one of the agents finds out that he can increase the benefit he obtains from the public goods, then he is entitled to a larger share of the surplus (since he contributes a greater amount to it) as long as this does not affect negatively the others. It is in this sense that it is called a solidarity axiom. An increase in the skill of one agent benefits the whole society, or at least does not hurt the other members. In particular, if one interprets the mappings $\pi_1, \ldots, \pi_n$ as the private technology used by the agents to exploit the public goods then, no agent will oppose technological advancement by others.

Thus, the axiom of solidarity is akin to the monotonicity axiom of Axiomatic Bargaining Theory. One may justify it on the basis that, since the technology to produce the public goods is jointly owned by all the agents, they are forced to cooperate in agreeing both on a single bundle of public goods and a financing plan for it. We postpone further discussion of this point to the next section.

We define next the *welfare egalitarian* correspondence $W : E \twoheadrightarrow Y \times X$. For each $\pi \in E$, the set $W(\pi)$ consists of those Pareto optimal allocations $(y; t) \in Y \times X$ satisfying $u_{\pi_1}(y; t) = \cdots = u_{\pi_n}(y; t)$. Recall that $\limsup_{||y|| \to \infty} \pi_i(y)/c(y) = 0$ for every $i = 1, \ldots, n$, and hence, there is (at least) one solution, say $\bar{y}$, to the maximization problem $\max\{\sum_{i=1}^n \pi_i(y) - c(y) : y \in Y\}$. Since, agents have quasi–linear preferences, the set

$$\{(v_1, \ldots, v_n) \in \mathbf{R}^n : v_1 + \cdots + v_n = \sum_{i=1}^n \pi_i(\bar{y}) - c(\bar{y})\}$$

is the set of utilities obtained by means of Pareto optimal allocations. The intersection of this set with the diagonal in $\mathbf{R}^n$ corresponds to the utility obtained by those allocations in $W(\pi)$. Therefore, $\emptyset \neq W(\pi) \subset \mathrm{P}(\pi)$ for every $\pi \in E$. This is in contrast with the framework of the Axiomatic Bargaining literature, where welfare egalitarian allocations are, in general, only weakly Pareto optimal, but not necessarily optimal in the strong sense. Due to the presence of quasi–linear preferences and the possibility of monetary transfers, the welfare egalitarian mechanism is (strongly) Pareto efficient in our setting.

Even though $W(\pi)$ might contain several allocations, the agents are indifferent among the allocations in $W(\pi)$. Clearly any selection from W satisfies axioms 2.3. The content of the next result is that this is essentially the only way to obtain a mechanism satisfying those properties.

Theorem 2.4 *A mechanism R satisfies axioms 2.3 if and only if $R(\pi) \in W(\pi)$ for every $\pi \in E$.*

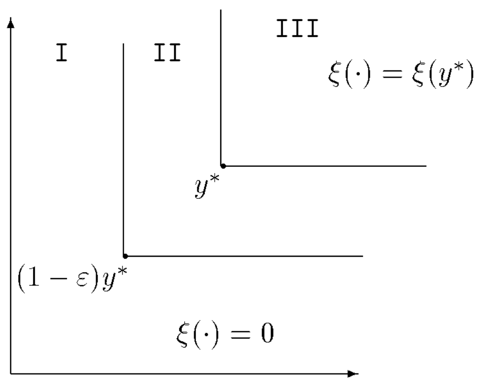

Figure 1:

Theorem 2.4 also admits a negative interpretation. One may ask whether it is possible that increasing the skill that some agents have to profit from the public goods benefits only (or perhaps mostly) those agents responsible for the larger surplus, without hurting the others. Theorem 2.4 shows that this is incompatible with Pareto efficiency and symmetry.

We address now the proof of Theorem 2.4. We will show that every mechanism satisfying 2.3 has to be a selection of the welfare egalitarian correspondence.

Proof of Theorem 2.4: Let R be a mechanism satisfying 2.3 and let $R(\pi) = (\bar{y}; \bar{r}) \in Y \times X$. Since $R(\pi)$ is Pareto optimal, then $\bar{y}$ is a solution to $\max_{y\in Y} \sum_{i=1}^n \pi_i(y) - c(y)$ and $c(\bar{y}) = \sum_{i=1}^n \bar{r}_i$. The proof proceeds in three steps.

