

# Tax evasion and the underground economy\*



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In this paper we present a partial equilibrium model of tax evasion in which evasion is modeled as a discrete variable, i.e. whether to join the underground economy or not. Several equilibrium concepts are used to analyze the game of the government against a single taxpayer.

## 1. Introduction

The problem of tax evasion arises in an economy in which every taxpayer regards the output of the public sector as independent of her actions and the taxpayer can be audited by the public authority and fined, if found guilty.

Earlier contributions to the study of tax evasion in a game-theoretical framework include Sandmo (1981), Reinganum and Wilde (1985, 1986), Graetz et al. (1986), Greenberg (1985), Benjamini and Maital (1985), Schlicht (1985), Mookherjee and P'ng (1986), Scotchmer (1987), Border and Sobel (1987), and Ortuño-Ortín (1991). Each of these papers focuses on the problem from a different angle, and obtains some conclusions on the welfare effects of tax evasion [a review of this literature can be found in Cowell (1985, 1990) and Mookherjee (1989)]. In all these papers, evasion is modeled as a continuous variable, i.e. the range of evasion goes from zero to the value of due taxes. A prominent example of this kind of situation is income tax evasion.

No matter how important these situations are, there are examples in which it is more natural to model evasion as a discrete choice model, reflecting an

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indivisibility in the choice of the potential tax-payers [see Cowell (1985, p. 174 footnote)]. Thus, for a given activity, the choice is either to be on the legal side, or to become a member of the underground economy. Examples of latter include barter transactions, black markets, illegal gambling and prostitution, secret workshops (shoes, garments, leather, furniture), street vendors, 'gypsy' cabs, rental of spare rooms, secret accounts in foreign countries, clandestine building, etc. In all these situations the parties concerned have no incentive to provide information (this lack of incentive is particularly acute if the workers involved are receiving unemployment benefits). Moreover, even in the income tax case, if someone decides to fill in a tax form underestimating his income, additional information is also transmitted: profession, address, etc. In some cases these signals may be so clear that it would be preferable not to fill this form in at all. This situation is particularly relevant in countries in which the fight against tax evasion is new and the public sector is relatively unorganized. For instance during the period 1983–1989 two million new taxpayers were discovered in Spain.

This paper presents a static partial equilibrium model of tax evasion in which there are *two* pure strategies for each player: for the taxpayer to evade or to pay due taxes (i.e. to be inside or outside the black economy) and for the government to monitor or not to monitor.<sup>1</sup> Thus our model – which is akin to those of externality causing behavior [see Becker (1968) and Mookherjee (1989)] or Inspector vs. Evader games [see Rapoport (1966)] – differs from previous work in the dichotomous choice of taxpayers and in that strategies for the government are interpreted as *monitoring* rather than *auditing*.

The rest of the paper goes as follows. In section 2 we analyze the game of the government against a single taxpayer, assuming that the government attempts to maximize net revenue [this assumption is also made by Graetz et al. (1986) and Border and Sobel (1987)]. It is shown that:

- a. There is no Nash equilibrium in pure strategies, but there is in mixed ones. This implies that the existing uncertainty is endogenous, caused by the randomized choices of the players.
- b. In expected terms the taxpayer pays the entire tax.
- c. An increase in the penalty for evasion increases the government's payoff but leaves untouched the taxpayer's expected utility.
- d. The government cannot expect any gain by announcing (and being committed to) its true policy (i.e. being a Stackelberg leader).
- e. If the taxpayer were assumed to be a leader and the government a follower, the payoffs for the latter would be larger than in a Nash

<sup>1</sup>Also players are considered to be risk-neutral. However it can be shown that most of our conclusions do not depend on this. See Corchón (1984).

Table 1

$\mathcal{P}$	$\mathcal{G}$ B <sub>1</sub>	B <sub>2</sub>
A <sub>1</sub>	$Y - T(1 + d), T(1 + d) - c$	$Y, 0$
A <sub>2</sub>	$Y - T, T - c$	$Y - T, T$

equilibrium and, given result (b) above, than in a Stackelberg equilibrium with the government as a leader.

- f. Maximin strategies call for honest behavior on the taxpayers' part and the same probability of monitoring as in a Nash equilibrium.
- g. If the penalty for evasion is sufficiently high, the Nash equilibrium is arbitrarily close to the core of the game.
- h. Bayesian equilibrium yields identical qualitative conclusions to those of Nash equilibrium.

