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Wages and Productivity Growth in a Competitive Industry

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Abstract

The model studies the evolution of productivity growth in a competitive industry. The exogenous wage rate determines the firms' engagement in labor productivity enhancing process innovation. There is a unique steady state of the industry dynamics, which is globally stable. In the steady state, the number of active firms, their unit labor cost and supply depend on the growth rate but not on the level of the wage rate. In addition to providing comparative statics of the steady state, the paper characterizes the industry's adjustment path.

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1 Introduction

How do wages affect the incentives for labor productivity enhancing innovation at the industry level? We address this question by studying the evolution of productivity growth in a competitive industry. Firms in this industry face an exogenous wage rate, which can be thought of as being determined in the aggregate labor market of the underlying economy. This wage affects the innovative performance of the industry as firms seek to reduce their labor cost by increasing labor productivity. The dynamics of innovation converge to a unique steady state, in which unit labor costs are constant over time. In the steady state, the number of active firms, their supply and unit labor cost turn out not to depend on the level of wages; they only depend on their rate of growth. From any initial configuration the industry characteristics monotonically approach the steady state as time evolves. Along the adjustment path, high but declining productivity growth rates are associated with entry of new firms and a decline in the size of firms. Exit induces an increase in market concentration when productivity growth is relatively low but increasing over time.

Technological innovations as a means to reduce labor costs seem to have been at the heart of economic growth for many decades. Our theoretical argument is in the same spirit as the empirical findings of Gordon (1987) who argues that a substantial component of accelerations and decelerations of productivity growth in Europe, Japan and the U.S. can be attributed to the behavior of the ratio of wages to labor productivity. A number of microeconometric studies have established a positive relationship between wages and the introduction of new technologies. The time series results of Doms, Dunne and Troske (1997) suggest that plants with high wage workforces are more likely to adopt new technologies. A possible explanation for this could be some complementarity between technology and skill: Wages are positively related to workforce skills and these skills allow new technologies to be adopted at lower costs. The alternative rationalization, which we model in this paper, is that higher wages will induce firms to substitute away from of firms and entry and exit over the product life cycle.[2] For simplicity, our study disregards firm heterogeneity and stochastic factors that may affect innovation. The industry variables monotonically approach their steady state values; entry and exit never occur simultaneously along the adjustment path.

The remainder of the paper is organized as follows. Section 2 presents a stylized model of a competitive industry. Section 3 describes its short-run equilibrium. The main results are contained in Section 4, which studies the industry's long-run behavior. The final Section offers concluding remarks. The results of Sections 3 and Section 4 employ a series of Lemmas that are relegated to an Appendix.

2 The Model

The model depicts the evolution of a competitive industry with free entry and exit. The firms produce a homogeneous good and take the market clearing price as fixed. Similarly, they behave competitively in the labor market by considering the wage rate as exogenous. Time is discrete and at each date there is a sufficiently large number of producers who have access to the current technology. Producers become active at date t by investing capital and engaging in process innovation to increase labor productivity. At date $t+1$ they employ labor to produce output. Given the intertemporal path of wages, the evolution of the industry is determined by the active firms' innovation behavior because this generates the available technology at the next date.

Formally, the model is specified as follows. To produce x units of output at date $t+1$, a firm has to invest the amount $C(x)+f$ at date t. Thus there is a fixed cost $f > 0$ and the variable capacity cost is $C(x)$. Production occurs at date $t+1$ and requires the labor input x/a_{t+1}, where a_{t+1} is the firm's labor productivity at $t+1$.

[2]See, e.g., Hopenhayn (1992), Jovanovic and MacDonald (1994) and Klepper (1996). Pakes and Ericson (1995) address this question in an imperfectly competitive industry.

We denote the total mass of active firms at date t by n_t. In each period, the industry faces the (inverse) demand function $P(\cdot)$ so that $P(\bar{x})$ is the market clearing price for the aggregate supply $\bar{x}$. The assumption that demand is stationary over time is not essential for our analysis. We discuss this issue below in Section 4.