Step 1: Choose another profile of utilities $u_\nu(y;t) = \nu(y) - t$ such that $\nu \leq \pi$ and $\nu_i(z) = \pi_i(\bar{y})$ is constant for every $z \geq \bar{y}$. Then, $u_\pi(R(\pi)) = u_\nu(R(\pi))$ and by 2.3 (iii) $u_\nu(R(\nu)) \leq u_\pi(R(\pi)) = u_\nu(R(\pi))$. Since $R(\nu) \in \mathrm{P}(\nu)$ then $u_\pi(R(\pi)) = u_\nu(R(\nu))$.

Step 2: Choose y^* large enough such that $y^* >> \bar{y}$ and $c(y^*) > n \max\{\nu_1(\bar{y}), \ldots, \nu_n(\bar{y})\}$. Let $\varepsilon > 0$ be a real number such that $(1-\varepsilon)y^* >> \bar{y}$ and choose a utility profile $u_\xi(y;t) = \xi(y) - t$ such that the following six conditions hold.

(i) $\xi_1 = \cdots = \xi_n$.

(ii) $\xi(z) = 0$ for every $z \not> (1-\varepsilon)y^*$.

(iii) $\xi(z) = \xi(y^*)$ for every $z \geq y^*$.

(iv) The solution to $\max_{z\in Y} \sum_{i=1}^n \xi_i(z) - c(z)$ is attained at the point y^*.

(v) $\xi(z) > \nu(z)$ for every $z \geq y^*$.

(vi) $\sum_{i=1}^n u_{v_i}(R(\nu)) = \sum_{i=1}^n u_{\xi_i}(R(\xi))$.

We indicate next (see figure 1) how it is possible to construct the functions $\xi_1, \ldots, \xi_n$. First, note that to obtain (iv) one only needs to make the functions $\xi_1 = \cdots = \xi_n$ "steep" enough in the region between $(1-\varepsilon)y^*$ and y^*. It follows from (ii) and (iii) that $R(\xi) = (y^*; r^*)$ for some $r^* \in X$ such that $c(y) = \sum_{i \in N} r_i^*$. Now define $\xi_1(y^*) = \cdots = \xi_n(y^*)$ so that $\sum_{i=1}^n u_{\xi_i}(R(\xi)) = \sum_{i=1}^n \xi_i(y^*) - c(y^*) = \sum_{i=1}^n u_{v_i}(R(\nu)) \geq 0$. In addition, $\xi_1(y^*) + \ldots + \xi_n(y^*) \geq c(y^*) > n \max\{\nu_1(\bar{y}), \ldots, \nu_n(\bar{y})\}$, so (v) also holds. Now (ii) and (iii) define the functions $\xi_1 = \cdots = \xi_n$ on the whole space. Clearly, $u_\xi(R(\xi)) = u_\xi(y^*; r^*))$ and it follows from (i) and axiom 2.3 (ii) that $r_1^* = \cdots = r_n^*$.

Step 3: Take the vector of functions

$$\beta(z) = (\beta_1(z), \ldots, \beta_n(z)) = (\max\{\nu_1(z), \xi_1(z)\}, \ldots, \max\{\nu_n(z), \xi_n(z)\})$$

and the profile of utilities $u_\beta(z;t) = \beta(z) - t$. We also choose $\varepsilon > 0$ in the previous step small enough so that the solution to

$$\left.\begin{array}{ll} \text{Max} & \sum_{i=1}^n \beta_i(y) - c(y) \\ \text{s.t.} & y \geq (1-\varepsilon)y^* \end{array}\right\}$$

is attained at the point y^* (recall that, by (v) in step 2, $\beta(z) = \xi(z)$ if z is close enough to y^*).

The theorem will follow if we can prove that $u_\beta(R(\beta)) = u_\nu(R(\nu)) = u_\xi(R(\xi))$. To see this, note first that by 2.3 (iii) we have that $u_\beta(R(\beta)) \geq u_\nu(R(\nu)$ and $u_\beta(R(\beta)) \geq u_\xi(R(\xi))$. Let $(\hat{y}; \hat{r}) = R(\beta)$. We consider three cases.