Needless to say, all these conclusions must be tested in more general models before any definite conclusion is drawn. In any case, our approach underlines the importance of mixed strategies. They are a natural device in our framework, since the classical argument in favor of them, namely that they defend players against the opponent's exploitation of the knowledge of her actions, seems perfectly applicable in our case. Moreover a mixed strategy equilibrium can be seen as the limit of Bayesian equilibria [Harsanyi (1973)].

Finally, section 3 gathers together our main conclusions.

## 2. The model and the main results

Consider a game with two players ( $\mathcal{P}$ , the taxpayer, and  $\mathcal{G}$ , the government), and two strategies for each of them (A<sub>1</sub>, A<sub>2</sub> and B<sub>1</sub>, B<sub>2</sub> respectively). We identify A<sub>1</sub> with evasion (A<sub>2</sub> no evasion) and B<sub>1</sub> with monitoring (B<sub>2</sub> no monitoring). We assume that utility is money, i.e. both players are risk-neutral. Let  $Y$  be the income of  $\mathcal{P}$ . The value of the tax due will be denoted by  $T$ . The fine will be proportional to the evaded tax. So if  $\mathcal{P}$  cheats and is caught she gets a fine of  $dT$ . We will take  $d$  and  $T$  as exogenously given. Let  $c$  be the cost of monitoring and conviction. We assume that  $c < dT$  (this may be interpreted as saying that the penalty for evasion includes the cost of monitoring). In order to make the problem interesting we will assume that  $T > c$  (i.e. the tax due is greater than the cost of monitoring) and  $Y > T$ . Table 1 summarizes our information about the game.

It follows from our assumptions that

$$Y > Y - T > Y - T(1 + d),$$

$$T(1+d)-c > T > T-c > 0.$$

Hence no dominant strategy exists.

### 2.1. Nash equilibrium

We first assume that both players have to move simultaneously and that information is complete. Both assumptions will be relaxed later on. Let us first look for a Nash equilibrium. It is easy to check that there is no Nash equilibrium in pure strategies. Therefore we introduce mixed strategies. Let  $p$  be the probability of  $\mathcal{P}$  taking  $A_1$  (i.e. she evades), and  $q$  the probability of  $\mathcal{G}$  taking  $B_1$  (i.e. monitoring). Then payoffs for  $\mathcal{P}$  and  $\mathcal{G}$  (denoted by  $M_p$  and  $M_g$  respectively) are

$$\begin{aligned} M_p &= p(q(Y - T(1+d)) + (1-q)Y) + (1-p)(q(Y - T) + (1-q)(Y - T)), \\ M_g &= q(p(T - c + dT) + (1-p)(T - c)) + (1-q)(1-p)T, \\ M_p &= p(T - qT(1+d)) + Y - T, & (1) \\ M_g &= q(pT(1+d) - c) + (1-p)T. & (2) \end{aligned}$$

Then, the probabilities associated with a Nash equilibrium are

$$\begin{aligned} p^* &= \frac{c}{T(1+d)}, \\ q^* &= \frac{1}{1+d}. \end{aligned}$$

The strategic aspects of the game are present in the values of  $p^*$  and  $q^*$ . For instance the probability of evading depends directly on  $c$  (the more costly it is to monitor, the more likely the evasion) and inversely on the penalty  $d$ . Also, the probability of monitoring depends inversely on the penalty, since a high penalty will discourage evasion, making monitoring useless [see Graetz et al. (1986) for similar conclusions]. Equilibrium payoffs are

$$\begin{aligned} M_p^* &= Y - T, \\ M_g^* &= T - \frac{c}{1+d}. \end{aligned}$$

Notice that:

1. An increase in the penalty rate  $d$  does not affect  $M_p^*$ , but it does affect (positively)  $M_g^*$ . Hence an increase of  $d$  implies a Pareto improvement. This result has been obtained (among others) by Graetz et al. (1986).
2. The expected utility of  $\mathcal{P}$  equals truthful payment of due taxes.
3. Taxation is not bounded by evasion (since  $\partial M_g/\partial T=1$ ), contradicting Edgeworth (1925) ('In fine the increase of taxation is limited by evasion').
4. An increase in  $T$  causes a fall in  $p$ , i.e. more taxes calls for more honest behavior. The reason for this is that if  $p^* > c/T(1+d)$ ,  $\mathcal{G}$  will always investigate. Therefore  $\mathcal{P}$  should decrease  $p$  in order to preserve the benefits of random evasion.
5. It is easily seen that  $p^* < q^*$ , and  $1-q^* > p^*$ . Therefore  $p^* < 1/2$ .