We maintain the following assumptions on the functions $K(\cdot), C(\cdot)$ and $P(\cdot)$: Inverse demand satisfies $P(0) = \bar{p} > 0$, $P' < 0$ and $P(\infty) = 0$. The cost functions satisfy $K(0) = 0, K'(0) = 0, K'(q) > 0, K''(q) > 0$ for all $q > 0$, and $C(0) = 0, C'(0) = 0, C'(x) > 0, C''(x) > 0$ for all $x > 0$. Moreover, let $K(q) \to \infty$ as $q \to \infty$ and $C(x)/x \to \infty$ as $x \to \infty$. In addition we require that

$$K''(q) \geq \frac{K'(q)^2}{2K(q)}, \quad C''(x) \geq \frac{2[C'(x)x - C(x)]}{x^2}. \tag{4}$$

Thus, the cost functions are assumed to be sufficiently convex in order to avoid problems with nonconvexities that typically arise in $R\&D$ models (see, e.g., Dasgupta and Stiglitz (1980)). It is easy to see that by the first inequality in (4) the elasticity of the marginal innovation cost $K'(q)$ is at least twice the elasticity of the cost $K(q)$. Similarly, the second inequality is identical to assuming that the elasticity of the difference between the marginal cost, $C'(x)$, and the average cost, $C(x)/x$, is at least one. Assumption (4) is satisfied for instance for $K(q) = \kappa q^\alpha$ and $C(x) = \chi x^\beta$ as long as $\alpha \geq 2$ and $\beta \geq 2$.

Finally, to ensure that the problem is non-trivial, we add the following assumption:

$$\delta\bar{p} > \min_{x \geq 0} \left[\frac{C(x) + f}{x}\right] \tag{5}$$

That is, demand is sufficiently high so that some producers are active whenever the wage rate is small enough.

labor through new technologies. Chennells and Van Reenen (1997) conclude from their analysis of British plant data that this substitution effect may indeed be important factor. In a dynamic factor demand model, Mohnen *et. al.* (1986) find that the long - run cross-price elasticity of *R&D* with respect to the price of labor is fairly large. Also Flaig and Stadler (1994) conclude from their estimation of a dynamic model of innovation behavior that the wage rate seems to be a major determinant for process innovations.

The impact of labor market conditions on productivity appears important for understanding the innovative performance of different industries and countries. While our analysis emphasizes the role of higher wages in creating substitution away from labor that boosts productivity, other studies have been concerned with the impact of unions on wages and innovation. Here the conventional wisdom follows Grout's (1984) argument that the union will appropriate some share of the rents from technological improvements. This tends to reduce the firm's incentive to innovate.[1] Our model abstracts from these issues by considering wages at the industry level as exogenous. The simplest theoretical justification for this is that the industry forms a small part of an economy in which the aggregate market for labor can be represented by a standard competitive model. Yet, our main insights do not necessarily rely on the competitive labor market paradigm. Our analysis shows that not the level of wages but their growth rate is important for long-run productivity growth. Therefore, the possible presence of industry wage differentials does not affect our results as long as the time path of the industry's wage follows the same trend as the competitive wage.

The partial equilibrium dynamics of a competitive industry have been studied first by Lucas and Prescott (1971). Since then, a number of models has been developed that focus on innovation under technological uncertainty in a competitive industry. These models investigate the stochastic evolution

[1] A short outline of the rent-sharing argument together with an empirical analysis can be found in Van Reenen (1996). Ulph and Ulph (1994) present a model with different conclusions.

n_t^* solve $p_{t+1} = P(n_t^* x_t^*)$. As $P(\cdot)$ is strictly decreasing, the solution is unique with $n_t^* > 0$. We now claim that (q_t^*, x_t^*, n_t^*) is the unique static equilibrium for the wage-productivity ratio c_t. As (q_t^*, x_t^*) minimizes the average cost $\varphi(\cdot|c_t)$, it also maximizes a firm's profits for given p_{t+1} and thus condition (i) of the definition of the static equilibrium is satisfied. Further, by definition of p_{t+1} a firm makes zero profits (condition (ii)). Finally, by the definition of n_t^*, the market clears (condition (iii)). Suppose next that $p_{t+1} \geq \bar{p}$. In this case, $n_t^* x_t^* > 0$ implies negative profits for all active firms. Hence, the unique static equilibrium is $n_t^* = q_t^* = x_t^* = 0$.

To prove the second part, note first that $\varphi(q_t^*, x_t^*|c_t)$ is strictly increasing in c_t by the Envelope Theorem. Further, $\varphi(q_t^*, x_t^*|c_t) > C(x_t^*)/x_t^*$ Thus, by Lemma 11 in the Appendix, $\varphi(q_t^*, x_t^*|c_t) \to \infty$ as $c_t \to \infty$. Further, by (5), $\varphi(q_t^*, x_t^*|0) < \delta\bar{p}$. Thus, by continuity of $\varphi(\cdot)$, there exists a $\bar{c}$ such that $\varphi(q_t^*, x_t^*|\bar{c}) = \delta\bar{p}$. By the above argument, $n_t^* > 0$ if and only if $c_t < \bar{c}$. Q.E.D.