(a) Assume first that $\hat{y} \geq y^*$. Then, by (v) we have that $\beta(\hat{y}) = \xi(\hat{y}) \geq \nu(\hat{y})$ and $u_\xi(R(\beta)) = u_\beta(R(\beta)) \geq u_\xi(R(\xi))$. Since $R(\beta)$ is also feasible in the economy u_ξ and $R(\xi) \in \mathrm{P}(\xi)$, then $u_\beta(R(\beta)) = u_\xi(R(\xi))$.

(b) Suppose now that $\hat{y} \not\geq (1-\varepsilon)y^*$. Then from (ii) we see that $\beta(\hat{y}) = \nu(\hat{y}) \geq \xi(\hat{y})$ and $u_\nu(\hat{y}, \hat{r}) = u_\beta(\hat{y}, \hat{r}) \geq u_\nu(R(\nu)) = u_\nu(\bar{y}, \bar{r})$. But, if some inequality is strict, then $R(\nu)$ would not be Pareto optimal. Hence, $u_\beta(R(\beta)) = u_\nu(R(\nu))$.

(c) Otherwise, $\hat{y} \geq (1-\varepsilon)y^*$ and $\hat{y} \not\geq y^*$ so $\hat{y}$ is in region II of figure 1. Then,

$$\begin{aligned} \sum_{i=1}^n u_{\xi_i}(R(\xi)) &= \sum_{i=1}^n \xi_i(y^*) - c(y^*) = \sum_{i=1}^n \beta_i(y^*) - c(y^*) \\ &\geq \sum_{i=1}^n \beta_i(\hat{y}) - c(\hat{y}) = \sum_{i=1}^n u_{\beta_i}(\hat{y}; \hat{r}) \end{aligned}$$

Since, $u_\beta(\hat{y},\hat{r}) \geq u_\xi(R(\xi))$, then $u_\beta(\hat{y},\hat{r}) = u_\xi(R(\xi))$, that is, $u_\beta(R(\beta)) = u_\xi(R(\xi))$.

In either of the three cases, the claim follows from $\sum_{i=1}^n u_{\xi_i}(R(\xi)) = \sum_{i=1}^n u_{\nu_i}(R(\nu))$ and $\beta_i(\hat{y}) = \max\{\xi_i(\hat{y}), \nu_i(\hat{y})\}$ for all $i = 1,\ldots,n$. Thus, $u_\pi(R(\pi)) = u_\xi(R(\xi))$ and Theorem 2.4 follows from (i) in step 2. □

3 Private goods

We argued in section 2 that axiom 2.3 (iii) can be justified in a cooperative setting and it is interpreted as some type of solidarity among the agents. In this section we elaborate further on this issue and present a different economic context in which the characterization result of the previous section does not translate. As we will see, in an economic environment in which agents do no have any incentives to coordinate their decisions, the solidarity axiom 2.3 (iii) does no longer determine a unique solution. Thus, imposing this requirement has bite only whenever some degree of cooperation among the agents is necessary.

Formally (though not conceptually), the model we consider now is a slight modification of the one studied in the previous section. We abandon now the setting of public goods and let $Y_i = \mathbf{R}_+^m$ be the space of (produced) private goods consumed by agent $i \in N$. That is, we assume that the sets $X_1,\ldots,X_n$, represent, as in Section , the spaces of some private good which can be used, by means of a *public technology* c, to produce a bundle of goods $y \in \sum_{i=1}^n Y_i$. The key difference with the previous section is that the new vector y is no longer a bundle of public goods, but it has to be divided $y = y^1 + \cdots + y^n$, with $y^i \in Y_i$ among the agents $i = 1,\ldots,n$ who consume them.

Thus the difference with the approach in Section is that, in the present context, a feasible allocation consists of a vector $(y^1,\ldots,y^n;t) \in Y_1 \times \cdots \times Y_n \times X$ such that $c(y^1+\cdots+y^n) = t_1+\cdots+t_n$. The rest of the model and the assumptions made in the last section are translated readily into this new scenario. The question now is whether the equivalent of Theorem 2.4 holds in the setting of private goods.