## 2.2. The Stackelberg solution

Sometimes it is argued that 'the government, via its agent the tax authority, acts as a 'leader', and that taxpayers act as recalcitrant followers' [Cowell (1990, p. 123)]. This corresponds to a situation in which  $\mathcal{G}$  moves first announcing (and being committed to) some strategy, and  $\mathcal{P}$  reacts in a rational way. This equilibrium is called a 'perfect Nash equilibrium' in modern jargon. Here we will show that the Nash equilibrium considered in the previous paragraph is also a Stackelberg equilibrium with  $\mathcal{G}$  as leader and  $\mathcal{P}$  as a follower.

For the time being assume that no mixed strategy is possible. Hence  $\mathcal{G}$  will maximize over the reaction function of  $\mathcal{P}$ . A quick inspection of table 1 shows that  $(B_1, A_2)$  and  $(B_2, A_1)$  are the possible outcomes. Hence  $\mathcal{G}$  will select  $B_1$  and the associated payoffs will be  $(Y-T, T-c)$ . Notice that the equilibrium payoff of  $\mathcal{G}$  is in this case  $T-c < \text{Nash payoff}$ . Now, let us introduce mixed strategies. If  $\mathcal{G}$  sets  $q > 1/(1+d)$  from (1) it follows that  $p=0$ . Then  $M_g = T - cq < T - c/(1+d) = \text{Nash payoff}$ . If  $\mathcal{G}$  sets  $q < 1/(1+d)$  by identical reasoning to before, the optimal reply of  $\mathcal{P}$  is  $p=1$ . Then,  $M_g = q(T - c + Td) < T - c/(1+d) = \text{Nash payoff}$ , so  $q = 1/(1+d)$  is the optimal choice.

Easy calculations show that in this case  $M_g = T - c/(1+d)$ , i.e. it does not pay for  $\mathcal{G}$  to be a Stackelberg leader. This is pretty surprising at first glance, since it implies that there is no incentive for  $\mathcal{G}$  to commit himself. However a little reflexion on the structure of the game will convince the reader that this is indeed the case. If for instance  $\mathcal{G}$  announces  $B_1$ ,  $\mathcal{P}$  will pay  $T$ . However  $\mathcal{G}$  can expect this to happen if an adequate randomized (and hence cheaper) monitoring is made. If  $\mathcal{G}$  announces  $B_2$ ,  $\mathcal{P}$  will not pay anything. If  $\mathcal{G}$  announces a randomized strategy which is not a Nash equilibrium for him, eqs. (1) and (2) show that  $\mathcal{P}$  will evade or will pay completely. The first case is certainly not intended by  $\mathcal{G}$ , and the latter can be achieved with a lower  $q$ , and hence with a lower expected cost. In other words, given the discontinuity

in  $\mathcal{P}$ 's behavior, the expected return of tax evasion should be zero, just like in the Nash equilibrium. This contrasts with Graetz et al. (1986) where  $\mathcal{G}$ 's lack of commitment causes a loss in revenue. This contrast is explained by the fact that it is essential for their model that a fraction of taxpayers are 'pathologically honest' [see Mookherjee (1989, p. 22)].

Finally, let us assume that  $\mathcal{P}$  is a leader and  $\mathcal{G}$  is a follower. This corresponds to a situation in which  $\mathcal{P}$  has to send her report before  $\mathcal{G}$  can take any action. Since  $T(1+d) > c$  the taxpayer will pay his entire taxes. Thus, equilibrium gives more utility to  $\mathcal{G}$  than any other non cooperative solution. This is due to the fact that under complete information  $\mathcal{G}$  can catch any evader with probability one, i.e. if information is also *perfect* the opportunities for tax evasion vanish. Notice that in this case mixed strategies do not make any sense.

### 2.3. Maximin behavior

In our case, Nash equilibrium can be objected to on grounds of being *unprofitable* [see Harsanyi (1966)], i.e. Nash equilibrium payoffs are equal to maximin payoffs but Nash strategies are different from maximin strategies. In this case, it can be argued that maximin behavior is more plausible [see Holler (1990, p. 322 and the references therein)]. Thus it seems sensible to analyze the consequences of assuming maximin behavior. From eqs. (1) and (2) it follows that maximin strategies are  $p^+ = 0$  and  $q^+ = 1/(1+d)$ , i.e.,  $\mathcal{P}$  is honest and  $\mathcal{G}$  monitors with the same probability as in a Nash equilibrium.