Proposition 1 establishes a unique static equilibrium for each wage - productivity ratio. If the wage rate is too high or the productivity of labor too low, then no firm enters the market because – even at the efficient scale – average costs exceed the chock-off price $P(0)$. If, however, the wage-productivity ratio is low enough, assumption (5) ensures that a positive measure of firms operates in the market. Firms choose their *R&D* expenditures such that their marginal benefit from the higher labor productivity tomorrow equals their marginal cost of innovation. Further, as firms are price takers, they choose the output level such that their marginal cost equals the market price. In addition, free entry in the industry implies that the firms' average cost is equal to the market price, and hence to their marginal cost. As a consequence, it is as if each firm were minimizing its average costs in equilibrium. Given that a firm's average cost is strictly convex in output and *R&D* expenditures, there is a unique output and innovation level that minimizes these costs for each wage-productivity ratio; moreover, the minimum average cost, and thus the market price, is unique. Finally, the number of firms adjusts such that demand equals supply. As demand is strictly decreasing, the number of firms

Each potential producer observes the process innovations performed by the active firms. As Klepper (1996), we assume that after one period he can costlessly incorporate these innovations into his own technology. Thus an active firm has a one-period monopoly over the technological improvements generated by its *R&D* activity in period t. We focus on labor productivity enhancing process innovation and assume that each firm can increase current productivity by the factor $(1+q)$ by investing the amount $K(q)$. Thus, if a_t describes the most advanced technology developed at date $t-1$, a firm's labor productivity at $t+1$ is

$$a_{t+1} = (1+q)a_t \tag{1}$$

if it invests the amount $K(q)$ in process innovation.

The exogenous wage rate w_t grows at the rate $\gamma > 0$ so that $w_{t+1} = (1+\gamma)w_t$, with $w_0 > 0$. One possible interpretation is that γ represents the average growth rate of labor productivity in the entire economy. Therefore, also wages grow at the rate γ in the equilibrium of the economy - wide labor market. Since the industry under consideration constitutes only a small part of the whole economy, its impact on the equilibrium wage rate can be taken to be negligible.[3]

Let $c_t \equiv w_t/a_t$. Then, after investing $K(q)$ at date t, the firm's labor cost per unit of output at the next date is

$$c_{t+1} = \frac{1+\gamma}{1+q}c_t. \tag{2}$$

The firm sells its output at the price p_{t+1}. Given the common discount factor $0 < \delta \leq 1$, its present value of profit is

$$\Pi(q, x|p_{t+1}, c_t) \equiv \delta\left[p_{t+1} - c_t\frac{1+\gamma}{1+q}\right]x - K(q) - C(x) - f. \tag{3}$$

[3]As we indicated in the Introduction, the industry's wage rate w_t does not have to be *identical* to the competitive wage rate. If ω_t represents the competitive wage rate, then our analysis remains valid as long as there is an $\alpha > 0$ such that $w_t = \alpha\omega_t$.

wage-productivity ratio. In other words, an increase in the unit labor cost raises the minimum efficiency scale, at which firms operate in a free entry equilibrium. Furthermore, since the minimum average cost is higher when a firm faces higher wages or its labor productivity is lower, the market clears at a higher price and total demand is reduced. Since bigger firms serve a smaller total demand, the number of active firms in the market is smaller when the wage-productivity ratio is higher.

Propositions 2 and 3 indicate that innovative investments are higher in a smaller, more concentrated market.[5] Yet, this observation does not imply a causal relationship since innovation, firm size and industry size are simultaneously determined. Another implication of Proposition 3 is that, as the wage-productivity ratio increases, aggregate employment in the industry decreases because industry output shrinks. As a result, higher productivity growth and lower aggregate employment are observed in the industry.[6] Note however that, as the size of firms increases, employment at the firm level may increase or decrease with the wage-productivity ratio. There is no clear relationship between productivity growth and employment at the plant level.

4 Equilibrium Dynamics

We now turn to the dynamics of innovation. In the previous Section it was shown how the state variable c_t affects the industry equilibrium in period t. As part of this equilibrium, the rate of productivity growth q_t^* is a function of c_t. Since q_t^* determines the change in the state variable from period t to $t+1$, the industry's dynamics are generated by the evolution of c_t. The industry starts in period $t = 0$ from the exogenously given labor productivity $\bar{a}_0$. Since the wage rate in this period is w_0, the initial value of the state variable

[5]For our model, it would seem natural to regard $1/n_t$ as a measure of the degree of concentration.

[6]This observation is in line with the model of Bean and Pissarides (1993) where an upward shift in the wage-setting schedule raises both equilibrium unemployment and productivity growth.