To see that this is not the case consider a linear technology, so that $c(y^1 + \cdots + y^n) = c(y^1) + \cdots + c(y^n)$. For each utility profile $u = (u_1,\ldots,u_n)$ and for each $i = 1,\ldots,n$ we let $(y^i(u_i), t_i(u_i)) \in Y_i \times X_i$ be the solution to agent i's maximization problem

$$\left.\begin{array}{rl} \max & u_i(y^i,t_i) \\ \text{s.t.} & c(y^i) = t_i \end{array}\right\}$$

Then, the mechanism $S : E \to Y_1 \times \cdots \times Y_n \times X$ assigning the allocation $S(u) =$

$(y^1(u_1), \ldots, y^n(u_n); t_1(u_1), \ldots, t_n(u_n))$ to every utility profile $u = (u_1, \ldots, u_n)$ verifies properties 2.3 but is not welfare egalitarian.

The difference between public versus private goods is that in the first case agents are forced to come up with some common identical bundle, consumed by all of them. In contrast, in the case of private goods, the linear technology allows each of them to behave individualistically; in such a way that the different solutions proposed by each of the agents are compatible. This example shows that one has to be careful when postulating the principles of Bargaining Theory within economic environments. As pointed out by the work of J. E. Roemer ([12]), some of those axioms might be reasonable in some settings but completely unjustified for others. In particular, the characterization results might hold only for some, very concrete family of models but not for all them. In this sense, the price paid for getting around Roemer's critique and making the principles of Axiomatic Bargaining Theory applicable to economic scenarios is a loss in its universal character.

4 Concluding comments

Motivated by some of the principles of Bargaining Theory we have postulated a set of axioms, which have an interpretation within specific economic environments. We have seen that this axioms determine the egalitarian correspondence, one of the standard solutions of Bargaining Theory and we have argued that the exact interpretation of this axioms might change depending on the family of economic models of interest. To finish we address the issue of clarifying the relationship between the contents and scope of the axioms postulated here and their Axiomatic Bargaining Theory counterpart.

We have limited our model to agents with quasi–linear preferences. In this environment it is an easy observation that, given a fixed profile of utilities $\pi \in E$, the set of attainable utilities is $A(\pi) = \{\pi(y) - t : (y;t) \in X \times Y, \quad c(y) \leq \sum_{i=1}^n t_i\}$. The Pareto frontier corresponds in utility space with the plane $\{(v_1, \ldots, v_n) \in \mathbf{R}^n : v_1 + \cdots + v_n = \sum_{i=1}^n \pi_i(\bar{y}) - c(\bar{y})\}$ where $\bar{y}$ is a solution to $\max\{\sum_{i=1}^n \pi_i(y) - c(y) : y \in Y\}$. From this one draws immediately the following three remarks.

Firstly, Axiomatic bargaining theory makes use of utility possibility sets which look like rectangles. Of course, the observation above excludes these domains from our setting. As a consequence, the proof used in the classical theory ([3]) does not carry over here. In addition, as we have remarked previously, in the present model, welfare egalitarian allocations are Pareto optimal in the strong sense, not only weakly Pareto optimal.

Secondly, in the setting of quasi–linear preferences, given two utility profiles π and ν it is always the case that either $A(\pi) \subset A(\nu)$ or $A(\nu) \subset A(\pi)$. Hence, the axiom of monotonicity as is used in Axiomatic Bargaining Theory applies to every pair of economic environments π and ν, whereas our axiom of solidarity applies only if $\pi \geq \nu$ or $\nu \geq \pi$.

Finally the axiom of symmetry in Bargaining Theory applies whenever the utility possibility set is symmetric. But this is always the case in our context. Hence, the axiom of symmetry as is used in bargaining theory cannot distinguish among the agents and would recommend to treat them equally regardless of them being very distinct. This shows clearly the strong demands of bargaining theory when applied to particular economic environments. Just the axiom of symmetry by itself would determine, in a trivial way, the egalitarian solution.

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