### 2.4. The cooperative approach

In our framework, it is easy to see that any non-cooperative equilibrium yields inefficient outcomes. We now consider cooperative solution concepts and relate the outcomes obtained by cooperation with those obtained under no cooperation.

Let us take the core as the solution concept. From section 2.3 above, it follows that  $\mathcal{G}$  can guarantee himself  $T - c/(1+d)$  and  $\mathcal{P}$ ,  $Y - T$ . Hence  $(A_2, B_2)$  is in the core of this game, as well the line  $(A, (A_2, B_2))$  in fig. 1. Notice that the core in pure strategies coincides with the Stackelberg equilibrium with  $\mathcal{P}$  as leader. Also when  $c/(1+d) \rightarrow 0$ , (i.e. either the monitoring cost tends to zero or the penalty becomes very high) we have that:

- a. The Nash equilibrium tends to the core; hence a high penalty will implement a core outcome through a Nash equilibrium, approximately, and

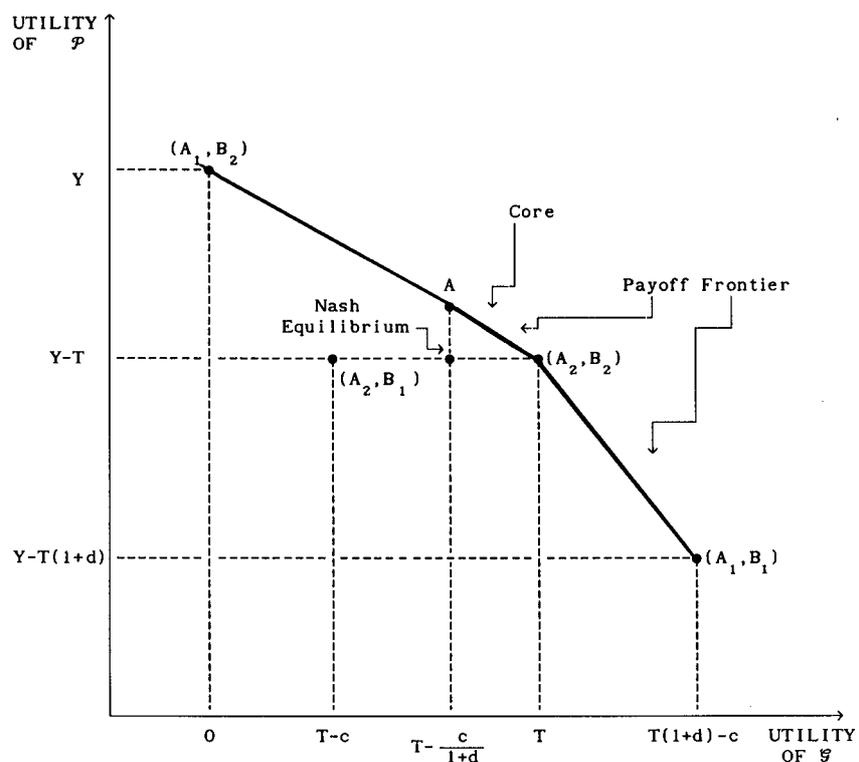


Fig. 1

b. the core shrinks towards  $(T, Y - T)$ .

### 2.5. Bayesian equilibrium

The previous analysis can be criticized since under complete information it is not at all clear which kind of tax evasion is captured by the model. There are two possible answers to this question. On the one hand, mixed strategies can be understood as the limit of Bayesian equilibrium, when uncertainty over types becomes negligible [see Harsanyi (1973)]. On the other hand, under simultaneous moves, those taxpayers who evade can get away with it in absence of monitoring. The same happens if  $\mathcal{S}$  moves first. So in these models tax evasion arises from wrong monitoring and not from incomplete information. This is also seen by noting that evasion is impossible if  $\mathcal{P}$  plays first. Here we will consider explicitly a game with incomplete information in order to see what difference it makes. It will be shown that conclusions

obtained under complete information are carried out with straightforward modifications. This reinforces our idea that tax evasion is due to imperfect, rather than, incomplete information.

For the sake of simplicity suppose that  $\mathcal{G}$  is uncertain about the true value of the tax due, but  $\mathcal{P}$  is not, i.e. we assume one-sided uncertainty. This can be formalized by saying that  $\mathcal{P}$  can be of types  $1, \dots, n$  with taxes due  $t_1, \dots, t_n$  and that  $\mathcal{P}$  knows her true type but  $\mathcal{G}$  does not. Let us assume that the probability of  $\mathcal{P}$  being of type  $i=1, \dots, n$  – denoted by  $\pi_i$  – is common knowledge. Let  $t = \sum_{i=1}^n \pi_i t_i$  be the expected value of taxes due.