3 Static Equilibrium

First, we consider the industry equilibrium at a particular date t. At this date, the wage rate w_t together with the labor productivity a_t determine the current wage-productivity ratio c_t. This parameter describes the state of the industry at date t. Of course, c_t depends on the evolution of wages and productivity in the past. Yet, in this Section we focus on the static aspects of firm behavior and consider c_t as exogenous.

Definition $(q_t^*, x_t^*, n_t^*|c_t)$ is a *static equilibrium* if

(i) (q_t^*, x_t^*) maximizes $\Pi(q, x|p_{t+1}, c_t)$ if $n_t^* > 0$; and $q_t^* = x_t^* = 0$ if $n_t^* = 0$;

(ii) $\Pi(q_t^*, x_t^*|p_{t+1}, c_t) = 0$ if $n_t^* > 0$; and $\Pi(q, x|p_{t+1}, c_t) \leq 0$ for all (q, x) if $n_t^* = 0$;

(iii) $p_{t+1} = P(n_t^* x_t^*)$.

At date t, a total mass of n_t^* firms enters the market to produce some output at $t+1$. Since all firms are identical, they choose the same output x_t^* and innovation rate q_t^*. The first equilibrium requirement is that the firms behave competitively by taking the market price p_{t+1} as fixed when choosing x_t^* and q_t^* so as to maximize profit. With free entry, profits cannot be positive. Condition (ii) states that each firm earns zero profit when the mass of active firms is positive. Otherwise, profits may be negative. Finally, as total output at date $t+1$ is $n_t^* x_t^*$, the third equilibrium condition ensures that the market clears at the price p_{t+1}.

Proposition 1 *For each c_t there is a unique static equilibrium $(q_t^*, x_t^*, n_t^*|c_t)$. Moreover, there is a $\bar{c} > 0$ such that $n_t^* > 0$ if and only if $c_t < \bar{c}$.*

Proof: Let (q_t^*, x_t^*) be the argmin of $\varphi(q_t, x_t|c_t)$ (see (9) in the Appendix). By Lemma 10 in the Appendix, (q_t^*, x_t^*) is unique since φ is strictly convex in (q_t, x_t). Define $p_{t+1} \equiv \varphi(q_t^*, x_t^*|c_t)/\delta$. Suppose first that $p_{t+1} < P(0) = \bar{p}$. Let and only if $\hat{c} < \bar{c}$. It follows from Lemma 11 in the Appendix that $q(\hat{c}) = \gamma$ and $\hat{c} < \bar{c}$ if and only if γ lies below some positive upper bound $\bar{\gamma}$. Q.E.D.

A steady state equilibrium is feasible only if the growth rate of wages is low enough so that the industry can afford to match it with labor saving innovations. If this is not the case, the evolution of productivity will lag behind the growth of wages and so average costs increase over time. Ultimately, this will drive the industry towards extinction as we show in Proposition 8 below. We will first deal with the more interesting case where $\gamma < \bar{\gamma}$. The following result shows that in this situation the time path of the industry will eventually approach the steady state, independently of the initial conditions.

Proposition 5 *If there is a steady state, it is stable. That is, as long as $\gamma < \bar{\gamma}$, the equilibrium sequence $(q_t^*, x_t^*, n_t^* | c_t^*)_t$ converges to $(\hat{q}, \hat{x}, \hat{n} | \hat{c})$ in the limit as $t \to \infty$.*

Proof: By Lemma 12 in the Appendix, $q_t^* < \gamma$ if $c_t^* < \hat{c}$ and $q_t^* > \gamma$ if $c_t^* > \hat{c}$. By (6) this implies

$$c_t^* < c_{t+1}^* \text{ if } c_t^* < \hat{c}; \quad c_t^* > c_{t+1}^* \text{ if } c_t^* > \hat{c}. \tag{7}$$

Lemma 13 in combination with (6) shows that c_{t+1}^* increases with c_t^*. This together with (7) yields

$$c_t^* < c_{t+1}^* < \hat{c} \text{ if } c_t^* < \hat{c}; \quad \hat{c} < c_{t+1}^* < c_t^* \text{ if } c_t^* > \hat{c}. \tag{8}$$

This proves that the sequence $(c_t^*)_t$ converges. By (6), therefore, $q_t^* \to \gamma$ so that $c_t^* \to \hat{c}$. By Proposition 1 and a simple continuity argument this implies that the equilibrium sequence converges to the steady state. Q.E.D.

Over time, productivity growth converges to the rate of wage growth. Thus, the steady increase in real wages determines the firms' persistent engagement in labor productivity enhancing innovations. Another important implication of the above result is that the long-run behavior of the industry is uniquely determined in equilibrium.