We first notice that the optimal strategy set by  $\mathcal{P}$  is independent of her type and identical to the Nash equilibrium case. This follows from the fact that fines are proportional to taxes due and therefore the utility function of  $\mathcal{P}$  if she were of type  $i$  is  $t_i p(1 - q(1 + d)) + Y_i - t_i$  where  $Y_i$  is her income. Thus if  $1 >$  (resp.  $<$ )  $q(1 + d)$  the optimal  $p$  is 1 (resp. 0). If  $1 = q(1 + d)$  any  $p$  maximizes the above expression. Since the optimal probability is independent of the type, we will denote it by  $p$  as before.

When playing against type  $i$   $\mathcal{G}$  can expect a payoff of  $q(pt_i(1 + d) - c) + (1 - p)t_i$ . Therefore expected payoffs for  $\mathcal{G}$  are

$$M_g = \sum_{i=1}^n \pi_i (q(pt_i(1 + d) - c) + (1 - p)t_i).$$

And from this equation we obtain the result that if  $t(1 + d) > c$ , in any Bayesian equilibrium with simultaneous reports and monitors

$$p' = \frac{c}{t(1 + d)},$$

$$q' = \frac{1}{1 + d}.$$

Then it is easy to show that all the conclusions obtained in sections 2.1–2.4 carry on under incomplete information. However – in contrast to all our previous results – the extension to non-risk-neutral agents is not straightforward. See Sanchez (1987) and Scotchmer (1987) for additional results with risk-neutral taxpayers. Finally if  $t(1 + d) < c$ , i.e. if expected profit from a monitoring is negative, equilibrium entails  $p=1$  and  $q=0$ .

### 3. Conclusions

In this paper we have considered a simple model of strategic interaction

between taxpayers and the government focusing our attention on the case in which taxpayers' decision is dichotomous (to be inside or outside the black economy) in a static framework. Moreover, we did not study how taxes are set, i.e. in terminology of Cowell (1990, p. 131) we focus on *pure enforcement*. However some conclusions appear to be robust enough to be worth mentioning.

First, the probability of monitoring is the same in Nash, Stackelberg (with  $\mathcal{G}$  as a leader), maximin and Bayesian equilibria.

Second, the probability of evading is the same (and positive) in Nash, Stackelberg (with  $\mathcal{G}$  as a leader), and Bayesian equilibria. However in both maximin and Stackelberg (with  $\mathcal{P}$  as a leader) the taxpayer is honest. This probability can be interpreted under some circumstances as the equilibrium relative size of the underground economy.

Third, the payoff of  $\mathcal{P}$  in every non-cooperative equilibrium is the same as if she were honest.

Fourth, a high penalty for evaders is (as in many models of tax evasion) socially desirable. Of course introducing a probability of error in convictions [see Bolton (1986)] may yield an appropriate upper bound to penalties [see Nalebuff and Scharfstein (1987) for cases in which this is not the case and an infinite penalty is socially desirable].

Fifth, tax evasion is due to imperfect, rather than incomplete, information.

Sixth, revenues from taxes are maximized when  $\mathcal{G}$  is a follower and  $\mathcal{P}$  is a Stackelberg leader.

How should the last result be interpreted? Since present-day governments can scarcely be considered as maximizing agents, the emphasis must be on the normative side. In this sense, our analysis recommends  $\mathcal{G}$  to behave as a follower (it must be remarked that there is nothing surprising in the fact that payoffs for the government are higher for a follower than for a leader: The same happens in models of price-making firms in the literature on industrial organization). However, this kind of solution makes sense if the decision of being a member of the black economy is taken before  $\mathcal{G}$  takes any action. This implies a delay which may be unacceptable because of legal constraints or because of high interest and/or inflation rates. Notice that an important consequence of all this is that when modeling tax evasion 'à la principal-agent' (in which  $\mathcal{G}$  acts as a leader) we are not exploring the best alternative from  $\mathcal{G}$ 's point of view. In other words, in general models we may regard evasion as arising from both incomplete and imperfect information. If  $\mathcal{G}$  acts as a follower, the opportunities for evasion arising from the latter disappear.

Finally we mention two possible extensions of our work: First, to model explicitly the interests of monitors [see Laffont and Tirole (1988)], and second to consider evasion and consumption in a unified framework (see Andersen (1977); Sandmo (1981) and Cowell (1981)).

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