Proposition 2 *Let $(q_t^*, x_t^*, n_t^*|c_t)$ be a static equilibrium. When $c_t < \bar{c}$, the rate of productivity growth q_t^* increases with the wage-productivity ratio c_t.*

Proof: This follows immediately from Lemma 3 in the Appendix. Q.E.D.

Higher labor costs per unit of output create a stronger incentive to substitute away from labor through productivity enhancing innovation. This is simply so because firms choose their *R&D* expenditures to equate the marginal benefit from the increase in labor productivity to the marginal cost of innovation. As the marginal benefit of innovation is proportional to the current wage-productivity ratio, the firm's optimal *R&D* expenditures are higher when the current wage is higher, or the current labor productivity is lower.[4]

Proposition 3 *Let $(q_t^*, x_t^*, n_t^*|c_t)$ be a static equilibrium. When $c_t < \bar{c}$, each firm's output x_t increases with c_t. The total mass of active firms n_t^* and aggregate industry output $n_t^* x_t^*$ strictly decrease with c_t.*

Proof: The first statement follows immediately from Lemma 3 in the Appendix. By Lemma 1, $\delta P(n_t^* x_t^*) = \varphi(q_t^*, x_t^*|c_t)$. Since by the Envelope Theorem, $\varphi(q_t^*, x_t^*|c_t)$ is strictly increasing in c_t, this implies that $n_t^* x_t^*$ decreases with c_t. As a consequence of the first statement, also n_t^* decreases with c_t. Q.E.D.

The rate of innovation and the level of output are complements for a profit maximizing firm. Intuitively, the total gain from a given reduction in unit labor costs increases with the number of goods produced. As we know already from Proposition 2, the higher the wage-productivity ratio, the higher is the firm's innovation rate. Accordingly, also output is positively related to the

[4]See the first order conditions (15) in the Appendix.

is $c_0 \equiv \bar{c}_0 = w_0/\bar{a}_0$. We consider the parameter $\bar{c}_0$ as exogenous and assume that it lies below the critical value $\bar{c}$ specified in Proposition 1 so that $n_0^* > 0$.

Definition $(q_t^*, x_t^*, n_t^*|c_t^*)_t$ is an *equilibrium sequence* if, for all $t = 0, 1, \dots,$ $(q_t^*, x_t^*, n_t^*|c_t^*)$ is a static equilibrium and

$$c_{t+1}^* = \frac{1+\gamma}{1+q_t^*} c_t^*, \text{ with } c_0 = \bar{c}_0, \tag{6}$$

It follows immediately from Proposition 1 that for any $\bar{c}_0$ the equilibrium path of the industry is fully determined. We are especially interested in the long-run behavior of the industry. Therefore, we look at the equilibrium outcome for large values of the time index t and investigate whether eventually the market will become stationary. The industry will be in a steady state if the number of active firms, their output and their innovation efforts remain constant over time.

Definition $(\hat{q}, \hat{x}, \hat{n}|\hat{c})$ is a *steady state* if it is a static equilibrium and $\hat{q} = \gamma$.

In a steady state, the state variable remains at the value $\hat{c}$ because wages and labor productivity grow at the same rate. As a result, also the number of active firms and their output do not change over time. Notice that, if there is a steady state, it is independent of the initial value $\bar{c}_0$ of the state variable. Instead the steady state endogenously determines the wage - productivity ratio $\hat{c}$. Also, the definition of a steady state implicitly presumes that $\hat{n} > 0$. This follows immediately from the static equilibrium condition (i) and $\gamma > 0$.

Proposition 4 *There is a $\bar{\gamma} > 0$ such that a steady state $(\hat{q}, \hat{x}, \hat{n}|\hat{c})$ exists if and only if $\gamma < \bar{\gamma}$. Moreover, if there is a steady state, it is unique.*

Proof: Since $\hat{n} > 0$, Lemma 9 in the Appendix implies that $\varphi(\gamma, \hat{x}|\hat{c}) \leq \varphi(q, x|\hat{c})$ for all (q, x). By Lemma 12 there is a unique $\hat{c}$ such that this condition is satisfied. By Proposition 1 there is thus a (unique) steady state if

Again, the intuition for this observation comes from Proposition 2. A higher rate of productivity growth can be supported only when higher unit labor costs force the firms to speed up innovation. In combination with Proposition 3, this implies that the steady state size of firms increases with γ. As a result of an increase in γ, a smaller number of firms operates in the industry producing a lower level of aggregate output.

We finally characterize the dynamic path of the industry.

Proposition 7 *Let $\gamma < \bar{\gamma}$. Then the equilibrium sequence $(q_t^*, x_t^*, n_t^* | c_t^*)_t$ satisfies*

$$q_t^* < q_{t+1}^* < \gamma,\ x_t^* < x_{t+1}^*,\ n_t^* > n_{t+1}^*,\ n_t^* x_t^* > n_{t+1}^* x_{t+1}^* \ \textit{if}\ \bar{c}_0 < \hat{c};$$

$$q_t^* > q_{t+1}^* > \gamma,\ x_t^* > x_{t+1}^*,\ n_t^* < n_{t+1}^*, n_t^* x_t^* < n_{t+1}^* x_{t+1}^* \ \textit{if}\ \bar{c}_0 > \hat{c}.$$

Proof: By the proof of Proposition 5, the equilibrium sequence satisfies (8). This in combination with Propositions 2 and 3 proves the statement of the Proposition. Q.E.D.

The industry monotonically approaches the steady state equilibrium. Depending on the initial state, the adjustment process exhibits either accelerations or decelerations of productivity growth. Changes in productivity growth are positively related with changes in firm size. Exit occurs in combination with relatively low but increasing rates of productivity growth. Along this path, the industry adjusts to a higher level of unit labor costs; total production and aggregate employment decrease while the output price increases. In contrast, new firms enter when the industry approaches a lower level of unit labor costs. In this case, the industry's production increases and the output price declines over time.

The literature on industrial dynamics associates a 'shakeout' in the number of producers with the maturity phase in the industry's product life cycle.

is independent of its initial productivity $\bar{a}_0$ and the level of the wage rate w_0. As time evolves, the industry's innovative efforts adjust labor productivity in such a way that it becomes proportional to the wage rate by the factor $1/\hat{c}$. The basic intuition for this phenomenon is derived in Proposition 2. The incentives for innovation are positively related to the wage-productivity ratio. This ratio reaches its steady state level when productivity and wages grow at the same rate. Above this level it induces productivity to grow faster than wages. The opposite happens when unit costs are below the steady state value. As a result, the endogenous pace of technical progress always moves the wage-productivity ratio towards the steady state.

In the long-run, the initial state of the industry becomes irrelevant not only for c_t but also for q_t, x_t and n_t. These variables tend towards their steady state values, which are independent of $\bar{a}_0$ and w_0. The level of wages, however, has a profound impact on the employment of labor. As $\hat{c}$ is a constant, a one percent increase in the level of wages raises also the long-run level of labor productivity by one percent. At the same time, the level of wages does not affect total industry output in the steady state. As an implication, employment falls by one percent. In other words, the long-run elasticity of employment with respect to the wage level equals minus unity.

In the long - run, it is not the level of wages but the growth rate of wages which determines the industry's unit labor cost. As the following Proposition shows, the latter is positively related to the growth rate of wages.

Proposition 6 *In the steady state $(\hat{q}, \hat{x}, \hat{n}|\hat{c})$ the wage - productivity ratio $\hat{c}$ increases with γ.*

Proof: Let $\gamma' < \gamma''$ and let $(\gamma', x', n'|c')$ and $(\gamma'', x'', n''|c'')$ be the corresponding steady states. Suppose $c' \geq c''$. Then Proposition 2 implies that $\gamma' \geq \gamma''$, a contradiction. Q.E.D.

But, ρ does not influence $\hat{q}, \hat{x}$ and $\hat{c}$. Employment either increases or decreases over time depending on ρ being larger or smaller than γ.

5 Concluding Remarks

Technical progress and a substantial increase in real wages are main attributes of the growth process in the advanced industrial nations. Our analysis presents a cost-push argument of productivity growth. The basic idea is that firms adjust their innovative activity to increasing labor costs. Higher labor costs create stronger incentives for process innovations that raise the productivity of labor. The more interesting issue, however, is the dynamic interaction between innovation and productivity. As current innovations aim at reducing the firms' labor cost, they also affect their future incentives for inventive activities. Our analysis shows that long - run productivity growth at the industry level is driven by the growth rate of wages. This rate determines the number of active firms, their labor costs per unit of output, the size of firms and the industry's output in the long -run. While these variables are independent of the level of the wage rate, the latter determines the level of labor productivity and employment within the industry.

The industry's adjustment path exhibits either entry or exit of firms. In contrast with a number of recent studies on industry dynamics, we ignore stochastic factors that induce firm heterogeneity. Yet, this restriction is mainly motivated by simplicity. In principle, our model could be enriched by a stochastic process so that entry and exit occur simultaneously.

Another interesting extension of our model is the consideration of imperfect competition. A Cournot or Bertrand framework could address the question of how strategic interactions between the firms affect productivity growth in the short - run and in the long - run. Stimulated by the work of Schumpeter (1947), a large part of the literature on *R&D* relates the pace of innovative activity to market structure. An imperfect competition version of our model could combine this approach with our cost-push argument.

As an empirical regularity of this phase (see, e.g., Gort and Klepper (1982) and Klepper (1996)), the number of producers steadily declines while their output increases. Also, the firms' efforts to improve the production process increase over time. Proposition 7 reflects these regularities when $\bar{c}_0 < \hat{c}$. This parameter constellation might apply to an industry in which previous technological breakthroughs have lead to a high productivity level. Once the industry matures, the process of innovation becomes more predictable and is driven mainly by continuous technological improvements.

Given the initial state $\bar{c}_0$, exit occurs when wages grow relatively fast. Indeed, as we indicated above, the industry will not be able reach a steady state when γ exceeds the critical level $\bar{\gamma}$. In this situation, the exit process eventually eliminates the entire industry.

Proposition 8 *Let $(q_t^*, x_t^*, n_t^* | c_t^*)_t$ be an equilibrium sequence. If $\gamma > \bar{\gamma}$, then there is a finite $T > 0$ such that $n_t^* = 0$ for all $t \geq T$.*

Proof: Suppose $n_T^* > 0$ for all finite T. Then the proof of Proposition 5 implies that c_t^* converges to some $\hat{c}$. By Proposition 4, $\hat{c} > \bar{c}$. By Proposition 1 this implies $n_t^* = 0$ for t sufficiently large, a contradiction. Since $n_T^* = 0$ implies $0 = q_t^* < \gamma$, one has $c_t^* > c_T^*$ for all $t > T$. Therefore $n_t^* = 0$ for all $t > T$. Q.E.D.

It is worth noting that in our model the demand function $P(\cdot)$ has no effect on how q_t, x_t and c_t are determined along the industry's equilibrium path. It only affects the number n_t of active firms, which adjusts to equate demand and supply. Thus, stationarity of demand is not essential for our analysis. Implicitly, stationarity presumes that the growth of wages and income in the economy does not affect industry demand. This could be justified by assuming that demand is derived from quasi-linear utility functions. If, however, demand does change over time, our analysis can easily be modified to take this into account. For instance, when demand grows at the rate ρ, then in the steady state also the number of active firms grows at the rate ρ.

Also, it would allow studying the impact of unionization on innovation. Rent sharing is likely to depress the short -run incentives for innovation. Yet, our results lead to the conjecture that unionization will not influence long - run productivity growth, unless wage bargaining affects not only the level but also the growth rate of industry wages.

6 Appendix

Define

$$\varphi(q,x|c) = \delta c\frac{1+\gamma}{1+q} + \frac{K(q)+C(x)+f}{x}. \tag{9}$$

Thus $\varphi(q,x|c)$ is the firm's average cost.

Lemma 9 *Let $(q_t, x_t, n_t|c_t)$ be a static equilibrium. If $n_t > 0$, then (q_t, x_t) minimizes $\varphi(q,x|c_t)$. Moreover, $\delta p_{t+1} = \varphi(q_t, x_t|c_t)$.*

Proof: By equilibrium condition (ii), $\delta p_{t+1} = \varphi(q_t, x_t|c_t)$. Suppose there exists (q', x') such that $\varphi(q', x'|c_t) < \varphi(q_t, x_t|c_t)$. Then $\Pi(q', x'|p_{t+1}, c_t) > 0 = \Pi(q_t, x_t|p_{t+1}, c_t)$. This yields a contradiction to condition (i). Q.E.D.

Lemma 10 *The function $\varphi(q,x|c)$ is strictly convex in (q,x) for all $(q,x) > 0$.*

Proof: We have

$$\varphi_{qq} = \frac{2\delta c(1+\gamma)}{(1+q)^3} + \frac{K''(q)}{x} > 0, \quad \varphi_{qx} = -\frac{K'(q)}{x^2}, \tag{10}$$

and

$$\varphi_{xx} = \frac{2(K(q)+f)}{x^3} + \frac{C''(x)x^2 + 2C(x) - 2C'(x)x}{x^3} > 0. \tag{11}$$

This implies

$$\varphi_{xx}\varphi_{qq} - \varphi_{qx}^2 > \frac{K''(q)2K(q) - K'(q)^2}{x^4} \geq 0. \tag{12}$$

By the inequalities in (10) - (12), $\varphi(q,x|c)$ is strictly convex. Q.E.D.

Lemma 11 *Let $(q(c), x(c))$ minimize $\varphi(q, x|c)$. Then $q(\cdot)$ and $x(\cdot)$ are continuous and strictly increasing in c. Moreover, $q(c) \to 0$ as $c \to 0$, $q(c) \to \infty$ as $c \to \infty$ and $x(c) \to \infty$ as $c \to \infty$.*

Proof: By strict convexity of $\varphi(\cdot)$ the values $(q(c), x(c))$ are unique. The assumptions on $K(\cdot)$ and $C(\cdot)$ ensure that $q(c) > 0$ and $x(c) > 0$. Since $\varphi(\cdot)$ is continuous in (q, x, c) for all (q, x, c), the functions $q(\cdot)$ and $x(\cdot)$ are continuous in c.

Let $(q', x') = \text{argmin } \varphi(q, x|c')$ and $(q'', x'') = \text{argmin } \varphi(q, x|c'')$. Then

$$\delta c' \frac{1+\gamma}{1+q'} + \frac{K(q') + C(x') + f}{x'} < \delta c' \frac{1+\gamma}{1+q''} + \frac{K(q'') + C(x'') + f}{x''}, \tag{13}$$

$$\delta c'' \frac{1+\gamma}{1+q''} + \frac{K(q'') + C(x'') + f}{x''} < \delta c'' \frac{1+\gamma}{1+q'} + \frac{K(q') + C(x') + f}{x'}.$$

Adding these inequalities yields

$$(c'' - c') \left(\frac{1+\gamma}{1+q''} - \frac{1+\gamma}{1+q'} \right) < 0. \tag{14}$$

Thus $c' < c''$ implies that $q'' > q'$. This proves that $q(\cdot)$ is strictly increasing. If $\varphi(q, x|c)$ attains a minimum at (q, x), q and x must satisfy the first order conditions

$$\delta c(1+\gamma)x = K'(q)(1+q)^2, \;\; C'(x)x - C(x) = K(q) + f. \tag{15}$$

The l.h.s. of the second equation is strictly increasing in x and the r.h.s. is strictly increasing in q. Therefore, as $q(\cdot)$ is strictly increasing, also $x(\cdot)$ is strictly increasing.

As $x(\cdot)$ is strictly increasing, the first equation in (15) implies that $q(c) \to 0$ as $c \to 0$ and $q(c) \to \infty$ as $c \to \infty$. The second equation in (15) therefore implies that also $x(c) \to \infty$ as $c \to \infty$. Q.E.D.

Lemma 12 *Let $(q(c), x(c))$ minimize $\varphi(q, x|c)$. Then there is a unique $\hat{c}$ such that $q(\hat{c}) = \gamma$. Moreover, $q(c) < \gamma$ if $c < \hat{c}$ and $q(c) > \gamma$ if $c > \hat{c}$*

Proof: By Lemma 11, one has $q(c) < \gamma$ for c sufficiently small and $q(c) > \gamma$ for c sufficiently large. Thus by continuity of $q(\cdot)$ there is a $\hat{c}$ such that $q(\hat{c}) = \gamma$. Uniqueness of $\hat{c}$ and the second statement follow from Lemma 11, as $q(\cdot)$ is strictly increasing. Q.E.D.

Lemma 13 *Let $(q(c), x(c))$ minimize $\varphi(q, x|c)$. Then $c(1+\gamma)/(1+q(c))$ is increasing in c.*

Proof: Define $z(c) \equiv c(1+\gamma)/(1+q(c))$. Then rearranging the first equation in (15) yields

$$z(c) = \frac{K'(q(c))(1+q(c))}{\delta x(c)}. \tag{16}$$

Thus we have $z'(c) > 0$ if

$$[K''(q)(1+q) + K'(q)]q'(c)x > x'(c)K'(q)(1+q). \tag{17}$$

Differentiation of the second equation in (15) yields

$$C''(x)xx'(c) = K'(q)q'(c). \tag{18}$$

Therefore (17) is equivalent to

$$[K''(q)(1+q) + K'(q)]C''(x)x^2 > [K'(q)]^2(1+q). \tag{19}$$

By assumption 4 this inequality is certainly satisfied if

$$\left[\frac{K'(q)(1+q)}{2K(q)} + 1\right][2C'(x)x - 2C(x)] > K'(q)(1+q). \tag{20}$$

Since by (15) $C'(x)x - C(x) = K(q) + f$, this inequality is equivalent to

$$\left[\frac{K'(q)(1+q)}{2K(q)} + 1\right] > \frac{K'(q)(1+q)}{2[K(q)+f]}. \tag{21}$$

As $f > 0$, (21) is certainly satisfied, which proves that $z'(c) > 0$. Q.E.D.

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